Destination λ-calculus

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March 11, 2024

1 Term and value syntax

var, x, y	Term-level variable name
k	Index for ranges

hdn, h	::=		Hole or destination name ($\mathbb{N}$)
hdns, H	::=		Set of hole names
	|	$\{h_1, .., h_k\}$	
term, t, u	::=		Term
	|	v	Value
	|	x	Variable
	|	$t \succ u$	Application
	|	$t\ ;\ u$	Pattern-match on unit
	|	$t \succ \mathbf{case}_m \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}$	Pattern-match on sum
	|	$t \succ \mathbf{case}_m (x_1 , x_2) \mapsto u$	Pattern-match on product
	|	$t \succ \mathbf{case}_m\ \mathrm{E}^n\, x \mapsto u$	Pattern-match on exponential
	|	$t \succ \mathbf{map}\ x \mapsto u$	Map over the right side of ampar t
	|	$\mathbf{to}_{\ltimes}\ t$	Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ t$	Extract value from trivial ampar
	|	$\mathbf{alloc}$	Return a fresh "identity" ampar object
	|	$t \lhd ()$	Fill destination with unit
	|	$t \lhd (\lambda x\ _m \mapsto u)$	Fill destination with function
	|	$t \lhd \mathsf{Inl}$	Fill destination with left variant
	|	$t \lhd \mathsf{Inr}$	Fill destination with right variant
	|	$t \lhd (,)$	Fill destination with product constructor
	|	$t \lhd \mathrm{E}^m$	Fill destination with exponential constructor
	|	$t \lhd\!\bullet\ u$	Fill destination with root of ampar u
val, v	::=		Term value
	|	$-h$	Hole
	|	$+h$	Destination
	|	$()$	Unit
	|	$\lambda^{v} x\ _m \mapsto t$	Lambda abstraction
	|	$\mathsf{Inl}\, v$	Left variant for sum
	|	$\mathsf{Inr}\, v$	Right variant for sum
	|	$\mathrm{E}^m\, v$	Exponential
	|	(v_1 , v_2)	Product
	|	$_H\langle v_1 ⹁ v_2 \rangle$	Ampar
eterm, j	::=		Pseudo-term
	|	$C[t]$	
ectx, C	::=		Evaluation context
	|	$\square$	Identity
	|	$^{op}_{H}\langle v_1 ⹁ C$	Open ampar
	|	$C \circ C'$	Compose evaluation contexts

2 Type system

type, T, U	::=		Type
	|	$\mathbf{1}$	Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	|	$!^{m}\, \mathsf{T}$	Exponential
	|	$\mathsf{T}_1 \ltimes \mathsf{T}_2$	Ampar type (consuming T_2 yields T_1)
	|	$\mathsf{T}_1\ {}_{m_1}\!\!\rightarrow \mathsf{T}_2$	Function
	|	$\lfloor \mathsf{T} \rfloor^{m}$	Destination
	|	$\mathsf{T}_1 \rightarrowtail \mathsf{T}_2$	Evaluation contexts
mode, m, n	::=		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_{\mathsf{k}}$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	$\mathit{1}$	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_{\mathsf{k}}$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_{\mathsf{k}}$	Semiring product
bndr, b	::=		Type assignment to either variable, destination or hole
	|	$\mathsf{x} :_{m} \mathsf{T}$	Variable
	|	$+\mathtt{h} :_{m} \lfloor \mathsf{T} \rfloor^{n}$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-\mathtt{h} : \mathsf{T}^{n}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)
ctx, Γ, Δ	::=		Typing context
	|	$\{\mathsf{b}_1, .., \mathsf{b}_{\mathsf{k}}\}$	List of bindings
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mo
	|	$-\Gamma$	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)

$\boxed{\Gamma \Vdash \mathsf{v} : \mathsf{T}}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \dfrac{}{\{-h : \mathsf{T}^{1a}\} \Vdash -h : \mathsf{T}}$$

$$\textsc{TyR-val-D}\quad \dfrac{\texttt{ctx\_Compatible}\ \Gamma\ \ +h :_{1\nu} \lfloor \mathsf{T} \rfloor^{n}}{\Gamma \Vdash +h : \lfloor \mathsf{T} \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \dfrac{}{\{\} \Vdash () : \mathsf{1}}$$

$$\textsc{TyR-val-F}\quad \dfrac{\begin{array}{c}\texttt{ctx\_DestOnly}\ \Gamma \\ \Gamma \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{t} : \mathsf{T}_2\end{array}}{\Gamma \Vdash \lambda^{\mathsf{v}} x_{m} \mapsto \mathsf{t} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}$$

$$\textsc{TyR-val-L}\quad \dfrac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-R}\quad \dfrac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-P}\quad \dfrac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-val-E}\quad \dfrac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{m \cdot \Gamma \Vdash \mathrm{E}^{m}\,\mathsf{v} : !^{m}\,\mathsf{T}}$$

