

metavariable, x, xs, y, uf, f, d

term, t, u ::=

- | x
- | v
- | $t\ u$
- | $t\ ;\ u$
- | $\text{case } t \text{ of } \{ () \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}$
- | $\text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \}$
- | $\text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{roll } x \mapsto u \}$
 $\textcolor{blue}{R}$
- | $\text{alloc } d . t$
 $\textcolor{blue}{A}$
- | $t \triangleleft ()$
- | $t \triangleleft u$
- | $t \triangleleft \text{Ur } u$
- | $t \triangleleft 1.d . u$
- | $t \triangleleft 2.d . u$
- | $t \triangleleft \langle d_1, d_2 \rangle . u$
- | $t \triangleleft \text{roll } d . u$
 $\textcolor{blue}{R}$
- | (t)
- | $t[\text{var_subs}]$

term

- variable
- value
- application
- effect execution
- pattern-matching on unit
- pattern-matching on exponentiated value
- pattern-matching on sum
- pattern-matching on product
- unroll for recursive types
- allocate data
- fill destination with unit
- fill terminal-type destination
- fill destination with exponential
- fill sum-type destination with variant 1
- fill sum-type destination with variant 2
- fill product-type destination
- fill destination with recursive type

S
M

var_sub, vs ::=

- | $x := t$

variable substitution

var_subs ::=

- | vs
- | $vs, \text{var_subs}$

variable substitutions

heap_val, h ::=

- | $()$
- | $\text{Ur } l$
- | $1.l$
- | $2.l$
- | $\langle l_1, l_2 \rangle$
- | $\text{roll } l$
 $\textcolor{blue}{R}$
- | $C\bar{l}$

M generic for all the cases above

val, v ::=

- | \bullet
- | $[l]$
- | $\lambda x:A . t$
- | h

unreducible value

- no-effect effect
- address of an allocated memory area
- lambda abstraction
- heap value

$label, l$::=

memory address

labels ::=

- | l
- | l, labels
- | \bar{l}

M

	\bar{l} , labels	M	
$label_set, L$	$::=$		set of used labels
	\emptyset		
	$\{labels\}$		
	$L_1 \sqcup L_2$		
	(L)	S	
heap_affect, ha	$::=$		heap cell
	$l \triangleleft v$		
	$\bar{l} \triangleleft \bar{v}$	M	generic for multiple occurrences
heap_affects	$::=$		heap cells
	ha		
	ha, heap_affects		
heap_context, \mathbb{H}	$::=$		heap contents
	\emptyset		
	$\{heap_affects\}$		
	$\mathbb{H}_1 \sqcup \mathbb{H}_2$		
type, A, B	$::=$		
	\perp		bottom type
	1		unit type
	R		recursive type bound to a name
	$A \otimes B$		product type
	$A \oplus B$		sum type
	$A \multimap B$		linear function type
	$[A]$		destination type
	$!A$		exponential
	(A)	S	
	$W[r := A]$	M	
type_with_hole, W	$::=$		
	\perp		bottom type
	r		type hole in recursive definition
	1		unit type
	R		recursive type bound to a name
	$W_1 \otimes W_2$		product type
	$W_1 \oplus W_2$		sum type
	$W_1 \multimap W_2$		linear function type
	$[W]$		destination type
	$!W$		exponential
	(W)	S	
rec.type_bound, R	$::=$		recursive type bound to a name
rec_type_def	$::=$		
	$\mu r. W$		

