Classification Supervised Learning (Part2)

Outline

- Naive Bayes
- Random Forests
- Support Vector Machine

Naive Bayes

What's Naive Bayes

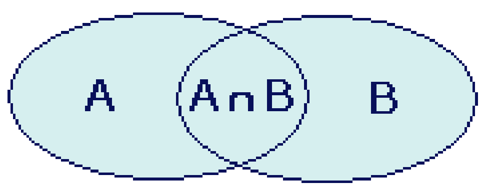
- **A family of probabilistic classifiers based on applying Bayes' theorem**
- **Naive Bayes classifiers are highly scalable**
  - **require parameters linear in the number of variables features**
- **Common used in document classification**

Thomas Bayes

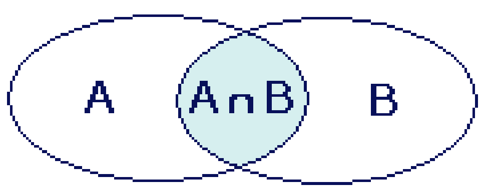
set operation | **probabilities**

$p(A) = 1$

if A is the population



$p(A \cup B) = p(A) + p(B) - p(A \cap B)$



$p(A \cap B) = p(A|B)\, p(B) = p(B|A)\, p(A)$

special case: independent events

$p(A \cap B) = p(A)\, p(B)$

independent probability

- **Two event are independent if**

$$P(A \cap B) = P(A) * P(B)$$

- **Example**
  - **Given a dice, if we toss the dice twice, what's probability that sum of the points is even?**

$$P(sum\ of\ points\ is\ even) = \frac{1}{2} * \frac{1}{2}$$

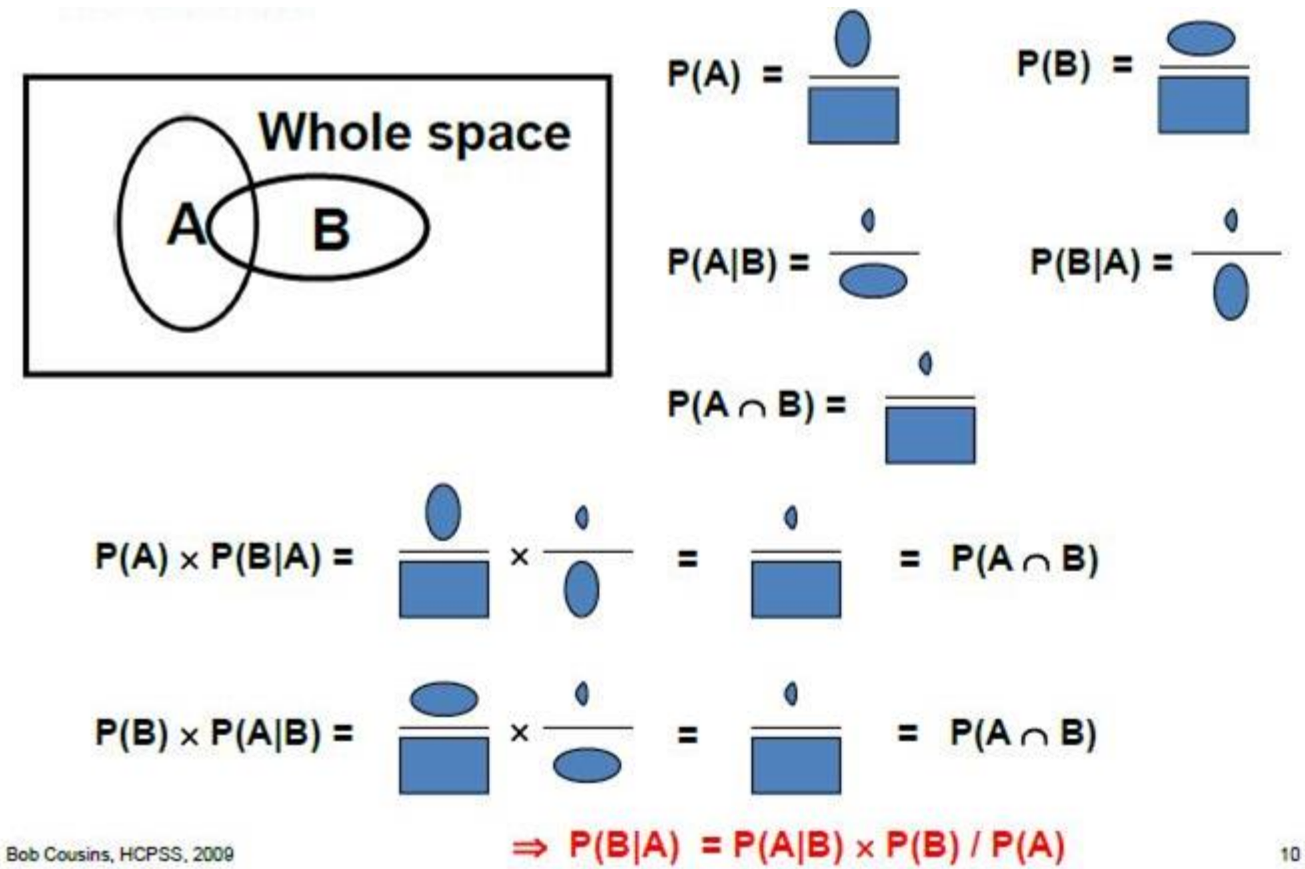
tossing the dice each time is independent event

Naive Bayes

- **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|\bar{A}) * P(\bar{A})}$$

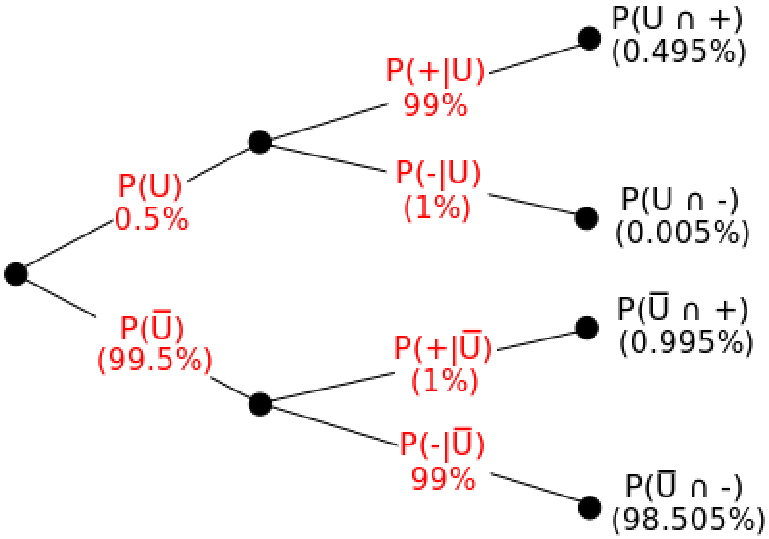

Whole space
A
B
P(A) =
P(B) =
P(A|B) =
P(B|A) =
P(A ∩ B) =
P(A) × P(B|A) = × = = P(A ∩ B)
P(B) × P(A|B) = × = = P(A ∩ B)
Bob Cousins, HCPSS, 2009
⇒ P(B|A) = P(A|B) × P(B) / P(A)
10

Example

- **Suppose that a test for using a particular drug is 99% sensitive and 99% specific. That is, the test will produce 99% true positive results for drug users and 99% true negative results for non-drug users.**
- **Suppose that 0.5% of people are users of the drug. What is the probability that a randomly selected individual with a positive test is a drug user?**

Example

$$
\begin{aligned}
P(\text{User} \mid +) &= \frac{P(+ \mid \text{User})P(\text{User})}{P(+)} \\
&= \frac{P(+ \mid \text{User})P(\text{User})}{P(+ \mid \text{User})P(\text{User}) + P(+ \mid \text{Non-user})P(\text{Non-user})} \\
&= \frac{0.99 \times 0.005}{0.99 \times 0.005 + 0.01 \times 0.995} \\
&\approx 33.2\%
\end{aligned}
$$



