Destination λ-calculus

Thomas Bagrel

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1 Term and value syntax

tvar, x, y, d	Term-level variable name
hvar, h	Hole or destination name
k	Index for ranges

val, v	::=		Term value
	|	$\langle v_1 \,,\, \overline{v_2} \rangle_{\Delta}$	Ampar
	|	$@h$	Destination
	|	$()$	Unit
	|	$\mathsf{Inl}\ v$	Left variant for sum
	|	$\mathsf{Inr}\ v$	Right variant for sum
	|	$(v_1 \,,\, v_2)$	Product
	|	$)\!)^{m}\ v$	Exponential
	|	$\lambda x\,.\,t$	Linear function

$\overline{\text{xval}}$, $\overline{v}$	::=		Pseudo-value that may contain holes
	|	v	Term value
	|	h	Hole
	|	$\mathsf{Inl}\ \overline{v}$	Left variant with val or hole
	|	$\mathsf{Inr}\ \overline{v}$	Right variant with val or hole
	|	$(\overline{v_1} \,,\, \overline{v_2})$	Product with val or hole
	|	$)\!)^{m}\ \overline{v}$	Exponential with val or hole

term, t, u	::=		Term
	|	v	Term value
	|	x	Variable
	|	$t \succ u$	Application
	|	$t \,;\, u$	Pattern-match on unit
	|	$t \succ \mathbf{case}\ \{\, \mathsf{Inl}\ x_1 \mapsto u_1 \,,\ \mathsf{Inr}\ x_2 \mapsto u_2 \,\}$	Pattern-match on sum
	|	$t \succ \mathbf{case}\ (x_1 \,,\, x_2) \mapsto u$	Pattern-match on product
	|	$t \succ \mathbf{case}\ )\!)^{m}\ x \mapsto u$	Pattern-match on exponential
	|	$t \succ \mathbf{mapL}\ x \mapsto u$	Map over the left side of the ampar
	|	$\mathbf{to}_{\ltimes}\ t$	Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ t$	Extract value from trivial ampar
	|	$\mathbf{alloc}_{\mathsf{T}}$	Return a fresh "identity" ampar object
	|	$t \triangleleft ()$	Fill destination with unit
	|	$t \triangleleft \mathsf{Inl}$	Fill destination with left variant
	|	$t \triangleleft \mathsf{Inr}$	Fill destination with right variant
	|	$t \triangleleft (,)$	Fill destination with product constructor
	|	$t \triangleleft)\!)^{m}$	Fill destination with exponential constructor
	|	$t \triangleleft\!\bullet\ u$	Fill destination with root of ampar u

