

termvar, x, y, d Term-level variable

$label, l$::= Label

$hole, h$::= Hole

mode, m ::= Mode

- | $-$ Placeholder for any mode
- | L Local
- | F Foreign
- | G Global
- | $\text{max_mode}(\Gamma)$
- | $\text{if mode_cond then } m_3 \text{ else } m_4$

mode_cond ::= Mode statement

- | $m_1 = m_2$
- | $m_1 \leq m_2$
- | $m \in \text{upper_modes}(\Gamma)$
- | $\exists m \in \text{upper_modes}(\Gamma)$

type, A, B ::= Type

- | 1 Unit
- | $A_1 \oplus A_2$ Sum
- | $A_1 \otimes A_2$ Product
- | $A_1 \ltimes A_2$ Ampar type (consuming A_1 yields A_2)
- | $m_1 A_1 \multimap A_2$ Linear function
- | A^\perp Destination
- | (A) S

term_value, v ::= Term value

- | l Label representing an Ampar
- | $@h$ Destination
- | $()$ Unit
- | $\text{Inl } v$ Left variant for sum
- | $\text{Inr } v$ Right variant for sum
- | (v_1, v_2) Product
- | $\lambda x. t$ Linear function
- | (v) S

extended_value, \underline{v} ::= Store value

- | v Term value
- | h Hole
- | $\text{Inl } \underline{v}$ Left variant with val or hole
- | $\text{Inr } \underline{v}$ Right variant with val or hole
- | $(\underline{v}_1, \underline{v}_2)$ Product with val or hole
- | (\underline{v}) S

store_affect, ha ::= Ampar (\underline{v}_2 is the root of the structure being

- | $l \mapsto \langle v_1, \underline{v}_2 \rangle$
- | $l \mapsto \langle \square, \underline{v}_2 \rangle$ Ampar (\underline{v}_2 is the root of the structure being

store_affects	$::=$ $\begin{array}{ l} \text{ha} \\ \text{ha, store_affects} \end{array}$	
store, S	$::=$ $\begin{array}{ l} [] \\ [\text{store_affects}] \\ S[e] \\ S_1 \sqcup S_2 \end{array}$	
term, t, u	$::=$ $\begin{array}{ l} v \\ x \\ t\ u \\ t \succ \text{case } () \mapsto u \\ t \succ \text{case } \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \} \\ t \succ \text{case } (x_1, x_2) \mapsto u \\ t \succ \text{mapL } x \mapsto u \\ \text{to}_x t \\ \text{from}_x t \\ \text{alloc} \\ t \triangleleft () \\ t \triangleleft \text{Inl} \\ t \triangleleft \text{Inr} \\ t \triangleleft (,) \\ t \triangleleft \bullet u \\ (t) \\ t[e] \end{array}$	<p>Term</p> <p>Term value</p> <p>Variable</p> <p>Application</p> <p>Pattern-match on unit</p> <p>Pattern-match on sum</p> <p>Pattern-match on product</p> <p>Map over the left side of the ampar</p> <p>Wrap t into a trivial ampar</p> <p>Extract value from trivial ampar</p> <p>Return a fresh "identity" ampar object</p> <p>Fill destination with unit</p> <p>Fill destination with left variant</p> <p>Fill destination with right variant</p> <p>Fill destination with product construct</p> <p>Fill destination with root of ampar u</p> <p>S</p> <p>M</p>
extended_term, \underline{t}	$::=$ $\begin{array}{ l} t \\ \underline{v} \end{array}$	Extended term
sub	$::=$ $\begin{array}{ l} x := v \\ h := \underline{v} \end{array}$	variable or label substitution
subs	$::=$ $\begin{array}{ l} \text{sub} \\ \text{sub, subs} \end{array}$	variable or substitutions
effect, e	$::=$ $\begin{array}{ l} \varepsilon \\ \text{subs} \end{array}$	empty effect
type_affect, ta	$::=$ $\begin{array}{ l} x :_m A \\ +l : A \\ -l : A \end{array}$	type affectation