type_affect, ta	$::=$ $\begin{array}{ l} x : A \\ l : A \\ \bar{l} : \bar{A} \end{array}$	type affectation var label generic for multiple occurrences
type_affects	$::=$ $\begin{array}{ l} ta \\ ta, type_affects \end{array}$	type affectations
typing_context, $\Gamma, \mathcal{U}, \Phi, \Psi$	$::=$ $\begin{array}{ l} \emptyset \\ \{type_affects\} \\ \Gamma_1 \sqcup \Gamma_2 \end{array}$	typing context
types, \bar{A}	$::=$ $\begin{array}{ l} \cdot \\ A \\ A \text{ types} \end{array}$	empty type list
command	$::=$ $\begin{array}{ l} \mathbb{H} \diamond_L t \end{array}$	
heap_constructor, C	$::=$ $\begin{array}{ l} \{()\} \\ \{U_r\} \\ \{1.\} \\ \{2.\} \\ \{\langle, \rangle\} \\ \{\text{roll } R\} \end{array}$	
judg	$::=$ $\begin{array}{ l} l \in \text{names}(\Phi) \\ l \notin \text{names}(\Phi) \\ type_affect \in \Gamma \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Phi) \sqcup \text{names}(\Psi) \subset L \\ l \in L \\ l \notin L \\ heap_affect \in \mathbb{H} \\ A = B \\ t = u \\ \Gamma = D \\ R \stackrel{\text{fix}}{=} \text{rec_type_def} \\ C : \bar{A} \subseteq A \\ \Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{command} : A \\ \Phi ; \Psi \vdash \mathbb{H} \\ \Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A \\ \text{command} \Downarrow \text{command}' \end{array}$	

terminals

::=

()
 \mapsto
 \star
 \otimes
 \oplus
 \circ
 $:=$
 \vdash
 \sqcup
 $;$
 \cap
 \emptyset
 \rightarrow
 \triangleright
 \neq
 \in
 \notin
 $\backslash n$
 \langle
 \rangle
1.
2.
Ur
 \triangleleft
 $|$
 \otimes
 \hookrightarrow
 $=$
 \Downarrow
 \dots
fix
 \equiv
 \perp
 \bullet
 \subset

formula

::=

| judgement

Ctx

::=

| $l \in \text{names}(\Phi)$
| $l \notin \text{names}(\Phi)$
| $\text{type_affect} \in \Gamma$
| $\text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset$

Γ_1 and Γ_2 are disjoint typing contexts with no clash

LabelSet

::=

| $\text{names}(\Phi) \sqcup \text{names}(\Psi) \subset L$
| $l \in L$
| $l = \text{fresh}(L \setminus L)$

Heap	$::=$ $ \quad \text{heap_affect} \in \mathbb{H}$
Eq	$::=$ $ \quad A = B$ $ \quad t = u$ $ \quad \Gamma = D$
Ty	$::=$ $ \quad R \stackrel{\text{fix}}{=} \text{rec_type_def}$ $ \quad C : \bar{A} \xrightarrow{c} A$ $ \quad \Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{command} : A$ $ \quad \Phi ; \Psi \vdash \mathbb{H}$ $ \quad \Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A$
Sem	$::=$ $ \quad \text{command} \Downarrow \text{command}'$
judgement	$::=$ $ \quad \text{Ctx}$ $ \quad \text{LabelSet}$ $ \quad \text{Heap}$ $ \quad \text{Eq}$ $ \quad \text{Ty}$ $ \quad \text{Sem}$
user_syntax	$::=$ $ \quad \text{metavariable}$ $ \quad \text{term}$ $ \quad \text{var_sub}$ $ \quad \text{var_subs}$ $ \quad \text{heap_val}$ $ \quad \text{val}$ $ \quad \textit{label}$ $ \quad \text{labels}$ $ \quad \textit{label_set}$ $ \quad \text{heap_affect}$ $ \quad \text{heap_affects}$ $ \quad \text{heap_context}$ $ \quad \text{type}$ $ \quad \text{type_with_hole}$ $ \quad \text{rec_type_bound}$ $ \quad \text{rec_type_def}$ $ \quad \text{type_affect}$ $ \quad \text{type_affects}$ $ \quad \text{typing_context}$ $ \quad \text{types}$ $ \quad \text{command}$ $ \quad \text{heap_constructor}$ $ \quad \text{judg}$

