

NAME: Solutions

This quiz contains 4 problems worth 30 points. You may not use books, notes, or a calculator. You have 30 minutes to take the quiz.

1. (5 points) List all strings over $X = \{0, 1\}$ of length 3 or less.

λ = null string

0

1

00

01

10

11

000

111

001

010

100

110

101

011

#20 from
§3.2

2. (10 points) Let $s_n = 2n - 3$ for $n \geq 1$.

- (a) List the first four terms of s_n .

$$s_1 = 2 \cdot 1 - 3 = -1$$

$$s_2 = 2 \cdot 2 - 3 = 1$$

$$s_3 = 2 \cdot 3 - 3 = 3$$

$$s_4 = 2 \cdot 4 - 3 = 5$$

-1, 1, 3, 5, ...

just like
#83 §3.2

- (b) List the first four terms of the sequence $b_k = \sum_{n=1}^k s_n$.

$$b_1 = -1$$

$$b_2 = -1 + 1 = 0$$

$$b_3 = -1 + 1 + 3 = 3$$

$$b_4 = -1 + 1 + 3 + 5 = 8$$

-1, 0, 3, 8, ...

like #69 §3.2
but easier.
No formula
required

- (c) List the first four terms of the sequence $b_k = \prod_{n=1}^k s_n$.

$$b_1 = -1$$

$$b_2 = (-1)(1) = -1$$

$$b_3 = (-1)(1)(3) = -3$$

$$b_4 = (-1)(1)(3)(5) = -15$$

-1, -1, -3, -15, ...

3. (12 points) Let $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d\}$ and $Z = \{A, B, C, D, E\}$. Determine whether each set forms a function or not and, if it is a function, whether it is one-to-one, onto, or a bijection

(a) $f : X \rightarrow Z$, defined as $f = \{(1, A), (2, C), (3, D), (4, B)\}$

function, one-to-one

(b) $f : Z \rightarrow Y$, defined as $f = \{(A, b), (B, b), (C, c), (D, d), (E, a)\}$

function, onto

(c) $f : X \rightarrow Y$, defined as $f = \{(1, a), (2, b), (3, c), (4, d), (3, d)\}$

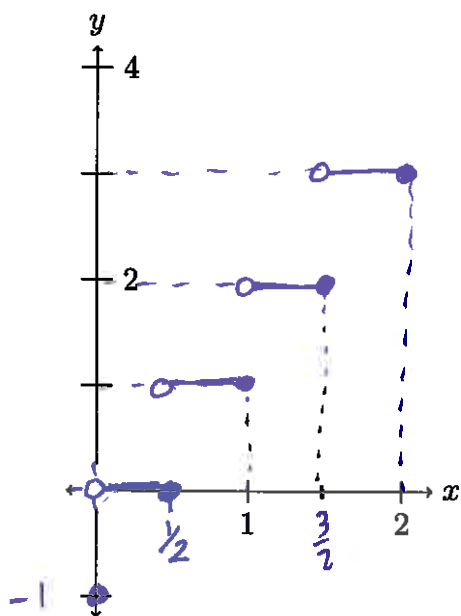
not a function

(d) $f : X \rightarrow Y$, defined as $f = \{(1, a), (2, a), (3, a), (4, a)\}$

function (not 1-1 or onto)

Just like #1-5
in §3.1

4. (4 points) Sketch $f(x) = \lceil 2x - 1 \rceil$ on the axes below.



easier than
#6 + 9 from
§3.1

x	$f(x)$
0	$\lceil 2 \cdot 0 - 1 \rceil = \lceil -1 \rceil = -1$
1	$\lceil 2 \cdot 1 - 1 \rceil = \lceil 1 \rceil = 1$
$\frac{1}{2}$	$\lceil 2 \cdot \frac{1}{2} - 1 \rceil = \lceil 0 \rceil = 0$

5. (4 points) Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ by $f(m, n) = m - n$. Determine whether f is one-to-one, onto, or both and justify your answer.

f is not one-to-one. $f(1, 0) = 1 - 0 = 1 = 2 - 1 = f(2, 1)$

f is onto. $\forall z \in \mathbb{Z}, f(z, 0) = z - 0 = z$

#16 from
text
§3.1