

# Destination $\lambda$ -calculus

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## 1 Term and value syntax

$\text{var}, x, y$  Term-level variable name  
 $k$  Index for ranges

$\text{hdn}, h$	$::=$ $  \quad h + h'$ $  \quad h[H \pm h']$ $  \quad \max(H)$	Hole or destination name ( $\mathbb{N}$ ) M M Shift by $h'$ if $h \in H$ M Maximum of a set of holes
$\text{hdns}, H$	$::=$ $  \quad \{h_1, \dots, h_k\}$ $  \quad H_1 \cup H_2$ $  \quad H \pm h'$ $  \quad \text{hnames}(\Gamma)$ $  \quad \text{hnames}(C)$	Set of hole names M Union of sets M Shift all names from $H$ by $h'$ . M Hole names of a context (requires $\text{ctx\_NoVar}(\Gamma)$ ) M Hole names of an evaluation context
term, $t, u$	$::=$ $  \quad v$ $  \quad x$ $  \quad t \succ t'$ $  \quad t ; u$ $  \quad t \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$ $  \quad t \succ \text{case}_m (x_1, x_2) \mapsto u$ $  \quad t \succ \text{case}_m E^n x \mapsto u$ $  \quad t \succ \text{map } x \mapsto t'$ $  \quad \text{to}_\times u$ $  \quad \text{from}_\times t$ $  \quad \text{alloc}$ $  \quad t \triangleleft ()$ $  \quad t \triangleleft \text{Inl}$ $  \quad t \triangleleft \text{Inr}$ $  \quad t \triangleleft E^m$ $  \quad t \triangleleft (,)$ $  \quad t \triangleleft (\lambda x_m \mapsto u)$ $  \quad t \triangleleft \bullet t'$ $  \quad t[x := v]$	Term Value Variable Application Pattern-match on unit Pattern-match on sum Pattern-match on product Pattern-match on exponential Map over the right side of ampar $t$ Wrap $u$ into a trivial ampar Extract value from trivial ampar Return a fresh "identity" ampar object Fill destination with unit Fill destination with left variant Fill destination with right variant Fill destination with exponential constructor Fill destination with product constructor Fill destination with function Fill destination with root of ampar $t'$ M
val, $v$	$::=$ $  \quad -h$ $  \quad +h$ $  \quad ()$ $  \quad \lambda^v x_m \mapsto u$ $  \quad \text{Inl } v$ $  \quad \text{Inr } v$ $  \quad E^m v$ $  \quad (v_1, v_2)$ $  \quad H \langle v_2 \circ v_1 \rangle$ $  \quad v[H \pm h']$	Term value Hole Destination Unit Lambda abstraction Left variant for sum Right variant for sum Exponential Product Ampar M Shift hole names inside $v$ by $h'$ if they belong to $H$ .

ectx, c	::=		Evaluation context component
		$\square \succ t'$	Application
		$v \succ \square$	Application
		$\square ; u$	Pattern-match on unit
		$\square \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$	Pattern-match on sum
		$\square \succ \text{case}_m (x_1, x_2) \mapsto u$	Pattern-match on product
		$\square \succ \text{case}_m E^m x \mapsto u$	Pattern-match on exponential
		$\square \succ \text{map } x \mapsto t'$	Map over the right side of ampar
		$\text{to}_x \square$	Wrap into a trivial ampar
		$\text{from}_x \square$	Extract value from trivial ampar
		$\square \triangleleft ()$	Fill destination with unit
		$\square \triangleleft \text{Inl}$	Fill destination with left variant
		$\square \triangleleft \text{Inr}$	Fill destination with right variant
		$\square \triangleleft E^m$	Fill destination with exponential constructor
		$\square \triangleleft (,)$	Fill destination with product constructor
		$\square \triangleleft (\lambda x_m \mapsto u)$	Fill destination with function
		$\square \triangleleft \bullet t'$	Fill destination with root of ampar
		$v \triangleleft \bullet \square$	Fill destination with root of ampar
		$\overset{\text{op}}{\mathbf{h}} \langle v_2, \square$	Open ampar. <b>Only new addition to term shapes</b>

ectxs, C	::=		Evaluation context stack
		$\square$	Represent the empty stack / "identity" evaluation context
		$C \circ c$	Push c on top of C
		$C[\mathbf{h} :=_{\mathbf{h}} v]$	M Fill <b>h</b> in C with value v (that may contain holes)