2 Type system

$\mathsf{typ},\ \mathsf{T},\ \mathsf{U}$	$::=$		Type
	$\mid$	$\mathbf{1}$	Unit
	$\mid$	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	$\mid$	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	$\mid$	$!^{m}\,\mathsf{T}$	Exponential
	$\mid$	$\mathsf{T}_1 \ltimes \mathsf{T}_2$	Ampar type (consuming T_1 yields T_2)
	$\mid$	$\mathsf{T}_1\ {}_{m_1}\!\!\to \mathsf{T}_2$	Linear function
	$\mid$	${}^{m}\lfloor \mathsf{T} \rfloor$	Destination
$moda,\ m,\ n$	$::=$		Modality (Semiring)
	$\mid$	$p\mid a$	Pair of a multiplicity and age
	$\mid$	$\underline{\nu}$	Neutral element of the product. Notation for $1\mid\nu$.
	$\mid$	$\underline{\uparrow}$	Same multiplicity, but one scope older. Notation for $1\mid\uparrow$.
	$\mid$	$\underline{\infty}$	Linear, infinitely old / static. Notation for $1\mid\infty$.
	$\mid$	$m_1 \cdot \ldots \cdot m_k$	Semiring product
$mul,\ p$	$::=$		Multiplicity (first component of modality)
	$\mid$	1	Linear. Neutral element of the product
	$\mid$	ω	Non-linear. Absorbing for the product
	$\mid$	$p_1 \cdot \ldots \cdot p_k$	Semiring product
$age,\ a$	$::=$		Age (second component of modality)
	$\mid$	ν	Born now. Neutral element of the product
	$\mid$	$\uparrow$	One scope older
	$\mid$	∞	Infinitely old / static. Absorbing for the product
	$\mid$	$a_1 \cdot \ldots \cdot a_k$	Semiring product
$\mathsf{pctx},\ \Gamma$	$::=$		Positive typing context
	$\mid$	$\{\mathsf{pas}_1, \ldots, \mathsf{pas}_k\}$	
	$\mid$	$m\cdot\Gamma$	Multiply each binding by m
	$\mid$	$\Gamma_1 \cup \Gamma_2$	
pas	$::=$		Positive type assignment
	$\mid$	$x :_{m} \mathsf{T}$	Variable
	$\mid$	$@h :_{m} {}^{n}\lfloor \mathsf{T} \rfloor$	Destination (m is its own modality; n is the modality for values it accepts)
$\mathsf{nctx},\ \Delta$	$::=$		Negative typing context
	$\mid$	$\{\mathsf{nas}_1, \ldots, \mathsf{nas}_k\}$	
	$\mid$	$m\cdot\Delta$	Multiply each binding by m
	$\mid$	$@^{-1}\Gamma$	maps each destination of Γ to a hole (requires `ctx_DestOnly` Γ)
	$\mid$	$\Delta_1 \cup \Delta_2$	
nas	$::=$		Negative type assignment
	$\mid$	$h :^{n} \mathsf{T}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \uplus \Delta \Vdash \bar{v} : \mathsf{T}}$ *(Typing of extended values (require both positive and negative contexts))*

$$\textsc{Ty-xval-H}\quad \frac{}{\{\} \uplus \{h :^{\underline{\nu}} \mathsf{T}\} \Vdash h : \mathsf{T}}$$

$$\textsc{Ty-xval-D}\quad \frac{}{\{@h :_{\underline{\nu}} {}^{n}\lfloor \mathsf{T} \rfloor\} \uplus \{\} \Vdash @h : {}^{n}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{Ty-xval-U}\quad \frac{}{\{\} \uplus \{\} \Vdash () : \mathbf{1}}$$

$$\textsc{Ty-xval-L}\quad \frac{\Gamma \uplus \Delta \Vdash \bar{v} : \mathsf{T}_1 \qquad \texttt{ctx\_Disjoint}\ \Gamma\ \Delta}{\Gamma \uplus \Delta \Vdash \mathsf{Inl}\,\bar{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

Ty-xval-R

$$\frac{\begin{array}{c}\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{\mathsf{v}} : \mathsf{T}_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \Delta\end{array}}{\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \mathsf{Inr}\,\overline{\mathsf{v}} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

Ty-xval-P

$$\frac{\begin{array}{c}\Gamma_1 \mathbin{\scriptstyle\cup} \Delta_1 \Vdash \overline{\mathsf{v}_1} : \mathsf{T}_1 \\ \Gamma_2 \mathbin{\scriptstyle\cup} \Delta_2 \Vdash \overline{\mathsf{v}_2} : \mathsf{T}_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\end{array}}{\Gamma_1 \cup \Gamma_2 \mathbin{\scriptstyle\cup} \Delta_1 \cup \Delta_2 \Vdash (\overline{\mathsf{v}_1} \,,\, \overline{\mathsf{v}_2}) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

Ty-xval-E

$$\frac{\begin{array}{c}\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{\mathsf{v}} : \mathsf{T} \\ \texttt{ctx\_Disjoint}\ \Gamma\ \Delta\end{array}}{m{\cdot}\Gamma \mathbin{\scriptstyle\cup} m{\cdot}\Delta \Vdash \rrparenthesis^{m}\,\overline{\mathsf{v}} : !^{m}\,\mathsf{T}}$$

