Destination λ-calculus

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1 Term and value syntax

tmv, x, y	Term-level variable name
hdmv	Hole or destination static name
k	Index for ranges

hddyn, d	::=		Hole or destination dynamic name
	|	$\star$	Root namespace
	|	d.1	Subnamespace 1
	|	d.2	Subnamespace 2
	|	d.3	Subnamespace 3

hdnm, h	::=		Hole or destination name
	|	d	Dynamic name
	|	hdmv	Static name

val, v	::=		Term value
	|	$\langle \mathsf{v}_1 \,,\, \overline{\mathsf{v}_2} \rangle_{\Delta}$	Ampar
	|	$+\mathtt{h}$	Destination
	|	$()$	Unit
	|	$\mathsf{Inl}\ \mathsf{v}$	Left variant for sum
	|	$\mathsf{Inr}\ \mathsf{v}$	Right variant for sum
	|	$(\mathsf{v}_1 , \mathsf{v}_2)$	Product
	|	$\rrparenthesis^{m}\ \mathsf{v}$	Exponential
	|	$\lambda \mathsf{x} \mapsto \mathsf{t}$	Linear function

$\overline{\mathsf{xval}}$, $\overline{\mathsf{v}}$	::=		Pseudo-value that may contain holes
	|	v	Term value
	|	$-\mathtt{h}$	Hole
	|	$\mathsf{Inl}\ \overline{\mathsf{v}}$	Left variant with val or hole
	|	$\mathsf{Inr}\ \overline{\mathsf{v}}$	Right variant with val or hole
	|	$(\overline{\mathsf{v}_1} , \overline{\mathsf{v}_2})$	Product with val or hole
	|	$\rrparenthesis^{m}\ \overline{\mathsf{v}}$	Exponential with val or hole

term, t, u	::=		Term
	|	v	Term value
	|	x	Variable
	|	$\mathsf{t} \succ \mathsf{u}$	Application
	|	$\mathsf{t}\ ;\ \mathsf{u}$	Pattern-match on unit
	|	$\mathsf{t} \succ \mathbf{case}\ \{\ \mathsf{Inl}\ \mathsf{x}_1 \mapsto \mathsf{u}_1\ ,\ \mathsf{Inr}\ \mathsf{x}_2 \mapsto \mathsf{u}_2\ \}$	Pattern-match on sum
	|	$\mathsf{t} \succ \mathbf{case}\ (\mathsf{x}_1 , \mathsf{x}_2) \mapsto \mathsf{u}$	Pattern-match on product
	|	$\mathsf{t} \succ \mathbf{case}\ \rrparenthesis^{m}\ \mathsf{x} \mapsto \mathsf{u}$	Pattern-match on exponential
	|	$\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}$	Map over the left side of the ampar
	|	$\mathbf{to}_{\rtimes}\ \mathsf{t}$	Wrap t into a trivial ampar
	|	$\mathbf{from}_{\rtimes}\ \mathsf{t}$	Extract value from trivial ampar
	|	$\mathbf{alloc}_{\top}$	Return a fresh "identity" ampar object
	|	$\mathsf{t} \triangleleft ()$	Fill destination with unit
	|	$\mathsf{t} \triangleleft \mathsf{Inl}$	Fill destination with left variant

$\mid$	$\mathsf{t} \triangleleft \mathsf{Inr}$	Fill destination with right variant
$\mid$	$\mathsf{t} \triangleleft (,)$	Fill destination with product constructor
$\mid$	$\mathsf{t} \triangleleft \rrparenthesis^{m}$	Fill destination with exponential constructor
$\mid$	$\mathsf{t} \triangleleft\!\bullet\, \mathsf{u}$	Fill destination with root of ampar u

