$\mathsf{metavariable}$, $x$, $xs$, $y$, $uf$, $f$, $d$

| | | | | |
|---|---|---|---|---|
| term, t, u | ::= | | | term |
| | \| | $x$ | | variable |
| | \| | $z$ | | value |
| | \| | $t\ u$ | | application |
| | \| | $t\ ;\ u$ | | pattern-matching on unit |
| | \| | $\mathsf{case}\ t\ \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}$ | | pattern-matching on sum |
| | \| | $\mathsf{case}\ t\ \mathsf{of}\ \{\langle x_1, x_2\rangle \mapsto u\}$ | | pattern-matching on product |
| | \| | $\mathsf{case}\ t\ \mathsf{of}\ \{\ \mathsf{Ur}\ x \mapsto u\}$ | | pattern-matching on exponentiated value |
| | \| | $\mathsf{case}\ t\ \mathsf{of}\ \{\mathsf{roll}_{R}\ x \mapsto u\}$ | | unroll for recursive types |
| | \| | $\mathsf{alloc}_{A}\ d\ .\ t$ | | allocate data |
| | \| | $t \triangleleft u$ | | fill terminal-type destination |
| | \| | $t \triangleleft 1.d'\ .\ u$ | | fill sum-type destination with variant 1 |
| | \| | $t \triangleleft 2.d'\ .\ u$ | | fill sum-type destination with variant 2 |
| | \| | $t \triangleleft \langle d_1, d_2\rangle\ .\ u$ | | fill product-type destination |
| | \| | $\star l$ | | note: $\star l$ is not a part of the user syntax |
| | \| | $(t)$ | S | |
| | \| | \n sp t \n spe | S | |
| | \| | \n sp t | S | |
| | \| | t[var_subs] | M | |
| var_sub, vs | ::= | | | variable substitution |
| | \| | $x := t$ | | |
| var_subs | ::= | | | variable substitutions |
| | \| | vs | | |
| | \| | vs, var_subs | | |
| data_val, v | ::= | | | data value |
| | \| | $()$ | | unit |
| | \| | $\lambda x{:}A\ .\ t$ | | lambda abstraction |
| | \| | $\mathsf{Ur}\ t$ | | exponential |
| | \| | $\mathsf{roll}_{R}\ t$ | | roll for recursive types |
| val, z | ::= | | | unreducible value |
| | \| | $v$ | | |
| | \| | $\lfloor A \rfloor^{l}$ | | note: $l$ is not a part of the user syntax |
| | \| | $1.l$ | | |
| | \| | $2.l$ | | |
| | \| | $\langle l_1, l_2\rangle$ | | |
| | \| | $C\bar{l}$ | M | |
| *label*, $l$ | ::= | | | label |
| label_stmt, s | ::= | | | label statement |
| | \| | $l \triangleleft v$ | | |
| | \| | $l \triangleleft 1.l'$ | | |
| | \| | $l \triangleleft 2.l'$ | | |
| | \| | $l \triangleleft \langle l_1, l_2\rangle$ | | |
| | \| | $l \triangleleft \oslash$ | | |

| | | | | |
|---|---|---|---|---|
| | \| | $l \triangleleft \mathsf{C}\bar{l}$ | M | TODO: hide. $l \triangleleft \mathsf{C}\bar{l}$ is an alias for any heap cons |
| | \| | $l \triangleleft \ldots$ | M | |
| label_stmts | ::= | | | label statements |
| | \| | s | | |
| | \| | s, label_stmts | | |
| heap_context, $\mathbb{H}$ | ::= | | | label statements |
| | \| | $\emptyset$ | | |
| | \| | {label_stmts} | | |
| | \| | $\mathbb{H}_1 \sqcup \mathbb{H}_2$ | | |
| type, A, B | ::= | | | |
| | \| | 1 | | unit type |
| | \| | R | | recursive type bound to a name |
| | \| | $\mathsf{A} \otimes \mathsf{B}$ | | product type |
| | \| | $\mathsf{A} \oplus \mathsf{B}$ | | sum type |
| | \| | $\mathsf{A} \multimap \mathsf{B}$ | | linear function type |
| | \| | $\lfloor \mathsf{A} \rfloor$ | | destination type |
| | \| | !A | | exponential |
| | \| | (A) | S | |
| | \| | $\mathsf{W}[\mathsf{r} := \mathsf{A}]$ | M | |
| type_with_hole, W | ::= | | | |
| | \| | r | | type hole in recursive definition |
| | \| | 1 | | unit type |
| | \| | R | | recursive type bound to a name |
| | \| | $\mathsf{W}_1 \otimes \mathsf{W}_2$ | | product type |
| | \| | $\mathsf{W}_1 \oplus \mathsf{W}_2$ | | sum type |
| | \| | $\mathsf{W}_1 \multimap \mathsf{W}_2$ | | linear function type |
| | \| | $\lfloor \mathsf{W} \rfloor$ | | destination type |
| | \| | !W | | exponential |
| | \| | (W) | S | |
| rec_type_bound, R | ::= | | | recursive type bound to a name |
| rec_type_def | ::= | | | |
| | \| | $\mu\, \mathsf{r} \,.\, \mathsf{W}$ | | |
| type_affect, ta | ::= | | | type affectation |
| | \| | x : A | | var |
| | \| | $l$ : A | | label |
| | \| | $\bar{l} : \bar{\mathsf{A}}$ | | labels |
| type_affects | ::= | | | type affectations |
| | \| | ta | | |
| | \| | ta, type_affects | | |
| typing_context, $\Gamma$, $\Delta$, $\mho$, $\Phi$ | ::= | | | typing context |