Ty-xval-A

$$\frac{\begin{array}{c}\Gamma_1 \mathbin{\scriptstyle\cup} \{\,\} \Vdash \mathsf{v}_1 : \mathsf{T}_1 \\ \Gamma_2 \mathbin{\scriptstyle\cup} @^{-1}\Gamma_1 \Vdash \overline{\mathsf{v}_2} : \mathsf{T}_2 \\ \texttt{pctx\_DestOnly}\ \Gamma_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\end{array}}{\Gamma_2 \mathbin{\scriptstyle\cup} \{\,\} \Vdash \langle \mathsf{v}_1 \,,\, \overline{\mathsf{v}_2} \rangle_{@^{-1}\Gamma_1} : \mathsf{T}_1 \rtimes \mathsf{T}_2}$$

Ty-xval-F

$$\frac{\begin{array}{c}\Gamma \cup \{\mathsf{x} :_{m} \mathsf{T}_1\} \vdash \mathsf{t} : \mathsf{T}_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \{\mathsf{x} :_{m} \mathsf{T}_1\}\end{array}}{\Gamma \mathbin{\scriptstyle\cup} \{\,\} \Vdash \lambda \mathsf{x} \,.\, \mathsf{t} : \mathsf{T}_1 \;{}_{m}\!\!\to \mathsf{T}_2}$$

$\boxed{\Gamma \vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms (only a positive context is needed))*

Ty-term-V

$$\frac{\Gamma \mathbin{\scriptstyle\cup} \{\,\} \Vdash \mathsf{v} : \mathsf{T}}{\Gamma \vdash \mathsf{v} : \mathsf{T}}$$

Ty-term-X0

$$\frac{}{\{\mathsf{x} :_{\underline{\nu}} \mathsf{T}\} \vdash \mathsf{x} : \mathsf{T}}$$

Ty-term-XInf

$$\frac{}{\{\mathsf{x} :_{\underline{\infty}} \mathsf{T}\} \vdash \mathsf{x} : \mathsf{T}}$$

Ty-term-App

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \;{}_{m}\!\!\to \mathsf{T}_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\end{array}}{m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

Ty-term-PatS

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Gamma_2 \cup \{\mathsf{x}_1 :_{m} \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U} \\ \Gamma_2 \cup \{\mathsf{x}_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x}_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x}_2 :_{m} \mathsf{T}_2\}\end{array}}{m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}\,\{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1 \,,\; \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\} : \mathsf{U}}$$

Ty-term-PatU

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathbf{1} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{U} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\end{array}}{\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t}\ ;\ \mathsf{u} : \mathsf{U}}$$

Ty-term-PatP

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Gamma_2 \cup \{\mathsf{x}_1 :_{m} \mathsf{T}_1, \mathsf{x}_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x}_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x}_2 :_{m} \mathsf{T}_2\} \\ \texttt{ctx\_Disjoint}\ \{\mathsf{x}_1 :_{m} \mathsf{T}_1\}\ \{\mathsf{x}_2 :_{m} \mathsf{T}_2\}\end{array}}{m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}\,(\mathsf{x}_1 \,,\, \mathsf{x}_2) \mapsto \mathsf{u} : \mathsf{U}}$$

Ty-term-PatE

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : !^{n}\,\mathsf{T} \\ \Gamma_2 \cup \{\mathsf{x} :_{m \cdot n} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x} :_{m \cdot n} \mathsf{T}\}\end{array}}{m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}\ \rrparenthesis^{n}\,\mathsf{x} \mapsto \mathsf{u} : \mathsf{U}}$$