2 Type system

typ, T, U	::=		Type
	|	$\mathsf{1}$	Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	|	$!^{m}\, \mathsf{T}$	Exponential
	|	$\mathsf{T}_1 \rtimes \mathsf{T}_2$	Ampar type (consuming T_1 yields T_2)
	|	$\mathsf{T}_1 \;{}_{m_1}\!\!\to \mathsf{T}_2$	Linear function
	|	${}^{m}\lfloor \mathsf{T} \rfloor$	Destination
md, m, n	::=		Mode (Semiring)
	|	$p\vert a$	Pair of a multiplicity and age
	|	$\underline{\nu}$	Neutral element of the product. Notation for $1\vert\nu$.
	|	$\underline{\uparrow}$	Same multiplicity, but one scope older. Notation for $1\vert\uparrow$.
	|	$\underline{\infty}$	Linear, infinitely old / static. Notation for $1\vert\infty$.
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	1	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	Semiring product
pctx, Γ	::=		Positive typing context
	|	$\{\mathsf{pas}_1, \ldots, \mathsf{pas}_k\}$	
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	
pas	::=		Positive type assignment
	|	$\mathtt{x} :_{m} \mathsf{T}$	Variable
	|	$+\mathtt{h} :_{m} {}^{n}\lfloor \mathsf{T} \rfloor$	Destination (m is its own modality; n is the modality for values it accepts)
nctx, Δ	::=		Negative typing context
	|	$\{\mathsf{nas}_1, \ldots, \mathsf{nas}_k\}$	
	|	$m \cdot \Delta$	Multiply each binding by m
	|	$+^{-1}\Gamma$	Maps each destination of Γ to a hole (requires `ctx_DestOnly` Γ)
	|	$\Delta_1 \uplus \Delta_2$	
nas	::=		Negative type assignment
	|	$-\mathtt{h} :^{n} \mathsf{T}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \cup \Delta \Vdash \bar{v} : \mathsf{T}}$ *(Typing of extended values (require both positive and negative contexts))*

$$\textsc{Ty-xval-H}\quad \frac{}{\{\,\} \cup \{-\mathtt{h} :^{\underline{\nu}} \mathsf{T}\} \Vdash -\mathtt{h} : \mathsf{T}}$$

$$\textsc{Ty-xval-D}\quad \frac{}{\{+\mathtt{h} :_{\underline{\nu}} {}^{n}\lfloor \mathsf{T} \rfloor\} \cup \{\,\} \Vdash +\mathtt{h} : {}^{n}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{Ty-xval-U}\quad \frac{}{\{\,\} \cup \{\,\} \Vdash () : \mathsf{1}}$$

$$\textsc{Ty-xval-L}\quad \frac{\Gamma \cup \Delta \Vdash \bar{v} : \mathsf{T}_1 \qquad \texttt{ctx\_Disjoint}\ \Gamma\ \Delta}{\Gamma \cup \Delta \Vdash \mathsf{Inl}\, \bar{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{Ty-xval-P}\quad \frac{\begin{array}{c} \Gamma_1 \cup \Delta_1 \Vdash \overline{v_1} : T_1 \\ \Gamma_2 \cup \Delta_2 \Vdash \overline{v_2} : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \ \Delta_2 \end{array}}{\Gamma_1 \uplus \Gamma_2 \cup \Delta_1 \uplus \Delta_2 \Vdash (\overline{v_1}\ ,\ \overline{v_2}) : T_1 \otimes T_2}$$

$$\textsc{Ty-xval-R}\quad \frac{\begin{array}{c} \Gamma \cup \Delta \Vdash \overline{v} : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \Delta \end{array}}{\Gamma \cup \Delta \Vdash \mathsf{Inr}\,\overline{v} : T_1 \oplus T_2}$$

$$\textsc{Ty-xval-E}\quad \frac{\begin{array}{c} \Gamma \cup \Delta \Vdash \overline{v} : T \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \Delta \end{array}}{m\!\cdot\!\Gamma \cup {}^{m}\!\cdot\!\Delta \Vdash \rrparenthesis^{m}\,\overline{v} : {!^{m}}\,T}$$

