```
metavariable, x, y
term, t, u
                                                                                                  _{\rm term}
                                                                                                      value
                                     V
                                                                                                      variable
                                     Х
                                     t u
                                                                                                      application
                                                                                                      effect sequencing
                                     case t of \{\star \mapsto u\}
                                                                                                      pattern-matching on unit
                                     \mathsf{case}\,\mathsf{t}\,\mathsf{of}\,\{\,\mathsf{Ur}\;\mathsf{x}\mapsto\mathsf{u}\}
                                                                                                      pattern-matching on exponentiated value
                                     case t of \{ \operatorname{Inl} x_1 \mapsto u_1, \operatorname{Inr} x_2 \mapsto u_2 \}
                                                                                                      pattern-matching on sum
                                     case t of \{\langle \mathsf{x}_1, \mathsf{x}_2 \rangle \mapsto \mathsf{u}\}
                                                                                                      pattern-matching on product
                                     case t of \{ @Rx \mapsto u \}
                                                                                                      unroll for recursive types
                                                                                                      allocate data
                                     alloc x.t
                                     t \triangleleft^p \star
                                                                                                      fill destination with unit
                                     t\triangleleft^p \lambda x : A.u
                                                                                                      fill destination with function
                                     \mathsf{t} \mathrel{\vartriangleleft^{p}} \mathsf{u}
                                     t ⊲<sup>p</sup> Ur y.u
                                                                                                      fill destination with exponential
                                     t \triangleleft^p Inl y.u
                                                                                                      fill sum-type destination with variant 1
                                                                                                      fill sum-type destination with variant 2
                                     t \triangleleft^p Inr y.u
                                     t\triangleleft^p \langle \mathsf{y}_1, \mathsf{y}_2 \rangle.u
                                                                                                      fill product-type destination
                                     t \triangleleft^p \mathbb{Q} R y.u
                                                                                                      fill destination with recursive type
                                                                                           S
                                     (t)
                                     t[subs]
                                                                                           Μ
hole, h
                             ::=
val, v
                                                                                                  unreducible value
                             ::=
                                                                                                      no-effect effect
                                     d
                                                                                                      data structure
data, d
                             ::=
                                      |h|
                                     \lambda x : A.t
                                     Ur d
                                     InId
                                     Inrd
                                     \langle \mathsf{d}_1, \mathsf{d}_2 \rangle
                                     @Rd
                                                                                           S
                                     (d)
multiplicity, p
                                                                                                  multiplicity
                                      1
                                     \omega
                                                                                                  substitution
sub
                             ::=
                                     x := t
                                     h := d
                                                                                                  substitutions
subs
                                     sub
```

```
sub, subs
data_with_hole, d
                                             d
                                             h
                                             Ur d
                                             InId
                                             Inr <u>d</u>
                                             \langle \underline{\mathsf{d}}_1,\underline{\mathsf{d}}_2 \rangle
                                             @Rd
                                                                     S
                                             (\underline{d})
store_affect, sa
                                                                            store cell
                                     ::=
                                             x : D = \underline{d}
store_affects
                                     ::=
                                                                            store cells
                                             sa
                                             sa, store\_affects
store, S
                                     ::=
                                                                            store contents
                                             {store_affects}
                                             \mathbb{S}_1 \sqcup \mathbb{S}_2
                                             S[subs]
                                                                     Μ
type, A
                                     ::=
                                             \perp
                                                                               bottom type
                                             D
                                                                               data type
data_type, D
                                             1
                                                                               unit type
                                             R
                                                                               recursive type bound to a name
                                            \mathsf{D}_1\otimes\mathsf{D}_2
                                                                               product type
                                            \mathsf{D}_1 {\oplus} \mathsf{D}_2
                                                                               sum type
                                             A_1 \multimap A_2
                                                                               linear function type
                                            |\mathsf{D}|^p
                                                                               destination type
                                             !D
                                                                               exponential
                                             (D)
                                                                     S
                                             \underline{D}[X := D]
                                                                     Μ
type_with_var, A
                                     ::=
                                             \perp
                                             D
data_type_with_var, D
                                             X