| | | | |
|---|---|---|---|
| | $\mid$ | $\emptyset$ | |
| | $\mid$ | $\{$type_affects$\}$ | |
| | $\mid$ | $\Gamma \sqcup \Delta$ | |
| types, $\bar{A}$ | $::=$ | | |
| | $\mid$ | $\bullet$ | empty type list |
| | $\mid$ | $A$ | |
| | $\mid$ | $A$ types | |
| heap_constructor, $C$ | $::=$ | | |
| | $\mid$ | $(1.)$ | |
| | $\mid$ | $(2.)$ | |
| | $\mid$ | $(\langle, \rangle)$ | |
| judg | $::=$ | | |
| | $\mid$ | $\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H}$ | |
| | $\mid$ | $\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \mid t : A$ | |
| | $\mid$ | $\Phi \,;\, \mho \,;\, \Gamma \vdash t : A$ | |
| | $\mid$ | $C : \bar{A} \xrightarrow{c} A$ | |
| | $\mid$ | $A = B$ | |
| | $\mid$ | $t = u$ | |
| | $\mid$ | $\Gamma = \Delta$ | |
| | $\mid$ | $\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'$ | |
| | $\mid$ | type_affect $\in \Gamma$ | |
| | $\mid$ | label_stmt $\in \mathbb{H}$ | |
| | $\mid$ | names $(\Gamma) \cap$ names $(\Delta) = \emptyset$ | |
| terminals | $::=$ | | |
| | $\mid$ | $\mapsto$ | |
| | $\mid$ | $\star$ | |
| | $\mid$ | $\circledast$ | |
| | $\mid$ | $\oplus$ | |
| | $\mid$ | $\multimap$ | |
| | $\mid$ | $:=$ | |
| | $\mid$ | $\vdash$ | |
| | $\mid$ | $\sqcup$ | |
| | $\mid$ | $;$ | |
| | $\mid$ | $\cap$ | |
| | $\mid$ | $\emptyset$ | |
| | $\mid$ | $\longrightarrow$ | |
| | $\mid$ | $\triangleright$ | |
| | $\mid$ | $\neq$ | |
| | $\mid$ | $\in$ | |
| | $\mid$ | $\notin$ | |
| | $\mid$ | \n | |
| | $\mid$ | $\langle$ | |
| | $\mid$ | $\rangle$ | |
| | $\mid$ | 1. | |
| | $\mid$ | 2. | |
| | $\mid$ | Ur | |

| | | | |
|---|---|---|---|
| | \| | $\lhd$ | |
| | \| | $\mid$ | |
| | \| | $\oslash$ | |
| | \| | $\stackrel{c}{\lhd}$ | |
| | \| | $=$ | |
| | \| | $\Downarrow$ | |
| | \| | $\ldots$ | |
| | \| | $\stackrel{\mathsf{fix}}{\equiv}$ | |
| formula | ::= | | |
| | \| | judgement | |
| Ctx | ::= | | |
| | \| | $\mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset$ | $\Gamma$ and $\Delta$ are disjoint typing contexts with no clashin |
| | \| | type_affect $\in \Gamma$ | |
| Heap | ::= | | |
| | \| | label_stmt $\in \mathbb{H}$ | |
| Eq | ::= | | |
| | \| | $\mathsf{A} = \mathsf{B}$ | |
| | \| | $\mathsf{t} = \mathsf{u}$ | |
| | \| | $\Gamma = \Delta$ | |
| Ty | ::= | | |
| | \| | $\mathsf{R} \stackrel{\mathsf{fix}}{=}$ rec_type_def | |
| | \| | $\Phi\ ;\ \mho\ ;\ \Gamma \vdash \mathbb{H}$ | $\mathbb{H}$ is a well-typed heap given heap typing context $\Phi$, u |
| | \| | $\Phi\ ;\ \mho\ ;\ \Gamma \vdash \mathbb{H} \mid \mathsf{t} : \mathsf{A}$ | |
| | \| | $\Phi\ ;\ \mho\ ;\ \Gamma \vdash \mathsf{t} : \mathsf{A}$ | t is a well-typed term of type A given heap typing con |
| | \| | $\mathsf{C} : \bar{\mathsf{A}} \stackrel{c}{\multimap} \mathsf{A}$ | Heap constructor C builds a value of type A given arg |
| Sem | ::= | | |
| | \| | $\mathbb{H} \mid \mathsf{t} \longrightarrow \mathbb{H}' \mid \mathsf{t}'$ | t reduces to $\mathsf{t}'$, with heap changing from $\mathbb{H}$ to $\mathbb{H}'$ |
| | \| | $\mathbb{H} \mid \mathsf{t} \Downarrow \mathbb{H}' \mid \mathsf{t}'$ | t reduces to $\mathsf{t}'$, with heap growing from $\mathbb{H}$ to $\mathbb{H}'$ |
| judgement | ::= | | |
| | \| | Ctx | |
| | \| | Heap | |
| | \| | Eq | |
| | \| | Ty | |
| | \| | Sem | |
| user_syntax | ::= | | |
| | \| | metavariable | |
| | \| | term | |
| | \| | var_sub | |
| | \| | var_subs | |
| | \| | data_val | |
| | \| | val | |

