1) a) SVM variant:

=> write in dual form:

- rewrite constraints:

$$\text{I)}\quad y_i(\vec{w}\cdot\vec{x}_i + b) - 1 + \xi_i \geq 0 \quad , \forall i \in \{1, \dots, n\}$$

$$\text{II)}\quad y_j^*(\vec{w}\cdot\vec{x}_j + b) - 1 + \xi_j^* \geq 0 \quad , \forall i \in \{1, \dots, k\}$$

$$\text{III)}\quad \xi_i \geq 0 \quad , \forall i \in \{1, \dots, n\}$$

$$\text{IV)}\quad \xi_j^* \geq 0 \quad , \forall i \in \{1, \dots, k\}$$

- Form Lagrange function:

$$\mathcal{L}(\vec{w}, b, \vec{\xi}, \vec{\xi}^*, \vec{\alpha}) = f(\vec{w}, b, \vec{\xi}, \vec{\xi}^*) - \vec{\alpha}\cdot(\text{constraints } = 0)$$

(*)

$$\mathcal{L}(\vec{w}, b, \vec{\xi}, \vec{\xi}^*, \vec{\alpha}) = \frac{1}{2}\vec{w}\cdot\vec{w} + C\sum_{i=1}^{n}\xi_i + C_-^*\sum_{j:y_j=-1}\xi_j^* + C_+^*\sum_{j:y_j=1}\xi_j^*$$
$$- \sum_{i=1}^{n}\alpha_i\left[y_i(\vec{w}\vec{x}_i + b) - 1 + \xi_i\right]$$
$$- \sum_{j=1}^{k}\alpha_j\left[y_j^*(\vec{w}\vec{x}_j + b) - 1 + \xi_j^*\right]$$
$$- \sum_{i=1}^{n}\alpha_i\xi_i$$
$$- \sum_{j=1}^{k}\alpha_j\xi_j^*$$

1) $\frac{\partial \mathcal{L}}{\partial \vec{w}} \overset{!}{=} 0$:

$$\vec{w} - \sum_{i=1}^{n} \alpha_i y_i \vec{x}_i - \sum_{j=1}^{k} \alpha_j y_j^* \vec{x}_j = 0$$

$$\vec{w} = \sum_{i=1}^{n} \alpha_i y_i \vec{x}_i + \sum_{j=1}^{k} \alpha_j y_j^* \vec{x}_j$$

$$\underline{\underline{\vec{w} = \sum_{i=1}^{n+k} \alpha_i y_i \vec{x}_i}}$$

2) $\frac{\partial \mathcal{L}}{\partial b} \overset{!}{=} 0$:

$$-\sum_{i=1}^{n} \alpha_i y_i - \sum_{j=1}^{k} \alpha_j y_j^* = 0$$

$$\Rightarrow \sum_{i=1}^{n+k} \alpha_i y_i = 0$$

3) $\frac{\partial \mathcal{L}}{\partial \xi_i} \overset{!}{=} 0$:

$$C \sum_{i=1}^{n} 1 + \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i = 0$$

$$\underline{\underline{C = 0}}$$

4) $\frac{\partial \mathcal{L}}{\partial \xi_j^*} \overset{!}{=} 0$:

$$C_-^* \sum 1 + C_+^* \sum 1 + \sum_{j=1}^{k} \alpha_j - \sum_{j=1}^{k} \alpha_j = 0$$

$$\underline{\underline{C_-^* + C_+^* = 0}}$$

$$\Rightarrow C_-^* = 0 \quad ; \quad C_+^* = 0$$

$\downarrow$ <u>insert in (*)</u> :

$$= \frac{1}{2} \sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \alpha_i \alpha_j y_i y_j \langle \vec{x}_i, \vec{x}_j \rangle - \sum_{i=1}^{n} \alpha_i \xi_i - \sum_{j=1}^{k} \alpha_j \xi_j^*$$
$$- \sum_{i=1}^{n} \alpha_i (1 - \xi_i) - \sum_{j=1}^{k} \alpha_j (1 - \xi_j^*)$$

$$= \underbrace{\frac{1}{2} \sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \alpha_i \alpha_j y_i y_j \langle \vec{x}_i, \vec{x}_j \rangle - \sum_{i=1}^{n+k} \alpha_i}_{\mathcal{L}(\vec{w}, b, \vec{\xi}, \vec{\xi}^*, \vec{\alpha})}$$

$$\Leftrightarrow \min_{\alpha} \{ \mathcal{L}(\alpha) \} = \max_{\alpha} \left(-\mathcal{L}(\alpha) \right)$$

- <u>Dual repr.:</u>

$$\max_{\alpha} \left\{ \sum_{i=1}^{n+k} \alpha_i - \frac{1}{2} \sum_{i=1}^{n+k} \sum_{j=1}^{n+k} \alpha_i \alpha_j y_i y_j \langle \vec{x}_i, \vec{x}_j \rangle \right\}$$