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Satellite Detumbling Project

Problem statement:

A spacecraft is deployed from a rocket in Low Earth Orbit with an initial body rate of $\omega = [\omega_{x0} \quad \omega_{y0} \quad \omega_{z0}]'$. Ground station sends a signal to the spacecraft to enter safe mode. That is, the spacecraft is commanded to stop tumbling $(\omega_{des} = [0 \quad 0 \quad 0]'\, rad/s)$ within one orbit (~90minutes). The spacecraft is using momentum wheels which saturate at $|M_{max}| = \pm 5_{Nm}$ and are assumed to actuate instantaneously. Ground station wants to use as little battery reserve power as possible. Design a controller (e.g. PID, LQR, etc.) to detumble the spacecraft, neglecting drag, given that:

$$\omega_0 = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix} rad/s$$

$$\vec{I}spacecraft = \begin{bmatrix} I_{xx} = 100 & 0 & 0 \\ 0 & I_{yy} = 100 & 0 \\ 0 & 0 & I_{zz} = 1{,}000 \end{bmatrix}$$

Analysis of effectiveness of solution:

A PID gain optimization script was used to optimize the gains of each PID controller used to control the spacecraft axis. These gains can be seen below,

$$PID\ Gain\ Matrix, \qquad G_M = \begin{bmatrix} P_{P,1} = 2.916 * 10^{-6} & P_{P,2} = 6.0834 * 10^{-5} & P_{P,3} = 1.0731 \\ P_{I,1} = 4.664 * 10^{-6} & P_{I,2} = 8.1814 * 10^{-6} & P_{I,3} = 1.0637 \\ P_{D,1} = 0.3948 & P_{D,2} = 0.3948 & P_{D,3} = 0.3948 \end{bmatrix}$$

With initial conditions,

$$Initial\ PID\ Gain\ Matrix, \qquad U_0 = \begin{bmatrix} P_{P,1} = 1 & P_{P,2} = 1 & P_{P,3} = 1 \\ P_{I,1} = 0 & P_{I,2} = 0 & P_{I,3} = 0 \\ P_{D,1} = 0.1 & P_{D,2} = 0.1 & P_{D,3} = 0.1 \end{bmatrix}$$

Multiple initial conditions were used to simulate the optimal gain matrix in attempts to find various local minima that may satisfy the function. Using the gains returned by the optimization, a total effort of 5000 Nm and 16 minutes and 40 seconds was obtained to detumble the satellite. Figure 1 shows the relationship between total effort and time. The relationship between total effort and time required to detumble the satellite is linear for the set of gains found.

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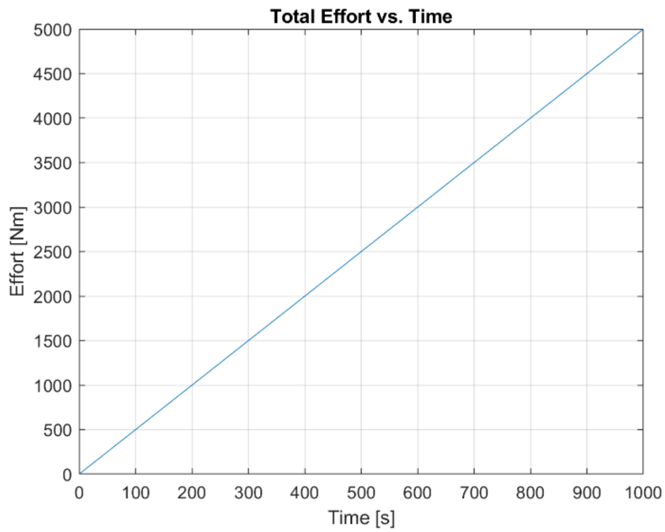

Figure 1 Total Effort of Spacecraft using Optimal Gain Matrix

The spacecraft was launched from the rocket with rates of $\omega_0 = |4 \quad -3 \quad 5| \frac{rad}{s}$. The transient response of the spacecraft body rates using the optimal gain matrix to these initial conditions can be seen in the figure below.