| *label*
| label_stmt
| label_stmts
| heap_context
| type
| type_with_hole
| rec_type_bound
| rec_type_def
| type_affect
| type_affects
| typing_context
| types
| heap_constructor
| judg
| terminals

$\boxed{\mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset}$ $\Gamma$ and $\Delta$ are disjoint typing contexts with no clashing variable names or labels

$\boxed{\mathsf{type\_affect} \in \Gamma}$

$\boxed{\mathsf{label\_stmt} \in \mathbb{H}}$

$\boxed{\mathsf{A} = \mathsf{B}}$

$\boxed{\mathsf{t} = \mathsf{u}}$

$\boxed{\Gamma = \Delta}$

$\boxed{\mathsf{R} \stackrel{\mathsf{fix}}{=} \mathsf{rec\_type\_def}}$

$\boxed{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H}}$ $\mathbb{H}$ is a well-typed heap given heap typing context $\Phi$, unrestricted typing context $\mho$ and linear

$$\frac{\begin{array}{l}\Phi_1 \,;\, \mho \,;\, \Gamma \vdash \mathbb{H}_1 \\ \Phi_2 \,;\, \mho \,;\, \Delta \vdash \mathbb{H}_2 \\ \mathsf{names}\,(\Phi_1) \cap \mathsf{names}\,(\Phi_2) = \emptyset \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi_1 \sqcup \Phi_2 \,;\, \mho \,;\, \Gamma \sqcup \Delta \vdash \mathbb{H}_1 \sqcup \mathbb{H}_2} \quad \textsc{TyHeap\_Union}$$

$$\frac{\begin{array}{l}\mathsf{C} : \bar{\mathsf{A}} \xrightarrow{\mathsf{c}} \mathsf{A} \\ \Phi \sqcup \{\bar{l} : \bar{\mathsf{A}}\} \,;\, \mho \,;\, \Gamma \vdash \mathbb{H}\end{array}}{\Phi \sqcup \{\bar{l} : \bar{\mathsf{A}}, l : \mathsf{A}\} \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \sqcup \{l \triangleleft \mathsf{C}\,\bar{l}\}} \quad \textsc{TyHeap\_Ctor}$$

$$\frac{}{\{l : \mathsf{A}\} \,;\, \mho \,;\, \emptyset \vdash \{l \triangleleft \oslash\}} \quad \textsc{TyHeap\_Null}$$

$$\frac{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathsf{v} : \mathsf{A}}{\Phi \sqcup \{l : \mathsf{A}\} \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \sqcup \{l \triangleleft \mathsf{v}\}} \quad \textsc{TyHeap\_Val}$$

$\boxed{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \,|\, \mathsf{t} : \mathsf{A}}$

$$\frac{\begin{array}{l}\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \\ \Phi \,;\, \mho \,;\, \Delta \vdash \mathsf{t} : \mathsf{A} \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathbb{H} \,|\, \mathsf{t} : \mathsf{A}} \quad \textsc{TyCommand\_Def}$$

