

4. (2 points) Negate the proposition If the sky is blue, then the pillow is soft. without using the words "It is not the case that ..." (Hint: Your answer should include the word AND)

What I am using:
 $\neg(p \rightarrow q) \equiv p \wedge \neg q$

The sky is blue and pillow is not soft.

5. (2 points each) Given that p and q are true, r and s are false. Determine the truth value of each proposition below.

(a) $\neg p \rightarrow (q \vee r) \equiv [F \rightarrow \text{whatever}] \equiv \boxed{T}$

(b) $\neg(p \vee \neg q) \leftrightarrow (r \wedge s) \equiv [\neg(T \vee F) \leftrightarrow (F \wedge F)] \equiv [F \leftrightarrow F] \equiv \boxed{T}$

(c) $p \rightarrow (q \rightarrow r) \equiv T \rightarrow (T \rightarrow F) \equiv T \rightarrow F \equiv \boxed{F}$

6. (5 points each) Determine whether each argument below is valid and justify your answer.

(a)
$$\begin{array}{l} p \rightarrow (q \vee r) \\ p \wedge \neg q \\ r \vee q \\ \hline \therefore q \end{array}$$

Invalid. Counter-example:

Choose $p = T$ $q = F$ $r = T$
All hypotheses are true. But the conclusion is false.

(b)
$$\begin{array}{l} p \\ p \rightarrow q \\ \neg q \vee r \\ \hline \therefore r \end{array}$$

Valid Since p is true and $p \rightarrow q$ is true, q must be true. Thus $\neg q$ is false.
If $\neg q$ is false and $\neg q \vee r$ is true, r must be true.

Alternately, see truth table on the next page.