Ty-term-Map

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \rtimes \mathsf{T}_2 \\ \underline{\uparrow}{\cdot}\Gamma_2 \cup \{\mathsf{x} :_{\underline{\nu}} \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{U} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \{\mathsf{x} :_{\underline{\nu}} \mathsf{T}_1\}\end{array}}{\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{mapL}\ \mathsf{x} \mapsto \mathsf{u} : \mathsf{U} \rtimes \mathsf{T}_2}$$

Ty-term-FillC

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_2 \rfloor \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \rtimes \mathsf{T}_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\end{array}}{\Gamma_1 \cup (\underline{\uparrow}{\cdot}n){\cdot}\Gamma_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_1}$$

Ty-term-FillU

$$\frac{\Gamma \vdash \mathsf{t} : {}^{n}\lfloor \mathbf{1} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft () : \mathbf{1}}$$

Ty-term-FillL

$$\frac{\Gamma \vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : {}^{n}\lfloor \mathsf{T}_1 \rfloor}$$

Ty-term-FillR

$$\frac{\Gamma \vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : {}^{n}\lfloor \mathsf{T}_2 \rfloor}$$

Ty-term-FillP

$$\frac{\Gamma \vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft (,) : {}^{n}\lfloor \mathsf{T}_1 \rfloor \otimes {}^{n}\lfloor \mathsf{T}_2 \rfloor}$$

Ty-term-FillE

$$\frac{\Gamma \vdash \mathsf{t} : {}^{m}\lfloor !^{n}\,\mathsf{T} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \rrparenthesis^{n} : {}^{m \cdot n}\lfloor \mathsf{T} \rfloor}$$

Ty-term-Alloc

$$\frac{}{\{\,\} \vdash \mathbf{alloc}_{\mathsf{T}} : {}^{\underline{\nu}}\lfloor \mathsf{T} \rfloor \rtimes \mathsf{T}}$$

Ty-term-ToA

$$\frac{\Gamma \vdash \mathsf{t} : \mathsf{T}}{\Gamma \vdash \mathbf{to}_{\rtimes}\ \mathsf{t} : \mathbf{1} \rtimes \mathsf{T}}$$

Ty-term-FromA

$$\frac{\Gamma \vdash \mathsf{t} : \mathbf{1} \rtimes \mathsf{T}}{\Gamma \vdash \mathbf{from}_{\rtimes}\ \mathsf{t} : \mathsf{T}}$$

$\boxed{\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}}$ *(Typing of commands (only a positive context is needed))*

Ty-cmd-C

$$\frac{\begin{array}{c}\Gamma_{11} \cup \Gamma_{12} \vdash \mathsf{v} : \mathsf{T} \\ \Gamma_2 \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{12} \Vdash e \\ \texttt{pctx\_DestOnly}\ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_{12}\ \Gamma_2\end{array}}{\Gamma_{11} \cup \Gamma_2 \vdash \mathsf{v} \diamond e : \mathsf{T}}$$

3 Effects and big-step semantics

$eff,\ e$	$::=$		Effect
	$\mid$	ε	
	$\mid$	$\mathtt{h} \coloneqq \overline{\mathsf{v}}$	
	$\mid$	$e_1, \ldots, e_{\mathsf{k}}$	Chain effects

$\boxed{\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash e}$ *(Typing of effects (require both positive and negative contexts))*

$$\frac{}{\{\} \mathbin{\scriptstyle\cup} \{\} \Vdash \varepsilon}\ \textsc{Ty-eff-N}$$

$$\frac{\begin{array}{c}\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{\mathsf{v}} : \mathsf{T} \\ \mathtt{ctx\_Disjoint}\ \Gamma\ \{@\mathtt{h} :_m {}^{n}\lfloor \mathsf{T} \rfloor\} \\ \mathtt{ctx\_Disjoint}\ \Gamma\ \Delta \\ \mathtt{ctx\_Disjoint}\ \{@\mathtt{h} :_m {}^{n}\lfloor \mathsf{T} \rfloor\}\ \Delta\end{array}}{(\uparrow\cdot m\cdot n)\cdot\Gamma \cup \{@\mathtt{h} :_m {}^{n}\lfloor \mathsf{T} \rfloor\} \mathbin{\scriptstyle\cup} (m\cdot n)\cdot\Delta \Vdash \mathtt{h} \coloneqq \overline{\mathsf{v}}}\ \textsc{Ty-eff-A}$$