## 2 Type system

type, T, U	::=		Type
		$1$	Unit
		$T_1 \oplus T_2$	Sum
		$T_1 \otimes T_2$	Product
		$!^m T$	Exponential
		$U \ltimes T$	Ampar type (consuming T yields U)
		$T \xrightarrow{m} U$	Function
		$[T]^m$	Destination

mode, m, n	::=		Mode (Semiring)
		$pa$	Pair of a multiplicity and age
		$\omega$	Error case (incompatible types, multiplicities, or ages)
		$m_1 \cdot \dots \cdot m_k$	M Semiring product

mul, p	::=		Multiplicity (first component of modality)
		$1$	Linear. Neutral element of the product
		$\omega$	Non-linear. Absorbing for the product
		$p_1 \cdot \dots \cdot p_k$	M Semiring product

age, a	::=		Age (second component of modality)
		$\nu$	Born now. Neutral element of the product
		$\uparrow$	One scope older
		$\infty$	Infinitely old / static. Absorbing for the product
		$a_1 \cdot \dots \cdot a_k$	M Semiring product

ctx, $\Gamma, \Delta, \Pi$	::=		Typing context
		$\{x :_m T\}$	
		$\{+h :_m [T]^n\}$	
		$\{-h : T^n\}$	
		$m \cdot \Gamma$	M Multiply each binding by m
		$\Gamma_1 \uplus \Gamma_2$	M Sum contexts $\Gamma_1$ and $\Gamma_2$ . Duplicate keys with incompatible values will be tagged

	$-\Gamma$	M	Transforms dest bindings into a hole bindings (requires $\text{ctx\_DestOnly } \Gamma$ and $\text{ctx\_LinOnly } \Gamma$ )
	$\Gamma[\mathbf{H}\pm\mathbf{h}']$	M	Shift hole/dest names by $\mathbf{h}'$ if they belong to $\mathbf{H}$

$\boxed{\Gamma \Vdash v : \mathbf{T}}$

(Typing of values (raw))

$\text{TYR-VAL-H} \quad \frac{}{\{-\mathbf{h} : \mathbf{T}^{\text{lv}}\} \Vdash -\mathbf{h} : \mathbf{T}}$	$\text{TYR-VAL-D} \quad \frac{\text{ctx\_CompatibleDH } \Gamma \quad \mathbf{h} \quad \text{lv} \quad [\mathbf{T}]^n}{\Gamma \Vdash +\mathbf{h} : [\mathbf{T}]^n}$	$\text{TYR-VAL-U} \quad \frac{}{\{\} \Vdash () : \mathbf{1}}$	$\text{TYR-VAL-F} \quad \frac{\text{ctx\_DestOnly } \Delta \quad \Delta \uplus \{\mathbf{x} : \textcolor{blue}{m}\mathbf{T}\} \vdash u : \mathbf{U}}{\Delta \Vdash \lambda^{\mathbf{v}} \mathbf{x}_m \mapsto u : \mathbf{T}_m \rightarrow \mathbf{U}}$
$\text{TYR-VAL-L} \quad \frac{\Gamma \Vdash v_1 : \mathbf{T}_1}{\Gamma \Vdash \text{Inl } v_1 : \mathbf{T}_1 \oplus \mathbf{T}_2}$	$\text{TYR-VAL-R} \quad \frac{\Gamma \Vdash v_2 : \mathbf{T}_2}{\Gamma \Vdash \text{Inr } v_2 : \mathbf{T}_1 \oplus \mathbf{T}_2}$	$\text{TYR-VAL-P} \quad \frac{\Gamma_1 \Vdash v_1 : \mathbf{T}_1 \quad \Gamma_2 \Vdash v_2 : \mathbf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (v_1, v_2) : \mathbf{T}_1 \otimes \mathbf{T}_2}$	$\text{TYR-VAL-E} \quad \frac{\Gamma \Vdash v' : \mathbf{T}}{n \cdot \Gamma \Vdash \mathbf{E}^n v' : \textcolor{blue}{!}^n \mathbf{T}}$
$\text{TYR-VAL-A} \quad \frac{\begin{array}{l} \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_DestOnly } \Delta_3 \\ \text{ctx\_LinOnly } \Delta_3 \\ \text{ctx\_ValidOnly } \Delta_3 \\ \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_Disjoint } \Delta_1 \Delta_3 \\ \text{ctx\_Disjoint } \Delta_2 \Delta_3 \\ \textcolor{teal}{I} \uparrow \cdot \Delta_1 \uplus \Delta_3 \Vdash v_1 : \mathbf{T} \\ \Delta_2 \uplus (-\Delta_3) \Vdash v_2 : \mathbf{U} \end{array}}{\Delta_1 \uplus \Delta_2 \Vdash \textcolor{brown}{hnames}(-\Delta_3) \langle v_2, v_1 \rangle : \mathbf{U} \ltimes \mathbf{T}}$			

