Test 2 in MATH 405 will take place on Monday April 4. The exam is closed book, and the only permissible aids are your brain and scratch paper. The topics on the exam are chapters 7-11.

While you will not be asked to explicitly state definitions and theorems from chapters 0-6, you will obviously need to use much of that material on this test.

Chapter 7

Here is where we learned the definition of *coset*. There is a Lemma about properties of cosets. One crucial part of the definition of a coset of a set H is that H is a *subgroup*. Why is this in the definition? What parts of the Lemma will fail to hold?
Be able to state Lagrange's Theorem. Do you understand why it is an If-Then and not an If-and-only-if? Know the terminology of *index* of a group. We have regularly used the first 4 corollaries following Lagrange's Theorem. We have regularly used the HK Theorem (Theorem 7.2). Note that while we often use it when $H \cap K = \{e\}$, we know this is not always the case. You want to think again about what this Theorem is saying in this instance.
Finally, you should think about Theorem 7.3 now that we have finished Chapter 11. Do you see it differently?
Stabilizer and orbit will not be emphasized on this test.

Chapter 8

Here we learned the definition of an *external direct product*. Recall that one of the nice things about this idea is that it allows us to construct lots of examples. Use some familiar groups and construct new groups using the external direct product. Think about the identity and the group operations. How do you find the order of an element? Can you find subgroups? Cosets of these subgroups? Can you construct a nonabelian group with normal subgroups of order 8 and 16?
Look carefully at the hypotheses of Theorems 8.1, 8.2 and Corollary 1 and 2. Do you understand why those hypotheses are needed? Can you construct examples to show they are needed? Do you understand how Corollary 2 differs from what we learned in Chapter 11?
As far as Theorem 8.3 and its corollary go, they help you construct and think about examples. I won't ask you to quote them. If I use the notation $U_t(st)$, I will remind you want it means.

Chapter 9

We learn the *definition* of a *normal* subgroup and we learned about a *Normal Subgroup Test*. You should have in your head examples of normal subgroups and subgroups that are not normal. You should have examples of nonabelian groups with nontrivial normal subgroups. What about infinite groups? Infinite nonabelian groups?

We learned the definition of a *factor* group. Think up your favorite factor group and play with it. Identity? Group operation? Nontrivial subgroups? Why does the definition of factor group require the subgroup be normal?

We have used Cauchy's Theorem for Finite Abelian Groups many times now. We used the G/Z Theorem, too, but one of the other important things to recall about it (and Theorem 9.4) is that it uses *what* normal subgroup?

We learned about the *internal direct product* of a group. Do you understand the difference in how one constructs/uses these? Find an example of a familiar group and see if you can write it as an internal direct product. Read Theorem 9.6 and make sure you understand how the two sides of the $\approx$ are different.

Chapter 10

The big definitions here are *homomorphism* and *kernel*. Think up examples of homomorphisms that are not isomorphisms or that are onto but not one-to-one or that are one-to-one but not onto. Can you find the kernel of these maps? What if you are given a set X and asked to construct a homomorphism with X as its kernel, how do you do that? Is it always possible to do that?
Think through the properties of Theorem 10.1 and 10.2.
Theorem 10.3 (the First Isomorphism Theorem) and its corollary and Theorem 10.4 have been used a lot.

Chapter 11

The Fundamental Theorem of Finite Abelian Groups. You should be able to list all the isomorphism classes of abelian groups of a particular (not too large) order.