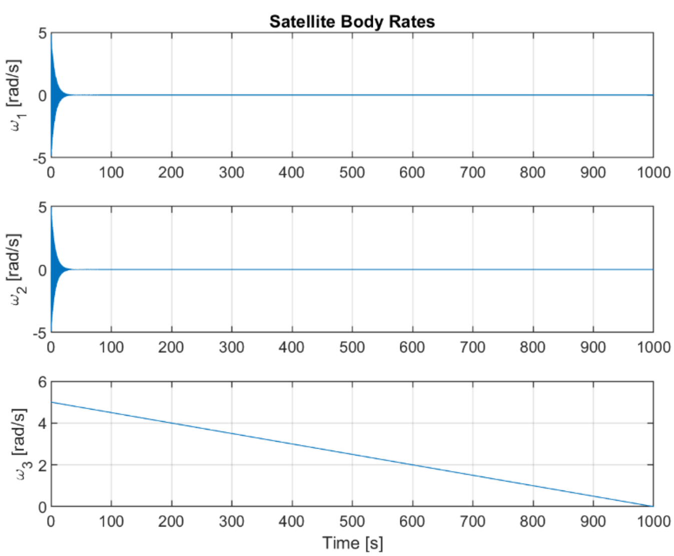

Figure 2 Body Rates of Spacecraft using Optimal Gain Matrix

From this figure the body rates in the z-coordinate appear to decrease linearly while the body rates in the x- and y-coordinates amplitudes appear to decrease exponential with time. The steady state error for the body rates can be seen below,

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$$1.7453 * 10^{-8} \approx 0 \ \frac{rad}{s}$$
$$e_{ss} = \ 1.7453 * 10^{-8} \approx 0 \ \frac{rad}{s}$$
$$1.00 * 10^{-4} \approx 0 \ \frac{rad}{s}$$

Although the program reported non-zero values for the body rates, in practice these values are negligible and can be approximated as zero.

Strategy:

The team implemented PID control to stabilize the spacecraft. A Simulink model was produced to simulate the spacecraft with initial body rates ω_0 and reaction wheels being used to control the system. The angular accelerations can be described with the following differential equations,

$$\dot{w}_1 = \frac{1}{I_1} * [M_1 + \ w_2 w_3 (I_2 - \ I_3)]$$

$$\dot{w}_2 = \frac{1}{I_2} * [M_2 + \ w_1 w_3 (I_3 - \ I_1)]$$

$$\dot{w}_3 = \frac{1}{I_3} * [M_3 + \ w_1 w_2 (I_1 - \ I_2)]$$

These values can then be integrated to give the angular velocities, and the initial conditions for velocity from the problem statement are applied in these blocks. The error being fed back to the PID blocks is the desired angular velocity (w = 0) minus the current angular velocities. In other words, the error being fed back is the negative of the current measured velocities. This results in a large expression with nine PID constants for each axis and to simplify this the errors stemming from the other axis are ignored. This leaves the following equations for M1, M2, and M3 where e is negative of the current angular velocity.

$$M_1 = \ K_{P1} e_1 + \ K_{I1} \int e_1 dt + \ K_{D1} \dot{e}_1$$

$$M_2 = \ K_{P2} e_2 + \ K_{I2} \int e_2 dt + \ K_{D2} \dot{e}_2$$

$$M_3 = \ K_{P3} e_3 + \ K_{I3} \int e_3 dt + \ K_{D3} \dot{e}_3$$

In order to quantify the cost of our solution, the moments were summed, and their absolute values are integrated to produce total control effort. Total control effort should be minimized along with time. Once the model was setup as shown below, the controller gains could be determined.