$\boxed{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathsf{t} : \mathsf{A}}$ t is a well-typed term of type A given heap typing context $\Phi$, unrestricted typing context

$$\frac{}{\Phi \,;\, \mho \,;\, \{\mathsf{x} : \mathsf{A}\} \vdash \mathsf{x} : \mathsf{A}} \quad \textsc{TyTerm\_Id}$$

$$\frac{}{\Phi\ ;\ \mho \sqcup \{x : A\}\ ;\ \emptyset \vdash x : A}\ \textsc{TyTerm\_Id'}$$

$$\frac{}{\Phi\ ;\ \mho\ ;\ \emptyset \vdash () : 1}\ \textsc{TyTerm\_Unit}$$

$$\frac{\Phi\ ;\ \mho\ ;\ \emptyset \vdash t : A}{\Phi\ ;\ \mho\ ;\ \emptyset \vdash \mathsf{Ur}\ t : !A}\ \textsc{TyTerm\_Exp}$$

$$\frac{}{\Phi\ ;\ \mho\ ;\ \emptyset \vdash \underset{\lfloor A \rfloor}{l} : \lfloor A \rfloor}\ \textsc{TyTerm\_LabelAsDest}$$

$$\frac{}{\Phi \sqcup \{l : A\}\ ;\ \mho\ ;\ \emptyset \vdash \star l : A}\ \textsc{TyTerm\_Deref}$$

$$\frac{C : \bar{A} \overset{c}{\rightharpoonup} A}{\Phi \sqcup \{\bar{l} : \bar{A}\}\ ;\ \mho\ ;\ \emptyset \vdash C\,\bar{l} : A}\ \textsc{TyTerm\_Ctor}$$

$$\frac{\begin{array}{l} R \overset{\text{fix}}{=} \mu\, r\,.\,W \\ \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : W[r := R] \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \vdash \underset{R}{\mathsf{roll}}\ t : R}\ \textsc{TyTerm\_Roll}$$

$$\frac{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \{x : A\} \vdash t : B}{\Phi\ ;\ \mho\ ;\ \Gamma \vdash \lambda x{:}A\,.\,t : A \multimap B}\ \textsc{TyTerm\_Lam}$$

$$\frac{\begin{array}{l} \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : A \multimap B \\ \Phi\ ;\ \mho\ ;\ \Delta \vdash u : A \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash t\, u : B}\ \textsc{TyTerm\_App}$$

$$\frac{\begin{array}{l} \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : 1 \\ \Phi\ ;\ \mho\ ;\ \Delta \vdash u : A \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash t\ ;\ u : A}\ \textsc{TyTerm\_PatU}$$

$$\frac{\begin{array}{l} \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : A_1 \otimes A_2 \\ \Phi\ ;\ \mho\ ;\ \Delta \sqcup \{x_1 : A_1, x_2 : A_2\} \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash \mathsf{case}\, t\, \mathsf{of}\ \{\langle x_1, x_2 \rangle \mapsto u\} : B}\ \textsc{TyTerm\_PatP}$$

$$\frac{\begin{array}{l} \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : A_1 \oplus A_2 \\ \Phi\ ;\ \mho\ ;\ \Delta \sqcup \{x_1 : A_1\} \vdash u_1 : B \\ \Phi\ ;\ \mho\ ;\ \Delta \sqcup \{x_2 : A_2\} \vdash u_2 : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash \mathsf{case}\, t\, \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} : B}\ \textsc{TyTerm\_PatS}$$

$$\frac{\begin{array}{l} \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : !A \\ \Phi\ ;\ \mho \sqcup \{x : A\}\ ;\ \Delta \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash \mathsf{case}\, t\, \mathsf{of}\ \{\,\mathsf{Ur}\ x \mapsto u\} : B}\ \textsc{TyTerm\_PatE}$$

$$\frac{\begin{array}{l} R \overset{\text{fix}}{=} \mu\, r\,.\,W \\ \Phi\ ;\ \mho\ ;\ \Gamma \vdash t : R \\ \Phi\ ;\ \mho\ ;\ \Delta \sqcup \{x : W[r := R]\} \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset \end{array}}{\Phi\ ;\ \mho\ ;\ \Gamma \sqcup \Delta \vdash \mathsf{case}\ t\ \mathsf{of}\ \{\underset{R}{\mathsf{roll}}\ x \mapsto u\} : B}\ \textsc{TyTerm\_PatR}$$

$$\frac{\Phi \,;\, \mho \,;\, \Gamma \sqcup \{d : \lfloor A \rfloor\} \vdash t : 1}{\Phi \,;\, \mho \,;\, \Gamma \vdash \mathsf{alloc}_{A}\ d \,.\, t : A}\ \textsc{TyTerm\_Alloc}$$

$$\frac{\begin{array}{l}\Phi \,;\, \mho \,;\, \Gamma \vdash t : \lfloor A \rfloor \\ \Phi \,;\, \mho \,;\, \Delta \vdash u : A \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi \,;\, \mho \,;\, \Gamma \sqcup \Delta \vdash t \triangleleft u : 1}\ \textsc{TyTerm\_FillL}$$

