TOTAL: ______/20

ECE 5548: Electronic Design I

Homework #2

Due: Wednesday, February 27th, 2019 (11pm)

Student Name: Thomas Collins

Note:

- **Please use this sheet as a cover page.**
- **Your work must be hand-written (no typing please).**
- **Homework must be submitted electronically through Canvas in a PDF format.**

Do the following problems from Nilsson & Riedel (11th edition)

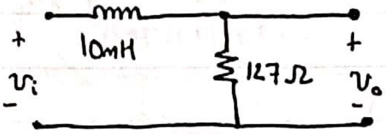
(Pages 565 – 568)

1. Low-pass filter: Solve at least two of the following problems: 14.1, 14.4, 14.7, 14.8
2. High-pass filter: Solve at least two of the following problems: 14.11, 14.13, 14.15, 14.16
3. Band-pass filter: Solve at least two of the following problems: 14.20, 14.21, 14.24, 14.30

Thomas Collins

ECE 548 Homework #2

Prob #14.1



(a) Find the cutoff frequency in hertz for the R-L circuit filter.

$\omega_c = R/L$

$\omega_c = 127 / (10\times10^{-3})$ $\qquad \omega_c = 12700$ rad/s

$12700 \text{ rad/s} \cdot \frac{1}{2\pi} =$

$\boxed{\omega_c = 2021.27 \text{ hz}}$

(b) Calculate $H(j\omega)$ at ω_c, $0.2\omega_c$, $5\omega_c$

$H(j\omega) = \frac{R/L}{j\omega + R/L}$ $\qquad H(j\omega_c) = \frac{R/L}{j\omega_c + R/L}$

$H(j\omega_c) = \frac{127\Omega/(10\times10^{-3}H)}{j(12700 \text{ rad/s}) + (127\Omega/(10\times10^{-3}H))}$

$H(j\omega_c) = \frac{12700}{j12700 + 12700}$ $\qquad H(j\omega_c) = 0.707 \angle 45°$

$\boxed{H(j\omega_c) = 0.707 \angle -45°}$

$H(j(0.2\omega_c)) = \frac{12700}{j(0.2)(12700) + 12700}$ $\qquad H(j(0.2\omega_c)) = \frac{12700}{j2540 + 12700}$

$\boxed{H(j(0.2\omega_c)) = 0.981 \angle -11.31°}$

$H(j(5\omega_c)) = \frac{12700}{j(5(12700)) + 12700}$ $\qquad H(j(5\omega_c)) = \frac{12700}{j63500 + 12700}$

$\boxed{H(j(5\omega_c)) = 0.196 \angle -78.90°}$

(c) If $v_i = 10\cos(\omega t)$ V, write the steady state expression for v_o when $\omega = \omega_c$, $\omega = 0.2\omega_c$, $\omega = 5\omega_c$

$\omega = \omega_c = 12700$

$v_i = 10\cos(12700t)$ V

$V_i = 10 \angle 0°$

$$V_o(j\omega_c) = \frac{R/L}{j\omega_c + R/L} V_i \qquad V_o(j\omega_c) = \frac{12700}{12700j + 12700}(10\angle 0^\circ)$$

$$V_o(j\omega_c) = (.707\angle -45^\circ)(10\angle 0^\circ)$$

$$V_o(j\omega_c) = 7.07\angle -45^\circ$$

$$\boxed{V_o(j\omega_c) = 7.07\cos(12700t - 45^\circ)\,V}$$

$\omega = 0.2\omega_c = 2540$

$V_i = 10\cos(2540t)\,V$

$V_i = 10\angle 0^\circ$

$$V_o(j(0.2\omega_c)) = \left(\frac{12700}{2540j + 12700}\right)(10\angle 0^\circ)$$

$$V_o(j(0.2\omega_c)) = (0.981\angle -11.31^\circ)(10\angle 0^\circ)$$

$$V_o(j(0.2\omega_c)) = 9.81\angle -11.31^\circ$$

$$\boxed{V_o(j(0.2\omega_c)) = 9.81\cos(2540t - 11.31^\circ)}$$

$\omega = 5\omega_c = 63500$

$v_i = 10\cos(63500t)\,V$

$v_i = 10\angle 0^\circ$

$$V_o(j(5\omega_c)) = \left(\frac{12700}{63500t + 12700}\right)(10\angle 0^\circ)$$

$$V_o(j(5\omega_c)) = (0.196\angle -78.90^\circ)(10\angle 0^\circ)$$

$$V_o(j(5\omega_c)) = 1.96\angle -78.90^\circ$$

$$\boxed{V_o(j(5\omega_c)) = 1.96\cos(63500t - 78.90^\circ)}$$

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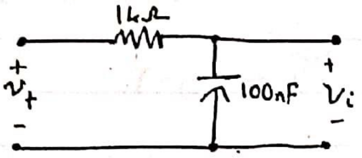
Problem # 14.7



