Destination λ-calculus

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1 Term and value syntax

var, x, y	Term-level variable name
k	Index for ranges

$\mathtt{hdn}$, $\mathtt{h}$	::=			Hole or destination name ($\mathbb{N}$)
	|	$\mathtt{h}+\mathtt{h}'$	M	Sum
	|	$\mathtt{max}(\mathtt{H})$	M	Maximum of a set of holes

$\mathtt{hdns}$, $\mathtt{H}$	::=			Set of hole names
	|	$\{\mathtt{h}_1, \ldots, \mathtt{h}_{\mathsf{k}}\}$		
	|	$\mathtt{H}_1 \cup \mathtt{H}_2$	M	Union of sets
	|	$\mathtt{H}\underline{\pm}\mathtt{h}$	M	Increase all names from $\mathtt{H}$ by $\mathtt{h}$.
	|	$\mathtt{hnames}(\Gamma)$	M	Hole names of a context (requires $\mathtt{ctx\_NoVar}(\Gamma)$)
	|	$\mathtt{hnames}(\mathsf{C})$	M	Hole names of an evaluation context

term, t, u	::=			Term
	|	v		Value
	|	x		Variable
	|	$\mathsf{t} \succ \mathsf{u}$		Application
	|	$\mathsf{t}\ ;\ \mathsf{u}$		Pattern-match on unit
	|	$\mathsf{t} \succ \mathbf{case}_{m}\ \{\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\ ,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\}$	bind x_1 in u_1 bind x_2 in u_2	Pattern-match on sum
	|	$\mathsf{t} \succ \mathbf{case}_{m}\ (\mathsf{x}_1\ ,\ \mathsf{x}_2) \mapsto \mathsf{u}$	bind x_1 in u bind x_2 in u	Pattern-match on product
	|	$\mathsf{t} \succ \mathbf{case}_{m}\ \mathrm{E}^{n}\,\mathsf{x} \mapsto \mathsf{u}$	bind x in u	Pattern-match on exponential
	|	$\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}$	bind x in u	Map over the right side of ampar t
	|	$\mathbf{to}_{\ltimes}\ \mathsf{t}$		Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ \mathsf{t}$		Extract value from trivial ampar
	|	$\mathbf{alloc}$		Return a fresh "identity" ampar object
	|	$\mathsf{t} \triangleleft ()$		Fill destination with unit
	|	$\mathsf{t} \triangleleft (\lambda \mathsf{x}\ {}_{m} \mapsto \mathsf{u})$	bind x in u	Fill destination with function
	|	$\mathsf{t} \triangleleft \mathsf{Inl}$		Fill destination with left variant
	|	$\mathsf{t} \triangleleft \mathsf{Inr}$		Fill destination with right variant
	|	$\mathsf{t} \triangleleft (,)$		Fill destination with product constructor
	|	$\mathsf{t} \triangleleft \mathrm{E}^{m}$		Fill destination with exponential constructor
	|	$\mathsf{t} \triangleleft\!\bullet\ \mathsf{u}$		Fill destination with root of ampar u
	|	$\mathsf{t}[\mathsf{x} := \mathsf{v}]$	M	

val, v	::=			Term value
	|	$-\mathtt{h}$		Hole
	|	$+\mathtt{h}$		Destination
	|	$()$		Unit
	|	$\lambda^{\mathsf{v}}\mathsf{x}\ {}_{m} \mapsto \mathsf{t}$	bind x in t	Lambda abstraction
	|	$\mathsf{Inl}\,\mathsf{v}$		Left variant for sum
	|	$\mathsf{Inr}\,\mathsf{v}$		Right variant for sum
	|	$\mathrm{E}^{m}\,\mathsf{v}$		Exponential
	|	$(\mathsf{v}_1\ ,\ \mathsf{v}_2)$		Product
	|	${}_{\mathtt{H}}\langle \mathsf{v}_1 \,{}_{9}\, \mathsf{v}_2 \rangle$		Ampar
	|	$\mathsf{v}\underline{\pm}\mathtt{h}$	M	Rename hole names inside v by shifting them by $\mathtt{h}$

