Destination λ-calculus

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1 Term and value syntax

var, x, y	Term-level variable name
k	Index for ranges

hdn, h	::=		Hole or destination name
	|	1	
	|	2	
	|	$h+h'$	Sum
	|	$\max(H)$	Maximum of a set of holes
hdns, H	::=		Set of hole or destination names
	|	$\{h_1, .., h_k\}$	Set of holes
	|	$H_1 \cup H_2$	Union of sets
	|	$H \pm h$	Increase all names from H by h.
	|	$\mathrm{hnames}(\Gamma)$	Hole names of a context (requires `ctx_NoVar`(Γ))
val, v	::=		Term value
	|	$-h$	Hole
	|	$+h$	Destination
	|	$()$	Unit
	|	$\lambda_v x \mapsto t$	Lambda abstraction
	|	$\mathsf{Inl}\ v$	Left variant for sum
	|	$\mathsf{Inr}\ v$	Right variant for sum
	|	$\mathrm{E}^{m}\ v$	Exponential
	|	(v_1, v_2)	Product
	|	$_{H}\langle v_1 ,\ v_2 \rangle$	Ampar
	|	$v \pm h$	Rename hole names inside v by shifting them by h
term, t, u	::=		Term
	|	v	Value
	|	x	Variable
	|	$t \succ u$	Application
	|	$t \mathbin{;} u$	Pattern-match on unit
	|	$t \succ \mathbf{case}\,\{ \mathsf{Inl}\, x_1 \mapsto u_1,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}$	Pattern-match on sum
	|	$t \succ \mathbf{case}\,(x_1, x_2) \mapsto u$	Pattern-match on product
	|	$t \succ \mathbf{case}\ \mathrm{E}^{m}\, x \mapsto u$	Pattern-match on exponential
	|	$t \succ \mathbf{map}\ x \mapsto u$	Map over the right side of the ampar
	|	$\mathbf{to}_{\ltimes}\ t$	Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ t$	Extract value from trivial ampar
	|	$\mathbf{alloc}_{T}$	Return a fresh "identity" ampar object
	|	$t \triangleleft ()$	Fill destination with unit
	|	$t \triangleleft \lambda x \mapsto u$	Fill destination with function
	|	$t \triangleleft \mathsf{Inl}$	Fill destination with left variant
	|	$t \triangleleft \mathsf{Inr}$	Fill destination with right variant
	|	$t \triangleleft (,)$	Fill destination with product constructor
	|	$t \triangleleft \mathrm{E}^{m}$	Fill destination with exponential constructor
	|	$t \triangleleft\!\bullet\, u$	Fill destination with root of ampar u

	$\vert$	$\mathsf{t}[\mathsf{x} \coloneqq \mathsf{v}]$	
$\mathsf{eterm},\ \mathsf{j}$	$::=$		Pseudo-term
	$\vert$	t	
	$\vert$	$\mathsf{C}[\mathsf{j}]$	
$\mathsf{ectx},\ \mathsf{C}$	$::=$		Evaluation context
	$\vert$	$[]$	Identity
	$\vert$	${}^{o}_{\mathtt{H}}\langle \mathsf{v}_1 \mathbin{,} \mathsf{C}$	Open ampar
	$\vert$	$\mathsf{C}[\mathsf{C}']$	Compose evaluation contexts
	$\vert$	$\mathsf{C}[\mathtt{h} \coloneqq_{\mathtt{H}} \mathsf{v}]$	Fill $\mathtt{h}$ with value v (that may contain holes)