How Naive Bayes Classifier Work

- **Assume**
  - **there are three attributes A1, A2, A3 and two class C0 and C1**

$$p(C_0|\ A_1, A_2, A_3) > p(C_1|\ A_1, A_2, A_3)$$

$$\Downarrow$$

Guess it is class 0

$$p(C_0|\ A_1, A_2, A_3) < p(C_1|\ A_1, A_2, A_3)$$

$$\Downarrow$$

Guess it is class 1

A1	A2	A3	Class
1	0	1	1
0	1	1	0
1	1	0	0
1	1	1	0
1	0	0	1

How Naive Bayes Classifier Work

$$p(C_0 | A_1, A_2, A_3) = \frac{p(A_1, A_2, A_3 | C_0) * p(C_0)}{p(A_1, A_2, A_3)}$$

Assume attributes are independent

$$p(C_0 | A_1, A_2, A_3) = \frac{p(A_1 | C_0) * p(A_2 | C_0) * p(A_3 | C_0) * p(C_0)}{p(A_1, A_2, A_3)}$$

$$p(C_1 | A_1, A_2, A_3) = \frac{p(A_1, A_2, A_3 | C_1) * p(C_1)}{p(A_1, A_2, A_3)}$$

Assume attributes are independent

$$p(C_1 | A_1, A_2, A_3) = \frac{p(A_1 | C_1) * p(A_2 | C_1) * p(A_3 | C_1) * p(C_1)}{p(A_1, A_2, A_3)}$$

How Naive Bayes Classifier Work

- **Assume**
  - **there are three attributes A1, A2, A3 and two class C0 and C1**

$$p(A_1|C_0) * p(A_2|C_0) * p(A_3|C_0) * p(C_0) > p(A_1|C_1) * p(A_2|C_1) * p(A_3|C_1) * p(C_1)$$

$$\Downarrow$$

Guess it is class 0

$$p(A_1|C_0) * p(A_2|C_0) * p(A_3|C_0) * p(C_0) < p(A_1|C_1) * p(A_2|C_1) * p(A_3|C_1) * p(C_1)$$

$$\Downarrow$$

Guess it is class 1

TibaMe

Example

Goal: predict if the text is about sport

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports

Example

- **If we want to predict if sentence "a very close game" is sports or not sports, we need to compare the following two term**

$p(sports|a\ very\ close\ game\ )$ $\quad\quad$ $p(Not\ sports|a\ very\ close\ game)$

$p(a\ very\ close\ game\ |sports) * p(sports)$ $\quad$ $p(a\ very\ close\ game\ |Not\ sports)\ p(Not\ sports)$

Example

$$p(sports|a\ very\ close\ game\ ) \qquad p(Not\ sports|a\ very\ close\ game)$$

use previous concept, we can compare the following term instead of origin one

$$p(a\ |sports) * p(very\ |sports) * p(close\ |sports) * p(game|sports) * p(sports)$$

$$p(a\ |Not\ sports) * p(very\ |Not\ sports) * p(close\ |Not\ sports) * p(game|Not\ sports) * p(Not\ sports)$$

TibaMe

Example

$$p(a\ |sports) * p(very\ |sports) * p(close\ |sports) * p(game|sports) * p(sports)$$

$$p(a\ |Not\ sports) * p(very\ |Not\ sports) * p(close\ |Not\ sports) * p(game|Not\ sports) * p(Not\ sports)$$

How to calculate each term?

Example

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports

$$p(sports) = \frac{3}{5}$$

$$p(Not\ sports) = \frac{2}{5}$$

How to calculate $p(word|Sports)$? The most intuitive way is like the following

$$p(game|Sports) = \frac{2}{11}$$

$$\vdots$$

$$\boxed{p(close\ |Sports) = 0}$$

We don't want this!!!

Example

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports

$$p(sports) = \frac{3}{5}$$

$$p(Not\ sports) = \frac{2}{5}$$

In order to deal with zero count problem, we use Laplace smoothing method to calculate $p(word|Sports)$

Example

Text	Category
“A great game”	Sports
“The election was over”	Not sports
“Very clean match”	Sports
“A clean but forgettable game”	Sports
“It was a close election”	Not sports

$$p(sports) = \frac{3}{5}$$

$$p(Not\ sports) = \frac{2}{5}$$

$$p(game|Sports) = \frac{2 + \boxed{1}}{11 + \boxed{14}}$$

add one to every count

add # of different words

Multinomial Naive Bayes

TibaMe

Example

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports

$$p(sports) = \frac{3}{5}$$

$$p(Not\ sports) = \frac{2}{5}$$

Word	P(word | sports)	P(word | not sports)
a	$\frac{2+1}{11+14}$	$\frac{1+1}{9+14}$
very	$\frac{1+1}{11+14}$	$\frac{0+1}{9+14}$
close	$\frac{0+1}{11+14}$	$\frac{1+1}{9+14}$
game	$\frac{2+1}{11+14}$	$\frac{0+1}{9+14}$

Example

$$p(a\ |sports) * p(very\ |sports) * p(close\ |sports) * p(game|sports) * p(sports)$$
$$= 0.0000276$$

$$p(a\ |Not\ sports) * p(very\ |Not\ sports) * p(close\ |Not\ sports) * p(game|Not\ sports)$$
$$* p(Not\ sports)$$
$$= 0.00000572$$

So, our classifier guess “a very nice game” is sports category

Different probability assumption

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports

$$\boxed{p(game|Sports) = \frac{2+1}{11+14}}$$

Why we calculate condition probability like this?

Different probability assumption

Actually, we can assume conditional probability as different probability distribution

$$p(attribute\ 1|class\ 1) = N(attribute1|\ \mu, \sigma)$$
$$where\ N\ is\ gaussian\ distribution$$

Assume conditional probability as gaussian distribution

TibaMe

Example

Height (ft)	Weight (lbs)	Shoe Size (in)	Gender
6.00	180	12	male
5.92	190	11	male
5.58	170	12	male
5.92	165	10	male
5.00	100	6	female
5.50	150	8	female
5.42	130	7	female
5.75	150	9	female

$$p(male) = p(female) = \frac{1}{2}$$

$$\begin{aligned} p(height \mid male) &= \mathcal{N}(height \mid \mu_{hm}, \sigma_{hm}) \\ p(weight \mid male) &= \mathcal{N}(weight \mid \mu_{wm}, \sigma_{wm}) \\ p(shoe \mid male) &= \mathcal{N}(shoe \mid \mu_{sm}, \sigma_{sm}) \end{aligned}$$

$$\begin{aligned} p(height \mid female) &= \mathcal{N}(height \mid \mu_{hf}, \sigma_{hf}) \\ p(weight \mid female) &= \mathcal{N}(weight \mid \mu_{wf}, \sigma_{wf}) \\ p(shoe \mid female) &= \mathcal{N}(shoe \mid \mu_{sf}, \sigma_{sf}) \end{aligned}$$

Example

$$\begin{aligned}
p(height \mid male) &= \mathcal{N}(height \mid \mu_{hm}, \sigma_{hm}) \\
p(weight \mid male) &= \mathcal{N}(weight \mid \mu_{wm}, \sigma_{wm}) \\
p(shoe \mid male) &= \mathcal{N}(shoe \mid \mu_{sm}, \sigma_{sm})
\end{aligned}$$

$$\begin{aligned}
p(height \mid female) &= \mathcal{N}(height \mid \mu_{hf}, \sigma_{hf}) \\
p(weight \mid female) &= \mathcal{N}(weight \mid \mu_{wf}, \sigma_{wf}) \\
p(shoe \mid female) &= \mathcal{N}(shoe \mid \mu_{sf}, \sigma_{sf})
\end{aligned}$$

	height mean	height variance	weight mean	weight variance	shoe size mean	shoe size variance
$male$	$\mu_{hm} = 5.855$	$\sigma^2_{hm} = .0350$	$\mu_{wm} = 176.25$	$\sigma^2_{wm} = 122.9$	$\mu_{sm} = 11.25$	$\sigma^2_{sm} = .9167$
$female$	$\mu_{hf} = 5.418$	$\sigma^2_{hf} = .0972$	$\mu_{wf} = 132.5$	$\sigma^2_{wf} = 558.3$	$\mu_{sf} = 7.5$	$\sigma^2_{sf} = 1.667$

TibaMe

Example

If a sample with height = 6, weight=130, and shoe=8, predict if it is male or female?

