Destination λ-calculus

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February 28, 2024

1 Term and value syntax

tmv, x, y	Term-level variable name
hdmv	Hole or destination static name
k	Index for ranges

hddyn, d	::=		Hole or destination dynamic name
	|	$\star$	Root namespace
	|	d.1	Subnamespace 1
	|	d.2	Subnamespace 2
	|	d.3	Subnamespace 3
hdnm, h	::=		Hole or destination name
	|	d	Dynamic name
	|	hdmv	Static name
val, v	::=		Term value
	|	$-$h	Hole
	|	$+$h	Destination
	|	$()$	Unit
	|	$\lambda x \mapsto t$	Lambda abstraction
	|	Inl v	Left variant for sum
	|	Inr v	Right variant for sum
	|	$)^{m}\ v$	Exponential
	|	$(v_1\ ,\ v_2)$	Product
	|	$\langle v_1\ ⨾\ v_2\rangle_{\Delta}$	Ampar
term, t, u	::=		Term
	|	v	Value
	|	x	Variable
	|	$t \succ u$	Application
	|	t ; u	Pattern-match on unit
	|	$t \succ$ **case** $\{$ Inl $x_1 \mapsto u_1$, Inr $x_2 \mapsto u_2 \}$	Pattern-match on sum
	|	$t \succ$ **case** $(x_1\ ,\ x_2) \mapsto u$	Pattern-match on product
	|	$t \succ$ **case** $)^{m}\ x \mapsto u$	Pattern-match on exponential
	|	$t \succ$ **map** $x \mapsto u$	Map over the right side of the ampar
	|	$\textbf{to}_{\ltimes}\ t$	Wrap t into a trivial ampar
	|	$\textbf{from}_{\ltimes}\ t$	Extract value from trivial ampar
	|	$\textbf{alloc}_{\top}$	Return a fresh "identity" ampar object
	|	$t \triangleleft ()$	Fill destination with unit
	|	$t \triangleleft$ Inl	Fill destination with left variant
	|	$t \triangleleft$ Inr	Fill destination with right variant
	|	$t \triangleleft (,)$	Fill destination with product constructor
	|	$t \triangleleft)^{m}$	Fill destination with exponential constructor
	|	$t \triangleleft\!\bullet\ u$	Fill destination with root of ampar u

2 Type system

type, T, U	::=		Type
	|	$\mathsf{1}$	Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	|	$!^{m}\,\mathsf{T}$	Exponential
	|	$\mathsf{T}_1 \ltimes \mathsf{T}_2$	Ampar type (consuming T_2 yields T_1)
	|	$\mathsf{T}_1 \;{}_{m_1}\!\!\rightarrow \mathsf{T}_2$	Function
	|	${}^{m}\lfloor \mathsf{T} \rfloor$	Destination
mode, m, n	::=		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	1	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	Semiring product
ctx, Γ, Δ	::=		Typing context
	|	$\{\mathsf{b}_1, .., \mathsf{b}_k\}$	List of bindings
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mod
	|	$\Gamma_1 \,{}^{-}\!\!\uplus^{+}\, \Gamma_2$	Sum contexts, but allow linear holes from Γ_1 to be compensated by linear dests from Γ_2
	|	$-\Gamma$	Transforms every hole binding into a dest binding (requires `ctx_DestOnly` Γ)
bndr, b	::=		Type assignment to either variable, destination or hole
	|	$\mathsf{x} :_{m} \mathsf{T}$	Variable
	|	$+\mathsf{h} :_{m} {}^{n}\lfloor \mathsf{T} \rfloor$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-\mathsf{h} :^{n} \mathsf{T}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \Vdash t : T}$ *(Typing of terms (raw))*

$$\textsc{TyR-term-H}\quad \frac{}{\{-h :^{1\nu} T\} \Vdash -h : T}$$

$$\textsc{TyR-term-D}\quad \frac{}{\{+h :_{1\nu} {}^{n}\lfloor T \rfloor\} \Vdash +h : {}^{n}\lfloor T \rfloor}$$

$$\textsc{TyR-term-U}\quad \frac{}{\{\} \Vdash () : 1}$$

$$\textsc{TyR-term-L}\quad \frac{\Gamma \Vdash v : T_1}{\Gamma \Vdash \mathsf{Inl}\, v : T_1 \oplus T_2}$$

