termvar, x, y, d Term holevar, h Hole	n-level variable	
term_value, v	::= $\langle v_1, \overline{v_2} \rangle_H$ $\mathbf{0h}$ () $\mathbf{Inl} \ v$ $\mathbf{Inr} \ v$ (v_1, v_2) $\mathbf{E}_m \ v$ $\lambda \mathbf{x} \cdot \mathbf{t}$ (v)	Term value Ampar Destination Unit Left variant for sum Right variant for sum Product Exponential Linear function
extended_value, v		Store value Term value Hole Left variant with val or hole Right variant with val or hole Product with val or hole Exponential with val or hole S M
term, t, u	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Term value Variable Application Pattern-match on unit Pattern-match on sum Pattern-match on product Pattern-match on exponential Map over the left side of the ampar Wrap t into a trivial ampar Extract value from trivial ampar Return a fresh "identity" ampar object Fill destination with unit Fill destination with left variant Fill destination with product constructor Fill destination with exponential constructor Fill destination with root of ampar u S M
sub		Variable substitution
effect, e	$ \begin{vmatrix} \varepsilon \\ \mathbf{h} := \overline{\mathbf{v}} \\ \mathbf{e}_1 \cdot \mathbf{e}_2 \\ \mathbf{e} \end{vmatrix} $	Effect No effect

type, A, B	::=	$egin{array}{l} 1 & & & & & & & & & & & & & & & & & & &$	S	Type Unit Sum Product Exponential Ampar type (consuming A ₁ yields A ₂) Linear function Destination
$multiplicity, \ m$::= 		S	Multiplicity (Semiring with product ·) Born now. Identity of the product One scope older Infinitely old / static. Absorbing for product Semiring product
typing_context, Δ	::= 	Г Н Г ⊔ Н		Typing context
pos_context, Γ	::= 	$ \begin{cases} pos_assigns \rbrace \\ \Gamma_1 \sqcup \Gamma_2 \\ -\mathrm{H} \\ m \cdot \Gamma \\ (\Gamma) \end{cases} $	S	Positive typing context
pos_assign, pa	::=	$+$ × : $_m$ A $+$ h : $_m$ A		Positive type assignment Variable Destination
pos_assigns	::=	pa pa, pos_assigns		Positive type assignments
neg_assign, na	::=	-h : _m A		Negative type assignment Hole
neg_assigns	::= 	na na, neg_assigns		Negative type assignments
neg_context, H	::=	$ \begin{cases} neg_assigns \rbrace \\ H_1 \sqcup H_2 \\ -\Gamma \\ m{\cdot}H \\ (H) \\ \end{cases} $	S	Negative typing context

```
Effect application
eff_app
                           ::=
                                      e, \overline{v}_H
                                      apply (eff_app)
                                      e · eff_app
terminals
                                       \rtimes
                                       \mapsto
                                       ()
                                      Inl
                                      Inr
                                      (,)
                                       ◁
                                       <▶
                                      :=
                                      \sqcup
                                      \emptyset
                                      \exists
                                      \neq \leq \leq \neq \subset
                                      \mathcal{N}
                                       \vdash
                                       \Vdash
                                         \Downarrow
formula
                                      judgement
Ctx
                           ::=
                                      \mathbf{x} \in \mathcal{N}\left(\Delta\right)
                                      \mathbf{h} \in \mathcal{N}(\Delta)
                                      \mathbf{x} \notin \mathcal{N}(\Delta)
                                      \mathbf{h} \notin \mathcal{N}(\Delta)
                                      fresh x
                                      fresh h
                                      \mathsf{pos}\_\mathsf{assign} \in \Gamma
                                      \mathsf{neg}\_\mathsf{assign}\,\in\,H
                                      \mathbf{onlyPositive}\left(\Delta\right)
                                      onlyNegative (\Delta)
Eq
                                      \mathbf{A}_1 = \mathbf{A}_2
                                      A_1 \neq A_2
                                      t = u