	$\begin{array}{ l} +h : A \\ -h : A \end{array}$	Hole Destination
type_affects	$\begin{array}{ l} ::= \\ ta \\ ta, type_affects \end{array}$	type affectations
typing_context, \mathcal{U} , Γ	$\begin{array}{ l} ::= \\ \{\} \\ \{type_affects\} \\ \Gamma_1 \sqcup \Gamma_2 \\ \Gamma_1 \uplus \Gamma_2 \\ \Gamma[m_1 \mapsto m_2] \\ (\Gamma) \end{array}$	typing context S
terminals	$\begin{array}{ l} ::= \\ \text{---} \circ \\ \times \\ \mapsto \\ () \\ \text{Inl} \\ \text{Inr} \\ (,) \\ \triangleleft \\ \triangleleft \blacktriangleright \\ \sqcup \\ \uplus \\ \{\} \\ \exists \\ \neq \\ \leq \\ \in \\ \notin \\ \subset \\ \mathcal{N} \\ \vdash \\ \\ \Downarrow \end{array}$	
formula	$\begin{array}{ l} ::= \\ \text{judgement} \end{array}$	
Ctx	$\begin{array}{ l} ::= \\ x \in \mathcal{N}(\Gamma) \\ l \in \mathcal{N}(\Gamma) \\ x \notin \mathcal{N}(\Gamma) \\ l \notin \mathcal{N}(\Gamma) \\ \text{fresh } x \\ \text{fresh } l \\ \text{fresh } h \end{array}$	

		$\text{type_affect} \in \Gamma$
		mode_cond
Eq	::=	
		$A_1 = A_2$
		$A_1 \neq A_2$
		$t = u$
		$t \neq u$
		$\Gamma_1 = \Gamma_2$
		$\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset$
Ty	::=	
		$\Gamma \vdash S \mid t : A$
		$\Gamma \vdash S$
		$\Gamma \vdash \underline{t} : A$
Sem	::=	
		$S \mid t \Downarrow S' \mid t'$
judgement	::=	
		Ctx
		Eq
		Ty
		Sem
user_syntax	::=	
		<i>termvar</i>
		<i>label</i>
		<i>hole</i>
		<i>mode</i>
		mode_cond
		type
		term_value
		extended_value
		store_affect
		store_affects
		store
		term
		extended_term
		sub
		subs
		<i>effect</i>
		type_affect
		type_affects
		typing_context
		terminals

$$\boxed{x \in \mathcal{N}(\Gamma)}$$

$$\boxed{l \in \mathcal{N}(\Gamma)}$$

$$\boxed{x \notin \mathcal{N}(\Gamma)}$$

$l \notin \mathcal{N}(\Gamma)$
fresh x
fresh l
fresh h
$\text{type_affect} \in \Gamma$
mode_cond
$A_1 = A_2$
$A_1 \neq A_2$
$t = u$
$t \neq u$
$\Gamma_1 = \Gamma_2$
$\mathcal{N}(\Gamma_1) \cap \mathcal{N}(\Gamma_2) = \emptyset$
$\Gamma \vdash S \mid t : A$

$$\frac{\Gamma_1 \vdash S \quad \Gamma_2 \vdash t : A}{\Gamma_1 \sqcup \Gamma_2 \vdash S \mid t : A} \text{TYCMD_CMD}$$

$$\Gamma \vdash S$$

$$\overline{\{\} \vdash []} \quad \text{TYHEAP_EMPTY}$$

$$\frac{\Gamma_1 \vdash S_1 \quad \Gamma_2 \vdash S_2}{\Gamma_1 \sqcup \Gamma_2 \vdash S_1 \sqcup S_2} \quad \text{TYHEAP_UNION}$$

$$\frac{\Gamma_1 \vdash v_1 : A_1 \quad \Gamma_2 \vdash v_2 : A_2}{(\Gamma_1 \sqcup \Gamma_2) \sqcup \{-l : A_1 \times A_2\} \vdash [l \mapsto \langle v_1, v_2 \rangle]} \quad \text{TYHEAP_CLOSEDAMPAR}$$

$$\frac{\Gamma_2 \vdash v_2 : A_2}{\Gamma_2 \vdash [l \mapsto \langle \square, v_2 \rangle]} \quad \text{TYHEAP_OPENAMPAR}$$