$$\textsc{TyR-term-R}\quad \frac{\Gamma \Vdash v : T_2}{\Gamma \Vdash \mathsf{Inr}\, v : T_1 \oplus T_2}$$

$$\textsc{TyR-term-P}\quad \frac{\Gamma_1 \Vdash v_1 : T_1 \qquad \Gamma_2 \Vdash v_2 : T_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (v_1 , v_2) : T_1 \otimes T_2}$$

$$\textsc{TyR-term-E}\quad \frac{\Gamma \Vdash v : T}{m\cdot\Gamma \Vdash \rrparenthesis^{m}\, v : !^{m}\, T}$$

$$\textsc{TyR-term-A}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash v_1 : T_1 \\ \Gamma_2 \Vdash v_2 : T_2 \qquad \texttt{ctx\_DestOnly}\ \Gamma_2 \\ \texttt{ctx\_SubsetEq}\ \ -\Gamma_2\ \Gamma_1\end{array}}{\Gamma_1 {}^{-}\!\uplus^{+} \Gamma_2 \Vdash \langle v_1 \mathbin{,} v_2 \rangle_{-\Gamma_1} : T_1 \ltimes T_2}$$

$$\textsc{TyR-term-F}\quad \frac{\Gamma \uplus \{x :_m T_1\} \Vdash t : T_2}{\Gamma \Vdash \lambda x \mapsto t : T_1\ {}_{m}\!\!\to T_2}$$

$$\textsc{TyR-term-Var}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ \{x :_{1\nu} T\}}{\Gamma \Vdash x : T}$$

$$\textsc{TyR-term-App}\quad \frac{\Gamma_1 \Vdash t : T_1 \qquad \Gamma_2 \Vdash u : T_1\ {}_{m}\!\!\to T_2}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash t \succ u : T_2}$$

$$\textsc{TyR-term-PatU}\quad \frac{\Gamma_1 \Vdash t : 1 \qquad \Gamma_2 \Vdash u : U}{\Gamma_1 \uplus \Gamma_2 \Vdash t\ ;\ u : U}$$

$$\textsc{TyR-term-PatS}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash t : T_1 \oplus T_2 \\ \Gamma_2 \uplus \{x_1 :_m T_1\} \Vdash u_1 : U \\ \Gamma_2 \uplus \{x_2 :_m T_2\} \Vdash u_2 : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash t \succ \mathsf{case}\,\{\,\mathsf{Inl}\, x_1 \mapsto u_1\, ,\ \mathsf{Inr}\, x_2 \mapsto u_2\,\} : U}$$

$$\textsc{TyR-term-PatP}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash t : T_1 \otimes T_2 \\ \Gamma_2 \uplus \{x_1 :_m T_1, x_2 :_m T_2\} \Vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash t \succ \mathsf{case}\,(x_1\, ,\ x_2) \mapsto u : U}$$

$$\textsc{TyR-term-PatE}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash t : !^{n}\, T \\ \Gamma_2 \uplus \{x :_{m\cdot n} T\} \Vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash t \succ \mathsf{case}\ \rrparenthesis^{n}\, x \mapsto u : U}$$

$$\textsc{TyR-term-Map}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash t : T_1 \ltimes T_2 \\ 1\!\uparrow\cdot\Gamma_2 \uplus \{x :_{1\nu} T_2\} \Vdash u : U\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash t \succ \mathsf{map}\ x \mapsto u : T_1 \ltimes U}$$

$$\textsc{TyR-term-FillC}\quad \frac{\Gamma_1 \Vdash t : {}^{n}\lfloor T_1 \rfloor \qquad \Gamma_2 \Vdash u : T_1 \ltimes T_2}{\Gamma_1 \uplus (1\!\uparrow\cdot n)\cdot\Gamma_2 \Vdash t \triangleleft\!\bullet\, u : T_2}$$

$$\textsc{TyR-term-FillU}\quad \frac{\Gamma \Vdash t : {}^{n}\lfloor 1 \rfloor}{\Gamma \Vdash t \triangleleft () : 1}$$

$$\textsc{TyR-term-FillL}\quad \frac{\Gamma \Vdash t : {}^{n}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \Vdash t \triangleleft \mathsf{Inl} : {}^{n}\lfloor T_1 \rfloor}$$