$$\textsc{TyR-val-A}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_DestOnly}\ \Gamma_2 \uplus \Gamma_3 \\ \texttt{ctx\_DestOnly}\ \Gamma_1 \\ \Gamma_1 \uplus (-\Gamma_3) \Vdash \mathsf{v}_1 : \mathsf{T}_1 \\ \Gamma_2 \uplus \Gamma_3 \Vdash \mathsf{v}_2 : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash {}_{\texttt{hnames}(-\Gamma_3)}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$\boxed{\Gamma \vdash \mathsf{j} : \mathsf{T}}$ *(Typing of (extended) terms)*

$$\textsc{Ty-eterm-Val}\quad \dfrac{\texttt{ctx\_NoHole}\ \Gamma \qquad \Gamma \Vdash \mathsf{v} : \mathsf{T}}{\Gamma \vdash \mathsf{v} : \mathsf{T}}$$

$$\textsc{Ty-eterm-Var}\quad \dfrac{\texttt{ctx\_Compatible}\ \Gamma\ \ x :_{1\nu} \mathsf{T}}{\Gamma \vdash x : \mathsf{T}}$$

$$\textsc{Ty-eterm-App}\quad \dfrac{\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}{m \cdot \Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-eterm-PatS}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U} \\ \Gamma_2 \uplus \{x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U}\end{array}}{m \cdot \Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,\{\,\mathsf{Inl}\,x_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,x_2 \mapsto \mathsf{u}_2\,\} : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatU}\quad \dfrac{\Gamma_1 \vdash \mathsf{t} : \mathsf{1} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{U}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t}\ ;\ \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatP}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \texttt{ctx\_Disjoint}\ \ \{x_1 :_{m} \mathsf{T}_1\}\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1, x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m \cdot \Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,(x_1\,,\,x_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatE}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m \cdot n} \mathsf{T}\} \\ \Gamma_1 \vdash \mathsf{t} : !^{n}\,\mathsf{T} \\ \Gamma_2 \uplus \{x :_{m \cdot n} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m \cdot \Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-eterm-Map}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{1\nu} \mathsf{T}_2\} \\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \ltimes \mathsf{T}_2 \\ 1\!\uparrow \cdot \Gamma_2 \uplus \{x :_{1\nu} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{map}\ x \mapsto \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{Ty-eterm-FillU}\quad \dfrac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{1} \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft () : \mathsf{1}}$$

$$\textsc{Ty-eterm-FillF}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m} \mathsf{T}_1\} \\ \Gamma_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2 \rfloor^{n} \\ \Gamma_2 \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (1\!\uparrow \cdot n) \cdot \Gamma_2 \vdash \mathsf{t} \triangleleft (\lambda x_{m} \mapsto \mathsf{u}) : \mathsf{1}}$$

$$\textsc{Ty-eterm-FillL}\quad \dfrac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : \lfloor \mathsf{T}_1 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillR}\quad \dfrac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillP}\quad \dfrac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft (,) : \lfloor \mathsf{T}_1 \rfloor^{n} \otimes \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillE}\quad \dfrac{\Gamma \vdash \mathsf{t} : \lfloor !^{m}\,\mathsf{T} \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathrm{E}^{m} : \lfloor \mathsf{T} \rfloor^{m \cdot n}}$$

$$\textsc{Ty-eterm-FillC}\quad \dfrac{\Gamma_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \rfloor^{n} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Gamma_1 \uplus (1\!\uparrow \cdot n) \cdot \Gamma_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-eterm-Alloc}\quad \dfrac{}{\{\} \vdash \mathsf{alloc} : \mathsf{T} \ltimes \lfloor \mathsf{T} \rfloor^{1\nu}}$$

$$\textsc{Ty-eterm-ToA}\quad \dfrac{\Gamma \vdash \mathsf{t} : \mathsf{T}}{\Gamma \vdash \mathsf{to}_{\ltimes}\,\mathsf{t} : \mathsf{T} \ltimes \mathsf{1}}$$

$$\textsc{Ty-eterm-FromA}\quad \dfrac{\Gamma \vdash \mathsf{t} : \mathsf{T} \ltimes \mathsf{1}}{\Gamma \vdash \mathsf{from}_{\ltimes}\,\mathsf{t} : \mathsf{T}}$$

$\boxed{\Gamma \Vdash \mathsf{C} : \mathsf{T}}$ *(Typing of evaluation contexts)*

$$\textsc{TyR-ectx-T}\quad \dfrac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \Gamma_3 \\ \texttt{ctx\_NoVar}\ \Gamma_2 \uplus \Gamma_3 \\ \texttt{ctx\_NoVar}\ \Gamma_1 \\ \Gamma_2 \uplus \Gamma_3 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_1 \vdash \mathsf{C}[\mathsf{t}] : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (-\Gamma_2) \Vdash \mathsf{C} : \mathsf{T}_1 \rightarrowtail \mathsf{T}_2}$$