$\boxed{\Pi \vdash t : \mathbf{T}}$

(Typing of terms)

$\text{TY-TERM-VAL} \quad \frac{\text{ctx\_DestOnly } \Delta \quad \Delta \Vdash v : \mathbf{T}}{\Delta \vdash v : \mathbf{T}}$	$\text{TY-TERM-VAR} \quad \frac{\text{ctx\_CompatibleVar } \Pi \times \textcolor{teal}{lv} \mathbf{T}}{\Pi \vdash \mathbf{x} : \mathbf{T}}$	$\text{TY-TERM-APP} \quad \frac{\Pi_1 \vdash t : \mathbf{T} \quad \Pi_2 \vdash t' : \mathbf{T}_{m \rightarrow \mathbf{U}}}{m \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ t' : \mathbf{U}}$		
$\text{TY-TERM-PATU} \quad \frac{\Pi_1 \vdash t : \mathbf{1} \quad \Pi_2 \vdash u : \mathbf{U}}{\Pi_1 \uplus \Pi_2 \vdash t ; u : \mathbf{U}}$	$\text{TY-TERM-PATS} \quad \frac{\begin{array}{l} \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x}_1 : \textcolor{teal}{m} \mathbf{T}_1 \} \\ \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x}_2 : \textcolor{teal}{m} \mathbf{T}_2 \} \\ \Pi_1 \vdash t : \mathbf{T}_1 \oplus \mathbf{T}_2 \\ \Pi_2 \uplus \{ \mathbf{x}_1 : \textcolor{teal}{m} \mathbf{T}_1 \} \vdash u_1 : \mathbf{U} \\ \Pi_2 \uplus \{ \mathbf{x}_2 : \textcolor{teal}{m} \mathbf{T}_2 \} \vdash u_2 : \mathbf{U} \end{array}}{m \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_m \{ \text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2 \} : \mathbf{U}}$			
$\text{TY-TERM-PATP} \quad \frac{\begin{array}{l} \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x}_1 : \textcolor{teal}{m} \mathbf{T}_1 \} \\ \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x}_2 : \textcolor{teal}{m} \mathbf{T}_2 \} \\ \text{ctx\_Disjoint } \{ \mathbf{x}_1 : \textcolor{teal}{m} \mathbf{T}_1 \} \{ \mathbf{x}_2 : \textcolor{teal}{m} \mathbf{T}_2 \} \\ \Pi_1 \vdash t : \mathbf{T}_1 \otimes \mathbf{T}_2 \\ \Pi_2 \uplus \{ \mathbf{x}_1 : \textcolor{teal}{m} \mathbf{T}_1 \} \uplus \{ \mathbf{x}_2 : \textcolor{teal}{m} \mathbf{T}_2 \} \vdash u : \mathbf{U} \end{array}}{m \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_m (\mathbf{x}_1, \mathbf{x}_2) \mapsto u : \mathbf{U}}$	$\text{TY-TERM-PATE} \quad \frac{\begin{array}{l} \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x} : \textcolor{teal}{m} \cdot n \mathbf{T} \} \\ \Pi_1 \vdash t : \textcolor{teal}{!