Destination λ-calculus

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1 Term and value syntax

$\mathtt{tvar}, \mathsf{x}, \mathsf{y}, \mathsf{d}$	Term-level variable
$\mathtt{hvar}, \mathtt{h}$	Hole
$\mathtt{n}$	

v	$::=$		Term value
	$\mid$	$\langle \mathsf{v}_1 \,{\boldsymbol{,}}\, \overline{\mathsf{v}_2} \rangle_{\Delta}$	Ampar
	$\mid$	$@\mathtt{h}$	Destination
	$\mid$	$()$	Unit
	$\mid$	$\mathsf{Inl}\,\mathsf{v}$	Left variant for sum
	$\mid$	$\mathsf{Inr}\,\mathsf{v}$	Right variant for sum
	$\mid$	$(\mathsf{v}_1\,,\,\mathsf{v}_2)$	Product
	$\mid$	$\rrparenthesis^{\mathrm{M}}\,\mathsf{v}$	Exponential
	$\mid$	$\lambda \mathsf{x}\,.\,\mathsf{t}$	Linear function
$\overline{\mathsf{v}}$	$::=$		Pseudo-value that may contain holes
	$\mid$	v	Term value
	$\mid$	$\mathtt{h}$	Hole
	$\mid$	$\mathsf{Inl}\,\overline{\mathsf{v}}$	Left variant with val or hole
	$\mid$	$\mathsf{Inr}\,\overline{\mathsf{v}}$	Right variant with val or hole
	$\mid$	$(\overline{\mathsf{v}_1}\,,\,\overline{\mathsf{v}_2})$	Product with val or hole
	$\mid$	$\rrparenthesis^{\mathrm{M}}\,\overline{\mathsf{v}}$	Exponential with val or hole
$\mathsf{t},\ \mathsf{u}$	$::=$		Term
	$\mid$	v	Term value
	$\mid$	x	Variable
	$\mid$	$\mathsf{t} \succ \mathsf{u}$	Application
	$\mid$	$\mathsf{t} \succ \mathbf{case}\ () \mapsto \mathsf{u}$	Pattern-match on unit
	$\mid$	$\mathsf{t} \succ \mathbf{case}\ \{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\}$	Pattern-match on sum
	$\mid$	$\mathsf{t} \succ \mathbf{case}\ (\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u}$	Pattern-match on product
	$\mid$	$\mathsf{t} \succ \mathbf{case}\ \rrparenthesis^{\mathrm{M}}\,\mathsf{x} \mapsto \mathsf{u}$	Pattern-match on exponential
	$\mid$	$\mathsf{t} \succ \mathbf{mapL}\ \mathsf{x} \mapsto \mathsf{u}$	Map over the left side of the ampar
	$\mid$	$\mathbf{to}_{\rtimes}\,\mathsf{t}$	Wrap t into a trivial ampar
	$\mid$	$\mathbf{from}_{\rtimes}\,\mathsf{t}$	Extract value from trivial ampar
	$\mid$	$\mathbf{alloc}_{\mathsf{T}}$	Return a fresh "identity" ampar object
	$\mid$	$\mathsf{t} \triangleleft ()$	Fill destination with unit
	$\mid$	$\mathsf{t} \triangleleft \mathsf{Inl}$	Fill destination with left variant
	$\mid$	$\mathsf{t} \triangleleft \mathsf{Inr}$	Fill destination with right variant
	$\mid$	$\mathsf{t} \triangleleft (,)$	Fill destination with product constructor
	$\mid$	$\mathsf{t} \triangleleft \rrparenthesis^{\mathrm{M}}$	Fill destination with exponential constructor
	$\mid$	$\mathsf{t} \triangleleft\!\bullet\, \mathsf{u}$	Fill destination with root of ampar u

