# Destination $\lambda$-calculus

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## 1 Term and value syntax

| | |
|---|---|
| $\mathsf{var}$, $\mathsf{x}$, $\mathsf{y}$ | Term-level variable name |
| $\mathsf{k}$ | Index for ranges |

| | | | | |
|---|---|---|---|---|
| $\mathtt{hdn}$, $\mathtt{h}$ | ::= | | | Hole or destination name ($\mathbb{N}$) |
| | \| | $\mathtt{h}+\mathtt{h}'$ | M | Sum |
| | \| | $\mathtt{max}(\mathtt{H})$ | M | Maximum of a set of holes |
| $\mathtt{hdns}$, $\mathtt{H}$ | ::= | | | Set of hole names |
| | \| | $\{\mathtt{h}_1, \ldots, \mathtt{h}_\mathsf{k}\}$ | | |
| | \| | $\mathtt{H}_1 \cup \mathtt{H}_2$ | M | Union of sets |
| | \| | $\mathtt{H} \pm \mathtt{h}$ | M | Increase all names from $\mathtt{H}$ by $\mathtt{h}$. |
| | \| | $\mathtt{hnames}(\Gamma)$ | M | Hole names of a context (requires $\mathtt{ctx\_NoVar}(\Gamma)$) |
| | \| | $\mathtt{hnames}(\mathsf{C})$ | M | Hole names of an evaluation context |
| $\mathsf{term}$, $\mathsf{t}$, $\mathsf{u}$ | ::= | | | Term |
| | \| | $\mathsf{v}$ | | Value |
| | \| | $\mathsf{x}$ | | Variable |
| | \| | $\mathsf{t} \succ \mathsf{u}$ | | Application |
| | \| | $\mathsf{t} \,;\, \mathsf{u}$ | | Pattern-match on unit |
| | \| | $\mathsf{t} \succ \mathbf{case}_m \{ \mathsf{Inl}\, \mathsf{x}_1 \mapsto \mathsf{u}_1 ,\ \mathsf{Inr}\, \mathsf{x}_2 \mapsto \mathsf{u}_2 \}$ | | Pattern-match on sum |
| | \| | $\mathsf{t} \succ \mathbf{case}_m (\mathsf{x}_1 , \mathsf{x}_2) \mapsto \mathsf{u}$ | | Pattern-match on product |
| | \| | $\mathsf{t} \succ \mathbf{case}_m\, \mathrm{E}^n\, \mathsf{x} \mapsto \mathsf{u}$ | | Pattern-match on exponential |
| | \| | $\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}$ | | Map over the right side of ampar $\mathsf{t}$ |
| | \| | $\mathbf{to}_{\ltimes}\ \mathsf{t}$ | | Wrap $\mathsf{t}$ into a trivial ampar |
| | \| | $\mathbf{from}_{\ltimes}\ \mathsf{t}$ | | Extract value from trivial ampar |
| | \| | $\mathbf{alloc}$ | | Return a fresh "identity" ampar object |
| | \| | $\mathsf{t} \triangleleft ()$ | | Fill destination with unit |
| | \| | $\mathsf{t} \triangleleft \mathsf{Inl}$ | | Fill destination with left variant |
| | \| | $\mathsf{t} \triangleleft \mathsf{Inr}$ | | Fill destination with right variant |
| | \| | $\mathsf{t} \triangleleft \mathrm{E}^m$ | | Fill destination with exponential constructor |
| | \| | $\mathsf{t} \triangleleft (,)$ | | Fill destination with product constructor |
| | \| | $\mathsf{t} \triangleleft (\lambda \mathsf{x}\ {}_m \mapsto \mathsf{u})$ | | Fill destination with function |
| | \| | $\mathsf{t} \triangleleft\!\bullet\, \mathsf{u}$ | | Fill destination with root of ampar $\mathsf{u}$ |
| | \| | $\mathsf{t}[\mathsf{x} \coloneqq \mathsf{v}]$ | M | |
| $\mathsf{val}$, $\mathsf{v}$ | ::= | | | Term value |
| | \| | $-\mathtt{h}$ | | Hole |
| | \| | $+\mathtt{h}$ | | Destination |
| | \| | $()$ | | Unit |
| | \| | $\lambda^{\mathsf{v}} \mathsf{x}\ {}_m \mapsto \mathsf{t}$ | | Lambda abstraction |
| | \| | $\mathsf{Inl}\, \mathsf{v}$ | | Left variant for sum |
| | \| | $\mathsf{Inr}\, \mathsf{v}$ | | Right variant for sum |
| | \| | $\mathrm{E}^m\, \mathsf{v}$ | | Exponential |
| | \| | $(\mathsf{v}_1 , \mathsf{v}_2)$ | | Product |
| | \| | ${}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle$ | | Ampar |
| | \| | $\mathsf{v} \pm \mathtt{h}$ | M | Rename hole names inside $\mathsf{v}$ by shifting them by $\mathtt{h}$ |