}^n \mathbf{T} \\ \Pi_2 \uplus \{ \mathbf{x} : \textcolor{teal}{m} \cdot n \mathbf{T} \} \vdash u : \mathbf{U} \end{array}}{m \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_m \mathbf{E}^n \mathbf{x} \mapsto u : \mathbf{U}}$	$\text{TY-TERM-MAP} \quad \frac{\begin{array}{l} \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x} : \textcolor{teal}{lv} \mathbf{T} \} \\ \Pi_1 \vdash t : \mathbf{U} \ltimes \mathbf{T} \\ \textcolor{teal}{I} \uparrow \cdot \Pi_2 \uplus \{ \mathbf{x} : \textcolor{teal}{lv} \mathbf{T} \} \vdash t' : \mathbf{T}' \end{array}}{\Pi_1 \uplus \Pi_2 \vdash t \succ \text{map } \mathbf{x} \mapsto t' : \mathbf{U} \ltimes \mathbf{T}'}$		
$\text{TY-TERM-TOA} \quad \frac{\Pi \vdash u : \mathbf{U}}{\Pi \vdash \text{to}_\ltimes u : \mathbf{U} \ltimes \mathbf{1}}$	$\text{TY-TERM-FROMA} \quad \frac{\Pi \vdash t : \mathbf{U} \ltimes \mathbf{1}}{\Pi \vdash \text{from}_\ltimes t : \mathbf{U}}$	$\text{TY-TERM-ALLOC} \quad \frac{}{\{\} \vdash \text{alloc} : \mathbf{U} \ltimes \lfloor \mathbf{U} \rfloor^{\textcolor{teal}{lv}}}$	$\text{TY-TERM-FILLU} \quad \frac{}{\Pi \vdash t \triangleleft () : \mathbf{1}}$	$\text{TY-TERM-FILLL} \quad \frac{}{\Pi \vdash t \triangleleft \text{Inl} : \lfloor \mathbf{T}_1 \rfloor^n}$
$\text{TY-TERM-FILLR} \quad \frac{\Pi \vdash t : \lfloor \mathbf{T}_1 \oplus \mathbf{T}_2 \rfloor^n}{\Pi \vdash t \triangleleft \text{Inr} : \lfloor \mathbf{T}_2 \rfloor^n}$	$\text{TY-TERM-FILLP} \quad \frac{\Pi \vdash t : \lfloor \mathbf{T}_1 \otimes \mathbf{T}_2 \rfloor^n}{\Pi \vdash t \triangleleft (,) : \lfloor \mathbf{T}_1 \rfloor^n \otimes \lfloor \mathbf{T}_2 \rfloor^n}$	$\text{TY-TERM-FILLE} \quad \frac{\Pi \vdash t : \lfloor \textcolor{teal}{!}^{n'} \mathbf{T} \rfloor^n}{\Pi \vdash t \triangleleft \mathbf{E}^{n'} : \lfloor \mathbf{T} \rfloor^{n' \cdot n}}$	$\text{TY-TERM-FILLF} \quad \frac{\begin{array}{l} \text{ctx\_Disjoint } \Pi_2 \{ \mathbf{x} : \textcolor{teal}{m} \mathbf{T} \} \\ \Pi_1 \vdash t : \lfloor \mathbf{T}_{m \rightarrow \mathbf{U}} \rfloor^n \\ \Pi_2 \uplus \{ \mathbf{x} : \textcolor{teal}{m} \mathbf{T} \} \vdash u : \mathbf{U} \end{array}}{\Pi_1 \uplus (\textcolor{teal}{I} \uparrow \cdot n) \cdot \Pi_2 \vdash t \triangleleft (\lambda \mathbf{x}_m \mapsto u) : \mathbf{1}}$	
$\text{TY-TERM-FILLC} \quad \frac{\Pi_1 \vdash t : \lfloor \mathbf{U} \rfloor^n \quad \Pi_2 \vdash t' : \mathbf{U} \ltimes \mathbf{T}}{\Pi_1 \uplus (\textcolor{teal}{I} \uparrow \cdot n) \cdot \Pi_2 \vdash t \triangleleft \bullet \cdot t' : \mathbf{T}}$				

$$\Delta \vdash C : \mathbf{T} \multimap \mathbf{U}_0$$

(Typing of evaluation contexts)