$$\textsc{TyR-term-FillR}\quad \frac{\Gamma \Vdash t : {}^{n}\lfloor T_1 \oplus T_2 \rfloor}{\Gamma \Vdash t \triangleleft \mathsf{Inr} : {}^{n}\lfloor T_2 \rfloor}$$

$$\textsc{TyR-term-FillP}\quad \frac{\Gamma \Vdash t : {}^{n}\lfloor T_1 \otimes T_2 \rfloor}{\Gamma \Vdash t \triangleleft (,) : {}^{n}\lfloor T_1 \rfloor \otimes {}^{n}\lfloor T_2 \rfloor}$$

$$\textsc{TyR-term-FillE}\quad \frac{\Gamma \Vdash t : {}^{m}\lfloor !^{n}\, T \rfloor}{\Gamma \Vdash t \triangleleft \rrparenthesis^{n} : {}^{m\cdot n}\lfloor T \rfloor}$$

$$\textsc{TyR-term-Alloc}\quad \frac{}{\{\} \Vdash \mathsf{alloc}_{T} : T \ltimes {}^{1\nu}\lfloor T \rfloor}$$

$$\textsc{TyR-term-ToA}\quad \frac{\Gamma \Vdash t : T}{\Gamma \Vdash \mathsf{to}_{\ltimes}\, t : T \ltimes 1}$$

$$\textsc{TyR-term-FromA}\quad \frac{\Gamma \Vdash t : T \ltimes 1}{\Gamma \Vdash \mathsf{from}_{\ltimes}\, t : T}$$

$\boxed{\Gamma \vdash t : T}$ *(Typing of terms (valid ones only))*

$$\textsc{Ty-term-T}\quad \frac{\begin{array}{c}\Gamma \Vdash t : T \\ \texttt{ctx\_Valid}\ \Gamma \qquad \texttt{ctx\_NoHole}\ \Gamma\end{array}}{\Gamma \vdash t : T}$$

3 Effects and big-step semantics

$eff,\ e$	$::=$		Effect
	$\mid$	ε	No-op effect
	$\mid$	f	Single hole assignment
	$\mid$	$e_1 \gg .. \gg e_k$	Chain effects
$\mathsf{hf},\ \mathsf{f}$	$::=$		Hole filling
	$\mid$	$\mathsf{h} := \mathsf{v}$	Fill h with value v (that may contain holes)

$\boxed{\Gamma \Vdash e}$ *(Typing of effects (raw))*

$$\frac{}{\{\} \Vdash \varepsilon}\ \textsc{TyR-eff-N}$$

$$\frac{\Gamma \uplus \Delta \Vdash \mathsf{v} : \mathsf{T} \qquad \texttt{ctx\_DestOnly}\ \Gamma \qquad \texttt{ctx\_HoleOnly}\ \Delta}{(1\!\uparrow\!\cdot m\cdot n)\cdot\Gamma \uplus \{+\mathsf{h} :_m {}^{n}\lfloor \mathsf{T}\rfloor\} \uplus (m\cdot n)\cdot\Delta \Vdash \mathsf{h} := \mathsf{v}}\ \textsc{TyR-eff-A}$$

$$\frac{\Gamma_1 \Vdash e_1 \qquad \Gamma_2 \Vdash e_2}{\Gamma_1 {}^{-}\!\uplus^{+} \Gamma_2 \Vdash e_1 \gg e_2}\ \textsc{TyR-eff-C}$$

$\boxed{\Gamma \vdash e}$ *(Typing of effects (valid ones only))*

$$\frac{\Gamma \Vdash e \qquad \texttt{ctx\_Valid}\ \Gamma}{\Gamma \vdash e}\ \textsc{Ty-eff-T}$$

$\boxed{\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}}$ *(Typing of commands (valid ones only))*

$$\frac{\Gamma_1 \vdash e \qquad \Gamma_2 \vdash \mathsf{v} : \mathsf{T} \qquad \texttt{ctx\_DestOnly}\ \Gamma_1 {}^{-}\!\uplus^{+} \Gamma_2}{\Gamma_1 {}^{-}\!\uplus^{+} \Gamma_2 \vdash \mathsf{v} \diamond e : \mathsf{T}}\ \textsc{Ty-cmd-C}$$