$$
\begin{aligned}
p(male \mid height, weight, shoe) &\propto p(male)\, p(height \mid male)\, p(weight \mid male)\, p(shoe \mid male) \\
&\propto p(male)\, \mathcal{N}(height \mid \mu_{hm}, \sigma_{hm})\, \mathcal{N}(weight \mid \mu_{wm}, \sigma_{wm})\, \mathcal{N}(shoe \mid \mu_{sm}, \sigma_{sm}) \\
&\propto \frac{1}{2}\, \mathcal{N}(6 \mid 5.855, \sqrt{.0350})\, \mathcal{N}(130 \mid 176.25, \sqrt{122.9})\, \mathcal{N}(8 \mid 11.25, \sqrt{.9167}) \\
&= .5 \times 1.579 \times 5.988 \cdot 10^{-6} \times 1.311 \cdot 10^{-3} \\
&= 6.120 \cdot 10^{-9}
\end{aligned}
$$

$$
\begin{aligned}
p(female \mid height, weight, shoe) &\propto p(female)\, p(height \mid female)\, p(weight \mid female)\, p(shoe \mid female) \\
&\propto p(female)\, \mathcal{N}(height \mid \mu_{hf}, \sigma_{hf})\, \mathcal{N}(weight \mid \mu_{wf}, \sigma_{wf})\, \mathcal{N}(shoe \mid \mu_{sf}, \sigma_{sf}) \\
&\propto \frac{1}{2}\, \mathcal{N}(6 \mid 5.418, \sqrt{.0972})\, \mathcal{N}(130 \mid 132.5, \sqrt{558.3})\, \mathcal{N}(8 \mid 7.5, \sqrt{1.667}) \\
&= .5 \times 2.235 \cdot 10^{-1} \times 1.679 \cdot 10^{-2} \times 2.867 \cdot 10^{-1} \\
&= 5.378 \cdot 10^{-4}
\end{aligned}
$$

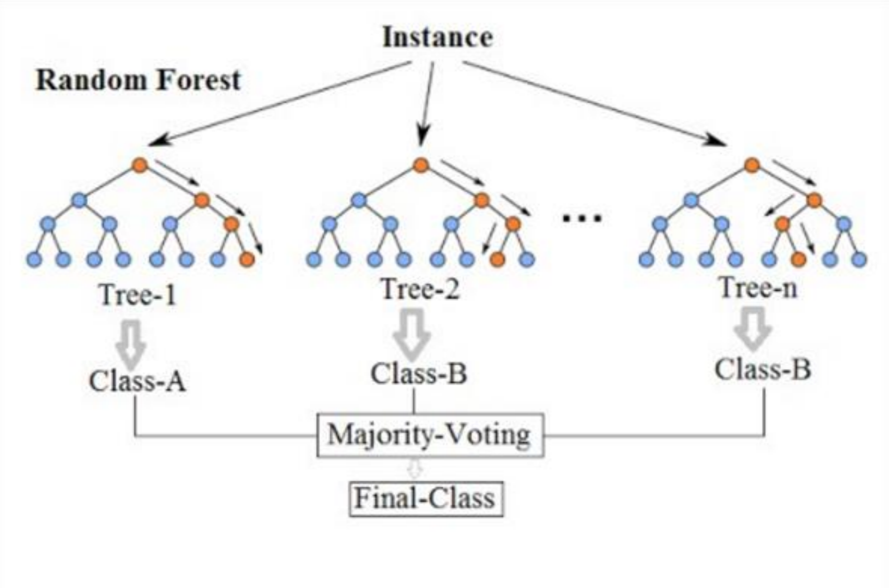
Example and Practice

- **Example**
  - **Naive Bayes**
    - example/supervised learning
- **Practice**
  - **Try to use decision tree to predict if someone in titanic would survive**
    - dataset/titanic/train.csv
    - practice/supervised learning
  - **More information about the dataset**
    - https://www.kaggle.com/c/titanic/data

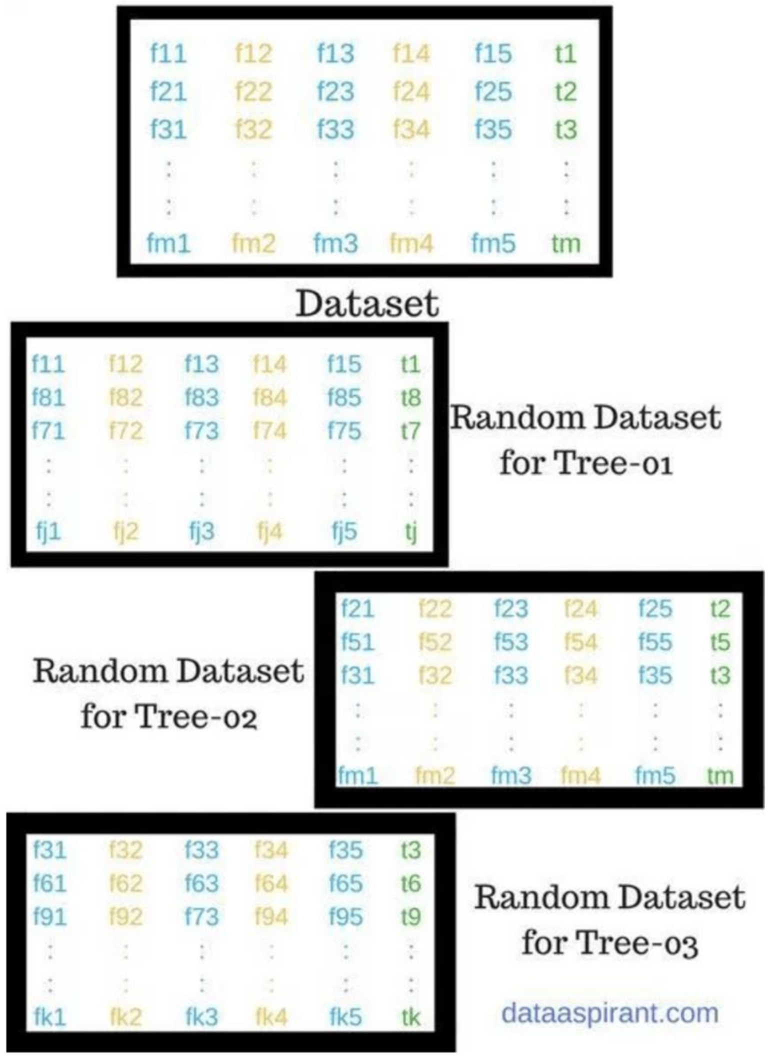
Random Forests

What's Random Forests

- An ensemble learning method for classification/regression by constructing a multiple decision trees at training time
  - usually use CART as tree
- processes of finding the root node and splitting the feature nodes will run randomly.