$$\frac{\begin{array}{l}\Phi \,;\, \mho \,;\, \Gamma \vdash t : \lfloor A_1 \oplus A_2 \rfloor \\ \Phi \,;\, \mho \,;\, \Delta \sqcup \{d' : \lfloor A_1 \rfloor\} \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi \,;\, \mho \,;\, \Gamma \sqcup \Delta \vdash t \triangleleft 1.d' \,.\, u : B}\ \textsc{TyTerm\_FillV1}$$

$$\frac{\begin{array}{l}\Phi \,;\, \mho \,;\, \Gamma \vdash t : \lfloor A_1 \oplus A_2 \rfloor \\ \Phi \,;\, \mho \,;\, \Delta \sqcup \{d' : \lfloor A_2 \rfloor\} \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi \,;\, \mho \,;\, \Gamma \vdash t \triangleleft 2.d' \,.\, u : B}\ \textsc{TyTerm\_FillV2}$$

$$\frac{\begin{array}{l}\Phi \,;\, \mho \,;\, \Gamma \vdash t : \lfloor A_1 \otimes A_2 \rfloor \\ \Phi \,;\, \mho \,;\, \Delta \sqcup \{d_1 : \lfloor A_1 \rfloor, d_2 : \lfloor A_2 \rfloor\} \vdash u : B \\ \mathsf{names}\,(\Gamma) \cap \mathsf{names}\,(\Delta) = \emptyset\end{array}}{\Phi \,;\, \mho \,;\, \Gamma \sqcup \Delta \vdash t \triangleleft \langle d_1, d_2 \rangle \,.\, u : B}\ \textsc{TyTerm\_FillP}$$

$\boxed{C : \bar{A} \overset{c}{\rightharpoonup} A}$ Heap constructor $C$ builds a value of type $A$ given arguments of type $\bar{A}$

$$\frac{}{(1.) : A \overset{c}{\rightharpoonup} A \oplus B}\ \textsc{TyCtor\_V1}$$

$$\frac{}{(2.) : B \overset{c}{\rightharpoonup} A \oplus B}\ \textsc{TyCtor\_V2}$$

$$\frac{}{(\langle , \rangle) : A \quad B \overset{c}{\rightharpoonup} A \otimes B}\ \textsc{TyCtor\_Pair}$$

$\boxed{\mathbb{H} \,|\, t \longrightarrow \mathbb{H}' \,|\, t'}$ $t$ reduces to $t'$, with heap changing from $\mathbb{H}$ to $\mathbb{H}'$

$$\frac{\mathbb{H} \,|\, t \longrightarrow \mathbb{H}' \,|\, t'}{\mathbb{H} \,|\, t\ u \longrightarrow \mathbb{H}' \,|\, t'\ u}\ \textsc{SemMut\_uApp}$$

$$\frac{}{\mathbb{H} \,|\, (\lambda\, x{:}A \,.\, t)\ u \longrightarrow \mathbb{H} \,|\, t[x := u]}\ \textsc{SemMut\_App}$$

$$\frac{\mathbb{H} \,|\, t \longrightarrow \mathbb{H}' \,|\, t'}{\mathbb{H} \,|\, t \,;\, u \longrightarrow \mathbb{H}' \,|\, t' \,;\, u}\ \textsc{SemMut\_uPatU}$$

$$\frac{}{\mathbb{H} \,|\, () \,;\, u \longrightarrow \mathbb{H} \,|\, u}\ \textsc{SemMut\_PatU}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \,|\, \star l \longrightarrow \mathbb{H} \sqcup \{l \triangleleft v\} \,|\, v}\ \textsc{SemMut\_DerefVal}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \,|\, \star l \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \,|\, C\bar{l}}\ \textsc{SemMut\_DerefCtor}$$

$$\frac{\mathbb{H} \,|\, t \longrightarrow \mathbb{H}' \,|\, t'}{\mathbb{H} \,|\, \mathsf{case}\ t\ \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H}' \,|\, \mathsf{case}\ t'\ \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}}\ \textsc{SemMut\_uPatS}$$

$$\frac{}{\mathbb{H} \,|\, \mathsf{case}\ 1.l\ \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \,|\, u_1[x_1 := \star l]}\ \textsc{SemMut\_PatSV1}$$