2 Type system

$\mathsf{type},\ \mathsf{T},\ \mathsf{U}$	$::=$		Type
	|	$\mathsf{1}$	Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	|	$!^{m}\,\mathsf{T}$	Exponential
	|	$\mathsf{T}_1 \ltimes \mathsf{T}_2$	Ampar type (consuming T_2 yields T_1)
	|	$\mathsf{T}_1\ {}_{m_1}\!\!\to \mathsf{T}_2$	Function
	|	$\lfloor \mathsf{T} \rfloor^{m}$	Destination
	|	$\mathsf{T}_1 \rightarrowtail \mathsf{T}_2$	Evaluation contexts
$\mathit{mode},\ m,\ n$	$::=$		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_{\mathsf{k}}$	Semiring product
$\mathit{mul},\ p$	$::=$		Multiplicity (first component of modality)
	|	$\mathit{1}$	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_{\mathsf{k}}$	Semiring product
$\mathit{age},\ a$	$::=$		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_{\mathsf{k}}$	Semiring product
$\mathsf{ctx},\ \Gamma,\ \Delta$	$::=$		Typing context
	|	$\{\mathsf{b}_1, \ldots, \mathsf{b}_{\mathsf{k}}\}$	List of bindings
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mo
	|	$-\Gamma$	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)
$\mathsf{bndr},\ \mathsf{b}$	$::=$		Type assignment to either variable, destination or hole
	|	$\mathsf{x} :_{m} \mathsf{T}$	Variable
	|	$+\mathsf{h} :_{m} \lfloor \mathsf{T} \rfloor^{n}$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-\mathsf{h} : \mathsf{T}^{n}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \Vdash \mathsf{v} : \mathsf{T}}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \frac{}{\{-\mathtt{h} : \mathsf{T}^{1a}\} \Vdash -\mathtt{h} : \mathsf{T}}$$

$$\textsc{TyR-val-D}\quad \frac{\mathtt{ctx\_Compatible}\ \Gamma\ \ {+}\mathtt{h} :_{1\nu} \lfloor \mathsf{T} \rfloor^{n}}{\Gamma \Vdash {+}\mathtt{h} : \lfloor \mathsf{T} \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \frac{}{\{\} \Vdash () : \mathbf{1}}$$

$$\textsc{TyR-val-F}\quad \frac{\mathtt{ctx\_DestOnly}\ \Gamma \qquad \Gamma \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{t} : \mathsf{T}_2}{\Gamma \Vdash \lambda_{\mathsf{v}} x \mapsto \mathsf{t} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}$$

$$\textsc{TyR-val-L}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-R}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-P}\quad \frac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-val-E}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{m{\cdot}\Gamma \Vdash \mathrm{E}^{m}\,\mathsf{v} : !^{m}\,\mathsf{T}}$$

$$\textsc{TyR-val-A}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\\ \mathtt{ctx\_DestOnly}\ \Gamma_2 \uplus \Gamma_3\\ \mathtt{ctx\_DestOnly}\ \Gamma_1\\ \Gamma_1 \uplus (-\Gamma_3) \Vdash \mathsf{v}_1 : \mathsf{T}_1\\ \Gamma_2 \uplus \Gamma_3 \Vdash \mathsf{v}_2 : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash_{\mathtt{hnames}(-\Gamma_3)} \langle \mathsf{v}_1\,{}_{\mathbf{9}}\,\mathsf{v}_2 \rangle : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$\boxed{\Gamma \vdash \mathsf{j} : \mathsf{T}}$ *(Typing of terms)*

$$\textsc{Ty-term-Val}\quad \frac{\mathtt{ctx\_NoHole}\ \Gamma \qquad \Gamma \Vdash \mathsf{v} : \mathsf{T}}{\Gamma \vdash \mathsf{v} : \mathsf{T}}$$

$$\textsc{Ty-term-Var}\quad \frac{\mathtt{ctx\_Compatible}\ \Gamma\ \ x :_{1\nu} \mathsf{T}}{\Gamma \vdash x : \mathsf{T}}$$