2 Type system

$T,\ U$	$::=$		Type
	$\mid$	1	Unit
	$\mid$	$T_1 \oplus T_2$	Sum
	$\mid$	$T_1 \otimes T_2$	Product
	$\mid$	$!^{M}\, T$	Exponential
	$\mid$	$T_1 \rtimes T_2$	Ampar type (consuming T_1 yields T_2)
	$\mid$	$T_1 \,{}_{M}\!\!\to T_2$	Linear function
	$\mid$	${}^{M}\lfloor T \rfloor$	Destination
$M,\ N$	$::=$		Multiplicity (Semiring with product $\cdot$)
	$\mid$	ν	Born now. Identity of the product
	$\mid$	$\uparrow$	One scope older
	$\mid$	∞	Infinitely old / static. Absorbing for product
	$\mid$	$M_1 \cdot \ldots \cdot M_n$	Semiring product
$PC,\ \Gamma$	$::=$		Positive typing context
	$\mid$	$\{PA_1, .., PA_n\}$	
	$\mid$	$M \cdot \Gamma$	Increase age of bindings by M
	$\mid$	$\Gamma_1 \sqcup \Gamma_2$	
PA	$::=$		Positive type assignment
	$\mid$	$x :_{M} T$	Variable
	$\mid$	$@h :_{M} {}^{N}\lfloor T \rfloor$	Destination (M is its own age; N is the age of values it accepts)
$NC,\ \Delta$	$::=$		Negative typing context
	$\mid$	$\{NA_1, .., NA_n\}$	
	$\mid$	$M \cdot \Delta$	Increase age of bindings by M
	$\mid$	$@^{-1}\Gamma$	Invert the sign of the context
	$\mid$	$\Delta_1 \sqcup \Delta_2$	
NA	$::=$		Negative type assignment
	$\mid$	$h :^{N} T$	Hole (N is the age of values it accepts, its own age is undefined)

$\boxed{\Gamma \,\mathsf{u}\, \Delta \Vdash \overline{v} : T}$ *(Typing of extended values (require both positive and negative contexts))*

$$\textsc{Ty-w-H}\quad \frac{}{\{\}\,\mathsf{u}\,\{h :^{\nu} T\} \Vdash h : T}$$

$$\textsc{Ty-w-D}\quad \frac{}{\{@h :_{\nu} {}^{N}\lfloor T \rfloor\}\,\mathsf{u}\,\{\} \Vdash @h : {}^{N}\lfloor T \rfloor}$$

$$\textsc{Ty-w-U}\quad \frac{}{\{\}\,\mathsf{u}\,\{\} \Vdash () : 1}$$

$$\textsc{Ty-w-L}\quad \frac{\begin{array}{c}\Gamma\,\mathsf{u}\,\Delta \Vdash \overline{v} : T_1 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{\Gamma\,\mathsf{u}\,\Delta \Vdash \mathsf{Inl}\,\overline{v} : T_1 \oplus T_2}$$

$$\textsc{Ty-w-P}\quad \frac{\begin{array}{c}\Gamma_1\,\mathsf{u}\,\Delta_1 \Vdash \overline{v_1} : T_1 \\ \Gamma_2\,\mathsf{u}\,\Delta_2 \Vdash \overline{v_2} : T_2 \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Delta_1) \cap \mathbf{dom}(\Delta_2) = \emptyset\end{array}}{\Gamma_1 \sqcup \Gamma_2\,\mathsf{u}\,\Delta_1 \sqcup \Delta_2 \Vdash (\overline{v_1}\,,\,\overline{v_2}) : T_1 \otimes T_2}$$

$$\textsc{Ty-w-R}\quad \frac{\begin{array}{c}\Gamma\,\mathsf{u}\,\Delta \Vdash \overline{v} : T_2 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{\Gamma\,\mathsf{u}\,\Delta \Vdash \mathsf{Inr}\,\overline{v} : T_1 \oplus T_2}$$

$$\textsc{Ty-w-E}\quad \frac{\begin{array}{c}\Gamma\,\mathsf{u}\,\Delta \Vdash \overline{v} : T \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{M \cdot \Gamma\,\mathsf{u}\,M \cdot \Delta \Vdash \mathclose{)\!)}^{M}\,\overline{v} : !^{M}\, T}$$

$$\textsc{Ty-w-A}\quad \frac{\begin{array}{c}\Gamma_1\,\mathsf{u}\,\{\} \Vdash v_1 : T_1 \\ \Gamma_2\,\mathsf{u}\,@^{-1}\Gamma_1 \Vdash \overline{v_2} : T_2 \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset\end{array}}{\Gamma_2\,\mathsf{u}\,\{\} \Vdash \langle v_1 \,,\, \overline{v_2}\rangle_{@^{-1}\Gamma_1} : T_1 \rtimes T_2}$$

$$\textsc{Ty-w-F}\quad \frac{\begin{array}{c}\Gamma \sqcup \{x :_{M} T_1\} \vdash t : T_2 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\{x :_{M} T_1\}) = \emptyset\end{array}}{\Gamma\,\mathsf{u}\,\{\} \Vdash \lambda x\,.\,t : T_1 \,{}_{M}\!\!\to T_2}$$

