

metavariable, x, xs, y, uf, f, d

term, t, u	$::=$ $ \quad x$ $ \quad z$ $ \quad t \ u$ $ \quad t \ ; \ u$ $ \quad \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}$ $ \quad \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}$ $ \quad \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}$ $ \quad \text{case } t \text{ of } \{ \text{roll } x \mapsto u \}$ $ \quad \text{alloc } d \ . t$ $ \quad t \triangleleft u$ $ \quad t \triangleleft 1.d' . u$ $ \quad t \triangleleft 2.d' . u$ $ \quad t \triangleleft \langle d_1, d_2 \rangle . u$ $ \quad \star l$ $ \quad (t)$ $ \quad \backslash n \text{ sp } t \backslash n \text{ spe}$ $ \quad \backslash n \text{ sp } t$ $ \quad t[\text{var_subs}]$	term variable value application pattern-matching on unit pattern-matching on sum pattern-matching on product pattern-matching on exponentiated value unroll for recursive types allocate data fill terminal-type destination fill sum-type destination with variant 1 fill sum-type destination with variant 2 fill product-type destination note: $\star l$ is not a part of the user syntax S S S M
var_sub, vs	$::=$ $ \quad x := t$	variable substitution
var_subs	$::=$ $ \quad vs$ $ \quad vs, \text{var_subs}$	variable substitutions
data_val, v	$::=$ $ \quad ()$ $ \quad \lambda x:A . t$ $ \quad \text{Ur } t$ $ \quad \text{roll } t$ $ \quad \text{roll } t$ $ \quad \text{roll } t$	data value unit lambda abstraction exponential roll for recursive types
val, z	$::=$ $ \quad v$ $ \quad [A]$ $ \quad 1.l$ $ \quad 2.l$ $ \quad \langle l_1, l_2 \rangle$ $ \quad C \bar{l}$	unreducible value note: l is not a part of the user syntax M
label, l	$::=$	label
label_stmt, s	$::=$ $ \quad l \triangleleft v$ $ \quad l \triangleleft 1.l'$ $ \quad l \triangleleft 2.l'$ $ \quad l \triangleleft \langle l_1, l_2 \rangle$ $ \quad l \triangleleft \emptyset$	label statement

	$\begin{array}{ l} l \triangleleft C\bar{l} \\ l \triangleleft \dots \end{array}$	$\begin{array}{l} M \\ M \end{array}$	<p>TODO: hide. $l \triangleleft C\bar{l}$ is an alias for any heap cons</p>
label_stmts	$\begin{array}{ l} s \\ s, \text{label_stmts} \end{array}$		label statements
heap_context, \mathbb{H}	$\begin{array}{ l} \emptyset \\ \{\text{label_stmts}\} \\ \mathbb{H}_1 \sqcup \mathbb{H}_2 \end{array}$		label statements
type, A, B	$\begin{array}{ l} 1 \\ R \\ A \otimes B \\ A \oplus B \\ A \multimap B \\ [A] \\ !A \\ (A) \\ W[r := A] \end{array}$	$\begin{array}{l} \\ \\ \\ \\ \\ \\ \\ S \\ M \end{array}$	<p>unit type recursive type bound to a name product type sum type linear function type destination type exponential</p>
type_with_hole, W	$\begin{array}{ l} r \\ 1 \\ R \\ W_1 \otimes W_2 \\ W_1 \oplus W_2 \\ W_1 \multimap W_2 \\ [W] \\ !W \\ (W) \end{array}$	$\begin{array}{l} \\ \\ \\ \\ \\ \\ \\ S \end{array}$	<p>type hole in recursive definition unit type recursive type bound to a name product type sum type linear function type destination type exponential</p>
rec_type_bound, R	$\begin{array}{ l} \end{array}$		recursive type bound to a name
rec_type_def	$\begin{array}{ l} \mu r. W \end{array}$		
type_affect, ta	$\begin{array}{ l} x : A \\ l : A \\ \bar{l} : \bar{A} \end{array}$		<p>type affectation var label labels</p>
type_affects	$\begin{array}{ l} \text{ta} \\ \text{ta}, \text{type_affects} \end{array}$		type affectations
typing_context, $\Gamma, \Delta, \mathcal{U}, \Phi$	$\begin{array}{ l} \end{array}$		typing context