$\boxed{\mathsf{v}_1\ \Gamma_1 \mid e_1 \Downarrow \mathsf{v}_2\ \Gamma_2 \mid e_2}$ *(Big-step evaluation of effects on values (with potential holes))*

$$\frac{}{\mathsf{v}_1\ \Gamma_1 \mid \varepsilon \Downarrow \mathsf{v}_1\ \Gamma_1 \mid \varepsilon}\ \textsc{Sem-eff-N}$$

$$\frac{\texttt{ctx\_HdnmNotMem}\ \mathsf{h}\ \Gamma_1 \qquad \mathsf{v}_1\ \Gamma_1 \mid e_1 \Downarrow \mathsf{v}_2\ \Gamma_2 \mid e_2}{\mathsf{v}_1\ \Gamma_1 \mid \mathsf{h} := \mathsf{v}' \gg e_1 \Downarrow \mathsf{v}_2\ \Gamma_2 \mid \mathsf{h} := \mathsf{v}' \gg e_2}\ \textsc{Sem-eff-S}$$

$$\frac{\Gamma_0 \Vdash \mathsf{v}_0 : \mathsf{T} \qquad \texttt{ctx\_Valid}\ \Gamma_0 \qquad \mathsf{v}_1[\mathsf{h} := \mathsf{v}_0]\ (\Gamma_1 \uplus n\cdot\Gamma_0) \mid e_1 \Downarrow \mathsf{v}_2\ \Gamma_2 \mid e_2}{\mathsf{v}_1\ \Gamma_1 \uplus \{-\mathsf{h} : {}^{n}\mathsf{T}\} \mid \mathsf{h} := \mathsf{v}_0 \gg e_1 \Downarrow \mathsf{v}_2\ \Gamma_2 \mid e_2}\ \textsc{Sem-eff-F}$$

$\boxed{\mathsf{t} \Downarrow_{\mathsf{d}} \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

$$\frac{}{\mathsf{v} \Downarrow_{\mathsf{d}} \mathsf{v} \diamond \varepsilon}\ \textsc{Sem-term-V}$$

$$\frac{\mathsf{t}_1 \Downarrow_{\mathsf{d}.1} \mathsf{v}_1 \diamond e_1 \qquad \mathsf{t}_2 \Downarrow_{\mathsf{d}.2} \lambda \mathsf{x} \mapsto \mathsf{u} \diamond e_2 \qquad \mathsf{u}[\mathsf{x} := \mathsf{v}_1] \Downarrow_{\mathsf{d}.3} \mathsf{v}_3 \diamond e_3}{\mathsf{t}_1 \succ \mathsf{t}_2 \Downarrow_{\mathsf{d}} \mathsf{v}_3 \diamond e_1 \gg e_2 \gg e_3}\ \textsc{Sem-term-App}$$

$$\frac{\mathsf{t}_1 \Downarrow_{\mathsf{d}.1} () \diamond e_1 \qquad \mathsf{t}_2 \Downarrow_{\mathsf{d}.2} \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1\ ;\ \mathsf{t}_2 \Downarrow_{\mathsf{d}} \mathsf{v}_2 \diamond e_1 \gg e_2}\ \textsc{Sem-term-PatU}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} \mathsf{Inl}\,\mathsf{v}_1 \diamond e_1 \qquad \mathsf{u}_1[\mathsf{x}_1 := \mathsf{v}_1] \Downarrow_{\mathsf{d}.2} \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathsf{case}\,\{\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\} \Downarrow_{\mathsf{d}} \mathsf{v}_2 \diamond e_1 \gg e_2}\ \textsc{Sem-term-PatL}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} \mathsf{Inr}\,\mathsf{v}_1 \diamond e_1 \qquad \mathsf{u}_2[\mathsf{x}_2 := \mathsf{v}_1] \Downarrow_{\mathsf{d}.2} \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathsf{case}\,\{\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\} \Downarrow_{\mathsf{d}} \mathsf{v}_2 \diamond e_1 \gg e_2}\ \textsc{Sem-term-PatR}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} (\mathsf{v}_1\,,\,\mathsf{v}_2) \diamond e_1 \qquad \mathsf{u}[\mathsf{x}_1 := \mathsf{v}_1][\mathsf{x}_2 := \mathsf{v}_2] \Downarrow_{\mathsf{d}.2} \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u} \Downarrow_{\mathsf{d}} \mathsf{v}_2 \diamond e_1 \gg e_2}\ \textsc{Sem-term-PatP}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} \langle \mathsf{v}_1 \,,\, \mathsf{v}_2\rangle_{\Delta} \diamond e_1 \qquad \mathsf{u}[\mathsf{x} := \mathsf{v}_1] \Downarrow_{\mathsf{d}.2} \mathsf{v}_3 \diamond e_2 \qquad \mathsf{v}_2\ \Delta \mid e_2 \Downarrow \mathsf{v}_4\ \Delta' \mid e_3}{\mathsf{t} \succ \mathsf{map}\ \mathsf{x} \mapsto \mathsf{u} \Downarrow_{\mathsf{d}} \langle \mathsf{v}_3 \,,\, \mathsf{v}_4\rangle_{\Delta'} \diamond e_1 \gg e_3}\ \textsc{Sem-term-Map}$$

