




termvar , x, y, d	Term-level variable	
holevar , h	Hole	
term_value , v	$::=$	Term value
	$\langle v_1, \overline{v_2} \rangle_H$	Ampar
	$@h$	Destination
	$()$	Unit
	$\text{Inl } v$	Left variant for sum
	$\text{Inr } v$	Right variant for sum
	(v_1, v_2)	Product
	$\rangle^m v$	Exponential
	$\lambda x. t$	Linear function
	(v)	S
$\overline{\text{extended_value}}$, \bar{v}	$::=$	Store value
	v	Term value
	h	Hole
	$\text{Inl } \bar{v}$	Left variant with val or hole
	$\text{Inr } \bar{v}$	Right variant with val or hole
	(\bar{v}_1, \bar{v}_2)	Product with val or hole
	$\rangle^m \bar{v}$	Exponential with val or hole
	(\bar{v})	S
	$\bar{v}[e]$	M
term , t, u	$::=$	Term
	v	Term value
	x	Variable
	$t \succ u$	Application
	$t \succ \text{case } () \mapsto u$	Pattern-match on unit
	$t \succ \text{case } \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$	Pattern-match on sum
	$t \succ \text{case } (x_1, x_2) \mapsto u$	Pattern-match on product
	$t \succ \text{case } \rangle^m x \mapsto u$	Pattern-match on exponential
	$t \succ \text{mapL } x \mapsto u$	Map over the left side of the ampar
	$\text{to}_x t$	Wrap t into a trivial ampar
	$\text{from}_x t$	Extract value from trivial ampar
	alloc_A	Return a fresh "identity" ampar object
	$t \triangleleft ()$	Fill destination with unit
	$t \triangleleft \text{Inl}$	Fill destination with left variant
	$t \triangleleft \text{Inr}$	Fill destination with right variant
	$t \triangleleft (,)$	Fill destination with product constructor
	$t \triangleleft \rangle^m$	Fill destination with exponential constructor
	$t \triangleleft \bullet u$	Fill destination with root of ampar u
	(t)	S
	$t[\text{sub}]$	M
sub	$::=$	Variable substitution
	$x := v$	
	$\text{sub}_1, \text{sub}_2$	
	sub	S
effect , e	$::=$	Effect
	ε	No effect
	$h := \bar{v}$	
	$e_1 \cdot e_2$	
	e	S

type, A, B	$::=$ $ $ 1 $ $ $A_1 \oplus A_2$ $ $ $A_1 \otimes A_2$ $ $ $!^m A$ $ $ $A_1 \ltimes A_2$ $ $ $A_1 \xrightarrow{m} A_2$ $ $ $^m[A]$ $ $ (A)	Type Unit Sum Product Exponential Ampar type (consuming A_1 yields A_2) Linear function Destination S
<i>multiplicity, m, n</i>	$::=$ $ $ ν $ $ \uparrow $ $ ∞ $ $ $m_1 \cdot m_2$ $ $ (m)	Multiplicity (Semiring with product \cdot) Born now. Identity of the product One scope older Infinitely old / static. Absorbing for product Semiring product S
typing_context, Δ	$::=$ $ $ Γ $ $ H $ $ $\Gamma \sqcup H$	Typing context
pos_context, Γ	$::=$ $ $ \emptyset $ $ $\{\text{pos_assigns}\}$ $ $ $\Gamma_1 \sqcup \Gamma_2$ $ $ $@H$ $ $ $^m \Gamma$ $ $ (Γ) $ $ $—$	Positive typing context Inverse the sign of the context Increase age of bindings by m S M
pos_assign, pa	$::=$ $ $ $x :^m A$ $ $ $@h :^m {}^n[A]$	Positive type assignment Variable Destination (m is its own age; n is the age of values it accepts)
pos_assigns	$::=$ $ $ pa $ $ $pa, \text{pos_assigns}$	Positive type assignments
neg_assign, na	$::=$ $ $ $h :^n A$	Negative type assignment Hole (n is the age of values it accepts, its own age is undefined)
neg_assigns	$::=$ $ $ na $ $ $na, \text{neg_assigns}$	Negative type assignments
neg_context, H	$::=$ $ $ \emptyset $ $ $\{\text{neg_assigns}\}$ $ $ $H_1 \sqcup H_2$ $ $ $@^{-1} \Gamma$ $ $ $^m H$ $ $ (H)	Negative typing context Inverse the sign of the context Increase age of bindings by m S