| | | | | |
|---|---|---|---|---|
| ectx, c | ::= | | | Evaluation context component |
| | \| | $\square \succ \mathsf{u}$ | | Application |
| | \| | $\mathsf{v} \succ \square$ | | Application |
| | \| | $\square \,;\, \mathsf{u}$ | | Pattern-match on unit |
| | \| | $\square \succ \mathbf{case}_{m}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}$ | | Pattern-match on sum |
| | \| | $\square \succ \mathbf{case}_{m}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u}$ | | Pattern-match on product |
| | \| | $\square \succ \mathbf{case}_{m}\,\mathrm{E}^{n}\,\mathsf{x} \mapsto \mathsf{u}$ | | Pattern-match on exponential |
| | \| | $\square \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}$ | | Map over the right side of ampar |
| | \| | $\mathbf{to}_{\ltimes}\ \square$ | | Wrap into a trivial ampar |
| | \| | $\mathbf{from}_{\ltimes}\ \square$ | | Extract value from trivial ampar |
| | \| | $\square \lhd ()$ | | Fill destination with unit |
| | \| | $\square \lhd \mathsf{Inl}$ | | Fill destination with left variant |
| | \| | $\square \lhd \mathsf{Inr}$ | | Fill destination with right variant |
| | \| | $\square \lhd \mathrm{E}^{m}$ | | Fill destination with exponential constructor |
| | \| | $\square \lhd (,)$ | | Fill destination with product constructor |
| | \| | $\square \lhd (\lambda \mathsf{x}\ {}_{m} \mapsto \mathsf{u})$ | | Fill destination with function |
| | \| | $\square \lhd\!\bullet\, \mathsf{u}$ | | Fill destination with root of ampar |
| | \| | $\mathsf{v} \lhd\!\bullet\, \square$ | | Fill destination with root of ampar |
| | \| | ${}^{\mathrm{op}}_{\mathtt{H}}\langle \mathsf{v}_1\ {}_{\circ}^{\circ}\ \square$ | | Open ampar. Only new addition to term shapes |

| | | | | |
|---|---|---|---|---|
| ectxs, C | ::= | | | Evaluation context stack |
| | \| | $\square$ | | Represent the empty stack / "identity" evaluation context |
| | \| | $\mathsf{C} \circ \mathsf{c}$ | | Push c on top of C |
| | \| | $\mathsf{C}[\mathtt{h} \coloneqq_{\mathtt{H}} \mathsf{v}]$ | M | Fill h in C with value v (that may contain holes) |

## 2 Type system

| | | | | |
|---|---|---|---|---|
| **type**, T, U | ::= | | | Type |
| | \| | $\mathbf{1}$ | | Unit |
| | \| | $\mathsf{T}_1 \oplus \mathsf{T}_2$ | | Sum |
| | \| | $\mathsf{T}_1 \otimes \mathsf{T}_2$ | | Product |
| | \| | $!^{m}\,\mathsf{T}$ | | Exponential |
| | \| | $\mathsf{T}_1 \ltimes \mathsf{T}_2$ | | Ampar type (consuming $\mathsf{T}_2$ yields $\mathsf{T}_1$) |
| | \| | $\mathsf{T}_1\ {}_{m_1}\!\!\rightarrow \mathsf{T}_2$ | | Function |
| | \| | $\lfloor \mathsf{T} \rfloor^{m}$ | | Destination |

| | | | | |
|---|---|---|---|---|
| *mode*, *m*, *n* | ::= | | | Mode (Semiring) |
| | \| | $pa$ | | Pair of a multiplicity and age |
| | \| | ☠ | | Error case (incompatible types, multiplicities, or ages) |
| | \| | $m_1 \cdot \ldots \cdot m_k$ | M | Semiring product |

| | | | | |
|---|---|---|---|---|
| *mul*, *p* | ::= | | | Multiplicity (first component of modality) |
| | \| | $1$ | | Linear. Neutral element of the product |
| | \| | $\omega$ | | Non-linear. Absorbing for the product |
| | \| | $p_1 \cdot \ldots \cdot p_k$ | M | Semiring product |

| | | | | |
|---|---|---|---|---|
| *age*, *a* | ::= | | | Age (second component of modality) |
| | \| | $\nu$ | | Born now. Neutral element of the product |
| | \| | $\uparrow$ | | One scope older |
| | \| | $\infty$ | | Infinitely old / static. Absorbing for the product |
| | \| | $a_1 \cdot \ldots \cdot a_k$ | M | Semiring product |

