Destination λ-calculus

Thomas Bagrel

March 11, 2024

1 Term and value syntax

var, x, y	Term-level variable name
k	Index for ranges

$\mathtt{hdn}$, $\mathtt{h}$	$::=$		Hole or destination name
	$\mid$	$\mathtt{1}$	
	$\mid$	$\mathtt{2}$	
$\mathtt{hdns}$, $\mathtt{H}$	$::=$		Set of hole or destination names
	$\mid$	$\{\mathtt{h}_1, .., \mathtt{h}_{\mathsf{k}}\}$	Set of holes
val, v	$::=$		Term value
	$\mid$	$-\mathtt{h}$	Hole
	$\mid$	$+\mathtt{h}$	Destination
	$\mid$	$()$	Unit
	$\mid$	$\lambda^{\mathsf{v}}\mathsf{x} \mapsto \mathsf{t}$	Lambda abstraction
	$\mid$	$\mathsf{Inl}\,\mathsf{v}$	Left variant for sum
	$\mid$	$\mathsf{Inr}\,\mathsf{v}$	Right variant for sum
	$\mid$	$\mathrm{E}^{m}\,\mathsf{v}$	Exponential
	$\mid$	$(\mathsf{v}_1\,,\,\mathsf{v}_2)$	Product
	$\mid$	$_{\mathtt{H}}\langle \mathsf{v}_1 \,{}_{\mathbf{9}}\, \mathsf{v}_2\rangle$	Ampar
term, t, u	$::=$		Term
	$\mid$	v	Value
	$\mid$	x	Variable
	$\mid$	$\mathsf{t} \succ \mathsf{u}$	Application
	$\mid$	$\mathsf{t}\,;\,\mathsf{u}$	Pattern-match on unit
	$\mid$	$\mathsf{t} \succ \mathbf{case}_{m}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}$	Pattern-match on sum
	$\mid$	$\mathsf{t} \succ \mathbf{case}_{m}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u}$	Pattern-match on product
	$\mid$	$\mathsf{t} \succ \mathbf{case}_{m}\,\mathrm{E}^{n}\,\mathsf{x} \mapsto \mathsf{u}$	Pattern-match on exponential
	$\mid$	$\mathsf{t} \succ \mathbf{map}\,\mathsf{x} \mapsto \mathsf{u}$	Map over the right side of the ampar
	$\mid$	$\mathbf{to}_{\ltimes}\,\mathsf{t}$	Wrap t into a trivial ampar
	$\mid$	$\mathbf{from}_{\ltimes}\,\mathsf{t}$	Extract value from trivial ampar
	$\mid$	$\mathbf{alloc}_{\mathsf{T}}$	Return a fresh "identity" ampar object
	$\mid$	$\mathsf{t} \triangleleft ()$	Fill destination with unit
	$\mid$	$\mathsf{t} \triangleleft \lambda\mathsf{x} \mapsto \mathsf{u}$	Fill destination with function
	$\mid$	$\mathsf{t} \triangleleft \mathsf{Inl}$	Fill destination with left variant
	$\mid$	$\mathsf{t} \triangleleft \mathsf{Inr}$	Fill destination with right variant
	$\mid$	$\mathsf{t} \triangleleft (,)$	Fill destination with product constructor
	$\mid$	$\mathsf{t} \triangleleft \mathrm{E}^{m}$	Fill destination with exponential constructor
	$\mid$	$\mathsf{t} \triangleleft\!\bullet\, \mathsf{u}$	Fill destination with root of ampar u
eterm, j	$::=$		Pseudo-term
	$\mid$	$\mathsf{C}[\mathsf{t}]$	
ectx, C	$::=$		Evaluation context
	$\mid$	$\square$	Identity

$\mid$	${}^{o}_{\mathtt{H}}\langle \mathsf{v}_1 \mathbin{,} \mathsf{C}$	Open ampar
$\mid$	$\mathsf{C} \circ \mathsf{C}'$	Compose evaluation contexts

2 Type system

type, T, U	::=		Type
	|	$\mathbf{1}$	Unit
	|	$T_1 \oplus T_2$	Sum
	|	$T_1 \otimes T_2$	Product
	|	$!^{m}\, T$	Exponential
	|	$T_1 \ltimes T_2$	Ampar type (consuming T_2 yields T_1)
	|	$T_1 \ {}_{m_1}\!\!\to T_2$	Function
	|	$\lfloor T \rfloor^{m}$	Destination
	|	$T_1 \rightarrowtail T_2$	Evaluation contexts
mode, m, n	::=		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	1	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	Semiring product
ctx, Γ, Δ	::=		Typing context
	|	$\{b_1, .., b_k\}$	List of bindings
	|	$m{\cdot}\Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mo
	|	$-\Gamma$	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)
bndr, b	::=		Type assignment to either variable, destination or hole
	|	$x :_{m} T$	Variable
	|	$+h :_{m} \lfloor T \rfloor^{n}$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-h : T^{n}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \Vdash v : T}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \frac{}{\{-h : T^{1a}\} \Vdash -h : T}$$