		—	M
eff_app	::=		Effect application
		e, \bar{v}_H	
		apply (eff_app)	
		$e \hat{=} \text{eff_app}$	
terminals	::=		
			
			
		\mapsto	
		$()$	
		Inl	
		Inr	
		$(,)$	
		\triangleleft	
			
		$:=$	
		\cdot	
		\sqcup	
		\emptyset	
		\exists	
		\neq	
		\leq	
		\in	
		\notin	
		\subset	
		\vdash	
		\Vdash	
		$ $	
		\Downarrow	
formula	::=		
		judgement	
Ctx	::=		
		$x \in \text{names}(\Delta)$	
		$h \in \text{names}(\Delta)$	
		$x \notin \text{names}(\Delta)$	
		$h \notin \text{names}(\Delta)$	
		fresh x	
		fresh h	
		$\text{pos_assign} \in \Gamma$	
		$\text{neg_assign} \in H$	
		onlyPositive (Δ)	
		onlyNegative (Δ)	
Eq	::=		
		$A_1 = A_2$	
		$A_1 \neq A_2$	
		$t = u$	
		$t \neq u$	
		$\Delta_1 = \Delta_2$	

		$\text{names}(\Delta_1) \cap \text{names}(\Delta_2) = \emptyset$	
Ty	::=	$\Delta \Vdash \bar{v} : \mathbf{A}$ $\Gamma \vdash t : \mathbf{A}$	
Sem	::=	$\text{eff_app}_1 = \text{eff_app}_2$ $t \Downarrow v \mid e$	(we assume effect lists are ε -terminated)
judgement	::=	Ctx Eq Ty Sem	
user_syntax	::=	termvar holevar $\frac{\text{term_value}}{\text{extended_value}}$ term sub effect type <i>multiplicity</i> typing_context pos_context pos_assign pos_assigns neg_assign neg_assigns neg_context eff_app terminals	

$x \in \text{names}(\Delta)$
$h \in \text{names}(\Delta)$
$x \notin \text{names}(\Delta)$
$h \notin \text{names}(\Delta)$
fresh x
fresh h
$\text{pos_assign} \in \Gamma$
$\text{neg_assign} \in H$
onlyPositive(Δ)
onlyNegative(Δ)
$A_1 = A_2$
$A_1 \neq A_2$
$t = u$
$t \neq u$
$\Delta_1 = \Delta_2$

$$\text{names}(\Delta_1) \cap \text{names}(\Delta_2) = \emptyset$$

$$\Delta \Vdash \bar{v} : \mathbf{A}$$

$$\frac{}{\emptyset \sqcup \{\mathbf{h} : \nu \mathbf{A}\} \Vdash \mathbf{h} : \mathbf{A}} \text{TYVALEXT_HOLE}$$

$$\frac{}{\{\textcircled{\mathbf{h}} : \nu \mathbf{A}\} \sqcup \emptyset \Vdash \textcircled{\mathbf{h}} : \mathbf{A}} \text{TYVALEXT_DEST}$$

$$\frac{}{\emptyset \sqcup \emptyset \Vdash () : \mathbf{1}} \text{TYVALEXT_UNIT}$$

$$\frac{\Gamma \sqcup \mathbf{H} \Vdash \bar{v} : \mathbf{A}_1}{\Gamma \sqcup \mathbf{H} \Vdash \text{Inl } \bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2} \text{TYVALEXT_INL}$$

$$\frac{\Gamma \sqcup \mathbf{H} \Vdash \bar{v} : \mathbf{A}_2}{\Gamma \sqcup \mathbf{H} \Vdash \text{Inr } \bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2} \text{TYVALEXT_INR}$$

$$\frac{\begin{array}{l} \Gamma_1 \sqcup \mathbf{H}_1 \Vdash \bar{v}_1 : \mathbf{A}_1 \\ \Gamma_2 \sqcup \mathbf{H}_2 \Vdash \bar{v}_2 : \mathbf{A}_2 \\ \text{names}(\Gamma_1 \sqcup \mathbf{H}_1) \cap \text{names}(\Gamma_2 \sqcup \mathbf{H}_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \mathbf{H}_1 \sqcup \mathbf{H}_2 \Vdash (\bar{v}_1, \bar{v}_2) : \mathbf{A}_1 \otimes \mathbf{A}_2} \text{TYVALEXT_PROD}$$

