ECE 369
Homework 1
Due September 10th, 2025

For questions with a final answer, please **box your answer**.	Name: Timothy Waldin List anyone you have collaborated with:

In this exam, you may use the following inference rules:

Name	Abbreviation	Rule
modus ponens	mp	$\frac{P \to Q \quad P}{Q}$
modus tollens	mt	$\frac{P \to Q \quad \neg Q}{\neg P}$
simplification	simpl	$\frac{P \land Q}{P}$
addition	add	$\frac{P}{P} \lor Q$
conjunction	conj	$\frac{P \quad Q}{P \land Q}$
hypothetical syllogism	hs	$\frac{P \to Q \quad Q \to R}{P \to R}$
disjunctive syllogism	ds	$\frac{P \lor Q \quad \neg P}{Q}$
disjunction elimination	de	$\frac{(P \lor Q) \quad P \to R \quad Q \to R}{R}$

And you may use the following equivalences ($\alpha \Leftrightarrow \beta$ should be read as "α is equivalent to β"):

Name	Abbreviation	Equivalence
Double negation	dn	$P \Leftrightarrow \neg\neg P$
Contrapositive	cp	$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$
Implication	impl	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$
Exportation	exp	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$
DeMorgan's Laws	dm	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
Associativity	assoc	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
Commutativity	comm	$P \wedge Q \Leftrightarrow Q \wedge P$ $P \vee Q \Leftrightarrow Q \vee P$
Distributivity	dist	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
Self-reference	self	$P \Leftrightarrow P \wedge P$ $P \Leftrightarrow P \vee P$

When you use the deduction theorem, introduce your assumption by labeling it "hyp." and discharge the assumption (turn it into an implication) by labeling it "X–Y, ded" where line X is where you introduced the assumption (the left hand side of the implication), and line Y is where you concluded the right-hand side of the implication. Drawing the vertical line is mandatory.

Problem 1

This problem will ask you to consider building several truth tables. For each boolean formula below, give a complete truth table for all possible combinations of the variables. As an example, here is the truth table for $(a \wedge b) \vee c$:

a	b	c	$(a \wedge b) \vee c$
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	1
0	1	1	1
1	1	1	1

For each subproblem, use the format above for your truth tables, draw a box around your truth table, and label which subproblem it is for. Feel free to add extra columns to your truth table to work out smaller parts of the formula. Just make sure your last column is for the overall formula.

(a) $(a \to b) \to (\neg a \vee b)$

a	b	$(a \to b) \to (\neg a \vee b)$
0	0	1
1	0	1
0	1	1
1	1	1

(b) $b \to (a \to \neg a)$

b	a	$b \to (a \to \neg a)$
0	0	1
1	0	1
0	1	1
1	1	0

(c) $b \to (a \to \neg b)$

b	a	$b \to (a \to \neg b)$
0	0	1
1	0	1
0	1	1
1	1	0

(d) $((a \to \neg b) \wedge a) \to \neg b$

a	b	$((a \to \neg b) \wedge a) \to \neg b$
0	0	1
1	0	1
0	1	1
1	1	1

Problem 2

In this problem, we want you to say whether the two formulas are equivalent. Justify your answer: if the two formulas are equivalent, draw the truth table demonstrating this (you must draw the entire truth table for full credit). If the two formulas are not equivalent, you can show a full truth table, or just give a row of the truth table that demonstrates the formulae are not equivalent.

(a) $\neg(a \vee b) \wedge c$ versus $\neg b \wedge (\neg a \wedge c)$ (10 points)

a	b	c	$\neg(a \vee b) \wedge c$
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	1
1	0	1	0
0	1	1	0
1	1	1	0

vs

b	a	c	$\neg b \wedge (\neg a \wedge c)$
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	1
1	0	1	0
0	1	1	0
1	1	1	0