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|---|---|---|---|---|
| bndr_var | ::= | | | |
| | \| | $\mathsf{x} :_{m} \mathsf{T}$ | | Variable binding |

| | | | | |
|---|---|---|---|---|
| bndr_dest | ::= | | | |
| | \| | $+\mathtt{h} :_{m} \lfloor \mathsf{T} \rfloor^{n}$ | | Dest binding ($m$ is its own modality; $n$ is the modality for values it accepts) |

| | | | | |
|---|---|---|---|---|
| bndr_hole | ::= | | | |
| | \| | $-\mathtt{h} : \mathsf{T}^{n}$ | | Hole binding ($n$ is the modality for values it accepts, it doesn't have a modality on i |

| | | | | |
|---|---|---|---|---|
| ctx, Γ, Δ, Π | ::= | | | Typing context |
| | \| | ⦇mvar mdest mhole⦈ | | Actual representation of contexts for Coq proofs (cannot hide) |
| | \| | $m{\cdot}\Gamma$ | M | Multiply each binding by $m$ |
| | \| | $\Gamma_1 \uplus \Gamma_2$ | M | Sum contexts $\Gamma_1$ and $\Gamma_2$. Duplicate keys with incompatible values will be |
| | \| | $-\Gamma$ | M | Transforms dest bindings into a hole bindings (requires `ctx_DestOnly` Γ a |

$\boxed{\Gamma \Vdash \mathsf{v} : \mathsf{T}}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \frac{}{\{-h : \mathsf{T}^{1\nu}\} \Vdash -h : \mathsf{T}}$$

$$\textsc{TyR-val-D}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ +h :_{1\nu} \lfloor \mathsf{T} \rfloor^{n}}{\Gamma \Vdash +h : \lfloor \mathsf{T} \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \frac{}{\{\} \Vdash () : \mathsf{1}}$$

$$\textsc{TyR-val-F}\quad \frac{\begin{array}{c}\texttt{ctx\_DestOnly}\ \Delta \\ \Delta \uplus \{x :_m \mathsf{T}_1\} \vdash \mathsf{t} : \mathsf{T}_2\end{array}}{\Delta \Vdash \lambda^{\mathsf{v}} x_m \mapsto \mathsf{t} : \mathsf{T}_1\ {}_m\!\!\to \mathsf{T}_2}$$

$$\textsc{TyR-val-L}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-R}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-P}\quad \frac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-val-E}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{n{\cdot}\Gamma \Vdash \mathrm{E}^{n}\,\mathsf{v} : !^{n}\,\mathsf{T}}$$

$$\textsc{TyR-val-A}\quad \frac{\begin{array}{c}\texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_3 \\ \texttt{ctx\_LinOnly}\ \Delta_3 \\ \texttt{ctx\_IsValid}\ \Delta_3 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_3 \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \Delta_3 \\ \Delta_1 \uplus (-\Delta_3) \Vdash \mathsf{v}_1 : \mathsf{T}_1 \\ \Delta_2 \uplus \Delta_3 \Vdash \mathsf{v}_2 : \mathsf{T}_2\end{array}}{\Delta_1 \uplus \Delta_2 \Vdash {}_{\texttt{hnames}(-\Delta_3)}\langle \mathsf{v}_1\,,\,\mathsf{v}_2\rangle : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$\boxed{\Pi \vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms)*

$$\textsc{Ty-term-Val}\quad \frac{\texttt{ctx\_DestOnly}\ \Delta \qquad \Delta \Vdash \mathsf{v} : \mathsf{T}}{\Delta \vdash \mathsf{v} : \mathsf{T}}$$

$$\textsc{Ty-term-Var}\quad \frac{\texttt{ctx\_Compatible}\ \Pi\ \ x :_{1\nu} \mathsf{T}}{\Pi \vdash x : \mathsf{T}}$$