	$\begin{array}{ l} \emptyset \\ \{\text{type_affects}\} \\ \Gamma \sqcup \Delta \end{array}$	
types, \bar{A}	$\begin{array}{ l} ::= \\ \bullet \\ A \\ A \text{ types} \end{array}$	empty type list
heap_constructor, C	$\begin{array}{ l} ::= \\ (1.) \\ (2.) \\ (\langle, \rangle) \end{array}$	
judg	$\begin{array}{ l} ::= \\ \Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \\ \Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A \\ \Phi ; \mathcal{U} ; \Gamma \vdash t : A \\ C : \bar{A} \xrightarrow{c} A \\ A = B \\ t = u \\ \Gamma = \Delta \\ \mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t' \\ \text{type_affect} \in \Gamma \\ \text{label_stmt} \in \mathbb{H} \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}$	
terminals	$\begin{array}{ l} ::= \\ \mapsto \\ \star \\ \oplus \\ \oplus \\ \circ \\ := \\ \vdash \\ \sqcup \\ ; \\ \cap \\ \emptyset \\ \longrightarrow \\ \triangleright \\ \neq \\ \in \\ \notin \\ \backslash n \\ \langle \\ \rangle \\ 1. \\ 2. \\ \text{Ur} \end{array}$	

	\triangleleft $ $ \bigcirc $\underline{\hookrightarrow}$ $=$ \Downarrow \dots $\underline{\text{fix}}$	
formula	$::=$ $ $ judgement	
Ctx	$::=$ $ $ $\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset$ $ $ $\text{type_affect} \in \Gamma$	Γ and Δ are disjoint typing contexts with no clashing
Heap	$::=$ $ $ $\text{label_stmt} \in \mathbb{H}$	
Eq	$::=$ $ $ $A = B$ $ $ $t = u$ $ $ $\Gamma = \Delta$	
Ty	$::=$ $ $ $R \stackrel{\text{fix}}{=} \text{rec_type_def}$ $ $ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H}$ $ $ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A$ $ $ $\Phi ; \mathcal{U} ; \Gamma \vdash t : A$ $ $ $C : \bar{A} \hookrightarrow A$	\mathbb{H} is a well-typed heap given heap typing context Φ , u t is a well-typed term of type A given heap typing con Heap constructor C builds a value of type A given arg
Sem	$::=$ $ $ $\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'$ $ $ $\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'$	t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}' t reduces to t' , with heap growing from \mathbb{H} to \mathbb{H}'
judgement	$::=$ $ $ Ctx $ $ Heap $ $ Eq $ $ Ty $ $ Sem	
user_syntax	$::=$ $ $ metavariable $ $ term $ $ var_sub $ $ var_subs $ $ data_val $ $ val	

$label$
 $label_stmt$
 $label_stmts$
 $heap_context$
 $type$
 $type_with_hole$
 rec_type_bound
 rec_type_def
 $type_affect$
 $type_affects$
 $typing_context$
 $types$
 $heap_constructor$
 $judg$
 $terminals$

$names(\Gamma) \cap names(\Delta) = \emptyset$ Γ and Δ are disjoint typing contexts with no clashing variable names or labels