$\frac{\text{TY-ECTXS-ID} \quad \{\} \vdash \square : \mathbf{U}_0 \multimap \mathbf{U}_0}{\{\} \vdash \square : \mathbf{U}_0 \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-APPFOC1} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{mode\_IsValid } m \\ \text{ctx\_ValidOnly } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \vdash t' : \mathbf{T} \xrightarrow{m} \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ t') : \mathbf{T} \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-APPFOC2} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_ValidOnly } m \cdot \Delta_1 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_1 \vdash v : \mathbf{T} \end{array}}{\Delta_2 \vdash C \circ (v \succ \square) : (\mathbf{T} \xrightarrow{m} \mathbf{U}) \multimap \mathbf{U}_0}$
$\frac{\text{TY-ECTXS-PATSFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{mode\_IsValid } m \\ \text{ctx\_ValidOnly } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x_1 : m \mathbf{T}_1\} \vdash u_1 : \mathbf{U} \\ \Delta_2 \uplus \{x_2 : m \mathbf{T}_2\} \vdash u_2 : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}) : (\mathbf{T}_1 \oplus \mathbf{T}_2) \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-PATPFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_Disjoint } \{x_1 : m \mathbf{T}_1\} \{x_2 : m \mathbf{T}_2\} \\ \text{mode\_IsValid } m \\ \text{ctx\_ValidOnly } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x_1 : m \mathbf{T}_1\} \uplus \{x_2 : m \mathbf{T}_2\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m (x_1, x_2) \mapsto u) : (\mathbf{T}_1 \otimes \mathbf{T}_2) \multimap \mathbf{U}_0}$	
$\frac{\text{TY-ECTXS-PATEFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{mode\_IsValid } m \\ \text{ctx\_ValidOnly } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x : m \cdot m' \mathbf{T}\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m E^{m'} x \mapsto u) : !^{m'} \mathbf{T} \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-MAPFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_ValidOnly } \Delta_2 \\ \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \ltimes \mathbf{T}' \multimap \mathbf{U}_0 \\ !\uparrow \cdot \Delta_2 \uplus \{x : !v \mathbf{T}\} \vdash t' : \mathbf{T}' \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{map } x \mapsto t') : (\mathbf{U} \ltimes \mathbf{T}) \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-TOAFoc} \quad \Delta \vdash C : (\mathbf{U} \ltimes \mathbf{1}) \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\text{to}_\ltimes \square) : \mathbf{U} \multimap \mathbf{U}_0}$
$\frac{\text{TY-ECTXS-FROMAFoc} \quad \Delta \vdash C : \mathbf{U} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\text{from}_\ltimes \square) : (\mathbf{U} \ltimes \mathbf{1}) \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLUFoc} \quad \Delta \vdash C : \mathbf{1} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft ()) : [\mathbf{1}]^n \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLLFoc} \quad \Delta \vdash C : [\mathbf{T}_1]^n \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft \text{Inl}) : [\mathbf{T}_1 \oplus \mathbf{T}_2]^n \multimap \mathbf{U}_0}$
$\frac{\text{TY-ECTXS-FILLRFoc} \quad \Delta \vdash C : [\mathbf{T}_2]^n \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft \text{Inr}) : [\mathbf{T}_1 \oplus \mathbf{T}_2]^n \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLPFoc} \quad \Delta \vdash C : ([\mathbf{T}_1]^n \otimes [\mathbf{T}_2]^n) \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft (,)) : [\mathbf{T}_1 \otimes \mathbf{T}_2]^n \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLEFoc} \quad \Delta \vdash C : [\mathbf{T}]^{m \cdot n} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft E^m) : [!^m \mathbf{T}]^n \multimap \mathbf{U}_0}$
$\frac{\text{TY-ECTXS-FILLFFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_ValidOnly } (!\uparrow \cdot n) \cdot \Delta_2 \\ \Delta_1 \uplus (!\uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{1} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x : m \mathbf{T}\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \triangleleft (\lambda x_m \mapsto u)) : [\mathbf{T} \xrightarrow{m} \mathbf{U}]^n \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLCFoc1} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_ValidOnly } (!\uparrow \cdot n) \cdot \Delta_2 \\ \Delta_1 \uplus (!\uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{T} \multimap \mathbf{U}_0 \\ \Delta_2 \vdash t' : \mathbf{U} \ltimes \mathbf{T} \end{array}}{\Delta_1 \vdash C \circ (\square \triangleleft \bullet t') : [\mathbf{U}]^n \multimap \mathbf{U}_0}$	$\frac{\text{TY-ECTXS-FILLCFoc2} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_ValidOnly } \Delta_1 \\ \text{mode\_IsValid } (!\uparrow \cdot n) \\ \Delta_1 \uplus (!\uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{T} \multimap \mathbf{U}_0 \\ \Delta_1 \vdash v : [\mathbf{U}]^n \end{array}}{\Delta_2 \vdash C \circ (v \triangleleft \bullet \square) : \mathbf{U} \ltimes \mathbf{T} \multimap \mathbf{U}_0}$
$\frac{\text{TY-ECTXS-AOPENFoc} \quad \begin{array}{l} \text{ctx\_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx\_Disjoint } \Delta_1 \Delta_3 \\ \text{hdns\_Disjoint } \text{hnames}(C) \text{ hnames}(-\Delta_3) \\ \text{ctx\_DestOnly } \Delta_1 \\ \text{ctx\_DestOnly } \Delta_2 \\ \text{ctx\_DestOnly } \Delta_3 \\ \text{ctx\_LinOnly } \Delta_3 \\ \text{ctx\_ValidOnly } \Delta_1 \\ \Delta_1 \uplus \Delta_2 \vdash C : (\mathbf{U} \ltimes \mathbf{T}') \multimap \mathbf{U}_0 \\ \Delta_2 \uplus -\Delta_3 \Vdash v_2 : \mathbf{U} \end{array}}{! \uparrow \cdot \Delta_1 \uplus \Delta_3 \vdash C \circ (\overset{\text{op}}{\text{hnames}}(-\Delta_3) \langle v_2, \square \rangle) : \mathbf{T}' \multimap \mathbf{U}_0}$		