                                                                               type variable in recursive definition
                                             1
                                                                               unit type
                                                                               recursive type bound to a name
                                             \underline{\mathsf{D}}_1 \otimes \underline{\mathsf{D}}_2
                                                                               product type
```

```
\underline{\mathsf{D}}_1 \oplus \underline{\mathsf{D}}_2
                                                                                                       sum type
                                                                    \underline{\mathsf{A}}_1 \multimap \underline{\mathsf{A}}_2
                                                                                                       linear function type
                                                                    \lfloor \underline{\mathsf{D}} \rfloor^p
                                                                                                       destination type
                                                                    !<u>D</u>
                                                                                                       exponential
                                                                                             S
                                                                    (\underline{\mathsf{D}})
rec_type_bound, R
                                                                                                   recursive type bound to a name
                                                           ::=
rec_type_def
                                                           ::=
                                                                    \mu X.\underline{D}
type_affect, ta
                                                                                                   type affectation
                                                           ::=
                                                                                                       variable
                                                                    x : A
                                                                    h^p: D
                                                                                                       hole
type_affects
                                                           ::=
                                                                                                   type affectations
                                                                    ta
                                                                    ta, type_affects
typing_context, \Gamma, \mho, H, H_P, H_N
                                                           ::=
                                                                                                   typing context
                                                                    {type_affects}
                                                                    \Gamma_1 \sqcup \Gamma_2
command
                                                           ::=
                                                                    S \mid t
terminals
                                                           ::=
                                                                    ()
                                                                    \mapsto
                                                                    :=
                                                                    \sqcup
                                                                    \emptyset
                                                                    \neq
                                                                    \in
                                                                    ∉
                                                                    \n
                                                                    Inl
                                                                    Inr
                                                                    Ur
                                                                    Dest
                                                                    ◁
```

```
_C_
formula
                                 ::=
                                            judgement
Ctx
                                            \mathbf{x} \in \mathcal{N}(\Gamma)
                                            \mathbf{x} \notin \mathcal{N}(\Gamma)
                                            \mathsf{type\_affect} \, \in \, \Gamma
                                            \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset
                                                                                                                                       \Gamma_1 and \Gamma_2 are disjoint typing contexts with no
                                            \begin{array}{l} p_1 = p_2 \implies \Gamma_1 = \Gamma_2 \\ p_1 = p_2 \implies (\Gamma_1 = \Gamma_2 \land \Gamma_3 = \Gamma_4) \end{array}
Store
                                            fresh h
                                            store\_affect \, \in \, \mathbb{S}
                                            \mathbf{x}\notin\mathcal{N}\left(\mathbb{S}\right)
Eq
                                 ::=
                                            A_1 = A_2
                                            A_1 \neq A_2
                                            t = u
                                            \Gamma = \mathsf{D}
Ту
                                 ::=
                                            R \stackrel{\text{fix}}{=} \text{rec\_type\_def}
                                            \mho \; ; \; \Gamma \vdash \mathsf{command} : \mathsf{A}
                                            \mathrm{H}_\mathrm{N} \vdash \underline{\mathsf{d}} \ ^{p} \! \colon \ \mathsf{D} \vdash \mathrm{H}_\mathrm{P}
                                                                                                                                       H_P stands for "provides", H_N for "needs"
                                            \mathbb{S} \vdash H
                                            \mho; H \sqcup \Gamma \vdash t : A
Sem
                                 ::=
                                            \mathsf{command} \ \Downarrow \ \mathsf{command'}
judgement
                                 ::=
                                            Ctx
                                            Store
                                            Eq
                                            Ту
                                            Sem
```

```
user_syntax
                                     metavariable
                                     term
                                     hole
                                     val
                                     data
                                     multiplicity
                                     sub
                                     subs
                                     data_with_hole
                                     store\_affect
                                     store_affects
                                     store
                                     type
                                     data_type
                                     type_with_var
                                     data_type_with_var
                                     rec_type_bound
                                     rec_type_def
                                     type_affect
                                     type_affects
                                     typing_context
                                     command
                                     terminals
\mathbf{x}\in\mathcal{N}\left(\Gamma\right)
\mathbf{x} \notin \mathcal{N}(\Gamma)
\mathsf{type\_affect} \in \Gamma
\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \Gamma_1 and \Gamma_2 are disjoint typing contexts with no clashing variable names or labels
p_1 = p_2 \implies \Gamma_1 = \Gamma_2
p_1 = p_2 \implies \overline{(\Gamma_1 = \Gamma_2 \land \Gamma_3 = \Gamma_4)}
fresh h
store\_affect \in S
\mathbf{x}\notin\mathcal{N}\left(\mathbb{S}\right)
A_1 = \overline{A_2}
A_1 \neq A_2
\Gamma = \mathsf{D}
R \stackrel{fix}{=} rec\_type\_def
\mho ; \Gamma \vdash \mathsf{command} : \mathsf{A}
                                                   \frac{\mho ; H \sqcup \Gamma \vdash t : A}{\mho ; \Gamma \vdash \mathbb{S} \mid t : A} \quad TyComm\_Def
```