$type_affect \in \Gamma$

$label_stmt \in \mathbb{H}$

$A = B$

$t = u$

$\Gamma = \Delta$

$R \stackrel{\text{fix}}{=} rec_type_def$

$\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H}$ \mathbb{H} is a well-typed heap given heap typing context Φ , unrestricted typing context \mathcal{U} and linear

$$\begin{array}{c}
\Phi_1 ; \mathcal{U} ; \Gamma \vdash \mathbb{H}_1 \\
\Phi_2 ; \mathcal{U} ; \Delta \vdash \mathbb{H}_2 \\
names(\Phi_1) \cap names(\Phi_2) = \emptyset \\
names(\Gamma) \cap names(\Delta) = \emptyset \\
\hline
\Phi_1 \sqcup \Phi_2 ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \mathbb{H}_1 \sqcup \mathbb{H}_2 \quad \text{TYHEAP_UNION}
\end{array}$$

$$\begin{array}{c}
C : \bar{A} \hookrightarrow A \\
\Phi \sqcup \{\bar{l} : \bar{A}\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \\
\hline
\Phi \sqcup \{\bar{l} : \bar{A}, l : A\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \quad \text{TYHEAP_CTOR}
\end{array}$$

$$\overline{\{l : A\} ; \mathcal{U} ; \emptyset \vdash \{l \triangleleft \emptyset\}} \quad \text{TYHEAP_NULL}$$

$$\begin{array}{c}
\Phi ; \mathcal{U} ; \Gamma \vdash v : A \\
\hline
\Phi \sqcup \{l : A\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \sqcup \{l \triangleleft v\} \quad \text{TYHEAP_VAL}
\end{array}$$

$\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A$

$$\begin{array}{c}
\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \\
\Phi ; \mathcal{U} ; \Delta \vdash t : A \\
names(\Gamma) \cap names(\Delta) = \emptyset \\
\hline
\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A \quad \text{TYCOMMAND_DEF}
\end{array}$$

$\Phi ; \mathcal{U} ; \Gamma \vdash t : A$ t is a well-typed term of type A given heap typing context Φ , unrestricted typing context

$$\overline{\Phi ; \mathcal{U} ; \{x : A\} \vdash x : A} \quad \text{TYTERM_ID}$$

$$\begin{array}{c}
\frac{}{\Phi ; \mathcal{U} \sqcup \{x : A\} ; \emptyset \vdash x : A} \text{TYTERM_ID'} \\
\\
\frac{}{\Phi ; \mathcal{U} ; \emptyset \vdash () : 1} \text{TYTERM_UNIT} \\
\\
\frac{\Phi ; \mathcal{U} ; \emptyset \vdash t : A}{\Phi ; \mathcal{U} ; \emptyset \vdash \text{Ur } t : !A} \text{TYTERM_EXP} \\
\\
\frac{}{\Phi ; \mathcal{U} ; \emptyset \vdash \textcolor{blue}{[A]} : \textcolor{blue}{[A]}} \text{TYTERM_LABELASDEST} \\
\\
\frac{}{\Phi \sqcup \{\textcolor{red}{l} : A\} ; \mathcal{U} ; \emptyset \vdash \star \textcolor{red}{l} : A} \text{TYTERM_DEREF} \\
\\
\frac{C : \bar{A} \xrightarrow{c} A}{\Phi \sqcup \{\bar{\textcolor{red}{l}} : \bar{A}\} ; \mathcal{U} ; \emptyset \vdash C \bar{\textcolor{red}{l}} : A} \text{TYTERM_CTOR} \\
\\
\frac{R \stackrel{\text{fix}}{=} \mu r . W}{\Phi ; \mathcal{U} ; \Gamma \vdash t : W[r := R]} \text{TYTERM_ROLL} \\
\frac{}{\Phi ; \mathcal{U} ; \Gamma \vdash \text{roll } t : R} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \sqcup \{x : A\} \vdash t : B}{\Phi ; \mathcal{U} ; \Gamma \vdash \lambda x : A . t : A \multimap B} \text{TYTERM_LAM} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \vdash t : A \multimap B \quad \Phi ; \mathcal{U} ; \Delta \vdash u : A}{\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset} \text{TYTERM_APP} \\
\frac{}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t u : B} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \vdash t : 1 \quad \Phi ; \mathcal{U} ; \Delta \vdash u : A \quad \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t ; u : A} \text{TYTERM_PATU} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \otimes A_2 \quad \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1, x_2 : A_2\} \vdash u : B \quad \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} : B} \text{TYTERM_PATP} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \oplus A_2 \quad \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1\} \vdash u_1 : B \quad \Phi ; \mathcal{U} ; \Delta \sqcup \{x_2 : A_2\} \vdash u_2 : B \quad \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} : B} \text{TYTERM_PATs} \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \vdash t : !A \quad \Phi ; \mathcal{U} \sqcup \{x : A\} ; \Delta \vdash u : B \quad \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} : B} \text{TYTERM_PATe} \\
\\
\frac{R \stackrel{\text{fix}}{=} \mu r . W \quad \Phi ; \mathcal{U} ; \Gamma \vdash t : R \quad \Phi ; \mathcal{U} ; \Delta \sqcup \{x : W[r := R]\} \vdash u : B \quad \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\text{roll } x \mapsto u\} : B} \text{TYTERM_PATR}
\end{array}$$