$\boxed{\Gamma \vdash t : T}$ *(Typing of terms (only a positive context is needed))*

$$\textsc{Ty-t-V}\quad \frac{\Gamma \uplus \{\,\} \Vdash v : T}{\Gamma \vdash v : T}$$

$$\textsc{Ty-t-X0}\quad \frac{}{\{x :_{\nu} T\} \vdash x : T}$$

$$\textsc{Ty-t-XInf}\quad \frac{}{\{x :_{\infty} T\} \vdash x : T}$$

$$\textsc{Ty-t-App}\quad \frac{\Gamma_1 \vdash t : T_1 \qquad \Gamma_2 \vdash u : T_1 {}_{M}\!\!\to T_2 \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{M{\cdot}\Gamma_1 \sqcup \Gamma_2 \vdash t \succ u : T_2}$$

$$\textsc{Ty-t-PatU}\quad \frac{\Gamma_1 \vdash t : 1 \qquad \Gamma_2 \vdash u : U \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathbf{case}\ () \mapsto u : U}$$

$$\textsc{Ty-t-PatS}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \oplus T_2 \\ \Gamma_2 \sqcup \{x_1 :_{M} T_1\} \vdash u_1 : U \\ \Gamma_2 \sqcup \{x_2 :_{M} T_2\} \vdash u_2 : U \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x_1 :_{M} T_1\}) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x_2 :_{M} T_2\}) = \emptyset\end{array}}{M{\cdot}\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathbf{case}\ \{\ \mathsf{Inl}\, x_1 \mapsto u_1\ ,\ \mathsf{Inr}\, x_2 \mapsto u_2\ \} : U}$$

$$\textsc{Ty-t-PatP}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \otimes T_2 \\ \Gamma_2 \sqcup \{x_1 :_{M} T_1, x_2 :_{M} T_2\} \vdash u : U \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x_1 :_{M} T_1\}) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x_2 :_{M} T_2\}) = \emptyset \\ \mathbf{dom}(\{x_1 :_{M} T_1\}) \cap \mathbf{dom}(\{x_2 :_{M} T_2\}) = \emptyset\end{array}}{M{\cdot}\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathbf{case}\ (x_1\ ,\ x_2) \mapsto u : U}$$

$$\textsc{Ty-t-PatE}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : !^{N}\, T \\ \Gamma_2 \sqcup \{x :_{M\cdot N} T\} \vdash u : U \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x :_{M\cdot N} T\}) = \emptyset\end{array}}{M{\cdot}\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathbf{case}\ \rrparenthesis^{N}\, x \mapsto u : U}$$

$$\textsc{Ty-t-Map}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \rtimes T_2 \\ \uparrow{\cdot}\Gamma_2 \sqcup \{x :_{\nu} T_1\} \vdash u : U \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{x :_{\nu} T_1\}) = \emptyset\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathbf{mapL}\ x \mapsto u : U \rtimes T_2}$$

$$\textsc{Ty-t-FillC}\quad \frac{\Gamma_1 \vdash t : {}^{N}\lfloor T_2 \rfloor \qquad \Gamma_2 \vdash u : T_1 \rtimes T_2 \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup (\uparrow{\cdot}N){\cdot}\Gamma_2 \vdash t \lhd\!\bullet\, u : T_1}$$

$$\textsc{Ty-t-FillU}\quad \frac{\Gamma \vdash t : {}^{N}\lfloor 1 \rfloor}{\Gamma \vdash t \lhd () : 1}$$

$$\textsc{Ty-t-FillL}\quad \frac{\Gamma \vdash t : {}^{N}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \vdash t \lhd \mathsf{Inl} : {}^{N}\lfloor T_1 \rfloor}$$

