

metavariable, x, xs, y, uf, f, d

term, t, u ::=

- | x
- | z
- | $t\ u$
- | $t\ ;\ u$
- | $\text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}$
- | $\text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}$
- | $\text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}$
- | $\text{alloc } d . t$
- | $\text{[A]} \rightarrow A$
- | $t \triangleleft u$
- | $t \triangleleft 1.d' ; u$
- | $t \triangleleft 2.d' ; u$
- | $t \triangleleft \langle d_1, d_2 \rangle ; u$
- | $\star l$
- | $\text{fold } t$
- | [R]
- | $\text{unfold } t$
- | [R]
- | (t)
- | $\backslash n\ \text{sp } t\ \backslash n\ \text{spe}$
- | $\backslash n\ \text{sp } t$
- | $t[\text{var_subs}]$

term

- variable
- value
- application
- pattern-matching on unit
- pattern-matching on sum
- pattern-matching on product
- pattern-matching on exponentiated value
- allocate data
- fill terminal-type destination
- fill sum-type destination with variant 1
- fill sum-type destination with variant 2
- fill product-type destination
- note: $\star l$ is not a part of the user syntax

S
S
S
M

var_sub, vs

::=

- | $x := t$

variable substitution

var_subs

::=

- | vs
- | $vs, \text{var_subs}$

variable substitutions

data_val, v

::=

- | $()$
- | $\lambda x:A . t$
- | $\text{Ur } t$

data value

- unit
- lambda abstraction
- exponential

val, z

::=

- | v
- | $\otimes l$
- | [A]^l

unreducible value

note: $\otimes l$ is not a part of the user syntax
note: l is not a part of the user syntax

label, l

::=

label

label_stmt, s

::=

- | $l \hookrightarrow 1.l'$
- | $l \hookrightarrow 2.l'$
- | $l \hookrightarrow \langle l_1, l_2 \rangle$
- | $l \hookrightarrow \emptyset$
- | $l \hookrightarrow v$
- | $l \hookrightarrow C\bar{l}$
- | $l \hookrightarrow \dots$

M
M

TODO: hide. $l \hookrightarrow C\bar{l}$ is an alias for any heap co

label_stmts	$::=$ $\begin{array}{ l} s \\ s, \text{label_stmts} \end{array}$	label statements
heap_context, \mathbb{H}	$::=$ $\begin{array}{ l} \emptyset \\ \{\text{label_stmts}\} \\ \mathbb{H}_1 \sqcup \mathbb{H}_2 \end{array}$	label statements
type, A, B	$::=$ $\begin{array}{ l} 1 \\ R \\ A \otimes B \\ A \oplus B \\ A \multimap B \\ [A] \\ !A \\ (A) \\ W[r := A] \end{array}$	unit type recursive type bound to a name product type sum type linear function type destination type exponential S M
type_with_hole, W	$::=$ $\begin{array}{ l} r \\ 1 \\ R \\ W_1 \otimes W_2 \\ W_1 \oplus W_2 \\ W_1 \multimap W_2 \\ [W] \\ !W \\ (W) \end{array}$	type hole in recursive definition unit type recursive type bound to a name product type sum type linear function type destination type exponential S
rec_type_bound, R	$::=$	recursive type bound to a name
rec_type_def	$::=$ $\begin{array}{ l} \mu r. W \end{array}$	
type_affect, ta	$::=$ $\begin{array}{ l} x : A \\ l : A \\ \bar{l} : \bar{A} \end{array}$	type affectation var label labels
type_affects	$::=$ $\begin{array}{ l} \text{ta} \\ \text{ta}, \text{type_affects} \end{array}$	type affectations
typing_context, $\Gamma, \Delta, \mathcal{U}, \Phi$	$::=$ $\begin{array}{ l} \emptyset \\ \{\text{type_affects}\} \\ \Gamma \sqcup \Delta \end{array}$	typing context