Heap constructor C builds a value of type A given a

\mathbb{H} is a well-typed heap given heap typing context Φ

t is a well-typed term of type A given heap typing c

| terminals

$$l \in \text{names}(\Phi)$$

$$l \notin \text{names}(\Phi)$$

$$\text{type_affect} \in \Gamma$$

$$\text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset$$

Γ_1 and Γ_2 are disjoint typing contexts with no clashing variable names or labels

$$\text{names}(\Phi) \sqcup \text{names}(\Psi) \subset L$$

$$l \in L$$

$$l = \text{fresh}(L \setminus L)$$

$$\text{heap_affect} \in \mathbb{H}$$

$$A = B$$

$$t = u$$

$$\Gamma = D$$

$$R \stackrel{\text{fix}}{=} \text{rec_type_def}$$

$$C : \bar{A} \hookrightarrow A \quad \text{Heap constructor } C \text{ builds a value of type } A \text{ given arguments of type } \bar{A}$$

$$\frac{}{\{()\} : \cdot \hookrightarrow 1} \text{TYCTOR_U}$$

$$\frac{}{\{1.\} : A \hookrightarrow A \oplus B} \text{TYCTOR_V1}$$

$$\frac{}{\{2.\} : B \hookrightarrow A \oplus B} \text{TYCTOR_V2}$$

$$\frac{}{\{\langle, \rangle\} : A \ B \hookrightarrow A \otimes B} \text{TYCTOR_P}$$

$$\frac{}{\{\text{Ur}\} : A \hookrightarrow !A} \text{TYCTOR_E}$$

$$\frac{R \stackrel{\text{fix}}{=} \mu r. W}{\{\text{roll } R\} : W[r := R] \hookrightarrow R} \text{TYCTOR_R}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{command} : A$$

$$\frac{\begin{array}{l} \text{names}(\Phi) \sqcup \text{names}(\Psi) \subset L \\ \Phi ; \Psi \vdash \mathbb{H} \\ \Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A \end{array}}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \diamond_L t : A} \text{TYCOMMAND_DEF}$$

$$\Phi ; \Psi \vdash \mathbb{H} \quad \mathbb{H} \text{ is a well-typed heap given heap typing context } \Phi$$

$$\frac{}{\emptyset ; \Psi \vdash \emptyset} \text{TYHEAP_EMPTY}$$

$$\frac{\begin{array}{l} \Phi ; \Psi \vdash \mathbb{H} \\ \Phi ; \Psi ; \emptyset ; \emptyset \vdash v : A \end{array}}{\Phi \sqcup \{l : A\} ; \Psi \vdash \mathbb{H} \sqcup \{l \triangleleft v\}} \text{TYHEAP_VAL}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A \quad t \text{ is a well-typed term of type } A \text{ given heap typing context } \Phi, \text{ unrestricted typing context } \Psi, \text{ and heap typing context } \mathcal{U}$$

$$\frac{}{\Phi ; \emptyset ; \mathcal{U} ; \emptyset \vdash \bullet : \perp} \text{TYTERM_NOEFF}$$

