Mathematical Definition of Requirements

This document is a refinement of:

- 2025-11-26-003-FlashlightCatTheoryRequirements

Preliminaries

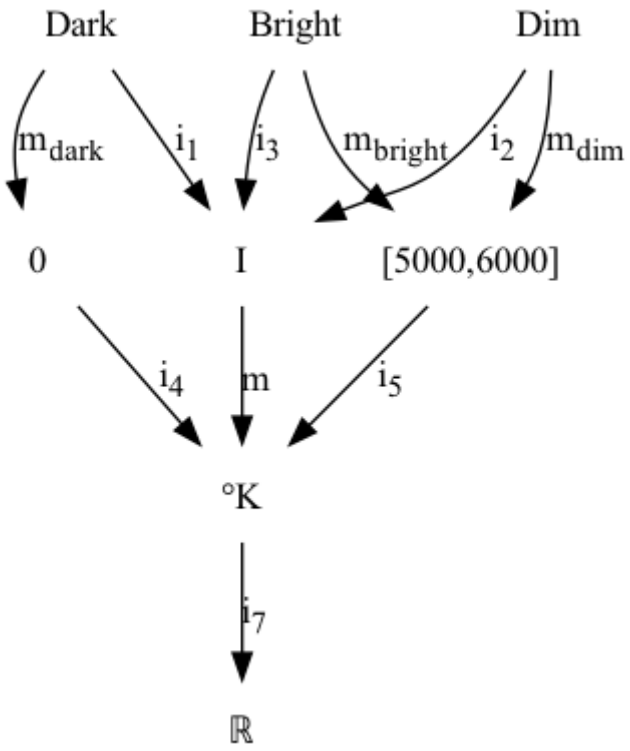
$$
\begin{aligned}
\mathcal{E} &\in Cat \\
\mathcal{E} &\subset Set \\
Objs(\mathcal{E}) &= \{A, Batt, Case, F, HeadCase, Housing, I, LED, LightHead, S, Switch, TailCase, Tailpiece, \mathbb{R}, \mathbb{B}\} \\
\mathbb{R} &= RealNumbers\ r \in [-\infty, +\infty] \\
\mathbb{B} &= \{true, false\} \\
A &= \{next!\} \\
F &= Flashlight \\
I &= \{Dark, Dim, Bright\} \\
S &= \{Off, Low, High\}
\end{aligned}
$$

Colour

m is a colour meter measuring colour temperature in °K of I, the outputs of the torch, i_1, i_2, i_3, i_4, i_5 are inclusion maps:

$$
\begin{aligned}
m &: I \to \mathbb{R} \\
i_1 &: Dark \hookrightarrow I \\
i_2 &: Dim \hookrightarrow I \\
i_3 &: Bright \hookrightarrow I \\
i_4 &: 0 \hookrightarrow \mathbb{R} \\
i_5 &: [5000, 6000] \hookrightarrow \mathbb{R} \\
m_{dark} &: Dark \to 0 \\
m_{dim} &: Dim \to [5000, 6000] \\
m_{bright} &: Bright \to [5000, 6000]
\end{aligned}
$$

$$
\begin{aligned}
m \circ i_1 &= i_4 \circ m_{dark} \\
m \circ i_2 &= i_5 \circ m_{dim} \\
m \circ i_3 &= i_5 \circ m_{bright}
\end{aligned}
$$