                                      t \neq u
                                      \Delta_1 = \Delta_2
```

 $\mathcal{N}(\Delta_1) \cap \mathcal{N}(\Delta_2) = \emptyset$

```
Ту
                                 ::=
                                           \Delta \; \Vdash \; \overline{\mathsf{v}} : \mathbf{A}
                                           \Gamma \, \vdash \, \mathsf{t} : \textcolor{red}{\textbf{A}}
 Sem
                                ::=
                                                                                               (we assume effect lists are \varepsilon-terminated)
                                           \mathsf{eff}\_\mathsf{app}_1 = \mathsf{eff}\_\mathsf{app}_2
                                           t \Downarrow v \mid e
 judgement
                                           \mathsf{Ctx}
                                           Eq
                                           Ту
                                           Sem
 user_syntax
                                ::=
                                           termvar
                                           holevar
                                           term_value
                                           extended_value
                                           term
                                           sub
                                           effect
                                           type
                                           multiplicity
                                           typing_context
                                           pos_context
                                           pos_assign
                                           pos_assigns
                                           neg_assign
                                           neg_assigns
                                           neg_context
                                           eff_app
                                           terminals
\mathsf{x} \in \mathcal{N}(\Delta)
\mathbf{h} \in \mathcal{N}(\Delta)
\mathbf{x} \notin \mathcal{N}(\Delta)
\mathbf{h} \notin \mathcal{N}\left(\Delta\right)
fresh x
fresh h
\mathsf{pos}\_\mathsf{assign} \, \in \, \Gamma
\mathsf{neg\_assign} \, \in \, H
\operatorname{onlyPositive}\left(\Delta\right)
onlyNegative (\Delta)
\overline{\mathbf{A}_1} = \overline{\mathbf{A}_2}
A_1 \neq A_2
t = u
t \neq u
\Delta_1 = \Delta_2
\mathcal{N}(\Delta_1) \cap \mathcal{N}(\Delta_2) = \emptyset
```

 $\Delta \Vdash \overline{\mathsf{v}} : \mathsf{A}$

 $\Gamma \vdash \mathsf{t} : \mathsf{A}$

```
\overline{\emptyset} \sqcup \{-\mathbf{h} :_{\nu} \mathbf{A}\} \Vdash \mathbf{h} : \mathbf{A}
 TyValExt_Hole
                                              \frac{}{\{+\mathbf{h}:_{m}\mathbf{A}\}\, \cup\, \emptyset\, \Vdash\, \mathbf{@h}: [\mathbf{A}]_{m}} \quad \text{TyValExt\_Dest}
                                                                         \overline{\emptyset \sqcup \emptyset \Vdash ():1} TyValExt_Unit
                                                             \frac{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \overline{\nu} : \textbf{A}_1}{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \textbf{Inl} \, \overline{\nu} : \textbf{A}_1 \! \oplus \! \textbf{A}_2} \quad \text{TyValext\_Inl}
                                                           \frac{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \overline{\nu} : \mathsf{A}_2}{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \mathsf{Inr} \, \overline{\nu} : \mathsf{A}_1 \oplus \mathsf{A}_2} \quad \mathsf{TYVALEXT\_INR}
                                         \Gamma_1 \,{\scriptscriptstyle \,\sqcup\,} \, H_1 \, \Vdash \, \overline{\mathsf{v}_1} : \textbf{A}_1
                                         \Gamma_2 \mathrel{{\scriptscriptstyle \sqcup}} H_2 \; \Vdash \; \overline{\mathsf{v}_2} : {\color{red}\mathsf{A}_2}
                          \frac{\mathcal{N}(\Gamma_1 \, {\scriptscriptstyle \sqcup}\, H_1) \cap \mathcal{N}(\Gamma_2 \, {\scriptscriptstyle \sqcup}\, H_2) = \emptyset}{\Gamma_1 \, {\scriptscriptstyle \sqcup}\, \Gamma_2 \, {\scriptscriptstyle \sqcup}\, H_1 \, {\scriptscriptstyle \sqcup}\, H_2 \, \Vdash \, (\overline{v_1}\,,\, \overline{v_2}) : \textbf{A}_1 \otimes \textbf{A}_2} \quad \text{TyValext\_Prod}