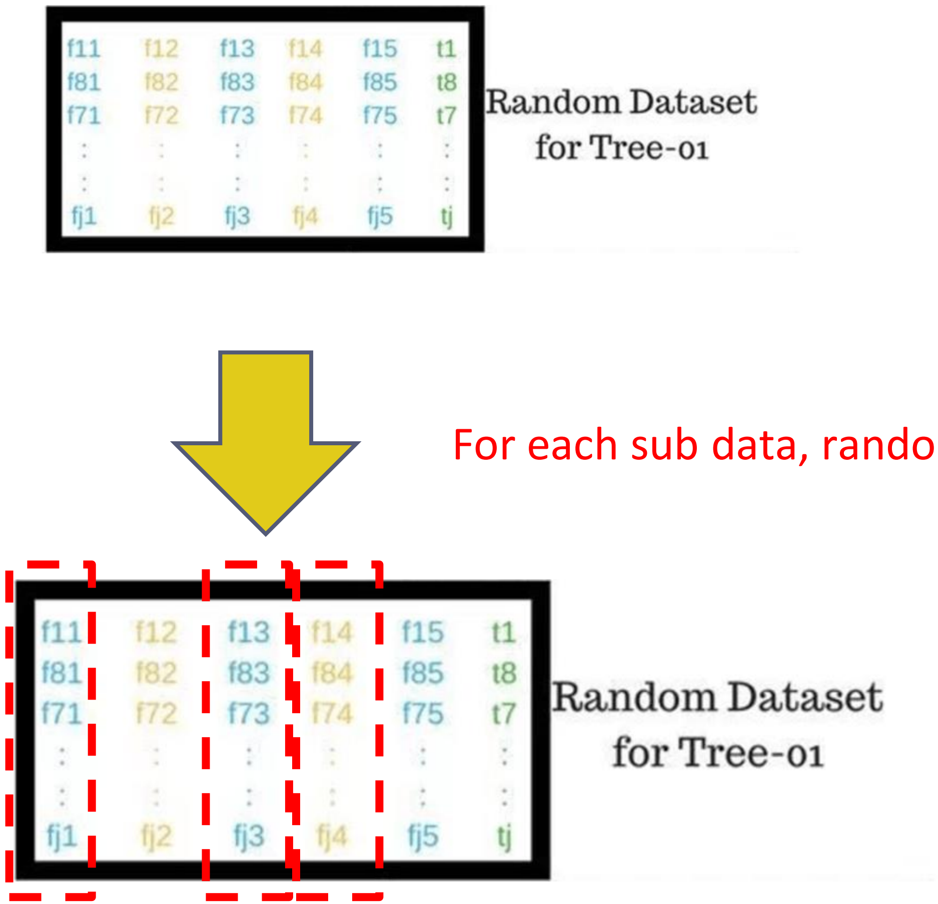


Example



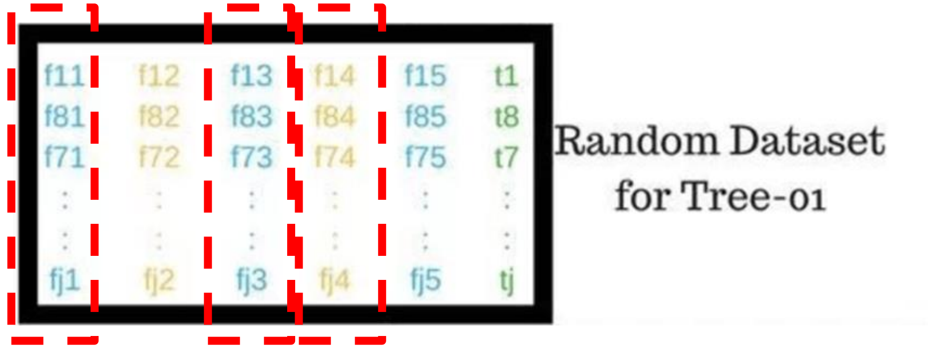
randomly select sub-data to create different tree

Example

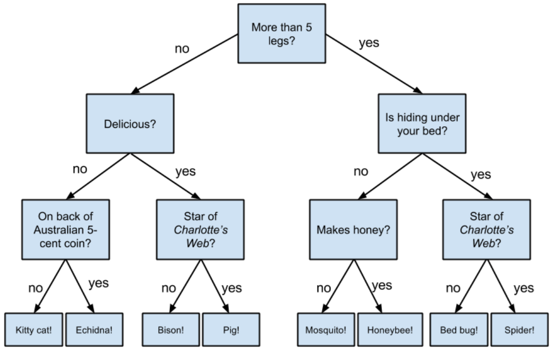


For each sub data, randomly select k features

Example

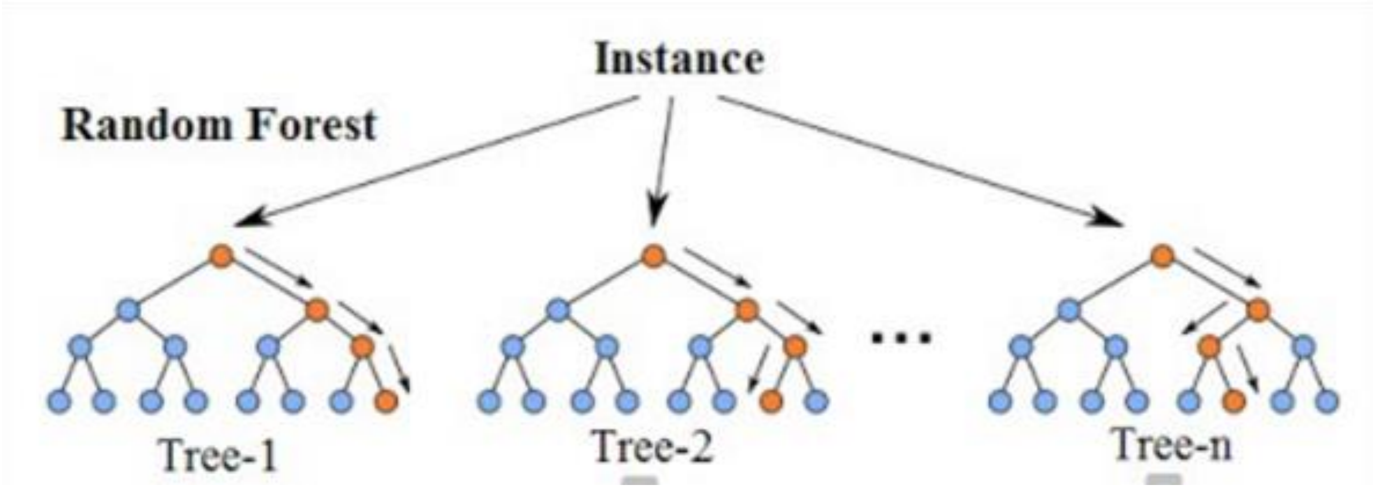


Use selected features to create CART



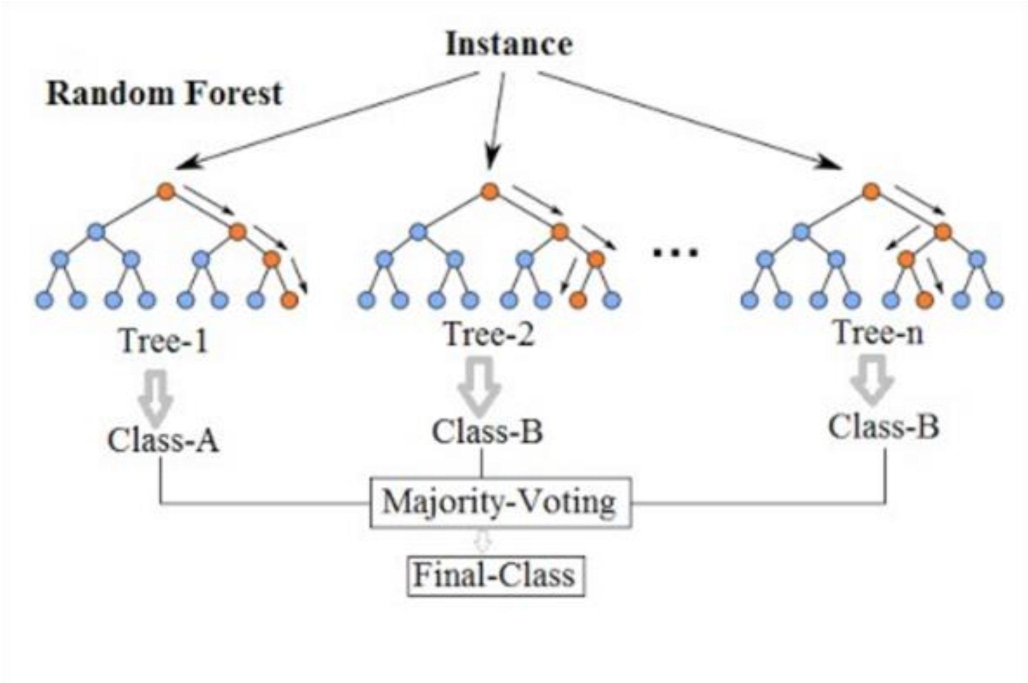
Example

Do the same procedure on each sub-data and create a forest



Example

When testing, collect each of decision tree's result and do majority-voting to determine the final prediction



