15-386/686 Neural Computation
Problem Set 1: Neuronal Model

Credit: 12 points. Out: 2/3, 2021.
Due 11:59 p.m. 2/17, 2021.

Part 1: Understanding membrane potential (6 points)

(a) [1 pt]

Nerst Equation: E = $58 * log_{10}\frac{C_{out}}{[C_{in}]}$
$E_K = 58 * log_{10}\frac{20}{500} = -81.1mV$
$E_{Na} = 58 * log_{10}\frac{460}{50} = 55.9mV$
$E_{Cl} = 58 * log_{10}\frac{540}{40} = 65.6mV$
E_{Na} is the closest to the observed 40 mV.

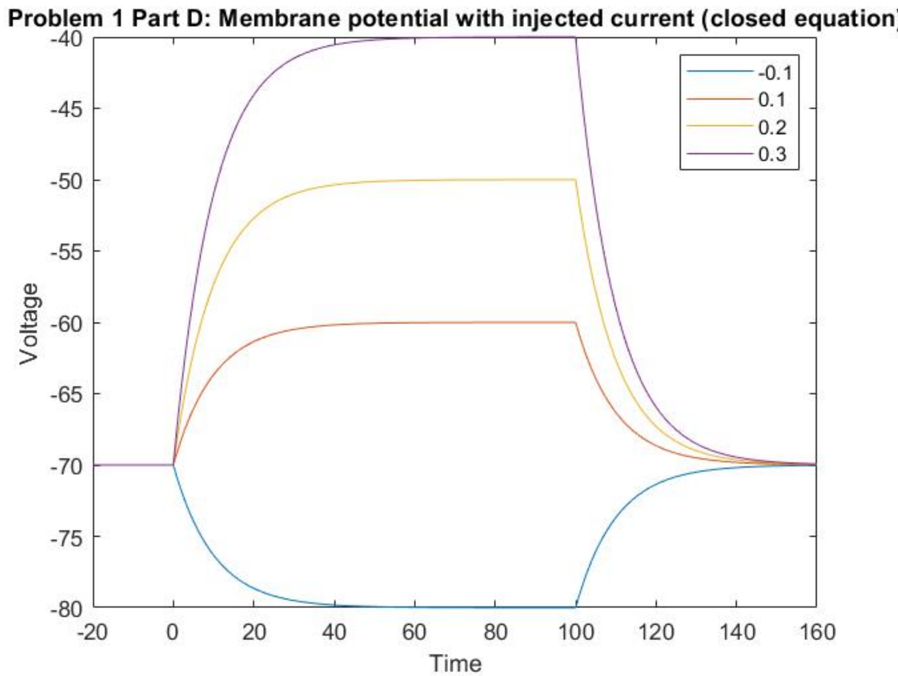
(b) [1.0 pt]

$V_m = \frac{RT}{F}log(\frac{P_K[K^+]_{out}+P_{Na}[Na^+]_{out}+P_{cl}[Cl^-]_{in}}{P_K[K^+]_{in}+P_{Na}[Na^+]_{in}+P_{cl}[Cl^-]_{out}}) = 58 * ln(\frac{1*20+0.03*460+0.1*40}{1*500+0.03*50+0.1*540}) = -67.7mV$

(c) [0.5 pt]

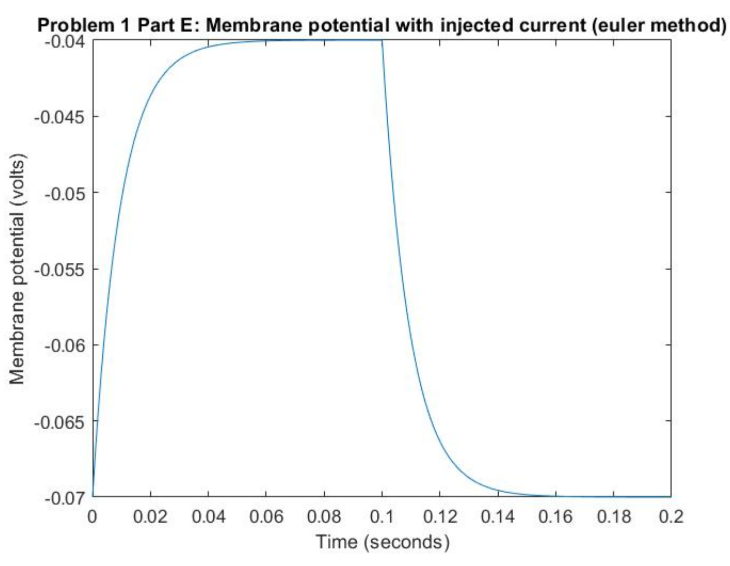
$V_m = \frac{RT}{F}log(\frac{P_K[K^+]_{in}+P_{Na}[Na^+]_{in}+P_{cl}[Cl^-]_{out}}{P_K[K^+]_{out}+P_{Na}[Na^+]_{out}+P_{cl}[Cl^-]_{in}}) = 58 * ln(\frac{1*500+0.03*50+0.1*540}{1*20+0.03*460+0.1*40}) = 67.7mV$