types, \bar{A}	$::=$ \bullet A A types	empty type list
heap_constructor, C	$::=$ $(1.)$ $(2.)$ (\langle, \rangle)	
judg	$::=$ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H}$ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A$ $\Phi ; \mathcal{U} ; \Gamma \vdash t : A$ $C : \bar{A} \xrightarrow{c} A$ $A = B$ $t = u$ $\Gamma = \Delta$ $\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'$ $\text{type_affect} \in \Gamma$ $\text{label_stmt} \in \mathbb{H}$ $\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset$	
terminals	$::=$ \mapsto \star \otimes \oplus \dashv $:=$ \vdash \sqcup $;$ \cap \emptyset \longrightarrow \triangleright \neq \in \notin $\backslash n$ \langle \rangle $1.$ $2.$ Ur \hookrightarrow $ $ \oslash \xrightarrow{c}	

	$ \begin{array}{ l} = \\ \Downarrow \\ \dots \\ \text{fix} \\ = \end{array} $	
formula	$ \begin{array}{ l} ::= \\ \text{judgement} \end{array} $	
Ctx	$ \begin{array}{ l} ::= \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \\ \text{type_affect} \in \Gamma \end{array} $	Γ and Δ are disjoint typing contexts with no clashing
Heap	$ \begin{array}{ l} ::= \\ \text{label_stmt} \in \mathbb{H} \end{array} $	
Eq	$ \begin{array}{ l} ::= \\ A = B \\ t = u \\ \Gamma = \Delta \end{array} $	
Ty	$ \begin{array}{ l} ::= \\ R \stackrel{\text{fix}}{=} \text{rec_type_def} \\ \Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \\ \Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A \\ \Phi ; \mathcal{U} ; \Gamma \vdash t : A \\ C : \bar{A} \stackrel{c}{\hookrightarrow} A \end{array} $	\mathbb{H} is a well-typed heap given heap typing context Φ , u t is a well-typed term of type A given heap typing con Heap constructor C builds a value of type A given arg
Sem	$ \begin{array}{ l} ::= \\ \mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t' \\ \mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t' \end{array} $	t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}' t reduces to t' , with heap growing from \mathbb{H} to \mathbb{H}'
judgement	$ \begin{array}{ l} ::= \\ \text{Ctx} \\ \text{Heap} \\ \text{Eq} \\ \text{Ty} \\ \text{Sem} \end{array} $	
user_syntax	$ \begin{array}{ l} ::= \\ \text{metavariable} \\ \text{term} \\ \text{var_sub} \\ \text{var_subs} \\ \text{data_val} \\ \text{val} \\ \text{label} \\ \text{label_stmt} \\ \text{label_stmts} \\ \text{heap_context} \end{array} $	

| type
 | type_with_hole
 | rec_type_bound
 | rec_type_def
 | type_affect
 | type_affects
 | typing_context
 | types
 | heap_constructor
 | judg
 | terminals

$$\boxed{\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset}$$

Γ and Δ are disjoint typing contexts with no clashing variable names or labels

$$\boxed{\text{type_affect} \in \Gamma}$$

$$\boxed{\text{label_stmt} \in \mathbb{H}}$$

$$\boxed{A = B}$$

$$\boxed{t = u}$$

$$\boxed{\Gamma = \Delta}$$

$$\boxed{R \stackrel{\text{fix}}{=} \text{rec_type_def}}$$

$$\boxed{\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H}}$$

\mathbb{H} is a well-typed heap given heap typing context Φ , unrestricted typing context \mathcal{U} and linear

$$\frac{\begin{array}{l} \Phi_1 ; \mathcal{U} ; \Gamma \vdash \mathbb{H}_1 \\ \Phi_2 ; \mathcal{U} ; \Delta \vdash \mathbb{H}_2 \\ \text{names}(\Phi_1) \cap \text{names}(\Phi_2) = \emptyset \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi_1 \sqcup \Phi_2 ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \mathbb{H}_1 \sqcup \mathbb{H}_2} \text{TYHEAP_UNION}$$

$$\frac{\begin{array}{l} C : \bar{A} \stackrel{c}{\hookrightarrow} A \\ \Phi \sqcup \{\bar{l} : \bar{A}\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \end{array}}{\Phi \sqcup \{\bar{l} : \bar{A}, l : A\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}\}} \text{TYHEAP_CTOR}$$