$$\textsc{Ty-xval-A}\quad \frac{\begin{array}{c} \Gamma_1 \cup \{\} \Vdash v_1 : T_1 \\ \Gamma_2 \cup {+^{-1}}\Gamma_1 \Vdash \overline{v_2} : T_2 \\ \texttt{pctx\_DestOnly}\ \Gamma_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \end{array}}{\Gamma_2 \cup \{\} \Vdash \langle v_1\ ,\ \overline{v_2}\rangle_{+^{-1}\Gamma_1} : T_1 \ltimes T_2}$$

$$\textsc{Ty-xval-F}\quad \frac{\begin{array}{c} \Gamma \uplus \{x :_{m} T_1\} \vdash t : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \{x :_{m} T_1\} \end{array}}{\Gamma \cup \{\} \Vdash \lambda x \mapsto t : T_1\ {}_{m}\!\!\to T_2}$$

$\boxed{\Gamma \vdash t : T}$ *(Typing of terms (only a positive context is needed))*

$$\textsc{Ty-term-V}\quad \frac{\Gamma \cup \{\} \Vdash v : T}{\Gamma \vdash v : T}$$

$$\textsc{Ty-term-X0}\quad \frac{}{\{x :_{\underline{\nu}} T\} \vdash x : T}$$

$$\textsc{Ty-term-XInf}\quad \frac{}{\{x :_{\underline{\infty}} T\} \vdash x : T}$$

$$\textsc{Ty-term-App}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : T_1 \qquad \Gamma_2 \vdash u : T_1\ {}_{m}\!\!\to T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \end{array}}{m\!\cdot\!\Gamma_1 \uplus \Gamma_2 \vdash t \succ u : T_2}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : T_1 \oplus T_2 \\ \Gamma_2 \uplus \{x_1 :_{m} T_1\} \vdash u_1 : U \\ \Gamma_2 \uplus \{x_2 :_{m} T_2\} \vdash u_2 : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} T_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} T_2\} \end{array}}{m\!\cdot\!\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}\,\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\} : U}$$

$$\textsc{Ty-term-PatU}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : 1 \qquad \Gamma_2 \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash t\ ;\ u : U}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : T_1 \otimes T_2 \\ \Gamma_2 \uplus \{x_1 :_{m} T_1, x_2 :_{m} T_2\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} T_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} T_2\} \\ \texttt{ctx\_Disjoint}\ \{x_1 :_{m} T_1\}\ \ \{x_2 :_{m} T_2\} \end{array}}{m\!\cdot\!\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}\,(x_1\ ,\ x_2) \mapsto u : U}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : {!^{n}}\,T \\ \Gamma_2 \uplus \{x :_{m\cdot n} T\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m\cdot n} T\} \end{array}}{m\!\cdot\!\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{case}\ \rrparenthesis^{n}\,x \mapsto u : U}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : T_1 \ltimes T_2 \\ \uparrow\!\cdot\!\Gamma_2 \uplus \{x :_{\underline{\nu}} T_1\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{\underline{\nu}} T_1\} \end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathsf{map}\ x \mapsto u : U \ltimes T_2}$$

$$\textsc{Ty-term-FillC}\quad \frac{\begin{array}{c} \Gamma_1 \vdash t : {}^{n}\lfloor T_2 \rfloor \qquad \Gamma_2 \vdash u : T_1 \ltimes T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \end{array}}{\Gamma_1 \uplus (\uparrow\!\cdot n)\!\cdot\!\Gamma_2 \vdash t \triangleleft\!\bullet\, u : T_1}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor 1 \rfloor}{\Gamma \vdash t \triangleleft () : 1}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \vdash t \triangleleft \mathsf{Inl} : {}^{n}\lfloor T_1 \rfloor}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \vdash t \triangleleft \mathsf{Inr} : {}^{n}\lfloor T_2 \rfloor}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \otimes T_2 \rfloor}{\Gamma \vdash t \triangleleft (,) : {}^{n}\lfloor T_1 \rfloor \otimes {}^{n}\lfloor T_2 \rfloor}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Gamma \vdash t : {}^{m}\lfloor {!^{n}}\,T \rfloor}{\Gamma \vdash t \triangleleft \rrparenthesis^{n} : {}^{m\cdot n}\lfloor T \rfloor}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathsf{alloc}_{T} : {}^{\nu}\lfloor T \rfloor \ltimes T}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{to}_{\ltimes}\, t : 1 \ltimes T}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Gamma \vdash t : 1 \ltimes T}{\Gamma \vdash \mathsf{from}_{\ltimes}\, t : T}$$