$$\begin{array}{c}
\frac{l \notin \text{names}(\Phi)}{\Phi ; \{l : A\} ; \mathcal{U} ; \emptyset \vdash [l] : [A]} \text{TYTERM_LDEST} \\
\\
\frac{\Phi ; \Psi ; \mathcal{U} ; \Gamma \sqcup \{x : A\} \vdash t : B}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \lambda x : A. t : A \multimap B} \text{TYTERM_LAM} \\
\\
\frac{C : \bar{A} \xrightarrow{c} A}{\Phi \sqcup \{\bar{l} : \bar{A}\} ; \emptyset ; \mathcal{U} ; \emptyset \vdash C\bar{l} : A} \text{TYTERM_HEAPVAL} \\
\\
\frac{}{\Phi ; \emptyset ; \mathcal{U} ; \{x : A\} \vdash x : A} \text{TYTERM_ID} \\
\\
\frac{}{\Phi ; \emptyset ; \mathcal{U} \sqcup \{x : A\} ; \emptyset \vdash x : A} \text{TYTERM_ID'} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : A \multimap B \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \vdash u : A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t u : B} \text{TYTERM_APP} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : \perp \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t ; u : B} \text{TYTERM_Witheff} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : 1 \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \vdash u : A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{() \mapsto u\} : A} \text{TYTERM_PATU} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : !A \\ \Phi ; \Psi_2 ; \mathcal{U} \sqcup \{x : A\} ; \Gamma_2 \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \} : B} \text{TYTERM_PATE} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : A_1 \oplus A_2 \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{x_1 : A_1\} \vdash u_1 : B \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{x_2 : A_2\} \vdash u_2 : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} : B} \text{TYTERM_PATs} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : A_1 \otimes A_2 \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{x_1 : A_1, x_2 : A_2\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \} : B} \text{TYTERM_PATP} \\
\\
\frac{\begin{array}{l} R \stackrel{\text{fix}}{=} \mu r. W \\ \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : R \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{x : W[r := R]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \text{roll } x \mapsto u \} : B} \text{TYTERM_PATR}
\end{array}$$

$$\begin{array}{c}
\frac{\Phi ; \Psi ; \mathcal{U} ; \Gamma \sqcup \{d : [A]\} \vdash t : \perp}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{alloc } d . t : A} \quad \text{TYTERM_ALLOC} \\
\\
\frac{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : [1]}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t \triangleleft () : \perp} \quad \text{TYTERM_FILLU} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \vdash u : A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft u : \perp} \quad \text{TYTERM_FILLL} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma \vdash t : [!A] \\ \Phi ; \emptyset ; \mathcal{U} ; \emptyset \vdash u : A \end{array}}{\Phi ; \Psi_1 ; \mathcal{U} ; \Gamma \vdash t \triangleleft \text{Ur } u : \perp} \quad \text{TYTERM_FILLE} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \oplus A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d' : [A_1]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft 1.d'.u : B} \quad \text{TYTERM_FILLV1} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \oplus A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d' : [A_2]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma \vdash t \triangleleft 2.d'.u : B} \quad \text{TYTERM_FILLV2} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \otimes A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d_1 : [A_1], d_2 : [A_2]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft \langle d_1, d_2 \rangle . u : B} \quad \text{TYTERM_FILLP} \\
\\
\frac{\begin{array}{l} R \stackrel{\text{fix}}{=} \mu r . W \\ \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [R] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d : [W[r := R]]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft \text{roll } d . u : B} \quad \text{TYTERM_FILLR}
\end{array}$$

command \Downarrow command'