$$\textsc{Ty-term-App}\quad \frac{\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-term-PatU}\quad \frac{\Gamma_1 \vdash \mathsf{t} : \mathbf{1} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{U}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t}\ ;\ \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x_1 :_{m} \mathsf{T}_1\}\\ \mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x_2 :_{m} \mathsf{T}_2\}\\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2\\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U}\\ \Gamma_2 \uplus \{x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\,\{\mathsf{Inl}\,x_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,x_2 \mapsto \mathsf{u}_2\} : \mathsf{U}}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x_1 :_{m} \mathsf{T}_1\}\\ \mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x_2 :_{m} \mathsf{T}_2\}\\ \mathtt{ctx\_Disjoint}\ \{x_1 :_{m} \mathsf{T}_1\}\ \{x_2 :_{m} \mathsf{T}_2\}\\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2\\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1, x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\,(x_1\,,\,x_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x :_{m\cdot n} \mathsf{T}\}\\ \Gamma_1 \vdash \mathsf{t} : !^{n}\,\mathsf{T}\\ \Gamma_2 \uplus \{x :_{m\cdot n} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\ \mathrm{E}^{m}\,x \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x :_{1\nu} \mathsf{T}_2\}\\ \Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \ltimes \mathsf{T}_2\\ 1\!\uparrow{\cdot}\Gamma_2 \uplus \{x :_{1\nu} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathbf{map}\ x \mapsto \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathbf{1} \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft () : \mathbf{1}}$$

$$\textsc{Ty-term-FillF}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_2\ \{x :_{m} \mathsf{T}_1\}\\ \Gamma_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2 \rfloor^{n}\\ \Gamma_2 \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (1\!\uparrow{\cdot}n){\cdot}\Gamma_2 \vdash \mathsf{t} \triangleleft \lambda x \mapsto \mathsf{u} : \mathbf{1}}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : \lfloor \mathsf{T}_1 \rfloor^{n}}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Gamma \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft (,) : \lfloor \mathsf{T}_1 \rfloor^{n} \otimes \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Gamma \vdash \mathsf{t} : \lfloor !^{m}\,\mathsf{T} \rfloor^{n}}{\Gamma \vdash \mathsf{t} \triangleleft \mathrm{E}^{m} : \lfloor \mathsf{T} \rfloor^{m\cdot n}}$$

$$\textsc{Ty-term-FillC}\quad \frac{\Gamma_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \rfloor^{n} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Gamma_1 \uplus (1\!\uparrow{\cdot}n){\cdot}\Gamma_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathbf{alloc}_{\mathsf{T}} : \mathsf{T} \ltimes \lfloor \mathsf{T} \rfloor^{1\nu}}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Gamma \vdash \mathsf{t} : \mathsf{T}}{\Gamma \vdash \mathbf{to}_{\ltimes}\ \mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Gamma \vdash \mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}{\Gamma \vdash \mathbf{from}_{\ltimes}\ \mathsf{t} : \mathsf{T}}$$

$\boxed{\Gamma \Vdash \mathsf{C} : \mathsf{T}}$ *(Typing of evaluation contexts)*

$$\textsc{TyR-ectx-T}\quad \frac{\begin{array}{c}\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2\\ \mathtt{ctx\_Disjoint}\ \Gamma_2\ \Gamma_3\\ \mathtt{ctx\_NoVar}\ \Gamma_2 \uplus \Gamma_3\\ \mathtt{ctx\_NoVar}\ \Gamma_1\\ \Gamma_2 \uplus \Gamma_3 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_1 \vdash \mathsf{C}[\mathsf{t}] : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (-\Gamma_2) \Vdash \mathsf{C} : \mathsf{T}_1 \rightarrowtail \mathsf{T}_2}$$

3 Effects and big-step semantics

$\boxed{\mathsf{j} \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h'}}\; \mathsf{j}'}$ *(Small-step evaluation of terms using evaluation contexts)*

Sem-eterm-App

$$\frac{}{\mathsf{C}[\mathsf{v} \succ (\lambda_{\mathsf{v}} \mathsf{x} \mapsto \mathsf{t})] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{t}[\mathsf{x} \coloneqq \mathsf{v}]]}$$

Sem-eterm-PatU

$$\frac{}{\mathsf{C}[() \,;\, \mathsf{t}_2] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{t}_2]}$$

Sem-eterm-PatL

$$\frac{}{\mathsf{C}[(\mathsf{Inl}\,\mathsf{v}) \succ \mathbf{case}\,\{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1 \,,\, \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{u}_1[\mathsf{x} \coloneqq \mathsf{v}]]}$$

