STIX Two

Mathematical expressions must be formatted using different rules than those applied to the surrounding text. When markup is used, the limits of the mathematical text are defined explicitly. In plain text it is possible to use a number of heuristics for identifying mathematical expressions. *Once recognized, they can be treated appropriately,* for example expressions input as plain text could be tagged with a rich-text math style. Such math style would connect in a straightforward way to appropriate MathML tags.

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

The size of *mathematical delimiters* or operators may change on the size of the enclosed text. In an **equation** such as

$$W^{3\beta}_{\delta_1\rho_1\sigma_2} = U^{3\beta}_{\delta_1\rho_1} + \frac{1}{8\pi^2} \int_{\alpha_1}^{\alpha_2} d\alpha'_2 \left[\frac{U^{2\beta}_{\delta_1\rho_1} - \alpha'_2 U^{1\beta}_{\rho_1\sigma_2}}{U^{0\beta}_{\rho_1\sigma_2}} \right],$$

the size of the bracket scales with the size of the enclosed expression, in this case a fraction, and the size of the integral could scale with the size of the integrand. The integral isn't scaled here, since common practice is to use one size for all larger integrals. This example also shows the positioning of multiple sub- and superscripts as well as the positioning of limit expressions on the integral. **Punctuation** following math in display is commonly placed on the local baseline or centerline. The example

$$\int_0^a \frac{x\,dx}{x^2 + a^2}$$

shows an increased space before the dx. In order to allow automatic formatting of this, the special character code U+2146 DOUBLE-STRUCK ITALIC SMALL D can be used. In this instance, it would not be rendered with an actual double struck glyph. The final example,

$$|x| = \begin{cases} x \; if x \geq 0 \\ -x \; if x < 0 \end{cases},$$

demonstrates regular text embedded in a mathematical formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

XITS

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them. Other propositions are true with only conditional necessity: "Socrates is sitting down", for instance, or "Plato is going for a walk" is necessarily true while (and only while) Socrates is in fact sitting down and Plato is in fact going for a walk, respectively. The same is true for phenomena like chariot races: the drivers' skillful maneuvers are necessary while I am observing them, but they were not necessary beforehand, since they are the result of the drivers' free will. Thus, things and events that are simply necessary are so because of their own nature; things and events that are conditionally necessary are so owing to extrinsic or accidental circumstances.

4. BOETHIUS ON PREDESTINATION AND FREE WILL

This argument is in fact based on an adaptation of the Aristotelian definition of knowledge: if I *know* something, then the object of my knowledge *necessarily*[150] is the way I know it to be, simply because that's the way knowledge (Greek *epistêmê*, Latin *scientia*, Arabic *'ilm*) is defined – at least in one of its many Aristotelian senses.[151]

4.2.1. The distinction between absolute and conditional necessity[152]

One Aristotelian text that is important in this regard is this one from the *De interpretatione* (19a23-6):

> *That what exists is when it is, and what does not exist is not when it is not, necessary.*[153]

For Aristotle, there can be *epistêmê* in this strict sense – the sense, that is, in which such knowledge is always true (*APo* II, 19, 100b18) – only of universals.[154] Indeed, the reason why knowledge is bereft of falsehood is that it is *necessary* for things to be in the way knowledge understands them to be.[155] This is clear, for instance, from a passage from the *Nicomachean Ethics* (VI, 3, 1139b20-25):

150 As Weidemann points out (1998, 198), Boethius' addition of the modal operator "necessarily" transforms Aristotle's consequentiality relation of *being* into a consequentiality relation of *necessity*.

151 "It is impossible for that of which there is knowledge in the absolute sense to be otherwise <than it is>," says Aristotle in the *Posterior Analytics* (I, 2 71b9-15).

152 Cf. Obertello 1989, 95ff.; Weidemann 1998; Bechtle 2006, 274f.

153 Το μεν ούν είναι το ον όταν η, και το μη ον μη είναι όταν μη η, ανάγκη. Cf. Frede 1972.