$$\begin{array}{c}
\frac{}{\mathbb{H} \diamond_L \bullet \Downarrow \mathbb{H} \diamond_L \bullet} \quad \text{SEMOP_NOEFF (value)} \\
\\
\frac{}{\mathbb{H} \diamond_L [l] \Downarrow \mathbb{H} \diamond_L [l]} \quad \text{SEMOP_LDEST (value)} \\
\\
\frac{}{\mathbb{H} \diamond_L \lambda x:A . t \Downarrow \mathbb{H} \diamond_L \lambda x:A . t} \quad \text{SEMOP_LAM (value)} \\
\\
\frac{}{\mathbb{H} \diamond_L C\bar{l} \Downarrow \mathbb{H} \diamond_L C\bar{l}} \quad \text{SEMOP_HEAPVAL (value)}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} \lambda x:A. t' \quad \mathbb{H}_1 \diamond_{L_1} u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2 \quad \mathbb{H}_2 \diamond_{L_2} t'[x := v_2] \Downarrow \mathbb{H}_3 \diamond_{L_3} v_3}{\mathbb{H}_0 \diamond_{L_0} t u \Downarrow \mathbb{H}_3 \diamond_{L_3} v_3} \text{SEMOP_APP} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} () \quad \mathbb{H}_1 \diamond_{L_1} u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ () \mapsto u \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATU} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} \text{Ur } l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} u[x := v_1] \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATE} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} 1.l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} u_1[x_1 := v] \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATV1} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} 2.l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} u_2[x_2 := v] \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATV2} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \sqcup \{ l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12} \} \diamond_{L_1} \langle l_1, l_2 \rangle \quad \mathbb{H}_1 \sqcup \{ l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12} \} \diamond_{L_1} u[x_1 := v_1, x_2 := v_2] \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATP} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} \text{roll } l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} u[x := v_1] \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} \text{case } t \text{ of } \{ \text{roll } x \mapsto u \} \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_PATR} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} \bullet \quad \mathbb{H}_1 \diamond_{L_1} u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} t ; u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2} \text{SEMOP_WITHEFF} \\
\\
\frac{l = \text{fresh}(\mathbb{L} \setminus L_0) \quad \mathbb{H}_0 \diamond_{L_0 \sqcup \{l\}} t[d := [l]] \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} \bullet}{\mathbb{H}_0 \diamond_{L_0} \text{alloc } d . t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \diamond_{L_1} v_1} \text{SEMOP_ALLOC} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l]}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft () \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft () \} \diamond_{L_1} \bullet} \text{SEMOP_FILLU}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \quad \mathbb{H}_1 \diamond_{L_1} u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft v_2\} \diamond_{L_2} \bullet} \text{SEMOP_FILLL} \\
\\
\frac{\mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \quad \mathbb{H}_1 \diamond_{L_1} u \Downarrow \mathbb{H}_2 \diamond_{L_2} v_2}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft \text{Ur } u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{Ur } l', l' \triangleleft v_2\} \diamond_{L_2} \bullet} \text{SEMOP_FILLE} \\
\\
\frac{\begin{array}{l} l' = \text{fresh}(\mathbb{L} \setminus L_1) \\ \mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \\ \mathbb{H}_1 \diamond_{L_1 \sqcup \{l'\}} u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \diamond_{L_2} v_2 \end{array}}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft 1.d.u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 1.l', l' \triangleleft v_1\} \diamond_{L_2} v_2} \text{SEMOP_FILLV1} \\
\\
\frac{\begin{array}{l} l' = \text{fresh}(\mathbb{L} \setminus L_1) \\ \mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \\ \mathbb{H}_1 \diamond_{L_1 \sqcup \{l'\}} u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \diamond_{L_2} v_2 \end{array}}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft 2.d.u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 2.l', l' \triangleleft v_1\} \diamond_{L_2} v_2} \text{SEMOP_FILLV2} \\
\\
\frac{\begin{array}{l} l_1 = \text{fresh}(\mathbb{L} \setminus L_1) \\ l_2 = \text{fresh}(\mathbb{L} \setminus (L_1 \sqcup \{l_1\})) \\ \mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \\ \mathbb{H}_1 \diamond_{L_1 \sqcup \{l_1, l_2\}} u[d_1 := [l_1], d_2 := [l_2]] \Downarrow \mathbb{H}_2 \sqcup \{l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} \diamond_{L_2} v_2 \end{array}}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft \langle d_1, d_2 \rangle . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} \diamond_{L_2} v_2} \text{SEMOP_FILLP} \\
\\
\frac{\begin{array}{l} l' = \text{fresh}(\mathbb{L} \setminus L_1) \\ \mathbb{H}_0 \diamond_{L_0} t \Downarrow \mathbb{H}_1 \diamond_{L_1} [l] \\ \mathbb{H}_1 \diamond_{L_1 \sqcup \{l'\}} u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \diamond_{L_2} v_2 \end{array}}{\mathbb{H}_0 \diamond_{L_0} t \triangleleft \text{roll } d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{roll } l', l' \triangleleft v_1\} \diamond_{L_2} v_2} \text{SEMOP_FILLR}
\end{array}$$

Definition rules: 50 good 0 bad
 Definition rule clauses: 151 good 0 bad