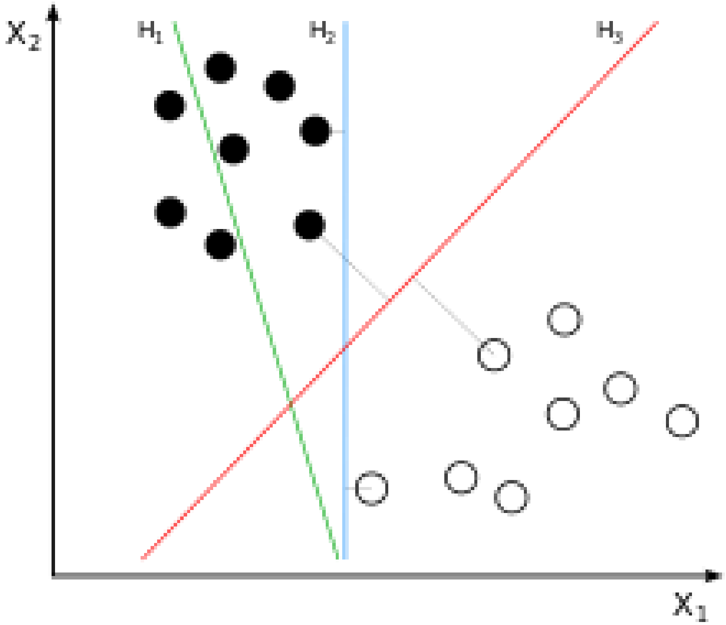
Example and Practice

- **Example**
  - **Random Forest**
    - example/supervised learning
- **Practice**
  - **Try to use random forest to predict different varieties of wheat**
    - dataset/seeds_dataset.csv
    - practice/supervised learning
  - **More information about the dataset**
    - https://archive.ics.uci.edu/ml/datasets/seeds#

Support Vector Machine

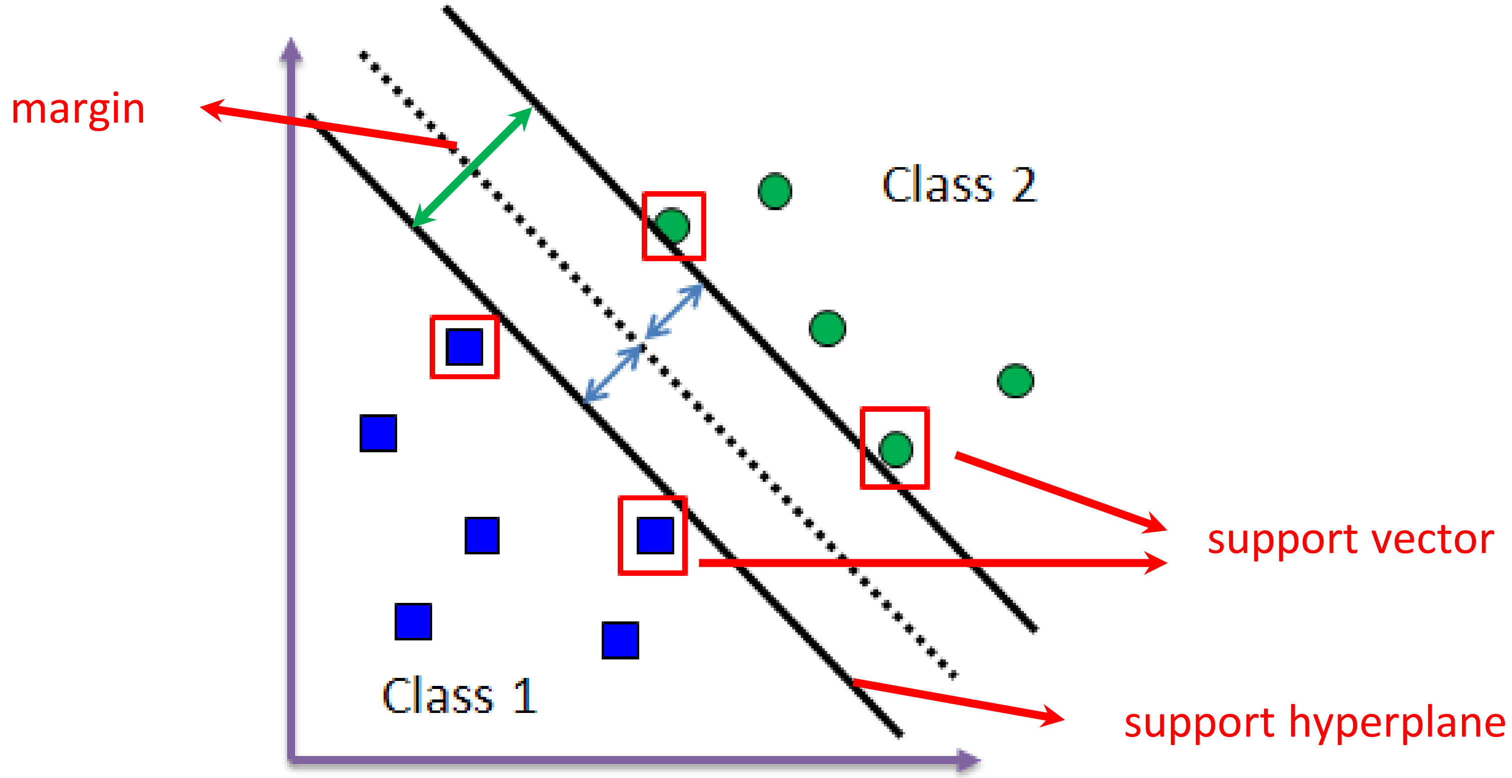
What's Support Vector Machine

- support vector machines (SVM) are supervised learning models
- Linear SVM find a hyperplane that separate data with maximum margin



- H_1 does not separate the classes
- H_2 does, but only with a small margin
- H_3 separates them with the maximum margin

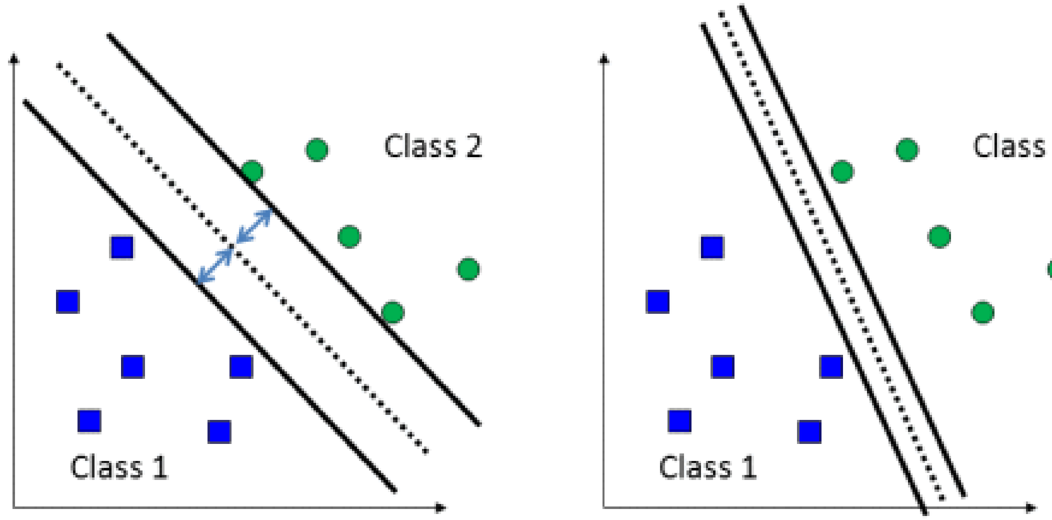
What's Support Vector



support vectors are points that affect hyperplane with maximum margin

What SVM do

Choose the one with large margin!



How to calculate margin/distance?

- What's margin/distance between hyperplane $2x - y + 2z = -5$ and point $(2, 0, 0)$

change all stuffs on one side

$$2x - y + 2z + 5 = 0$$

calculate distance

$$\frac{|2 * 2 - 0 - 2 * 0 + 5|}{\sqrt{2^2 + (-1)^2 + 2^2}} = 3$$

How to calculate margin/distance?