 $H_N \vdash \underline{d}$  \*P:  $D \vdash H_P$  H<sub>P</sub> stands for "provides",  $H_N$  for "needs"

$$\overline{\emptyset \vdash h} \stackrel{p}{:} D \vdash \overline{\{h} \stackrel{p}{:} D\} \qquad TYDH\_H$$

$$\overline{\{h} \stackrel{p}{:} D \vdash \overline{\{h} \stackrel{p}{:} D\}} \qquad TYDH\_DEST$$

$$\overline{\emptyset \vdash \star} \stackrel{p}{:} 1 \vdash \overline{\emptyset} \qquad TYDH\_U$$

$$\emptyset : H_N \circ \{x : A_1\} \vdash t : A_2$$

$$p = \omega \implies H_N = \emptyset$$

$$\overline{H_N \vdash \lambda \times : D \cdot t \cdot P : A_1 \multimap A_2 \vdash \emptyset} \qquad TYDH\_FN$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D \vdash H_P$$

$$\overline{H_N \vdash Ur \neq P : D \vdash H_P} \qquad TYDH\_E$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D_1 \vdash H_P$$

$$\overline{H_N \vdash Inl \neq P : D_1 \vdash H_P} \qquad TYDH\_INL$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D_1 \vdash H_P$$

$$\overline{H_N \vdash Inl \neq P : D_1 \vdash H_P} \qquad TYDH\_INR$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D_1 \vdash H_P$$

$$\overline{H_N \vdash Inl \neq P : D_1 \vdash H_P} \qquad TYDH\_INR$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D_1 \vdash H_P$$

$$\overline{S} \vdash H \vdash H_N$$

$$\overline{0 \vdash \emptyset} \qquad TYSTORE\_EMPTY$$

$$S \vdash H \sqcup H_N$$

$$\overline{H_N \vdash d} \stackrel{p}{:} D \vdash H_P$$

$$\overline{S} \sqcup \{x : D = d\} \vdash H \sqcup H_P$$

$$\overline{S} \sqcup \{x : D = d\} \vdash H \sqcup H_P$$

$$\overline{S} \sqcup \{x : D = d\} \vdash H \sqcup H_P$$

$$\overline{S} \sqcup \{x : D \vdash d\} \vdash H_1 \vdash D_1 \vdash D_1 \vdash TYTERM\_DEST$$

$$\overline{S} : \{h \vdash P \vdash D\} = \emptyset \vdash [h] : [D] \vdash TYTERM\_U$$

$$\frac{\emptyset : H \sqcup \{x : A_1\} \vdash t : A_2}{\overline{S} : H \sqcup \emptyset \vdash A \times : A_1 \cdot t : A_1 \multimap A_2} \qquad TYTERM\_FN$$

$$\frac{\emptyset : \emptyset \sqcup \emptyset \vdash A \sqcup D_1}{\overline{S} : H \sqcup \emptyset \vdash A \sqcup D_1} \qquad TYTERM\_E$$

$$\frac{\emptyset : \emptyset \sqcup \emptyset \vdash A \sqcup D_1}{\overline{S} : H \sqcup \emptyset \vdash A \sqcup D_1} \qquad TYTERM\_INL$$

$$\frac{\emptyset : H \sqcup \emptyset \vdash A \sqcup D_2}{\overline{S} : H \sqcup \emptyset \vdash A \sqcup D_1} \qquad TYTERM\_INR$$