$\boxed{\Gamma \vdash v\ \diamond\ e : T}$ *(Typing of commands (only a positive context is needed))*

$$\textsc{Ty-cmd-C}\quad \frac{\begin{array}{c} \Gamma_{11} \uplus \Gamma_{12} \vdash v : T \\ \Gamma_2 \cup {+^{-1}}\Gamma_{12} \Vdash e \\ \texttt{pctx\_DestOnly}\ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_{12}\ \ \Gamma_2 \end{array}}{\Gamma_{11} \uplus \Gamma_2 \vdash v\ \diamond\ e : T}$$

3 Effects and big-step semantics

$$
\begin{array}{lcll}
eff,\ e & ::= & & \text{Effect} \\
 & | & \varepsilon & \\
 & | & \mathsf{has} & \\
 & | & e_1 \gg .. \gg e_k & \text{Chain effects}
\end{array}
$$

$\boxed{\Gamma \cup \Delta \Vdash e}$ *(Typing of effects (require both positive and negative contexts))*

$$
\textsc{Ty-eff-N}\quad \frac{}{\{\,\} \cup \{\,\} \Vdash \varepsilon}
$$

$$
\textsc{Ty-eff-A}\quad \frac{\begin{array}{c}\Gamma \cup \Delta \Vdash \overline{v} : \mathsf{T} \\ \texttt{ctx\_Disjoint}\ \Gamma\ \{+\mathsf{h} :_m {}^n\lfloor \mathsf{T} \rfloor\} \\ \texttt{ctx\_Disjoint}\ \Gamma\ \Delta \\ \texttt{ctx\_Disjoint}\ \{+\mathsf{h} :_m {}^n\lfloor \mathsf{T} \rfloor\}\ \Delta\end{array}}{(\uparrow\cdot m\cdot n)\cdot\Gamma \uplus \{+\mathsf{h} :_m {}^n\lfloor \mathsf{T} \rfloor\} \cup {}^{(m\cdot n)}\cdot\Delta \Vdash \mathsf{h} \coloneqq \overline{v}}
$$

$$
\textsc{Ty-eff-P}\quad \frac{\begin{array}{c}\Gamma_1 \cup \Delta_1 \uplus +^{-1}\Gamma_{22} \Vdash e_1 \\ \Gamma_{21} \uplus \Gamma_{22} \cup \Delta_2 \Vdash e_2 \\ \texttt{pctx\_DestOnly}\ \Gamma_{22} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{21} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{22} \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_{21}\ \Gamma_{22} \\ \texttt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2\end{array}}{\Gamma_1 \uplus \Gamma_{21} \cup \Delta_1 \uplus \Delta_2 \Vdash e_1 \gg e_2}
$$

