

Torch In More Detail

Current Source

$$\begin{aligned}Poly &\in Cat \\ Interface &= Iy^A \\ Interface &\in Poly\end{aligned}$$

$$\begin{aligned}StateSystem &= Sy^S \\ StateSystem &\in Poly\end{aligned}$$

$$\begin{aligned}\Phi &= StateSystem \rightarrow Interface \\ &= Sy^S \rightarrow Iy^A\end{aligned}$$

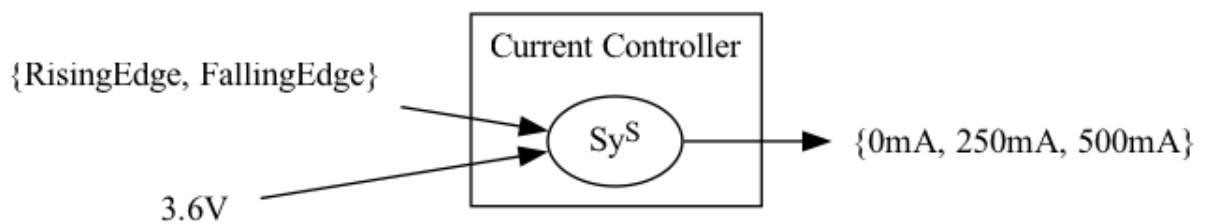
$$\begin{aligned}V &= \{3.6V\} \\ A &= V \times \{RisingEdge, FallingEdge\} \\ I &= \{0mA, 250mA, 500mA\} \\ S &= \{Off, Low, High\}\end{aligned}$$

$$\begin{aligned}\Phi^\# : S \times A &\rightarrow S \\ \Phi_0^\# &= (s, a) \rightarrow Off \\ \Phi_1 : S &\rightarrow I\end{aligned}$$

$$\begin{aligned}\Phi_1(Off) &= 0mA \\ \Phi_1(Low) &= 250mA \\ \Phi_1(High) &= 500mA\end{aligned}$$

$$\begin{aligned}\Phi^\#(Off, 3.6V, FallingEdge) &= Low \\ \Phi^\#(Low, 3.6V, FallingEdge) &= High \\ \Phi^\#(High, 3.6V, FallingEdge) &= Off \\ \Phi^\#(Off, 3.6V, RisingEdge) &= Off \\ \Phi^\#(Low, 3.6V, RisingEdge) &= Low \\ \Phi^\#(High, 3.6V, RisingEdge) &= High\end{aligned}$$

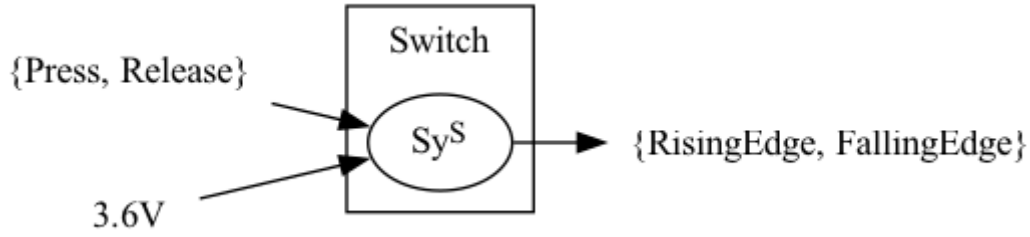
Diagrams of the State System



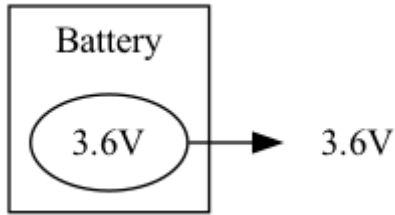
The Switch

$$\begin{aligned}\Phi &= Ty^T \rightarrow Jy^B \\ T &= \{Conducting, NotConducting\} \\ V &= C \\ B &: V \times \{Press, Release\} \\ B &= \{\{3.6V, Press\}, \{3.6V, Release\}\} \\ J &= \{RisingEdge, FallingEdge\}\end{aligned}$$

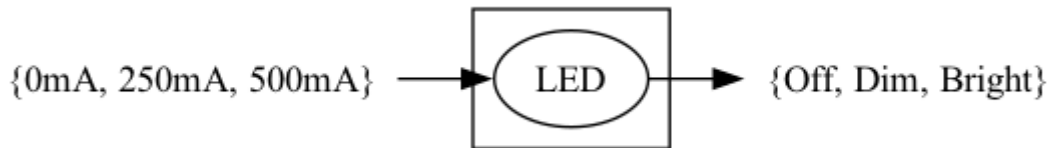
$$\begin{aligned}\Phi^\# &: T \times B \rightarrow T \\ \Phi_0^\# &= (t, b) \rightarrow NotConducting \\ \Phi_1 &: T \rightarrow J\end{aligned}$$

$$\begin{aligned}\Phi_1(Conducting) &= RisingEdge \\ \Phi_1(NotConducting) &= FallingEdge \\ \Phi^\#(-, 3.6V, Press) &= Conducting \\ \Phi^\#(-, 3.6V, Release) &= NotConducting\end{aligned}$$