                                                      \frac{\Gamma \, \sqcup \, \mathbf{H} \, \Vdash \, \overline{\mathbf{v}} : \mathbf{A}}{m \cdot \Gamma \, \sqcup \, m \cdot \mathbf{H} \, \Vdash \, \mathbf{E}_m \, \overline{\mathbf{v}} : !_m \mathbf{A}} \quad \mathbf{TyValExt\_Exp}
                                                              -\mathbf{H} \sqcup \emptyset \Vdash \mathsf{v}_1 : \mathsf{A}_1
                                        \frac{\Gamma_2 \sqcup H \Vdash \overline{v_2} : \mathbf{A}_2}{\Gamma_2 \sqcup \emptyset \Vdash \langle \mathsf{v}_1, \overline{\mathsf{v}_2} \rangle_H : \mathbf{A}_1 \rtimes \mathbf{A}_2} \quad \text{TyValext\_Ampar}
                                           \frac{\Gamma \sqcup \{+\times :_{m} \mathsf{A}_{1}\} \vdash \mathsf{t} : \mathsf{A}_{2}}{\Gamma \sqcup \emptyset \Vdash \lambda \times . \mathsf{t} : \mathsf{A}_{1} \xrightarrow{m} \mathsf{A}_{2}} \quad \text{TyValext\_Lambda}
                                                                          \frac{\Gamma \sqcup \emptyset \Vdash \mathsf{v} : \mathsf{A}}{\Gamma \vdash \mathsf{v} : \mathsf{A}} \quad \mathsf{TYTERM\_VAL}
                                                             \overline{\{+\mathsf{x}:_{\nu}\mathsf{A}\}} \vdash \mathsf{x}:\mathsf{A} TYTERM_VARNOW
                                                              \frac{}{\{+\times :_{\infty} A\} \vdash \times : A} TYTERM_VARINF
                                                                  \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1
                                                                   \Gamma_2 \vdash \mathsf{u} : \mathsf{A}_1 \xrightarrow{m} \mathsf{A}_2
                                                          \frac{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash t \; \succ \; \mathsf{u} : \mathsf{A}_2} \quad \mathsf{TYTERM\_APP}
                                                          \Gamma_1 \vdash \mathsf{t} : \mathsf{1}
                                                          \Gamma_2 \vdash \mathsf{u} : \mathsf{B}
                                        \frac{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \, \vdash \, t \; \succ \! \mathsf{case} \, () \mapsto \, \mathsf{u} : \mathsf{B}} \quad \mathsf{TYTERM\_PATUNIT}
                                               \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1 \oplus \mathsf{A}_2
                                               \Gamma_2 \sqcup \{+\mathsf{x}_1 :_m \mathsf{A}_1\} \vdash \mathsf{u}_1 : \mathsf{B}
                                               \Gamma_2 \sqcup \{+\mathsf{x}_2 :_m \mathsf{A}_2\} \vdash \mathsf{u}_2 : \mathsf{B}
                                              \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset
                                                                                                                                                                                                           TyTerm_PatSum
m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{case} \{ \mathsf{Inl} \times_1 \mapsto \mathsf{u}_1 \,, \; \mathsf{Inr} \times_2 \mapsto \mathsf{u}_2 \} : \mathsf{B}
                         \Gamma_1 \vdash \mathsf{t} : \mathbf{A}_1 \otimes \mathbf{A}_2
                         \Gamma_2 \sqcup \{+\mathsf{x}_1 :_m \mathsf{A}_1, +\mathsf{x}_2 :_m \mathsf{A}_2\} \vdash \mathsf{u} : \mathsf{B}
```

 $\frac{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{case}(\mathsf{x}_1, \mathsf{x}_2) \mapsto \mathsf{u} : \mathsf{B}} \quad \text{TyTerm_PatProd}$

 $\mathcal{N}(\Gamma_1)\cap\mathcal{N}(\Gamma_2)=\emptyset$

```
\Gamma_1 \vdash \mathsf{t} : !_{m'} \mathsf{A}
                                                                                                                                \Gamma_2 \sqcup \{+\mathsf{x}:_{m\cdot m'} \mathsf{A}_1\} \vdash \mathsf{u}: \mathsf{B}
                                                                                                                \frac{1}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case} \, \mathsf{E}_{m'} \, \mathsf{x} \mapsto \, \mathsf{u} : \mathsf{B}} \quad \mathsf{TYTERM\_PATEXP}
