MATH 156: Precalculus
Fall 2015
Test 2: Review and Sample Problems

Test 2 will be Monday 2 November during our regular class time. The test will cover Chapter 2 Sections 5-8, Chapter 3 Sections 1-3, 6-7, Chapter 4 Sections 1-6. Calculators, notes, books or other aids will not be allowed.

Chapter 2 Section 5
How to recognize linear functions. How to write the equation of a linear function.
Most important skill: Given a verbal description of a function, you must know how to write its equation and use it.

Chapter 2 Section 6
Transformations of functions. Vertical and horizontal shifts. Reflections about x- and y-axis. Vertical and horizontal stretching and shrinking. You should be able to recognize symmetry in graphs.
Most important skill: You must be able to recognize and graph a simple transformation of a known function. See the list at the end of the review for the list of functions you must know.

Chapter 2 Section 7
Combining functions. Given $f(x)$ and $g(x)$, know how to find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $(f/g)(x)$, and $(f \circ g)(x)$, find their domains, and use them.

Chapter 2 Section 8
One-to-one functions and inverses. Know how to determine whether or not a function is one-to-one. Know how to find the inverse of a one-to-one function. Know the inverse function property and how to use it.

Chapter 3 Section 1
Quadratic functions and models. Be able to place a quadratic function into **standard form** and pick off the **vertex**. Using the standard form, be able to sketch the transformed quadratic, find any intercepts, any maximum or minimum, and identify its domain and range.

Chapter 3 Section 2
Polynomial functions and their graphs. Given a polynomial function, be able to (i) identify its end-behavior, (ii) use the multiplicity of roots and intercepts to sketch an approximate graph. Know what the leading terms implies about the number of roots and number of local extrema.

Chapter 3 Section 3
Know how to do long division and rewrite an expression $p(x)/d(x)$ using the quotient and remainder.

Chapter 3 Section 6
Given a rational function, be able to find any asymptotes, including slant asymptotes, find intercepts, and determine end behavior.

Chapter 3 Section 7
Be able to solve rational and polynomial inequalities.

Chapter 4 Section 1
Exponential functions. Know how to sketch the graph of an exponential function and how to use an exponential function. Know how to use the formula for compound interest.

Chapter 4 Section 2
The natural exponential function. The same as Chapter 4 Section 1 but with base e.

Chapter 4 Section 3
Know the definition of a logarithmic function and rules about how to calculate with them, including simplifying and expanding. Know how to graph a logarithmic function. Know the standard notation associated with logarithms (such as those base 10 and base e.)

Chapter 4 Section 4
Laws of logarithms. Know them. Be aware of common mistakes and don't make them! Know the change of base formula.

Chapter 4 Section 5
Exponential and Logarithmic equations. Have facility solving equations with logarithms. Recall that our techniques include: rewriting a logarithmic equation in exponential form, rewriting an exponential equation in logarithmic form, taking the logarithm of both sides, rewriting terms in an exponential equation with the same base, identify a hidden quadratic equation.

Chapter 4 Section 6
Be able to use formulas for doubling, exponential growth, and radioactive decay.

Functions you must know how to graph:
$y = mx + b$,
$y = x^2$, $y = x^3$, $y = x^4$, $y = x^5, \cdots$
$y = \sqrt{x}$, $y = \sqrt[3]{x}$, $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}, \cdots$
$y = 1/x$, $y = 1/x^2$, $y = 1/x^3$, $y = 1/x^4, \cdots$
$y = a^x$
$y = \log_a x$
$y = |x|$

Sample Problems

Here are the formulas that will be on the front of your test:
$n(t) = n_0 2^{t/a}$
$n(t) = n_0 e^{rt}$
$m(t) = m_0 2^{-t/h}$
$m(t) = m_0 e^{rt}$ where $r = (\ln 2)/h$,
$\log_b x = (\log_a x)/(\log_a b)$
$A(t) = P(1 + \frac{r}{n})^{nt}$
$A(t) = Pe^{rt}$

These are all odd problems from your book. At the end of the list you will find the numbers identified so you can check your answers. I just selected TWO sample problems from each section. You must also look over difficult problems from (1) quizzes, (2) homework, (3) in-class problems from lectures and worksheets.