$$\textsc{Ty-term-App}\quad \frac{\Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_m\!\!\to \mathsf{T}_2}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-term-PatU}\quad \frac{\Pi_1 \vdash \mathsf{t} : \mathsf{1} \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{U}}{\Pi_1 \uplus \Pi_2 \vdash \mathsf{t}\ ;\ \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \{x_1 :_m \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Pi_2\ \{x_2 :_m \mathsf{T}_2\} \\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Pi_2 \uplus \{x_1 :_m \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U} \\ \Pi_2 \uplus \{x_2 :_m \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_m\,\{\,\mathsf{Inl}\,x_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,x_2 \mapsto \mathsf{u}_2\,\} : \mathsf{U}}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \{x_1 :_m \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Pi_2\ \{x_2 :_m \mathsf{T}_2\} \\ \texttt{ctx\_Disjoint}\ \{x_1 :_m \mathsf{T}_1\}\ \{x_2 :_m \mathsf{T}_2\} \\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Pi_2 \uplus \{x_1 :_m \mathsf{T}_1\} \uplus \{x_2 :_m \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_m\,(x_1\,,\,x_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \{x :_{m\cdot n} \mathsf{T}\} \\ \Pi_1 \vdash \mathsf{t} : !^{n}\,\mathsf{T} \\ \Pi_2 \uplus \{x :_{m\cdot n} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_m\,\mathrm{E}^{n}\,x \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \{x :_{1\nu} \mathsf{T}_2\} \\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \ltimes \mathsf{T}_2 \\ 1{\uparrow}{\cdot}\Pi_2 \uplus \{x :_{1\nu} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{map}\ x \mapsto \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Pi \vdash \mathsf{t} : \mathsf{T}}{\Pi \vdash \mathsf{to}_{\ltimes}\ \mathsf{t} : \mathsf{T} \ltimes \mathsf{1}}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Pi \vdash \mathsf{t} : \mathsf{T} \ltimes \mathsf{1}}{\Pi \vdash \mathsf{from}_{\ltimes}\ \mathsf{t} : \mathsf{T}}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathsf{alloc} : \mathsf{T} \ltimes \lfloor \mathsf{T} \rfloor^{1\nu}}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{1} \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft () : \mathsf{1}}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathsf{Inl} : \lfloor \mathsf{T}_1 \rfloor^{n}}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathsf{Inr} : \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft (,) : \lfloor \mathsf{T}_1 \rfloor^{n} \otimes \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor !^{n'}\,\mathsf{T} \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathrm{E}^{n'} : \lfloor \mathsf{T} \rfloor^{n'\cdot n}}$$

$$\textsc{Ty-term-FillF}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \{x :_m \mathsf{T}_1\} \\ \Pi_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1\ {}_m\!\!\to \mathsf{T}_2 \rfloor^{n} \\ \Pi_2 \uplus \{x :_m \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{T}_2\end{array}}{\Pi_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Pi_2 \vdash \mathsf{t} \triangleleft (\lambda x_m \mapsto \mathsf{u}) : \mathsf{1}}$$

$$\textsc{Ty-term-FillC}\quad \frac{\Pi_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \rfloor^{n} \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Pi_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Pi_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_2}$$

$\boxed{\Delta \dashv C : T_1 \rightarrowtail T_2}$ *(Typing of evaluation contexts)*

Ty-ectxs-Id

$$\frac{}{\{\} \dashv \square : U_0 \rightarrowtail U_0}$$

Ty-ectxs-AppFoc1

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{mode\_IsValid}\ m\\ \texttt{ctx\_IsValid}\ \Delta_2\\ m{\cdot}\Delta_1 \uplus \Delta_2 \dashv C : T_2 \rightarrowtail U_0\\ \Delta_2 \vdash u : T_1\ {}_m\!\!\rightarrow T_2\end{array}}{\Delta_1 \dashv C \circ (\square \succ u) : T_1 \rightarrowtail U_0}$$

Ty-ectxs-AppFoc2

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ m{\cdot}\Delta_1\\ m{\cdot}\Delta_1 \uplus \Delta_2 \dashv C : T_2 \rightarrowtail U_0\\ \Delta_1 \vdash v : T_1\end{array}}{\Delta_2 \dashv C \circ (v \succ \square) : (T_1\ {}_m\!\!\rightarrow T_2) \rightarrowtail U_0}$$

Ty-ectxs-PatUFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ \Delta_2\\ \Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0\\ \Delta_2 \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square\ ;\ u) : 1 \rightarrowtail U_0}$$

