MATH F156X TEST 2 – VERSION A FALL 2015

Your Name (print clearly)

Grading Sheet

Monday, November 2

Page	Total Points	Score
1	15	
2	18	
3	20	
4	15	
5	20	
6	12	
extra credit	5	
Total	100	

Instructions and information:

- Please turn off cell phones or any other thing that will go BEEP.
- Calculators are **not** allowed on this test.
- Read the directions for each problem. You must always show your work to receive partial credit.
- Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader where to look.
- Raise your hand (or come up to the front) if you have a question.

Formulas

$n(t) = n_0 2^{t/a}$
$m(t) = m_0 2^{-t/h}$
$A(t) = P(1 + \frac{r}{n})^{nt}$
$\log_b x = (\log_a x)/(\log_a b)$

$n(t) = n_0 e^{rt}$
$m(t) = m_0 e^{rt}$ where $r = (\ln 2)/h$,
$A(t) = Pe^{rt}$

1. The owner of a toy factory estimates that it costs \$1300 to produce 100 toys in one day and \$1900 to produce 300 toys in one day.

   (a) (3 points) Assuming that the relationship between cost and the number of toys produced is linear, find a linear function C that models the cost of producing x toys in one day.

   $\frac{\Delta C}{\Delta x} = \frac{1900-1300}{300-100} = \frac{600}{200} = 3$

   $C - 1300 = 3(x-100) = 3x - 300$ → So $C(x) = 3x + 1000$

   (b) (2 points) At what rate does the factory's cost increase for every additional toy produced?

   \$3/toy

2. (5 points each) Let $f(x) = \sqrt{16 - x^2}$ and $g(x) = \sqrt{x+2}$.

   (a) Find f/g and state its domain.

   $\frac{f}{g} = \frac{\sqrt{16-x^2}}{\sqrt{x+2}}$ domain: $(-2, 4]$

   Need $16 - x^2 \geq 0$

   $(4-x)(4+x) \geq 0$

   − + − (sign chart)

   $-4 \quad 4$

   $[-4, 4]$ ← both

   Need $x + 2 > 0$

   $x > -2$

   $(-2, \infty)$ ↗ both

   (b) Find $f \circ g$ and state its domain.

   $f \circ g = f(g(x)) = f(\sqrt{x+2}) = \sqrt{16 - x - 2} = \sqrt{14 - x}$; domain: $[-2, 14]$

   Need $14 - x \geq 0$

   $14 \geq x$

   ↑ So that $f \circ g$ makes sense

   Need $x + 2 \geq 0$

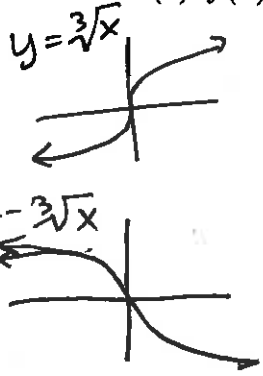
   $x \geq -2$

   ↑ So that g makes sense.

   both!

3. (6 points each) Sketch the graphs below. Label any asymptotes and intercepts.

(a) $f(x) = 2 - \sqrt[3]{x}$

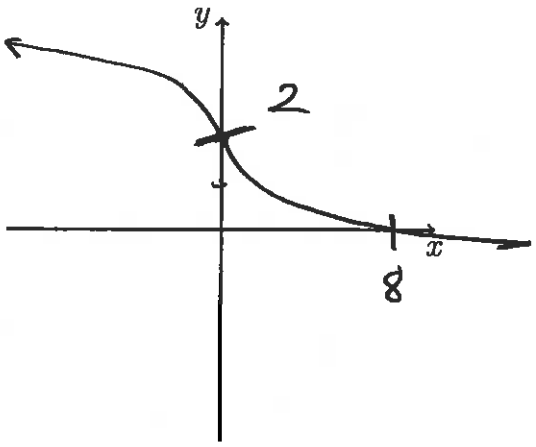


x intercept:

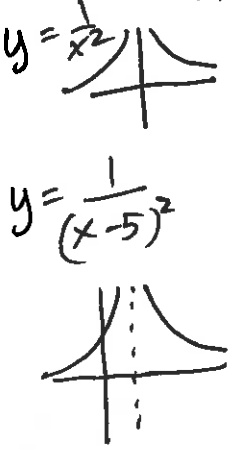
$0 = 2 - \sqrt[3]{x}$

$2 = \sqrt[3]{x}$

$x = 8$

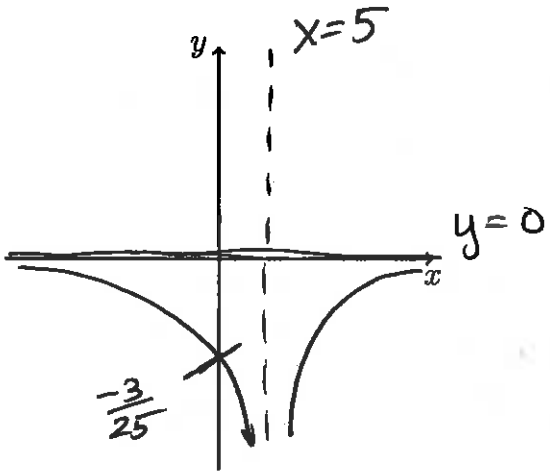


(b) $f(x) = \frac{-3}{(x-5)^2}$



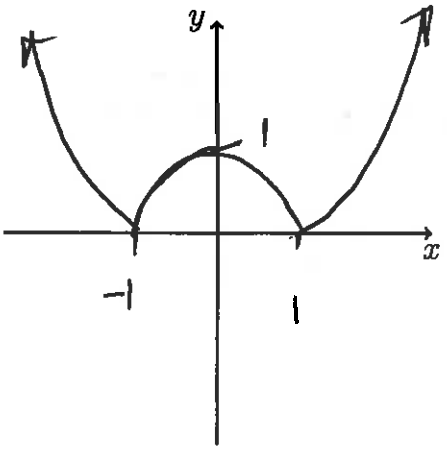
y-intercept:

$y = \frac{-3}{(0-5)^2} = \frac{-3}{25}$



(c) $f(x) = |x^2 - 1|$

$y = x^2 - 1$



4. (5 points) Find the inverse of $h(x) = \frac{(2-x^3)^5}{7}$.

$x = \frac{1}{7}(2-y^3)^5$

$7x = (2-y^3)^5$

$(7x)^{\frac{1}{5}} = 2-y^3$

$y^3 = 2-(7x)^{1/5}$

$y = \sqrt[3]{2-(7x)^{1/5}}$

5. Let $g(x) = 5x^2 - 15x + 2$.

(a) (6 points) Express g in standard form.

$g(x) = 5(x^2-3x+\frac{9}{4}-\frac{9}{4})+2$

$\frac{b}{2} = \frac{-3}{2}$, $(\frac{b}{2})^2 = \frac{9}{4}$

$= 5(x-\frac{3}{2})^2 - \frac{45}{4} + \frac{8}{4}$

$= 5(x-\frac{3}{2})^2 - \frac{37}{4} = g(x)$

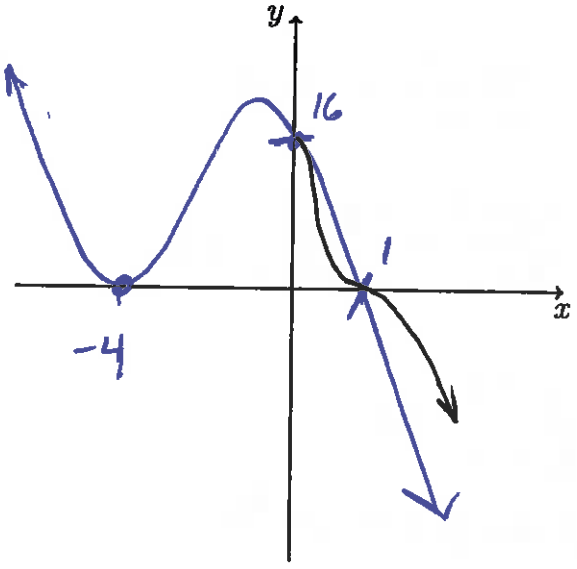
(b) (2 points) Find the vertex of the graph of g.

vertex $= (\frac{3}{2}, \frac{-37}{4})$

(c) (2 points) Determine the range of g.

g opens up. So $[\frac{-37}{4}, \infty)$

6. (5 points)
Sketch the graph of $P(x) = -(x+4)^2(x-1)^3$ on the axes. Make sure you label all intercepts and exhibits proper end behavior.



Zeros: $x=-4$, $x=1$

y-intercept: Set $x=0$:

$P(0) = -(4)^2(-1)^3 = 16$

for large values of $|x|$:

$P(x) \approx -x^2 \cdot x^3 = -x^5$

$x \to \infty$, $P \to -\infty$

$x \to -\infty$ $P \to \infty$

7. (5 points) Find the quotient, $Q(x)$, and remainder $R(x)$, of $\frac{6x^2-17x+7}{2x-3}$.

$$
\begin{array}{r|l}
2x-3 & 3x-4 \\
\hline
 & 6x^2-17x+7 \\
 & -(6x^2-9x) \\
\hline
 & -8x+7 \\
 & -(-8x+12) \\
\hline
 & -5
\end{array}
$$

$Q(x) = 3x-4$

$R(x) = -5$

8. (5 points) Let $r(x) = \frac{x^2-25}{3x^2+17x+10} = \frac{(x-5)(x+5)}{(3x+2)(x+5)}$

(a) Find any horizontal asymptotes or state that none exist.

$y = \frac{1}{3}$ (leading coefficients)

(b) Find any vertical asymptotes or state that none exist.

$3x+2=0$

$x=-2/3$

Note $x=-5$ corresponds to a hole in the graph.

9. (5 points) Solve the rational inequality $\frac{6x-7}{5x-2} \geq 1$. Give your answer in interval notation.

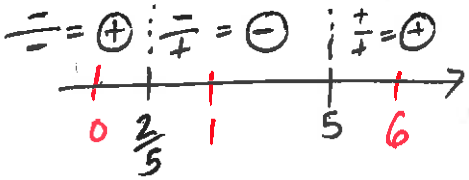
$\frac{6x-7}{5x-2} - 1 \geq 0$

$\frac{6x-7}{5x-2} - \left(\frac{5x-2}{5x-2}\right) \geq 0$

$\frac{x-5}{5x-2} \geq 0$

Zeros @ $x=5$, $x=2/5$

$\frac{-}{-} = \oplus$; $\frac{-}{+} = \ominus$; $\frac{+}{+} = \oplus$



ANS: $(-\infty, \frac{2}{5}) \cup [5, \infty)$

We don't include $\frac{2}{5}$ because it makes the denominator zero.