$$\textsc{TyR-val-D}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ +h :_{1\nu} \lfloor T \rfloor^{n}}{\Gamma \Vdash +h : \lfloor T \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \frac{}{\{\} \Vdash () : 1}$$

$$\textsc{TyR-val-F}\quad \frac{\texttt{ctx\_DestOnly}\ \Gamma \qquad \Gamma \uplus \{x :_m T_1\} \vdash t : T_2}{\Gamma \Vdash \lambda^{v} x \mapsto t : T_1\ {}_m\!\!\to T_2}$$

$$\textsc{TyR-val-L}\quad \frac{\Gamma \Vdash v : T_1}{\Gamma \Vdash \mathsf{Inl}\, v : T_1 \oplus T_2}$$

$$\textsc{TyR-val-R}\quad \frac{\Gamma \Vdash v : T_2}{\Gamma \Vdash \mathsf{Inr}\, v : T_1 \oplus T_2}$$

$$\textsc{TyR-val-P}\quad \frac{\Gamma_1 \Vdash v_1 : T_1 \qquad \Gamma_2 \Vdash v_2 : T_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (v_1\,,\,v_2) : T_1 \otimes T_2}$$

$$\textsc{TyR-val-E}\quad \frac{\Gamma \Vdash v : T}{m\cdot\Gamma \Vdash \mathrm{E}^{m}\, v : !^{m}\, T}$$

$$\textsc{TyR-val-A}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\\ \texttt{ctx\_DestOnly}\ \Gamma_2 \uplus \Gamma_3\\ \texttt{ctx\_DestOnly}\ \Gamma_1\\ \Gamma_1 \uplus (-\Gamma_3) \Vdash v_1 : T_1\\ \Gamma_2 \uplus \Gamma_3 \Vdash v_2 : T_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash {}_{\texttt{hnames}(-\Gamma_3)}\langle v_1 \,{}_{\mathbf{,}}\, v_2\rangle : T_1 \ltimes T_2}$$

$\boxed{\Gamma \vdash j : T}$ *(Typing of terms)*

$$\textsc{Ty-term-Val}\quad \frac{\texttt{ctx\_NoHole}\ \Gamma \qquad \Gamma \Vdash v : T}{\Gamma \vdash v : T}$$

$$\textsc{Ty-term-Var}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ x :_{1\nu} T}{\Gamma \vdash x : T}$$

$$\textsc{Ty-term-App}\quad \frac{\Gamma_1 \vdash t : T_1 \qquad \Gamma_2 \vdash u : T_1\ {}_m\!\!\to T_2}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ u : T_2}$$

$$\textsc{Ty-term-PatU}\quad \frac{\Gamma_1 \vdash t : 1 \qquad \Gamma_2 \vdash u : U}{\Gamma_1 \uplus \Gamma_2 \vdash t\ ;\ u : U}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_m T_1\}\\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_m T_2\}\\ \Gamma_1 \vdash t : T_1 \oplus T_2\\ \Gamma_2 \uplus \{x_1 :_m T_1\} \vdash u_1 : U\\ \Gamma_2 \uplus \{x_2 :_m T_2\} \vdash u_2 : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}_m\,\{\mathsf{Inl}\,x_1 \mapsto u_1\,,\ \mathsf{Inr}\,x_2 \mapsto u_2\} : U}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_m T_1\}\\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_m T_2\}\\ \texttt{ctx\_Disjoint}\ \ \{x_1 :_m T_1\}\ \ \{x_2 :_m T_2\}\\ \Gamma_1 \vdash t : T_1 \otimes T_2\\ \Gamma_2 \uplus \{x_1 :_m T_1, x_2 :_m T_2\} \vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}_m\,(x_1\,,\,x_2) \mapsto u : U}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m\cdot n} T\}\\ \Gamma_1 \vdash t : !^{n}\, T\\ \Gamma_2 \uplus \{x :_{m\cdot n} T\} \vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}_m\ \mathrm{E}^{n}\, x \mapsto u : U}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{1\nu} T_2\}\\ \Gamma_1 \vdash t : T_1 \ltimes T_2\\ 1{\uparrow}\cdot\Gamma_2 \uplus \{x :_{1\nu} T_2\} \vdash u : U\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{map}\ x \mapsto u : T_1 \ltimes U}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Gamma \vdash t : \lfloor 1 \rfloor^{n}}{\Gamma \vdash t \triangleleft () : 1}$$

$$\textsc{Ty-term-FillF}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_m T_1\}\\ \Gamma_1 \vdash t : \lfloor T_1\ {}_m\!\!\to T_2 \rfloor^{n}\\ \Gamma_2 \uplus \{x :_m T_1\} \vdash u : T_2\end{array}}{\Gamma_1 \uplus (1{\uparrow}\cdot n)\cdot\Gamma_2 \vdash t \triangleleft \lambda x \mapsto u : 1}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Gamma \vdash t : \lfloor T_1 \oplus T_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathsf{Inl} : \lfloor T_1 \rfloor^{n}}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Gamma \vdash t : \lfloor T_1 \oplus T_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathsf{Inr} : \lfloor T_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Gamma \vdash t : \lfloor T_1 \otimes T_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft (,) : \lfloor T_1 \rfloor^{n} \otimes \lfloor T_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Gamma \vdash t : \lfloor !^{m}\, T \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathrm{E}^{m} : \lfloor T \rfloor^{m\cdot n}}$$

