

<i>termvar</i> , <i>x</i> , <i>y</i> , <i>d</i>	Term-level variable	
<i>hole</i> , <i>h</i>	::=	Hole
<i>term_value</i> , <i>v</i>	::=	Term value
	$\langle v_1, \bar{v}_2 \rangle_H$	Ampar
	$@h$	Destination
	$()$	Unit
	$\text{Inl } v$	Left variant for sum
	$\text{Inr } v$	Right variant for sum
	(v_1, v_2)	Product
	$\lambda x. t$	Linear function
	(v)	S
$\overline{\text{extended_value}}$, \bar{v}	::=	Store value
	v	Term value
	h	Hole
	$\text{Inl } \bar{v}$	Left variant with val or hole
	$\text{Inr } \bar{v}$	Right variant with val or hole
	(\bar{v}_1, \bar{v}_2)	Product with val or hole
	(\bar{v})	S
	$\bar{v}[e]$	M
<i>term</i> , <i>t</i> , <i>u</i>	::=	Term
	v	Term value
	x	Variable
	$t \ u$	Application
	$t \succ \text{case } () \mapsto u$	Pattern-match on unit
	$t \succ \text{case } \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$	Pattern-match on sum
	$t \succ \text{case } (x_1, x_2) \mapsto u$	Pattern-match on product
	$t \succ \text{mapL } x \mapsto u$	Map over the left side of the ampar
	$\text{to}_x t$	Wrap t into a trivial ampar
	$\text{from}_x t$	Extract value from trivial ampar
	alloc_A	Return a fresh "identity" ampar object
	$t \triangleleft ()$	Fill destination with unit
	$t \triangleleft \text{Inl}$	Fill destination with left variant
	$t \triangleleft \text{Inr}$	Fill destination with right variant
	$t \triangleleft (,)$	Fill destination with product constructor
	$t \triangleleft \bullet u$	Fill destination with root of ampar u
	(t)	S
	$t[\text{sub}]$	M
$\overline{\text{extended_term}}$, \bar{t}	::=	Extended term
	t	
	\bar{v}	
<i>sub</i>	::=	Variable substitution
	$x := v$	
	$\text{sub}_1, \text{sub}_2$	
	sub	S
<i>effect</i> , <i>e</i>	::=	Effect
	ε	No effect
	$h := \bar{v}$	
	$e_1 \cdot e_2$	

	e	S
type, A, B	$::=$ 1 $A_1 \oplus A_2$ $A_1 \otimes A_2$ $A_1 \ltimes A_2$ $A_1 \multimap A_2$ A^D (A)	Type Unit Sum Product Ampar type (consuming A_1 yields A_2) Linear function Destination S
mode, m	$::=$ L F G $\text{max_mode}(\Gamma)$ $\text{if mode_cond then } m_3 \text{ else } m_4$	Mode Local Foreign Global
mode_cond	$::=$ $m_1 = m_2$ $m \in \text{upper_modes}(\Gamma)$ $\exists m \in \text{upper_modes}(\Gamma)$	Mode statement
typing_context, Δ	$::=$ Γ H $\Gamma \sqcup H$	Typing context
pos_context, Γ	$::=$ $\{\}$ $\{\text{pos_assigns}\}$ $\Gamma_1 \sqcup \Gamma_2$ $\neg H$ $\Gamma[m_1 \mapsto m_2]$ (Γ)	Positive typing context S
pos_assign, pa	$::=$ $x \multimap_m A$ $+h : A$	Positive type assignment Destination
pos_assigns	$::=$ pa $pa, \text{pos_assigns}$	Positive type assignments
neg_assign, na	$::=$ $\neg h : A$	Negative type assignment Hole
neg_assigns	$::=$ na $na, \text{neg_assigns}$	Negative type assignments
neg_context, H	$::=$ $\{\}$	Negative typing context