Ty-ectxs-PatSFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{mode\_IsValid}\ m\\ \texttt{ctx\_IsValid}\ \Delta_2\\ m{\cdot}\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0\\ \Delta_2 \uplus \{x_1 :_m T_1\} \vdash u_1 : U\\ \Delta_2 \uplus \{x_2 :_m T_2\} \vdash u_2 : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m\ \{\, \mathsf{Inl}\, x_1 \mapsto u_1\ ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\}) : (T_1 \oplus T_2) \rightarrowtail U_0}$$

Ty-ectxs-PatPFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_Disjoint}\ \{x_1 :_m T_1\}\ \{x_2 :_m T_2\}\\ \texttt{mode\_IsValid}\ m\\ \texttt{ctx\_IsValid}\ \Delta_2\\ m{\cdot}\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0\\ \Delta_2 \uplus \{x_1 :_m T_1\} \uplus \{x_2 :_m T_2\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m\ (x_1\ ,\ x_2) \mapsto u) : (T_1 \otimes T_2) \rightarrowtail U_0}$$

Ty-ectxs-PatEFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{mode\_IsValid}\ m\\ \texttt{ctx\_IsValid}\ \Delta_2\\ m{\cdot}\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0\\ \Delta_2 \uplus \{x :_{m\cdot m'} T\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m\ \mathrm{E}^{m'}\ x \mapsto u) : !^{m'}\ T \rightarrowtail U_0}$$

Ty-ectxs-MapFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ \Delta_2\\ \Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0\\ 1\!\uparrow{\cdot}\Delta_2 \uplus \{x :_{1\nu} T_2\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{map}\ x \mapsto u) : (T_1 \ltimes T_2) \rightarrowtail U_0}$$

Ty-ectxs-ToAFoc

$$\frac{\Delta \dashv C : (T \ltimes 1) \rightarrowtail U_0}{\Delta \dashv C \circ (\mathbf{to}_{\ltimes}\ \square) : T \rightarrowtail U_0}$$

Ty-ectxs-FromAFoc

$$\frac{\Delta \dashv C : T \rightarrowtail U_0}{\Delta \dashv C \circ (\mathbf{from}_{\ltimes}\ \square) : (T \ltimes 1) \rightarrowtail U_0}$$

Ty-ectxs-FillUFoc

$$\frac{\Delta \dashv C : 1 \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft ()) : \lfloor 1 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillLFoc

$$\frac{\Delta \dashv C : \lfloor T_1 \rfloor^n \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathsf{Inl}) : \lfloor T_1 \oplus T_2 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillRFoc

$$\frac{\Delta \dashv C : \lfloor T_2 \rfloor^n \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathsf{Inr}) : \lfloor T_1 \oplus T_2 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillPFoc

$$\frac{\Delta \dashv C : (\lfloor T_1 \rfloor^n \otimes \lfloor T_2 \rfloor^n) \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft (,)) : \lfloor T_1 \otimes T_2 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillEFoc

$$\frac{\Delta \dashv C : \lfloor T \rfloor^{m\cdot n} \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathrm{E}^m) : \lfloor !^m\ T \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillFFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ (1\!\uparrow{\cdot}n){\cdot}\Delta_2\\ \Delta_1 \uplus (1\!\uparrow{\cdot}n){\cdot}\Delta_2 \dashv C : 1 \rightarrowtail U_0\\ \Delta_2 \uplus \{x :_m T_1\} \vdash u : T_2\end{array}}{\Delta_1 \dashv C \circ (\square \triangleleft (\lambda x\ {}_m \mapsto u)) : \lfloor T_1\ {}_m\!\!\rightarrow T_2 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-FillCFoc1

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ (1\!\uparrow{\cdot}n){\cdot}\Delta_2\\ \Delta_1 \uplus (1\!\uparrow{\cdot}n){\cdot}\Delta_2 \dashv C : T_2 \rightarrowtail U_0\\ \Delta_2 \vdash u : T_1 \ltimes T_2\end{array}}{\Delta_1 \dashv C \circ (\square \triangleleft\!\bullet\, u) : \lfloor T_1 \rfloor^n \rightarrowtail U_0}$$

Ty-ectxs-AOpenFoc

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_3\\ \texttt{hdns\_Disjoint}\ \mathtt{hnames}(C)\ \mathtt{hnames}(-\Delta_3)\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_3\\ \texttt{ctx\_LinOnly}\ \Delta_3\\ \texttt{ctx\_IsValid}\ \Delta_1\\ \Delta_1 \uplus \Delta_2 \dashv C : (T_1 \ltimes U) \rightarrowtail U_0\\ \Delta_1 \uplus -\Delta_3 \Vdash v_1 : T_1\end{array}}{1\!\uparrow{\cdot}\Delta_2 \uplus \Delta_3 \dashv C \circ ({}^{\mathrm{op}}_{\mathtt{hnames}(-\Delta_3)}\langle v_1\ ,\ \square) : U \rightarrowtail U_0}$$