$$\frac{}{\mathbb{H} \,|\, \mathsf{case}\ 2.l\ \mathsf{of}\ \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \,|\, u_2[x_2 := \star l]}\ \textsc{SemMut\_PatSV2}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \mathsf{case}\ t\ \mathsf{of}\ \{\langle x_1, x_2\rangle \mapsto u\} \longrightarrow \mathbb{H}' \mid \mathsf{case}\ t'\ \mathsf{of}\ \{\langle x_1, x_2\rangle \mapsto u\}}\ \textsc{SemMut\_uPatP}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \langle l_1, l_2\rangle\ \mathsf{of}\ \{\langle x_1, x_2\rangle \mapsto u\} \longrightarrow \mathbb{H} \mid u[x_1 := \star l_1, x_2 := \star l_2]}\ \textsc{SemMut\_PatP}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \mathsf{case}\ t\ \mathsf{of}\ \{\, \mathsf{Ur}\ x \mapsto u\} \longrightarrow \mathbb{H}' \mid \mathsf{case}\ t'\ \mathsf{of}\ \{\, \mathsf{Ur}\ x \mapsto u\}}\ \textsc{SemMut\_uPatE}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \mathsf{Ur}\ t\ \mathsf{of}\ \{\, \mathsf{Ur}\ x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]}\ \textsc{SemMut\_PatE}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \mathsf{case}\ t\ \mathsf{of}\ \{\underset{R}{\mathsf{roll}}\ x \mapsto u\} \longrightarrow \mathbb{H}' \mid \mathsf{case}\ t'\ \mathsf{of}\ \{\underset{R}{\mathsf{roll}}\ x \mapsto u\}}\ \textsc{SemMut\_uPatR}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \underset{R}{\mathsf{roll}}\ t\ \mathsf{of}\ \{\underset{R}{\mathsf{roll}}\ x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]}\ \textsc{SemMut\_PatR}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft u \longrightarrow \mathbb{H}' \mid t' \triangleleft u}\ \textsc{SemMut\_uFillL}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \overset{l}{\lfloor A \rfloor} \triangleleft t \longrightarrow \mathbb{H}' \mid \overset{l}{\lfloor A \rfloor} \triangleleft t'}\ \textsc{SemMut\_uFillL'}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 1.d'\,.\,u \longrightarrow \mathbb{H}' \mid t' \triangleleft 1.d'\,.\,u}\ \textsc{SemMut\_uFillV1}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 2.d'\,.\,u \longrightarrow \mathbb{H}' \mid t' \triangleleft 2.d'\,.\,u}\ \textsc{SemMut\_uFillV2}$$

$$\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \langle d_1, d_2\rangle\,.\,u \longrightarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2\rangle\,.\,u}\ \textsc{SemMut\_uFillP}$$

$$\frac{}{\mathbb{H} \mid \underset{A}{\mathsf{alloc}}\ d\,.\,t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid t[d := \overset{l}{\lfloor A \rfloor}]\ ;\ \star l}\ \textsc{SemMut\_Alloc}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid \overset{l}{\lfloor A \rfloor} \triangleleft v \longrightarrow \mathbb{H} \sqcup \{l \triangleleft v\} \mid ()}\ \textsc{SemMut\_FillLV}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid \overset{l}{\lfloor A \rfloor} \triangleleft C\bar{l} \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid ()}\ \textsc{SemMut\_FillLCtor}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid \overset{l}{\lfloor A_1 \oplus A_2 \rfloor} \triangleleft 1.d'\,.\,t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 1.l', l' \triangleleft \varnothing\} \mid t[d' := \overset{l'}{\lfloor A_1 \rfloor}]}\ \textsc{SemMut\_FillV1}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid \overset{l}{\lfloor A_1 \oplus A_2 \rfloor} \triangleleft 2.d'\,.\,t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 2.l', l' \triangleleft \varnothing\} \mid t[d' := \overset{l'}{\lfloor A_2 \rfloor}]}\ \textsc{SemMut\_FillV2}$$

$$\frac{}{\mathbb{H} \sqcup \{l \triangleleft \varnothing\} \mid \overset{l}{\lfloor A_1 \otimes A_2 \rfloor} \triangleleft \langle d_1, d_2\rangle\,.\,t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \langle l_1, l_2\rangle, l_1 \triangleleft \varnothing, l_2 \triangleleft \varnothing\} \mid t[d_1 := \overset{l_1}{\lfloor A_1 \rfloor}, d_2 := \overset{l_2}{\lfloor A_2 \rfloor}]}\ \textsc{SemMut\_FillP}$$

$\boxed{\mathbb{H} \mid t \ \Downarrow\ \mathbb{H}' \mid t'}$ $t$ reduces to $t'$, with heap growing from $\mathbb{H}$ to $\mathbb{H}'$

$$\frac{}{\mathbb{H} \mid \underset{R}{\mathsf{roll}}\ t \ \Downarrow\ \mathbb{H} \mid t}\ \textsc{SemImm\_Roll}$$

$$\frac{\mathbb{H} \mid t \ \Downarrow\ \mathbb{H}' \mid t'}{\mathbb{H} \mid t\ u \ \Downarrow\ \mathbb{H}' \mid t'\ u}\ \textsc{SemImm\_uApp}$$