		{neg__assigns}	
		$H_1 \sqcup H_2$	
		$\neg\Gamma$	
		(H)	S

terminals	::=	
		\dashv
		\times
		\mapsto
		()
		Inl
		Inr
		(,)
		\triangleleft
		\blacktriangleleft
		\vdash
		\sqcup
		\boxplus
		{}
		\exists
		\neq
		\leq
		\in
		\notin
		\subset
		\mathcal{N}
		\vdash
		$ $
		\Downarrow

formula	::=	
		judgement

Ctx	::=	
		$x \in \mathcal{N}(\Delta)$
		$h \in \mathcal{N}(\Delta)$
		$x \notin \mathcal{N}(\Delta)$
		$h \notin \mathcal{N}(\Delta)$
		fresh x
		fresh h
		pos__assign $\in \Gamma$
		neg__assign $\in H$
		onlyPositive (Δ)
		onlyNegative (Δ)
		mode__cond

Eq	::=	
		$A_1 = A_2$
		$A_1 \neq A_2$
		$t = u$
		$t \neq u$
		$\Delta_1 = \Delta_2$
		$\mathcal{N}(\Delta_1) \cap \mathcal{N}(\Delta_2) = \emptyset$

Eff	$::=$ $ \quad e_2, \overline{v_2} H_2 = \mathbf{apply}(e_1, \overline{v_1} H_1)$	Apply effect
Ty	$::=$ $ \quad \Delta \vdash \bar{t} : \mathbf{A}$	
Sem	$::=$ $ \quad t \Downarrow t' \mid e$	
judgement	$::=$ $ \quad \text{Ctx}$ $ \quad \text{Eq}$ $ \quad \text{Eff}$ $ \quad \text{Ty}$ $ \quad \text{Sem}$	
user_syntax	$::=$ $ \quad \text{termvar}$ $ \quad \text{hole}$ $ \quad \frac{\text{term_value}}{\text{extended_value}}$ $ \quad \frac{\text{term}}{\text{extended_term}}$ $ \quad \text{sub}$ $ \quad \text{effect}$ $ \quad \text{type}$ $ \quad \text{mode}$ $ \quad \text{mode_cond}$ $ \quad \text{typing_context}$ $ \quad \text{pos_context}$ $ \quad \text{pos_assign}$ $ \quad \text{pos_assigns}$ $ \quad \text{neg_assign}$ $ \quad \text{neg_assigns}$ $ \quad \text{neg_context}$ $ \quad \text{terminals}$	

$$x \in \mathcal{N}(\Delta)$$

$$h \in \mathcal{N}(\Delta)$$

$$x \notin \mathcal{N}(\Delta)$$

$$h \notin \mathcal{N}(\Delta)$$

$$\mathbf{fresh} \, x$$

$$\mathbf{fresh} \, h$$

$$\text{pos_assign} \in \Gamma$$

$$\text{neg_assign} \in H$$

$$\mathbf{onlyPositive}(\Delta)$$

$$\mathbf{onlyNegative}(\Delta)$$

$$\text{mode_cond}$$

$$A_1 = A_2$$

$$A_1 \neq A_2$$

$$t = u$$

$t \neq u$
$\Delta_1 = \Delta_2$
$\mathcal{N}(\Delta_1) \cap \mathcal{N}(\Delta_2) = \emptyset$
$e_2, \overline{v_2} H_2 = \mathbf{apply}(e_1, \overline{v_1} H_1)$
$\Delta \vdash \bar{t} : \mathbf{A}$