$$\boxed{\vdash C[t] : \mathbf{T}}$$

(Typing of extended terms (pair of evaluation context and term))

$$\begin{array}{c} \text{TY-ETERM-CLOSEDETERM} \\ \text{ctx\_ValidOnly } \Delta \\ \text{ctx\_DestOnly } \Delta \\ \hline \Delta \dashv C : \mathbf{T} \multimap \mathbf{U}_0 \quad \Delta \vdash t : \mathbf{T} \\ \hline \vdash C[t] : \mathbf{U}_0 \end{array}$$

### 3 Small-step semantics

$C[t] \longrightarrow C'[t']$			<i>(Small-step evaluation of terms using evaluation contexts)</i>		
SEM-ETERM-APPFOC1 $\text{term\_NotVal } t$		$\frac{}{C[t \succ t'] \longrightarrow (C \circ (\Box \succ t'))[t]}$	SEM-ETERM-APPUNFOC1		$\frac{}{(C \circ (\Box \succ t'))[v] \longrightarrow C[v \succ t']}$
			SEM-ETERM-APPFOC2 $\text{term\_NotVal } t'$		$\frac{}{C[v \succ t'] \longrightarrow (C \circ (v \succ \Box))[t']}$
SEM-ETERM-APPUNFOC2		$\frac{}{(C \circ (v \succ \Box))[v'] \longrightarrow C[v \succ v']}$	SEM-ETERM-APPRED		$\frac{}{C[v \succ (\lambda^v x_m \mapsto u)] \longrightarrow C[u[x := v]]}$
			SEM-ETERM-PATUFOC $\text{term\_NotVal } t$		$\frac{}{C[t ; u] \longrightarrow (C \circ (\Box ; u))[t]}$
SEM-ETERM-PATUUNFOC		$\frac{}{(C \circ (\Box ; u))[v] \longrightarrow C[v ; u]}$	SEM-ETERM-PATURED		$\frac{}{C[(\Box ; u)] \longrightarrow C[u]}$
SEM-ETERM-PATSFOC		$\frac{\text{term\_NotVal } t}{C[t \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow (C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[t]}$			
SEM-ETERM-PATSUNFOC		$\frac{}{(C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[v] \longrightarrow C[v \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}]}$			
SEM-ETERM-PATLRED		$\frac{}{C[(\text{Inl } v_1) \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow C[u_1[x_1 := v_1]]}$			
SEM-ETERM-PATRRED		$\frac{}{C[(\text{Inr } v_2) \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow C[u_2[x_2 := v_2]]}$			
SEM-ETERM-PATPFOC		$\frac{\text{term\_NotVal } t}{C[t \succ \text{case}_m (x_1, x_2) \mapsto u] \longrightarrow (C \circ (\Box \succ \text{case}_m (x_1, x_2) \mapsto u))[t]}$			
SEM-ETERM-PATPUNFOC		$\frac{}{(C \circ (\Box \succ \text{case}_m (x_1, x_2) \mapsto u))[v] \longrightarrow C[v \succ \text{case}_m (x_1, x_2) \mapsto u]}$			