1. Sketch $f(x) = (1/2)\sqrt{x+4} - 3$.
2. The fox population in a certain region has a relative growth rate of 8% per year. It is estimated that the population in 2013 was 18000.
   (a) Find a function $n(t) = n_0 e^{rt}$ that models the population t years after 2013.
   (b) Use the function from part (a) to estimate the fox population in the year 2021.
3. Solve $\frac{x+2}{x+3} < \frac{x-1}{x-2}$
4. Sketch the graph of the polynomial function $P(x) = x^3(x+2)(x-3)^2$. Make sure your graph shows all intercepts and exhabits the proper end behavior.
5. Find intercepts, asymptotes, and domain for $f(x) =$.
6. Change $\log_3 15$ to base e.
7. For $f(x) = -4x^2 - 12x + 1$, (a) express f in standard form, (b) find the vertex and x- and y-intercepts of f, (c) Find the domain and range of f, and (d) determine whether f has a maximum or minimum and identify it.
8. Find $f \circ g$, $f \circ f$, $g \circ f$, and $g \circ g$ for $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{1}{x}$ and determine their domains.
9. Evaluate (a) $\log_8 0.25$ (b) $\ln e^4$ (c) $\ln(1/e)$
10. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May her driving cost was \$380 for 480 miles and in June her cost was \$460 for 800 miles. Assume that there is a linear relationship between the monthly cost C of driving a car and the distance x driven.
    (a) Find a linear function C that models the cost of driving x miles per month.
    (b) Draw a graph of C. What is the slope of this line? (c) At what rate does Lynn's cost increase for eery additional mile she drives?
11. Solve the exponential equation $2^x - 10(2^{-x}) + 3 = 0$.
12. Sketch $h(x) = 2^{x-4} + 1$ including domain, range, asymptotes, and intercepts.

13. Find the quotient and remainder of $\frac{x^6+x^4+x^2+1}{x^2+1}$.

14. Determine if the function $f(x) = x^2 + x$ is even or odd or neither.

15. Expand $\log \sqrt{\frac{x^2+4}{(x^2+1)(x^3-7)^2}}$.

16. If \$600 is invested at an interest rate of 2.5% per year, find the amount of the investment at the end of 10 years for the following compounding methods. (c) quarterly (d) continuously.

17. Determine if the function $f(x) = x^6 - 3$ on $0 \leq x \leq 5$ is one-to-one.

18. Express the function as a composition of functions $H(x) = |1 - x^3|$.

19. Solve the equation $\log_9(x - 5) + \log_9(x + 3) = 1$.

20. Show that $c = 1/2$ is a zero of $P(x) = 2x^3 + 7x^2 + 6x - 5$ and show that $x - c$ is a factor of $P(x)$.

21. The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample. (a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years. (b) Find a function $m(t) = m_0 e^{rt}$ where $r = (ln2)/h$, that models the mass remaining after t years. (c) How much of the sample will remain after 4000 years?

22. Solve the exponential equation $\frac{50}{1+e^{-x}} = 4$.

23. Graph the family of polynomials on the same set of axes: $P(x) = x^4 + c$ for $c = -1, 0, 1, 2$.

24. Find the slant asymptotes and the vertical asymptotes of $r(x) = \frac{x^3+x^2}{x^2-4}$.

25. Sketch $y = \log_3(x - 1) - 2$ and $y = |\ln x|$.

26. Look at the graph on page 208 with problem #71 and sketch all the graphs from parts (a)-(f). (Sorry, it was so much easier than reproducing the problem...)

27. Find a function f whose graph is a parabola with vertex $(2, -3)$ and that passes through the point $(3, 1)$.

28. Find the domain of $h(x) = \sqrt[4]{x^4 - 1}$.

29. Write the equation of the linear function f with rate of change 3 and initial value of -1.

30. Sketch $y = e^{x+1} - 3$ include domain, range, intercepts, asymptotes.

31. Use the Inverse Function Property to show that $f(x) = \frac{x+2}{x-2}$ and $g(x) = \frac{2x+2}{x-1}$ are inverses of each other.

32. Solve $\log_2(1/2) = x$ and $logx = -3$.

33. Use the Laws of Logarithms to combine $(1/3)\log(x + 2)^3 + (1/2)[\log x^4 - \log(x^2 - x - 6)^2]$.

Numbers of Sample problems.

1. 2.6.51 Sketch $f(x) = (1/2)\sqrt{x+4} - 3$.

2. 4.6.5 The fox population in a certain region has a relative growth rate of 8% per year. It is estimated that the population in 2013 was 18000.
   (a) Find a function $n(t) = n_0 e^{rt}$ that models the population t years after 2013.
   (b) Use the function from part (a) to estimate the fox population in the year 2021.