Apply effect

$$\begin{array}{c}
\frac{}{\{\} \sqcup \{-h : \mathbf{A}\} \vdash h : \mathbf{A}} \text{TyTERMEXT_HOLE} \\
\\
\frac{}{\{+h : \mathbf{A}\} \sqcup \{\} \vdash @h : \mathbf{A}^D} \text{TyTERMEXT_DEST} \\
\\
\frac{}{\{\} \sqcup \{\} \vdash () : \mathbf{1}} \text{TyTERMEXT_UNIT} \\
\\
\frac{\Gamma \sqcup H \vdash \bar{v} : \mathbf{A}_1}{\Gamma \sqcup H \vdash \text{Inl } \bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2} \text{TyTERMEXT_INL} \\
\\
\frac{\Gamma \sqcup H \vdash \bar{v} : \mathbf{A}_2}{\Gamma \sqcup H \vdash \text{Inr } \bar{v} : \mathbf{A}_1 \oplus \mathbf{A}_2} \text{TyTERMEXT_INR} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup H_1 \vdash \bar{v}_1 : \mathbf{A}_1 \\ \Gamma_2 \sqcup H_2 \vdash \bar{v}_2 : \mathbf{A}_2 \\ \mathcal{N}(\Gamma_1 \sqcup H_1) \cap \mathcal{N}(\Gamma_2 \sqcup H_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup H_1 \sqcup H_2 \vdash (\bar{v}_1, \bar{v}_2) : \mathbf{A}_1 \otimes \mathbf{A}_2} \text{TyTERMEXT_PROD} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup -H \sqcup \{\} \vdash v_1 : \mathbf{A}_1 \\ \Gamma_2 \sqcup H \vdash \bar{v}_2 : \mathbf{A}_2 \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash \langle v_1, \bar{v}_2 \rangle_H : \mathbf{A}_1 \ltimes \mathbf{A}_2} \text{TyTERMEXT_AMPAR} \\
\\
\frac{\Gamma \sqcup \{\times :_{\mathbf{m}_1} \mathbf{A}_1\} \sqcup \{\} \vdash t : \mathbf{A}_2}{\Gamma \sqcup \{\} \vdash \lambda \times. t : \mathbf{A}_1 \multimap \mathbf{A}_2} \text{TyTERMEXT_LAMBDA} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_1 \multimap \mathbf{A}_2 \\ \Gamma_2 \sqcup \{\} \vdash u : \mathbf{A}_1 \\ \mathbf{m}_1 \in \mathbf{upper_modes}(\Gamma_2) \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t u : \mathbf{A}_2} \text{TyTERMEXT_APP} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{1} \\ \Gamma_2 \sqcup \{\} \vdash u : \mathbf{B} \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \succ \mathbf{case} () \mapsto u : \mathbf{B}} \text{TyTERMEXT_PATUNIT} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_1 \oplus \mathbf{A}_2 \\ \exists \mathbf{m} \in \mathbf{upper_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{\times_1 :_{\mathbf{m}} \mathbf{A}_1\} \sqcup \{\} \vdash u_1 : \mathbf{B} \\ \Gamma_2 \sqcup \{\times_2 :_{\mathbf{m}} \mathbf{A}_2\} \sqcup \{\} \vdash u_2 : \mathbf{B} \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \succ \mathbf{case} \{ \text{Inl } \times_1 \mapsto u_1, \text{Inr } \times_2 \mapsto u_2 \} : \mathbf{B}} \text{TyTERMEXT_PATSUM} \\
\\
\frac{\begin{array}{l} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_1 \otimes \mathbf{A}_2 \\ \exists \mathbf{m} \in \mathbf{upper_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{\times_1 :_{\mathbf{m}} \mathbf{A}_1, \times_2 :_{\mathbf{m}} \mathbf{A}_2\} \sqcup \{\} \vdash u : \mathbf{B} \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \succ \mathbf{case} (\times_1, \times_2) \mapsto u : \mathbf{B}} \text{TyTERMEXT_PATPROD}
\end{array}$$

$$\begin{array}{c}
\Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_1 \times \mathbf{A}_2 \\
\exists \mathbf{m}' \in \mathbf{upper_modes}(\Gamma_1 \sqcup \Gamma_2) \\
\mathbf{m} = \text{if } \mathbf{F} \in \mathbf{upper_modes}(\Gamma_1) \text{ then } \mathbf{F} \text{ else } \mathbf{L} \\
\Gamma_2[\mathbf{L} \mapsto \mathbf{F}] \sqcup \{\mathbf{x} :_{\mathbf{m}} \mathbf{A}_1\} \sqcup \{\} \vdash u : \mathbf{B} \\
\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \\
\hline
\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \succ \text{mapL } \mathbf{x} \mapsto u : \mathbf{B} \times \mathbf{A}_2 \quad \text{TYTERMEXT_MAPAMPAR}
\end{array}$$