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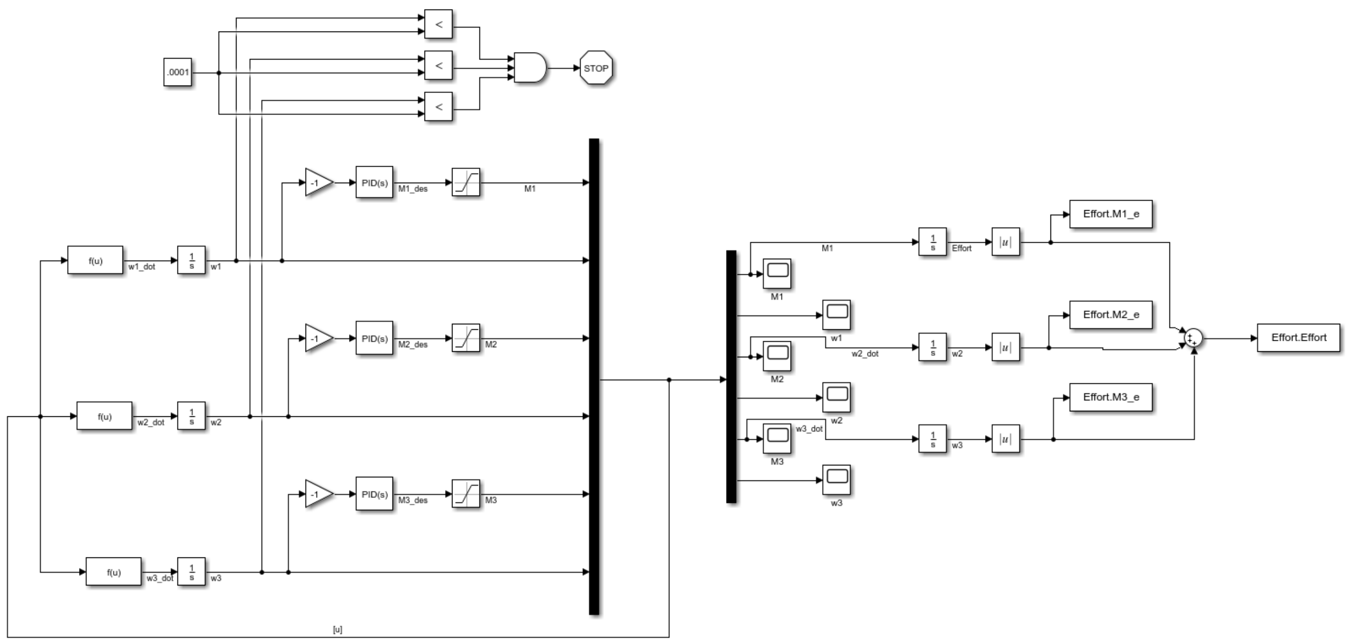

Figure 3 Simulink Map

The PID gains were left variable to be changed iteratively to "tune" the gains to produce the desired output. Starting from all ones, the gain values were slightly changed, and their corresponding outputs closely monitored until everything stabilized in enough time. This method may seem simple, but it is commonly used as simpler simulations such as these do not require much run time, so iterative guess and check problem solving is a valid approach.

For situations requiring more sophisticated simulation, one might use numerical optimization to target suitable values more quickly. The objective of this scenario was to minimize the amount of effort and time required to detumble the spacecraft. This can be reformulated into the minimization problem below,

$$\min_{PID\ Gain} |M_x| + |M_y| + |M_z| + t \qquad s.t. 0 < \ G_M < 250$$

Here the cost function is the function being minimized. In order to use this logic needed to be added to the Simulink model to stop the model from running if the body rates fell below a specified threshold. The threshold chosen was $1 * 10^{-4}$ and the sub model can be seen below.

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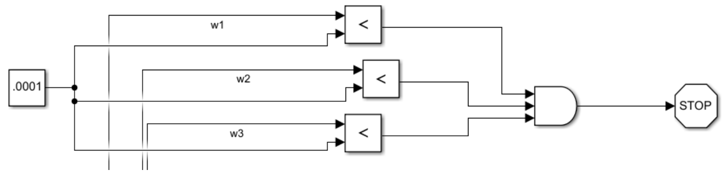

Figure 4 Simulink Logic for Minimizing Detumbling time

After formulating the problem, the team utilized MATLAB's fmincon function and to find local minima of a constrained nonlinear multivariable function. This is still an iterative method, but it can find solutions to more complex problems much faster and accurately.

After completion of the project, it was realized that if one non-linear dynamic was solved the remaining two would become linear and finding optimal gains could be more efficient. If optimizing the gains were to be attempted again, additional Simulink logic would be added to determine which non-linear dynamic is more efficient to solve and bring closer to zero first.

<u>Design Parameters:</u>

The team was tasked with detumbling a spacecraft within one ninety-minute orbit, minimizing time and total effort was a secondary priority. This spacecraft held with it a few design parameters we must abide by and navigate around. Primarily, the momentum wheels must not exceed $|M_{max}| = \pm 5_{Nm}$ of torque. Initial momentum values of $I_{xx} = 100\ kg * m^2$, $I_{yy} = 100\ kg * m^2$, and $I_{zz} = 1{,}000\ kg * m^2$ were recorded. Initial body rates of $\omega_0 = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix} rad/s$ and final body rate values of $\omega_{des} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} rad/s$. These parameters were met with initial conditions as stated in the problem statement.

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Effort Report:

Thomas Collins contributed 33%

X

Thomas Collins

Zachary Raboin contributed 33%

X

Zachary Raboin

Lucas Simmonds contributed 33%

X

Lucas Simmonds