$$\frac{}{\{\bar{l} : A\} ; \mathcal{U} ; \emptyset \vdash \{l \hookrightarrow \emptyset\}} \text{TYHEAP_NULL}$$

$$\frac{\Phi ; \mathcal{U} ; \Gamma \vdash v : A}{\Phi \sqcup \{l : A\} ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \sqcup \{l \hookrightarrow v\}} \text{TYHEAP_VAL}$$

$$\boxed{\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A}$$

$$\frac{\begin{array}{l} \Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \\ \Phi ; \mathcal{U} ; \Delta \vdash t : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A} \text{TYCOMMAND_DEF}$$

$$\boxed{\Phi ; \mathcal{U} ; \Gamma \vdash t : A}$$

t is a well-typed term of type A given heap typing context Φ , unrestricted typing context

$$\frac{}{\Phi ; \mathcal{U} ; \{x : A\} \vdash x : A} \text{TYTERM_ID}$$

$$\frac{}{\Phi ; \mathcal{U} \sqcup \{x : A\} ; \emptyset \vdash x : A} \text{TYTERM_ID'}$$

$$\frac{}{\Phi ; \mathcal{U} ; \emptyset \vdash () : 1} \text{TYTERM_UNIT}$$

$$\begin{array}{c}
\frac{\Phi ; \mathcal{U} ; \emptyset \vdash t : A}{\Phi ; \mathcal{U} ; \emptyset \vdash \text{Ur } t : !A} \quad \text{TYTERM_EXP} \\
\\
\frac{}{\Phi ; \mathcal{U} ; \emptyset \vdash \textcolor{red}{l}_{[A]} : [A]} \quad \text{TYTERM_LABELASDEST} \\
\\
\frac{}{\Phi \sqcup \{\textcolor{red}{l} : A\} ; \mathcal{U} ; \emptyset \vdash \textcolor{red}{\oplus} \textcolor{red}{l} : A} \quad \text{TYTERM_DEREFVAL} \\
\\
\frac{}{\Phi \sqcup \{\textcolor{red}{l} : A\} ; \mathcal{U} ; \emptyset \vdash \textcolor{red}{\star} \textcolor{red}{l} : A} \quad \text{TYTERM_DEREF} \\
\\
\frac{\begin{array}{c} R \stackrel{\text{fix}}{=} \mu r . W \\ \Phi ; \mathcal{U} ; \Gamma \vdash t : W[r := R] \end{array}}{\Phi ; \mathcal{U} ; \Gamma \vdash \text{fold } t : R} \quad \text{TYTERM_FOLD} \\
\\
\frac{\begin{array}{c} R \stackrel{\text{fix}}{=} \mu r . W \\ \Phi ; \mathcal{U} ; \Gamma \vdash t : R \end{array}}{\Phi ; \mathcal{U} ; \Gamma \vdash \text{unfold } t : W[r := R]} \quad \text{TYTERM_UNFOLD} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \sqcup \{x : A\} \vdash t : B \\ \Phi ; \mathcal{U} ; \Gamma \vdash \lambda x : A . t : A \multimap B \end{array}}{} \quad \text{TYTERM_LAM} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A \multimap B \\ \Phi ; \mathcal{U} ; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t u : B} \quad \text{TYTERM_APP} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : 1 \\ \Phi ; \mathcal{U} ; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t ; u : A} \quad \text{TYTERM_PATU} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \otimes A_2 \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1, x_2 : A_2\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} : B} \quad \text{TYTERM_PATP} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \oplus A_2 \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1\} \vdash u_1 : B \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_2 : A_2\} \vdash u_2 : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} : B} \quad \text{TYTERM_PATs} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : !A \\ \Phi ; \mathcal{U} \sqcup \{x : A\} ; \Delta \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} : B} \quad \text{TYTERM_PATE} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \sqcup \{d : [A]\} \vdash t : 1 \\ \Phi ; \mathcal{U} ; \Gamma \vdash \text{alloc } d . t : A \end{array}}{} \quad \text{TYTERM_ALLOC} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : [A] \\ \Phi ; \mathcal{U} ; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft u : 1} \quad \text{TYTERM_FILLL}
\end{array}$$