- **Any Hyperplane in N-dimension can model**

$$w^T x - b = 0$$

$$w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Example

- **What's distance between the following 5-D hyperplane and point (2, 3, 4,1,1)**

$$H: w^T x - b = 0 \; where \; w = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \\ 1 \end{bmatrix} and \; b = -5$$

Example

change all stuffs on one side

$$w^T x - b = 0$$

$$w = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \\ 1 \end{bmatrix} and\ b = -5$$

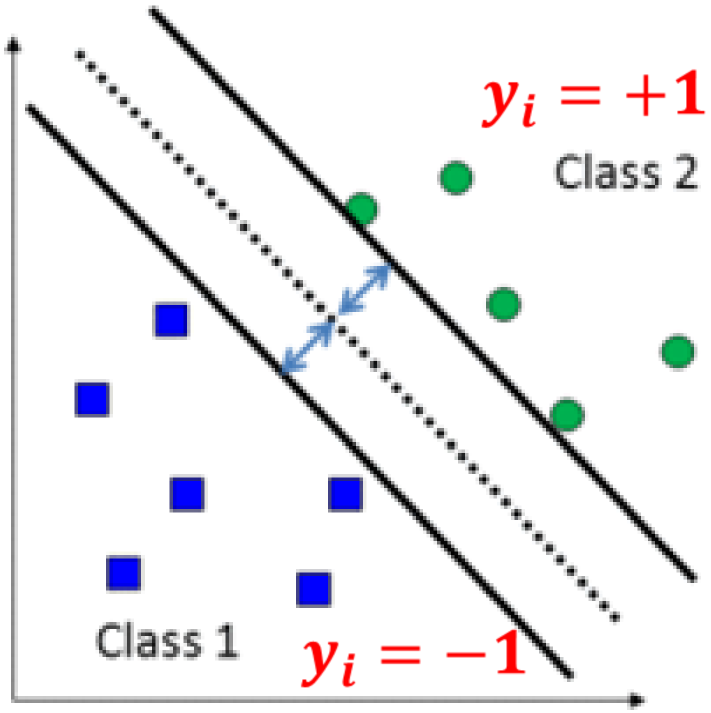
calculate distance

$$\frac{|1 * 2 + (-1) * 3 + 2 * 4 + 3 * 1 + 1 * 1 + 5|}{\sqrt{1^2 + (-1)^2 + 2^2 + 3^2 + 1^2}} = 4$$

SVM

$$\{x_i, y_i\}, i = 1, \dots\dots, n$$
$$x_i \in R^d, y^i \in \{+1, -1\}$$

label



SVM

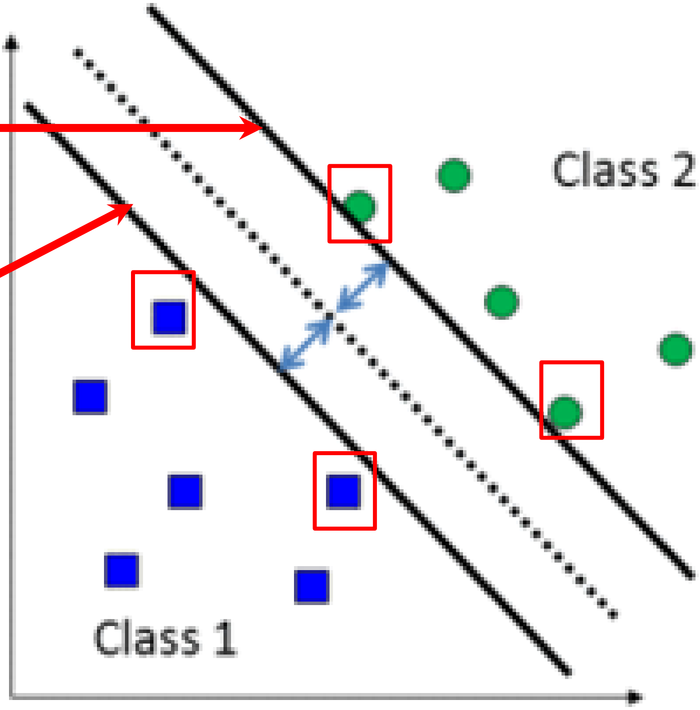
Assume

support hyperplane(green) is

$$f(x_g) = w^T x_g - b = 1$$

support hyperplane(blue) is

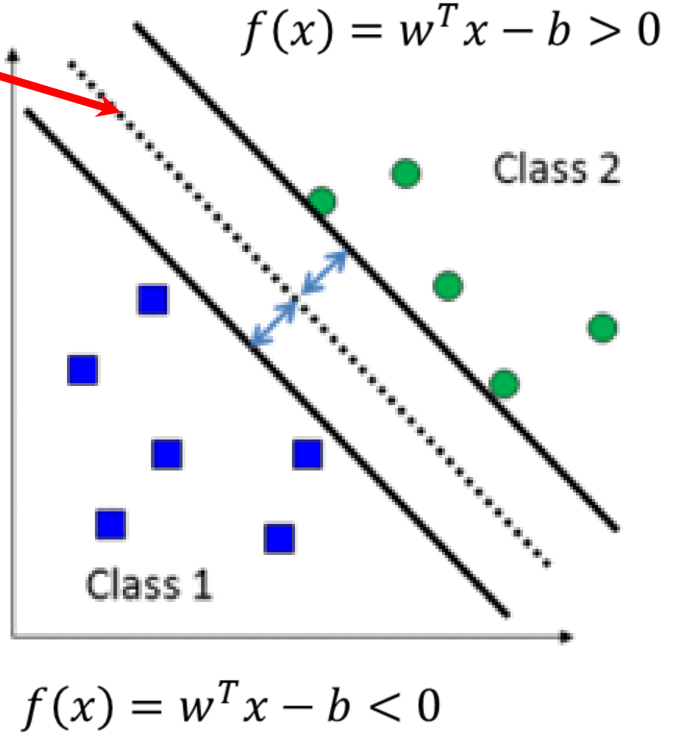
$$f(x_b) = w^T x_b - b = -1$$



SVM

$$f(x) = w^T x - b = 0$$

$if\ f(x_i) = w^T x_i - b < 0$	$if\ f(x_i) = w^T x_i - b > 0$
Class 1	Class 2
$y_i = -1$	$y_i = +1$

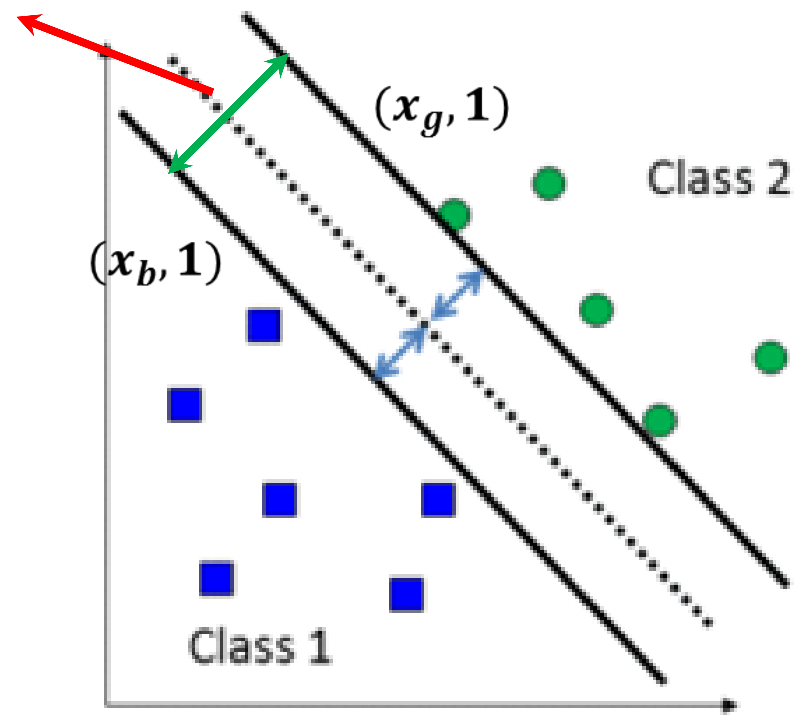


SVM

Goal(what we want):

margin

$$margin = \frac{|w^T x_g - b|}{\|w\|} + \frac{|w^T x_b - b|}{\|w\|} = \frac{2}{\|w\|}$$