                                                                                                                           \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1 \rtimes \mathsf{A}_2
                                                                                                                           \uparrow \cdot \Gamma_2 \sqcup \{+\mathbf{x} :_{\nu} \mathbf{A}_1\} \vdash \mathbf{u} : \mathbf{B}
                                                                                                        \frac{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \; \succ \! \mathsf{mapL} \times \! \mapsto \, \mathsf{u} : \mathsf{B} \rtimes \mathsf{A}_2}
                                                                                                                                                                                                                                                         TyTerm_MapAmpar
                                                                                                                                        \Gamma_1 \vdash \mathsf{t} : |\mathsf{A}_2|_m
                                                                                                                                        \Gamma_2 \vdash \mathsf{u} : \mathsf{A}_1 \rtimes \mathsf{A}_2
                                                                                                                           \frac{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup (\uparrow \cdot m) \cdot \Gamma_2 \, \vdash \, \mathsf{t} \triangleleft_{\bullet} \, \mathsf{u} : \mathsf{A}_1} \quad \mathsf{TYTERM\_FILLCOMP}
                                                                                                                                                   \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{1} \rfloor_m}{\Gamma \vdash \mathsf{t} \triangleleft () : \mathsf{1}} \quad \mathsf{TYTERM\_FILLUNIT}
                                                                                                                                             \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{A}_1 \oplus \mathsf{A}_2 \rfloor_m}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : |\mathsf{A}_1|_m} \quad \mathsf{TYTERM\_FILLINL}
                                                                                                                                            \frac{\Gamma \vdash \mathsf{t} : [\mathsf{A}_1 \oplus \mathsf{A}_2]_m}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : [\mathsf{A}_2]_m} \quad \mathsf{TYTERM\_FILLINR}
                                                                                                                            \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{A}_1 \otimes \mathsf{A}_2 \rfloor_m}{\Gamma \vdash \mathsf{t} \triangleleft (,) : |\mathsf{A}_1|_m \otimes |\mathsf{A}_2|_m} \quad \mathsf{TYTERM\_FILLPROD}
                                                                                                                                         \frac{\Gamma \vdash \mathsf{t} : \lfloor !_{m'} \mathsf{A} \rfloor_{m}}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{E}_{m} : |\mathsf{A}|_{m \cdot m'}} \quad \mathsf{TYTERM\_FILLEXP}
                                                                                                                                              \overline{\emptyset \, \vdash \, \mathsf{alloc}_{\mathsf{A}} : \lfloor \mathsf{A} \rfloor_{\nu} \rtimes \mathsf{A}} \quad \mathrm{TYTERM\_ALLOC}
                                                                                                                                               \frac{\Gamma \vdash \mathsf{t} : \mathsf{A}}{\Gamma \vdash \mathsf{to}_{\bowtie} \mathsf{t} : 1 \rtimes \mathsf{A}} \quad \mathsf{TYTERM\_ToAMPAR}
                                                                                                                                           \frac{\Gamma \vdash t: 1 \rtimes \textbf{A}}{\Gamma \vdash \textbf{from}_{\rtimes} t: \textbf{A}} \quad \text{TyTerm\_FromAmpar}