$$\textsc{Ty-t-FillR}\quad \frac{\Gamma \vdash t : {}^{N}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \vdash t \lhd \mathsf{Inr} : {}^{N}\lfloor T_2 \rfloor}$$

$$\textsc{Ty-t-FillP}\quad \frac{\Gamma \vdash t : {}^{N}\lfloor T_1 \otimes T_2 \rfloor}{\Gamma \vdash t \lhd (,) : {}^{N}\lfloor T_1 \rfloor \otimes {}^{N}\lfloor T_2 \rfloor}$$

$$\textsc{Ty-t-FillE}\quad \frac{\Gamma \vdash t : {}^{M}\lfloor !^{N}\, T \rfloor}{\Gamma \vdash t \lhd \rrparenthesis^{N} : {}^{M\cdot N}\lfloor T \rfloor}$$

$$\textsc{Ty-t-Alloc}\quad \frac{}{\{\,\} \vdash \mathbf{alloc}_{T} : {}^{\nu}\lfloor T \rfloor \rtimes T}$$

$$\textsc{Ty-t-ToA}\quad \frac{\Gamma \vdash t : T}{\Gamma \vdash \mathbf{to}_{\rtimes}\ t : 1 \rtimes T}$$

$$\textsc{Ty-t-FromA}\quad \frac{\Gamma \vdash t : 1 \rtimes T}{\Gamma \vdash \mathbf{from}_{\rtimes}\ t : T}$$

$\boxed{\Gamma \vdash v \diamond e : T}$ *(Typing of commands (only a positive context is needed))*

$$\textsc{Ty-c-C}\quad \frac{\begin{array}{c}\Gamma_{11} \sqcup \Gamma_{12} \vdash v : T \\ \Gamma_2 \uplus @^{-1}\Gamma_{12} \Vdash e \\ \mathbf{dom}(\Gamma_{11}) \cap \mathbf{dom}(\Gamma_{12}) = \emptyset \\ \mathbf{dom}(\Gamma_{11}) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_{12}) \cap \mathbf{dom}(\Gamma_2) = \emptyset\end{array}}{\Gamma_{11} \sqcup \Gamma_2 \vdash v \diamond e : T}$$

3 Effects and big-step semantics

$$\begin{array}{llll} e & ::= & & \text{Effect} \\ & \mid & \varepsilon & \\ & \mid & h := \overline{v} & \\ & \mid & e_1, \ldots, e_n & \text{Chain effects} \end{array}$$

$\boxed{\Gamma \mathbin{u} \Delta \Vdash e}$ *(Typing of effects (require both positive and negative contexts))*

$$\textsc{Ty-e-N}\quad \frac{}{\{\} \mathbin{u} \{\} \Vdash \varepsilon}$$

$$\textsc{Ty-e-A}\quad \frac{\begin{array}{c} \Gamma \mathbin{u} \Delta \Vdash \overline{v} : \mathsf{T} \\ \mathsf{dom}(\Gamma) \cap \mathsf{dom}(\{@h :_{M} {}^{N}\lfloor \mathsf{T} \rfloor\}) = \emptyset \\ \mathsf{dom}(\Gamma) \cap \mathsf{dom}(\Delta) = \emptyset \\ \mathsf{dom}(\{@h :_{M} {}^{N}\lfloor \mathsf{T} \rfloor\}) \cap \mathsf{dom}(\Delta) = \emptyset \end{array}}{(\uparrow \cdot M \cdot N) \cdot \Gamma \sqcup \{@h :_{M} {}^{N}\lfloor \mathsf{T} \rfloor\} \mathbin{u} {}^{(M \cdot N) \cdot}\Delta \Vdash h := \overline{v}}$$