(d) [1 pt]



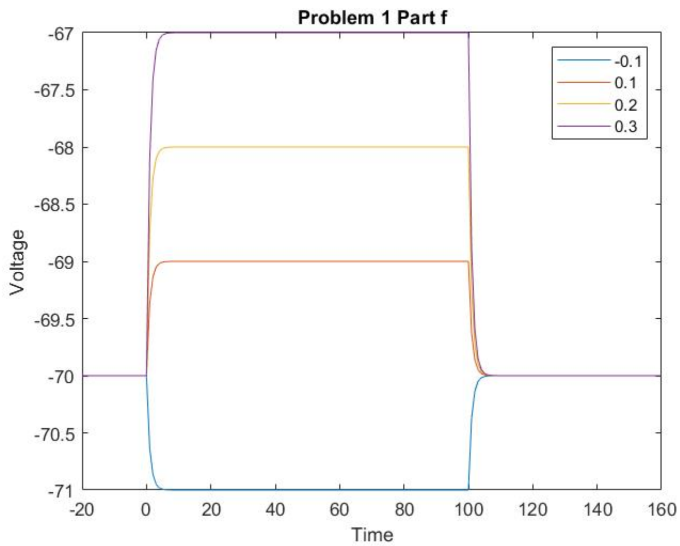
Code included in code folder.

(e) [1.5 pt]

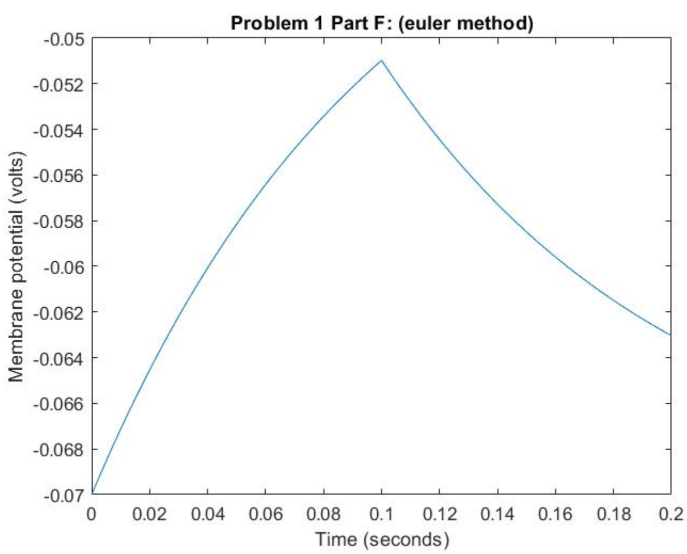


The two plots look identical in terms of shape, as well as the size if looking at first plot's 0.3 nA data.

(f) [1.0 pt]



Changed to 10 M Ohms:

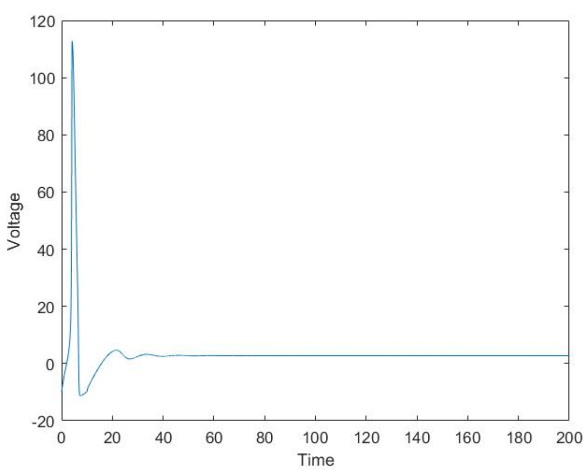


Changed to 1000 pF:

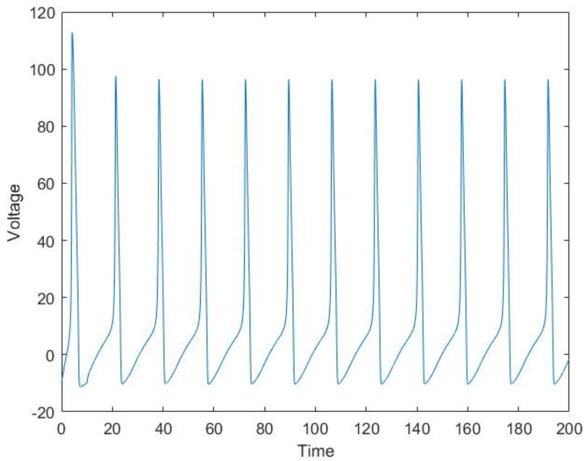
When I changed R to 10 M Ohms, the shape of the curve remained largely the same, except with a steeper rise in the beginning. This is likely due to decreased time constant (tau) with the decreased membrane resistance, causing the exponential growth to be more pronounced. This is consistant with the equation. However, in the original graph in 1d, the voltage peaks at -40 mV. In this modified graph, the voltage peaks at -67 mV when injected with the same 0.3 nA current. This trend is observed at all the different injected currents. We can conclude that when we decrease the membrane resistance, the change in membrane potential decreases as well, which follows the Ohm's Law and the equation.
When I changed C to 1000 pF, I observed first a decreased peak in membrane potential from -0.04 V to -0.051 V when injected the same amount of current. It also has a steeper rise but slower decline in voltage, unlike the original shape. Note that I used the euler method for this observation as the equation is directly dependent on C. We see that when membrane capacitance increases, the change in membrane potential decreases, which makes sense according to the equation.

