termvar, x, y, d	Term-level variable		
$\underline{label}$, $\underline{\ell}$	::=		Label
hole, h	::=		Hole
mode, m	::=		Mode
	|	L	Local
	|	F	Foreign
	|	G	Global
	|	max_mode(Γ)	
	|	if mode_cond then m_3 else m_4	
mode_cond	::=		Mode statement
	|	$m_1 = m_2$	
	|	$m_1 \leq m_2$	
	|	$m \in$ upper_modes (Γ)	
	|	$\exists m \in$ upper_modes (Γ)	
type, A, B	::=		Type
	|	1	Unit
	|	$A_1 \oplus A_2$	Sum
	|	$A_1 \otimes A_2$	Product
	|	$A_1 \rtimes A_2$	Ampar type (consuming A_1 yields A_2)
	|	$_{m_1} A_1 \multimap A_2$	Linear function
	|	$A^{\perp}$	Destination
	|	(A)	S
term_value, v	::=		Term value
	|	$\underline{\ell}$	Label representing an Ampar
	|	$@h$	Destination
	|	()	Unit
	|	Inl v	Left variant for sum
	|	Inr v	Right variant for sum
	|	(v_1 , v_2)	Product
	|	$\lambda x . t$	Linear function
	|	(v)	S
$\overline{\text{extended\_value}}$, $\overline{v}$	::=		Store value
	|	v	Term value
	|	h	Hole
	|	Inl $\overline{v}$	Left variant with val or hole
	|	Inr $\overline{v}$	Right variant with val or hole
	|	$(\overline{v_1} , \overline{v_2})$	Product with val or hole
	|	$(\overline{v})$	S
store_affect, ha	::=		
	|	$\underline{\ell} \mapsto \langle v_1 , \overline{v_2} \rangle$	Closed ampar ($\overline{v_2}$ = root of the incomplete stru
	|	$\underline{\ell} \mapsto \langle \square , \overline{v_2} \rangle$	Open ampar ($\overline{v_2}$ = root of the incomplete struc
store_affects	::=		

	| ha		
	| ha, store_affects		
store, S	::=		
	| []		
	| [store_affects]		
	| S[subs]		
	| $S_1 \sqcup S_2$		
term, t, u	::=		Term
	| v		Term value
	| x		Variable
	| t u		Application
	| t $\succ$**case** $() \mapsto$ u		Pattern-match on unit
	| t $\succ$**case** { **Inl** $x_1 \mapsto u_1$, **Inr** $x_2 \mapsto u_2$ }		Pattern-match on sum
	| t $\succ$**case** $(x_1, x_2) \mapsto$ u		Pattern-match on product
	| t $\succ$**mapL** $x \mapsto$ u		Map over the left side of the ampar
	| **to**$_{\bowtie}$ t		Wrap t into a trivial ampar
	| **from**$_{\bowtie}$ t		Extract value from trivial ampar
	| **alloc**		Return a fresh "identity" ampar objec
	| t $\lhd$ ()		Fill destination with unit
	| t $\lhd$ **Inl**		Fill destination with left variant
	| t $\lhd$ **Inr**		Fill destination with right variant
	| t $\lhd$ (,)		Fill destination with product construc
	| t $\lhd\bullet$ u		Fill destination with root of ampar u
	| (t)	S	
	| t[subs]	M	
$\overline{\text{extended\_term}}$, $\bar{t}$	::=		Extended term
	| t		
	| $\bar{v}$		
sub	::=		variable or label substitution
	| x := v		
	| $h := \bar{v}$		
subs	::=		variable or substitutions
	| sub		
	| sub, subs		
type_affect, ta	::=		type affectation
	| $x :_{\mathtt{m}}$ **A**		
	| $+\underline{\ell}$: **A**		
	| $-\underline{\ell}$: **A**		
	| $+h$: **A**		Hole
	| $-h$: **A**		Destination
type_affects	::=		type affectations
	| ta		

| | ta, type_affects

typing_context, $\mathfrak{V}$, Γ ::= typing context
| | {}
| | {type_affects}
| | $\Gamma_1 \sqcup \Gamma_2$
| | $\Gamma_1 \uplus \Gamma_2$
| | $\Gamma[\mathtt{m}_1 \mapsto \mathtt{m}_2]$
| | (Γ) S