$$\frac{\begin{array}{c}\Gamma_1 \mathbin{\scriptstyle\cup} \Delta_1 \cup @^{-1}\Gamma_{22} \Vdash e_1 \\ \Gamma_{21} \cup \Gamma_{22} \mathbin{\scriptstyle\cup} \Delta_2 \Vdash e_2 \\ \mathtt{pctx\_DestOnly}\ \Gamma_{22} \\ \mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{21} \\ \mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{22} \\ \mathtt{ctx\_Disjoint}\ \Gamma_1\ \Delta_1 \\ \mathtt{ctx\_Disjoint}\ \Gamma_1\ \Delta_2 \\ \mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Gamma_{22} \\ \mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_1 \\ \mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_2 \\ \mathtt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_1 \\ \mathtt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_2 \\ \mathtt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\end{array}}{\Gamma_1 \cup \Gamma_{21} \mathbin{\scriptstyle\cup} \Delta_1 \cup \Delta_2 \Vdash e_1, e_2}\ \textsc{Ty-eff-P}$$

$\boxed{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid e_1 \ \rightsquigarrow\ \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2}$ *(Big-step evaluation of effects on extended values)*

$$\frac{}{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \varepsilon \ \rightsquigarrow\ \overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \varepsilon}\ \textsc{Sem-eff-N}$$

$$\frac{\begin{array}{c}\mathtt{ctx\_HvarNotMem}\ \mathtt{h}\ \Delta_1 \\ \overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid e_1 \ \rightsquigarrow\ \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2\end{array}}{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}'}, e_1 \ \rightsquigarrow\ \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}'}, e_2}\ \textsc{Sem-eff-S}$$

$$\frac{\begin{array}{c}\Gamma_0 \mathbin{\scriptstyle\cup} \Delta_0 \Vdash \overline{\mathsf{v}_0} : \mathsf{T} \\ \mathtt{ctx\_Disjoint}\ \Gamma_0\ \Delta_0 \\ \mathtt{ctx\_Disjoint}\ \Delta_1\ \{\mathtt{h} :^{n} \mathsf{T}\} \\ \mathtt{ctx\_Disjoint}\ \Delta_1\ \Delta_0 \\ \overline{\mathsf{v}_1}[\mathtt{h} \coloneqq \overline{\mathsf{v}_0}]\ {}_{(\Delta_1 \cup n\cdot \Delta_0)} \mid e_1 \ \rightsquigarrow\ \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2\end{array}}{\overline{\mathsf{v}_1}\ {}_{\Delta_1 \cup \{\mathtt{h} :^{n} \mathsf{T}\}} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}_0}, e_1 \ \rightsquigarrow\ \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2}\ \textsc{Sem-eff-F}$$

$\boxed{\mathsf{t} \Downarrow \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

$$\frac{}{\mathsf{v} \Downarrow \mathsf{v} \diamond \varepsilon}\ \textsc{Sem-term-V}$$

$$\frac{\begin{array}{c}\mathsf{t}_1 \Downarrow \mathsf{v}_1 \diamond e_1 \qquad \mathsf{t}_2 \Downarrow \lambda \mathsf{x}\,.\,\mathsf{u} \diamond e_2 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_3\end{array}}{\mathsf{t}_1 \succ \mathsf{t}_2 \Downarrow \mathsf{v}_3 \diamond e_1, e_2, e_3}\ \textsc{Sem-term-App}$$