Part 2: Numerical Integration of Spiking Neuron models (6 points)

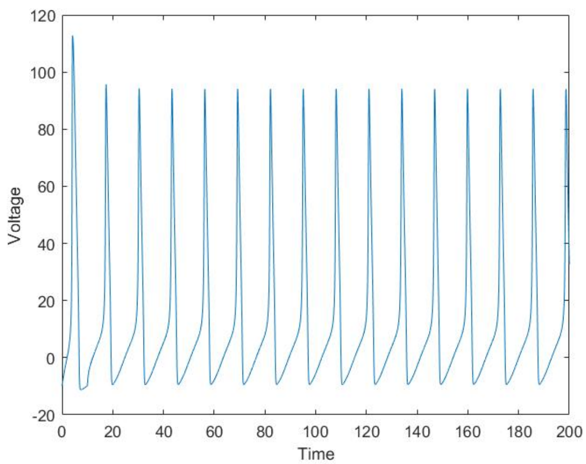
(a) [1 pt]



$I_{ext} = 4$

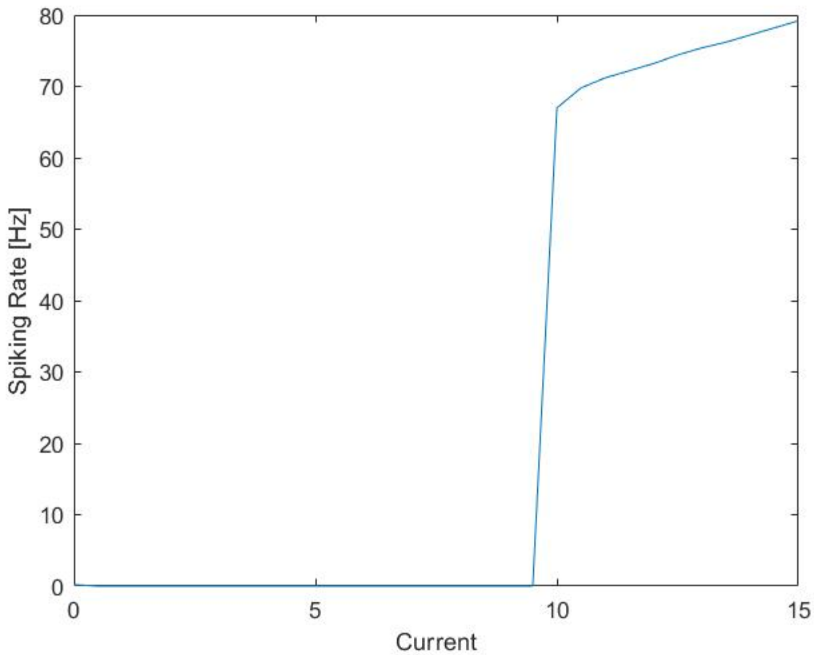


$I_{ext} = 7$



$I_{ext} = 14$

(b) [1 pt]

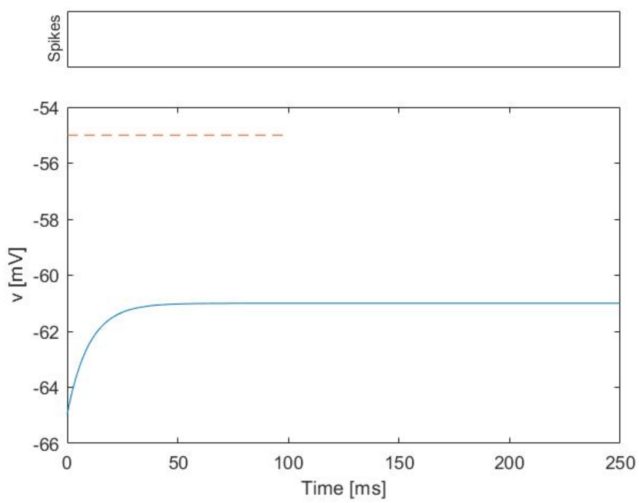


(c) [1 pt]

