

By the end of this section you must know how to:

- find the slope of a line given
    - its equation, OR
    - its graph, OR
    - two points on the line
  - find the equation of a line given
    - two points on the line OR
    - its slope and a point on the line OR
    - its slope and an intercept ( $x$  or  $y$ ) OR
    - a point and a line parallel to it OR
    - a point and a line perpendicular to it OR
    - its graph
  - graph a line given its equation
  - find the  $x$ - and  $y$ -intercepts of a line (if they exist)
  - write and recognize the equation of a vertical line and a horizontal line
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DEFINITION: The slope  $m$  of a line through  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is

EXAMPLE 1: Given the points  $A(2, -3)$ ,  $B(10, 5)$ ,  $C(-5, -3)$  and  $D(2, 7)$ , find the slope of a line through:

1.  $A$  and  $B$
2.  $A$  and  $C$
3.  $A$  and  $D$

USE OLD TO MAKE NEW: Use the DEFINITION OF SLOPE ABOVE to show that

if you KNOW (1) the slope  $m$  and (2) a point on the line  $(x_1, x_2)$  you can write the equation of the line. (This is called *point-slope form*.)

EXAMPLE 2: Write the equation of the line with a slope of 5 and containing the point  $(-20, 4)$ .

NOTATION: A line in the form  $y = mx + b$  is called *slope-intercept form*.

EXAMPLE: Rewrite the equation from example 2 so that it is in slope-intercept form.

QUESTIONS: What is the  $y$ -intercept of the line from example 2? The  $x$ -intercept?

GRAPH THE LINE FROM EXAMPLE 2.

EXAMPLE 3: Write the slope-intercept form of the equation of the line containing the points  $(-4, 3)$  and  $(1, 13)$ .

USE OLD TO MAKE NEW: What slope to horizontal lines have and why?

USE OLD TO MAKE NEW: What slope to vertical lines have and why?

EXAMPLE 4: Write an equation of the horizontal line through the point  $(3, 5)$ .

EXAMPLE 5: Write an equation of the vertical line through point  $(3, 5)$ . (HINT: Look again the the answer to Example 4 and observe that vertical lines should have the same sort of “form” as horizontal lines....)

QUESTIONS: If two lines are parallel, what is the relationship between their slopes? What if they are perpendicular?

EXAMPLE: Let line  $L_1$  have equation  $2x - 3y = 12$ .

1. Write an equation of a line parallel to line  $L_1$  containing the point  $(1, 7)$ .
2. Write an equation of a line perpendicular to the line  $L_1$  containing the point  $(1, 7)$ .

YOU CONSTRUCT EXAMPLES:

1. Give the equation of a line that has no  $x$ -intercepts and sketch it.

2. Give the equation of a line that has no  $y$ -intercepts and sketch it.

3. Give the equation of a line with negative slope and sketch it.

LAST EXAMPLE The relationship between Fahrenheit (F) and Celsius (C) temperature is given by  $F = \frac{9}{5}C + 32$ .

1. Explain how you know  $F$  and  $C$  have a linear relationship.
2. Give a detailed interpretation of what the  $9/5$  means in this equation.
3. Give a detailed interpretation of what the 32 means in this equation.
4. Water boils at  $212^\circ F$ . Use the equation to determine the temperature in degrees Celsius at which water boils.
5. Find the temperature at which the scales agree.