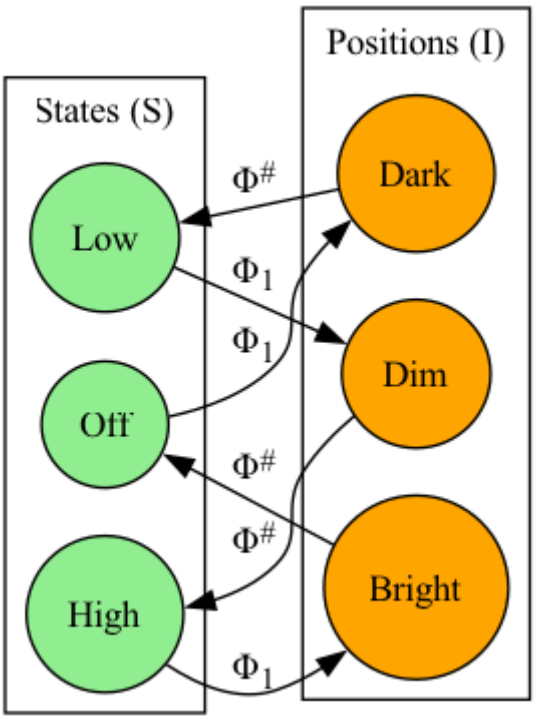


Behaviour

A morphism Φ in $Poly$ defines the behaviour:

$$
\begin{aligned}
Poly &\in Cat \\
Interface &= Iy^A \\
Interface &\in Poly \\
StateSystem &= Sy^S \\
StateSystem &\in Poly \\
\Phi &= StateSystem \to Interface \\
&= Sy^S \to Iy^A \\
\Phi^\# &: S \times A \to S \\
\Phi_0^\# &= (s, a) \to Off \\
\Phi_1 &: S \to I \\
\Phi_1(Off) &= Dark \\
\Phi_1(Low) &= Dim \\
\Phi_1(High) &= Bright \\
\Phi^\#(Off, next!) &= Low \\
\Phi^\#(Low, next!) &= High \\
\Phi^\#(High, next!) &= Off
\end{aligned}
$$

Map of the State System



Structure

Categorical products define the structure:

$$
\begin{aligned}
F &= Housing \times LightHead \\
head &: F \to LightHead \\
housing &: F \to Housing
\end{aligned}
$$

$$
\begin{aligned}
Housing &= Tailpiece \times Case \times Batt \\
tail &: Housing \to Tailpiece \\
case &: Housing \to Case \\
battery &: Housing \to Batt
\end{aligned}
$$

$$
\begin{aligned}
Tailpiece &= Switch \times TailCase \\
switch &: Tailpiece \to Switch \\
tailcase &: Tailpiece \to TailCase
\end{aligned}
$$

$$
\begin{aligned}
LightHead &= LED \times HeadCase \\
led &: LightHead \to LED \\
headcase &: LightHead \to HeadCase
\end{aligned}
$$

Replace Battery

The battery is replaceable:

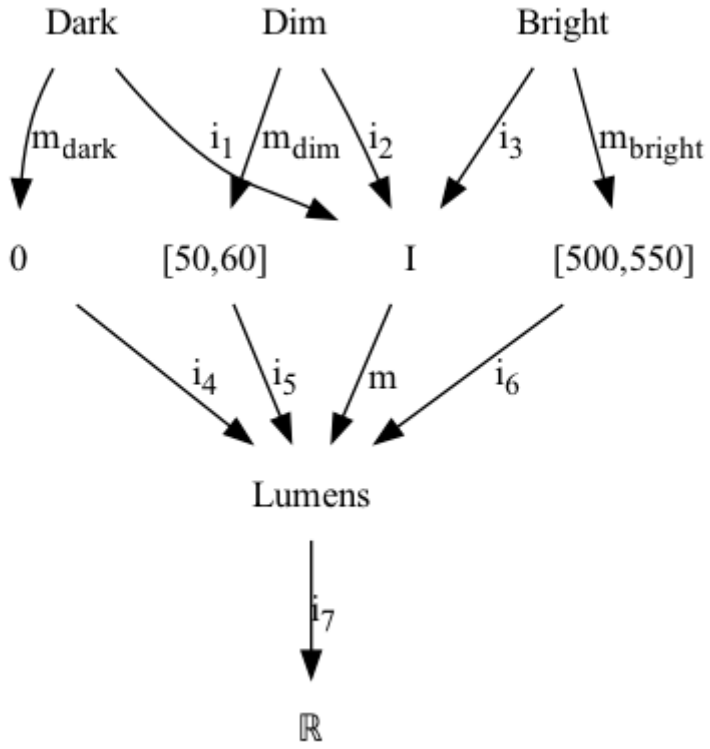
$$
\begin{aligned}
HousingAndBattery &= Housing \times Batt \\
h1 &: HousingAndBattery \to Housing \\
b1 &: HousingAndBattery \to Battery \\
replaceBattery &: HousingAndBattery \to HousingAndBattery \\
replaceBattery(h, b_{new}) &= (h', b_{old}) \\
battery(h) = b_{old} &\wedge battery(h') = b_{new}
\end{aligned}
$$