     eff_app_1 = eff_app_2
                                                                                                    (we assume effect lists are \varepsilon-terminated)
                                                                                                                                              \overline{\mathsf{apply}\left(\varepsilon,\,\overline{\vee}_{\,\mathrm{H}}\right)=\varepsilon,\,\overline{\vee}_{\,\mathrm{H}}}\quad\mathrm{EffApp}\_\mathrm{NoEff}
                                                                                                       \frac{\mathbf{h} \notin \mathcal{N}\left(H\right)}{\text{apply}\left(\mathbf{h} \coloneqq \overline{v'} \cdot e, \, \overline{v}_{H}\right) = \mathbf{h} \coloneqq \overline{v'} \, \, \widehat{\cdot} \, \, \text{apply}\left(e, \, \overline{v}_{H}\right)}
                                                                                                                                                                                                                                                                                      EffApp_Skip
                                                                                 \overline{\text{apply}\left(\mathbf{\underline{h}}\text{:=}\left(\right)\,\cdot\,\mathbf{e},\,\overline{\mathbf{v}}_{\,H\sqcup\left\{-\mathbf{\underline{h}}\text{:}_{\nu}\mathbf{1}\right\}}\right)=\text{apply}\left(\mathbf{e},\,\overline{\mathbf{v}}|\mathbf{\underline{h}}\text{:=}\left(\right)|_{\,H}\right)}\quad EffApp\_FillUnit
                                                                                                                                                                                                                                                                                                                                EffApp_FillInl
                                               \overline{\mathsf{apply}\,(\textcolor{red}{\mathtt{h}} \coloneqq \mathsf{Inl}\,\textcolor{red}{\mathtt{h}'}\,\cdot\, e,\, \overline{\mathsf{v}}_{\,\mathrm{H}\sqcup\{-\mathtt{h}:_{\nu}\mathsf{A}_{1}\oplus\mathsf{A}_{2}\}}) = \mathsf{apply}\,(e,\, \overline{\mathsf{v}}[\textcolor{red}{\mathtt{h}} \coloneqq \mathsf{Inl}\,\textcolor{red}{\mathtt{h}'}]_{\,\mathrm{H}\sqcup\{-\mathtt{h}':_{\nu}\mathsf{A}_{1}\}})}
                                              \overline{\text{apply}\left( \overset{\textbf{h}}{\overset{\text{...}}{\text{...}}} \text{Inr} \overset{\textbf{h}'}{\text{...}} \cdot \text{e}, \, \overline{\text{v}}_{H \sqcup \{-\overset{\textbf{h}:}{\text{...}}} \textbf{A}_1 \oplus \textbf{A}_2 \}} \right) = \text{apply}\left( \text{e}, \, \overline{\text{v}} [\overset{\textbf{h}:=\text{Inr}}{\text{...}} \overset{\textbf{h}'}{\text{...}}]_{H \sqcup \{-\overset{\textbf{h}':}{\text{...}}} \textbf{A}_2 \}} \right)} \quad \text{EffApp\_FillInr}
               \overline{\text{apply}\left(\textbf{h} \coloneqq \left(\textbf{h}_{1} \,,\, \textbf{h}_{2}\right) \,\cdot\, \textbf{e},\, \overline{\textbf{v}}_{\, \textbf{H} \sqcup \left\{-\textbf{h}: \nu \textbf{A}_{1} \otimes \textbf{A}_{2}\right\}}\right) = \text{apply}\left(\textbf{e},\, \overline{\textbf{v}}[\textbf{h} \coloneqq \left(\textbf{h}_{1} \,,\, \textbf{h}_{2}\right)]_{\, \textbf{H} \sqcup \left\{-\textbf{h}_{1}: \nu \textbf{A}_{1}, -\textbf{h}_{2}: \nu \textbf{A}_{2}\right\}}\right)} \quad \text{EffApp\_FillProd}}
                                          \Gamma' \, {\scriptstyle \sqcup} \, \operatorname{H}' \, \Vdash \, \overline{\mathsf{v}'} : \mathsf{A}
\frac{\mathcal{N}(H \sqcup \{-\textbf{h}:_{\nu} \textbf{A}\}) \cap \mathcal{N}(H') = \emptyset}{\text{apply}\,(\textbf{h}:=\overline{v'}\,\cdot\,e,\,\overline{v}_{\,H\sqcup\{-\textbf{h}:_{\nu} \textbf{A}\}}) = \text{apply}\,(e,\,\overline{v}[\textbf{h}:=\overline{v'}]_{\,H\sqcup H'})}
                                                                                                                                                                                                                  EffApp_FillComp (Encompasses all other Fill rules)
```

Definition rules: 49 good 0 bad Definition rule clauses: 117 good 0 bad