$$\frac{\mathsf{t}_1 \Downarrow () \diamond e_1 \qquad \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1 \ ;\ \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_1, e_2}\ \textsc{Sem-term-PatU}$$

$$\frac{\begin{array}{c}\mathsf{t} \Downarrow \mathsf{Inl}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_1[\mathsf{x}_1 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\ ,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}\ \textsc{Sem-term-PatL}$$

$$\frac{\begin{array}{c}\mathsf{t} \Downarrow \mathsf{Inr}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_2[\mathsf{x}_2 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\ ,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}\ \textsc{Sem-term-PatR}$$

$$\frac{\begin{array}{c}\mathsf{t} \Downarrow (\mathsf{v}_1\ ,\ \mathsf{v}_2) \diamond e_1 \\ \mathsf{u}[\mathsf{x}_1 \coloneqq \mathsf{v}_1][\mathsf{x}_2 \coloneqq \mathsf{v}_2] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1\ ,\ \mathsf{x}_2) \mapsto \mathsf{u} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}\ \textsc{Sem-term-PatP}$$

$$\frac{\begin{array}{c}\mathsf{t} \Downarrow \langle \mathsf{v}_1\, ,\, \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_1 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_2 \\ \overline{\mathsf{v}_2}\ {}_{\Delta} \mid e_2 \ \rightsquigarrow\ \overline{\mathsf{v}_4}\ {}_{\Delta'} \mid e_3\end{array}}{\mathsf{t} \succ \mathbf{mapL}\ \mathsf{x} \mapsto \mathsf{u} \Downarrow \langle \mathsf{v}_3\, ,\, \overline{\mathsf{v}_4} \rangle_{\Delta'} \diamond e_1, e_3}\ \textsc{Sem-term-Map}$$

$$\frac{\mathtt{hvar\_Fresh}\ \mathtt{h}}{\mathbf{alloc}_{\mathsf{T}} \Downarrow \langle @\mathtt{h}\, ,\, \mathtt{h} \rangle_{\{\mathtt{h} :^{\underline{\nu}} \mathsf{T}\}} \diamond \varepsilon}\ \textsc{Sem-term-Alloc}$$

$$\frac{\mathsf{t} \Downarrow \mathsf{v} \diamond e}{\mathbf{to}_{\ltimes}\ \mathsf{t} \Downarrow \langle ()\, ,\, \mathsf{v} \rangle_{\{\}} \diamond e}\ \textsc{Sem-term-ToA}$$

$$\frac{\mathsf{t} \Downarrow \langle ()\, ,\, \mathsf{v} \rangle_{\{\}} \diamond e}{\mathbf{from}_{\ltimes}\ \mathsf{t} \Downarrow \mathsf{v} \diamond e}\ \textsc{Sem-term-FromA}$$

$$\frac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e}{\mathsf{t} \triangleleft () \Downarrow () \diamond e, \mathtt{h} \coloneqq ()}\ \textsc{Sem-term-FillU}$$

$$\frac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e \qquad \mathtt{hvar\_Fresh}\ \mathtt{h}'}{\mathsf{t} \triangleleft \mathsf{Inl} \Downarrow @\mathtt{h}' \diamond e, \mathtt{h} \coloneqq \mathsf{Inl}\,\mathtt{h}'}\ \textsc{Sem-term-FillL}$$

$$\frac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e}{\mathsf{t} \triangleleft \mathsf{Inr} \Downarrow @\mathtt{h}' \diamond e, \mathtt{h} \coloneqq \mathsf{Inr}\,\mathtt{h}'}\ \textsc{Sem-term-FillR}$$

$$\frac{\begin{array}{c}\mathsf{t} \Downarrow @\mathtt{h} \diamond e \qquad \mathtt{hvar\_Fresh}\ \mathtt{h}_1 \\ \mathtt{hvar\_Fresh}\ \mathtt{h}_2\end{array}}{\mathsf{t} \triangleleft (,) \Downarrow (@\mathtt{h}_1\ ,\ @\mathtt{h}_2) \diamond e, \mathtt{h} \coloneqq (\mathtt{h}_1\ ,\ \mathtt{h}_2)}\ \textsc{Sem-term-FillP}$$