Lumen Output

m is a light meter with an integrating sphere measuring lumens:

$$
\begin{aligned}
m &: I \to \mathbb{R} \\
i_1 &: Dark \hookrightarrow I \\
i_2 &: Dim \hookrightarrow I \\
i_3 &: Bright \hookrightarrow I \\
i_4 &: 0 \hookrightarrow \mathbb{R} \\
i_5 &: [50, 60] \hookrightarrow \mathbb{R} \\
i_6 &: [500, 550] \hookrightarrow \mathbb{R} \\
m(Dark) &= 0 \\
m(Dim) &\to [50, 60] \\
m(Bright) &\to [500, 550]
\end{aligned}
$$

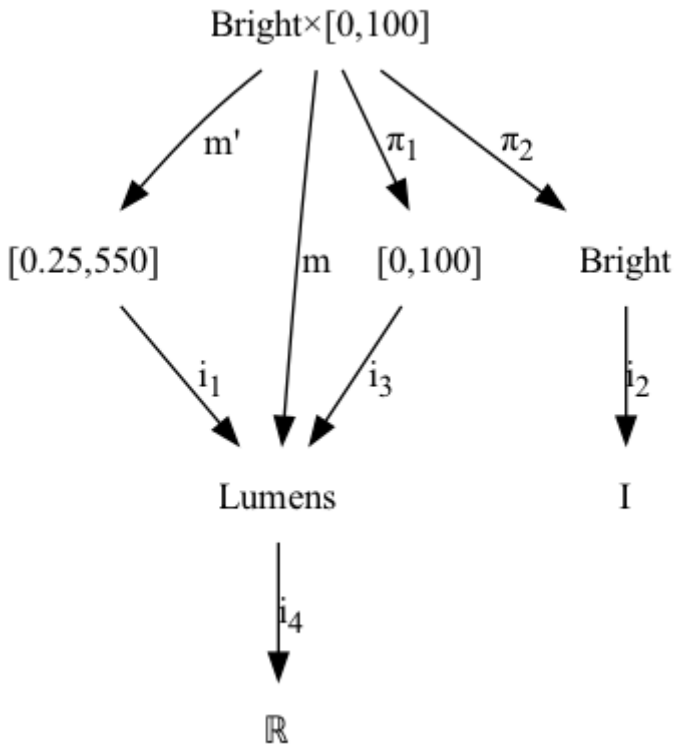
$$
\begin{aligned}
i_4 \circ m_{dark} &= m \circ i_1 \\
i_5 \circ m_{dim} &= m \circ i_2 \\
i_6 \circ m_{brignt} &= m \circ i_3
\end{aligned}
$$



Beam Distance

m is a light meter at a distance measuring lumens, the i and π morphisms are inclusions and projections respectively:

$$
\begin{aligned}
m &: Bright \times [0, 100] \to \mathbb{R} \\
i_1 &\circ m' = m
\end{aligned}
$$