$$\frac{}{\mathbb{H} \mid (\lambda x{:}A\,.\,t)\ u \ \Downarrow\ \mathbb{H} \mid t[x := u]}\ \textsc{SemImm\_App}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{t}\ ;\ \mathsf{u} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'\ ;\ \mathsf{u}} \ \textsc{SemImm\_uPatU}$$

$$\frac{}{\mathbb{H} \mid ()\ ;\ \mathsf{u} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}} \ \textsc{SemImm\_PatU}$$

$$\frac{}{\mathbb{H} \sqcup \{l \lhd \mathsf{v}\} \mid \star l \ \Downarrow\ \mathbb{H} \sqcup \{l \lhd \mathsf{v}\} \mid \mathsf{v}} \ \textsc{SemImm\_DerefVal}$$

$$\frac{}{\mathbb{H} \sqcup \{l \lhd \mathsf{C}\bar{l}\} \mid \star l \ \Downarrow\ \mathbb{H} \sqcup \{l \lhd \mathsf{C}\bar{l}\} \mid \mathsf{C}\bar{l}} \ \textsc{SemImm\_DerefCtor}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{case}\ \mathsf{t}\ \mathsf{of}\ \{1.\mathsf{x}_1 \mapsto \mathsf{u}_1, 2.\mathsf{x}_2 \mapsto \mathsf{u}_2\} \ \Downarrow\ \mathbb{H}' \mid \mathsf{case}\ \mathsf{t}'\ \mathsf{of}\ \{1.\mathsf{x}_1 \mapsto \mathsf{u}_1, 2.\mathsf{x}_2 \mapsto \mathsf{u}_2\}} \ \textsc{SemImm\_uPatS}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ 1.l\ \mathsf{of}\ \{1.\mathsf{x}_1 \mapsto \mathsf{u}_1, 2.\mathsf{x}_2 \mapsto \mathsf{u}_2\} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}_1[\mathsf{x}_1 := \star l]} \ \textsc{SemImm\_PatSV1}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ 2.l\ \mathsf{of}\ \{1.\mathsf{x}_1 \mapsto \mathsf{u}_1, 2.\mathsf{x}_2 \mapsto \mathsf{u}_2\} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}_2[\mathsf{x}_2 := \star l]} \ \textsc{SemImm\_PatSV2}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{case}\ \mathsf{t}\ \mathsf{of}\ \{\langle \mathsf{x}_1, \mathsf{x}_2 \rangle \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H}' \mid \mathsf{case}\ \mathsf{t}'\ \mathsf{of}\ \{\langle \mathsf{x}_1, \mathsf{x}_2 \rangle \mapsto \mathsf{u}\}} \ \textsc{SemImm\_uPatP}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \langle l_1, l_2 \rangle\ \mathsf{of}\ \{\langle \mathsf{x}_1, \mathsf{x}_2 \rangle \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}[\mathsf{x}_1 := \star l_1, \mathsf{x}_2 := \star l_2]} \ \textsc{SemImm\_PatP}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{case}\ \mathsf{t}\ \mathsf{of}\ \{\ \mathsf{Ur}\ \mathsf{x} \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H}' \mid \mathsf{case}\ \mathsf{t}'\ \mathsf{of}\ \{\ \mathsf{Ur}\ \mathsf{x} \mapsto \mathsf{u}\}} \ \textsc{SemImm\_uPatE}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \mathsf{Ur}\ \mathsf{t}\ \mathsf{of}\ \{\ \mathsf{Ur}\ \mathsf{x} \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}[\mathsf{x} := \mathsf{t}]} \ \textsc{SemImm\_PatE}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{case}\ \mathsf{t}\ \mathsf{of}\ \{\mathsf{roll}_{\mathsf{R}}\ \mathsf{x} \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H}' \mid \mathsf{case}\ \mathsf{t}'\ \mathsf{of}\ \{\mathsf{roll}_{\mathsf{R}}\ \mathsf{x} \mapsto \mathsf{u}\}} \ \textsc{SemImm\_uPatR}$$

$$\frac{}{\mathbb{H} \mid \mathsf{case}\ \mathsf{roll}_{\mathsf{R}}\ \mathsf{t}\ \mathsf{of}\ \{\mathsf{roll}_{\mathsf{R}}\ \mathsf{x} \mapsto \mathsf{u}\} \ \Downarrow\ \mathbb{H} \mid \mathsf{u}[\mathsf{x} := \mathsf{t}]} \ \textsc{SemImm\_PatR}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{t} \lhd \mathsf{u} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}' \lhd \mathsf{u}} \ \textsc{SemImm\_uFillL}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \overset{l}{\lfloor \mathsf{A} \rfloor} \lhd \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \overset{l}{\lfloor \mathsf{A} \rfloor} \lhd \mathsf{t}'} \ \textsc{SemImm\_uFillL'}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{t} \lhd 1.\mathsf{d}' \centerdot \mathsf{u} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}' \lhd 1.\mathsf{d}' \centerdot \mathsf{u}} \ \textsc{SemImm\_uFillV1}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{t} \lhd 2.\mathsf{d}' \centerdot \mathsf{u} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}' \lhd 2.\mathsf{d}' \centerdot \mathsf{u}} \ \textsc{SemImm\_uFillV2}$$

