Worksheet §5.1: The Unit Circle

The unit circle in the xy-plane has equation: $\boxed{x^2+y^2=1}$

The circumference of the unit circle is: $C=2\pi r$, $r=1$

So $\boxed{C=2\pi}$

---

1. Determine which of the following points lie on the unit circle: $P(\frac{-\sqrt{11}}{6}, \frac{5}{6})$, $Q(0,-1)$, $R(\frac{2\sqrt{5}}{5}, \frac{2}{5})$, ~~$S(\frac{-3}{4}, \frac{-2\sqrt{7}}{4})$~~.

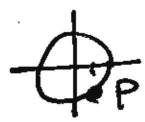
$S=\left(\frac{1}{2}, \frac{\sqrt{6}}{2\sqrt{2}}\right)$

see attached for Q, R, S : $\left(-\frac{\sqrt{11}}{6}\right)^2+\left(\frac{5}{6}\right)^2=\frac{11+25}{36}=\frac{36}{36}=1$

So P is on the unit circle.

2. Find the missing coordinate of P using the fact that P lies on the unit circle in the given quadrant.

(a) $P(1/2, y)$, quadrant IV

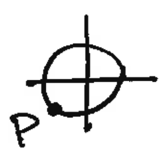


So $y<0$

$\left(\frac{1}{2}\right)^2+y^2=1$. So $y^2=\frac{3}{4}$, $y=-\frac{\sqrt{3}}{2}$

(b) $P(x, -2/7)$, quadrant III

So $x<0$

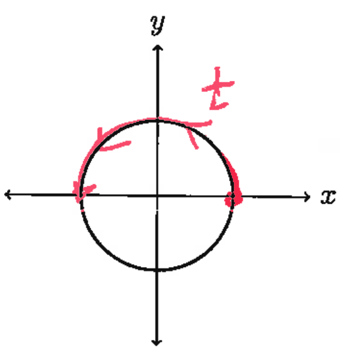


$x^2+\left(\frac{-2}{7}\right)^2=1$

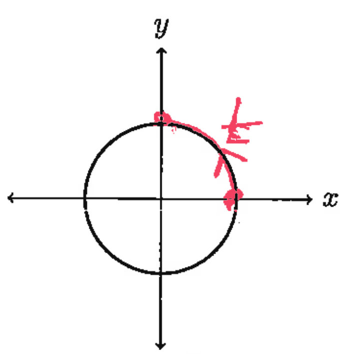
$x^2=1-\frac{4}{49}=\frac{45}{49}$

$x=-\frac{3\sqrt{5}}{\sqrt{7}}$

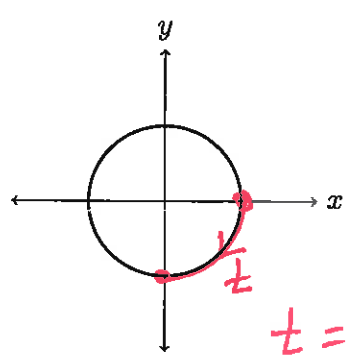
In our text, t, will always represent a distance along the unit circle starting at $(1,0)$ in the clockwise direction if $t>0$ and a counterclockwise direction if $t<0$.