(a) Find the cutoff frequency (in hertz) of the low pass filter

$\omega_c = 1/RC = 1/[(1\times10^3)(100\times10^{-9})]$

$\omega_c = 10{,}000$ rad/s

$10{,}000 \cdot \frac{1}{2\pi} = 1591.55\ Hz$

$\boxed{\omega_c = 1591.55 Hz}$

(b) Calculate $H(j\omega)$ at $\omega_c = \omega_c,\ \omega_c\cdot(0.1),\ \omega_c\cdot(10)$

$H(j\omega_c) = \frac{1/RC}{j\omega_c + 1/RC} = \frac{10000}{10000j + 10000}$

$\boxed{H(j\omega_c) = 0.707 \angle -45°}$ [Same process as prob # 14.1]

$H(0.1\omega_c j) = \frac{1/RC}{0.1\omega_c j + 1/RC} = \frac{10000}{j1000 + 10000}$

$\boxed{H(j(0.1\omega_c)) = 0.995 \angle -5.71}$

$H(j\omega_c(10)) = \frac{1/RC}{10\omega_c j + 1/RC} = \frac{10000}{100{,}000j + 10000}$

$\boxed{H(j(10\omega_c)) = 0.0999 \angle -84.29°}$

(c) If $v_i = 200\cos(\omega t)$ mV, write the steady state expression for v_o when $\omega = \omega_c,\ 0.1\omega_c,\ 10\omega_c$

$v_i(\omega_c) = 200\cos(10000t)$ mV

$v_i(\omega_c) = 200\angle 0°$

$V_o(j\omega_c) = \frac{1/RC}{j\omega_c + 1/RC} V_i = \frac{10000}{10000j + 10000j}(200\angle 0°)$

$V_o(j\omega_c) = (0.707\angle -45°)(200\angle 0°)$

$V_o(j\omega_c) = 141.4\angle -45°$

$\boxed{V_o(j\omega_c) = 141.4\cos(10000t - 45°)\ mV}$

$V_i(0.1w_c) = 200\cos(1000t)\ mV$

$V_i(0.1w_c) = 200\angle 0°$

$$V_o(0.1w_c \cdot j) = \frac{1/RC}{0.1w_c j + 1/RC} V_i = \frac{10000}{1000j + 1000}(200\angle 0°)$$

$V_o(0.1w_c \cdot j) = (0.995\angle -5.71)(200\angle 0°)$

$$\boxed{V_o(0.1w_c \cdot j) = 199\cos(1000t - 5.71°)\ mV}$$

$V_i(10w_c) = 200\cos(100{,}000t)\ mV$

$V_i(10w_c) = 200\angle 0°$

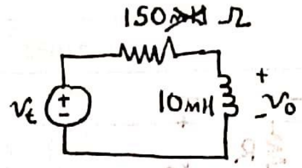
$$V_o(10w_c) = \frac{1/RC}{10w_c j + 1/RC} V_i = \frac{10000}{100000j + 10000}(200\angle 0°)$$

$V_o(10w_c) = (0.0999\angle -84.29°)(200\angle 0°)$

$V_o(10w_c) = 19.98\angle -84.29°$

$$\boxed{V_o(10w_c) = 19.98\cos(100000t - 84.29°)\ mV}$$

Problem # 14.11



(a) What is the transfer function $H(s) = V_o(s) / V_i(s)$ of this filter

$$V_o = \left(\frac{SL}{SL + R}\right) V_i = \left(\frac{s(10\times10^{-3})}{s(10\times10^{-3}) + 150}\right) V_i$$

$$\frac{V_o}{V_i} = \frac{s(10\times10^{-3})}{s(10\times10^{-3}) + 150} \qquad \frac{V_o}{V_i} = \frac{S}{S + 15000} \qquad w/ \ \frac{V_o}{V_i} = H(s)$$

$$\boxed{H(j\omega) = \frac{j\omega}{j\omega + 15000}}$$

(b) What is the cut off frequency of this filter?

