Your Name (print clearly)

Solutions

Monday, November 2

Page	Total Points	Score
1	15	
2	20	
3	18	
4	15	
5	20	
6	12	
extra credit	5	
Total	100	

Instructions and information:

- Please turn off cell phones or any other thing that will go BEEP.
- Calculators are **not** allowed on this test.
- Read the directions for each problem. You must always show your work to receive partial credit.
- Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- If you need more room, use the backs of the pages and indicate to the grader where to look.
- Raise your hand (or come up to the front) if you have a question.

Formulas

$n(t) = n_0 2^{t/a}$
$m(t) = m_0 2^{-t/h}$
$A(t) = P(1 + \frac{r}{n})^{nt}$
$\log_b x = (\log_a x)/(\log_a b)$

$n(t) = n_0 e^{rt}$
$m(t) = m_0 e^{rt}$ where $r = (\ln 2)/h$,
$A(t) = Pe^{rt}$

3. (5 points) Find the inverse of $h(x) = \frac{(2-x^3)^5}{7}$.

$x = \frac{1}{7}(2-y^3)^5$

$7x = (2-y^3)^5$

$(7x)^{\frac{1}{5}} = 2-y^3$

$y^3 = 2-(7x)^{1/5}$

$y = \sqrt[3]{2-(7x)^{1/5}}$

4. Let $g(x) = 5x^2 - 15x + 2$.

$\frac{b}{2} = \frac{-3}{2}$, $\left(\frac{b}{2}\right)^2 = \frac{9}{4}$

(a) (6 points) Express g in standard form.

$g(x) = 5(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) + 2$

$= 5\left(x - \frac{3}{2}\right)^2 - \frac{45}{4} + \frac{8}{4}$

$45 - 8 = 37$

So, $g(x) = 5\left(x - \frac{3}{2}\right)^2 - \frac{37}{4}$

(b) (2 points) Find the vertex of the graph of g.

$\left(\frac{3}{2}, -\frac{37}{4}\right)$

(c) (2 points) Determine the range of g.

g opens up. So $\left[\frac{-37}{4}, \infty\right)$

5. (5 points)
Sketch the graph of $P(x) = -(x+1)^3(x-3)^2$ on the axes. Make sure you label all intercepts and exhibits proper end behavior.

Zeros: $x = -1$, $x = 3$.

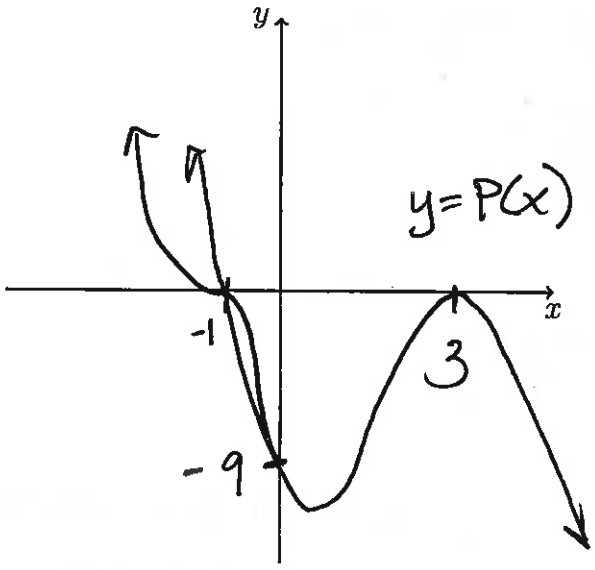
$P(x) \approx -x^3 \cdot x^2 = -x^5$ for $|x|$ large

So as $x \to \infty$, $P \to -\infty$

as $x \to -\infty$, $P \to \infty$.

y-intercept: Set $x = 0$

$P(0) = -(1)^3(-3)^2 = -9$



10. (5 points) Expand $\log_2 x\sqrt{\frac{y}{z}}$ using the Laws of Logarithms.

$$\log_2 x + \tfrac{1}{2}\log_2 y - \tfrac{1}{2}\log_2 z$$

11. (5 points each) Solve.