$t=\pi$



$t=\pi/2$



$t=-\frac{\pi}{2}$

#1 cont'd

$Q(0,-1)$: $\quad 0^2+(-1)^2=1 \checkmark \qquad$ Q on circle

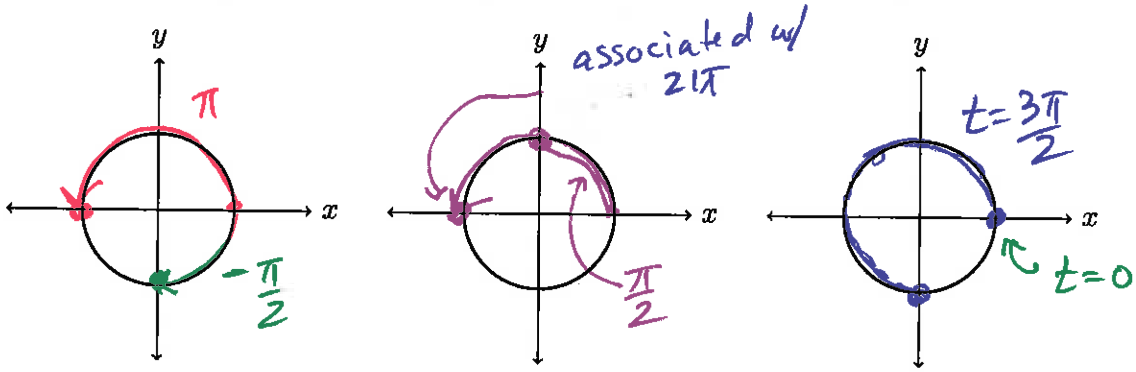
$R\left(\frac{2\sqrt{5}}{5}, \frac{2}{5}\right)$: $\left(\frac{2\sqrt{5}}{5}\right)^2+\left(\frac{2}{5}\right)^2 = \frac{4\cdot 5}{25}+\frac{4}{25} = \frac{24}{25} \neq 1$

R not on cycle.

$\cancel{S\left(\frac{-3}{4}, \frac{-2\sqrt{7}}{4}\right): \left(\frac{-3}{4}\right)^2+\left(\frac{-2\sqrt{7}}{4}\right)^2 = \frac{9}{16}+\frac{4\cdot 7}{16} = \frac{9+28}{16} =}$

$S\left(\frac{1}{2}, \frac{\sqrt{6}}{2\sqrt{2}}\right)$: $\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{6}}{2\sqrt{2}}\right)^2 = \frac{1}{4}+\frac{6}{8} = \frac{1}{4}+\frac{3}{4} = 1 \checkmark$

S is on circle.



3. Use the unit circles above to draw and label the terminal point determined by the given value of t:

$t=\pi$, $(x,y)=$ $(-1,0)$ $\qquad$ $t=\pi/2$, $(x,y)=(0,1)$

$t=-\pi/2$, $(x,y)=(0,-1)$ $\qquad$ $t=0$, $(x,y)=(1,0)$

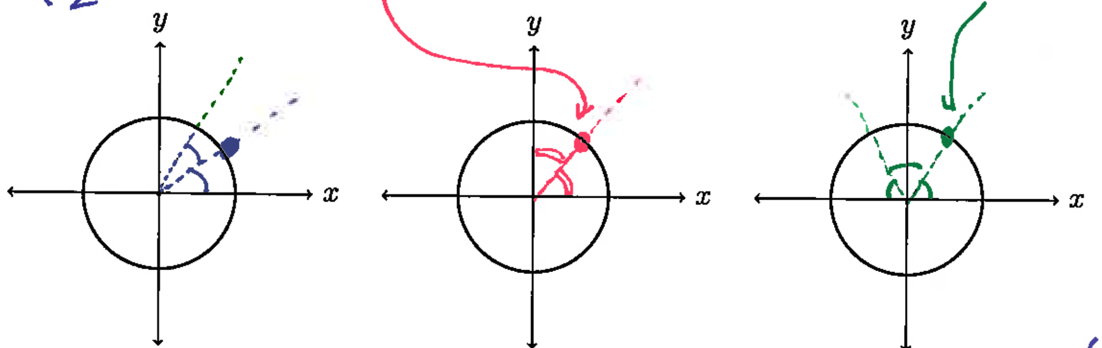
$21\pi = 10\cdot 2\pi + \pi$ ← $t=21\pi$, $(x,y)=(-1,0)$ $\qquad$ $t=7\pi/2$, $(x,y)=(0,-1)$ → $\frac{7\pi}{2}=\frac{4\pi}{2}+\frac{3\pi}{2}=2\pi+\frac{3\pi}{2}$

Three very special terminal points

$t=\pi/6$ $\quad P=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$t=\pi/4$ $\quad P\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$t=\pi/3$ $\quad P\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