$$\begin{array}{c}
\frac{\Phi ; \mathcal{U} ; \Gamma \sqcup \{\mathbf{d} : \mathbf{[A]}\} \vdash t : \mathbf{1}}{\Phi ; \mathcal{U} ; \Gamma \vdash \text{alloc } \mathbf{d} . t : \mathbf{A}} \quad \text{TYTERM_ALLOC} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : \mathbf{[A]} \\ \Phi ; \mathcal{U} ; \Delta \vdash u : \mathbf{A} \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft u : \mathbf{1}} \quad \text{TYTERM_FILLL} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : \mathbf{[A_1 \oplus A_2]} \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{\mathbf{d}' : \mathbf{[A_1]}\} \vdash u : \mathbf{B} \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft \mathbf{1.d' . u} : \mathbf{B}} \quad \text{TYTERM_FILLV1} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : \mathbf{[A_1 \oplus A_2]} \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{\mathbf{d}' : \mathbf{[A_2]}\} \vdash u : \mathbf{B} \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \vdash t \triangleleft \mathbf{2.d' . u} : \mathbf{B}} \quad \text{TYTERM_FILLV2} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : \mathbf{[A_1 \otimes A_2]} \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{\mathbf{d_1} : \mathbf{[A_1]}, \mathbf{d_2} : \mathbf{[A_2]}\} \vdash u : \mathbf{B} \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft \langle \mathbf{d_1}, \mathbf{d_2} \rangle . u : \mathbf{B}} \quad \text{TYTERM_FILLP}
\end{array}$$

$\boxed{\mathbf{C} : \bar{\mathbf{A}} \hookrightarrow \mathbf{A}}$ Heap constructor \mathbf{C} builds a value of type \mathbf{A} given arguments of type $\bar{\mathbf{A}}$

$$\begin{array}{c}
\frac{}{(1.) : \mathbf{A} \hookrightarrow \mathbf{A \oplus B}} \quad \text{TYCTOR_V1} \\
\\
\frac{}{(2.) : \mathbf{B} \hookrightarrow \mathbf{A \oplus B}} \quad \text{TYCTOR_V2} \\
\\
\frac{}{(\langle, \rangle) : \mathbf{A} \ \mathbf{B} \hookrightarrow \mathbf{A \otimes B}} \quad \text{TYCTOR_PAIR}
\end{array}$$

$\boxed{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}$ t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}'