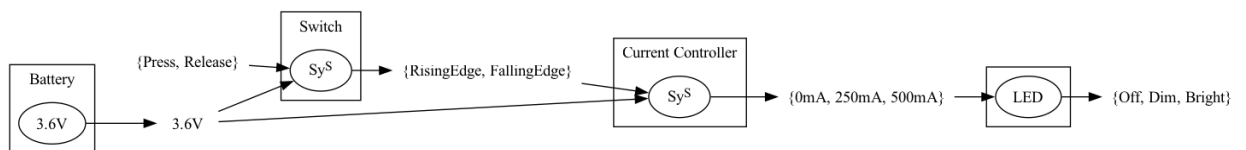
The Battery

$$C = 3.6V$$


LED

$$\begin{aligned}L &= \{Off, Dim, Bright\} \\ f &: I \rightarrow L \\ f(0mA) &= Off \\ f(250mA) &= Dim \\ f(500mA) &= Bright\end{aligned}$$


Combined System



System as Combined by Gemini

Based on the mathematical definitions provided, you are describing a discrete dynamical system composed of two state machines wired in series, culminating in a physical output.

Here is the logical reconstruction of the total system.

Ambiguities and Assumptions

Before defining the combined system, I have resolved the following ambiguities using standard systems theory assumptions:

- Naming:** You used the symbol Φ for both the "Button" system and the "Controller" system. In the output, I have renamed them Φ_{btn} and Φ_{ctrl} respectively.
- Timing/Composition:** There is an ambiguity regarding *when* the Controller receives the signal from the Button.
 - Assumption:* I have modeled this as a synchronous composition where the **new state** of the button determines the input to the controller.
 - Reasoning:* If the controller read the *old* state of the button, the system would react to a button press before the button state actually updated.
- Voltage Injection:** The Controller input A requires a voltage V , but the Button output J only provides an "Edge".
 - Assumption:* The constant $C = 3.6V$ is applied structurally to bridge the connection between the Button output and Controller input.

Combined System Definition

The combined system, Ψ , represents a **single-button dimmable lighting controller**. The system changes light intensity (Off \rightarrow Low \rightarrow High) specifically upon the **release** of the button (Falling Edge).

Let Ψ be the Composite System in Cat

$$\Psi : (T \times S)y^{(T \times S)} \rightarrow Ly^B$$

1. Composite State and Interface Spaces

$$\begin{aligned} \text{Inputs (User): } B &= (3.6V, Press), (3.6V, Release) \\ \text{States (Internal): } \Sigma &= T \times S \\ &\quad \Sigma = Conducting, NotConducting \times Off, Low, High \\ \text{Outputs (Physical): } L &= Off, Dim, Bright \end{aligned}$$

2. Internal Composition Logic

We define the projection of signals between the two sub-systems:

$$\begin{aligned} \text{Let } \gamma : J \rightarrow A \text{ be the wiring map} \\ \gamma(j) &= (3.6V, j) \end{aligned}$$

3. Composite Functions

The system Ψ is defined by a readout function Ψ_1 and an update function $\Psi^\#$.

The Readout Function (Ψ_1):

This maps the internal state to the physical light brightness.

$$\begin{aligned}\Psi_1 : \Sigma &\rightarrow L \\ \Psi_1(t, s) &= f(\Phi_{ctrl,1}(s))\end{aligned}$$

Explicitly:

$$\begin{aligned}\Psi_1(-, Off) &= Off \\ \Psi_1(-, Low) &= Dim \\ \Psi_1(-, High) &= Bright\end{aligned}$$

The Update Function ($\Psi^\#$):

This maps the current combined state and user input to the next combined state.

$$\begin{aligned}\Psi^\# : \Sigma \times B &\rightarrow \Sigma \\ \Psi^\#((t, s), b) &= (t_{next}, s_{next})\end{aligned}$$

Where the state transitions are derived sequentially:

$$\begin{aligned}t_{next} &= \Phi_{btn}^\#(t, b) \\ s_{next} &= \Phi_{ctrl}^\#(s, \gamma(\Phi_{btn,1}(t_{next})))\end{aligned}$$

4. System Behavior Table (Truth Table)

Evaluating $\Psi^\#$ for the specific interactions defined in your input:

Interaction 1: Pressing the Button

$$\begin{aligned}\text{Input } b &= (3.6V, Press) \\ \text{Resulting } t_{next} &= Conducting \\ \text{Intermediate Signal } j &= RisingEdge \\ \text{Controller Reaction } s_{next} &= s \quad (\text{Since } \Phi_{ctrl}^\#(-, RisingEdge) \text{ holds state}) \\ \implies \Psi^\#((- , s), Press) &= (Conducting, s) \quad \textbf{(No Light Change)}\end{aligned}$$

Interaction 2: Releasing the Button

$$\begin{aligned}\text{Input } b &= (3.6V, Release) \\ \text{Resulting } t_{next} &= NotConducting \\ \text{Intermediate Signal } j &= FallingEdge \\ \text{Controller Reaction } s_{next} &= \text{Next State in Cycle } (Off \rightarrow Low \rightarrow High) \\ \implies \Psi^\#((- , Off), Release) &= (NotConducting, Low) \\ \implies \Psi^\#((- , Low), Release) &= (NotConducting, High) \\ \implies \Psi^\#((- , High), Release) &= (NotConducting, Off)\end{aligned}$$

Next Step

Would you like me to generate a **diagram** of this system (e.g., a state transition diagram or a block diagram) to visualize the wiring between the button and the controller?