SEM-ETERM-PATPREd		$\frac{}{C[(v_1, v_2) \succ \text{case}_m (x_1, x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]}$	SEM-ETERM-PATEFOC $\text{term\_NotVal } t$		$\frac{}{C[t \succ \text{case}_m E^n x \mapsto u] \longrightarrow (C \circ (\Box \succ \text{case}_m E^n x \mapsto u))[t]}$
SEM-ETERM-PATEUNFOC		$\frac{}{(C \circ (\Box \succ \text{case}_m E^n x \mapsto u))[v] \longrightarrow C[v \succ \text{case}_m E^n x \mapsto u]}$	SEM-ETERM-PATERED		$\frac{}{C[E^n v' \succ \text{case}_m E^n x \mapsto u] \longrightarrow C[u[x := v']]}]$
SEM-ETERM-MAPFOC $\text{term\_NotVal } t$		$\frac{}{C[t \succ \text{map } x \mapsto t'] \longrightarrow (C \circ (\Box \succ \text{map } x \mapsto t'))[t]}$	SEM-ETERM-MAPUNFOC		$\frac{}{(C \circ (\Box \succ \text{map } x \mapsto t'))[v] \longrightarrow C[v \succ \text{map } x \mapsto t']}$
SEM-ETERM-MAPREDAOPENFOC $h' = \max(hnames(C))$		$\frac{}{C[h \langle v_2, v_1 \rangle \succ \text{map } x \mapsto t'] \longrightarrow (C \circ (\overset{\text{op}}{H \pm h} \langle v_2 [H \pm h'], \Box \rangle))[t' [x := v_1 [H \pm h']]]}$	SEM-ETERM-AOPENUNFOC		$\frac{}{(C \circ \overset{\text{op}}{H} \langle v_2, \Box \rangle)[v_1] \longrightarrow C[h \langle v_2, v_1 \rangle]}$
SEM-ETERM-ALLOCRED		$\frac{}{C[\text{alloc}] \longrightarrow C[\{1\} \langle +1, -1 \rangle]}$	SEM-ETERM-TOAFOC $\text{term\_NotVal } u$		$\frac{}{C[\text{to}_x u] \longrightarrow (C \circ (\text{to}_x \Box))[u]}$
			SEM-ETERM-TOAUNFOC		$\frac{}{(C \circ (\text{to}_x \Box))[v_2] \longrightarrow C[\text{to}_x v_2]}$
SEM-ETERM-TOARED		$\frac{}{C[\text{to}_x v_2] \longrightarrow C[\{1\} \langle v_2, () \rangle]}$	SEM-ETERM-FROMAFOC $\text{term\_NotVal } t$		$\frac{}{C[\text{from}_x t] \longrightarrow (C \circ (\text{from}_x \Box))[t]}$
			SEM-ETERM-FROMAUNFOC		$\frac{}{(C \circ (\text{from}_x \Box))[v] \longrightarrow C[\text{from}_x v]}$
SEM-ETERM-FROMARED		$\frac{}{C[\text{from}_x \{1\} \langle v_2, () \rangle] \longrightarrow C[v_2]}$	SEM-ETERM-FILLUFOC $\text{term\_NotVal } t$		$\frac{}{C[t \triangleleft ()] \longrightarrow (C \circ (\Box \triangleleft ())) [t]}$
			SEM-ETERM-FILLUUNFOC		$\frac{}{(C \circ (\Box \triangleleft ())) [v] \longrightarrow C[v \triangleleft ()]}$