Ty-ectxs-FillCFoc2

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\\ \texttt{ctx\_DestOnly}\ \Delta_1\\ \texttt{ctx\_DestOnly}\ \Delta_2\\ \texttt{ctx\_IsValid}\ \Delta_1\\ \texttt{mode\_IsValid}\ (1\!\uparrow{\cdot}n)\\ \Delta_1 \uplus (1\!\uparrow{\cdot}n){\cdot}\Delta_2 \dashv C : T_2 \rightarrowtail U_0\\ \Delta_1 \vdash v : \lfloor T_1 \rfloor^n\end{array}}{\Delta_2 \dashv C \circ (v \triangleleft\!\bullet\, \square) : T_1 \ltimes T_2 \rightarrowtail U_0}$$

$\boxed{\vdash \mathsf{C}[\mathsf{t}] : \mathbf{T}}$ *(Typing of extended terms (pair of evaluation context and term))*

$$\frac{\begin{array}{c}\text{TY-ETERM-CLOSEDETERM} \\ \texttt{ctx\_IsValid}\ \Delta \\ \texttt{ctx\_DestOnly}\ \Delta \\ \Delta \dashv \mathsf{C} : \mathbf{T} \rightarrowtail \mathbf{U}_0 \qquad \Delta \vdash \mathsf{t} : \mathbf{T}\end{array}}{\vdash \mathsf{C}[\mathsf{t}] : \mathbf{U}_0}$$

# 3 Small-step semantics

$\boxed{C[t] \longrightarrow C'[t']}$ *(Small-step evaluation of terms using evaluation contexts)*

$$\textsc{Sem-eterm-AppFoc1}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ u] \longrightarrow (C \circ (\Box \succ u))[t]}$$

$$\textsc{Sem-eterm-AppUnfoc1}\quad \frac{}{(C \circ (\Box \succ u))[v] \longrightarrow C[v \succ u]}$$

$$\textsc{Sem-eterm-AppFoc2}\quad \frac{\texttt{term\_NotVal}\ u}{C[v \succ u] \longrightarrow (C \circ (v \succ \Box))[u]}$$

$$\textsc{Sem-eterm-AppUnfoc2}\quad \frac{}{(C \circ (v \succ \Box))[v'] \longrightarrow C[v \succ v']}$$

$$\textsc{Sem-eterm-AppRed}\quad \frac{}{C[v \succ (\lambda^{v} x_{m} \mapsto t)] \longrightarrow C[t[x := v]]}$$

$$\textsc{Sem-eterm-PatUFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t\ ;\ u] \longrightarrow (C \circ (\Box\ ;\ u))[t]}$$

$$\textsc{Sem-eterm-PatUUnfoc}\quad \frac{}{(C \circ (\Box\ ;\ u))[v] \longrightarrow C[v\ ;\ u]}$$

$$\textsc{Sem-eterm-PatURed}\quad \frac{}{C[()\ ;\ u] \longrightarrow C[u]}$$

$$\textsc{Sem-eterm-PatLFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}] \longrightarrow (C \circ (\Box \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}))[t]}$$

$$\textsc{Sem-eterm-PatLUnfoc}\quad \frac{}{(C \circ (\Box \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}))[v] \longrightarrow C[v \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}]}$$

$$\textsc{Sem-eterm-PatLRed}\quad \frac{}{C[(\mathsf{Inl}\,v) \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}] \longrightarrow C[u_1[x := v]]}$$

$$\textsc{Sem-eterm-PatRFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}] \longrightarrow (C \circ (\Box \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}))[t]}$$

$$\textsc{Sem-eterm-PatRUnfoc}\quad \frac{}{(C \circ (\Box \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}))[v] \longrightarrow C[v \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}]}$$

$$\textsc{Sem-eterm-PatRRed}\quad \frac{}{C[(\mathsf{Inr}\,v) \succ \mathsf{case}_{m}\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\}] \longrightarrow C[u_2[x := v]]}$$

$$\textsc{Sem-eterm-PatPFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ \mathsf{case}_{m}(x_1\ ,\ x_2) \mapsto u] \longrightarrow (C \circ (\Box \succ \mathsf{case}_{m}(x_1\ ,\ x_2) \mapsto u))[t]}$$