eterm, j	::=			Pseudo-term
	|	$C[t]$		

ectx, C	::=			Evaluation context
	|	$\square$		Identity
	|	$C \succ u$		Application
	|	$v \succ C$		Application
	|	$C \mathbin{;} u$		Pattern-match on unit
	|	$C \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 , \mathsf{Inr}\, x_2 \mapsto u_2 \}$	bind x_1 in u_1 bind x_2 in u_2	Pattern-match on sum
	|	$C \succ \mathbf{case}_{m} (x_1 , x_2) \mapsto u$	bind x_1 in u bind x_2 in u	Pattern-match on product
	|	$C \succ \mathbf{case}_{m}\ \mathrm{E}^{n}\, x \mapsto u$	bind x in u	Pattern-match on exponential
	|	$C \succ \mathbf{map}\ x \mapsto u$	bind x in u	Map over the right side of ampar
	|	$\mathbf{to}_{\ltimes}\ C$		Wrap into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ C$		Extract value from trivial ampar
	|	$C \triangleleft ()$		Fill destination with unit
	|	$C \triangleleft (\lambda x\ {}_{m} \mapsto u)$	bind x in u	Fill destination with function
	|	$C \triangleleft \mathsf{Inl}$		Fill destination with left variant
	|	$C \triangleleft \mathsf{Inr}$		Fill destination with right variant
	|	$C \triangleleft (,)$		Fill destination with product constructor
	|	$C \triangleleft \mathrm{E}^{m}$		Fill destination with exponential constructor
	|	$C \triangleleft\!\bullet u$		Fill destination with root of ampar
	|	$v \triangleleft\!\bullet C$		Fill destination with root of ampar
	|	${}^{\mathrm{op}}_{\mathrm{H}}\langle v_1 \,{}_{9}\, C$		Open ampar. Only new addition to term shapes
	|	$C \circ C'$	M	Compose evaluation contexts
	|	$C[h \coloneqq_{\mathrm{H}} v]$	M	Fill h with value v (that may contain holes)

2 Type system

type, T, U	::=			Type
	|	1		Unit
	|	$T_1 \oplus T_2$		Sum
	|	$T_1 \otimes T_2$		Product
	|	$!^{m}\, T$		Exponential
	|	$T_1 \ltimes T_2$		Ampar type (consuming T_2 yields T_1)
	|	$T_1\ {}_{m_1}\!\!\to T_2$		Function
	|	$\lfloor T \rfloor^{m}$		Destination

mode, m, n	::=			Mode (Semiring)
	|	pa		Pair of a multiplicity and age
	|	☠		Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	M	Semiring product

mul, p	::=			Multiplicity (first component of modality)
	|	1		Linear. Neutral element of the product
	|	ω		Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	M	Semiring product

age, a	::=			Age (second component of modality)
	|	ν		Born now. Neutral element of the product
	|	$\uparrow$		One scope older
	|	∞		Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	M	Semiring product

bndr, b	::=			Type assignment to either variable, destination or hole
	|	$x :_{m} T$		Variable
	|	$+h :_{m} \lfloor T \rfloor^{n}$		Destination (m is its own modality; n is the modality for values it accepts)
	|	$-h : T^{n}$		Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

ctx, Γ, Δ	::=			Typing context
	|	$\{\mathsf{b}_1, .., \mathsf{b}_k\}$		List of bindings
	|	$m{\cdot}\Gamma$	M	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	M	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mod
	|	$-\Gamma$	M	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)

$\boxed{\Gamma \Vdash \mathsf{v} : \mathsf{T}}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \frac{}{\{-h : \mathsf{T}^{1\nu}\} \Vdash -h : \mathsf{T}}$$

$$\textsc{TyR-val-D}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ +h :_{1\nu} \lfloor \mathsf{T} \rfloor^{n}}{\Gamma \Vdash +h : \lfloor \mathsf{T} \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \frac{}{\{\,\} \Vdash () : \mathsf{1}}$$

$$\textsc{TyR-val-F}\quad \frac{\begin{array}{c}\texttt{ctx\_DestOnly}\ \Gamma \\ \Gamma \uplus \{x :_{m} \mathsf{T}_1\} \vdash t : \mathsf{T}_2\end{array}}{\Gamma \Vdash \lambda^{\mathsf{v}} x\ {}_{m} \mapsto t : \mathsf{T}_1\ {}_{m}\!\!\rightarrow \mathsf{T}_2}$$