$$\frac{}{\mathsf{alloc}_{\mathsf{T}} \Downarrow_{\mathsf{d}} \langle +\mathsf{d} \,,\, -\mathsf{d}\rangle_{\{-\mathsf{d} : {}^{1\nu}\mathsf{T}\}} \diamond \varepsilon}\ \textsc{Sem-term-Alloc}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}} \mathsf{v} \diamond e}{\mathsf{to}_{\ltimes}\ \mathsf{t} \Downarrow_{\mathsf{d}} \langle () \,,\, \mathsf{v}\rangle_{\{\}} \diamond e}\ \textsc{Sem-term-ToA}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}} \langle () \,,\, \mathsf{v}\rangle_{\{\}} \diamond e}{\mathsf{from}_{\ltimes}\ \mathsf{t} \Downarrow_{\mathsf{d}} \mathsf{v} \diamond e}\ \textsc{Sem-term-FromA}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}} +\mathsf{h} \diamond e}{\mathsf{t} \triangleleft () \Downarrow_{\mathsf{d}} () \diamond e \gg \mathsf{h} := ()}\ \textsc{Sem-term-FillU}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} +\mathsf{h} \diamond e}{\mathsf{t} \triangleleft \mathsf{Inl} \Downarrow_{\mathsf{d}} +\mathsf{d}.2 \diamond e \gg \mathsf{h} := \mathsf{Inl}\ -\mathsf{d}.2}\ \textsc{Sem-term-FillL}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} +\mathsf{h} \diamond e}{\mathsf{t} \triangleleft \mathsf{Inr} \Downarrow_{\mathsf{d}} +\mathsf{d}.2 \diamond e \gg \mathsf{h} := \mathsf{Inr}\ -\mathsf{d}.2}\ \textsc{Sem-term-FillR}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} +\mathsf{h} \diamond e}{\mathsf{t} \triangleleft (,) \Downarrow_{\mathsf{d}} (+\mathsf{d}.2\,,\,+\mathsf{d}.3) \diamond e \gg \mathsf{h} := (-\mathsf{d}.2\,,\,-\mathsf{d}.3)}\ \textsc{Sem-term-FillP}$$

$$\frac{\mathsf{t} \Downarrow_{\mathsf{d}.1} +\mathsf{h} \diamond e_1 \qquad \mathsf{u} \Downarrow_{\mathsf{d}.2} \langle \mathsf{v}_1 \,,\, \mathsf{v}_2\rangle_{\Delta} \diamond e_2}{\mathsf{t} \triangleleft\!\bullet\, \mathsf{u} \Downarrow_{\mathsf{d}} \mathsf{v}_1 \diamond e_1 \gg e_2 \gg \mathsf{h} := \mathsf{v}_2}\ \textsc{Sem-term-FillC}$$

4 Type safety

Theorem 1 (Type safety). *If* `ctx_DestOnly` Γ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