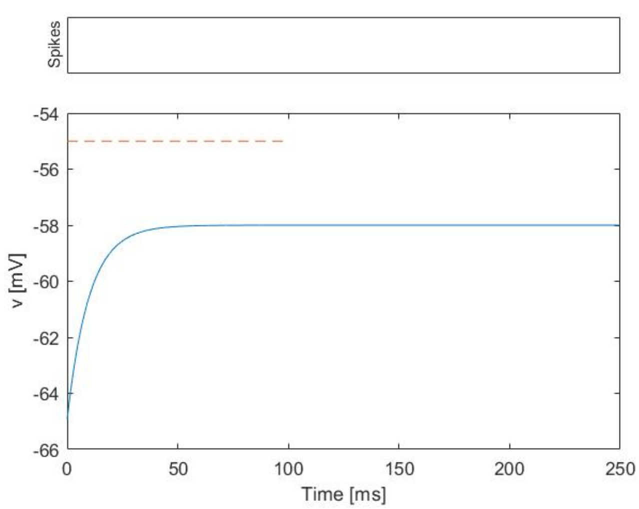
See 2b for the F-I curve. One thing to note is that the neuron does not really fire until the current is at around 9, and then it increases dramatically. This shows that there is a threshold that the neuron must reach. For the neuron to fire, the current must be strong enough to push the membrane potential above the threshold. This is because the voltage-gated channels that control the action potential only opens at a certain voltage. Without reaching this voltage, the channels will not open and an action potential will not happen. This feature is desirable because it can make sure that the neuron only responds to a strong enough signal and not to the random, weaker noises. It is necessary in order to regulate neuron firing.

The other feature to note is the linear end after the threshold is reached when the current is 9-15. From this point on, the spiking rate increases linearly as the strength of injected current increases. This implies that the stronger the signal, the more the neuron can fire. This is probably because with the larger, constant injection, the neuron stays depolarized enough for it to fire faster. This feature is also desirable because it allows for diverse methods of signally and more ways to convey information.

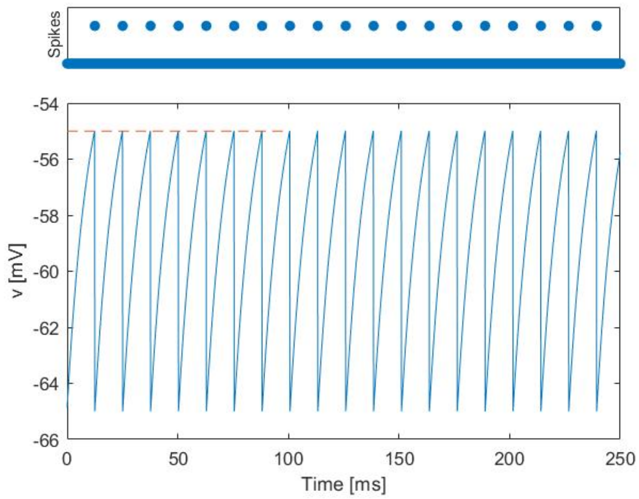
(d) [1 pt]