terminals ::=
| | $\multimap$
| | $\rtimes$
| | $\mapsto$
| | ()
| | Inl
| | Inr
| | (,)
| | $\triangleleft$
| | $\blacktriangleleft$
| | $\sqcup$
| | $\uplus$
| | {}
| | $\exists$
| | $\neq$
| | $\leq$
| | $\in$
| | $\notin$
| | $\subset$
| | $\mathcal{N}$
| | $\vdash$
| | $\mid$
| | $\Downarrow$

formula ::=
| | judgement

Ctx ::=
| | $\mathtt{x} \in \mathcal{N}(\Gamma)$
| | $\underline{\ell} \in \mathcal{N}(\Gamma)$
| | $\mathtt{x} \notin \mathcal{N}(\Gamma)$
| | $\underline{\ell} \notin \mathcal{N}(\Gamma)$
| | fresh $\mathtt{x}$
| | fresh $\underline{\ell}$
| | fresh h
| | type_affect $\in \Gamma$
| | mode_cond

Eq ::=
| | $\mathtt{A}_1 = \mathtt{A}_2$

	$\mathsf{A}_1 \neq \mathsf{A}_2$
	$\mathsf{t} = \mathsf{u}$
	$\mathsf{t} \neq \mathsf{u}$
	$\Gamma_1 = \Gamma_2$
	$\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset$

Ty	::=	
	|	$\Gamma \vdash \mathsf{S} \mid \mathsf{t} : \mathsf{A}$
	|	$\Gamma \vdash \mathsf{S}$
	|	$\Gamma \vdash \bar{\mathsf{t}} : \mathsf{A}$

Sem	::=	
	|	$\mathsf{S} \mid \mathsf{t} \Downarrow \mathsf{S}' \mid \mathsf{t}'$

judgement	::=	
	|	Ctx
	|	Eq
	|	Ty
	|	Sem

user_syntax	::=	
	|	termvar
	|	$\underline{\ell abel\ell}$
	|	$hole$
	|	`mode`
	|	`mode_cond`
	|	**type**
	|	term_value
	|	extended_value
	|	store_affect
	|	store_affects
	|	store
	|	term
	|	extended_term
	|	sub
	|	subs
	|	type_affect
	|	type_affects
	|	typing_context
	|	terminals

$\mathsf{x} \in \mathcal{N}(\Gamma)$

$\underline{\ell} \in \mathcal{N}(\Gamma)$

$\mathsf{x} \notin \mathcal{N}(\Gamma)$

$\underline{\ell} \notin \mathcal{N}(\Gamma)$

fresh x

fresh $\underline{\ell}$

fresh h

type_affect $\in \Gamma$

$\boxed{\texttt{mode\_cond}}$

$\boxed{\mathsf{A}_1 = \mathsf{A}_2}$

$\boxed{\mathsf{A}_1 \neq \mathsf{A}_2}$

$\boxed{\mathsf{t} = \mathsf{u}}$

$\boxed{\mathsf{t} \neq \mathsf{u}}$

$\boxed{\Gamma_1 = \Gamma_2}$

$\boxed{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}$

$\boxed{\Gamma \vdash \mathsf{S} \mid \mathsf{t} : \mathsf{A}}$

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{S} \\ \Gamma_2 \vdash \mathsf{t} : \mathsf{A}\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{S} \mid \mathsf{t} : \mathsf{A}}\ \textsc{TyCmd\_Cmd}$$