154 Cf. *Metaph*. K 1, 1059b26; 2, 1060b20; B 6, 1003a15; M 9, 1086b5.10; 1086b 33; *Anal. pr.* 31 87b33, *De an*. 2.5417b23; *EN* 7, 6, 1140b31; 1180b15. The Narrator begins by speaking not of knowledge but of opinion, only to slip into talking about knowledge by virtue of the (Platonic!) equivalence true opinion = knowledge.

155 Cf. *Cons*. 5.3.21: *Ea namque causa est cur mendacio scientia careat, quod se ita rem quamque habere necesse est uti eam sese habere scientia comprehendit.*

> *We all suppose that what we know is not capable of being otherwise (...) therefore the object of knowledge is of necessity. Therefore it is eternal, for things that are of necessity in the unqualified sense are all eternal*[156]*; and things that are eternal are ungenerated and imperishable.*

The reason this distinction is important is as follows: the Narrator reasons that (1) necessarily, if an event p will happen, then God foresees it ($N(p \longrightarrow F(G, p))$); and (2) necessarily, if God foresees p, it will happen ($N(F(G, p) \longrightarrow p)$). Note that the necessity here bears upon the entire implication: it is a *necessitas consequentiae*. It has been argued[157] that Boethius now makes a simple logical mistake, inferring from (1) and (2) that (3) if p, then necessarily God foresees p ($p \longrightarrow NF(G, p)$), and (4) if God foresees p, then necessarily p ($F(G, p) \longrightarrow Np$), where in both the latter cases the necessity bears upon the consequent (*necessitas consequentis*).

Yet it is not the case that it is necessary now that (p) be true, and it is also not the case that it is necessary that (~p) be true, i.e.

$$\sim(Np) \wedge \sim(N{\sim}p)$$

I believe this analysis is mistaken. Boethius does believe both (3) and (4) are true, but they are true only *conditionally*, where the condition is God's knowledge. In other words, the necessity imposed by God's knowledge of a future event is of the same kind as that which necessitates that Socrates be sitting when I know he is sitting: such conditional necessity (*kath' hupothesin* in Greek[158]; *secundum praecessionem* in the Latin of Chalcidius[159]) imposes no constraint upon Socrates, but simply concerns the nature of knowledge.[160] As Boethius will claim, such future events can be said to be necessary with regard to God's knowledge but free with regard to their own nature.

Вященника ἐσσήν (так Иосиф Флавий воспроизводит греческими буквами библейский термин !v,x, хошен) соответствует греческий термин λόγιον, «прорицание» (ср. Септуагинту: λογεῖον).

These considerations go some way toward explaining the key point of how God can know future events, which are by their nature indeterminate, in a determinate way. The reason why this seems counter-intuitive to us is

156 Cf. *De Caelo* I, 12, 281a28-282a4.

157 Graeser 1992; Marenbon 2003a, 533ff.

158 Cf. Eustratius, *In EN VI*, p. 293, 1-2 Heylbut (CAG 20): ὡς εἶναι τὰ ἁπλῶς ἐξ ἀνάγκης πάντα ἀίδια. ἁπλῶς δὲ λέγομεν ἐξ ἀνάγκης ὅσα μὴ καθ' ὑπόθεσιν ἐξ ἀνάγκης, οἷον τὸ καθῆσθαί τινα ἔστ' ἂν κάθηται ὁ καθήμενος, ἐξ ἀνάγκης εἶναι λέγομεν τὸ καθῆσθαι αὐτόν, ἀλλ' οὐχὶ ἁπλῶς ἀλλ' ἐξ ὑποθέσεως ("thus, all things that are simply by necessity are perpetual [*aidia*]. We call 'simply by necessity' whatever is not hypothetically (*kath' hupothesin*) by necessity: for instance, the fact of sitting: as long as the seated person is sitting, we say that the fact that he is sitting is necessary, yet not simply but by hypothesis (*ex hupotheseôs*)".

159 Chalcidius, *In Tim*., p. 186, 15 Waszink.

160 In the words of Bächli 2001, it is an "epistemological necessity".