 $\mathbb{S} \vdash \mathbf{H}$ 

```
\emptyset; H_1 \cup \emptyset \vdash d_1 : D_1
                                                           \frac{\emptyset \ ; \ H_2 \sqcup \emptyset \vdash \mathsf{d}_2 : \mathsf{D}_2}{\mho \ ; \ H_1 \sqcup H_2 \sqcup \emptyset \vdash \langle \mathsf{d}_1, \mathsf{d}_2 \rangle : \mathsf{D}_1 \otimes \mathsf{D}_2} \quad \mathsf{TYTERM\_P}
                                                                                 R \stackrel{\text{fix}}{=} \mu X \cdot \underline{D}
                                                                             \frac{\emptyset ; \overset{\cdot}{H} \cup \emptyset \vdash d : \underline{D}[X := R]}{\mho ; \overset{\cdot}{H} \cup \emptyset \vdash @R d : R} \quad TyTerm\_R
                                                                                     \overline{\mho \; ; \; \emptyset \; \sqcup \; \{ \times \; : \; \mathsf{A} \} \; \vdash \; \times \; : \; \mathsf{A}} \qquad \mathsf{TYTERM\_ID}
                                                                            \overline{\mho \sqcup \{ \mathsf{x} : \mathsf{A} \} \; ; \; \emptyset \sqcup \emptyset \vdash \mathsf{x} : \mathsf{A}} \quad \mathrm{TYTERM\_ID'}
                                                                           \mho : H_1 \sqcup \Gamma_1 \vdash t : A_1 \multimap A_2
                                                              \frac{\mho \ ; \ H_2 \ \lrcorner \ \Gamma_2 \vdash u : \mathsf{A}_1}{\mho \ ; \ H_1 \ \sqcup \ H_2 \ \lrcorner \ \Gamma_1 \ \sqcup \ \Gamma_2 \vdash \mathsf{t} \ u : \mathsf{A}_2} \quad \text{TYTERM\_APP}
                                                                             \mho : H_1 \sqcup \Gamma_1 \vdash t : \bot
                                                 \frac{\sigma \; ; \; H_2 \; \sqcup \; \Gamma_2 \vdash u : \mathsf{A}_2}{\sigma \; ; \; H_1 \; \sqcup \; H_2 \; \sqcup \; \Gamma_1 \; \sqcup \; \Gamma_2 \vdash t \; \; ; \; u : \mathsf{A}_2} \quad \text{TyTerm\_EffSeQ}
                                                                                    \mho \ ; \ H_1 \mathbin{{\scriptscriptstyle \sqcup}} \Gamma_1 \vdash t : \mathbf{1}
                               \frac{\mho \ ; \ H_2 \sqcup \Gamma_2 \vdash u : \mathsf{A}}{\mho \ ; \ H_1 \sqcup H_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{case} \ \mathsf{t} \ \mathsf{of} \ \{ \star \mapsto \mathsf{u} \} : \mathsf{A}}
                                                                                                                                                                                                                         TyTerm_PatU
                                                                 \mho; H_1 \sqcup \Gamma_1 \vdash t : !D
                          \frac{\mho \sqcup \{\mathsf{x} : \mathsf{D}\} \; ; \; \mathrm{H}_2 \sqcup \Gamma_2 \vdash \mathsf{u} : \mathsf{A}}{\mho \; ; \; \mathrm{H}_1 \sqcup \mathrm{H}_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{case} \, \mathsf{t} \, \mathsf{of} \, \{\, \mathsf{Ur} \, \mathsf{x} \mapsto \mathsf{u}\} : \mathsf{A}} \quad \mathsf{TYTERM\_PATE}
                                                            \mho ; H_1 \sqcup \Gamma_1 \vdash t : D_1 \oplus D_2
                                                            \mho : H_2 \sqcup \Gamma_2 \sqcup \{x_1 : D_1\} \vdash u_1 : A
                                                            \mho ; H_2 \,{\scriptstyle \sqcup}\, \Gamma_2 \,{\scriptstyle \sqcup}\, \{\mathsf{x}_2 : \mathsf{D}_2\} \vdash \mathsf{u}_2 : \mathsf{A}
\overline{\mho \; ; \; H_1 \sqcup H_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{case}\, \mathsf{t}\, \mathsf{of}\, \big\{\, \mathsf{Inl}\, \mathsf{x}_1 \mapsto \mathsf{u}_1, \, \mathsf{Inr}\, \mathsf{x}_2 \mapsto \mathsf{u}_2 \big\} : \mathsf{A}}
                                                                                                                                                                                                                                                              TYTERM_PATS