$$\begin{array}{c}
\frac{\begin{array}{l} \Phi ; \mathcal{U} ; \Gamma \vdash t : [A_1 \oplus A_2] \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{d' : [A_1]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft 1.d' ; u : B} \text{TYTERM_FILLV1} \\
\\
\frac{\begin{array}{l} \Phi ; \mathcal{U} ; \Gamma \vdash t : [A_1 \oplus A_2] \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{d' : [A_2]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \vdash t \triangleleft 2.d' ; u : B} \text{TYTERM_FILLV2} \\
\\
\frac{\begin{array}{l} \Phi ; \mathcal{U} ; \Gamma \vdash t : [A_1 \otimes A_2] \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{d_1 : [A_1], d_2 : [A_2]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t \triangleleft \langle d_1, d_2 \rangle ; u : B} \text{TYTERM_FILLP}
\end{array}$$

$\boxed{C : \bar{A} \multimap A}$ Heap constructor C builds a value of type A given arguments of type \bar{A}

$$\begin{array}{c}
\frac{}{(1.) : A \multimap A \oplus B} \text{TYCTOR_V1} \\
\\
\frac{}{(2.) : B \multimap A \oplus B} \text{TYCTOR_V2} \\
\\
\frac{}{(\langle, \rangle) : A \multimap B \multimap A \otimes B} \text{TYCTOR_PAIR}
\end{array}$$

$\boxed{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}$ t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}'

$$\begin{array}{c}
\frac{}{\mathbb{H} \mid \text{fold}_{\mathcal{R}} t \longrightarrow \mathbb{H} \mid t} \text{SEMMUT_FOLD} \\
\\
\frac{}{\mathbb{H} \mid \text{unfold}_{\mathcal{R}} t \longrightarrow \mathbb{H} \mid t} \text{SEMMUT_UNFOLD} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t u \longrightarrow \mathbb{H}' \mid t' u} \text{SEMMUT_UAPP} \\
\\
\frac{}{\mathbb{H} \mid (\lambda x:A. t) u \longrightarrow \mathbb{H} \mid t[x := u]} \text{SEMMUT_APP} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t ; u \longrightarrow \mathbb{H}' \mid t' ; u} \text{SEMMUT_UPATU} \\
\\
\frac{}{\mathbb{H} \mid () ; u \longrightarrow \mathbb{H} \mid u} \text{SEMMUT_PATU} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow v\} \mid \star l \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow v\} \mid v} \text{SEMMUT_DEREFVAL} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}\} \mid \star l \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}\} \mid \star l} \text{SEMMUT_DEREFCOR} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}} \text{SEMMUT_UPATS} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow 1.l'\} \mid \text{case } \star l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow 1.l'\} \mid u_1[x_1 := \star l']} \text{SEMMUT_PATSV1} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow 2.l'\} \mid \text{case } \star l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow 2.l'\} \mid u_2[x_2 := \star l']} \text{SEMMUT_PATSV2} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}} \text{SEMMUT_UPATP}
\end{array}$$