$$\textsc{Sem-eterm-PatPUnfoc}\quad \frac{}{(C \circ (\Box \succ \mathsf{case}_{m}(x_1\ ,\ x_2) \mapsto u))[v] \longrightarrow C[v \succ \mathsf{case}_{m}(x_1\ ,\ x_2) \mapsto u]}$$

$$\textsc{Sem-eterm-PatPRed}\quad \frac{}{C[(v_1\ ,\ v_2) \succ \mathsf{case}_{m}(x_1\ ,\ x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]}$$

$$\textsc{Sem-eterm-PatEFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto u] \longrightarrow (C \circ (\Box \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto u))[t]}$$

$$\textsc{Sem-eterm-PatEUnfoc}\quad \frac{}{(C \circ (\Box \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto u))[v] \longrightarrow C[v \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto u]}$$

$$\textsc{Sem-eterm-PatERed}\quad \frac{}{C[\mathrm{E}^{n}\,v \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto u] \longrightarrow C[u[x := v]]}$$

$$\textsc{Sem-eterm-MapFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[t \succ \mathsf{map}\ x \mapsto u] \longrightarrow (C \circ (\Box \succ \mathsf{map}\ x \mapsto u))[t]}$$

$$\textsc{Sem-eterm-MapUnfoc}\quad \frac{}{(C \circ (\Box \succ \mathsf{map}\ x \mapsto u))[v] \longrightarrow C[v \succ \mathsf{map}\ x \mapsto u]}$$

$$\textsc{Sem-eterm-MapRedAOpenFoc}\quad \frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C))}{C[{}_{\mathtt{H}}\langle v_1 , v_2\rangle \succ \mathsf{map}\ x \mapsto u] \longrightarrow (C \circ ({}^{\mathrm{op}}_{\mathtt{H}\pm\mathtt{h}'}\langle v_1{\pm}\mathtt{h}' , \Box\rangle))[u[x := v_2{\pm}\mathtt{h}']]}$$

$$\textsc{Sem-eterm-AOpenUnfoc}\quad \frac{}{(C \circ {}^{\mathrm{op}}_{\mathtt{H}}\langle v_1 , \Box\rangle)[v_2] \longrightarrow C[{}_{\mathtt{H}}\langle v_1 , v_2\rangle]}$$

$$\textsc{Sem-eterm-AllocRed}\quad \frac{}{C[\mathsf{alloc}] \longrightarrow C[{}_{\{1\}}\langle +1 , -1\rangle]}$$

$$\textsc{Sem-eterm-ToAFoc}\quad \frac{\texttt{term\_NotVal}\ t}{C[\mathsf{to}_{\ltimes}\ t] \longrightarrow (C \circ (\mathsf{to}_{\ltimes}\ \Box))[t]}$$

$$\textsc{Sem-eterm-ToAUnfoc}\quad \frac{}{(C \circ (\mathsf{to}_{\ltimes}\ \Box))[v] \longrightarrow C[\mathsf{to}_{\ltimes}\ v]}$$

Sem-eterm-ToARed
$$\frac{}{C[\mathsf{to}_{\ltimes}\ v] \longrightarrow C[_{\{\}}\langle v , ()\rangle]}$$

Sem-eterm-FromAFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[\mathsf{from}_{\ltimes}\ t] \longrightarrow (C \circ (\mathsf{from}_{\ltimes}\ \square))[t]}$$

Sem-eterm-FromAUnfoc
$$\frac{}{(C \circ (\mathsf{from}_{\ltimes}\ \square))[v] \longrightarrow C[\mathsf{from}_{\ltimes}\ v]}$$

Sem-eterm-FromARed
$$\frac{}{C[\mathsf{from}_{\ltimes}\ _{\{\}}\langle v , ()\rangle] \longrightarrow C[v]}$$

Sem-eterm-FillUFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft ()] \longrightarrow (C \circ (\square \triangleleft ()))[t]}$$

Sem-eterm-FillUUnfoc
$$\frac{}{(C \circ (\square \triangleleft ()))[v] \longrightarrow C[v \triangleleft ()]}$$

Sem-eterm-FillURed
$$\frac{}{C[+\mathtt{h} \triangleleft ()] \longrightarrow C[\mathtt{h} :=_{\{\}}\ ()][()]}$$

Sem-eterm-FillLFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft \mathsf{Inl}] \longrightarrow (C \circ (\square \triangleleft \mathsf{Inl}))[t]}$$

Sem-eterm-FillLUnfoc
$$\frac{}{(C \circ (\square \triangleleft \mathsf{Inl}))[v] \longrightarrow C[v \triangleleft \mathsf{Inl}]}$$

