Categorical Definition of Requirements

To-do

- [x] Define the set of requirements R. ✅ 2025-11-28
- [x] Add cat theory elements - products, limits, composed morphisms. ✅ 2025-12-02
- [x] Make the definitions below more cat-theory-like. ✅ 2025-11-28

Preliminary Definitions

Let E be a category.

$Objs(E)$ is the set of Electro-mechanical devices plus $\{\mathbb{R}, \mathbb{B}\}$.

Let F be the object representing the Flashlight, $F \in Objs(E)$.

Let $\mathbb{R}$ be the real numbers (measurements), and $\mathbb{B}$ be the Boolean object $\{true, false\}$.

FR-001: White Light

Colour temperature measurement in °K using a colour meter device $cmeter$:

Given $I = \{Dark, Dim, Bright\}$ representing the torch output.

$$\begin{aligned} cmeter : I \to \mathbb{R} \\ cmeter(Dark) = 0 \\ cmeter(Dim) \in [5000, 6000] \\ cmeter(Bright) \in [5000, 6000] \end{aligned}$$

FR-002 & FR-003: Controls

Let $S = \{Off, Low, High\}$ be the set of internal states of F.

Let $\Phi = Sy^S \to Iy^A$ be the dynamic system representing the behaviour of F.

$A = \{next!\}$ representing the user input - $next!$ represent the input from a single switch.

Initial State: $\Phi_0 : S \times A \to S \quad = (-, -) \to Off$
Inspect State:

$$\begin{aligned} \Phi_1 : S \to I = \\ Off = Dark \\ Low = Dim \\ High = Bright \end{aligned}$$

Next State:

$$\begin{aligned} \Phi^{\#} : S \times A \to S = (Off, next!) = Low \\ (Low, next!) = High \\ (High, next!) = Off \end{aligned}$$

FR-004: Replaceable Battery

See IR-002

PR-001 & PR-002: Lumen Output

Let $lmeter$ be a Lux meter with an integrating sphere producing a Lumens measurement $\mathbb{R}$.

$$lmeter : I \rightarrow \mathbb{R}$$
$$lmeter(Dark) = 0$$
$$lmeter(Dim) \in [50, 60]$$
$$lmeter(Bright) \in [500, 550]$$

500 Lumens is Approximately 5250 Candelas.

PR-003: Beam Distance

Let $lmeter$ be a Lux meter *without* an integrating sphere, producing a Lux measurement $\mathbb{R}$ at distance d metres from F.

$$lmeter : I \times \mathbb{R} \rightarrow \mathbb{R}$$
$$lmeter(Bright, 400) \geq 0.25$$

Rule of thumb: the usable distance is approximately 25% of the distance where the lux value drops off to 0.25, so measuring at 400m should give a usable distance of 100m.

PR-004 & PR-005: Run Time

I is the current consumption in mA.
C is the battery capacity in mAh.

$$T = \frac{C}{I}$$

$$T_{high} = \frac{C}{I_{high}} \wedge T_{high} \geq 2$$
$$T_{low} = \frac{C}{I_{low}} T_{low} \wedge T_{low} \geq 10$$

PhR-001: Power Source

See FR-004

Phr-002: Material and Durability

$housing : F \rightarrow H$ where H is the torch housing.
$H \subset Aluminium \ \wedge \ H \subset Anodised$

For normal usage:

Y is the years of life, H is the hours of use, C number of operational cycles. Batteries excluded from durability.
Constraints:

$$Y \geq 5$$
$$H \geq 2000$$
$$C \geq 100000$$

PhR-003: Weight

$scale : F \rightarrow \mathbb{R}$ is a weight measurement function returning a weight in grams.
$scale(f) \leq 200$

PhR-004: Dimensions

Let $length : F \to \mathbb{R}$ be a function that returns the length of F in units of cm.
Let $width : F \to \mathbb{R}$ be a function that returns the width of F in units of cm.
Let $height : F \to \mathbb{R}$ be a function that returns the height of F in units of cm.