(a) $10^{1-x} = 7$

Yes. These are base 10.

$$\log(10^{1-x}) = \log 7$$

$$1 - x = \log 7$$

$$x = 1 - \log 7$$

(b) $e^{2x} + e^x - 20 = 0$

$$(e^x)^2 + e^x - 20 = 0$$

$$(e^x + 5)(e^x - 4) = 0$$

$e^x = -5$ or $e^x = 4$

But $e^x > 0$, so $e^x = -5$ makes no sense!

So $e^x = 4$.

So $x = \ln 4$

(c) $\log_6 x + \log_6(x+1) = 1$

$$\log_6(x(x+1)) = 1$$

So $x^2 + x = 6$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

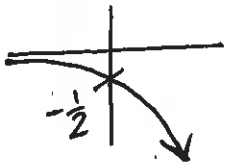
$x = -3$ or $x = 2$

but $x = -3$ not possible.

So $x = 2$

12. (6 points each) Sketch the graphs below. Label any asymptotes and intercepts.

(a) $h(x) = 4 - 2^{x-1}$

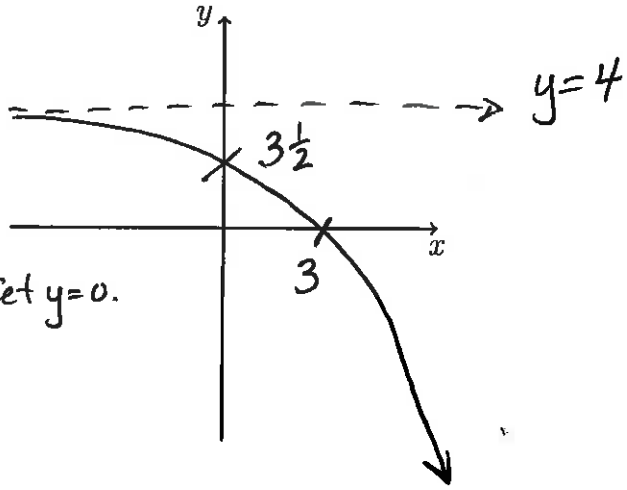


x-intercept: Set $y = 0$.

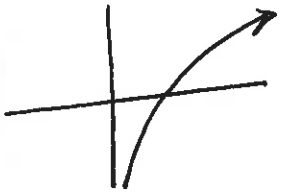
$2^{x-1} = 4 = 2^2$

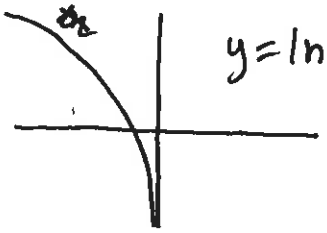
$x - 1 = 2$

$x = 3$



(b) $f(x) = 1 + \ln(-x)$





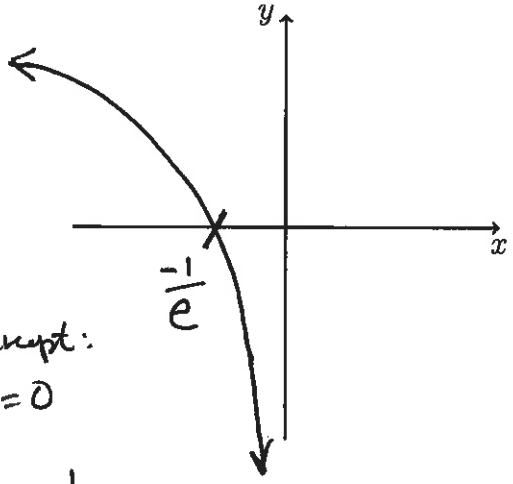
x-intercept:

Set $y = 0$

$\ln(-x) = -1$

$e^{-1} = -x$

$x = -e^{-1}$



EXTRA CREDIT (5 points) A certain population of fish has a relative growth rate of 2.5% per year. How long will it take for the population to double? (Yes. You do have enough information to complete t his problem.)

$n = n_0 e^{.025t}$

Find t when $n = 2n_0$.

So $2n_0 = n_0 e^{.025t}$.

So $2 = e^{.025t}$

So $\ln 2 = 0.025t$

Finally, $t = \frac{\ln 2}{.025}$