Name: Solutions

Section 6.1

Let X be the set of integers from 4 to 300 inclusive.

1. How many elements are in X?

$$300 - 3 = \underline{\underline{297}}$$

2. How many elements of X are even?

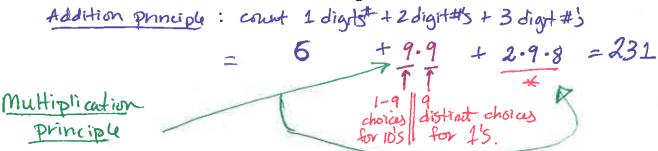Starts with even and ends with even, so just more than half:

$$\frac{297+1}{2} = \underline{\underline{149}}$$

3. How many elements of X are odd?

$$297 - 149 = \underline{\underline{148}}$$

4. How many elements of X are divisible by 5?

5,10,15,...,300 is the list.

This is $\frac{1}{5}$ of integers from 1 to 300.

Ans: $\frac{300}{5} = \underline{\underline{60}}$

5. How many elements of X are greater than 72?

$$300 - 72 = \underline{\underline{228}}$$

$*$ • 100's digit only choice 1 or 2
• 10's digit has 9 choices
• 1's digit has 8 choices

6. How many elements of X have all distinct digits?

Addition principle : count 1 digit#'s + 2 digit#'s + 3 digit#'s

$$= \quad 6 \quad + \quad 9 \cdot 9 \quad + \quad 2 \cdot 9 \cdot 8 \quad = 231$$

Multiplication Principle

1-9 choices for 10's ‖ 9 distinct choices for 1's.

7. How many elements of X contain the digit 7?

1 digit #'s : 1 (7)

2 digit #'s w/ 7 at end : 9 · (17, 27, ..., 97)

2 digit #'s w/ 7 at beginning + not counted already : 9 (70, 71, ..., 76, 78, 79)] total $\underline{18}$

3 digit #'s w/ 7 : 2·(18 + 1) (1xy, 2xy, 107, 207)

$\underline{\text{Final Answer}}$

$1 + 18 + 2(18+1) = 57$

8. How many elements of X do not contain the digit 0?

1 digit : 5 (5, 6, ..., 9)

2 digit : 9·9 = 81 (9 choices for 10's and 1's)

3 digit : 2·9·9 (2 choices for 10's, 9 for 10's + 1's)

$\underline{\text{ANS}}$: $5 + 81 + 2·81 = 248$

9. How many elements of X are greater than 101 and do not contain the digit 6?

List of types :

300

2xy

1xy

Now $x, y \in \{0, 1, ..., 5, 7, 8, 9\}$. So 9 choices for x and 9 choices for y. But this includes 100 and 101.

$\underline{\text{ANS}}$: $1 + 2·9·9 - 2$

$= 161$

10. How many elements of X have digits in strictly increasing order? (examples: 125, 48, 3)

1 digit : 5

2 digit : 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 8·9/2 = 36
(8, (9), (78, 79) (67, 68, 69)) ... 3x ↓ 2x 1x

$\underline{\text{ANS}}$

$5 + 36 + 49 = 90$

next page ✗ 3 digit : $(1 + 2 + ... + 7) + (1 + 2 + ... + 6) = \frac{7·8}{2} + \frac{6·7}{2} = 28 + 21 = 49$

11. How many elements of X are of the form abc where $a \neq 0$, $a < b$ and $c < b$?

types :

1xy

2xy

type 1xy = 2 + 3 + 4 + ... + 9 = $\frac{10·9}{2} - 1 = 44$

type 2xy = 3 + 4 + 5 + ... + 9 = 44 - 2 = 42

$\underline{\text{ANS}}$: 86

12. How many 3-digit #'s in X have a 5 in the tens position or have a 5 in the ones position.

A = #'s w/ 5 in tens position,

B = #'s w/ 5 in ones position,

$|A| = 2·1·10 = 20$

$|B| = 2·10·1 = 20$

$|A \cap B| = 2·1·1 = 2$

So $|A \cup B| = 20 + 20 - 2 = 38$

✳

type: $1 \times y$

$x = 2,\ y \in \{3, 4, \ldots, 9\}$ 7 choices

$x = 3,\ y \in \{4, \ldots, 9\}$ 6 choices

$\vdots$ $\vdots$

$x = 8,\ y = 9$ 1 choice

# of this type: $1 + 2 + \ldots + 6 + 7 = 28$

type: $2 \times y$

$x = 3\ y \in \{4, 5, \ldots, 9\}$ 6 choices

$x = 4\ y \in \{5, 6, \ldots, 9\}$ 5 choices

$\vdots$

$x = 8\ y = 9$ 1 choice

# of this type: $1 + 2 + \ldots + 6 = 21$