$\boxed{\overline{v_1}\ {}_{\Delta_1} \mid e_1 \Downarrow \overline{v_2}\ {}_{\Delta_2} \mid e_2}$ *(Big-step evaluation of effects on extended values)*

$$
\textsc{Sem-eff-N}\quad \frac{}{\overline{v_1}\ {}_{\Delta_1} \mid \varepsilon \Downarrow \overline{v_1}\ {}_{\Delta_1} \mid \varepsilon}
$$

$$
\textsc{Sem-eff-S}\quad \frac{\begin{array}{c}\texttt{ctx\_hdmvNotMem}\ \mathsf{h}\ \Delta_1 \\ \overline{v_1}\ {}_{\Delta_1} \mid e_1 \Downarrow \overline{v_2}\ {}_{\Delta_2} \mid e_2\end{array}}{\overline{v_1}\ {}_{\Delta_1} \mid \mathsf{h} \coloneqq \overline{v'} \gg e_1 \Downarrow \overline{v_2}\ {}_{\Delta_2} \mid \mathsf{h} \coloneqq \overline{v'} \gg e_2}
$$

$$
\textsc{Sem-eff-F}\quad \frac{\begin{array}{c}\Gamma_0 \cup \Delta_0 \Vdash \overline{v_0} : \mathsf{T} \\ \texttt{ctx\_Disjoint}\ \Gamma_0\ \Delta_0 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \{-\mathsf{h} :^n \mathsf{T}\} \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_0 \\ \overline{v_1}[\mathsf{h} \coloneqq \overline{v_0}]\ {}_{(\Delta_1 \uplus {}^{n}\cdot\Delta_0)} \mid e_1 \Downarrow \overline{v_2}\ {}_{\Delta_2} \mid e_2\end{array}}{\overline{v_1}\ {}_{\Delta_1 \uplus \{-\mathsf{h}:^n\mathsf{T}\}} \mid \mathsf{h} \coloneqq \overline{v_0} \gg e_1 \Downarrow \overline{v_2}\ {}_{\Delta_2} \mid e_2}
$$

$\boxed{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

$$
\textsc{Sem-term-V}\quad \frac{}{\mathsf{v}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond \varepsilon}
$$

$$
\textsc{Sem-term-App}\quad \frac{\begin{array}{c}\mathsf{t}_1\ {}_{\mathsf{d}.1}\!\Downarrow\ \mathsf{v}_1 \diamond e_1 \\ \mathsf{t}_2\ {}_{\mathsf{d}.2}\!\Downarrow\ \lambda \mathsf{x} \mapsto \mathsf{u} \diamond e_2 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1]\ {}_{\mathsf{d}.3}\!\Downarrow\ \mathsf{v}_3 \diamond e_3\end{array}}{\mathsf{t}_1 \succ \mathsf{t}_2\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_3 \diamond e_1 \gg e_2 \gg e_3}
$$

$$
\textsc{Sem-term-PatU}\quad \frac{\mathsf{t}_1\ {}_{\mathsf{d}.1}\!\Downarrow\ () \diamond e_1 \qquad \mathsf{t}_2\ {}_{\mathsf{d}.2}\!\Downarrow\ \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1\ ;\ \mathsf{t}_2\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}
$$

$$
\textsc{Sem-term-PatL}\quad \frac{\begin{array}{c}\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ \mathsf{Inl}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_1[\mathsf{x}_1 \coloneqq \mathsf{v}_1]\ {}_{\mathsf{d}.2}\!\Downarrow\ \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}
$$

$$
\textsc{Sem-term-PatR}\quad \frac{\begin{array}{c}\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ \mathsf{Inr}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_2[\mathsf{x}_2 \coloneqq \mathsf{v}_1]\ {}_{\mathsf{d}.2}\!\Downarrow\ \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}
$$

$$
\textsc{Sem-term-PatP}\quad \frac{\begin{array}{c}\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ (\mathsf{v}_1\,,\,\mathsf{v}_2) \diamond e_1 \\ \mathsf{u}[\mathsf{x}_1 \coloneqq \mathsf{v}_1][\mathsf{x}_2 \coloneqq \mathsf{v}_2]\ {}_{\mathsf{d}.2}\!\Downarrow\ \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}
$$