$$\Gamma \vdash \underline{t} : A$$

$$\overline{\{+l : A\} \vdash l : A} \quad \text{TYTERM_AMPAR}$$

$$\overline{\{-h : A\} \vdash @h : A^\perp} \quad \text{TYTERM_DEST}$$

$$\overline{\{+h : A\} \vdash h : A} \quad \text{TYTERM_HOLE}$$

$$\overline{\{\} \vdash () : 1} \quad \text{TYTERM_UNIT}$$

$$\frac{\Gamma \vdash \underline{v} : A_1}{\Gamma \vdash \text{Inl } \underline{v} : A_1 \oplus A_2} \quad \text{TYTERM_INL}$$

$$\frac{\Gamma \vdash \underline{v} : A_2}{\Gamma \vdash \text{Inr } \underline{v} : A_1 \oplus A_2} \quad \text{TYTERM_INR}$$

$$\frac{\Gamma_1 \vdash v_1 : A_1 \quad \Gamma_2 \vdash v_2 : A_2}{\Gamma_1 \sqcup \Gamma_2 \vdash (v_1, v_2) : A_1 \otimes A_2} \quad \text{TYTERM_PROD}$$

$$\begin{array}{c}
\frac{\Gamma \sqcup \{x :_{m_1} A_1\} \vdash t : A_2}{\Gamma \vdash \lambda x. t :_{m_1} A_1 \multimap A_2} \text{TYTERM_LAMBDA} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t :_{m_1} A_1 \multimap A_2 \\ \Gamma_2 \vdash u : A_1 \\ m_1 \in \text{upper_modes}(\Gamma_2) \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t u : A_2} \text{TYTERM_APP} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t : 1 \\ \Gamma_2 \vdash u : B \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{case}() \mapsto u : B} \text{TYTERM_PATUNIT} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t : A_1 \oplus A_2 \\ \exists m \in \text{upper_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{x_1 :_m A_1\} \vdash u_1 : B \\ \Gamma_2 \sqcup \{x_2 :_m A_2\} \vdash u_2 : B \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{case} \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \} : B} \text{TYTERM_PATSUM} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t : A_1 \otimes A_2 \\ \exists m \in \text{upper_modes}(\Gamma_1) \\ \Gamma_2 \sqcup \{x_1 :_m A_1, x_2 :_m A_2\} \vdash u : B \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{case}(x_1, x_2) \mapsto u : B} \text{TYTERM_PATPROD} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t : A_1 \ltimes A_2 \\ \exists m' \in \text{upper_modes}(\Gamma_1 \sqcup \Gamma_2) \\ m = \text{if } F \in \text{upper_modes}(\Gamma_1) \text{ then } F \text{ else } L \\ \Gamma_2[L \mapsto F] \sqcup \{x :_m A_1\} \vdash u : B \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \succ \text{mapL } x \mapsto u : B \ltimes A_2} \text{TYTERM_MAPAMPAR} \\
\\
\frac{}{\{\} \vdash \text{alloc} : A^\perp \ltimes A} \text{TYTERM_ALLOC} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash \text{to}_\ltimes t : 1 \ltimes A} \text{TYTERM_TOAMPAR} \\
\\
\frac{\Gamma \vdash t : 1 \ltimes A}{\Gamma \vdash \text{from}_\ltimes t : A} \text{TYTERM_FROMAMPAR} \\
\\
\frac{\Gamma \vdash t : 1^\perp}{\Gamma \vdash t \triangleleft () : 1} \text{TYTERM_FILLUNIT} \\
\\
\frac{\Gamma \vdash t : (A_1 \oplus A_2)^\perp}{\Gamma \vdash t \triangleleft \text{Inl} : A_1^\perp} \text{TYTERM_FILLINL} \\
\\
\frac{\Gamma \vdash t : (A_1 \oplus A_2)^\perp}{\Gamma \vdash t \triangleleft \text{Inr} : A_2^\perp} \text{TYTERM_FILLINR} \\
\\
\frac{\Gamma \vdash t : (A_1 \otimes A_2)^\perp}{\Gamma \vdash t \triangleleft (,) : A_1^\perp \otimes A_2^\perp} \text{TYTERM_FILLPROD} \\
\\
\frac{\begin{array}{c} \Gamma_1 \vdash t : A_2^\perp \\ \Gamma_2 \vdash u : A_1 \ltimes A_2 \\ L \in \text{upper_modes}(\Gamma_1) \\ F \in \text{upper_modes}(\Gamma_2) \end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft \bullet u : A_1} \text{TYTERM_FILLCOMPL}
\end{array}$$