$$\omega_c = \frac{R}{L} \qquad \omega_c = \frac{150\,\Omega}{(10\times10^{-3})\,H}$$

$$\boxed{\begin{array}{l} \omega_c = 15000 \text{ rad/s} \\ \omega_c = 2387.32 \text{ Hz} \end{array}}$$

(c) What is the magnitude of the filters transfer function at $s = j\omega_c$

$$H(j\omega_c) = \frac{j\omega_c}{j\omega_c + 15000} = \frac{15000}{j15000 + 15000}$$

$$\boxed{|H(j\omega_c)| = 0.707}$$

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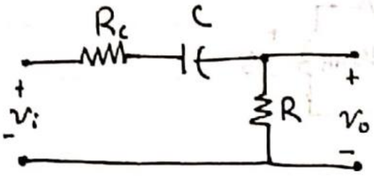
Problem # 14.16



(a) Derive the expression for $H(s)$ where $H(s) = V_o / V_i$

$$V_o(s) = \frac{R}{R_c + R + 1/sC} V_i(s) \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R}{R_c + R + 1/sC}$$

$$\boxed{H(s) = \frac{sRC}{sC(R_c+R)+1}}$$

(b) At what frequency will the magnitude of $H(j\omega)$ be a max.

$$H(j\omega) = \frac{j\omega RC}{j\omega C(R_c+R)+1} \Rightarrow \frac{RC}{C(R_c+R) + \frac{1}{j\omega}}$$

$$H(j(0)) = \frac{RC}{C(R_c+R)+\frac{1}{j(0)}} \qquad H(j(\infty)) = \frac{RC}{C(R_c+R)+\frac{1}{j(\infty)}}$$

$$\boxed{H(j\omega) \text{ is a max at } \omega = \infty}$$

(c) What is the maximum value of $H(j\omega)$

$$H(j\omega) \rightarrow H(j(\infty)) = \frac{RC}{C(R_c+R)+\frac{1}{j(\infty)}}$$

$$H(j(\infty)) = \frac{RC}{C(R_c+R)+0} \qquad H(j(\infty)) = \frac{RC}{C(R_c+R)}$$

$$\boxed{H(j(\infty)) = \frac{R}{R_c+R}}$$

(d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum divided by $\sqrt{2}$

$$|H(j\omega_c)| = \frac{|H(j\omega)|_{max}}{\sqrt{2}} \Rightarrow \left| \frac{j\omega_c RC}{j\omega_c C(R_c+R)+1} \right| = \frac{\frac{RC}{C(R_c+R)}}{\sqrt{2}}$$

$$\frac{\sqrt{(j\omega_c RC)^2}}{\sqrt{(j\omega_c C(R_c+R)+1)^2}} = \frac{RC}{\sqrt{2}\, C(R_c+R)}$$

$$(\omega_c RC)^2 = \left(\frac{RC}{\sqrt{2}\, C(R_c+R)}\right)^2 \left(\sqrt{(j\omega_c C(R_c+R)+1)^2}\right)^2$$

Problem # 14.16 (cont.)

$$\omega_c^2 R^2 C^2 = \frac{R^2C^2}{2C^2(R_c+R)^2}\left(\left(\omega_c C(R_c+R)\right)^2+1\right)$$

$$\omega_c^2 = \frac{\left(\left(\omega_c C(R_c+R)\right)^2+1\right)}{2C^2(R_c+R)^2}$$

$$2C^2(R_c+R)^2\omega_c^2 = \left(\omega_c C(R_c+R)\right)^2+1$$

$$2C^2(R_c+R)^2\omega_c^2 = \omega_c^2 C^2(R_c+R)^2+1$$

$$C^2(R_c+R)^2\omega_c^2 = 1$$

$$\omega_c^2 = \frac{1}{C^2(R_c+R)^2}$$

$$\boxed{\omega_c = \frac{1}{C(R_c+R)}}$$

(e) Assume a resistance of 12.5kΩ is connected in series with a 5nF cap in the circuit. Calculate ω_c; $H(j\omega_c)$ $H(j(0.2)\omega_c)$ and $H(j(5\omega_c))$

