

How and What to write for a Proof by Induction

Problem: Use Mathematical Induction to prove

MUST $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$ for $n \geq 1$.

What you write down (in blue)

(my) Comments/explanations

$S(n)$ is $1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

Explicitly identify $S(n)$
ALWAYS write this.

Basis Step: $S(1)$ is $1^3 = 1 = \left[\frac{1 \cdot 2}{2} \right]^2 = \left[\frac{1(1+1)}{2} \right]^2$,
which is true.

"Basis Step" is ALWAYS Present

Always check this case explicitly.

Always state explicitly that it is true

Inductive Step: Assume $S(n)$,
 $1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$, is true

ALWAYS identify the Inductive Step

Explicitly ~~the~~ state the Inductive hypothesis

Now,

LHS $= (1^3 + 2^3 + \dots + n^3) + (n+1)^3 = \left[\frac{n(n+1)}{2} \right]^2 + (n+1)^3$

ALWAYS start with one side of $S(n+1)$ and work to the other. DO NOT EVER use both sides simultaneously. IDENTIFY the side you are starting with unchanged. State when you use the Inductive hypothesis.

$= \frac{n^2(n+1)^2 + 4(n+1)^3}{4}$

Common denominator

$= \frac{(n+1)^2 [n^2 + 4n + 4]}{4}$

factor out $(n+1)^2$

$= \frac{(n+1)^2 (n+2)^2}{4}$

factor $n^2 + 4n + 4$

$= \left[\frac{(n+1)(n+2)}{2} \right]^2 = \text{RHS}$

Explain your steps

End by obtaining and identifying the other side.