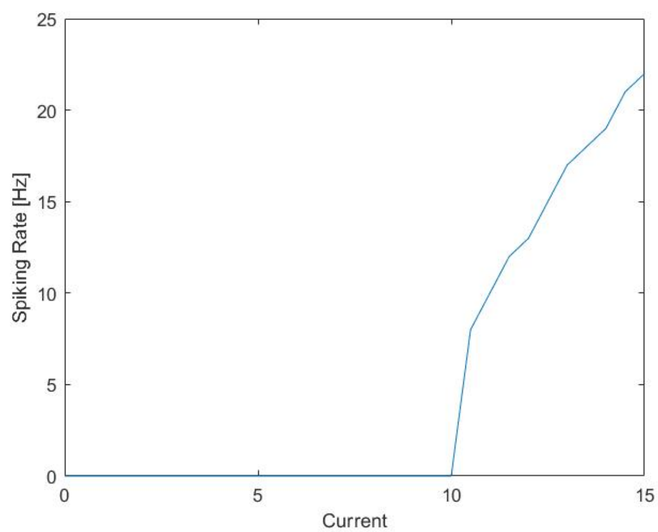
$I_{ext} = 4$



$I_{ext} = 7$



$I_{ext} = 14$



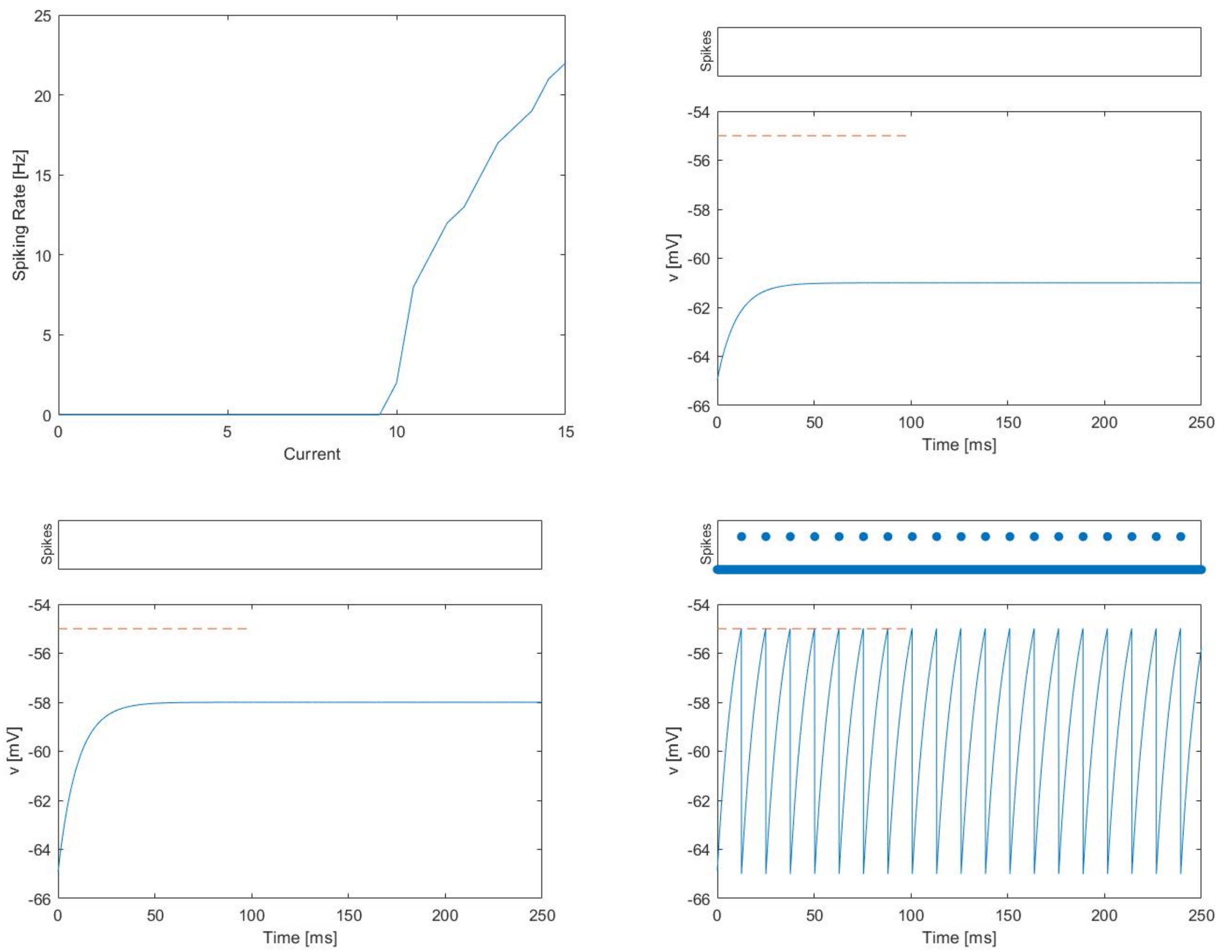
F-I Curve

See 2b for HH model's f-I curve. LIF model's f-I curve starts at a slightly higher current at 10, which means that it needs a larger current to cause the neuron to fire. It also has lower firing rates in general compared to the HH model with peaks at 22 and 80 Hz respectively. In addition, LIF model's f-I curve shows a steeper increase at higher current levels after the threshold has been reached. This implies that its firing rate is affected more by the injected current than HH model.

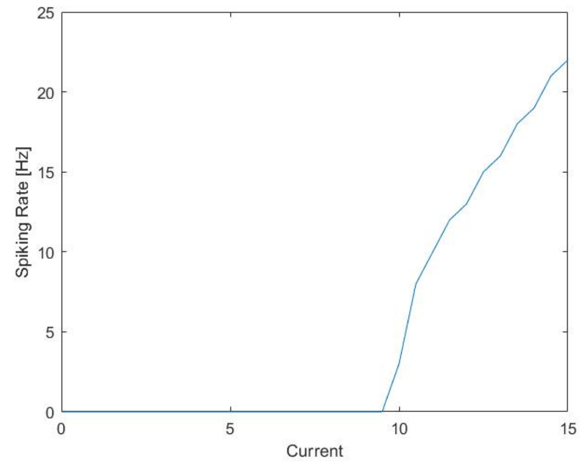
(e) [1 pt]