$$\frac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e_1 \qquad \mathsf{u} \Downarrow \langle \mathsf{v}_1\, ,\, \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_2}{\mathsf{t} \triangleleft\!\bullet\, \mathsf{u} \Downarrow \mathsf{v}_1 \diamond e_1, e_2, \mathtt{h} \coloneqq \overline{\mathsf{v}_2}}\ \textsc{Sem-term-FillC}$$

4 Type safety

Theorem 1 (Type safety)**.** *If* $\texttt{pctx\_DestOnly}\ \Gamma$ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t}\ \Downarrow\ \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

Theorem 2 (Type safety for complete programs)**.** *If* $\{\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t}\ \Downarrow\ \mathsf{v} \diamond \varepsilon$ *and* $\{\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v}\ \Downarrow\ \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$
  (3) $\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t}\ \Downarrow\ \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \cup \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \cup \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u}\ \Downarrow\ \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \cup \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \cup \Gamma_{22}$.
  Using Lemma ?? on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x} \,.\, \mathsf{t}'$ and (14) $\Gamma_{21} \cup \Gamma_{23} \cup \{\mathsf{x} :_m \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma ?? on (14) and (5) we get (15) $m{\cdot}(\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1]\ \Downarrow\ \mathsf{v}_3 \diamond e_3$ where (17) $m{\cdot}(\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{23} \Vdash e_2$

  $\texttt{ctx\_Disjoint}\ \Gamma_{12}\ \Gamma_{22}$ comes naturally from (3), (7) and (12).

  We must show:
  $\texttt{ctx\_Disjoint}\ \Gamma_{12}\ \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  $\texttt{ctx\_Disjoint}\ \Gamma_{22}\ \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  $\texttt{ctx\_Disjoint}\ \Gamma_{13}\ \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \cup \Gamma_{22} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1, e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $m{\cdot}(\Gamma_{111} \cup \Gamma_{131}) \cup \Gamma_{211} \cup \Gamma_{231} \cup \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $m{\cdot}(\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{\mathsf{k}1} \cup \Gamma_{\mathsf{k}2} = \Gamma_{\mathsf{k}}$

  We have
  (18) $\Gamma_{12} \cup \Gamma_{22} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1, e_2$
  (20) $m{\cdot}(\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \mathbin{\scriptstyle\cup} @^{-1}\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $@^{-1}\Gamma_{23}$, we get (22) $\Gamma_{12} \cup \Gamma_{22} \mathbin{\scriptstyle\cup} @^{-1}(\Gamma_{131} \cup \Gamma_{231}) \cup @^{-1}(\Gamma_{132} \cup \Gamma_{232}) \Vdash e_1, e_2$

  We want Γ_{132} from (22) to cancel $m{\cdot}\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{@\mathsf{h} :_{\underline{\nu}} {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash @\mathsf{h}' \diamond \mathsf{h} \coloneqq \mathsf{Inl}\,\mathsf{h}' : {}^{n}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} \coloneqq \mathsf{Inl}\,\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(@\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\ (\mathsf{x}_1\ ,\ \mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ @\mathsf{h}_1)\ ;\ \mathsf{x}_2) \succ (\lambda \mathsf{x}_2 \,.\, @\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ \mathsf{x}_2))$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all BigStep_Fill⟨*Ctor*⟩ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TyCmd_Cmd), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;

- or are those of pre-existing holes coming from the extended value $\overline{\mathsf{v}_2}$ of an ampar, when BigStep_FillComp is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program. And v_1 is exactly the accompanying value returned by the evaluation of BigStep_FillComp

TODO: prove that this property is preserved by typing rules □