James
Ding

7. $F(x) = (5x^6 + 2x^3)^4$

$F'(x) = \boxed{4(5x^6 + 2x^3)^3 \cdot [30x^5 + 6x^2]}$

21. $g(u) = \left(\dfrac{u^3 - 1}{u^3 + 1}\right)^8$

$g'(u) = 8\left(\dfrac{u^3 - 1}{u^3 + 1}\right)^7 \cdot \dfrac{\cancel{}}{\cancel{}} \cdot \dfrac{d}{du}\left[\dfrac{u^3 - 1}{u^3 + 1}\right]$

9. $f(x) = \sqrt{5x + 1}$

$= (5x + 1)^{1/2}$

$f'(x) = \dfrac{1}{2}(5x + 1)^{-1/2} \cdot 5$

$\boxed{= \dfrac{5}{2}(5x + 1)^{-1/2}}$

$\dfrac{d}{du}\left[\dfrac{u^3 - 1}{u^3 + 1}\right] = \dfrac{(u^3 + 1)\frac{d}{du}(u^3 - 1) - (u^3 - 1)\cdot\frac{d}{du}(u^3 + 1)}{(u^3 + 1)^2}$

$= \dfrac{(u^3 + 1)(3u^2) - (u^3 - 1)(3u^2)}{(u^3 + 1)^2}$

$\dfrac{\cancel{}\cancel{}}{(u^3 + 1)^2}$

11. $A(t) = \dfrac{1}{(\cos t + \tan t)^2} = (\cos t + \tan t)^{-2}$

$\boxed{= -2(\cos t + \tan t)^{-3}[-\sin t + \sec^2 t]}$

$= \dfrac{6u^2}{(u^3 + 1)^2}$

$g'(u) = 48\left(\dfrac{u^3 - 1}{u^3 + 1}\right)^7 \left(\dfrac{1}{u^3 + 1}\right)^2 u^2$

$= \dfrac{48u^2(u^3 - 1)^7}{(u^3 + 1)^9}$

13. $f(\theta) = \cos(\theta^2)$

$f'(\theta) = -\sin(\theta^2) \cdot 2\theta$

$\boxed{= -2\theta \sin(\theta^2)}$

33. $y = \sin\left(\sqrt{1 + x^2}\right)$

$y' = \cos\left(\sqrt{1 + x^2}\right) \cdot \dfrac{d}{dx}\sqrt{1 + x^2}$

$= \cos\left(\sqrt{1 + x^2}\right)\dfrac{1}{2}(1 + x^2)^{-1/2} \cdot 2x$

$= 2x\cos\left(\sqrt{1 + x^2}\right) \cdot \dfrac{1}{2}(1 + x^2)^{-1/2}$

$= x\cos\left(\sqrt{1 + x^2}\right)(1 + x^2)^{-1/2}$

17. $f(x) = (2x - 3)^4(x^2 + x + 1)^5$ $y = u^5$ $u = x^2 + x + 1$

$f'(x) = (2x - 3)^4 \dfrac{d}{dx}\left[(x^2 + x + 1)^5\right] + (x^2 + x + 1)^5 \dfrac{d}{dx}\left[(2x - 3)^4\right]$

$= (2x - 3)^4 \cdot 5(x^2 + x + 1)^4(2x + 1) + (x^2 + x + 1)^5 \cdot 4(2x - 3)^3 \cdot 2$

$= 5(2x - 3)^4(x^2 + x + 1)^4(2x + 1) + 8(x^2 + x + 1)^5(2x - 3)^3$

$= (2x - 3)^3(x^2 + x + 1)^4\left(5(2x - 3)(2x + 1) + 8(x^2 + x + 1)\right)$

$\boxed{= (2x - 3)^3(x^2 + x + 1)^4(28x^2 - 12x - 7)}$

41. $y = \sqrt{x + \sqrt{x}}$

$y' = \dfrac{1}{2}(x + \sqrt{x})^{-1/2} \dfrac{d}{dx}(x + \sqrt{x})$

$\boxed{= \dfrac{1}{2}(x + \sqrt{x})^{-1/2}\left(1 + \dfrac{1}{2}x^{-1/2}\right)}$