$$\begin{array}{c}
\frac{}{\{\} \sqcup \{\} \vdash \text{alloc}_{\mathbf{A}} : \mathbf{A}^{\mathbf{D}} \times \mathbf{A}} \quad \text{TYTERMEXT_ALLOC} \\
\frac{\Gamma \sqcup \{\} \vdash t : \mathbf{A}}{\Gamma \sqcup \{\} \vdash \text{to}_{\times} t : \mathbf{1} \times \mathbf{A}} \quad \text{TYTERMEXT_TOAMPAR} \\
\frac{\Gamma \sqcup \{\} \vdash t : \mathbf{1} \times \mathbf{A}}{\Gamma \sqcup \{\} \vdash \text{from}_{\times} t : \mathbf{A}} \quad \text{TYTERMEXT_FROMAMPAR} \\
\frac{\Gamma \sqcup \{\} \vdash t : \mathbf{1}^{\mathbf{D}}}{\Gamma \sqcup \{\} \vdash t \triangleleft () : \mathbf{1}} \quad \text{TYTERMEXT_FILLUNIT} \\
\frac{\Gamma \sqcup \{\} \vdash t : (\mathbf{A}_1 \oplus \mathbf{A}_2)^{\mathbf{D}}}{\Gamma \sqcup \{\} \vdash t \triangleleft \text{Inl} : \mathbf{A}_1^{\mathbf{D}}} \quad \text{TYTERMEXT_FILLINL} \\
\frac{\Gamma \sqcup \{\} \vdash t : (\mathbf{A}_1 \oplus \mathbf{A}_2)^{\mathbf{D}}}{\Gamma \sqcup \{\} \vdash t \triangleleft \text{Inr} : \mathbf{A}_2^{\mathbf{D}}} \quad \text{TYTERMEXT_FILLINR} \\
\frac{\Gamma \sqcup \{\} \vdash t : (\mathbf{A}_1 \otimes \mathbf{A}_2)^{\mathbf{D}}}{\Gamma \sqcup \{\} \vdash t \triangleleft (,) : \mathbf{A}_1^{\mathbf{D}} \otimes \mathbf{A}_2^{\mathbf{D}}} \quad \text{TYTERMEXT_FILLPROD} \\
\frac{\begin{array}{c} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_2^{\mathbf{D}} \\ \Gamma_2 \sqcup \{\} \vdash u : \mathbf{A}_1 \times \mathbf{A}_2 \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \\ \mathbf{L} \in \mathbf{upper_modes}(\Gamma_1) \\ \mathbf{F} \in \mathbf{upper_modes}(\Gamma_2) \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \triangleleft \bullet u : \mathbf{A}_1} \quad \text{TYTERMEXT_FILLCOMPL} \\
\frac{\begin{array}{c} \Gamma_1 \sqcup \{\} \vdash t : \mathbf{A}_2^{\mathbf{D}} \\ \Gamma_2 \sqcup \{\} \vdash u : \mathbf{A}_1 \times \mathbf{A}_2 \\ \mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset \\ \mathbf{F} \in \mathbf{upper_modes}(\Gamma_1) \\ \mathbf{G} \in \mathbf{upper_modes}(\Gamma_2) \end{array}}{\Gamma_1 \sqcup \Gamma_2 \sqcup \{\} \vdash t \triangleleft \bullet u : \mathbf{A}_1} \quad \text{TYTERMEXT_FILLCOMPF}
\end{array}$$

