Limits Review

Section 1.5

21. $$\lim_{x\to 0} \frac{\sin x}{x+\tan x} = 0.5$$

(table via desmos)

24. $$\lim_{x\to 5^+} \frac{x+1}{x-5}$$

$= \frac{5+1}{5^+-5}$

$= \frac{6}{0^+}$

$\boxed{= \infty}$ ∎

33. $$\lim_{x\to -2^+} \frac{x-1}{x^2(x+2)}$$

$= \frac{-2-1}{(-2)^2(-2^++2)}$

$= \frac{-3}{4(0^+)}$

$= -\frac{3}{4}\cdot\frac{1}{0^+}$

$\boxed{= -\infty}$ ∎

35. $$\lim_{x\to \frac{\pi}{2}^+} \frac{1}{x}\sec x$$

$= \lim_{x\to\pi/2^+} \frac{1}{x} \cdot \lim_{x\to\pi/2^+} \sec x$

$= \frac{2}{\pi} \cdot \lim_{x\to\pi/2^+} \frac{1}{\cos x}$

$= \frac{2}{\pi}\cdot\frac{1}{0^-}$

$\boxed{= -\infty}$ ∎

46. $f(x) = \tan\left(\frac{1}{x}\right)$

a. $\tan(x)$ is a function that has a period of π, and is continuous between $\left(k-\frac{\pi}{2}, k+\frac{\pi}{2}\right)$; where k is any integer.

When $a = \pi, 2\pi, 3\pi$, or $k\pi$, $\tan(a) = 0$.

$\therefore$ when $x = \frac{1}{\pi} \cdots \frac{1}{k\pi}$, the $\frac{1}{x}$ part evaluates to $\pi \ldots k\pi$

Thus, $f(x) = 0$ when $x = \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \ldots$ ∎

b. As mentioned in a, $\tan(x)$ is continuous between $\left(k-\frac{\pi}{2}, k+\frac{\pi}{2}\right)$ and has a period of π

When $b = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$, or $k\pi+\frac{\pi}{4}$, $\tan(b) = 1$

$\therefore$ when $x = \frac{4}{\pi} \cdots \frac{1}{k\pi+\pi/4}$, the $\frac{1}{x}$ part evaluates to $\frac{\pi}{4}\ldots$

Thus, $f(x) = 1$ when $\frac{4}{\pi}, \frac{4}{5\pi}, \frac{4}{9\pi}, \ldots$ ∎

c. $$\lim_{x\to 0^+} \frac{1}{x} = \infty$$

Given that $\tan(x)$ is always continuous, if it's input goes to infinity, more cycles/oscillations occur.

$\therefore \lim_{x\to 0^+} \tan\left(\frac{1}{x}\right) = \text{DNE}$ ∎

Chapter 1 Review

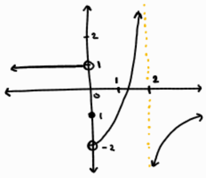
23. a. i. 3 ii. 0 iii. DNE

iv. 2 v. ∞ vi. $-\infty$

b. $x=0, x=2$

c. $x = -3, 0, 2, 4$

24.



25. $$\lim_{x\to 0} \cos(x+\sin x)$$

$= \cos(\sin 0)$

$= \cos 0$

$\boxed{= 1}$ ∎

29. $$\lim_{h\to 0} \frac{(h-1)^3+1}{h}$$

$f(x) = x^3$

Solve for $f'(-1)$

$= \lim_{h\to 0} \frac{h^3-3h^2+3h-1+1}{h}$

$= \lim_{h\to 0} \frac{h^3-3h^2+3h}{h}$

$= \lim_{h\to 0} h^2-3h+3$

$\boxed{= 3}$ ∎

33. $$\lim_{u\to 1} \frac{u^4-1}{u^3+5u^2-6u}$$

$= \lim_{u\to 1} \frac{(u^2-1)(u^2+1)}{u(u^2+5u-6)}$

$= \lim_{u\to 1} \frac{(u^2+1)(u+1)(u-1)}{u(u+6)(u-1)}$

$= \lim_{u\to 1} \frac{(u^2+1)(u+1)}{u(u+6)}$

$\boxed{= \frac{4}{7}}$ ∎

35. $$\lim_{s\to 16} \frac{4-\sqrt{s}}{s-16}$$

$= \lim_{s\to 16} \frac{4-\sqrt{s}}{s-16}\cdot\frac{4+\sqrt{s}}{4+\sqrt{s}}$

$= \lim_{s\to 16} \frac{16-s}{(s-16)(4+\sqrt{s})}$

$= \lim_{s\to 16} \frac{-(s-16)}{(s-16)(4+\sqrt{s})}$

$= \lim_{s\to 16} -\frac{1}{4+\sqrt{s}}$

$\boxed{= -\frac{1}{8}}$ ∎

37. $$\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}$$

$= \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$

$= \lim_{x\to 0} \frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})}$

$= \lim_{x\to 0} \frac{1-1+x^2}{x(1+\sqrt{1-x^2})}$

$= \lim_{x\to 0} \frac{x^2}{x(1+\sqrt{1-x^2})}$

$= \lim_{x\to 0} \frac{x}{1+\sqrt{1-x^2}}$

$\boxed{= 0}$ ∎

39. $2x-1 \le f(x) \le x^2$ $\quad x\in(0,3)$

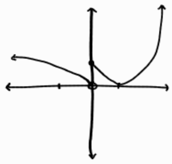
By squeeze theorem, $\lim_{x\to 1} f(x) = 1$ ∎

45. a. i. 3 ii. 0 iii. DNE

iv. 0 v. 0 vi. 0

b. $x=0$

c.



49. $x^5-x^3+3x-5=0$ $\quad x\in(1,2)$

Let $f(x) = x^5-x^3+3x-5$

IVT states that if a function is continuous between $[a,b]$, there exists a value $c\in(a,b)$ s.t. $f(a)<f(c)<f(b)$

- $f(x)$ is continuous at $\mathbb{R}$ (polynomial)

$f(1) = -2$ $\quad f(2) = 25$

$\therefore$ By IVT, there exists a value c between a and b s.t. $f(c) = 0$ ∎

51. $|f(x)| \le g(x)$

$-g(x) \le f(x) \le g(x)$

If $\lim_{x\to a} g(x) = 0$, by squeeze theorem, $\lim_{x\to a} f(x)$ must also equal 0 ∎

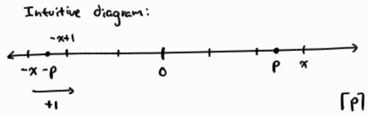
52. $f(x) = \lceil x\rceil + \lceil -x\rceil$

$$\begin{cases} \ldots & \\ -1+2 = 1 & x\in(-2,-1] \\ 0+1 = 1 & x\in(-1,0] \\ 1+0 = 1 & x\in(0,1] \\ 2-1 = 1 & x\in(1,2] \\ \ldots & \end{cases}$$

Furthermore, this function can be generalized to:

$x + (-x+1) = 1$

Intuitive diagram:



$\lceil p\rceil = x$

$\lceil -p\rceil = -x+1$

$\therefore f(x) = 1$

a. $\lim_{x\to a} f(x)$ exists at $\mathbb{R}$

b. None