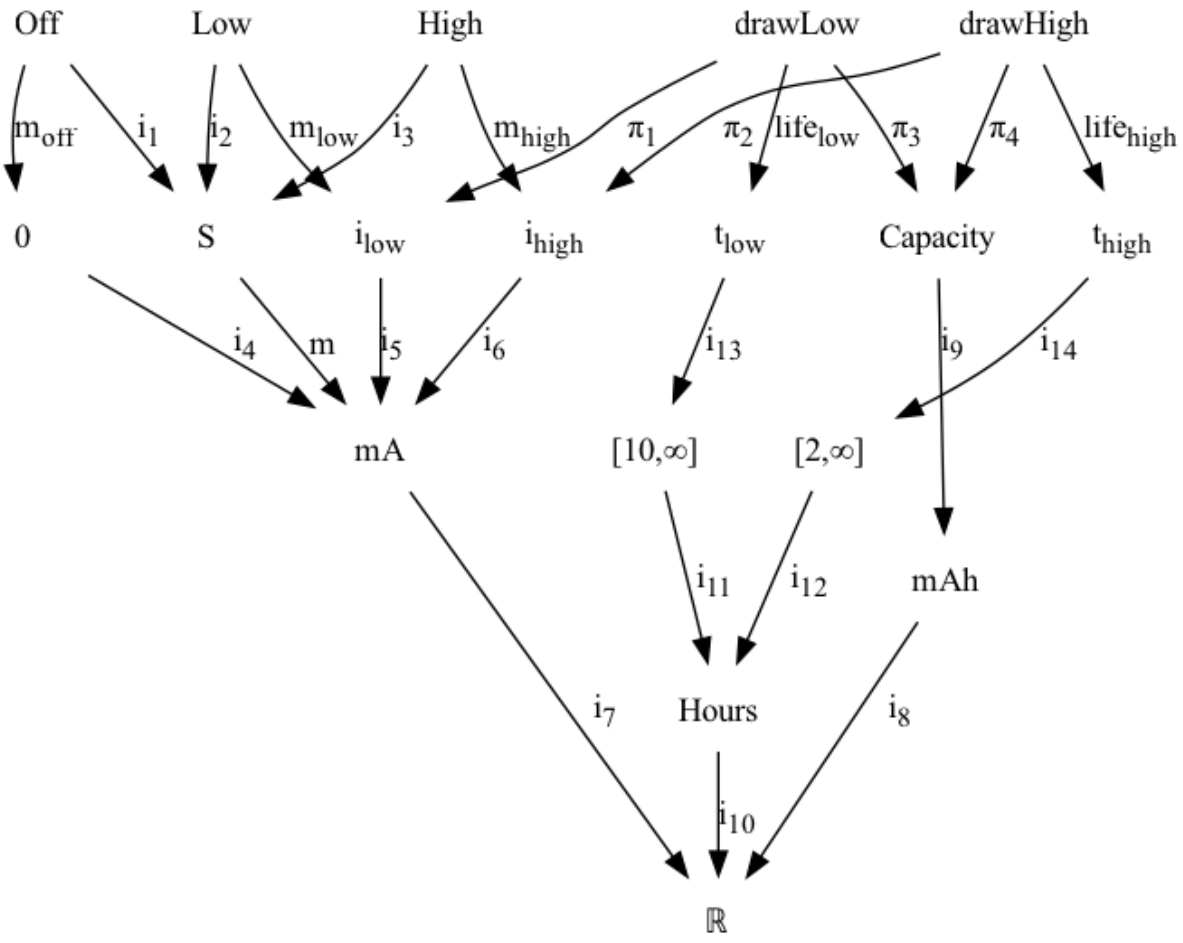


Run Time

m is an ammeter function measuring current drawn from the battery in mA:

$$
\begin{aligned}
m : S \to mA \\
i_4 \circ m_{off} = m \circ i_1 \\
i_5 \circ m_{low} = m \circ i_2 \\
i_6 \circ m_{high} = m \circ i_3
\end{aligned}
$$

$$
\begin{aligned}
c : Capacity \\
c = battery\ capacity\ in\ mAh \\
t : Hours \\
t_{low} = \frac{c}{i_{low}} \wedge t_{low} \in [10, \infty] \\
t_{high} = \frac{c}{i_{high}} \wedge t_{high} \in [2, \infty]
\end{aligned}
$$