$$\begin{array}{c}
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \langle l_1, l_2 \rangle\} \mid \text{case } \oplus l \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow \langle l_1, l_2 \rangle\} \mid u[x_1 := \star l_1, x_2 := \star l_2]} \quad \text{SEMMUT_PATP} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UPATE} \\
\overline{\mathbb{H} \mid \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{Ur } x \mapsto u\}} \\
\overline{\mathbb{H} \mid \text{case Ur } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := t]} \quad \text{SEMMUT_PATE} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UFILL} \\
\overline{\mathbb{H} \mid t \triangleleft u \longrightarrow \mathbb{H}' \mid t' \triangleleft u} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UFILL}' \\
\overline{\mathbb{H} \mid \underline{A} \triangleleft t \longrightarrow \mathbb{H}' \mid \underline{A} \triangleleft t'} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UFILLV1} \\
\overline{\mathbb{H} \mid t \triangleleft 1.d' ; u \longrightarrow \mathbb{H}' \mid t' \triangleleft 1.d' ; u} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UFILLV2} \\
\overline{\mathbb{H} \mid t \triangleleft 2.d' ; u \longrightarrow \mathbb{H}' \mid t' \triangleleft 2.d' ; u} \\
\\
\overline{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UFILLP} \\
\overline{\mathbb{H} \mid t \triangleleft \langle d_1, d_2 \rangle ; u \longrightarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2 \rangle ; u} \\
\\
\overline{\mathbb{H} \mid \text{alloc } \underline{d}.t \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid t[\underline{d} := \underline{A}] ; \star l} \quad \text{SEMMUT_ALLOC} \\
\overline{\mathbb{H} \mid \text{alloc } \underline{d}.t \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid t[\underline{d} := \underline{A}] ; \star l} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid \underline{A} \triangleleft v \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow v\} \mid ()} \quad \text{SEMMUT_FILLV} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \circ, l' \hookrightarrow C\bar{l}\} \mid \underline{A} \triangleleft \oplus l' \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}, l' \hookrightarrow C\bar{l}\} \mid ()} \quad \text{SEMMUT_FILLCTOR} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid \underline{A_1 \oplus A_2} \triangleleft 1.d' ; t \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow 1.l', l' \hookrightarrow \circ\} \mid t[\underline{d}' := \underline{A_1}]} \quad \text{SEMMUT_FILLV1} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid \underline{A_1 \oplus A_2} \triangleleft 2.d' ; t \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow 2.l', l' \hookrightarrow \circ\} \mid t[\underline{d}' := \underline{A_2}]} \quad \text{SEMMUT_FILLV2} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow \circ\} \mid \underline{A_1 \otimes A_2} \triangleleft \langle d_1, d_2 \rangle ; t \longrightarrow \mathbb{H} \sqcup \{l \hookrightarrow \langle l_1, l_2 \rangle, l_1 \hookrightarrow \circ, l_2 \hookrightarrow \circ\} \mid t[\underline{d}_1 := \underline{A_1}, \underline{d}_2 := \underline{A_2}]} \quad \text{SEMMUT.} \\
\\
\boxed{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'} \quad t \text{ reduces to } t', \text{ with heap growing from } \mathbb{H} \text{ to } \mathbb{H}' \\
\\
\overline{\mathbb{H} \mid \text{fold } t \Downarrow \mathbb{H} \mid t} \quad \text{SEMMUT_FOLD} \\
\overline{\mathbb{H} \mid \text{unfold } t \Downarrow \mathbb{H} \mid t} \quad \text{SEMMUT_UNFOLD} \\
\\
\overline{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UAPP} \\
\overline{\mathbb{H} \mid t u \Downarrow \mathbb{H}' \mid t' u} \\
\\
\overline{\mathbb{H} \mid (\lambda x:\underline{A}.t) u \Downarrow \mathbb{H} \mid t[x := u]} \quad \text{SEMMUT_APP} \\
\\
\overline{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'} \quad \text{SEMMUT_UPATU} \\
\overline{\mathbb{H} \mid t ; u \Downarrow \mathbb{H}' \mid t' ; u} \\
\\
\overline{\mathbb{H} \mid () ; u \Downarrow \mathbb{H} \mid u} \quad \text{SEMMUT_PATU} \\
\\
\overline{\mathbb{H} \sqcup \{l \hookrightarrow v\} \mid \star l \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow v\} \mid v} \quad \text{SEMMUT_DEREFVAL}
\end{array}$$