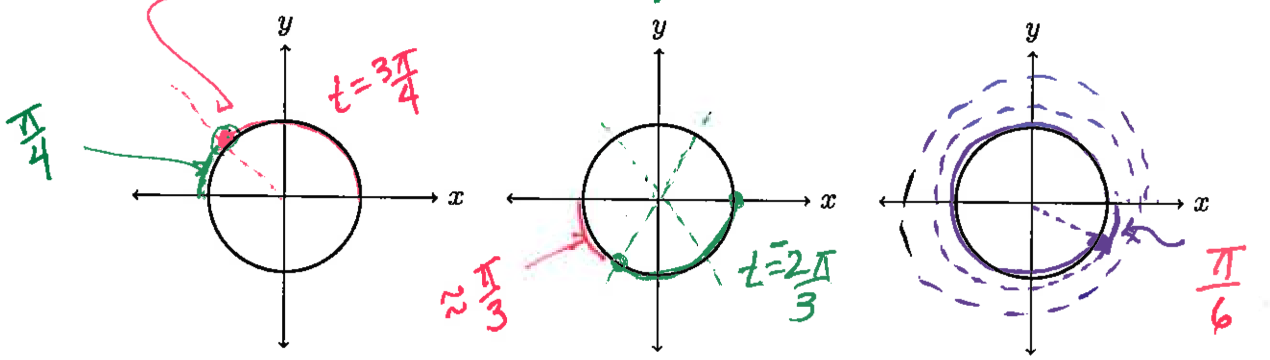


Three more instructive examples

$t=3\pi/4$ $\quad P=\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$t=-2\pi/3$ $\quad P=\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

$t=23\pi/6 = 2\pi+\pi+\frac{5\pi}{6}$ $\qquad$ $6\overline{)23}$, 3, 18, 5

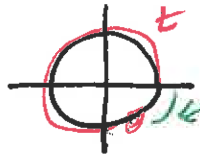


One last definition:

$\bar{t}$ is called the reference number associated with t and is the shortest distance along the unit circle between the terminal point and the x-axis.

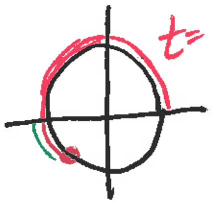
4. Find the reference number and terminal point for each value of t.

(a) $t = 5\pi/3$



$\bar{t} = \frac{\pi}{3}$ $\quad P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

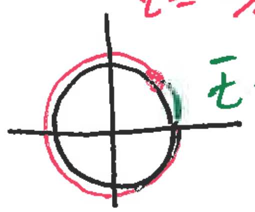
(b) $t = 7\pi/6$



$t = \frac{7\pi}{6}$

$\bar{t} = \frac{\pi}{6}$ $\quad P\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

(c) $t = -7\pi/4$



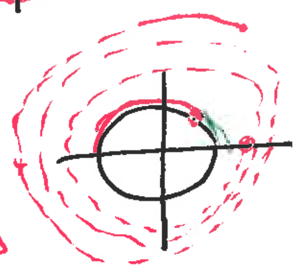
$t = -\frac{7\pi}{4}$

$\bar{t} = \frac{\pi}{4}$ $\quad P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

(d) $t = -17\pi/3$

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ 15 \\ \hline 2 \end{array}$$

$\frac{17\pi}{3} = \underbrace{5\pi}_{\text{2.5 times around}} + \frac{2\pi}{3}$



$t = -\frac{17\pi}{3}$

$\bar{t} = \frac{\pi}{3}$ $\quad P = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

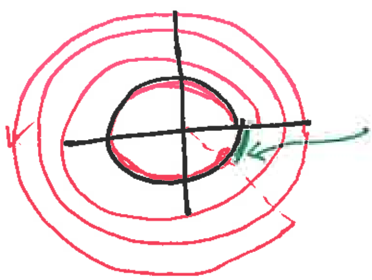
(e) $t = \pi$

$\bar{t} = 0$ $\quad P(-1, 0)$

(f) $t = \frac{35\pi}{6}$

$$\begin{array}{r} 5 \\ 6\overline{)35} \\ 30 \\ \hline 5 \end{array}$$

$\frac{35\pi}{6} = 5\pi + \frac{5\pi}{6}$



$\bar{t} = \frac{\pi}{6}$ $\quad P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

5. Explain what a *radian* is.

A radian is the measure of the angle corresponding to an arc length of 1 on unit circle