termvar, x, y, d	Term-level variable			
$\ell abel$, ℓ	::=			Label
$hole$, h	::=			Hole
term_value, v	::=			Term value
	|	ℓ		Label representing an Ampar
	|	$@h$		Destination
	|	$()$		Unit
	|	$\mathbf{Inl}\ v$		Left variant for sum
	|	$\mathbf{Inr}\ v$		Right variant for sum
	|	$(v_1\ ,\ v_2)$		Product
	|	$\lambda x\ .\ t$		Linear function
	|	(v)	S	
$\overline{\text{extended\_value}}$, $\bar{v}$	::=			Store value
	|	v		Term value
	|	h		Hole
	|	$\mathbf{Inl}\ \bar{v}$		Left variant with val or hole
	|	$\mathbf{Inr}\ \bar{v}$		Right variant with val or hole
	|	$(\bar{v}_1\ ,\ \bar{v}_2)$		Product with val or hole
	|	$(\bar{v})$	S	
term, t, u	::=			Term
	|	v		Term value
	|	x		Variable
	|	$t\ u$		Application
	|	$t\ \succ\mathbf{case}\ () \mapsto\ u$		Pattern-match on unit
	|	$t\ \succ\mathbf{case}\ \{\ \mathbf{Inl}\ x_1\ \mapsto\ u_1\ ,\ \mathbf{Inr}\ x_2\ \mapsto\ u_2\ \}$		Pattern-match on sum
	|	$t\ \succ\mathbf{case}\ (x_1\ ,\ x_2) \mapsto\ u$		Pattern-match on product
	|	$t\ \succ\mathbf{mapL}\ x \mapsto\ u$		Map over the left side of the ampar
	|	$\mathbf{to}_{\ltimes}\ t$		Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ t$		Extract value from trivial ampar
	|	$\mathbf{alloc}$		Return a fresh "identity" ampar object
	|	$t \triangleleft ()$		Fill destination with unit
	|	$t \triangleleft\ \mathbf{Inl}$		Fill destination with left variant
	|	$t \triangleleft\ \mathbf{Inr}$		Fill destination with right variant
	|	$t \triangleleft (,)$		Fill destination with product constructor
	|	$t \triangleleft\!\bullet\ u$		Fill destination with root of ampar u
	|	(t)	S	
	|	$t[\text{subs}]$	M	
$\overline{\text{extended\_term}}$, $\bar{t}$	::=			Extended term
	|	t		
	|	$\bar{v}$		
sub	::=			Variable or label substitution
	|	$x := v$		
	|	$h := \bar{v}$		
subs	::=			Substitutions
	|	sub		
	|	sub, subs		

store, S	::=			
	|	$[]$		
	|	$[\,\mathsf{store\_assigns}\,]$		
	|	$\mathsf{S}[\mathsf{subs}]$		
	|	$\mathsf{S}_1 \sqcup \mathsf{S}_2$		
store_assign, ha	::=			
	|	$\ell \mapsto \langle \mathsf{v}_1 \,,\, \overline{\mathsf{v}_2} \rangle$		Closed ampar ($\overline{\mathsf{v}_2}$ = root of the incomplete struct)
	|	$\ell \mapsto \langle \square \,,\, \overline{\mathsf{v}_2} \rangle$		Open ampar ($\overline{\mathsf{v}_2}$ = root of the incomplete struct)
store_assigns	::=			
	|	ha		
	|	ha, store_assigns		
type, **A**, **B**	::=			Type
	|	$\mathbf{1}$		Unit
	|	$\mathsf{A}_1 \oplus \mathsf{A}_2$		Sum
	|	$\mathsf{A}_1 \otimes \mathsf{A}_2$		Product
	|	$\mathsf{A}_1 \rtimes \mathsf{A}_2$		Ampar type (consuming A_1 yields A_2)
	|	${}_{\mathtt{m}_1}\mathsf{A}_1 \multimap \mathsf{A}_2$		Linear function
	|	$\mathsf{A}^{\perp}$		Destination
	|	(A)	S	
`mode`, `m`	::=			Mode
	|	$\mathtt{L}$		Local
	|	$\mathtt{F}$		Foreign
	|	$\mathtt{G}$		Global
	|	$\mathtt{max\_mode}(\Gamma)$		
	|	**if** $\mathtt{mode\_cond}$ **then** $\mathtt{m}_3$ **else** $\mathtt{m}_4$		
`mode_cond`	::=			Mode statement
	|	$\mathtt{m}_1 = \mathtt{m}_2$		
	|	$\mathtt{m} \in \textbf{upper\_modes}\,(\Gamma)$		
	|	$\exists\, \mathtt{m} \in \textbf{upper\_modes}\,(\Gamma)$		
typing_context, $\mho$, Γ	::=			Typing context
	|	$\{\}$		
	|	$\{\mathsf{type\_assigns}\}$		
	|	$\Gamma_1 \sqcup \Gamma_2$		
	|	$\Gamma_1 \uplus \Gamma_2$		
	|	$\Gamma[\mathtt{m}_1 \mapsto \mathtt{m}_2]$		
	|	(Γ)	S	
type_assign, ta	::=			Type assignment
	|	$\mathsf{x} :_{\mathtt{m}} \mathsf{A}$		
	|	$+\ell : \mathsf{A}$		
	|	$-\ell : \mathsf{A}$		
	|	$+h : \mathsf{A}$		Hole
	|	$-h : \mathsf{A}$		Destination
type_assigns	::=			Type assignments
	|	ta		
	|	ta, type_assigns		

