Name: James Ding

Date: 8/22/2023

Limits review

1. Compute the following limits

   a. $\lim\limits_{x\to1}\frac{x^3-1}{x-1}$ $= \lim\limits_{x\to1}\frac{(x-1)(x^2-x+1)}{x-1} = \lim\limits_{x\to1} x^2-x+1 = \boxed{1}$ ■

   b. $\lim\limits_{x\to0}\frac{(x+2)^2-4}{x}$ $= \lim\limits_{x\to0}\frac{(x^2+2x+4)-4}{x} = \lim\limits_{x\to0}\frac{x^2+2x}{x} = \lim\limits_{x\to0} x+2 = \boxed{2}$ ■

   c. $\lim\limits_{x\to1}\frac{1-\sqrt{x}}{1-x}$ $= \lim\limits_{x\to1}\frac{1^2-x}{(1-x)(1+\sqrt{x})} = \lim\limits_{x\to1}\frac{1}{1+\sqrt{x}} = \boxed{\frac{1}{2}}$ ■

   d. $\lim\limits_{x\to1^+}(x-\lceil x \rceil)$

   e. $\lim\limits_{x\to1^-}(x-\lceil x \rceil)$

   $$\begin{cases} \cdots & \cdots \\ x-1 & ;\ x\in[1,2) \\ x-2 & ;\ x\in[2,3) \\ \cdots \end{cases}$$

   $\therefore \lim\limits_{x\to1^+} x-2 = \boxed{1}$ ■

   $\therefore \lim\limits_{x\to1^-} x-1 = \boxed{0}$ ■

   f. $\lim\limits_{x\to1}\frac{|x-1|}{x-1}$

   $$\frac{|x-1|}{x-1} = \begin{cases} \frac{x-1}{x-1} & ;\ x>1 \\ \text{undefined} & ;\ x=1 \\ -\frac{x-1}{x-1} & ;\ x<1 \end{cases} = \begin{cases} 1 & ;\ x>1 \\ \text{undef.} & ;\ x=1 \\ -1 & ;\ x<1 \end{cases}$$

   $\therefore \lim\limits_{x\to1}\frac{|x-1|}{x-1} = \boxed{\text{DNE}}$ ■

   g. $\lim\limits_{x\to0}\frac{\sin(x)}{x}$ $=\boxed{1}$ ■

   h. $\lim\limits_{x\to0}\frac{\sin(3x)}{3x}$ let $\theta = 3x$ $\lim\limits_{\theta\to0}\frac{\sin\theta}{\theta} = \boxed{1}$ ■

   i. $\lim\limits_{x\to0}\frac{\sin(3x)}{x}$ $= \frac{3}{3}\cdot\lim\limits_{x\to0}\frac{\sin(3x)}{x} = 3\lim\limits_{x\to0}\frac{\sin(3x)}{3x} = 3(1) = \boxed{3}$ ■

2. What value of k would make it so that $\lim\limits_{n\to3} f(n)$ exists?

   $$f(n) = \begin{cases} (n-5)^2, & n\geq 3 \\ kn+3, & n<3 \end{cases}$$

   Condition: $\lim\limits_{n\to3^-} f(n) = \lim\limits_{n\to3^+} f(n)$

   $\lim\limits_{n\to3^+} f(n) = (3-5)^2 = 4$

   $\therefore$ we want $kn+3=4$

   $3k+3=4$

   $3k=1$

   $\boxed{k=\frac{1}{3}}$ ■

3. Find $\lim\limits_{\theta\to0}\sin(\theta)\cos\left(\frac{1}{\theta^2}\right)$

4. What is wrong with the following calculation? What is the correct way of solving it?

   $$\lim_{x\to0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right) = \lim_{x\to0^+}\frac{1}{x} - \lim_{x\to0^+}\frac{1}{x^2} = +\infty - (+\infty) = 0$$

   You cannot do $\infty-\infty=0$, as it is an indeterminate form.

   $\lim\limits_{x\to0^+}\left[\frac{1}{x}-\frac{1}{x^2}\right]$

   $= \lim\limits_{x\to0^+}\frac{x-1}{x^2}$

   $= \lim\limits_{x\to0^+}[x-1]\cdot\lim\limits_{x\to0^+}\frac{1}{x^2}$

   $\boxed{=-\infty}$ ■

5. Suppose that f has domain $\mathbb{R}$ and is continuous. Prove or disprove that if the range of f contains both positive and negative numbers, then f must have a root.

   Given: • $f(x)$ is continuous over $(-\infty,\infty)$.

   • Has a positive / negative number

   Proof by IVT.

   Since f is continuous over $(-\infty,\infty)$ and 0 is some number within that range, there exist a number c within the domain s.t. $f(c)=0$.

6. Where are the discontinuities for $y = \ln(tan(x))$? (Hint: Remember that $\tan(x)$ is a periodic function.)

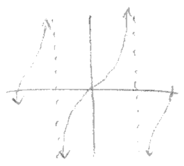
7. Find a function f that is continuous on all of $\mathbb{R}$ except $-1, 0$, and 1.

8. Let f be continuous on $[-1,1]$. Where $f(-1)$=-10 and $f(1) = 10$. Does a value $-1 < c < 1$ exist such that $f(c) = 11$? Why/why not?

9. Let h be a function on $[-1,1]$. Where $h(-1) = -10$ and $h(1) = 10$. Does a value $-1 < c < 1$ exist such that $h(c) = 0$? Why/why not?

6. Observations of $\tan(x)$

- Has a period of π
- Undefined every $\frac{\pi}{2} + \pi k$



Observations of $\ln(x)$

- Only defined when $x > 0$

Only valid for half a period of π

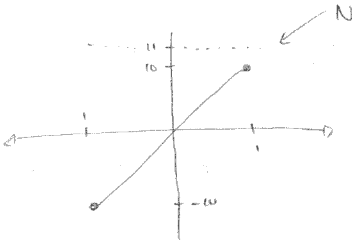
Therefore, $\ln(\tan(x))$ is discontinuous

$$\left[\pi k - \frac{\pi}{2},\ 2\pi k\right]$$

where k is any integer ■

7. $f(x) = \boxed{\dfrac{x^3}{x(x^2-1)}}$ ■

8. No. Proof with counter:



Notice how 11 is not in the range.

IVT can prove a value between $[-10, 10]$ exists but since 11 is not within the range, there is no guarantee.

Therefore, by contradiction, this claim is wrong. ■

9. See #8 explanation

Therefore, by IVT, 0 must exist within $(-1,1)$ as all conditions for IVT are met.