$$\textsc{Ty-e-P}\quad \frac{\begin{array}{c} \Gamma_1 \mathbin{u} \Delta_1 \sqcup @^{-1}\Gamma_{22} \Vdash e_1 \\ \Gamma_{21} \sqcup \Gamma_{22} \mathbin{u} \Delta_2 \Vdash e_2 \\ \mathsf{dom}(\Gamma_1) \cap \mathsf{dom}(\Gamma_{21}) = \emptyset \\ \mathsf{dom}(\Gamma_1) \cap \mathsf{dom}(\Gamma_{22}) = \emptyset \\ \mathsf{dom}(\Gamma_1) \cap \mathsf{dom}(\Delta_1) = \emptyset \\ \mathsf{dom}(\Gamma_1) \cap \mathsf{dom}(\Delta_2) = \emptyset \\ \mathsf{dom}(\Gamma_{21}) \cap \mathsf{dom}(\Gamma_{22}) = \emptyset \\ \mathsf{dom}(\Gamma_{21}) \cap \mathsf{dom}(\Delta_1) = \emptyset \\ \mathsf{dom}(\Gamma_{21}) \cap \mathsf{dom}(\Delta_2) = \emptyset \\ \mathsf{dom}(\Gamma_{22}) \cap \mathsf{dom}(\Delta_1) = \emptyset \\ \mathsf{dom}(\Gamma_{22}) \cap \mathsf{dom}(\Delta_2) = \emptyset \\ \mathsf{dom}(\Delta_1) \cap \mathsf{dom}(\Delta_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_{21} \mathbin{u} \Delta_1 \sqcup \Delta_2 \Vdash e_1, e_2}$$

$\boxed{\overline{v_1}\,\Delta_1 \mid e_1 \rightsquigarrow \overline{v_2}\,\Delta_2 \mid e_2}$ *(Big-step evaluation of effects on extended values)*

$$\textsc{Sem-e-N}\quad \frac{}{\overline{v_1}\,\Delta_1 \mid \varepsilon \rightsquigarrow \overline{v_1}\,\Delta_1 \mid \varepsilon}$$

$$\textsc{Sem-e-S}\quad \frac{\begin{array}{c} h \notin \mathsf{dom}(\Delta_1) \\ \overline{v_1}\,\Delta_1 \mid e_1 \rightsquigarrow \overline{v_2}\,\Delta_2 \mid e_2 \end{array}}{\overline{v_1}\,\Delta_1 \mid h := \overline{v'}, e_1 \rightsquigarrow \overline{v_2}\,\Delta_2 \mid h := \overline{v'}, e_2}$$

$$\textsc{Sem-e-F}\quad \frac{\begin{array}{c} \Gamma'_1 \mathbin{u} \Delta'_1 \Vdash \overline{v'} : \mathsf{T} \\ \mathsf{dom}(\Gamma'_1) \cap \mathsf{dom}(\Delta'_1) = \emptyset \\ \mathsf{dom}(\Delta_1) \cap \mathsf{dom}(\{h :^{N} \mathsf{T}\}) = \emptyset \\ \mathsf{dom}(\Delta_1) \cap \mathsf{dom}(\Delta'_1) = \emptyset \\ \overline{v_1}[h := \overline{v'}]\,(\Delta_1 \sqcup {}^{N\cdot}\Delta'_1) \mid e_1 \rightsquigarrow \overline{v_2}\,\Delta_2 \mid e_2 \end{array}}{\overline{v_1}\,\Delta_1 \sqcup \{h :^{N} \mathsf{T}\} \mid h := \overline{v'}, e_1 \rightsquigarrow \overline{v_2}\,\Delta_2 \mid e_2}$$

$\boxed{t \Downarrow v \diamond e}$ *(Big-step evaluation into commands)*

$$\textsc{Sem-t-V}\quad \frac{}{v \Downarrow v \diamond \varepsilon}$$

$$\textsc{Sem-t-App}\quad \frac{t_1 \Downarrow v_1 \diamond e_1 \qquad t_2 \Downarrow \lambda x \,.\, u \diamond e_2 \qquad u[x := v_1] \Downarrow v_3 \diamond e_3}{t_1 \succ t_2 \Downarrow v_3 \diamond e_1, e_2, e_3}$$

$$\textsc{Sem-t-PatU}\quad \frac{t_1 \Downarrow () \diamond e_1 \qquad t_2 \Downarrow v_2 \diamond e_2}{t_1 \succ \mathsf{case}\ () \mapsto t_2 \Downarrow v_2 \diamond e_1, e_2}$$

$$\textsc{Sem-t-PatL}\quad \frac{t \Downarrow \mathsf{Inl}\, v_1 \diamond e_1 \qquad u_1[x_1 := v_1] \Downarrow v_2 \diamond e_2}{t \succ \mathsf{case}\ \{\mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2\} \Downarrow v_2 \diamond e_1, e_2}$$