3 Small-step semantics

$\boxed{j \longrightarrow j'}$ *(Small-step evaluation of terms using evaluation contexts)*

$$\textsc{Sem-eterm-App}\quad \frac{}{C[v \succ (\lambda^{v} x_{m} \mapsto t)] \longrightarrow C[t[x := v]]}$$

$$\textsc{Sem-eterm-PatU}\quad \frac{}{C[() \;;\; t_2] \longrightarrow C[t_2]}$$

$$\textsc{Sem-eterm-PatL}\quad \frac{}{C[(\mathsf{Inl}\, v) \succ \mathbf{case}_{m}\,\{\, \mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\}] \longrightarrow C[u_1[x := v]]}$$

$$\textsc{Sem-eterm-PatR}\quad \frac{}{C[(\mathsf{Inr}\, v) \succ \mathbf{case}_{m}\,\{\, \mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\}] \longrightarrow C[u_2[x := v]]}$$

$$\textsc{Sem-eterm-PatP}\quad \frac{}{C[(v_1\,,\, v_2) \succ \mathbf{case}_{m}\,(x_1\,,\, x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]}$$

$$\textsc{Sem-eterm-PatE}\quad \frac{}{C[\mathrm{E}^{n}\, v \succ \mathbf{case}_{m}\, \mathrm{E}^{n}\, x \mapsto u] \longrightarrow C[u[x := v]]}$$

$$\textsc{Sem-eterm-MapOpen}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C))}{C[{}_{\mathtt{H}}\langle v_1 \,,\, v_2\rangle \succ \mathbf{map}\; x \mapsto u] \longrightarrow (C \circ ({}^{o}_{\mathtt{H}\stackrel{\pm}{=}h'}\langle v_1 \stackrel{\pm}{=} h' \,,\, \square\rangle))[u[x := v_2 \stackrel{\pm}{=} h']]}$$

$$\textsc{Sem-eterm-MapClose}\quad \frac{}{(C \circ {}^{o}_{\mathtt{H}}\langle v_1 \,,\, \square\rangle)[v_2] \longrightarrow C[{}_{\mathtt{H}}\langle v_1 \,,\, v_2\rangle]}$$

$$\textsc{Sem-eterm-Alloc}\quad \frac{}{\mathbf{alloc} \longrightarrow {}_{\{1\}}\langle +1 \,,\, -1\rangle}$$

$$\textsc{Sem-eterm-ToA}\quad \frac{}{C[\mathbf{to}_{\ltimes}\; v] \longrightarrow C[{}_{\{\,\}}\langle v \,,\, ()\rangle]}$$

$$\textsc{Sem-eterm-FromA}\quad \frac{}{C[\mathbf{from}_{\ltimes}\; {}_{\{\,\}}\langle v \,,\, ()\rangle] \longrightarrow v}$$

$$\textsc{Sem-eterm-FillU}\quad \frac{}{C[+h \triangleleft ()] \longrightarrow C[h :=_{\{\,\}}\; ()][()]}$$

$$\textsc{Sem-eterm-FillF}\quad \frac{}{C[+h \triangleleft (\lambda x_{m} \mapsto u)] \longrightarrow C[h :=_{\{\,\}}\; \lambda^{v} x_{m} \mapsto u][()]}$$

$$\textsc{Sem-eterm-FillL}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inl}] \longrightarrow C[h :=_{\{h'+1\}}\; \mathsf{Inl}\; -(h'+1)][+(h'+1)]}$$

$$\textsc{Sem-eterm-FillR}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inr}] \longrightarrow C[h :=_{\{h'+1\}}\; \mathsf{Inr}\; -(h'+1)][+(h'+1)]}$$

$$\textsc{Sem-eterm-FillE}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathrm{E}^{m}] \longrightarrow C[h :=_{\{h'+1\}}\; \mathrm{E}^{m}\; -(h'+1)][+(h'+1)]}$$

$$\textsc{Sem-eterm-FillP}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft (,)] \longrightarrow C[h :=_{\{h'+1,h'+2\}}\; (-(h'+1)\,,\, -(h'+2))][(+(h'+1)\,,\, +(h'+2))]}$$

$$\textsc{Sem-eterm-FillC}\quad \frac{h' \;=\; \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft\!\bullet\; {}_{\mathtt{H}}\langle v_1 \,,\, v_2\rangle] \longrightarrow C[h :=_{(\mathtt{H}\stackrel{\pm}{=}h')}\; v_1 \stackrel{\pm}{=} h'][v_2 \stackrel{\pm}{=} h']}$$