$$\frac{\Gamma \sqcup \mathbf{H} \Vdash \bar{v} : \mathbf{A}}{\mathbf{m} \cdot \Gamma \sqcup \mathbf{m} \cdot \mathbf{H} \Vdash \mathbf{!}^{\mathbf{m}} \bar{v} : \mathbf{!}^{\mathbf{m}} \mathbf{A}} \text{TYVALEXT_EXP}$$

$$\frac{\begin{array}{l} \textcircled{\mathbf{H}} \sqcup \emptyset \Vdash v_1 : \mathbf{A}_1 \\ \Gamma_2 \sqcup \mathbf{H} \Vdash \bar{v}_2 : \mathbf{A}_2 \end{array}}{\Gamma_2 \sqcup \emptyset \Vdash \langle v_1, \bar{v}_2 \rangle_{\mathbf{H}} : \mathbf{A}_1 \ltimes \mathbf{A}_2} \text{TYVALEXT_AMPAR}$$

$$\frac{\Gamma \sqcup \{\mathbf{x} : \mathbf{m} \mathbf{A}_1\} \vdash \mathbf{t} : \mathbf{A}_2}{\Gamma \sqcup \emptyset \Vdash \lambda \mathbf{x} . \mathbf{t} : \mathbf{A}_1 \multimap \mathbf{A}_2} \text{TYVALEXT_LAMBDA}$$

$$\Gamma \vdash \mathbf{t} : \mathbf{A}$$

$$\frac{\Gamma \sqcup \emptyset \Vdash v : \mathbf{A}}{\Gamma \vdash v : \mathbf{A}} \text{TYTERM_VAL}$$

$$\frac{}{\{\mathbf{x} : \nu \mathbf{A}\} \vdash \mathbf{x} : \mathbf{A}} \text{TYTERM_VARNOW}$$

$$\frac{}{\{\mathbf{x} : \infty \mathbf{A}\} \vdash \mathbf{x} : \mathbf{A}} \text{TYTERM_VARINF}$$

$$\frac{\begin{array}{l} \Gamma_1 \vdash \mathbf{t} : \mathbf{A}_1 \\ \Gamma_2 \vdash \mathbf{u} : \mathbf{A}_1 \multimap \mathbf{A}_2 \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \end{array}}{\mathbf{m} \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathbf{t} \succ \mathbf{u} : \mathbf{A}_2} \text{TYTERM_APP}$$

$$\frac{\begin{array}{l} \Gamma_1 \vdash \mathbf{t} : \mathbf{1} \\ \Gamma_2 \vdash \mathbf{u} : \mathbf{B} \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash \mathbf{t} \succ \text{case } () \mapsto \mathbf{u} : \mathbf{B}} \text{TYTERM_PATUNIT}$$

$$\frac{\begin{array}{l} \Gamma_1 \vdash \mathbf{t} : \mathbf{A}_1 \oplus \mathbf{A}_2 \\ \Gamma_2 \sqcup \{\mathbf{x}_1 : \mathbf{m} \mathbf{A}_1\} \vdash \mathbf{u}_1 : \mathbf{B} \\ \Gamma_2 \sqcup \{\mathbf{x}_2 : \mathbf{m} \mathbf{A}_2\} \vdash \mathbf{u}_2 : \mathbf{B} \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \end{array}}{\mathbf{m} \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathbf{t} \succ \text{case } \{\text{Inl } \mathbf{x}_1 \mapsto \mathbf{u}_1, \text{Inr } \mathbf{x}_2 \mapsto \mathbf{u}_2\} : \mathbf{B}} \text{TYTERM_PATSUM}$$

$$\frac{\begin{array}{l} \Gamma_1 \vdash \mathbf{t} : \mathbf{A}_1 \otimes \mathbf{A}_2 \\ \Gamma_2 \sqcup \{\mathbf{x}_1 : \mathbf{m} \mathbf{A}_1, \mathbf{x}_2 : \mathbf{m} \mathbf{A}_2\} \vdash \mathbf{u} : \mathbf{B} \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \end{array}}{\mathbf{m} \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathbf{t} \succ \text{case } (\mathbf{x}_1, \mathbf{x}_2) \mapsto \mathbf{u} : \mathbf{B}} \text{TYTERM_PATPROD}$$

