3. $f(x) = \sqrt{x}$; $a = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$; $f'(a) = \frac{1}{4}$; $f(a) = 2$

$$L(x) = 2 + \frac{1}{4}(x-4)$$

8. $f(x) = (1+x)^{-3}$; $a = 0$

$f'(x) = -3(x+1)^{-4}$; $f'(a) = -3$; $f(a) = 1$

$L(x) = 1 - 3x$ ✓

$|(1+x)^3 - (1-3x)| < 0.1$

$-0.1 < (1+x)^3 - (1-3x) < 0.1$

$$-0.1162 < x < 0.144$$

12. a. $\frac{d}{du}\left[\frac{1+2u}{1+3u}\right]$

$= \frac{(1+3u)(2) - (1+2u)(3)}{(1+3u)^2}$

$= \frac{2 + 6u - 3 - 6u}{(1+3u)^2}$

$= -\frac{1}{(1+3u)^2}$

$$dy = -\frac{1}{(1+3u)^2} du$$

b. $\frac{d}{d\theta}\left[\theta^2 \sin 2\theta\right]$

$= \theta^2 \cos 2\theta \cdot 2 + \sin 2\theta \cdot 2\theta$

$= 2\theta^2 \cos 2\theta + 2\theta \sin 2\theta$

$= 2\theta(\theta \cos 2\theta + \sin 2\theta)$

$$dy = 2\theta(\theta \cos 2\theta + \sin 2\theta)\, d\theta$$

13. a. $\frac{d}{dt} \tan\sqrt{t}$

$= \sec^2 \sqrt{t} \cdot \frac{1}{2} t^{-1/2}$

$= \sec^2 \sqrt{t} \cdot \frac{1}{2\sqrt{t}}$

$= \frac{\sec^2 \sqrt{t}}{2\sqrt{t}}$

$$dy = \frac{\sec^2 \sqrt{t}}{2\sqrt{t}}\, dt$$

b. $\frac{d}{dv}\left[\frac{1-v^2}{1+v^2}\right]$

$= \frac{(1+v^2)(-2v) - (1-v^2)(2v)}{(1+v^2)^2}$

$= \frac{-2v - 2v^3 - (2v - 2v^3)}{(1+v^2)^2}$

$= \frac{-4v}{(1+v^2)^2}$

$$dy = -\frac{4v}{(1+v^2)^2}\, dv$$

16. a. $\frac{d}{dx} \cos \pi x$

$= -\sin \pi x \cdot \pi$

$$dy = -\pi \sin \pi x \, dx$$

b. $dy = -\pi \sin\left(\frac{\pi}{3}\right)(-0.02)$

$$\approx 0.5441$$

18. a. $\frac{d}{dx}\left[\frac{x+1}{x-1}\right]$

$= \frac{(x-1) - (x+1)}{(x-1)^2}$

$= \frac{x - 1 - x - 1}{(x-1)^2}$

$= -\frac{2}{(x-1)^2}$

$$dy = -\frac{2}{(x-1)^2}\, dx$$

18. b. $dy = -\frac{2}{(2-1)^2} \cdot (0.05) = \boxed{-0.1}$

22. $\Delta y = f(x + \Delta x) - f(x)$

$= f(1 + 0.5) - f(1)$
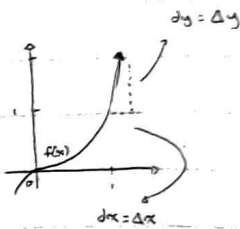
$= (1.5)^3 - 1$

$$= 2.375$$

$\frac{d}{dx} x^3 = 3x^2$

$dy = 3x^2 dx$

$= 3(1)^2 (0.5)$

$$= 1.5$$

23. $f(x) = x^4$; $a = 2$

$f'(x) = 4x^3$; $f'(a) = 32$; $f(a) = 16$

$L(x) = 16 + 32(x-2)$

$L(1.999) = 16 + 32(1.999 - 2)$

$$= 15.968$$

24. $f(x) = \frac{1}{x}$; $a = 4$

$f'(x) = -x^{-2}$; $f'(a) = -\frac{1}{16}$; $f(a) = \frac{1}{4}$

$L(x) = \frac{1}{4} - \frac{1}{16}(x-4)$

$L(4.002) = \frac{1}{4} - \frac{1}{16}(4.002 - 4)$

$$\approx 0.2498$$

26. $f(x) = \sqrt{x}$; $a = 100$

$f'(x) = \frac{1}{2}x^{-1/2}$; $f'(a) = \frac{1}{20}$; $f(a) = 10$

$L(x) = 10 + \frac{1}{20}(x - 100)$

$L(100.5) = 10 + \frac{1}{20}(100.5 - 100)$

$\boxed{= 10.025}$

32. a. $f(x) = \pi x^2$; $a = 24$

$f'(x) = 2\pi x$; $f'(a) = 48\pi$; $f(a) = 576\pi$

$L(x) = 576\pi + 48\pi(x - 24)$

$L(24.2) = 576\pi + 48\pi(24.2 - 24)$

≈ 1837.72

Error: $1837.72 - \pi(24)^2 = \boxed{30.1593 \text{ cm}^2}$

b. $\dfrac{1837.72 - \pi 24^2}{\pi 24^2} = 0.01667$

$\boxed{0.01667 \quad \text{or} \quad 1.667\%}$

34. $\dfrac{d}{dv}\left[\dfrac{2}{3}\pi v^3\right]$

$= 2\pi v^2$

$\partial V = 2\pi v^2 \, dv$

$= 2\pi(25)^2(0.05) = \boxed{196.35 \text{ cm}^3}$

36. a. $\sin\theta = \dfrac{20}{h}$

$h = 20\csc\theta$

$\dfrac{d}{d\theta}\left[20\csc\theta\right]$

$= -20\csc\theta\cot\theta$

$\partial h = -20\csc\theta\cot\theta \, d\theta$

$= -20\csc\dfrac{\pi}{6}\cot\dfrac{\pi}{6}\left(\pm\dfrac{\pi}{180}\right)$

$\boxed{\approx \pm 1.2092}$

b. $\dfrac{-20\csc\frac{\pi}{6}\cot\frac{\pi}{6}\left(\pm\frac{\pi}{180}\right)}{20/\sin\frac{\pi}{6}} = \pm 0.03023 = \boxed{3.023\%}$

38. $dF = \boxed{4kR^3 \, dR}$ $\longleftarrow$ 4 as coefficient

$= 4kR^3(0.05)$

$\boxed{= 0.2kR^3}$

42. a. $f(x) = g(x)$; $a = 2$

$f'(x) = \sqrt{x^2 + 5}$; $f'(a) = 3$; $f(a) = -4$

$L(x) = -4 + 3(x - 2)$

$L(1.95) = \boxed{-4.15}$

$L(2.05) = \boxed{-3.85}$

b. Too big. Square root tangents are always too large