Material

Mappings from the torch to its construction materials and coating:

$$material : F \to Material$$
$$material(f) = Aluminium$$

$$coating : F \to Caterial$$
$$coating(f) = Anodised$$

Durability

The torch still works after 10^5 operations:

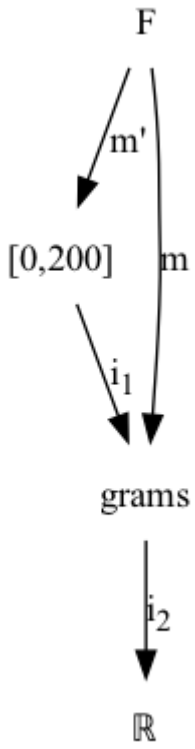
$$(\Phi^{\triangleleft 100000})(Off, next!) = Low$$

Weight

m is a weighing machine in grams:

$$[0, 200] \hookrightarrow grams \hookrightarrow \mathbb{R}$$
$$m : F \to grams$$
$$m' : F \to [0, 200]$$

$$\forall f \in F, m(f) = (i_1 \circ m')(f)$$

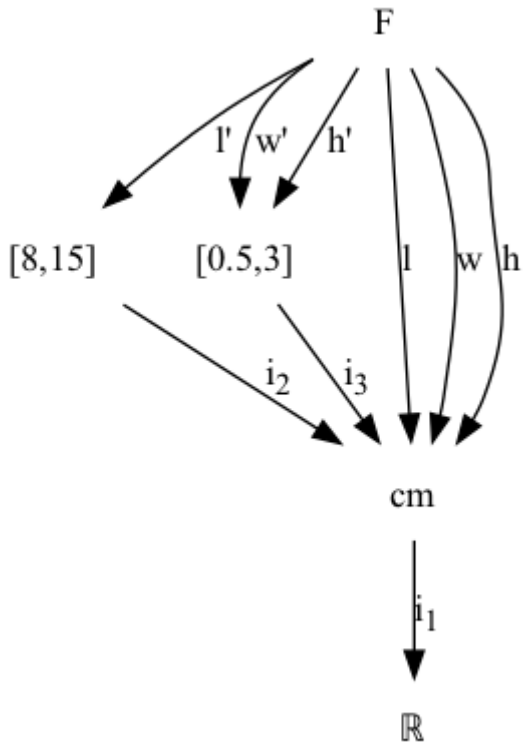


Dimensions

Three functions measure the dimensions of a torch in cm:

$$
\begin{aligned}
[8, 15] &\hookrightarrow cm \hookrightarrow \mathbb{R} \\
[0.5, 3] &\hookrightarrow cm \hookrightarrow \mathbb{R} \\
l &: F \to cm \\
w &: F \to cm \\
h &: F \to cm \\
l' &: F \to [8, 15] \\
w' &: F \to [0.5, 3] \\
h' &: F \to [0.5, 3]
\end{aligned}
$$

$$
\begin{aligned}
\forall f \in F, l(f) &= (i_2 \circ l')(f) \\
\forall f \in F, w(f) &= (i_3 \circ w')(f) \\
\forall f \in F, h(f) &= (i_3 \circ h')(f)
\end{aligned}
$$