$$
\textsc{Sem-term-Map}\quad \frac{\begin{array}{c}\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ \langle \mathsf{v}_1\,,\,\overline{\mathsf{v}_2}\rangle_{\Delta} \diamond e_1 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1]\ {}_{\mathsf{d}.2}\!\Downarrow\ \mathsf{v}_3 \diamond e_2 \\ \overline{\mathsf{v}_2}\ {}_{\Delta} \mid e_2 \Downarrow \overline{\mathsf{v}_4}\ {}_{\Delta'} \mid e_3\end{array}}{\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \langle \mathsf{v}_3\,,\,\overline{\mathsf{v}_4}\rangle_{\Delta'} \diamond e_1 \gg e_3}
$$

$$
\textsc{Sem-term-Alloc}\quad \frac{}{\mathbf{alloc}_{\mathsf{T}}\ {}_{\mathsf{d}}\!\Downarrow\ \langle +\mathsf{d}\,,\,-\mathsf{d}\rangle_{\{-\mathsf{d}:^{\nu}\mathsf{T}\}} \diamond \varepsilon}
$$

$$
\textsc{Sem-term-ToA}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}{\mathbf{to}_{\ltimes}\ \mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \langle ()\,,\,\mathsf{v}\rangle_{\{\,\}} \diamond e}
$$

$$
\textsc{Sem-term-FromA}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \langle ()\,,\,\mathsf{v}\rangle_{\{\,\}} \diamond e}{\mathbf{from}_{\ltimes}\ \mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}
$$

$$
\textsc{Sem-term-FillU}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd ()\ {}_{\mathsf{d}}\!\Downarrow\ () \diamond e \gg \mathsf{h} \coloneqq ()}
$$

$$
\textsc{Sem-term-FillL}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd \mathsf{Inl}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{d}.2 \diamond e \gg \mathsf{h} \coloneqq \mathsf{Inl}\ -\mathsf{d}.2}
$$

$$
\textsc{Sem-term-FillR}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd \mathsf{Inr}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{d}.2 \diamond e \gg \mathsf{h} \coloneqq \mathsf{Inr}\ -\mathsf{d}.2}
$$

$$
\textsc{Sem-term-FillP}\quad \frac{\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd (,)\ {}_{\mathsf{d}}\!\Downarrow\ (+\mathsf{d}.2\,,\,+\mathsf{d}.3) \diamond e \gg \mathsf{h} \coloneqq (-\mathsf{d}.2\,,\,-\mathsf{d}.3)}
$$

$$
\textsc{Sem-term-FillC}\quad \frac{\begin{array}{c}\mathsf{t}\ {}_{\mathsf{d}.1}\!\Downarrow\ +\mathsf{h} \diamond e_1 \\ \mathsf{u}\ {}_{\mathsf{d}.2}\!\Downarrow\ \langle \mathsf{v}_1\,,\,\overline{\mathsf{v}_2}\rangle_{\Delta} \diamond e_2\end{array}}{\mathsf{t} \lhd\!\bullet\ \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_1 \diamond e_1 \gg e_2 \gg \mathsf{h} \coloneqq \overline{\mathsf{v}_2}}
$$

4 Type safety

Theorem 1 (Type safety)**.** *If* `pctx_DestOnly` Γ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \;{}_{\mathsf{d}}\!\Downarrow\; \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

Theorem 2 (Type safety for complete programs)**.** *If* $\{\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \;{}_{\star}\!\Downarrow\; \mathsf{v} \diamond \varepsilon$ *and* $\{\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v} \;{}_{\mathsf{d}}\!\Downarrow\; \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \;{}_{m}\!\!\to \mathsf{T}_2$
  (3) `ctx_Disjoint` $\Gamma_1\ \Gamma_2$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t} \;{}_{\mathsf{d.1}}\!\Downarrow\; \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \uplus \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12} \cup +^{-1}\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \uplus \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u} \;{}_{\mathsf{d.2}}\!\Downarrow\; \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1 \;{}_{m}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \uplus \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1 \;{}_{m}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22} \cup +^{-1}\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \uplus \Gamma_{22}$.
  Using Lemma **??** on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x} \mapsto \mathsf{t}'$ and (14) $\Gamma_{21} \uplus \Gamma_{23} \uplus \{\mathsf{x} :_m \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma **??** on (14) and (5) we get (15) $m{\cdot}(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] \;{}_{\mathsf{d.3}}\!\Downarrow\; \mathsf{v}_3 \diamond e_3$ where (17) $m{\cdot}(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12} \cup +^{-1}\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22} \cup +^{-1}\Gamma_{23} \Vdash e_2$

  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{22}$ comes naturally from (3), (7) and (12).