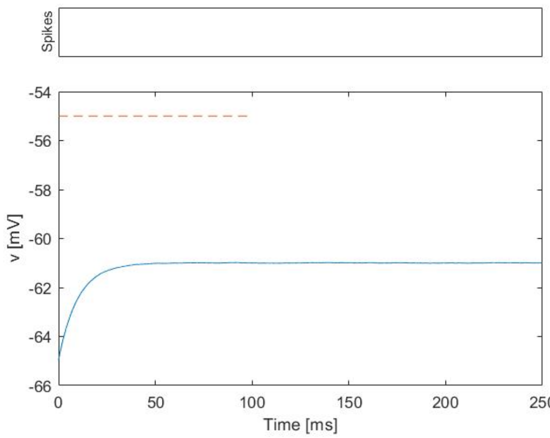
Graphs below are arranged as:
top left: F-I Curve
top right: I = 4
bottom left: I = 7
bottom right: I = 14

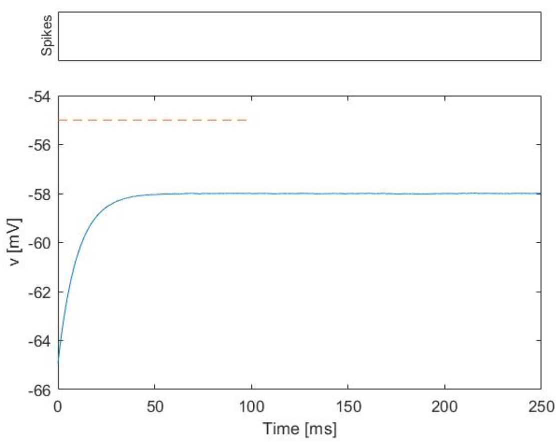
sigma = 0.01

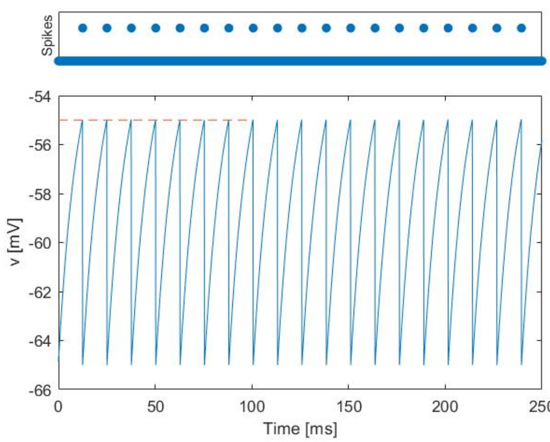


sigma = 0.1

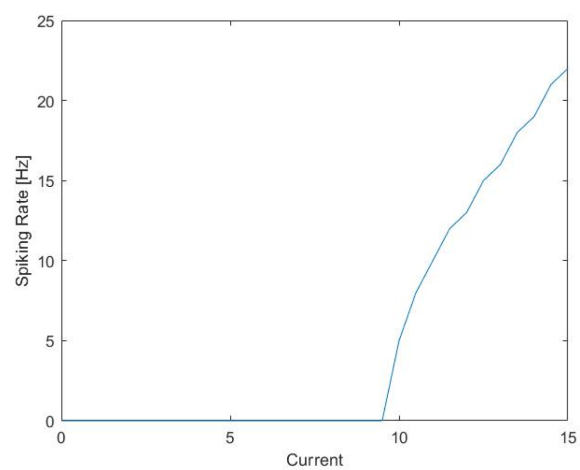

Spiking Rate [Hz]
Current
0
5
10
15
20
25

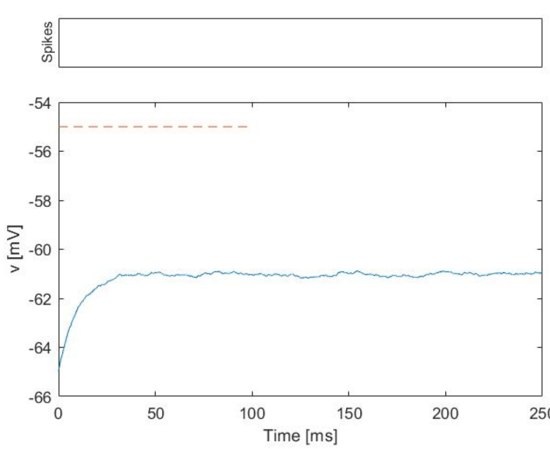

Spikes
v [mV]
Time [ms]
-54
-56
-58
-60
-62
-64
-66
0
50
100
150
200
250

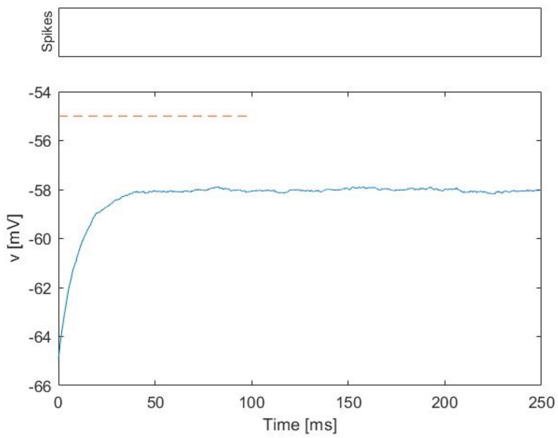

Spikes
v [mV]
Time [ms]
-54
-56
-58
-60
-62
-64
-66
0
50
100
150
200
250

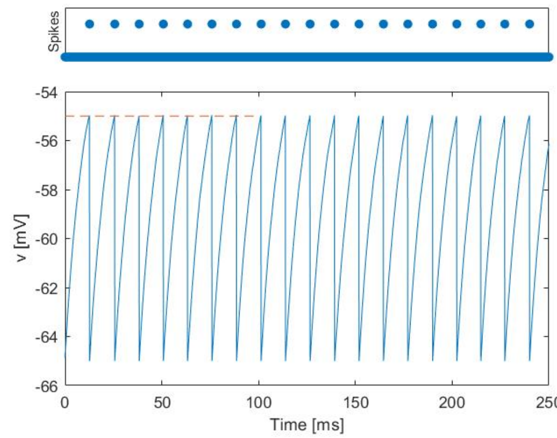

Spikes
v [mV]
Time [ms]
-54
-56
-58
-60
-62
-64
-66
0
50
100
150
200
250

sigma = 1


Spiking Rate [Hz]
Current
0
5
10
15
20
25


Spikes
v [mV]
Time [ms]
-54
-56
-58
-60
-62
-64
-66
0
50
100
150
200
250





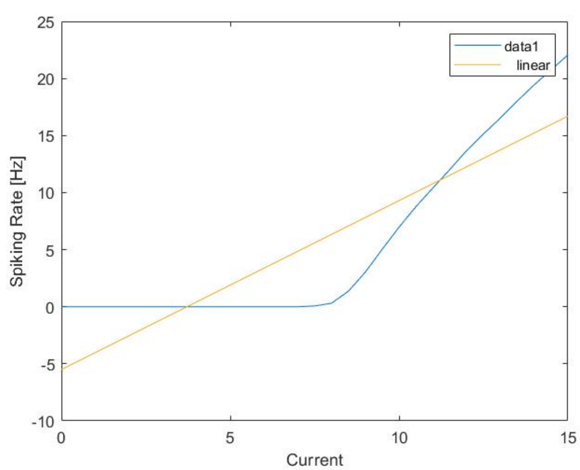
See the original f-I curve in 2d. When noise is added, the f-I curve becomes more linear compared to the original plot. As for the spike train plots, some small fluctuations are seen in all sigma values for current = 4 or 7. However, it looks like at a higher current, there isn't a visible change in spikes. As shown in