3. 3.7.35 Solve $\frac{x+2}{x+3} < \frac{x-1}{x-2}$

4. 3.2.29 Sketch the graph of the polynomial function $P(x) = x^3(x+2)(x-3)^2$. Make sure your graph shows all intercepts and exhibits the proper end behavior.

5. 3.6.61 Find intercepts, asymptotes, and domain for $f(x) =$.

6. 4.4.61 Change $\log_3 15$ to base e.

7. 3.1.23 For $f(x) = -4x^2 - 12x + 1$, (a) express f in standard form, (b) find the vertex and x- and y-intercepts of f, (c) Find the domain and range of f, and (d) determine whether f has a maximum or minimum and identify it.

8. 2.7.57 Find $f \circ g$, $f \circ f$, $g \circ f$, and $g \circ g$ for $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{1}{x}$ and determine their domains.

9. 4.3.33 Evaluate (a) $\log_8 0.25$ (b) $\ln e^4$ (c) $\ln(1/e)$

10. 2.5.49 The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May her driving cost was \$380 for 480 miles and in June her cost was \$460 for 800 miles. Assume that there is a linear relationship between the monthly cost C of driving a car and the distance x driven.
    (a) Find a linear function C that models the cost of driving x miles per month.
    (b) Draw a graph of C. What is the slope of this line? (c) At what rate does Lynn's cost increase for eery additional mile she drives?

11. 4.5.43 Solve the exponential equation $2^x - 10(2^{-x}) + 3 = 0$.

12. 4.1.37 Sketch $h(x) = 2^{x-4} + 1$ including domain, range, asymptotes, and intercepts.

13. 3.3.23 Find the quotient and remainder of $\frac{x^6+x^4+x^2+1}{x^2+1}$.

14. 2.6.85 Determine if the function $f(x) = x^2 + x$ is even or odd or neither.

15. 4.4.47 Expand $\log \sqrt{\frac{x^2+4}{(x^2+1)(x^3-7)^2}}$.

16. 4.2.35 If \$600 is invested at an interest rate of 2.5% per year, find the amount of the investment at the end of 10 years for the following compounding methods. (c) quarterly (d) continuously.

17. 2.8.21 Determine if the function $f(x) = x^6 - 3$ on $0 \le x \le 5$ is one-to-one.

18. 2.7.67 Express the function as a composition of functions $H(x) = |1 - x^3|$.

19. 4.5.65 Solve the equation $\log_9(x-5) + \log_9(x+3) = 1$.

20. 3.3.55 Show that $c = 1/2$ is a zero of $P(x) = 2x^3 + 7x^2 + 6x - 5$ and show that $x - c$ is a factor of $P(x)$.

21. 4.6.17 The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample. (a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years. (b) Find a function $m(t) = m_0 e^{rt}$ where $r = (ln2)/h$, that models the mass remaining after t years. (c) How much of the sample will remain after 4000 years?

22. 4.5.37 Solve the exponential equation $\frac{50}{1+e^{-x}} = 4$.

23. 3.2.75 Graph the family of polynomials on the same set of axes: $P(x) = x^4 + c$ for $c = -1, 0, 1, 2$.

24. 3.6.75 Find the slant asymptotes and the vertical asymptotes of $r(x) = \frac{x^3+x^2}{x^2-4}$.

25. 4.3.69 and 71 Sketch $y = \log_3(x - 1) - 2$ and $y = |\ln x|$.

26. 2.6.71 Look at the graph on page 208 with problem #71 and sketch all the graphs from parts (a)-(f). (Sorry, it was so much easier than reproducing the problem...)

27. 3.1.47 Find a function f whose graph is a parabola with vertex $(2, -3)$ and that passes through the point $(3, 1)$.

28. 3.7.43 Find the domain of $h(x) = \sqrt[4]{x^4 - 1}$.

29. 2.5.27 Write the equation of the linear function f with rate of change 3 and initial value of -1.

30. 4.2.15 Sketch $y = e^{x+1} - 3$ include domain, range, intercepts, asymptotes.

31. 2.8.47 Use the Inverse Function Property to show that $f(x) = \frac{x+2}{x-2}$ and $g(x) = \frac{2x+2}{x-1}$ are inverses of each other.

32. 4.3.41 Solve $\log_2(1/2) = x$ and $logx = -3$.

33. 4.4.57