$$\frac{\mathbb{H} \mid \mathsf{t} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}'}{\mathbb{H} \mid \mathsf{t} \lhd \langle \mathsf{d}_1, \mathsf{d}_2 \rangle \centerdot \mathsf{u} \ \Downarrow\ \mathbb{H}' \mid \mathsf{t}' \lhd \langle \mathsf{d}_1, \mathsf{d}_2 \rangle \centerdot \mathsf{u}} \ \textsc{SemImm\_uFillP}$$

$$\frac{\mathbb{H} \mid \mathsf{t}[\mathsf{d} := \overset{l}{\lfloor \mathsf{A} \rfloor}] \ \Downarrow\ \mathbb{H}' \sqcup \{l \lhd \ldots\} \mid ()}{\mathbb{H} \mid \mathsf{alloc}_{\mathsf{A}}\ \mathsf{d} \centerdot \mathsf{t} \ \Downarrow\ \mathbb{H}' \sqcup \{l \lhd \ldots\} \mid \star l} \ \textsc{SemImm\_Alloc}$$

— *Changes start from there* —

$$\frac{}{\mathbb{H} \mid \overset{l}{\lfloor \mathsf{A} \rfloor} \lhd \mathsf{v} \ \Downarrow\ \mathbb{H} \sqcup \{l \lhd \mathsf{v}\} \mid ()} \ \textsc{SemImm\_FillLV}$$

$$\frac{}{\mathbb{H} \mid \overset{l}{\lfloor \mathsf{A} \rfloor} \lhd \mathsf{C}\bar{l} \ \Downarrow\ \mathbb{H} \sqcup \{l \lhd \mathsf{C}\bar{l}\} \mid ()} \ \textsc{SemImm\_FillLCtor}$$

$$\frac{\mathbb{H} \,|\, \mathtt{t}[\mathsf{d}' := {}_{\lfloor \mathsf{A}_1 \rfloor}^{\,l'}] \ \Downarrow\ \mathbb{H}' \sqcup \{ l' \triangleleft \ldots \} \,|\, ()}{\mathbb{H} \,|\, {}_{\lfloor \mathsf{A}_1 \oplus \mathsf{A}_2 \rfloor}^{\,l} \triangleleft 1.\mathsf{d}' \mathbin{\boldsymbol{.}} \mathtt{t} \ \Downarrow\ \mathbb{H}' \sqcup \{ l' \triangleleft \ldots, l \triangleleft 1.l' \} \,|\, ()} \ \textsc{SemImm\_FillV1}$$

$$\frac{\mathbb{H} \,|\, \mathtt{t}[\mathsf{d}' := {}_{\lfloor \mathsf{A}_2 \rfloor}^{\,l'}] \ \Downarrow\ \mathbb{H}' \sqcup \{ l' \triangleleft \ldots \} \,|\, ()}{\mathbb{H} \,|\, {}_{\lfloor \mathsf{A}_1 \oplus \mathsf{A}_2 \rfloor}^{\,l} \triangleleft 2.\mathsf{d}' \mathbin{\boldsymbol{.}} \mathtt{t} \ \Downarrow\ \mathbb{H}' \sqcup \{ l' \triangleleft \ldots, l \triangleleft 2.l' \} \,|\, ()} \ \textsc{SemImm\_FillV2}$$

$$\frac{\mathbb{H} \,|\, \mathtt{t}[\mathsf{d}_1 := {}_{\lfloor \mathsf{A}_1 \rfloor}^{\,l_1}, \mathsf{d}_2 := {}_{\lfloor \mathsf{A}_2 \rfloor}^{\,l_2}] \ \Downarrow\ \mathbb{H}' \sqcup \{ l_1 \triangleleft \ldots, l_2 \triangleleft \ldots \} \,|\, ()}{\mathbb{H} \,|\, {}_{\lfloor \mathsf{A}_1 \otimes \mathsf{A}_2 \rfloor}^{\,l} \triangleleft \langle \mathsf{d}_1, \mathsf{d}_2 \rangle \mathbin{\boldsymbol{.}} \mathtt{t} \ \Downarrow\ \mathbb{H}' \sqcup \{ l_1 \triangleleft \ldots, l_2 \triangleleft \ldots, l \triangleleft \langle l_1, l_2 \rangle \} \,|\, ()} \ \textsc{SemImm\_FillP}$$

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Definition rules:         81 good    0 bad
Definition rule clauses: 155 good    0 bad
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