the f-I curve, the firing rate increases at lower current levels and decreases at high current levels, with more drastic changes at a higher noise level. This is due to the variability and randomness added to the original deterministic model by noise that pushes the membrane potential up or down the threshold. It implies that with more noise, action potential become less of an all or nothing response with a less pronounced threshold. This is not desirable if the neuron requires precise and accurate firing, but can be desirable if the neuron needs a higher firing rate but not necessary precision.

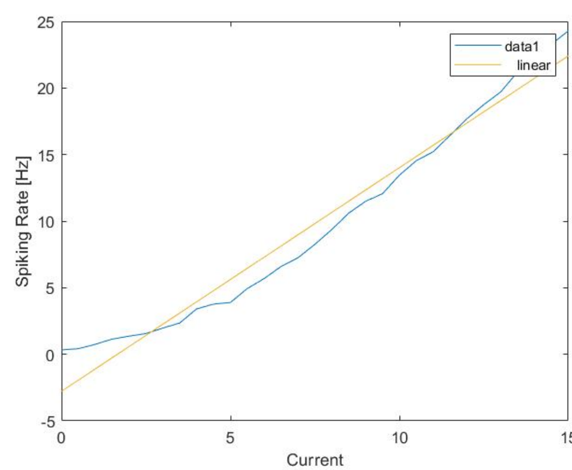
(f) [1 pt]

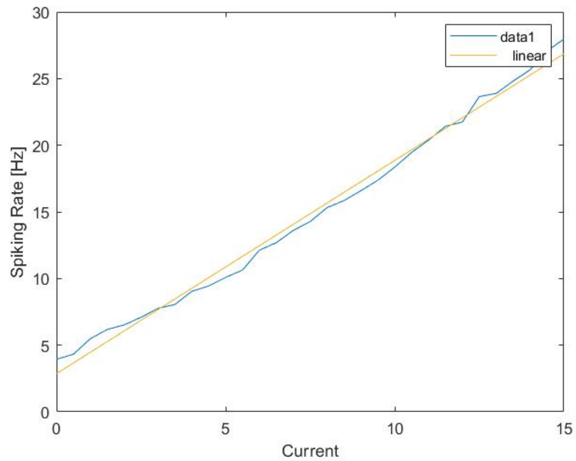
My question is how much noise will it take for the f-I curve to lose its threshold shape and becomes linear. I am looking for a minimum value of sigma over 0-15 injected current. My approach is to increment sigma by 20 to find where it gets close to being linear, then using a smaller increment. After experimenting, most graphs do not change much within 5 increments of sigma, so I am looking for the minimum of sigma (multiple of 5) that gives me a linear plot. The plots are included below.