$$\begin{array}{c}
\frac{\Gamma_1 \vdash t : !^{m'} A \quad \Gamma_2 \sqcup \{x : m \cdot m' A_1\} \vdash u : B \quad \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{case} \rangle^{m'} x \mapsto u : B} \text{TYTERM_PATEXP} \\
\\
\frac{\Gamma_1 \vdash t : A_1 \times A_2 \quad \uparrow \Gamma_2 \sqcup \{x : \nu A_1\} \vdash u : B \quad \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{mapL} x \mapsto u : B \times A_2} \text{TYTERM_MAPAMPAR} \\
\\
\frac{\Gamma_1 \vdash t : m[A_2] \quad \Gamma_2 \vdash u : A_1 \times A_2 \quad \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup (\uparrow \cdot m) \cdot \Gamma_2 \vdash t \triangleleft \bullet u : A_1} \text{TYTERM_FILLCOMP} \\
\\
\frac{\Gamma \vdash t : m[1]}{\Gamma \vdash t \triangleleft () : 1} \text{TYTERM_FILLUNIT} \\
\\
\frac{\Gamma \vdash t : m[A_1 \oplus A_2]}{\Gamma \vdash t \triangleleft \text{Inl} : m[A_1]} \text{TYTERM_FILLINL} \\
\\
\frac{\Gamma \vdash t : m[A_1 \oplus A_2]}{\Gamma \vdash t \triangleleft \text{Inr} : m[A_2]} \text{TYTERM_FILLINR} \\
\\
\frac{\Gamma \vdash t : m[A_1 \otimes A_2]}{\Gamma \vdash t \triangleleft (,) : m[A_1] \otimes m[A_2]} \text{TYTERM_FILLPROD} \\
\\
\frac{\Gamma \vdash t : m[!^{m'} A]}{\Gamma \vdash t \triangleleft \rangle^{m'} : m \cdot m' [A]} \text{TYTERM_FILLEXP} \\
\\
\frac{}{\emptyset \vdash \text{alloc}_A : \nu[A] \times A} \text{TYTERM_ALLOC} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash \text{to}_{\times} t : 1 \times A} \text{TYTERM_TOAMPAR} \\
\\
\frac{\Gamma \vdash t : 1 \times A}{\Gamma \vdash \text{from}_{\times} t : A} \text{TYTERM_FROMAMPAR}
\end{array}$$

$\text{eff_app}_1 = \text{eff_app}_2$

(we assume effect lists are ε -terminated)

$$\begin{array}{c}
\frac{}{\text{apply}(\varepsilon, \bar{v}_H) = \varepsilon, \bar{v}_H} \text{EFFAPP_NOEFF} \\
\\
\frac{\mathbf{h} \notin \text{names}(H)}{\text{apply}(\mathbf{h} := \bar{v}_2 \cdot e, \bar{v}_1 H) = \mathbf{h} := \bar{v}_2 \hat{\cdot} \text{apply}(e, \bar{v}_1 H)} \text{EFFAPP_SKIP} \\
\\
\frac{_ \sqcup H' \Vdash \bar{v}_2 : A \quad \text{names}(H \sqcup \{\mathbf{h} : m A\}) \cap \text{names}(H') = \emptyset}{\text{apply}(\mathbf{h} := \bar{v}_2 \cdot e, \bar{v}_1 H \sqcup \{\mathbf{h} : m A\}) = \text{apply}(e, \bar{v}_1 [\mathbf{h} := \bar{v}_2] H \sqcup m \cdot H')} \text{EFFAPP_FILLCOMP} \quad (\text{Encompasses all other FILL rules}) \\
\\
\boxed{t \Downarrow v \mid e}
\end{array}$$

$$\begin{array}{c}
\frac{}{v \Downarrow v \mid \varepsilon} \text{BIGSTEP_VAL} \\
\\
\frac{t_1 \Downarrow v_1 \mid e_1 \quad t_2 \Downarrow \lambda x \cdot u \mid e_2 \quad u[x := v_1] \Downarrow v_3 \mid e_3}{t_1 \succ t_2 \Downarrow v_3 \mid e_1 \cdot e_2 \cdot e_3} \text{BIGSTEP_APP} \\
\\
\frac{t_1 \Downarrow () \mid e_1 \quad t_2 \Downarrow v_2 \mid e_2}{t_1 \succ \text{case} () \mapsto t_2 \Downarrow v_2 \mid e_1 \cdot e_2} \text{BIGSTEP_PATUNIT}
\end{array}$$

