

How and What to write for a Direct Proof

Problem : Prove that if x and y are rational numbers, then xy is rational.

What you MUST write down

my comments + explanations

Proof : Assume x and y are rational numbers.

Identify the IF clause and begin by explicitly assuming it. ALWAYS begin with ASSUME or Let.

Since x and y are rational, we know from the definition that there exist integers a, b, c, d so that

$$x = \frac{a}{b} \text{ and } y = \frac{c}{d} \text{ and } b \neq 0 \text{ and } d \neq 0.$$

Before proceeding further, YOU SURE-AS-SHOOTIN' BETTER CLARIFY ANY NEEDED DEFINITIONS. In this case, the needed definition is rational number:

$$x \in \mathbb{Q} \text{ means } x = \frac{a}{b} \text{ where } a, b \in \mathbb{Z}, b \neq 0.$$

$$\text{Now, } xy = \left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}.$$

Now, USE the definitions & STATE that you are doing so.

Observe that ac and bd are integers and $bd \neq 0$. So xy is rational.

Proceed toward the conclusion: In this case: xy is rational.

USE the definition of rational number a SECOND time

Things Indicating a FULL BLOWN Disaster.

1. No words. No capital letters. No periods. Arrows. Proofs are WRITING in the ENGLISH sense.
2. Assuming the conclusion. In this case beginning, $xy = \frac{a}{b}$.
3. No clearly stated application of a definition (or definitions.)