terminals ::=
| $\multimap$
| $\rtimes$
| $\mapsto$
| ()
| Inl
| Inr
| (,)
| $\triangleleft$
| $\triangleleft\!\!\bullet$
| $\sqcup$
| $\uplus$
| {}
| $\exists$
| $\neq$
| $\leq$
| $\in$
| $\notin$
| $\subset$
| $\mathcal{N}$
| $\vdash$
| $\mid$
| $\Downarrow$

formula ::=
| judgement

Ctx ::=
| $x \in \mathcal{N}(\Gamma)$
| $\ell \in \mathcal{N}(\Gamma)$
| $x \notin \mathcal{N}(\Gamma)$
| $\ell \notin \mathcal{N}(\Gamma)$
| **fresh** x
| **fresh** ℓ
| **fresh** h
| type_assign $\in \Gamma$
| `mode_cond`

Eq ::=
| $\mathbf{A}_1 = \mathbf{A}_2$
| $\mathbf{A}_1 \neq \mathbf{A}_2$
| $t = u$
| $t \neq u$
| $\Gamma_1 = \Gamma_2$
| $\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset$

Ty ::=
| $\Gamma \vdash S \mid t : \mathbf{A}$
| $\Gamma \vdash S$
| $\Gamma \vdash \bar{t} : \mathbf{A}$

Sem ::=
| $S \mid t \Downarrow S' \mid t'$

judgement ::=
| Ctx
| Eq
| Ty
| Sem

user_syntax ::=
| termvar
| *label*
| *hole*
| term_value
| extended_value
| term
| extended_term
| sub
| subs
| store
| store_assign
| store_assigns
| **type**
| mode
| mode_cond
| typing_context
| type_assign
| type_assigns
| terminals

$\boxed{x \in \mathcal{N}(\Gamma)}$

$\boxed{\ell \in \mathcal{N}(\Gamma)}$

$\boxed{x \notin \mathcal{N}(\Gamma)}$

$\boxed{\ell \notin \mathcal{N}(\Gamma)}$

$\boxed{\textbf{fresh}\ x}$

$\boxed{\textbf{fresh}\ \ell}$

$\boxed{\textbf{fresh}\ h}$

$\boxed{\text{type\_assign} \in \Gamma}$

$\boxed{\text{mode\_cond}}$

$\boxed{\mathbf{A}_1 = \mathbf{A}_2}$

$\boxed{\mathbf{A}_1 \neq \mathbf{A}_2}$

$\boxed{t = u}$

$\boxed{t \neq u}$

$\boxed{\Gamma_1 = \Gamma_2}$

$\boxed{\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset}$

$\boxed{\Gamma \vdash S \mid t : \mathbf{A}}$

$$\frac{\begin{array}{c}\Gamma_1 \vdash S \\ \Gamma_2 \vdash t : \mathbf{A}\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash S \mid t : \mathbf{A}} \quad \text{TYCMD\_CMD}$$