$\boxed{\Gamma \vdash \mathsf{S}}$

$$\frac{}{\{\} \vdash []}\ \textsc{TyHeap\_Empty}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{S}_1 \\ \Gamma_2 \vdash \mathsf{S}_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{S}_1 \sqcup \mathsf{S}_2}\ \textsc{TyHeap\_Union}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{v}_1 : \mathsf{A}_1 \\ \Gamma_2 \vdash \overline{\mathsf{v}_2} : \mathsf{A}_2\end{array}}{(\Gamma_1 \uplus \Gamma_2) \sqcup \{-\underline{\ell} : \mathsf{A}_1 \rtimes \mathsf{A}_2\} \vdash [\underline{\ell} \mapsto \langle \mathsf{v}_1 \, , \overline{\mathsf{v}_2} \rangle]}\ \textsc{TyHeap\_ClosedAmpar}$$

$$\frac{\Gamma_2 \vdash \overline{\mathsf{v}_2} : \mathsf{A}_2}{\Gamma_2 \vdash [\underline{\ell} \mapsto \langle \square \, , \overline{\mathsf{v}_2} \rangle]}\ \textsc{TyHeap\_OpenAmpar}$$

$\boxed{\Gamma \vdash \bar{\mathsf{t}} : \mathsf{A}}$

$$\frac{}{\{+\underline{\ell} : \mathsf{A}\} \vdash \underline{\ell} : \mathsf{A}}\ \textsc{TyTerm\_Ampar}$$

$$\frac{}{\{-h : \mathsf{A}\} \vdash @h : \mathsf{A}^{\perp}}\ \textsc{TyTerm\_Dest}$$

$$\frac{}{\{+h : \mathsf{A}\} \vdash h : \mathsf{A}}\ \textsc{TyTerm\_Hole}$$

$$\frac{}{\{\} \vdash () : \mathbf{1}}\ \textsc{TyTerm\_Unit}$$

$$\frac{\Gamma \vdash \bar{\mathsf{v}} : \mathsf{A}_1}{\Gamma \vdash \mathsf{Inl}\, \bar{\mathsf{v}} : \mathsf{A}_1 \oplus \mathsf{A}_2}\ \textsc{TyTerm\_Inl}$$

$$\frac{\Gamma \vdash \bar{\mathsf{v}} : \mathsf{A}_2}{\Gamma \vdash \mathsf{Inr}\, \bar{\mathsf{v}} : \mathsf{A}_1 \oplus \mathsf{A}_2}\ \textsc{TyTerm\_Inr}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash \overline{\mathsf{v}_1} : \mathsf{A}_1 \\ \Gamma_2 \vdash \overline{\mathsf{v}_2} : \mathsf{A}_2\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash (\overline{\mathsf{v}_1} \, , \overline{\mathsf{v}_2}) : \mathsf{A}_1 \otimes \mathsf{A}_2}\ \textsc{TyTerm\_Prod}$$

$$\frac{\Gamma \sqcup \{\mathsf{x} :_{\mathsf{m}_1} \mathsf{A}_1\} \vdash \mathsf{t} : \mathsf{A}_2}{\Gamma \vdash \lambda \mathsf{x} \, . \, \mathsf{t} :_{\mathsf{m}_1} \mathsf{A}_1 \multimap \mathsf{A}_2}\ \textsc{TyTerm\_Lambda}$$

$$\frac{\Gamma_1 \vdash t :_{m_1} A_1 \multimap A_2 \quad \Gamma_2 \vdash u : A_1 \quad m_1 \in \mathsf{upper\_modes}(\Gamma_2)}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ u : A_2}\ \textsc{TyTerm\_App}$$

$$\frac{\Gamma_1 \vdash t : 1 \quad \Gamma_2 \vdash u : B}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{case}\,() \mapsto u : B}\ \textsc{TyTerm\_PatUnit}$$

$$\frac{\Gamma_1 \vdash t : A_1 \oplus A_2 \quad \exists m \in \mathsf{upper\_modes}(\Gamma_1) \quad \Gamma_2 \sqcup \{x_1 :_m A_1\} \vdash u_1 : B \quad \Gamma_2 \sqcup \{x_2 :_m A_2\} \vdash u_2 : B}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{case}\,\{\,\mathsf{Inl}\,x_1 \mapsto u_1\,,\ \mathsf{Inr}\,x_2 \mapsto u_2\,\} : B}\ \textsc{TyTerm\_PatSum}$$

$$\frac{\Gamma_1 \vdash t : A_1 \otimes A_2 \quad \exists m \in \mathsf{upper\_modes}(\Gamma_1) \quad \Gamma_2 \sqcup \{x_1 :_m A_1, x_2 :_m A_2\} \vdash u : B}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{case}\,(x_1\,,\,x_2) \mapsto u : B}\ \textsc{TyTerm\_PatProd}$$