Theorem 2 (Type safety for complete programs). *If* $\{\,\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \ {}_{\star}\!\Downarrow\ \mathsf{v} \diamond \varepsilon$ *and* $\{\,\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v} \ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $m\cdot\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \ {}_{m}\!\!\to \mathsf{T}_2$
  (3) `ctx_Disjoint` $\Gamma_1\ \Gamma_2$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t} \ {}_{\mathsf{d.1}}\!\Downarrow\ \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \uplus \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12} \uplus -\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \uplus \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u} \ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1 \ {}_{m}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \uplus \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1 \ {}_{m}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22} \uplus -\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \uplus \Gamma_{22}$.
  Using Lemma **??** on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x} \mapsto \mathsf{t}'$ and (14) $\Gamma_{21} \uplus \Gamma_{23} \uplus \{\mathsf{x} :_{m} \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma **??** on (14) and (5) we get (15) $m\cdot(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} := \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} := \mathsf{v}_1] \ {}_{\mathsf{d.3}}\!\Downarrow\ \mathsf{v}_3 \diamond e_3$ where (17) $m\cdot(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12} \uplus -\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22} \uplus -\Gamma_{23} \Vdash e_2$

  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{22}$ comes naturally from (3), (7) and (12).

  We must show:
  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  `ctx_Disjoint` $\Gamma_{22}\ \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  `ctx_Disjoint` $\Gamma_{13}\ \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \uplus \Gamma_{22} \uplus -\Gamma_{13} \uplus -\Gamma_{23} \Vdash e_1 \gg e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $m\cdot(\Gamma_{111} \uplus \Gamma_{131}) \uplus \Gamma_{211} \uplus \Gamma_{231} \uplus \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $m\cdot(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \uplus -$
  $\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{\mathsf{k}\,1} \uplus \Gamma_{\mathsf{k}\,2} = \Gamma_{\mathsf{k}}$

  We have
  (18) $\Gamma_{12} \uplus \Gamma_{22} \uplus -\Gamma_{13} \uplus -\Gamma_{23} \Vdash e_1 \gg e_2$
  (20) $m\cdot(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \uplus -\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $-\Gamma_{23}$, we get (22) $\Gamma_{12} \uplus \Gamma_{22} \uplus -(\Gamma_{131} \uplus \Gamma_{231}) \uplus -(\Gamma_{132} \uplus \Gamma_{232}) \Vdash e_1 \gg e_2$

  We want Γ_{132} from (22) to cancel $m\cdot\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{+\mathsf{h} :_{1\nu} {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash +\mathsf{h}' \diamond \mathsf{h} := \mathsf{Inl}\ -\mathsf{h}' : {}^{n}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} := \mathsf{Inl}\ -\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1 , \mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2) \succ (\lambda \mathsf{x}_2 \mapsto +\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ \mathsf{x}_2))$

  $+\mathsf{h}_0 \triangleleft (,) \ {}_{\mathsf{d}}\!\Downarrow\ (+\mathsf{d.2} , +\mathsf{d.3}) \diamond \mathsf{h}_0 := (-\mathsf{d.2} , -\mathsf{d.3})$

  $(\mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2)[\mathsf{x}_1 := +\mathsf{d.2}][\mathsf{x}_2 := +\mathsf{d.3}] \ {}_{\mathsf{d}'}\!\Downarrow\ +\mathsf{d.3} \diamond \mathsf{d.2} := +\mathsf{h}_1$

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1 , \mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2) \ {}_{\mathsf{d}''}\!\Downarrow\ +\mathsf{d.3} \diamond \mathsf{h}_0 := (-\mathsf{d.2} , -\mathsf{d.3}) \gg \mathsf{d.2} := +\mathsf{h}_1$

  $(+\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\ltimes}\ \mathsf{x}_2))[\mathsf{x}_2 := +\mathsf{d.3}] \ {}_{\mathsf{d}'''}\!\Downarrow\ () \diamond \mathsf{h}_3 := +\mathsf{d.3}$

  $\mathsf{t} \ {}_{\mathsf{d}''''}\!\Downarrow\ () \diamond \mathsf{h}_0 := (-\mathsf{d.2} , -\mathsf{d.3}) \gg \mathsf{d.2} := +\mathsf{h}_1 \gg \mathsf{h}_3 := +\mathsf{d.3}$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all $\textsc{BigStep\_Fill}\langle \mathit{Ctor} \rangle$ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in $\textsc{TyCmd\_Cmd}$), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;
- or are those of pre-existing holes coming from the extended value v_2 of an ampar, when $\textsc{BigStep\_FillComp}$ is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program · And v_1 is exactly the accompanying value returned by the evaluation of $\textsc{BigStep\_FillComp}$

TODO: prove that this property is preserved by typing rules □