$\boxed{\Gamma \vdash S}$

$$\frac{}{\{\} \vdash []} \quad \text{TYHEAP\_EMPTY}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash S_1 \\ \Gamma_2 \vdash S_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash S_1 \sqcup S_2}\ \textsc{TyHeap\_Union}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash v_1 : \mathbf{A}_1 \\ \Gamma_2 \vdash \overline{v_2} : \mathbf{A}_2\end{array}}{(\Gamma_1 \uplus \Gamma_2) \sqcup \{-\underline{\ell} : \mathbf{A}_1 \rtimes \mathbf{A}_2\} \vdash [\,\underline{\ell} \mapsto \langle v_1 , \overline{v_2}\rangle\,]}\ \textsc{TyHeap\_ClosedAmpar}$$

$$\frac{\Gamma_2 \vdash \overline{v_2} : \mathbf{A}_2}{\Gamma_2 \vdash [\,\underline{\ell} \mapsto \langle \square , \overline{v_2}\rangle\,]}\ \textsc{TyHeap\_OpenAmpar}$$

$\boxed{\Gamma \vdash \bar{t} : \mathbf{A}}$

$$\frac{}{\{+\underline{\ell} : \mathbf{A}\} \vdash \underline{\ell} : \mathbf{A}}\ \textsc{TyTerm\_Ampar}$$

$$\frac{}{\{-h : \mathbf{A}\} \vdash @h : \mathbf{A}^{\perp}}\ \textsc{TyTerm\_Dest}$$

$$\frac{}{\{+h : \mathbf{A}\} \vdash h : \mathbf{A}}\ \textsc{TyTerm\_Hole}$$

$$\frac{}{\{\} \vdash () : \mathbf{1}}\ \textsc{TyTerm\_Unit}$$

$$\frac{\Gamma \vdash \bar{v} : \mathbf{A}_1}{\Gamma \vdash \mathsf{Inl}\,\bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2}\ \textsc{TyTerm\_Inl}$$

$$\frac{\Gamma \vdash \bar{v} : \mathbf{A}_2}{\Gamma \vdash \mathsf{Inr}\,\bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2}\ \textsc{TyTerm\_Inr}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash \overline{v_1} : \mathbf{A}_1 \\ \Gamma_2 \vdash \overline{v_2} : \mathbf{A}_2\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash (\overline{v_1} , \overline{v_2}) : \mathbf{A}_1 \otimes \mathbf{A}_2}\ \textsc{TyTerm\_Prod}$$

$$\frac{\Gamma \sqcup \{x :_{\mathtt{m}_1} \mathbf{A}_1\} \vdash t : \mathbf{A}_2}{\Gamma \vdash \lambda x \,.\, t :_{\mathtt{m}_1} \mathbf{A}_1 \multimap \mathbf{A}_2}\ \textsc{TyTerm\_Lambda}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash t :_{\mathtt{m}_1} \mathbf{A}_1 \multimap \mathbf{A}_2 \\ \Gamma_2 \vdash u : \mathbf{A}_1 \\ \mathtt{m}_1 \in \mathbf{upper\_modes}(\Gamma_2)\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ u : \mathbf{A}_2}\ \textsc{TyTerm\_App}$$

$$\frac{\begin{array}{c}\Gamma_1 \vdash t : \mathbf{1} \\ \Gamma_2 \vdash u : \mathbf{B}\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ \succ\!\mathbf{case}\ () \mapsto\ u : \mathbf{B}}\ \textsc{TyTerm\_PatUnit}$$

$$\frac{\begin{array}{l}\Gamma_1 \vdash t : \mathbf{A}_1 \oplus \mathbf{A}_2 \\ \exists\, \mathtt{m} \in \mathbf{upper\_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{x_1 :_{\mathtt{m}} \mathbf{A}_1\} \vdash u_1 : \mathbf{B} \\ \Gamma_2 \sqcup \{x_2 :_{\mathtt{m}} \mathbf{A}_2\} \vdash u_2 : \mathbf{B}\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ \succ\!\mathbf{case}\ \{\,\mathsf{Inl}\,x_1 \mapsto u_1 ,\ \mathsf{Inr}\,x_2 \mapsto u_2\,\} : \mathbf{B}}\ \textsc{TyTerm\_PatSum}$$