$$\textsc{Sem-t-PatR}\quad \frac{t \Downarrow \mathsf{Inr}\, v_1 \diamond e_1 \qquad u_2[x_2 := v_1] \Downarrow v_2 \diamond e_2}{t \succ \mathsf{case}\ \{\mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2\} \Downarrow v_2 \diamond e_1, e_2}$$

$$\textsc{Sem-t-PatP}\quad \frac{t \Downarrow (v_1 , v_2) \diamond e_1 \qquad u[x_1 := v_1][x_2 := v_2] \Downarrow v_2 \diamond e_2}{t \succ \mathsf{case}\ (x_1 , x_2) \mapsto u \Downarrow v_2 \diamond e_1, e_2}$$

$$\textsc{Sem-t-Map}\quad \frac{\begin{array}{c} t \Downarrow \langle v_1 ⨾ \overline{v_2} \rangle_{\Delta} \diamond e_1 \\ u[x := v_1] \Downarrow v_3 \diamond e_2 \\ \overline{v_2}\,\Delta \mid e_2 \rightsquigarrow \overline{v_4}\,\Delta' \mid e_3 \end{array}}{t \succ \mathsf{mapL}\ x \mapsto u \Downarrow \langle v_3 ⨾ \overline{v_4} \rangle_{\Delta'} \diamond e_1, e_3}$$

$$\textsc{Sem-t-Alloc}\quad \frac{\mathsf{fresh}\ h}{\mathsf{alloc}_{\mathsf{T}} \Downarrow \langle @h ⨾ h \rangle_{\{h :^{\nu} \mathsf{T}\}} \diamond \varepsilon}$$

$$\textsc{Sem-t-ToA}\quad \frac{t \Downarrow v \diamond e}{\mathsf{to}_{\ltimes}\ t \Downarrow \langle () ⨾ v \rangle_{\{\}} \diamond e}$$

$$\textsc{Sem-t-FromA}\quad \frac{t \Downarrow \langle () ⨾ v \rangle_{\{\}} \diamond e}{\mathsf{from}_{\ltimes}\ t \Downarrow v \diamond e}$$

$$\textsc{Sem-t-FillU}\quad \frac{t \Downarrow @h \diamond e}{t \triangleleft () \Downarrow () \diamond e, h := ()}$$

$$\textsc{Sem-t-FillL}\quad \frac{t \Downarrow @h \diamond e \qquad \mathsf{fresh}\ h'}{t \triangleleft \mathsf{Inl} \Downarrow @h' \diamond e, h := \mathsf{Inl}\ h'}$$

$$\textsc{Sem-t-FillR}\quad \frac{t \Downarrow @h \diamond e}{t \triangleleft \mathsf{Inr} \Downarrow @h' \diamond e, h := \mathsf{Inr}\ h'}$$

$$\textsc{Sem-t-FillP}\quad \frac{t \Downarrow @h \diamond e \qquad \mathsf{fresh}\ h_1 \qquad \mathsf{fresh}\ h_2}{t \triangleleft (,) \Downarrow (@h_1 , @h_2) \diamond e, h := (h_1 , h_2)}$$

$$\textsc{Sem-t-FillC}\quad \frac{t \Downarrow @h \diamond e_1 \qquad u \Downarrow \langle v_1 ⨾ \overline{v_2} \rangle_{\Delta} \diamond e_2}{t \triangleleft\!\bullet\, u \Downarrow v_1 \diamond e_1, e_2, h := \overline{v_2}}$$

4 Type safety

Theorem 1 (Type safety)**.** *If* $\textsf{destOnly}\,\Gamma$ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