$$\begin{array}{c}
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t u \longrightarrow \mathbb{H}' \mid t' u} \quad \text{SEMMUT_UAPP} \\
\\
\frac{}{\mathbb{H} \mid (\lambda \mathbf{x} : \mathbf{A} . t) u \longrightarrow \mathbb{H} \mid t[\mathbf{x} := u]} \quad \text{SEMMUT_APP} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t ; u \longrightarrow \mathbb{H}' \mid t' ; u} \quad \text{SEMMUT_UPATU} \\
\\
\frac{}{\mathbb{H} \mid () ; u \longrightarrow \mathbb{H} \mid u} \quad \text{SEMMUT_PATU} \\
\\
\frac{}{\mathbb{H} \sqcup \{\mathbf{l} \triangleleft \mathbf{v}\} \mid \star \mathbf{l} \longrightarrow \mathbb{H} \sqcup \{\mathbf{l} \triangleleft \mathbf{v}\} \mid \mathbf{v}} \quad \text{SEMMUT_DEREFVAL} \\
\\
\frac{}{\mathbb{H} \sqcup \{\mathbf{l} \triangleleft \mathbf{C}\bar{\mathbf{l}}\} \mid \star \mathbf{l} \longrightarrow \mathbb{H} \sqcup \{\mathbf{l} \triangleleft \mathbf{C}\bar{\mathbf{l}}\} \mid \mathbf{C}\bar{\mathbf{l}}} \quad \text{SEMMUT_DEREFCOR} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{1.\mathbf{x_1} \mapsto u_1, 2.\mathbf{x_2} \mapsto u_2\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{1.\mathbf{x_1} \mapsto u_1, 2.\mathbf{x_2} \mapsto u_2\}} \quad \text{SEMMUT_UPATS} \\
\\
\frac{}{\mathbb{H} \mid \text{case } 1.\mathbf{l} \text{ of } \{1.\mathbf{x_1} \mapsto u_1, 2.\mathbf{x_2} \mapsto u_2\} \longrightarrow \mathbb{H} \mid u_1[\mathbf{x_1} := \star \mathbf{l}]} \quad \text{SEMMUT_PATSV1} \\
\\
\frac{}{\mathbb{H} \mid \text{case } 2.\mathbf{l} \text{ of } \{1.\mathbf{x_1} \mapsto u_1, 2.\mathbf{x_2} \mapsto u_2\} \longrightarrow \mathbb{H} \mid u_2[\mathbf{x_2} := \star \mathbf{l}]} \quad \text{SEMMUT_PATSV2}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}} \quad \text{SEMMUT_UPATP} \\
\\
\frac{\mathbb{H} \mid \text{case } \langle l_1, l_2 \rangle \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H} \mid u[x_1 := \star l_1, x_2 := \star l_2]}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H} \mid u[x_1 := \star l_1, x_2 := \star l_2]} \quad \text{SEMMUT_PATP} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{Ur } x \mapsto u\}} \quad \text{SEMMUT_UPATE} \\
\\
\frac{\mathbb{H} \mid \text{case Ur } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]} \quad \text{SEMMUT_PATE} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{roll}_R x \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{roll}_R x \mapsto u\}} \quad \text{SEMMUT_UPATR} \\
\\
\frac{\mathbb{H} \mid \text{case roll}_R t \text{ of } \{\text{roll}_R x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{roll}_R x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]} \quad \text{SEMMUT_PATR} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft u \longrightarrow \mathbb{H}' \mid t' \triangleleft u} \quad \text{SEMMUT_UFILLL} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \underline{[A]} \triangleleft t \longrightarrow \mathbb{H}' \mid \underline{[A]} \triangleleft t'} \quad \text{SEMMUT_UFILLL'} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 1.d'.u \longrightarrow \mathbb{H}' \mid t' \triangleleft 1.d'.u} \quad \text{SEMMUT_UFILLV1} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 2.d'.u \longrightarrow \mathbb{H}' \mid t' \triangleleft 2.d'.u} \quad \text{SEMMUT_UFILLV2} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \langle d_1, d_2 \rangle . u \longrightarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2 \rangle . u} \quad \text{SEMMUT_UFILLP} \\
\\
\frac{\mathbb{H} \mid \text{alloc}_A d . t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid t[d := \underline{[A]}] ; \star l}{\mathbb{H} \mid \text{alloc}_A d . t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid t[d := \underline{[A]}] ; \star l} \quad \text{SEMMUT_ALLOC} \\
\\
\frac{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A]} \triangleleft v \longrightarrow \mathbb{H} \sqcup \{l \triangleleft v\} \mid ()}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A]} \triangleleft v \longrightarrow \mathbb{H} \sqcup \{l \triangleleft v\} \mid ()} \quad \text{SEMMUT_FILLV} \\
\\
\frac{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A]} \triangleleft C\bar{l} \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid ()}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A]} \triangleleft C\bar{l} \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid ()} \quad \text{SEMMUT_FILLCTOR} \\
\\
\frac{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \oplus A_2]} \triangleleft 1.d'.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 1.l', l' \triangleleft \emptyset\} \mid t[d' := \underline{[A_1]}]}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \oplus A_2]} \triangleleft 1.d'.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 1.l', l' \triangleleft \emptyset\} \mid t[d' := \underline{[A_1]}]} \quad \text{SEMMUT_FILLV1} \\
\\