IPX4

w is a function measuring the quantity of liquid water inside the torch, and s is a function that sprays water onto a torch from all directions for 10 minutes:

$$w : F \to ml$$
$$s : F \to F$$

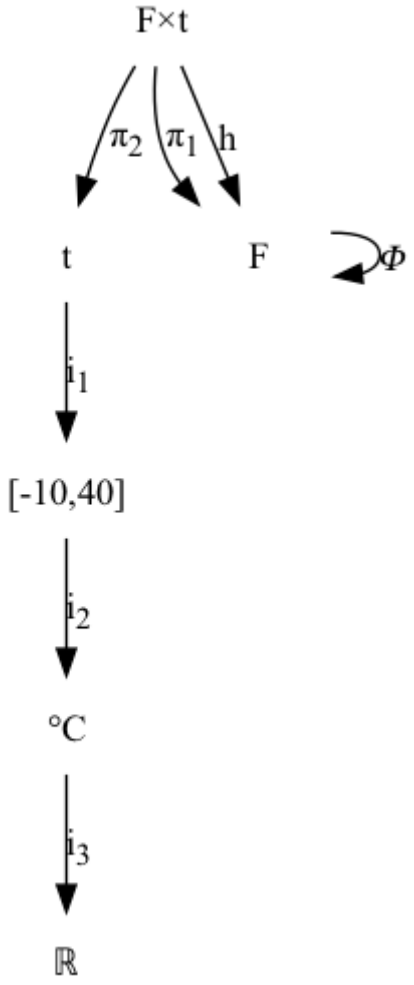
$$\forall f \in F, (w \circ s)(f) = (i_1 \circ w' \circ s)(f)$$

Operating Temperatures

h is a function to make a torch $f \in F$ have a temperature of $t \in [-10, 40]$, and Φ *models* F.

$$\begin{gathered} \Phi \models F \\ h : F \times t \to F' \\ \forall f \in F, \wedge \forall t \in [-10, 40], \Phi(h(f, t)) = \Phi(h(f, 20)) \end{gathered}$$

I.e. A torch in any of the temperatures in the specified range operates in the same way as a torch at room temperature.

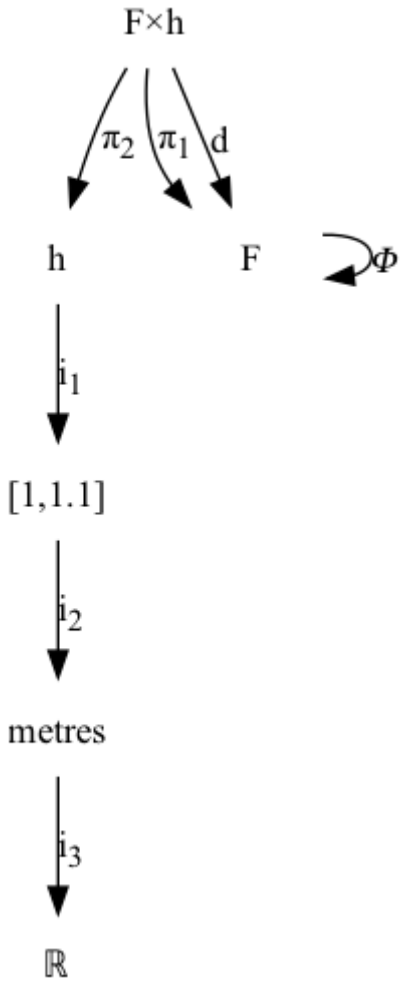


Drop Resistance

Let $d : F \times h \to F$ be a function that drops a torch F from a height h in metres onto a concrete surface.

$$\begin{gathered} \Phi \models F \\ d : F \times h \to F' \\ \forall f \in F, \wedge \forall h \in [1, 1.1], \Phi(d(f, h)) = \Phi(f) \end{gathered}$$