$length(f) \leq 15 \wedge length(f) \geq 8$
$width(f) \leq 3 \wedge width(f) \geq 0.5$
$height(f) \leq 3 \wedge height(f) \geq 0.5$

PhR-005: IPX4

Let $water : F \to \mathbb{R}$ be a function that measures the amount of liquid water inside the torch.
Let $spray : F \to F'$ be a water spray from all directions for 10 minutes.

$water(spray(F)) = 0$

IR-001: User Interface

Let $tail : H \to T$ be a function that returns the tail cap T from the housing H
Let $switch : T \to S$ be a function that returns the switch S from the tail cap T.

$s = switch(tail(housing(f)) \wedge s \in P$ where P is the set of momentary action push button switches.

The sets A and I from FR-002 and FR-003 also form part of the interface.

The switch $s \in S$ implements A and the light source $l \in L$, where L is the set of LEDs, implements I.

IR-002: Battery Access

Also FR-004.

Let $case : F \to C \times T$ be a function that returns the case C and the tail cap T from a torch F.
Let $battery : C \to B$ be a function that returns the battery B from a case C.

$replace_{batt} : F \times B \to F' \times B' : replace_{batt}(f, b_{new}) = (f', b_{old})$
and
$battery(case(f) = b_{old}$
$battery(case(f') = b_{new}$

ER-001: Operating Temperature Range

Let F be a torch that satisfies the requirements R, i.e. $F \vdash R$.
Let $cold : F \to F$ be a function that cools F to -10°C.
Let $hot : F \to F$ be a function that heats F to 40°C.

$F \vdash R \wedge cold(F) \vdash R \wedge hot(F) \vdash R$

ER-002: Drop Resistance

Let $drop : F \times \mathbb{R} \to F$ be a function that drops a torch F from a height $\mathbb{R}$ in metres onto a concrete surface.

$F \vdash R \wedge drop(F, 1) \vdash R$

$F \vdash R \wedge drop(drop(F,1),1) \vdash R$ is not necessarily true.

SR-001: Non-Incendive

Let $heat\_test : F \times \mathbb{R} \times \mathbb{R} \times S \rightarrow \mathbb{B}$ be a function that tests a torch $f \in F$ at a temperature $\mathbb{R}$ in °C for a duration $\mathbb{R}$ minutes, in state S, and returns $true$ if fire is detected, and $false$ if it is not.

$heat\_test(f, 40, 180, High) = false$

I.e. the highest output for the longest duration in the hottest expected environment should not cause a fire.

FR-005: Beam Width/Angle

Let $beam\_width : F \times \mathbb{R} \rightarrow \mathbb{R}$ be a function that measures the minimum beam width in metres of a torch F at distance $\mathbb{R}$ in metres.

$w = beam\_width(f, 3) \wedge w \geq 1 \wedge w \leq 3$ (constraint C_1)

Let α be the beam angle measured in degrees at the light source, then:

$C_1 \Rightarrow \alpha \in [18, 53]$

BR-001: Costs, Profit, Etc

$$\begin{aligned}
&Revenue = Price \times Sales\\
&Costs = Fixed + Variable\\
&Fixed = Design + Salaries + Rent\\
&Variable = Sales \times (Materials + Labour + Shipping)\\
&Profit = Revenue - Costs
\end{aligned}$$

$$\begin{aligned}
&Sales \geq 5000 \wedge Price \geq 15 \Rightarrow Revenue \geq 75000\\
&Profit = 0.2 \times Revenue \Rightarrow Profit \geq 15000 \Rightarrow Costs \leq 0.8 \times Revenue
\end{aligned}$$

Finally

- [x] Don't forget the To-do list ✅ 2025-11-27

Notes/Observations

- A measurable requirement can be constrained - the measurement is produced by an activity which can be modelled in HQDM.