$$\frac{\Gamma_1 \vdash t : A_1 \rtimes A_2 \quad \exists m' \in \mathsf{upper\_modes}(\Gamma_1 \sqcup \Gamma_2) \quad m = \mathsf{if}\ \mathtt{F} \in \mathsf{upper\_modes}(\Gamma_1)\ \mathsf{then}\ \mathtt{F}\ \mathsf{else}\ \mathtt{L} \quad \Gamma_2[\mathtt{L} \mapsto \mathtt{F}] \sqcup \{x :_m A_1\} \vdash u : B}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \mathsf{mapL}\,x \mapsto u : B \rtimes A_2}\ \textsc{TyTerm\_MapAmpar}$$

$$\frac{}{\{\} \vdash \mathsf{alloc} : A^\perp \rtimes A}\ \textsc{TyTerm\_Alloc}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{to}_\rtimes\, t : 1 \rtimes A}\ \textsc{TyTerm\_ToAmpar}$$

$$\frac{\Gamma \vdash t : 1 \rtimes A}{\Gamma \vdash \mathsf{from}_\rtimes\, t : A}\ \textsc{TyTerm\_FromAmpar}$$

$$\frac{\Gamma \vdash t : 1^\perp}{\Gamma \vdash t \triangleleft () : 1}\ \textsc{TyTerm\_FillUnit}$$

$$\frac{\Gamma \vdash t : (A_1 \oplus A_2)^\perp}{\Gamma \vdash t \triangleleft \mathsf{Inl} : A_1{}^\perp}\ \textsc{TyTerm\_FillInl}$$

$$\frac{\Gamma \vdash t : (A_1 \oplus A_2)^\perp}{\Gamma \vdash t \triangleleft \mathsf{Inr} : A_2{}^\perp}\ \textsc{TyTerm\_FillInr}$$

$$\frac{\Gamma \vdash t : (A_1 \otimes A_2)^\perp}{\Gamma \vdash t \triangleleft (,) : A_1{}^\perp \otimes A_2{}^\perp}\ \textsc{TyTerm\_FillProd}$$

$$\frac{\Gamma_1 \vdash t : A_2{}^\perp \quad \Gamma_2 \vdash u : A_1 \rtimes A_2 \quad \mathtt{L} \in \mathsf{upper\_modes}(\Gamma_1) \quad \mathtt{F} \in \mathsf{upper\_modes}(\Gamma_2)}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft\!\bullet u : A_1}\ \textsc{TyTerm\_FillCompL}$$

$$\frac{\Gamma_1 \vdash t : A_2{}^\perp \quad \Gamma_2 \vdash u : A_1 \rtimes A_2 \quad \mathtt{F} \in \mathsf{upper\_modes}(\Gamma_1) \quad \mathtt{G} \in \mathsf{upper\_modes}(\Gamma_2)}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft\!\bullet u : A_1}\ \textsc{TyTerm\_FillCompF}$$

$\boxed{S \mid t \Downarrow S' \mid t'}$

$$\frac{}{S_0 \mid v \Downarrow S_0 \mid v}\ \textsc{BigStep\_Val}$$

$$\frac{\begin{array}{c}S_0 \mid t_1 \Downarrow S_1 \mid \lambda x.\, u \\ S_1 \mid t_2 \Downarrow S_2 \mid v_2 \\ S_2 \mid u[x := v_2] \Downarrow S_3 \mid v_3\end{array}}{S_0 \mid t_1\, t_2 \Downarrow S_3 \mid v_3}\ \textsc{BigStep\_App}$$