$$\omega_c = \frac{1}{(5\times10^{-9})(12.5k\Omega+7.5k\Omega)}$$

$$\omega_c = 10000$$

$$H(j\omega_c) = \frac{j\omega_c RC}{j\omega_c C(R_c+R)+1}$$

$$H(j\omega_c) = \frac{10000j(12.5k)(5nF)}{10000j(5nF)(20k\Omega)+1}$$

$$H(j\omega_c) = \frac{0.625j}{1+1j}$$

$$\boxed{H(j\omega_c) = 0.442\angle 45^\circ \quad @\ \omega_c = 10000\ rad/s}$$

$$0.2\omega_c = 2000$$

$$H(j(0.2)\omega_c) = \frac{(0.2)j\omega_c RC}{(0.2)j\omega_c C(R_c+R)+1}$$

$$H(j(0.2)\omega_c) = \frac{2000j(12.5k\Omega)(5nF)}{(2000j)(5nF)(20k\Omega)+1}$$

$$H(j(0.2)\omega_c) = \frac{0.125j}{0.2j+1}$$

$$\boxed{H(j\omega_c(0.2)) = 0.123\angle 11.31^\circ}$$

$$5(\omega_c) = 50000$$

$$H(5\omega_c j) = \frac{50000j(12.5k\Omega)(5nF)}{(50000j)(5nF)(20k\Omega)+1}$$

$$H(5\omega_c j) = \frac{3.125j}{5j+1}$$

$$\boxed{H(5\omega_c j) = 0.613\angle 78.69}$$

Problem #14.20

Calculate the center frequency, the bandwidth and the quality factor of a bandpass filter that has an upper cutoff frequency of 121 krad/s and a lower cutoff frequency of 100 krad/s.

$$\omega_o = \sqrt{\omega_{c1} \cdot \omega_{c2}}$$

$$= \sqrt{(121\,krad/s)(100\,krad/s)}$$

$$\boxed{\omega_o = 110\,krad/s}$$

$$\beta = \omega_{c1} - \omega_{c2}$$

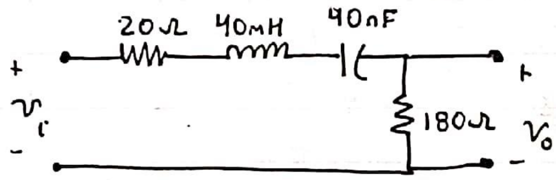
$$\beta = 121\,krad/s - 100\,krad/s$$

$$\boxed{\beta = 21\,krad/s}$$

$$Q = \omega_o/\beta = \frac{110\,krad/s}{21\,krad/s}$$

$$\boxed{Q = 5.238}$$

Problem #14.24



(a) $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(40mH)(40nF)}}$$ $$\boxed{f_o = 3978.87\,Hz}$$

(b) $$Q = \frac{\sqrt{\frac{L}{C}}}{R+R_1} = \frac{\sqrt{\frac{40mH}{40nF}}}{20\Omega + 180\Omega}$$ $$\boxed{Q = 5}$$

(c) $$\omega_{c1} = -\frac{R+R_1}{2L} + \sqrt{\left(\frac{R+R_1}{2L}\right)^2 + \frac{1}{LC}} = -\frac{180\Omega+20\Omega}{2(40mH)} + \sqrt{\left(\frac{180\Omega+20\Omega}{2(40mH)}\right)^2 + \frac{1}{40mH + 40nF}}$$

$$\boxed{\omega_{c2} = 22.624\,krad/s}$$

$$f_{c2} = \frac{\omega_{c2}}{2\pi} \qquad f_{c2} = \frac{22.624\,krad/s}{2\pi} \qquad \boxed{f_{c2} = 3.6\,kHz}$$

(d) $$\omega_{c2} = \frac{R+R_1}{2L} + \sqrt{\left(\frac{R+R_1}{2L}\right)^2 + \frac{1}{LC}} = \frac{180\Omega+20\Omega}{2(40mH)} + \sqrt{\left(\frac{180\Omega+20\Omega}{2(40mH)}\right)^2 + \frac{1}{40mH+40nF}}$$

$$\boxed{\omega_{c2} = 27.624\,krad/s} \qquad f_{c2} = \frac{\omega_{c2}}{2\pi} \qquad \boxed{f_{c2} = 4.396\,kHz}$$

(e) $$\beta = \omega_{c2} - \omega_{c1} \qquad \beta = 27.624\,krad/s - 22.624\,krad/s \qquad \boxed{\beta = 5\,krad/s}$$

$$f_\beta = \frac{\beta}{2\pi} \qquad f_\beta = \frac{5\,krad/s}{2\pi} \qquad \boxed{f_\beta = 796\,Hz}$$