\frac{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \oplus A_2]} \triangleleft 2.d'.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 2.l', l' \triangleleft \emptyset\} \mid t[d' := \underline{[A_2]}]}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \oplus A_2]} \triangleleft 2.d'.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 2.l', l' \triangleleft \emptyset\} \mid t[d' := \underline{[A_2]}]} \quad \text{SEMMUT_FILLV2} \\
\\
\frac{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \otimes A_2]} \triangleleft \langle d_1, d_2 \rangle . t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft \emptyset, l_2 \triangleleft \emptyset\} \mid t[d_1 := \underline{[A_1]}, d_2 := \underline{[A_2]}]}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \underline{[A_1 \otimes A_2]} \triangleleft \langle d_1, d_2 \rangle . t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft \emptyset, l_2 \triangleleft \emptyset\} \mid t[d_1 := \underline{[A_1]}, d_2 := \underline{[A_2]}]} \quad \text{SEMMUT_FILLP} \\
\\
\boxed{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'} \quad t \text{ reduces to } t', \text{ with heap growing from } \mathbb{H} \text{ to } \mathbb{H}'
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H} \mid \text{roll}_R t \Downarrow \mathbb{H} \mid t}{\mathbb{H} \mid \text{roll}_R t \Downarrow \mathbb{H} \mid t} \quad \text{SEMMUT_ROLL} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t u \Downarrow \mathbb{H}' \mid t' u} \quad \text{SEMMUT_UAPP} \\
\\
\frac{\mathbb{H} \mid (\lambda x:A. t) u \Downarrow \mathbb{H} \mid t[x := u]}{\mathbb{H} \mid (\lambda x:A. t) u \Downarrow \mathbb{H} \mid t[x := u]} \quad \text{SEMMUT_APP}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t ; u \Downarrow \mathbb{H}' \mid t' ; u} \text{SEMIMM_UPATU} \\
\\
\frac{}{\mathbb{H} \mid () ; u \Downarrow \mathbb{H} \mid u} \text{SEMIMM_PATU} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \mid \star l \Downarrow \mathbb{H} \sqcup \{l \triangleleft v\} \mid v} \text{SEMIMM_DEREFVAL} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid \star l \Downarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid C\bar{l}} \text{SEMIMM_DEREFTOR} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}} \text{SEMIMM_UPATS} \\
\\
\frac{}{\mathbb{H} \mid \text{case } 1.l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H} \mid u_1[x_1 := \star l]} \text{SEMIMM_PATSV1} \\
\\
\frac{}{\mathbb{H} \mid \text{case } 2.l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H} \mid u_2[x_2 := \star l]} \text{SEMIMM_PATSV2} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}} \text{SEMIMM_UPATP} \\
\\
\frac{}{\mathbb{H} \mid \text{case } \langle l_1, l_2 \rangle \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \Downarrow \mathbb{H} \mid u[x_1 := \star l_1, x_2 := \star l_2]} \text{SEMIMM_PATP} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{Ur \ x \mapsto u\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{Ur \ x \mapsto u\}} \text{SEMIMM_UPATE} \\
\\
\frac{}{\mathbb{H} \mid \text{case } Ur \ t \text{ of } \{Ur \ x \mapsto u\} \Downarrow \mathbb{H} \mid u[x := t]} \text{SEMIMM_PATE} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{roll}_R \ x \mapsto u\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{roll}_R \ x \mapsto u\}} \text{SEMIMM_UPATR} \\
\\
\frac{}{\mathbb{H} \mid \text{case } \text{roll}_R \ t \text{ of } \{\text{roll}_R \ x \mapsto u\} \Downarrow \mathbb{H} \mid u[x := t]} \text{SEMIMM_PATR} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft u \Downarrow \mathbb{H}' \mid t' \triangleleft u} \text{SEMIMM_UFILLL} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \overset{l}{[A]} \triangleleft t \Downarrow \mathbb{H}' \mid \overset{l}{[A]} \triangleleft t'} \text{SEMIMM_UFILLL'} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 1.d' . u \Downarrow \mathbb{H}' \mid t' \triangleleft 1.d' . u} \text{SEMIMM_UFILLV1} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 2.d' . u \Downarrow \mathbb{H}' \mid t' \triangleleft 2.d' . u} \text{SEMIMM_UFILLV2} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \langle d_1, d_2 \rangle . u \Downarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2 \rangle . u} \text{SEMIMM_UFILLP} \\
\\
\frac{\mathbb{H} \mid t[d := \overset{l}{[A]}] \Downarrow \mathbb{H}' \sqcup \{l \triangleleft \dots\} \mid ()}{\mathbb{H} \mid \text{alloc}_A \ d . t \Downarrow \mathbb{H}' \sqcup \{l \triangleleft \dots\} \mid \star l} \text{SEMIMM_ALLOC} \quad \text{--- Changes start from there ---} \\
\\
\frac{}{\mathbb{H} \mid \overset{l}{[A]} \triangleleft v \Downarrow \mathbb{H} \sqcup \{l \triangleleft v\} \mid ()} \text{SEMIMM_FILLV} \\
\\
\frac{}{\mathbb{H} \mid \overset{l}{[A]} \triangleleft C\bar{l} \Downarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid ()} \text{SEMIMM_FILLLCTOR}
\end{array}$$