$$\begin{array}{c}
\frac{
\begin{array}{c}
t \Downarrow \text{Inl } v_1 \mid e_1 \\
u_1[x_1 := v_1] \Downarrow v_2 \mid e_2
\end{array}
}{t \succ \text{case} \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \} \Downarrow v_2 \mid e_1 \cdot e_2} \text{BIGSTEP_PATINL} \\
\\
\frac{
\begin{array}{c}
t \Downarrow \text{Inr } v_1 \mid e_1 \\
u_2[x_2 := v_1] \Downarrow v_2 \mid e_2
\end{array}
}{t \succ \text{case} \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \} \Downarrow v_2 \mid e_1 \cdot e_2} \text{BIGSTEP_PATINR} \\
\\
\frac{
\begin{array}{c}
t \Downarrow (v_1, v_2) \mid e_1 \\
u[x_1 := v_1, x_2 := v_2] \Downarrow v_2 \mid e_2
\end{array}
}{t \succ \text{case} (x_1, x_2) \mapsto u \Downarrow v_2 \mid e_1 \cdot e_2} \text{BIGSTEP_PATPROD} \\
\\
\frac{
\begin{array}{c}
t \Downarrow \langle v_1, \overline{v_2} \rangle_H \mid e_1 \\
u[x := v_1] \Downarrow v_3 \mid e_2 \\
e_3, \overline{v_4}_{H'} = \text{apply}(e_2, \overline{v_2}_H)
\end{array}
}{t \succ \text{mapL } x \mapsto u \Downarrow \langle v_3, \overline{v_4} \rangle_{H'} \mid e_1 \cdot e_3} \text{BIGSTEP_MAPAMPAR} \\
\\
\frac{
\text{fresh } h \\
\text{alloc}_A \Downarrow \langle @h, h \rangle_{\{h:\nu A\}} \mid \varepsilon
}{\text{BIGSTEP_ALLOC}} \\
\\
\frac{
t \Downarrow v \mid e
}{\text{to}_x t \Downarrow \langle (), v \rangle_\emptyset \mid e} \text{BIGSTEP_TOAMPAR} \\
\\
\frac{
t \Downarrow \langle (), v \rangle_\emptyset \mid e
}{\text{from}_x t \Downarrow v \mid e} \text{BIGSTEP_FROMAMPAR} \\
\\
\frac{
t \Downarrow @h \mid e
}{t \triangleleft () \Downarrow () \mid e \cdot h := ()} \text{BIGSTEP_FILLUNIT} \\
\\
\frac{
\begin{array}{c}
t \Downarrow @h \mid e \\
\text{fresh } h'
\end{array}
}{t \triangleleft \text{Inl} \Downarrow @h' \mid e \cdot h := \text{Inl } h'} \text{BIGSTEP_FILLINL} \\
\\
\frac{
\begin{array}{c}
t \Downarrow @h \mid e \\
\text{fresh } h'
\end{array}
}{t \triangleleft \text{Inr} \Downarrow @h' \mid e \cdot h := \text{Inr } h'} \text{BIGSTEP_FILLINR} \\
\\
\frac{
\begin{array}{c}
t \Downarrow @h \mid e \\
\text{fresh } h_1 \\
\text{fresh } h_2
\end{array}
}{t \triangleleft (,) \Downarrow (@h_1, @h_2) \mid e \cdot h := (h_1, h_2)} \text{BIGSTEP_FILLPROD} \\
\\
\frac{
\begin{array}{c}
t \Downarrow @h \mid e_1 \\
u \Downarrow \langle v_1, \overline{v_2} \rangle_H \mid e_2
\end{array}
}{t \triangleleft \bullet u \Downarrow v_1 \mid e_1 \cdot e_2 \cdot h := \overline{v_2}} \text{BIGSTEP_FILLCOMP}
\end{array}$$

Definition rules: 45 good 0 bad
 Definition rule clauses: 113 good 0 bad