                                              \mho \ ; \ H_1 \mathbin{{\scriptscriptstyle \sqcup}} \Gamma_1 \vdash \mathsf{t} : \mathsf{D}_1 \otimes \mathsf{D}_2
                      \frac{\mho \; ; \; H_2 \sqcup \Gamma_2 \sqcup \{ \mathsf{x}_1 : \mathsf{D}_1, \mathsf{x}_2 : \mathsf{D}_2 \} \vdash \mathsf{u} : \mathsf{A}}{\mho \; ; \; H_1 \sqcup H_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{case} \, \mathsf{t} \, \mathsf{of} \, \left\{ \langle \mathsf{x}_1, \mathsf{x}_2 \rangle \mapsto \mathsf{u} \right\} : \mathsf{A}} \quad \mathsf{TYTERM\_PATP}
                                                 R \stackrel{\text{fix}}{=} \mu X.D
                                                \mho \ ; \ H_1 \mathbin{{\scriptstyle \;\sqcup\;}} \Gamma_1 \vdash t : {\textstyle \mathsf{R}}
                          \frac{\mho \; ; \; H_2 \; \sqcup \; \Gamma_2 \; \sqcup \; \{ \mathsf{X} : \; \underline{\mathsf{D}}[\mathsf{X} := \mathsf{R}] \} \vdash \mathsf{u} : \mathsf{A}}{\mho \; ; \; H_1 \; \sqcup \; H_2 \; \sqcup \; \Gamma_1 \; \sqcup \; \Gamma_2 \vdash \mathsf{case} \, \mathsf{t} \, \mathsf{of} \, \left\{ @\mathsf{R} \, \mathsf{X} \mapsto \mathsf{u} \right\} : \mathsf{A}} \quad \mathsf{TYTERM\_PATR}
                                                      \frac{\sigma ; H \sqcup \Gamma \sqcup \{x : \lfloor \mathsf{D} \rfloor^{1}\} \vdash \mathsf{t} : \bot}{\sigma ; H \sqcup \Gamma \vdash \mathsf{alloc} \times \mathsf{.t} : \mathsf{D}} \quad \mathsf{TYTERM\_ALLOC}
                                                                            \frac{\mho \; ; \; \mathbf{H} \; \sqcup \; \Gamma \vdash \mathbf{t} \; : \; \lfloor \mathbf{1} \rfloor^p}{\mho \; ; \; \mathbf{H} \; \sqcup \; \Gamma \vdash \mathbf{t} \; \vartriangleleft^p \; \star \; : \; \bot} \quad \mathsf{TYTERM\_FILLU}
                                                       \mho : H_1 \sqcup \Gamma_1 \vdash t : |A_1 \multimap A_2|^p
                                                       \mho \ ; \ H_2 \mathbin{{\scriptscriptstyle \sqcup}} \Gamma_2 \mathbin{{\scriptscriptstyle \sqcup}} \{ \mathsf{x} : \mathsf{A}_1 \} \vdash \mathsf{u} : \mathsf{A}_2
                               \frac{p = \omega \implies (H_2 = \emptyset \land \Gamma_2 = \emptyset)}{\mho ; H_1 \sqcup H_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \triangleleft^p \lambda \times : \mathsf{A_1.u} : \bot}
                                                                                                                                                                                                                  TyTerm_FillFn
                                                          \mho ; H_1 \sqcup \Gamma_1 \vdash \mathsf{t} : |\mathsf{D}|^p
                                                          \mho ; H_2 \sqcup \Gamma_2 \vdash u : D
                                                   \frac{p = \omega \implies (H_2 = \emptyset \land \Gamma_2 = \emptyset)}{\mho \; ; \; H_1 \sqcup H_2 \sqcup \Gamma_1 \sqcup \Gamma_2 \vdash t \vartriangleleft^p \; \sqcup : \bot} \quad \text{TyTerm\_FillL}
```

command ↓ command

$$\begin{array}{c} \text{fresh } h \\ S_0 \sqcup \{ \mathsf{x} : \mathsf{D} = h \} \mid \mathsf{t}[\mathsf{x} := \lfloor h \rfloor] \quad \Downarrow \quad S_1 \sqcup \{ \mathsf{x} : \mathsf{D} = d \} \mid \bullet \\ \hline S_0 \mid \mathsf{alloc} \times \mathsf{t} \quad \Downarrow \quad S_1 \mid \mathsf{d} \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad S_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{S}_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{S}_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{J} \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{J} \mid \lfloor h \rfloor \\ \hline \\ S_1 \mid h := \ln h' \mid \mathsf{l} \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{S}_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{S}_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{S}_1 \mid \lfloor h \rfloor \\ \hline \\ S_0 \mid \mathsf{t} \quad \Downarrow \quad \mathsf{I} \mid \mathsf{l} \mid \mathsf{l$$

Definition rules: 57 good 0 bad Definition rule clauses: 152 good 0 bad