$$\frac{\mathbb{H} \mid \mathfrak{t}[\mathbf{d}' := \textcolor{blue}{A_1}] \Downarrow \mathbb{H}' \sqcup \{l' \triangleleft \dots\} \mid ()}{\mathbb{H} \mid \textcolor{blue}{A_1 \oplus A_2} \triangleleft \mathbf{1}.\mathbf{d}' \cdot \mathfrak{t} \Downarrow \mathbb{H}' \sqcup \{l' \triangleleft \dots, l \triangleleft \mathbf{1}.l'\} \mid ()} \text{SEMIMM_FILLV1}$$

$$\frac{\mathbb{H} \mid \mathfrak{t}[\mathbf{d}' := \textcolor{blue}{A_2}] \Downarrow \mathbb{H}' \sqcup \{l' \triangleleft \dots\} \mid ()}{\mathbb{H} \mid \textcolor{blue}{A_1 \oplus A_2} \triangleleft \mathbf{2}.\mathbf{d}' \cdot \mathfrak{t} \Downarrow \mathbb{H}' \sqcup \{l' \triangleleft \dots, l \triangleleft \mathbf{2}.l'\} \mid ()} \text{SEMIMM_FILLV2}$$

$$\frac{\mathbb{H} \mid \mathfrak{t}[\mathbf{d}_1 := \textcolor{blue}{A_1}, \mathbf{d}_2 := \textcolor{blue}{A_2}] \Downarrow \mathbb{H}' \sqcup \{l_1 \triangleleft \dots, l_2 \triangleleft \dots\} \mid ()}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \textcolor{blue}{A_1 \otimes A_2} \triangleleft \langle \mathbf{d}_1, \mathbf{d}_2 \rangle \cdot \mathfrak{t} \Downarrow \mathbb{H}' \sqcup \{l_1 \triangleleft \dots, l_2 \triangleleft \dots, l \triangleleft \langle l_1, l_2 \rangle\} \mid ()} \text{SEMIMM_FILLP}$$

Definition rules: 81 good 0 bad

Definition rule clauses: 155 good 0 bad