$$\frac{\begin{array}{l}\Gamma_1 \vdash t : \mathbf{A}_1 \otimes \mathbf{A}_2 \\ \exists\, \mathtt{m} \in \mathbf{upper\_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{x_1 :_{\mathtt{m}} \mathbf{A}_1, x_2 :_{\mathtt{m}} \mathbf{A}_2\} \vdash u : \mathbf{B}\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ \succ\!\mathbf{case}\ (x_1 , x_2) \mapsto\ u : \mathbf{B}}\ \textsc{TyTerm\_PatProd}$$

$$\frac{\begin{array}{l}\Gamma_1 \vdash t : \mathbf{A}_1 \rtimes \mathbf{A}_2 \\ \exists\, \mathtt{m}' \in \mathbf{upper\_modes}(\Gamma_1 \sqcup \Gamma_2) \\ \mathtt{m} = \mathbf{if}\ \mathtt{F} \in \mathbf{upper\_modes}(\Gamma_1)\ \mathbf{then}\ \mathtt{F}\ \mathbf{else}\ \mathtt{L} \\ \Gamma_2[\mathtt{L} \mapsto \mathtt{F}] \sqcup \{x :_{\mathtt{m}} \mathbf{A}_1\} \vdash u : \mathbf{B}\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t\ \succ\!\mathbf{mapL}\ x \mapsto\ u : \mathbf{B} \rtimes \mathbf{A}_2}\ \textsc{TyTerm\_MapAmpar}$$

$$\frac{}{\{\} \vdash \mathsf{alloc} : A^{\perp} \rtimes A}\quad \textsc{TyTerm\_Alloc}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathsf{to}_{\rtimes}\ t : 1 \rtimes A}\quad \textsc{TyTerm\_ToAmpar}$$

$$\frac{\Gamma \vdash t : 1 \rtimes A}{\Gamma \vdash \mathsf{from}_{\rtimes}\ t : A}\quad \textsc{TyTerm\_FromAmpar}$$

$$\frac{\Gamma \vdash t : 1^{\perp}}{\Gamma \vdash t \triangleleft () : 1}\quad \textsc{TyTerm\_FillUnit}$$

$$\frac{\Gamma \vdash t : (A_1 \oplus A_2)^{\perp}}{\Gamma \vdash t \triangleleft \mathsf{Inl} : {A_1}^{\perp}}\quad \textsc{TyTerm\_FillInl}$$

$$\frac{\Gamma \vdash t : (A_1 \oplus A_2)^{\perp}}{\Gamma \vdash t \triangleleft \mathsf{Inr} : {A_2}^{\perp}}\quad \textsc{TyTerm\_FillInr}$$

$$\frac{\Gamma \vdash t : (A_1 \otimes A_2)^{\perp}}{\Gamma \vdash t \triangleleft (,) : {A_1}^{\perp} \otimes {A_2}^{\perp}}\quad \textsc{TyTerm\_FillProd}$$

$$\frac{\begin{array}{l}\Gamma_1 \vdash t : {A_2}^{\perp} \\ \Gamma_2 \vdash u : A_1 \rtimes A_2 \\ \mathtt{L} \in \mathsf{upper\_modes}\,(\Gamma_1) \\ \mathtt{F} \in \mathsf{upper\_modes}\,(\Gamma_2)\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft\!\bullet\ u : A_1}\quad \textsc{TyTerm\_FillCompL}$$

$$\frac{\begin{array}{l}\Gamma_1 \vdash t : {A_2}^{\perp} \\ \Gamma_2 \vdash u : A_1 \rtimes A_2 \\ \mathtt{F} \in \mathsf{upper\_modes}\,(\Gamma_1) \\ \mathtt{G} \in \mathsf{upper\_modes}\,(\Gamma_2)\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft\!\bullet\ u : A_1}\quad \textsc{TyTerm\_FillCompF}$$

$\boxed{S \mid t \Downarrow S' \mid t'}$

$$\frac{}{S_0 \mid v \Downarrow S_0 \mid v}\quad \textsc{BigStep\_Val}$$

$$\frac{\begin{array}{c}S_0 \mid t_1 \Downarrow S_1 \mid \lambda x\,.\,u \\ S_1 \mid t_2 \Downarrow S_2 \mid v_2 \\ S_2 \mid u[x := v_2] \Downarrow S_3 \mid v_3\end{array}}{S_0 \mid t_1\, t_2 \Downarrow S_3 \mid v_3}\quad \textsc{BigStep\_App}$$