  We must show:
  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  `ctx_Disjoint` $\Gamma_{22}\ \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  `ctx_Disjoint` $\Gamma_{13}\ \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \uplus \Gamma_{22} \cup +^{-1}\Gamma_{13} \uplus +^{-1}\Gamma_{23} \Vdash e_1 \gg e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $m{\cdot}(\Gamma_{111} \uplus \Gamma_{131}) \uplus \Gamma_{211} \uplus \Gamma_{231} \uplus \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $m{\cdot}(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \cup +$
  e_3 where (21) $\Gamma_{\mathsf{k}1} \uplus \Gamma_{\mathsf{k}2} = \Gamma_{\mathsf{k}}$

  We have
  (18) $\Gamma_{12} \uplus \Gamma_{22} \cup +^{-1}\Gamma_{13} \uplus +^{-1}\Gamma_{23} \Vdash e_1 \gg e_2$
  (20) $m{\cdot}(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \cup +^{-1}\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $+^{-1}\Gamma_{23}$, we get (22) $\Gamma_{12} \uplus \Gamma_{22} \cup +^{-1}(\Gamma_{131} \uplus \Gamma_{231}) \uplus +^{-1}(\Gamma_{132} \uplus \Gamma_{232}) \Vdash e_1 \gg e_2$

  We want Γ_{132} from (22) to cancel $m{\cdot}\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{+\mathsf{h} :_{\underline{\nu}} {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash +\mathsf{h}' \diamond \mathsf{h} \coloneqq \mathsf{Inl}\ -\mathsf{h}' : {}^{n}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} \coloneqq \mathsf{Inl}\ -\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2) \succ (\lambda \mathsf{x}_2 \mapsto +\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))$

  $+\mathsf{h}_0 \triangleleft (,) \;{}_{\mathsf{d}}\!\Downarrow\; (+\mathsf{d.2}\,,\,+\mathsf{d.3}) \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3})$

  $(\mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2)[\mathsf{x}_1 \coloneqq +\mathsf{d.2}][\mathsf{x}_2 \coloneqq +\mathsf{d.3}] \;{}_{\mathsf{d}'}\!\Downarrow\; +\mathsf{d.3} \diamond \mathsf{d.2} \coloneqq +\mathsf{h}_1$

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2) \;{}_{\mathsf{d}''}\!\Downarrow\; +\mathsf{d.3} \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3}) \gg \mathsf{d.2} \coloneqq +\mathsf{h}_1$

  $(+\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))[\mathsf{x}_2 \coloneqq +\mathsf{d.3}] \;{}_{\mathsf{d}'''}\!\Downarrow\; () \diamond \mathsf{h}_3 \coloneqq +\mathsf{d.3}$

  $\mathsf{t} \;{}_{\mathsf{d}''''}\!\Downarrow\; () \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3}) \gg \mathsf{d.2} \coloneqq +\mathsf{h}_1 \gg \mathsf{h}_3 \coloneqq +\mathsf{d.3}$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all BigStep_Fill$\langle Ctor \rangle$ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TyCmd_Cmd), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;

- or are those of pre-existing holes coming from the extended value $\overline{\mathsf{v}_2}$ of an ampar, when BigStep_FillComp is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program $\cdot$ And v_1 is exactly the accompanying value returned by the evaluation of BigStep_FillComp

TODO: prove that this property is preserved by typing rules □