Note :

$$f(x_g) = w^T x_g - b = 1$$

$$f(x_b) = w^T x_b - b = -1$$

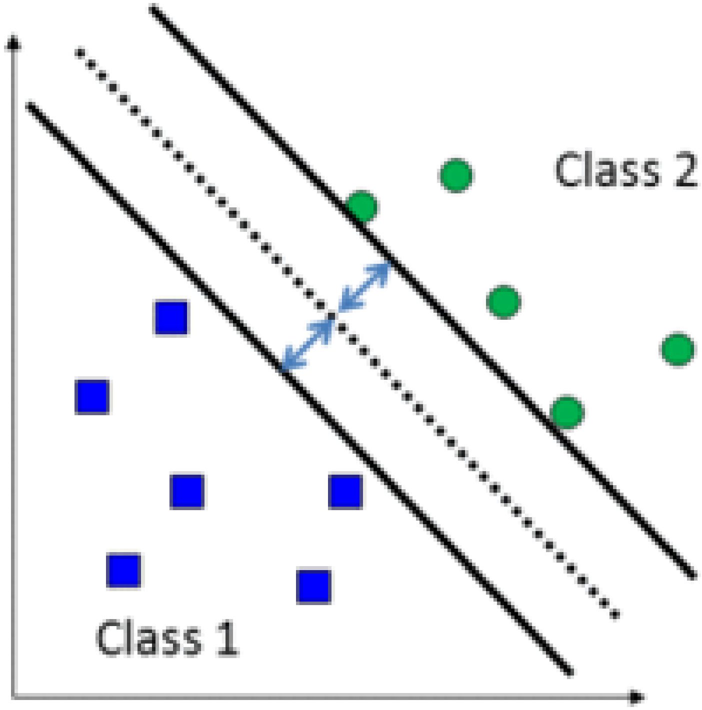
Constrain:

$$w^T x_i - b \leq -1 \ \ \forall y_i = -1$$

$$w^T x_i - b \geq -1 \ \ \forall y_i = +1$$

combine

$$y_i(w^T x_i - b) - 1 \geq 0$$



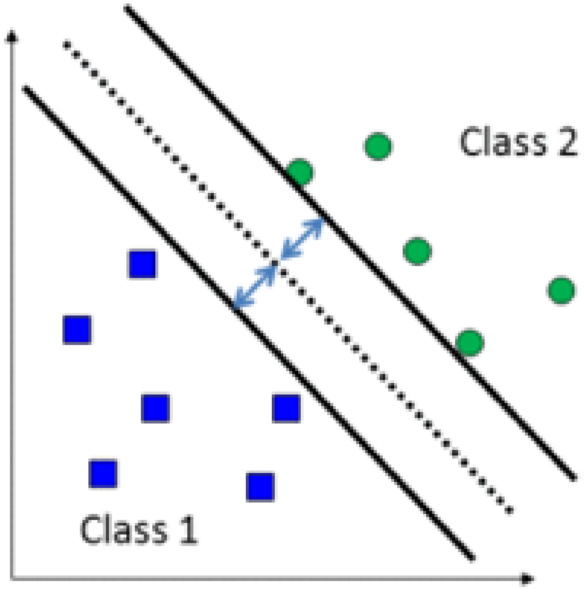
SVM

$$\max \frac{2}{\|w\|}$$

$$\text{subject to } y_i(w^T x_i - b) - 1 \geq 0 \ \forall i$$

$$\boxed{\begin{gathered} \min \frac{1}{2}\|w\|^2 \\ \text{subject to } y_i(w^T x_i - b) \geq 1 \ \forall i \end{gathered}}$$

What SVM solve in math



SVM

$$\min \frac{1}{2}\|w\|^2$$

$$\text{subject to } y_i(w^T x_i - b) \geq 1 \ \forall i$$

How to solve actually?

Please reference:
http://www.cmlab.csie.ntu.edu.tw/~cyy/learning/tutorials/SVM2.pdf

Hard Cost V.S. Soft Cost

Hard Cost

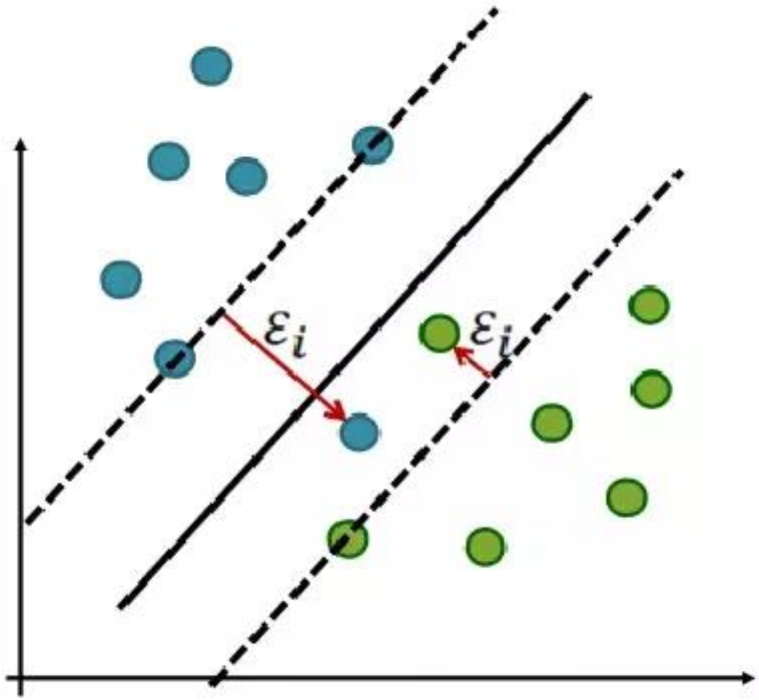
$$min \frac{1}{2} \|w\|^2$$

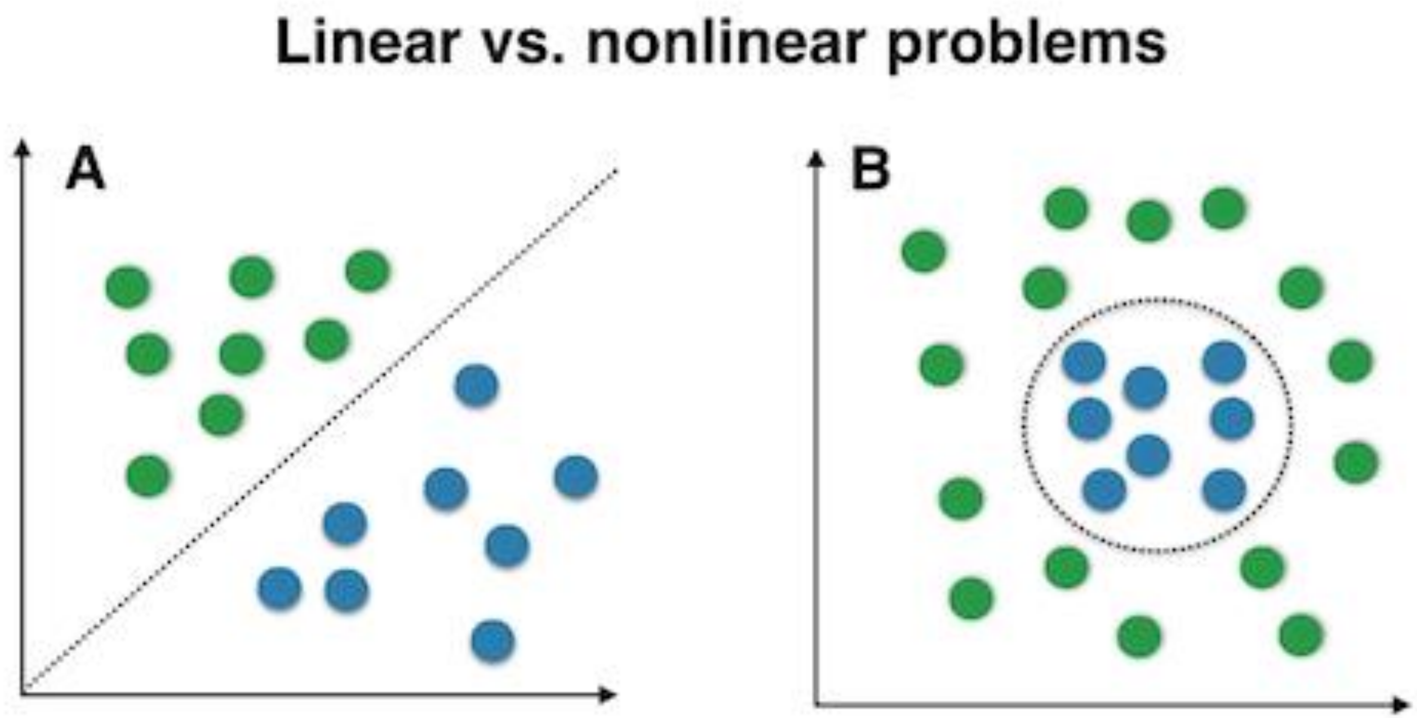
$$subject\ to\ y_i(w^T x_i - b) \geq 1\ \forall i$$