Theorem 2 (Type safety for complete programs)**.** *If* $\{\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond \varepsilon$ *and* $\{\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v} \Downarrow \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $\mathrm{M}{\cdot}\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 {}_{\mathrm{M}}\!\!\to \mathsf{T}_2$
  (3) $\textsf{dom}(\Gamma_1) \cap \textsf{dom}(\Gamma_2) = \emptyset$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t} \Downarrow \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \sqcup \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12}\,\mathsf{u}\,@^{-1}\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \sqcup \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u} \Downarrow \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1 {}_{\mathrm{M}}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \sqcup \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1 {}_{\mathrm{M}}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22}\,\mathsf{u}\,@^{-1}\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \sqcup \Gamma_{22}$.
  Using Lemma **??** on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x}\,.\,\mathsf{t}'$ and (14) $\Gamma_{21} \sqcup \Gamma_{23} \sqcup \{\mathsf{x} :_{\mathrm{M}} \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma **??** on (14) and (5) we get (15) $\mathrm{M}{\cdot}(\Gamma_{11} \sqcup \Gamma_{13}) \sqcup (\Gamma_{21} \sqcup \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_3$ where (17) $\mathrm{M}{\cdot}(\Gamma_{11} \sqcup \Gamma_{13}) \sqcup (\Gamma_{21} \sqcup \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12}\,\mathsf{u}\,@^{-1}\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22}\,\mathsf{u}\,@^{-1}\Gamma_{23} \Vdash e_2$

  $\textsf{dom}(\Gamma_{12}) \cap \textsf{dom}(\Gamma_{22}) = \emptyset$ comes naturally from (3), (7) and (12).

  We must show:
  $\textsf{dom}(\Gamma_{12}) \cap \textsf{dom}(\Gamma_{23}) = \emptyset$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  $\textsf{dom}(\Gamma_{22}) \cap \textsf{dom}(\Gamma_{13}) = \emptyset$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  $\textsf{dom}(\Gamma_{13}) \cap \textsf{dom}(\Gamma_{23}) = \emptyset$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \sqcup \Gamma_{22}\,\mathsf{u}\,@^{-1}\Gamma_{13} \sqcup @^{-1}\Gamma_{23} \Vdash e_1, e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $\mathrm{M}{\cdot}(\Gamma_{111} \sqcup \Gamma_{131}) \sqcup \Gamma_{211} \sqcup \Gamma_{231} \sqcup \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $\mathrm{M}{\cdot}(\Gamma_{112} \sqcup \Gamma_{132}) \sqcup \Gamma_{212} \sqcup \Gamma_{232}\,\mathsf{u}\,@^{-1}\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{\mathsf{n}\,1} \sqcup \Gamma_{\mathsf{n}\,2} = \Gamma_{\mathsf{n}}$

  We have
  (18) $\Gamma_{12} \sqcup \Gamma_{22}\,\mathsf{u}\,@^{-1}\Gamma_{13} \sqcup @^{-1}\Gamma_{23} \Vdash e_1, e_2$
  (20) $\mathrm{M}{\cdot}(\Gamma_{112} \sqcup \Gamma_{132}) \sqcup \Gamma_{212} \sqcup \Gamma_{232}\,\mathsf{u}\,@^{-1}\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $@^{-1}\Gamma_{23}$, we get (22) $\Gamma_{12} \sqcup \Gamma_{22}\,\mathsf{u}\,@^{-1}(\Gamma_{131} \sqcup \Gamma_{231}) \sqcup @^{-1}(\Gamma_{132} \sqcup \Gamma_{232}) \Vdash e_1, e_2$

  We want Γ_{132} from (22) to cancel $\mathrm{M}{\cdot}\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by M), then should we also delay the RHS of its associated effect? In other terms, if we have $\{@\mathsf{h} :_{\nu} {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash @\mathsf{h}' \diamond \mathsf{h} \coloneqq \textsf{Inl}\,\mathsf{h}' : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay M (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} \coloneqq \textsf{Inl}\,\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(@\mathsf{h}_0 \triangleleft (,)\ \succ\textsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\textsf{to}_{\rtimes}\ @\mathsf{h}_1)\ \succ\textsf{case}\,() \mapsto \mathsf{x}_2)\ \succ\ (\lambda \mathsf{x}_2\,.\,@\mathsf{h}_3 \triangleleft\!\bullet (\textsf{to}_{\rtimes}\ \mathsf{x}_2))$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all BigStep_Fill⟨*Ctor*⟩ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TyCmd_Cmd), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;

- or are those of pre-existing holes coming from the extended value $\overline{v_2}$ of an ampar, when BigStep_FillComp is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program. And v_1 is exactly the accompanying value returned by the evaluation of BigStep_FillComp

TODO: prove that this property is preserved by typing rules □