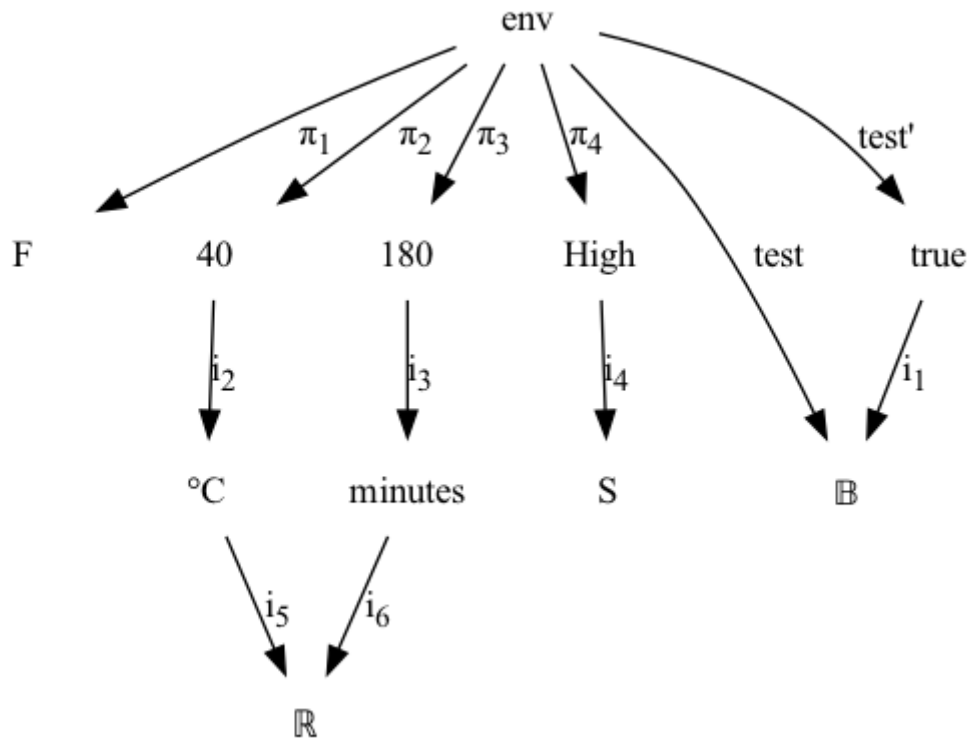
I.e. a dropped torch functions in the same way as a non-dropped torch.



Heat Test

env is an environment at a certain temperature for a specified duration, $test$ is a function that checks for any indications of fire or smoke.

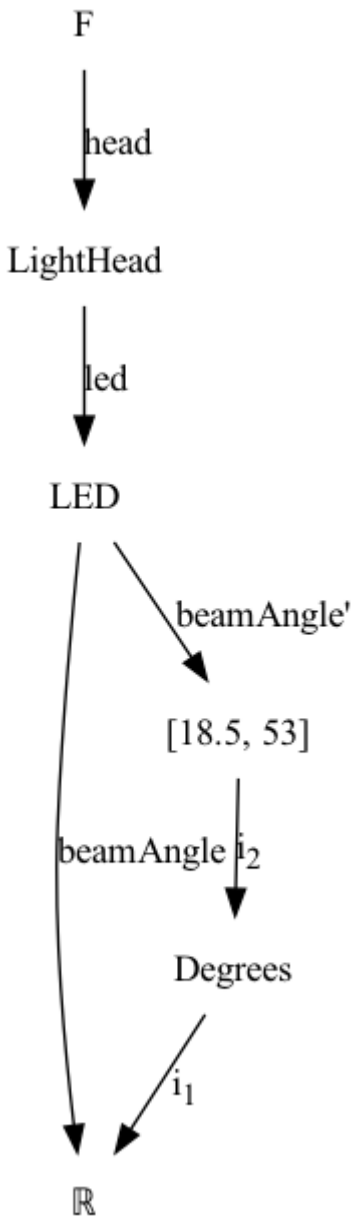
$$\forall f \in F : f = \pi_1(env)$$
$$(i_1 \circ test')(env) = test(env)$$



Beam Angle

$beamAngle$ is a function that measures the beam angle of a LED:

$$beamAngle : LED \to \mathbb{R}$$
$$beamAngle' : LED \to [18.5, 53]$$
$$\forall f \in F, (beamAngle \circ led \circ head)(f) = (i_1 \circ i_2 \circ beamAngle' \circ led \circ head)(f)$$



Category $\mathcal{E}$ Diagram

TBD