$$\textsc{TyR-val-L}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\, \mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-R}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\, \mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-P}\quad \frac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-val-E}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{n{\cdot}\Gamma \Vdash \mathrm{E}^{n}\, \mathsf{v} : !^{n}\, \mathsf{T}}$$

$$\textsc{TyR-val-A}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_DestOnly}\ \Gamma_2 \uplus \Gamma_3 \\ \texttt{ctx\_DestOnly}\ \Gamma_1 \\ \Gamma_1 \uplus (-\Gamma_3) \Vdash \mathsf{v}_1 : \mathsf{T}_1 \\ \Gamma_2 \uplus \Gamma_3 \Vdash \mathsf{v}_2 : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash {}_{\texttt{hnames}(-\Gamma_3)}\langle \mathsf{v}_1\,,\, \mathsf{v}_2 \rangle : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$\boxed{\Gamma \vdash j : \mathsf{T}}$ *(Typing of (extended) terms)*

$$\textsc{Ty-eterm-Val}\quad \frac{\texttt{ctx\_NoHole}\ \Gamma \qquad \Gamma \Vdash \mathsf{v} : \mathsf{T}}{\Gamma \vdash \mathsf{v} : \mathsf{T}}$$

$$\textsc{Ty-eterm-Var}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ x :_{1\nu} \mathsf{T}}{\Gamma \vdash x : \mathsf{T}}$$

$$\textsc{Ty-eterm-App}\quad \frac{\Gamma_1 \vdash t : \mathsf{T}_1 \qquad \Gamma_2 \vdash u : \mathsf{T}_1\ {}_{m}\!\!\rightarrow \mathsf{T}_2}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash t \succ u : \mathsf{T}_2}$$

$$\textsc{Ty-eterm-PatU}\quad \frac{\Gamma_1 \vdash t : \mathsf{1} \qquad \Gamma_2 \vdash u : \mathsf{U}}{\Gamma_1 \uplus \Gamma_2 \vdash t\, ;\, u : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatS}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \Gamma_1 \vdash t : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1\} \vdash u_1 : \mathsf{U} \\ \Gamma_2 \uplus \{x_2 :_{m} \mathsf{T}_2\} \vdash u_2 : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_{m}\,\{\, \mathsf{Inl}\, x_1 \mapsto u_1\,,\ \mathsf{Inr}\, x_2 \mapsto u_2 \,\} : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatP}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} \mathsf{T}_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \texttt{ctx\_Disjoint}\ \ \{x_1 :_{m} \mathsf{T}_1\}\ \ \{x_2 :_{m} \mathsf{T}_2\} \\ \Gamma_1 \vdash t : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Gamma_2 \uplus \{x_1 :_{m} \mathsf{T}_1, x_2 :_{m} \mathsf{T}_2\} \vdash u : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_{m}\,(x_1\,,\, x_2) \mapsto u : \mathsf{U}}$$

$$\textsc{Ty-eterm-PatE}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m \cdot n} \mathsf{T}\} \\ \Gamma_1 \vdash t : !^{n}\, \mathsf{T} \\ \Gamma_2 \uplus \{x :_{m \cdot n} \mathsf{T}\} \vdash u : \mathsf{U}\end{array}}{m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_{m}\, \mathrm{E}^{n}\, x \mapsto u : \mathsf{U}}$$

$$\textsc{Ty-eterm-Map}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{1\nu} \mathsf{T}_2\} \\ \Gamma_1 \vdash t : \mathsf{T}_1 \ltimes \mathsf{T}_2 \\ 1{\uparrow}{\cdot}\Gamma_2 \uplus \{x :_{1\nu} \mathsf{T}_2\} \vdash u : \mathsf{U}\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{map}\ x \mapsto u : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{Ty-eterm-FillU}\quad \frac{\Gamma \vdash t : \lfloor \mathsf{1} \rfloor^{n}}{\Gamma \vdash t \triangleleft () : \mathsf{1}}$$

$$\textsc{Ty-eterm-FillF}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m} \mathsf{T}_1\} \\ \Gamma_1 \vdash t : \lfloor \mathsf{T}_1\ {}_{m}\!\!\rightarrow \mathsf{T}_2 \rfloor^{n} \\ \Gamma_2 \uplus \{x :_{m} \mathsf{T}_1\} \vdash u : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Gamma_2 \vdash t \triangleleft (\lambda x\ {}_{m} \mapsto u) : \mathsf{1}}$$