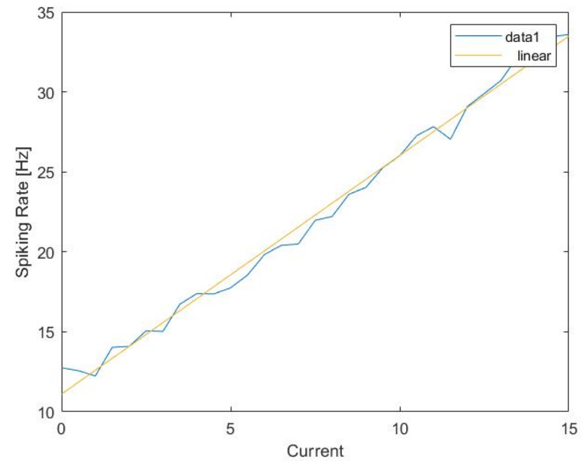
I found that the plots get very close to being linear at the sigma value of around 165-170. Further increase or decrease of sigma seems to yield equal or even worse results. I think it's interesting that it takes much more noise to bring the spiking rates at a lower current level to linear whereas at a high current level, the plot becomes linear fairly soon. This makes sense since the noise would have to reach a high enough value to reach the action potential threshold at a lower current level. In contrast, the original plot already shows increased firing rate with increased level of injected current.

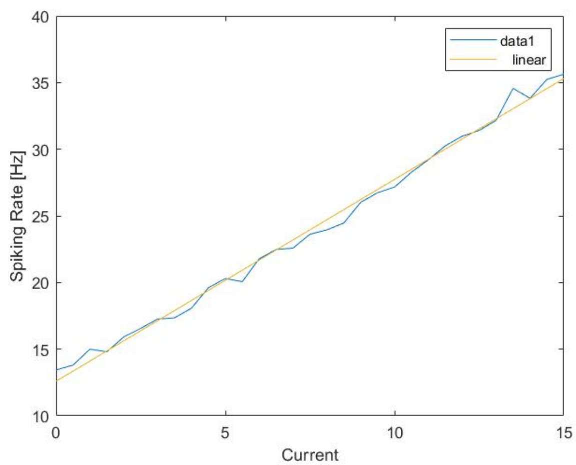
Note: not all graphs are included in the report but the ones that are closed to my conclusion value are all included. The rest are there to show a general trend. The ones shown are in order of their sigma value: 10, 50, 90, 150, 165, 170, 190.

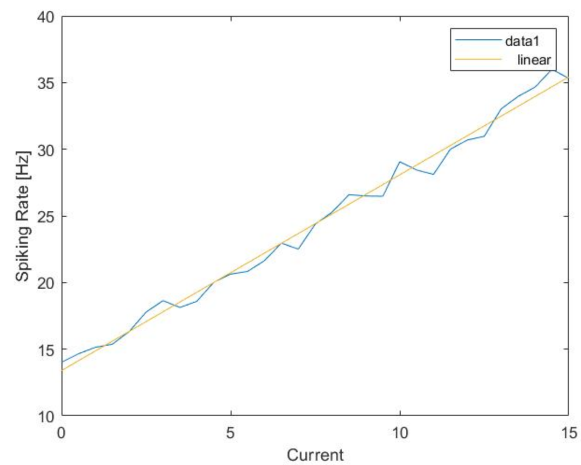


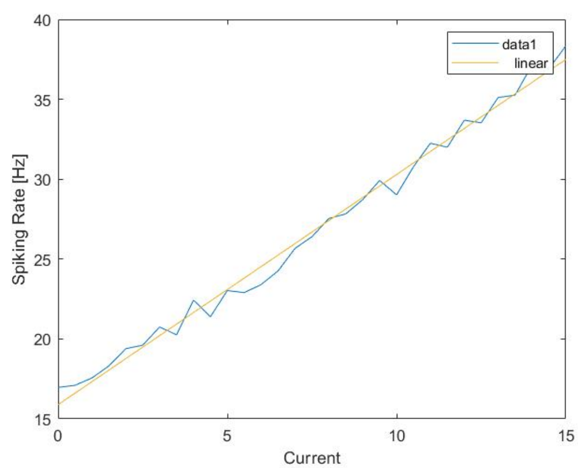



data1
linear
Spiking Rate [Hz]
Current


data1
linear
Spiking Rate [Hz]
Current


data1
linear
Spiking Rate [Hz]
Current


data1
linear
Spiking Rate [Hz]
Current


data1
linear
Spiking Rate [Hz]
Current