Soft Cost

$$min \frac{1}{2} \|w\|^2 + C \sum \varepsilon_i$$

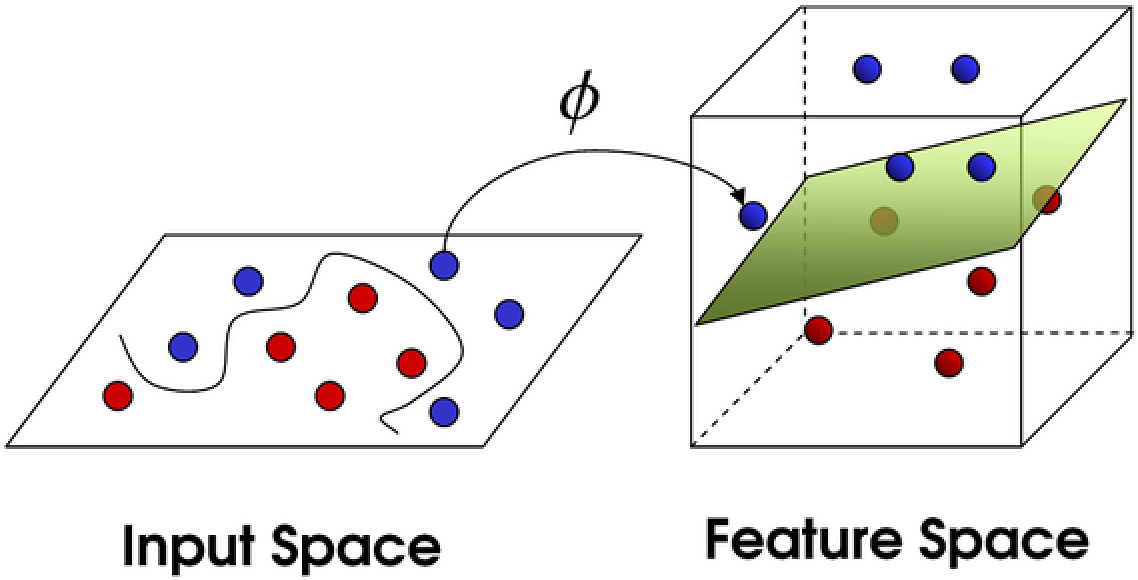
$$subject\ to\ y_i(w^T x_i - b) \geq 1 - \varepsilon_i$$
$$\varepsilon_i \geq 0\ \forall i$$




Linear vs. nonlinear problems
A
B

SVM Kernel Trick

- **Usually, data can't be linear separable**
  - **map data to higher dimension**
  - **https://www.youtube.com/watch?v=3liCbRZPrZA**



SVM Kernel Trick

$$\Phi : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} x_1^2 \\ x_2^2 \\ \sqrt{2}x_1x_2 \end{pmatrix} \qquad \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$\begin{aligned}
\boxed{\Phi(\mathbf{x})^\top \Phi(\mathbf{z})} &= \left(x_1^2, x_2^2, \sqrt{2}x_1x_2\right) \begin{pmatrix} z_1^2 \\ z_2^2 \\ \sqrt{2}z_1z_2 \end{pmatrix} \\
&= x_1^2z_1^2 + x_2^2z_2^2 + 2x_1x_2z_1z_2 \\
&= (x_1z_1 + x_2z_2)^2 \\
&= (\mathbf{x}^\top\mathbf{z})^2
\end{aligned}$$

kernel

SVM Kernel Trick

$$\min \frac{1}{2}\|w\|^2$$

$$subject\ to\ y_i(w^T x_i - b) \geq 1\ \forall i$$

primal problem

$$\max \sum_{i=1}^{m} \alpha_i - \frac{1}{2}\sum_{i,j=1}^{m} \alpha_i \alpha_j y_i y_j (x_i)^T x_j$$

$$subject\ to\ \alpha_i \geq 0\ \forall i$$

$$\sum_{i=1}^{m} \alpha_i y_i = 0$$

dual problem

SVM Kernel Trick

$$\emptyset(x_i)^T\emptyset(x_j)$$

$$max \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{m} \alpha_i \alpha_j y_i y_j \boxed{(x_i)^T x_j}$$

$$subject\ to\ \alpha_i \geq 0\ \forall i$$

$$\sum_{i=1}^{m} \alpha_i y_i = 0$$

dual problem

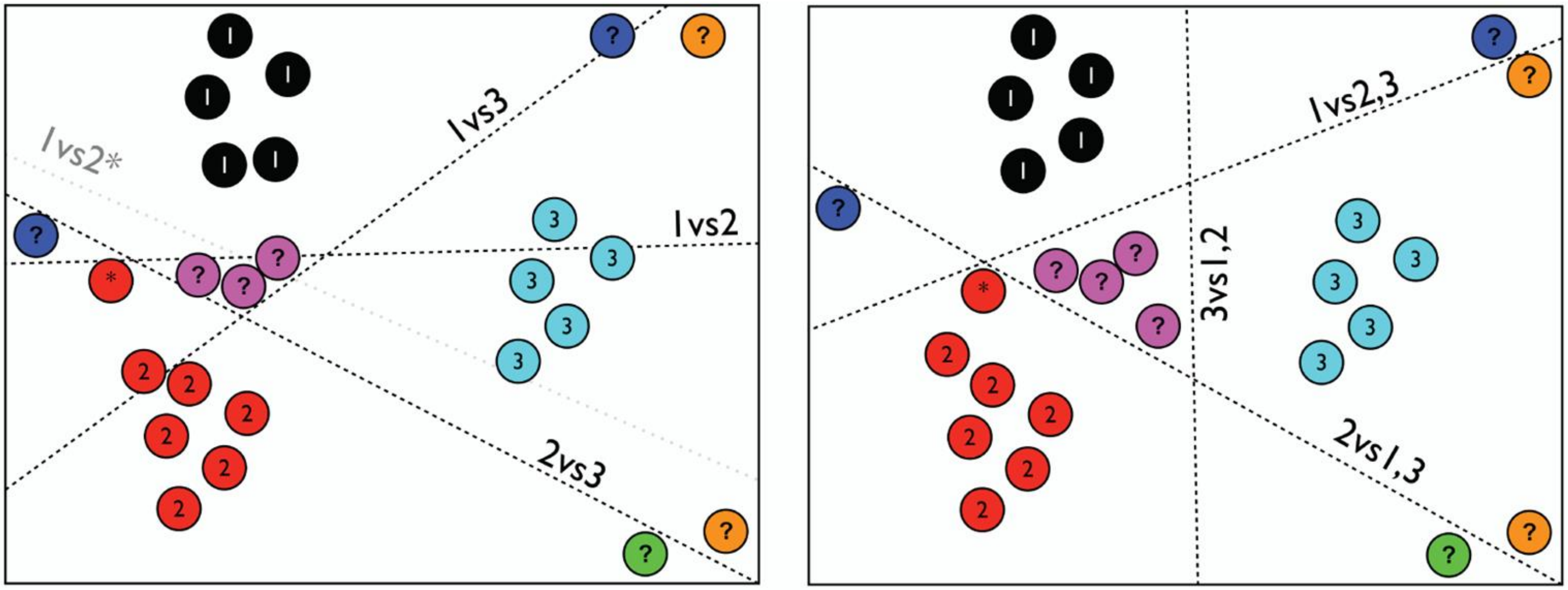
Common Kernel in SVM

Kernel name	Kernel function
Linear kernel	$K(x, y) = x \times y$
Polynomial kernel	$K(x, y) = (x \times y + 1)^d$
RBF kernel	$K(x, y) = e^{-\gamma\|x-y\|^2}$

Multi-class in SVM

- If there are k class
  - Method 1: one-against-rest(One-vs-All)
    - Make k SVM binary classifier and use m-th of binary SVM predict if the data belong to m-th class
  - Method 2: one-against-one(OvO)
    - Make $\frac{n(n-1)}{2}$ binary classifier (n is # of class) and each of binary SVM predict if the data belong to one of any two class

Multi-class in SVM



(a) 1-vs-1 (b) 1-vs-All

Example and Practice

- **Example**
  - **SVM**
    - example/supervised learning
- **Practice**
  - **Try to use random forest to predict different varieties of wheat**
    - dataset/pima-indians-diabetes.csv
    - practice/supervised learning
  - **More information about the dataset**
    - https://www.kaggle.com/uciml/pima-indians-diabetes-database/data