Sem-eterm-FillLRed
$$\frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C) \cup \{\mathtt{h}\})}{C[+\mathtt{h} \triangleleft \mathsf{Inl}] \longrightarrow C[\mathtt{h} :=_{\{\mathtt{h}'+1\}}\ \mathsf{Inl}\ -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillRFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft \mathsf{Inr}] \longrightarrow (C \circ (\square \triangleleft \mathsf{Inr}))[t]}$$

Sem-eterm-FillRUnfoc
$$\frac{}{(C \circ (\square \triangleleft \mathsf{Inr}))[v] \longrightarrow C[v \triangleleft \mathsf{Inr}]}$$

Sem-eterm-FillRRed
$$\frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C) \cup \{\mathtt{h}\})}{C[+\mathtt{h} \triangleleft \mathsf{Inr}] \longrightarrow C[\mathtt{h} :=_{\{\mathtt{h}'+1\}}\ \mathsf{Inr}\ -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillEFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft \mathrm{E}^{m}] \longrightarrow (C \circ (\square \triangleleft \mathrm{E}^{m}))[t]}$$

Sem-eterm-FillEUnfoc
$$\frac{}{(C \circ (\square \triangleleft \mathrm{E}^{m}))[v] \longrightarrow C[v \triangleleft \mathrm{E}^{m}]}$$

Sem-eterm-FillERed
$$\frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C) \cup \{\mathtt{h}\})}{C[+\mathtt{h} \triangleleft \mathrm{E}^{m}] \longrightarrow C[\mathtt{h} :=_{\{\mathtt{h}'+1\}}\ \mathrm{E}^{m}\ -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillPFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft (,)] \longrightarrow (C \circ (\square \triangleleft (,)))[t]}$$

Sem-eterm-FillPUnfoc
$$\frac{}{(C \circ (\square \triangleleft (,)))[v] \longrightarrow C[v \triangleleft (,)]}$$

Sem-eterm-FillPRed
$$\frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C) \cup \{\mathtt{h}\})}{C[+\mathtt{h} \triangleleft (,)] \longrightarrow C[\mathtt{h} :=_{\{\mathtt{h}'+1,\mathtt{h}'+2\}}\ (-(\mathtt{h}'+1) , -(\mathtt{h}'+2))][(+(\mathtt{h}'+1) , +(\mathtt{h}'+2))]}$$

Sem-eterm-FillFFoc
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft (\lambda x_{m} \mapsto u)] \longrightarrow (C \circ (\square \triangleleft (\lambda x_{m} \mapsto u)))[t]}$$

Sem-eterm-FillFUnfoc
$$\frac{}{(C \circ (\square \triangleleft (\lambda x_{m} \mapsto u)))[v] \longrightarrow C[v \triangleleft (\lambda x_{m} \mapsto u)]}$$

Sem-eterm-FillFRed
$$\frac{}{C[+\mathtt{h} \triangleleft (\lambda x_{m} \mapsto u)] \longrightarrow C[\mathtt{h} :=_{\{\}}\ \lambda^{v} x_{m} \mapsto u][()]}$$

Sem-eterm-FillCFoc1
$$\frac{\texttt{term\_NotVal}\ t}{C[t \triangleleft\!\bullet u] \longrightarrow (C \circ (\square \triangleleft\!\bullet u))[t]}$$

Sem-eterm-FillCUnfoc1
$$\frac{}{(C \circ (\square \triangleleft\!\bullet u))[v] \longrightarrow C[v \triangleleft\!\bullet u]}$$

Sem-eterm-FillCFoc2
$$\frac{}{C[v \triangleleft\!\bullet u] \longrightarrow (C \circ (v \triangleleft\!\bullet \square))[u]}$$

Sem-eterm-FillCUnfoc2
$$\frac{}{(C \circ (v \triangleleft\!\bullet \square))[v'] \longrightarrow C[v \triangleleft\!\bullet v']}$$

Sem-eterm-FillCRed
$$\frac{\mathtt{h}' = \mathtt{max}(\mathtt{hnames}(C) \cup \{\mathtt{h}\})}{C[+\mathtt{h} \triangleleft\!\bullet_{\mathtt{H}} \langle v_1 , v_2\rangle] \longrightarrow C[\mathtt{h} :=_{(\mathtt{H} \underline{\pm} \mathtt{h}')}\ v_1 \underline{\pm} \mathtt{h}'][v_2 \underline{\pm} \mathtt{h}']}$$