$$\textsc{Ty-eterm-FillL}\quad \frac{\Gamma \vdash t : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathsf{Inl} : \lfloor \mathsf{T}_1 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillR}\quad \frac{\Gamma \vdash t : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathsf{Inr} : \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillP}\quad \frac{\Gamma \vdash t : \lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor^{n}}{\Gamma \vdash t \triangleleft (,) : \lfloor \mathsf{T}_1 \rfloor^{n} \otimes \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-eterm-FillE}\quad \frac{\Gamma \vdash t : \lfloor !^{n'}\, \mathsf{T} \rfloor^{n}}{\Gamma \vdash t \triangleleft \mathrm{E}^{n'} : \lfloor \mathsf{T} \rfloor^{n' \cdot n}}$$

$$\textsc{Ty-eterm-FillC}\quad \frac{\Gamma_1 \vdash t : \lfloor \mathsf{T}_1 \rfloor^{n} \qquad \Gamma_2 \vdash u : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Gamma_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Gamma_2 \vdash t \triangleleft\!\bullet u : \mathsf{T}_2}$$

$$\textsc{Ty-eterm-Alloc}\quad \frac{}{\{\,\} \vdash \mathbf{alloc} : \mathsf{T} \ltimes \lfloor \mathsf{T} \rfloor^{1\nu}}$$

$$\textsc{Ty-eterm-ToA}\quad \frac{\Gamma \vdash t : \mathsf{T}}{\Gamma \vdash \mathbf{to}_{\ltimes}\ t : \mathsf{T} \ltimes \mathsf{1}}$$

$$\textsc{Ty-eterm-FromA}\quad \frac{\Gamma \vdash t : \mathsf{T} \ltimes \mathsf{1}}{\Gamma \vdash \mathbf{from}_{\ltimes}\ t : \mathsf{T}}$$

$\boxed{\Gamma \Vdash C : \mathsf{T}_1 \rightarrowtail \mathsf{T}_2}$ *(Typing of evaluation contexts)*

$$\textsc{TyR-ectx-T}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \Gamma_3 \\ \texttt{ctx\_NoVar}\ \Gamma_2 \uplus \Gamma_3 \\ \texttt{ctx\_NoVar}\ \Gamma_1 \\ \Gamma_2 \uplus \Gamma_3 \vdash t : \mathsf{T}_1 \qquad \Gamma_1 \vdash C[t] : \mathsf{T}_2\end{array}}{\Gamma_1 \uplus (-\Gamma_2) \Vdash C : \mathsf{T}_1 \rightarrowtail \mathsf{T}_2}$$

3 Small-step semantics

$\boxed{\mathsf{j} \longrightarrow \mathsf{j}'}$ *(Small-step evaluation of terms using evaluation contexts)*

Sem-eterm-App
$$\frac{}{\mathsf{C}[\mathsf{v} \succ (\lambda^{\mathsf{v}} \mathsf{x}_{\,m} \mapsto \mathsf{t})] \longrightarrow \mathsf{C}[\mathsf{t}[\mathsf{x} := \mathsf{v}]]}$$

Sem-eterm-PatU
$$\frac{}{\mathsf{C}[() \ ; \mathsf{t}_2] \longrightarrow \mathsf{C}[\mathsf{t}_2]}$$

Sem-eterm-PatL
$$\frac{}{\mathsf{C}[(\mathsf{Inl}\,\mathsf{v}) \succ \mathbf{case}_m \{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1 \,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\}] \longrightarrow \mathsf{C}[\mathsf{u}_1[\mathsf{x} := \mathsf{v}]]}$$

Sem-eterm-PatR
$$\frac{}{\mathsf{C}[(\mathsf{Inr}\,\mathsf{v}) \succ \mathbf{case}_m \{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1 \,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\}] \longrightarrow \mathsf{C}[\mathsf{u}_2[\mathsf{x} := \mathsf{v}]]}$$

Sem-eterm-PatP
$$\frac{}{\mathsf{C}[(\mathsf{v}_1 \,,\, \mathsf{v}_2) \succ \mathbf{case}_m (\mathsf{x}_1 \,,\, \mathsf{x}_2) \mapsto \mathsf{u}] \longrightarrow \mathsf{C}[\mathsf{u}[\mathsf{x}_1 := \mathsf{v}_1][\mathsf{x}_2 := \mathsf{v}_2]]}$$