$$\textsc{Ty-term-FillC}\quad \frac{\Gamma_1 \vdash t : \lfloor T_1 \rfloor^{n} \qquad \Gamma_2 \vdash u : T_1 \ltimes T_2}{\Gamma_1 \uplus (1{\uparrow}\cdot n)\cdot\Gamma_2 \vdash t \triangleleft\!\bullet u : T_2}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathsf{alloc}_{T} : T \ltimes \lfloor T \rfloor^{1\nu}}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{to}_{\ltimes}\ t : T \ltimes 1}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Gamma \vdash t : T \ltimes 1}{\Gamma \vdash \mathsf{from}_{\ltimes}\ t : T}$$

$\boxed{\Gamma \Vdash C : T}$ *(Typing of evaluation contexts)*

$$\textsc{TyR-ectx-T}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \Gamma_3\\ \texttt{ctx\_NoVar}\ \Gamma_2 \uplus \Gamma_3\\ \texttt{ctx\_NoVar}\ \Gamma_1\\ \Gamma_2 \uplus \Gamma_3 \vdash t : T_1 \qquad \Gamma_1 \vdash C[t] : T_2\end{array}}{\Gamma_1 \uplus (-\Gamma_2) \Vdash C : T_1 \rightarrowtail T_2}$$

3 Effects and big-step semantics

$\boxed{j \longrightarrow j'}$ *(Small-step evaluation of terms using evaluation contexts)*

Sem-eterm-App

$$\overline{C[v \succ (\lambda^{v} x \mapsto t)] \longrightarrow C[t[x := v]]}$$

Sem-eterm-PatU

$$\overline{C[() \,;\, t_2] \longrightarrow C[t_2]}$$

Sem-eterm-PatL

$$\overline{C[(\mathsf{Inl}\, v) \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}] \longrightarrow C[u_1[x := v]]}$$

Sem-eterm-PatR

$$\overline{C[(\mathsf{Inr}\, v) \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}] \longrightarrow C[u_2[x := v]]}$$

Sem-eterm-PatP

$$\overline{C[(v_1 , v_2) \succ \mathbf{case}_{m} (x_1 , x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]}$$

Sem-eterm-PatE

$$\overline{C[\mathrm{E}^{n}\, v \succ \mathbf{case}_{m}\, \mathrm{E}^{n}\, x \mapsto u] \longrightarrow C[u[x := v]]}$$

Sem-eterm-MapOpen

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C))}{C[{}_{\mathtt{H}}\langle v_1 , v_2 \rangle \succ \mathbf{map}\ x \mapsto u] \longrightarrow (C \circ ({}^{o}_{\mathtt{H} \overset{\pm}{=} h'} \langle v_1 \overset{\pm}{=} h' , \square \rangle))[u[x := v_2 \overset{\pm}{=} h']]}$$

Sem-eterm-MapClose

$$\overline{(C \circ {}^{o}_{\mathtt{H}}\langle v_1 , \square \rangle)[v_2] \longrightarrow C[{}_{\mathtt{H}}\langle v_1 , v_2 \rangle]}$$

Sem-eterm-Alloc

$$\overline{\mathbf{alloc}_{T} \longrightarrow {}_{\{1\}}\langle +1 , -1 \rangle}$$

Sem-eterm-ToA

$$\overline{C[\mathbf{to}_{\ltimes}\, v] \longrightarrow C[{}_{\{\,\}}\langle v , () \rangle]}$$

Sem-eterm-FromA

$$\overline{C[\mathbf{from}_{\ltimes}\, {}_{\{\,\}}\langle v , () \rangle] \longrightarrow v}$$

Sem-eterm-FillU

$$\overline{C[+h \triangleleft ()] \longrightarrow C[h :=_{\{\,\}} ()][()]}$$

Sem-eterm-FillL

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inl}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inl}\, -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillR

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inr}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inr}\, -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillE

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathrm{E}^{m}] \longrightarrow C[h :=_{\{h'+1\}} \mathrm{E}^{m}\, -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillP

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft (,)] \longrightarrow C[h :=_{\{h'+1, h'+2\}} (-(h'+1) , -(h'+2))][(+(h'+1) , +(h'+2))]}$$

Sem-eterm-FillC

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft\!\bullet\, {}_{\mathtt{H}}\langle v_1 , v_2 \rangle] \longrightarrow C[h :=_{(\mathtt{H} \overset{\pm}{=} h')} v_1 \overset{\pm}{=} h'][v_2 \overset{\pm}{=} h']}$$