$$\begin{array}{c}
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}\} \mid \star l \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}\} \mid \otimes l} \text{SEMIMM_DEREFCTOR} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}} \text{SEMIMM_UPATS} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow 1.l'\} \mid \text{case } \otimes l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow 1.l'\} \mid u_1[x_1 := \star l']} \text{SEMIMM_PATSV1} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow 2.l'\} \mid \text{case } \otimes l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow 2.l'\} \mid u_2[x_2 := \star l']} \text{SEMIMM_PATSV2} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}} \text{SEMIMM_UPATP} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \hookrightarrow \langle l_1, l_2 \rangle\} \mid \text{case } \otimes l \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow \langle l_1, l_2 \rangle\} \mid u[x_1 := \star l_1, x_2 := \star l_2]} \text{SEMIMM_PATP} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} \Downarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{Ur } x \mapsto u\}} \text{SEMIMM_UPATE} \\
\\
\frac{}{\mathbb{H} \mid \text{case Ur } t \text{ of } \{\text{Ur } x \mapsto u\} \Downarrow \mathbb{H} \mid u[x := t]} \text{SEMIMM_PATE} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft u \Downarrow \mathbb{H}' \mid t' \triangleleft u} \text{SEMIMM_UFILL} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \underline{[A]} \triangleleft t \Downarrow \mathbb{H}' \mid \underline{[A]} \triangleleft t'} \text{SEMIMM_UFILL'} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 1.d' ; u \Downarrow \mathbb{H}' \mid t' \triangleleft 1.d' ; u} \text{SEMIMM_UFILLV1} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 2.d' ; u \Downarrow \mathbb{H}' \mid t' \triangleleft 2.d' ; u} \text{SEMIMM_UFILLV2} \\
\\
\frac{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \langle d_1, d_2 \rangle ; u \Downarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2 \rangle ; u} \text{SEMIMM_UFILLP} \\
\\
\frac{\mathbb{H} \mid t[d := \underline{[A]}] \Downarrow \mathbb{H}' \sqcup \{l \hookrightarrow \dots\} \mid ()}{\mathbb{H} \mid \text{alloc } d \text{ t} \Downarrow \mathbb{H}' \sqcup \{l \hookrightarrow \dots\} \mid \star l} \text{SEMIMM_ALLOC} \quad \text{--- Changes start from there ---} \\
\\
\frac{}{\mathbb{H} \mid \underline{[A]} \triangleleft v \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow v\} \mid ()} \text{SEMIMM_FILLV} \\
\\
\frac{}{\mathbb{H} \sqcup \{l' \hookrightarrow C\bar{l}\} \mid \underline{[A]} \triangleleft \otimes l' \Downarrow \mathbb{H} \sqcup \{l \hookrightarrow C\bar{l}, l' \hookrightarrow C\bar{l}\} \mid ()} \text{SEMIMM_FILLCTOR} \\
\\
\frac{\mathbb{H} \mid t[d' := \underline{[A_1]}] \Downarrow \mathbb{H}' \sqcup \{l' \hookrightarrow \dots\} \mid ()}{\mathbb{H} \mid \underline{[A_1 \oplus A_2]} \triangleleft 1.d' ; t \Downarrow \mathbb{H}' \sqcup \{l' \hookrightarrow \dots, l \hookrightarrow 1.l'\} \mid ()} \text{SEMIMM_FILLV1} \\
\\
\frac{\mathbb{H} \mid t[d' := \underline{[A_2]}] \Downarrow \mathbb{H}' \sqcup \{l' \hookrightarrow \dots\} \mid ()}{\mathbb{H} \mid \underline{[A_1 \oplus A_2]} \triangleleft 2.d' ; t \Downarrow \mathbb{H}' \sqcup \{l' \hookrightarrow \dots, l \hookrightarrow 2.l'\} \mid ()} \text{SEMIMM_FILLV2} \\
\\
\frac{\mathbb{H} \mid t[d_1 := \underline{[A_1]}, d_2 := \underline{[A_2]}] \Downarrow \mathbb{H}' \sqcup \{l_1 \hookrightarrow \dots, l_2 \hookrightarrow \dots\} \mid ()}{\mathbb{H} \sqcup \{l \hookrightarrow \odot\} \mid \underline{[A_1 \otimes A_2]} \triangleleft \langle d_1, d_2 \rangle ; t \Downarrow \mathbb{H}' \sqcup \{l_1 \hookrightarrow \dots, l_2 \hookrightarrow \dots, l \hookrightarrow \langle l_1, l_2 \rangle\} \mid ()} \text{SEMIMM_FILLP}
\end{array}$$

Definition rules: 80 good 0 bad
 Definition rule clauses: 149 good 0 bad