Sem-eterm-PatE
$$\frac{}{\mathsf{C}[\mathrm{E}^{n}\,\mathsf{v} \succ \mathbf{case}_m \mathrm{E}^{n}\,\mathsf{x} \mapsto \mathsf{u}] \longrightarrow \mathsf{C}[\mathsf{u}[\mathsf{x} := \mathsf{v}]]}$$

Sem-eterm-MapOpen
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}))}{\mathsf{C}[{}_{\mathtt{H}}\langle \mathsf{v}_1 \,, \mathsf{v}_2 \rangle \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}] \longrightarrow (\mathsf{C} \circ ({}^{\mathrm{op}}_{\mathtt{H} \pm \mathtt{h}'}\langle \mathsf{v}_1 \pm \mathtt{h}' \,, \square \rangle))[\mathsf{u}[\mathsf{x} := \mathsf{v}_2 \pm \mathtt{h}']]}$$

Sem-eterm-MapClose
$$\frac{}{(\mathsf{C} \circ {}^{\mathrm{op}}_{\mathtt{H}}\langle \mathsf{v}_1 \,, \square \rangle)[\mathsf{v}_2] \longrightarrow \mathsf{C}[{}_{\mathtt{H}}\langle \mathsf{v}_1 \,, \mathsf{v}_2 \rangle]}$$

Sem-eterm-Alloc
$$\frac{}{\mathbf{alloc} \longrightarrow {}_{\{1\}}\langle +1 \,, -1 \rangle}$$

Sem-eterm-ToA
$$\frac{}{\mathsf{C}[\mathbf{to}_{\ltimes}\ \mathsf{v}] \longrightarrow \mathsf{C}[{}_{\{\,\}}\langle \mathsf{v} \,, () \rangle]}$$

Sem-eterm-FromA
$$\frac{}{\mathsf{C}[\mathbf{from}_{\ltimes}\ {}_{\{\,\}}\langle \mathsf{v} \,, () \rangle] \longrightarrow \mathsf{v}}$$

Sem-eterm-FillU
$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft ()] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\,\}} ()][()]}$$

Sem-eterm-FillF
$$\frac{}{\mathsf{C}[+\mathtt{h} \triangleleft (\lambda \mathsf{x}_{\,m} \mapsto \mathsf{u})] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\,\}} \lambda^{\mathsf{v}} \mathsf{x}_{\,m} \mapsto \mathsf{u}][()]}$$

Sem-eterm-FillL
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}) \cup \{\mathtt{h}\})}{\mathsf{C}[+\mathtt{h} \triangleleft \mathsf{Inl}] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\mathtt{h}'+1\}} \mathsf{Inl} -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillR
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}) \cup \{\mathtt{h}\})}{\mathsf{C}[+\mathtt{h} \triangleleft \mathsf{Inr}] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\mathtt{h}'+1\}} \mathsf{Inr} -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillE
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}) \cup \{\mathtt{h}\})}{\mathsf{C}[+\mathtt{h} \triangleleft \mathrm{E}^{m}] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\mathtt{h}'+1\}} \mathrm{E}^{m} -(\mathtt{h}'+1)][+(\mathtt{h}'+1)]}$$

Sem-eterm-FillP
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}) \cup \{\mathtt{h}\})}{\mathsf{C}[+\mathtt{h} \triangleleft (,)] \longrightarrow \mathsf{C}[\mathtt{h} :=_{\{\mathtt{h}'+1, \mathtt{h}'+2\}} (-(\mathtt{h}'+1) \,, -(\mathtt{h}'+2))][(+(\mathtt{h}'+1) \,, +(\mathtt{h}'+2))]}$$

Sem-eterm-FillC
$$\frac{\mathtt{h}' \ = \ \mathtt{max}(\mathtt{hnames}(\mathsf{C}) \cup \{\mathtt{h}\})}{\mathsf{C}[+\mathtt{h} \triangleleft\!\bullet {}_{\mathtt{H}}\langle \mathsf{v}_1 \,, \mathsf{v}_2 \rangle] \longrightarrow \mathsf{C}[\mathtt{h} :=_{(\mathtt{H} \pm \mathtt{h}')} \mathsf{v}_1 \pm \mathtt{h}'][\mathsf{v}_2 \pm \mathtt{h}']}$$