Sem-eterm-PatR

$$\frac{}{\mathsf{C}[(\mathsf{Inr}\,\mathsf{v}) \succ \mathbf{case}\,\{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1 \,,\, \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{u}_2[\mathsf{x} \coloneqq \mathsf{v}]]}$$

Sem-eterm-PatP

$$\frac{}{\mathsf{C}[(\mathsf{v}_1 \,,\, \mathsf{v}_2) \succ \mathbf{case}\,(\mathsf{x}_1 \,,\, \mathsf{x}_2) \mapsto \mathsf{u}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{u}[\mathsf{x}_1 \coloneqq \mathsf{v}_1][\mathsf{x}_2 \coloneqq \mathsf{v}_2]]}$$

Sem-eterm-PatE

$$\frac{}{\mathsf{C}[\mathrm{E}^{n}\,\mathsf{v} \succ \mathbf{case}\;\mathrm{E}^{m}\,\mathsf{x} \mapsto \mathsf{u}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}]]}$$

Sem-eterm-MapOpen

$$\frac{}{\mathsf{C}[{}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle \succ \mathbf{map}\;\mathsf{x} \mapsto \mathsf{u}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{max(H)+h}}\; \mathsf{C}[{}^{o}_{\mathtt{H}\overset{\pm}{=}\mathtt{h}}\langle \mathsf{v}_1 \overset{\pm}{=}\mathtt{h} \,,\, \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_2 \overset{\pm}{=}\mathtt{h}] \rangle]}$$

Sem-eterm-MapClose

$$\frac{}{\mathsf{C}[{}^{o}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[{}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle]}$$

Sem-eterm-Alloc

$$\frac{}{\mathbf{alloc}_{\mathsf{T}} \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; {}_{\{\mathtt{1}\}}\langle +\mathtt{1} \,,\, -\mathtt{1} \rangle}$$

Sem-eterm-ToA

$$\frac{}{\mathsf{C}[\mathbf{to}_{\ltimes}\,\mathsf{v}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[{}_{\{\,\}}\langle \mathsf{v} \,,\, () \rangle]}$$

Sem-eterm-FromA

$$\frac{}{\mathsf{C}[\mathbf{from}_{\ltimes}\,{}_{\{\,\}}\langle \mathsf{v} \,,\, () \rangle] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{v}}$$

Sem-eterm-FillU

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft ()] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h}}\; \mathsf{C}[\mathtt{h} \coloneqq_{\{\,\}} ()][()]}$$

Sem-eterm-FillL

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft \mathsf{Inl}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h+1}}\; \mathsf{C}[\mathtt{h} \coloneqq_{\{\mathtt{h+1}\}} \mathsf{Inl}\; -(\mathtt{h+1})][+(\mathtt{h+1})]}$$

Sem-eterm-FillR

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft \mathsf{Inr}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h+1}}\; \mathsf{C}[\mathtt{h} \coloneqq_{\{\mathtt{h+1}\}} \mathsf{Inr}\; -(\mathtt{h+1})][+(\mathtt{h+1})]}$$

Sem-eterm-FillE

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft \mathrm{E}^{m}] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h+1}}\; \mathsf{C}[\mathtt{h} \coloneqq_{\{\mathtt{h+1}\}} \mathrm{E}^{m}\; -(\mathtt{h+1})][+(\mathtt{h+1})]}$$

Sem-eterm-FillP

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft (,)] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{h+2}}\; \mathsf{C}[\mathtt{h} \coloneqq_{\{\mathtt{h+1},\mathtt{h+2}\}} (-(\mathtt{h+1}) \,,\, -(\mathtt{h+2}))][(+(\mathtt{h+1}) \,,\, +(\mathtt{h+2}))]}$$

Sem-eterm-FillC

$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft\!\bullet\; {}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle] \;{}_{\mathtt{h}}{\longrightarrow}_{\mathtt{max(H)+h}}\; \mathsf{C}[\mathtt{h} \coloneqq_{(\mathtt{H}\overset{\pm}{=}\mathtt{h})} \mathsf{v}_1 \overset{\pm}{=}\mathtt{h}][\mathsf{v}_2 \overset{\pm}{=}\mathtt{h}]}$$