SEM-ETERM-FILLURED	SEM-ETERM-FILLFOC <b>term_NotVal</b> t	SEM-ETERM-FILLUNFOC
$\overline{C[+h \triangleleft ()] \rightarrow C[h := \{\} ()][()]}$	$\overline{C[t \triangleleft \text{Inl}] \rightarrow (C \circ (\Box \triangleleft \text{Inl}))[t]}$	$\overline{(C \circ (\Box \triangleleft \text{Inl}))[v] \rightarrow C[v \triangleleft \text{Inl}]}$
SEM-ETERM-FILLURED $h' = \max(\text{hnames}(C) \cup \{h\})$	SEM-ETERM-FILLRFOC <b>term_NotVal</b> t	
$\overline{C[+h \triangleleft \text{Inl}] \rightarrow C[h := \{h'+1\} \text{Inl} - (h'+1)][+(h'+1)]}$	$\overline{C[t \triangleleft \text{Inr}] \rightarrow (C \circ (\Box \triangleleft \text{Inr}))[t]}$	
SEM-ETERM-FILLRUNFOC	SEM-ETERM-FILLRRED $h' = \max(\text{hnames}(C) \cup \{h\})$	
$\overline{(C \circ (\Box \triangleleft \text{Inr}))[v] \rightarrow C[v \triangleleft \text{Inr}]}$	$\overline{C[+h \triangleleft \text{Inr}] \rightarrow C[h := \{h'+1\} \text{Inr} - (h'+1)][+(h'+1)]}$	
SEM-ETERM-FILLEFOC <b>term_NotVal</b> t	SEM-ETERM-FILLEUNFOC	
$\overline{C[t \triangleleft E^m] \rightarrow (C \circ (\Box \triangleleft E^m))[t]}$	$\overline{(C \circ (\Box \triangleleft E^m))[v] \rightarrow C[v \triangleleft E^m]}$	
SEM-ETERM-FILLERED $h' = \max(\text{hnames}(C) \cup \{h\})$	SEM-ETERM-FILLPFOC <b>term_NotVal</b> t	SEM-ETERM-FILLPUNFOC
$\overline{C[+h \triangleleft E^m] \rightarrow C[h := \{h'+1\} E^m - (h'+1)][+(h'+1)]}$	$\overline{C[t \triangleleft (,)] \rightarrow (C \circ (\Box \triangleleft (,)))[t]}$	$\overline{(C \circ (\Box \triangleleft (,)))[v] \rightarrow C[v \triangleleft (,)]}$
SEM-ETERM-FILLPRED $h' = \max(\text{hnames}(C) \cup \{h\})$		
$\overline{C[+h \triangleleft (,)] \rightarrow C[h := \{h'+1, h'+2\} (- (h'+1), - (h'+2))][+(h'+1), +(h'+2)]}$		
SEM-ETERM-FILLFFOC <b>term_NotVal</b> t	SEM-ETERM-FILLFUNFOC	
$\overline{C[t \triangleleft (\lambda x_m \mapsto u)] \rightarrow (C \circ (\Box \triangleleft (\lambda x_m \mapsto u)))[t]}$	$\overline{(C \circ (\Box \triangleleft (\lambda x_m \mapsto u)))[v] \rightarrow C[v \triangleleft (\lambda x_m \mapsto u)]}$	
SEM-ETERM-FILLFRED	SEM-ETERM-FILLCFOC1 <b>term_NotVal</b> t	SEM-ETERM-FILLCUNFOC1
$\overline{C[+h \triangleleft (\lambda x_m \mapsto u)] \rightarrow C[h := \{\} \lambda^v x_m \mapsto u][()]}$	$\overline{C[t \triangleleft \bullet t'] \rightarrow (C \circ (\Box \triangleleft \bullet t'))[t]}$	$\overline{(C \circ (\Box \triangleleft \bullet t'))[v] \rightarrow C[v \triangleleft \bullet t']}$
SEM-ETERM-FILLCFOC2 <b>term_NotVal</b> t'	SEM-ETERM-FILLCUNFOC2	SEM-ETERM-FILLCRED $h' = \max(\text{hnames}(C) \cup \{h\})$
$\overline{C[v \triangleleft \bullet t'] \rightarrow (C \circ (v \triangleleft \bullet \Box))[t']}$	$\overline{(C \circ (v \triangleleft \bullet \Box))[v'] \rightarrow C[v \triangleleft \bullet v']}$	$\overline{C[+h \triangleleft \bullet_H \langle v_2, v_1 \rangle] \rightarrow C[h :=_{(H \pm h')} v_2 [H \pm h']][v_1 [H \pm h']}]}$