Since their truth tables are the same,

$$\neg(a \vee b) \wedge c \Leftrightarrow \neg b \wedge (\neg a \wedge c)$$

(b) $\neg a \wedge \neg b$ versus $\neg(\neg b \vee a)$ (5 points)

a	b	$\neg a \wedge \neg b$
0	0	1
1	0	0
0	1	0
1	1	0

vs

b	a	$\neg(\neg b \vee a)$
0	0	0
1	0	1
0	1	0
1	1	0

Since their truth tables are not the same,

$$\neg a \wedge \neg b \not\Leftrightarrow \neg(\neg b \vee a)$$

(c) $a \vee b$ versus $\neg(\neg a \wedge \neg b)$ (5 points)

a	b	$a \vee b$
0	0	0
1	0	1
0	1	1
1	1	1

vs

a	b	$\neg(\neg a \wedge \neg b)$
0	0	0
1	0	1
0	1	1
1	1	1

Since their truth tables are the same,

$$a \vee b \Leftrightarrow \neg(\neg a \wedge \neg b)$$

Problem 3

Translate these English sentences into propositions and logical formulas. Please make sure your proposition refers to an English sentence that has no negation.

(a) The Boba tea shop will give customer discount if the customer is a student in ECE20869 and it's Thursday. Bob is a student in ECE20869. Bob always walks past the Boba tea shop if he came to ECE20869 class. Bob always buys a Boba tea if he walks pass the Boba tea shop and there is a discount. Bob does not come to class if it's raining. ECE20869 meets on Tuesdays and Thursdays. Bob came to ECE20869 class today but did not buy a boba tea. Therefore, today is Tuesday and today is not raining.

Propositions:	**Logical formulas:**
• a: The boba tea shop gives customer discount	(a) $(b \land g) \to a$
• b: Bob is a student in ECE20869	(b) b
• c: Bob came to ECE20869 class	(c) $c \to d$
• d: Bob walks past the boba tea shop	(d) $(d \land a) \to e$
• e: Bob buys boba tea	(e) $f \to \neg c$
• f: It is raining	(f) $c \to (g \lor h)$
• g: Today is Thursday	(g) $c \land \neg e$
• h: Today is Tuesday	(h) Therefore: $h \land \neg f$

(b) Alice lives in 2nd street. Bob likes eating apple. If Bob lives in 2nd street, Bob will meet Alice this afternoon. If Bob meets Alice and if Alice likes eating apple, Bob will give Alice an apple. Alice did not receive an apple from Bob this afternoon, therefore either Alice does not like eating apple or Bob did not live in 2nd street.

Propositions:	**Logical formulas:**
• p: Alice lives in 2nd street	(a) p
• q: Bob likes eating apple	(b) q
• r: Bob lives in 2nd street	(c) $r \to s$
• s: Bob meets Alice this afternoon	(d) $(s \land t) \to u$
• t: Alice likes eating apple	(e) $\neg u$
• u: Bob gives Alice an apple	(f) Therefore: $\neg t \lor \neg r$

Problem 4

Prove the following statements step by step: $\{\neg X, Y, Y \to (X \lor Z)\} \vdash Z$

1.	$\neg X$	Premise
2.	Y	Premise
3.	$Y \to (X \lor Z)$	Premise
4.	$X \lor Z$	2, 3 mp
5.	Z	1, 4 ds

Problem 5

Prove the following statement: $\{A \to (B \to C), D \to A, B\} \vdash D \to C$

(a) Prove it without using the deduction rule. You are allowed to use any rule specified on page 1.

1.	$A \to (B \to C)$	Premise
2.	$D \to A$	Premise
3.	B	Premise
4.	$D \to (B \to C)$	1, 2 hs
5.	$(D \to (B \to C)) \Leftrightarrow ((D \wedge B) \to C)$	exp
6.	$(D \wedge B) \to C$	4, 5 equiv
7.	$D \to C$	3, 6 (B as constant)

(b) Prove it using the deduction rule (Hint: You should assume the antecedent (left hand side) of the implication as your hypothesis. Then try to prove the consequent (right hand side))

1.	$A \to (B \to C)$	Premise
2.	$D \to A$	Premise
3.	B	Premise
4.	D	hyp
5.	A	2, 4 mp
6.	$B \to C$	1, 5 mp
7.	C	3, 6 mp
8.	$D \to C$	4-7, ded

Problem 6

Prove the following statements, step by step, using deduction: $\{\neg P \to \neg Q, P \to R\} \vdash Q \to R$

1.	$\neg P \to \neg Q$	Premise
2.	$P \to R$	Premise
3.	Q	hyp
4.	$Q \to P$	1, cp
5.	P	3, 4 mp
6.	R	2, 5 mp
7.	$Q \to R$	3-6, ded