$$\frac{\begin{array}{c}S_0 \mid t_1 \Downarrow S_1 \mid () \\ S_1 \mid t_2 \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t_1 \ \succ \mathsf{case}\,() \mapsto\ t_2 \Downarrow S_2 \mid v_2}\ \textsc{BigStep\_PatUnit}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid \mathsf{Inl}\, v_1 \\ S_1 \mid u_1[x_1 := v_1] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t\ \succ \mathsf{case}\,\{\,\mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2\,\} \Downarrow S_2 \mid v_2}\ \textsc{BigStep\_PatInl}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid \mathsf{Inr}\, v_1 \\ S_1 \mid u_2[x_2 := v_1] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t\ \succ \mathsf{case}\,\{\,\mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2\,\} \Downarrow S_2 \mid v_2}\ \textsc{BigStep\_PatInr}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid (v_1\,,\, v_2) \\ S_1 \mid u[x_1 := v_1, x_2 := v_2] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t\ \succ \mathsf{case}\,(x_1\,,\, x_2) \mapsto\ u \Downarrow S_2 \mid v_2}\ \textsc{BigStep\_PatProd}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \sqcup [\,\underline{\ell} \mapsto \langle v_1\,,\, \overline{v_1}\rangle\,] \mid \underline{\ell} \\ S_1 \sqcup [\,\underline{\ell} \mapsto \langle \square\,,\, \overline{v_1}\rangle\,] \mid u[x := v_1] \Downarrow S_2 \sqcup [\,\underline{\ell} \mapsto \langle \square\,,\, \overline{v_2}\rangle\,] \mid v_2\end{array}}{S_0 \mid t\ \succ \mathsf{mapL}\, x \mapsto\ u \Downarrow S_2 \sqcup [\,\underline{\ell} \mapsto \langle v_2\,,\, \overline{v_2}\rangle\,] \mid \underline{\ell}}\ \textsc{BigStep\_MapAmpar}$$

$$\frac{\begin{array}{c}\mathsf{fresh}\ h \\ \mathsf{fresh}\ \underline{\ell}\end{array}}{S_0 \mid \mathsf{alloc} \Downarrow S_0 \sqcup [\,\underline{\ell} \mapsto \langle @h\,,\, h\rangle\,] \mid \underline{\ell}}\ \textsc{BigStep\_Alloc}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid v \\ \mathsf{fresh}\ \underline{\ell}\end{array}}{S_0 \mid \mathsf{to}_{\rtimes}\, t \Downarrow S_1 \sqcup [\,\underline{\ell} \mapsto \langle ()\,,\, v\rangle\,] \mid \underline{\ell}}\ \textsc{BigStep\_ToAmpar}$$

$$\frac{S_0 \mid t \Downarrow S_1 \sqcup [\,\underline{\ell} \mapsto \langle ()\,,\, v\rangle\,] \mid \underline{\ell}}{S_0 \mid \mathsf{from}_{\rtimes}\, t \Downarrow S_1 \mid v}\ \textsc{BigStep\_FromAmpar}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h}{S_0 \mid t \triangleleft () \Downarrow S_1[h := ()] \mid ()}\ \textsc{BigStep\_FillUnit}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid @h \\ \mathsf{fresh}\ h'\end{array}}{S_0 \mid t \triangleleft \mathsf{Inl} \Downarrow S_1[h := \mathsf{Inl}\, h'] \mid @h'}\ \textsc{BigStep\_FillInl}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid @h \\ \mathsf{fresh}\ h'\end{array}}{S_0 \mid t \triangleleft \mathsf{Inr} \Downarrow S_1[h := \mathsf{Inr}\, h'] \mid @h'}\ \textsc{BigStep\_FillInr}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid @h \\ \mathsf{fresh}\ h_1 \\ \mathsf{fresh}\ h_2\end{array}}{S_0 \mid t \triangleleft (,) \Downarrow S_1[h := (h_1\,,\, h_2)] \mid (@h_1\,,\, @h_2)}\ \textsc{BigStep\_FillProd}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid @h \\ S_1 \mid u \Downarrow S_2 \sqcup [\,\underline{\ell} \mapsto \langle v_1\,,\, \overline{v_1}\rangle\,] \mid \underline{\ell}\end{array}}{S_0 \mid t \triangleleft\bullet\, u \Downarrow S_2[h := \overline{v_1}] \mid v_1}\ \textsc{BigStep\_FillComp}$$

```
Definition rules:        42 good    0 bad
Definition rule clauses: 112 good    0 bad
```