$$t \Downarrow t' \mid e$$

$$\begin{array}{c}
\frac{}{v \Downarrow v \mid \varepsilon} \quad \text{BIGSTEP_VAL} \\
\frac{\begin{array}{c} t_1 \Downarrow \lambda \mathbf{x} . u \mid e_1 \\ t_2 \Downarrow v_2 \mid e_2 \\ u[\mathbf{x} := v_2] \Downarrow v_3 \mid e_3 \end{array}}{t_1 t_2 \Downarrow v_3 \mid e_1 \cdot e_2 \cdot e_3} \quad \text{BIGSTEP_APP} \\
\frac{\begin{array}{c} t_1 \Downarrow () \mid e_1 \\ t_2 \Downarrow v_2 \mid e_2 \end{array}}{t_1 \succ \text{case } () \mapsto t_2 \Downarrow v_2 \mid e_1 \cdot e_2} \quad \text{BIGSTEP_PATUNIT} \\
\frac{\begin{array}{c} t \Downarrow \text{Inl } v_1 \mid e_1 \\ u_1[\mathbf{x}_1 := v_1] \Downarrow v_2 \mid e_2 \end{array}}{t \succ \text{case } \{\text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2\} \Downarrow v_2 \mid e_1 \cdot e_2} \quad \text{BIGSTEP_PATINL} \\
\frac{\begin{array}{c} t \Downarrow \text{Inr } v_1 \mid e_1 \\ u_2[\mathbf{x}_2 := v_1] \Downarrow v_2 \mid e_2 \end{array}}{t \succ \text{case } \{\text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2\} \Downarrow v_2 \mid e_1 \cdot e_2} \quad \text{BIGSTEP_PATINR}
\end{array}$$

$$\begin{array}{c}
\frac{t \Downarrow (v_1, v_2) \mid e_1 \quad u[x_1 := v_1, x_2 := v_2] \Downarrow v_2 \mid e_2}{t \succ \text{case}(x_1, x_2) \mapsto u \Downarrow v_2 \mid e_1 \cdot e_2} \quad \text{BIGSTEP_PATPROD} \\
\\
\frac{t \Downarrow \langle v_1, \overline{v_2} \rangle_H \mid e_1 \quad u[x := v_1] \Downarrow v_3 \mid e_2 \quad e_3, \overline{v_4}_{H'} = \text{apply}(e_1 \cdot e_2, \overline{v_2}_H)}{t \succ \text{mapL } x \mapsto u \Downarrow \langle v_3, \overline{v_4} \rangle_{H'} \mid e_3} \quad \text{BIGSTEP_MAPAMPAR} \\
\\
\frac{\text{fresh } h}{\text{alloc } \mathbf{A} \Downarrow \langle @h, h \rangle_{\{-h:A\}} \mid \varepsilon} \quad \text{BIGSTEP_ALLOC} \\
\\
\frac{t \Downarrow v \mid e}{\text{to}_{\times} t \Downarrow \langle (), v \rangle_{\{\}} \mid e} \quad \text{BIGSTEP_TOAMPAR} \\
\\
\frac{t \Downarrow \langle (), v \rangle_{\{\}} \mid e}{\text{from}_{\times} t \Downarrow v \mid e} \quad \text{BIGSTEP_FROMAMPAR} \\
\\
\frac{t \Downarrow @h \mid e}{t \triangleleft () \Downarrow () \mid e \cdot h := ()} \quad \text{BIGSTEP_FILLUNIT} \\
\\
\frac{t \Downarrow @h \mid e \quad \text{fresh } h'}{t \triangleleft \text{Inl} \Downarrow @h' \mid e \cdot h := \text{Inl } h'} \quad \text{BIGSTEP_FILLINL} \\
\\
\frac{t \Downarrow @h \mid e}{t \triangleleft \text{Inr} \Downarrow @h' \mid e \cdot h := \text{Inr } h'} \quad \text{BIGSTEP_FILLINR} \\
\\
\frac{t \Downarrow @h \mid e \quad \text{fresh } h_1 \quad \text{fresh } h_2}{t \triangleleft (,) \Downarrow (@h_1, @h_2) \mid e \cdot h := (h_1, h_2)} \quad \text{BIGSTEP_FILLPROD} \\
\\
\frac{t \Downarrow @h \mid e_1 \quad u \Downarrow \langle v_1, \overline{v_2} \rangle_H \mid e_2}{t \triangleleft \bullet u \Downarrow v_1 \mid e_1 \cdot e_2 \cdot h := \overline{v_2}} \quad \text{BIGSTEP_FILLCOMP}
\end{array}$$

Definition rules: 37 good 0 bad
 Definition rule clauses: 109 good 0 bad