$$\frac{\begin{array}{c}S_0 \mid t_1 \Downarrow S_1 \mid () \\ S_1 \mid t_2 \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t_1 \succ \mathsf{case}\ () \mapsto t_2 \Downarrow S_2 \mid v_2}\quad \textsc{BigStep\_PatUnit}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid \mathsf{Inl}\, v_1 \\ S_1 \mid u_1[x_1 := v_1] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t \succ \mathsf{case}\,\{\, \mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\} \Downarrow S_2 \mid v_2}\quad \textsc{BigStep\_PatInl}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid \mathsf{Inr}\, v_1 \\ S_1 \mid u_2[x_2 := v_1] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t \succ \mathsf{case}\,\{\, \mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\} \Downarrow S_2 \mid v_2}\quad \textsc{BigStep\_PatInr}$$

$$\frac{\begin{array}{c}S_0 \mid t \Downarrow S_1 \mid (v_1\,,\,v_2) \\ S_1 \mid u[x_1 := v_1, x_2 := v_2] \Downarrow S_2 \mid v_2\end{array}}{S_0 \mid t \succ \mathsf{case}\,(x_1\,,\,x_2) \mapsto u \Downarrow S_2 \mid v_2}\quad \textsc{BigStep\_PatProd}$$

$$\frac{\begin{array}{l}S_0 \mid t \Downarrow S_1 \sqcup [\,\ell \mapsto \langle v_1 \,{,}\, \overline{v_1}\rangle\,] \mid \ell \\ S_1 \sqcup [\,\ell \mapsto \langle \Box \,{,}\, \overline{v_1}\rangle\,] \mid u[x := v_1] \Downarrow S_2 \sqcup [\,\ell \mapsto \langle \Box \,{,}\, \overline{v_2}\rangle\,] \mid v_2\end{array}}{S_0 \mid t \succ \mathsf{mapL}\, x \mapsto u \Downarrow S_2 \sqcup [\,\ell \mapsto \langle v_2 \,{,}\, \overline{v_2}\rangle\,] \mid \ell}\quad \textsc{BigStep\_MapAmpar}$$

$$\frac{\mathsf{fresh}\ h \qquad \mathsf{fresh}\ \ell}{S_0 \mid \mathsf{alloc} \Downarrow S_0 \sqcup [\,\ell \mapsto \langle @h \,,\, h\rangle\,] \mid \ell} \quad \text{BIGSTEP\_ALLOC}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid v \qquad \mathsf{fresh}\ \ell}{S_0 \mid \mathsf{to}_{\rtimes}\ t \Downarrow S_1 \sqcup [\,\ell \mapsto \langle () \,,\, v\rangle\,] \mid \ell} \quad \text{BIGSTEP\_TOAMPAR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \sqcup [\,\ell \mapsto \langle () \,,\, v\rangle\,] \mid \ell}{S_0 \mid \mathsf{from}_{\rtimes}\ t \Downarrow S_1 \mid v} \quad \text{BIGSTEP\_FROMAMPAR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h}{S_0 \mid t \triangleleft () \Downarrow S_1[h := ()] \mid ()} \quad \text{BIGSTEP\_FILLUNIT}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \qquad \mathsf{fresh}\ h'}{S_0 \mid t \triangleleft \mathsf{Inl} \Downarrow S_1[h := \mathsf{Inl}\ h'] \mid @h'} \quad \text{BIGSTEP\_FILLINL}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \qquad \mathsf{fresh}\ h'}{S_0 \mid t \triangleleft \mathsf{Inr} \Downarrow S_1[h := \mathsf{Inr}\ h'] \mid @h'} \quad \text{BIGSTEP\_FILLINR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \qquad \mathsf{fresh}\ h_1 \qquad \mathsf{fresh}\ h_2}{S_0 \mid t \triangleleft (,) \Downarrow S_1[h := (h_1 \,,\, h_2)] \mid (@h_1 \,,\, @h_2)} \quad \text{BIGSTEP\_FILLPROD}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \qquad S_1 \mid u \Downarrow S_2 \sqcup [\,\ell \mapsto \langle v_1 \,,\, \overline{v_1}\rangle\,] \mid \ell}{S_0 \mid t \triangleleft\!\bullet\, u \Downarrow S_2[h := \overline{v_1}] \mid v_1} \quad \text{BIGSTEP\_FILLCOMP}$$

```
Definition rules:        42 good    0 bad
Definition rule clauses: 112 good    0 bad
```