$$\boxed{S \mid t \Downarrow S' \mid t'}$$

$$\frac{\Gamma_1 \vdash t : A_2^\perp \quad \Gamma_2 \vdash u : A_1 \rtimes A_2 \quad \textcolor{violet}{F} \in \text{upper_modes}(\Gamma_1) \quad \textcolor{violet}{G} \in \text{upper_modes}(\Gamma_2)}{\Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft_\bullet u : A_1} \quad \text{TYTERM_FILLCOMP}F$$

$$\frac{}{S_0 \mid v \Downarrow S_0 \mid v} \quad \text{BIGSTEP_VAL}$$

$$\frac{S_0 \mid t_1 \Downarrow S_1 \mid \lambda \textcolor{violet}{x}.u \quad S_1 \mid t_2 \Downarrow S_2 \mid v_2 \quad S_2 \mid u[\textcolor{violet}{x} := v_2] \Downarrow S_3 \mid v_3}{S_0 \mid t_1 t_2 \Downarrow S_3 \mid v_3} \quad \text{BIGSTEP_APP}$$

$$\frac{S_0 \mid t_1 \Downarrow S_1 \mid () \quad S_1 \mid t_2 \Downarrow S_2 \mid v_2}{S_0 \mid t_1 \succ \text{case} () \mapsto t_2 \Downarrow S_2 \mid v_2} \quad \text{BIGSTEP_PATUNIT}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid \text{Inl } v_1 \quad S_1 \mid u_1[\textcolor{violet}{x}_1 := v_1] \Downarrow S_2 \mid v_2}{S_0 \mid t \succ \text{case} \{ \text{Inl } \textcolor{violet}{x}_1 \mapsto u_1, \text{Inr } \textcolor{violet}{x}_2 \mapsto u_2 \} \Downarrow S_2 \mid v_2} \quad \text{BIGSTEP_PATINL}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid \text{Inr } v_1 \quad S_1 \mid u_2[\textcolor{violet}{x}_2 := v_1] \Downarrow S_2 \mid v_2}{S_0 \mid t \succ \text{case} \{ \text{Inl } \textcolor{violet}{x}_1 \mapsto u_1, \text{Inr } \textcolor{violet}{x}_2 \mapsto u_2 \} \Downarrow S_2 \mid v_2} \quad \text{BIGSTEP_PATINR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid (v_1, v_2) \quad S_1 \mid u[\textcolor{violet}{x}_1 := v_1, \textcolor{violet}{x}_2 := v_2] \Downarrow S_2 \mid v_2}{S_0 \mid t \succ \text{case} (\textcolor{violet}{x}_1, \textcolor{violet}{x}_2) \mapsto u \Downarrow S_2 \mid v_2} \quad \text{BIGSTEP_PATPROD}$$

$$\frac{S_0 \mid t \Downarrow S_1 \sqcup [\textcolor{violet}{l} \mapsto \langle v_1, \underline{v}_1 \rangle] \mid \textcolor{brown}{l} \quad S_1 \sqcup [\textcolor{violet}{l} \mapsto \langle \square, \underline{v}_1 \rangle] \mid u[\textcolor{violet}{x} := v_1] \Downarrow S_2 \sqcup [\textcolor{violet}{l} \mapsto \langle \square, \underline{v}_2 \rangle] \mid v_2}{S_0 \mid t \succ \text{mapL } \textcolor{violet}{x} \mapsto u \Downarrow S_2 \sqcup [\textcolor{violet}{l} \mapsto \langle v_2, \underline{v}_2 \rangle] \mid \textcolor{brown}{l}} \quad \text{BIGSTEP_MAPAMPAR}$$

$$\frac{\text{fresh } h \quad \text{fresh } \textcolor{brown}{l}}{S_0 \mid \text{alloc} \Downarrow S_0 \sqcup [\textcolor{violet}{l} \mapsto \langle @h, h \rangle] \mid \textcolor{brown}{l}} \quad \text{BIGSTEP_ALLOC}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid v \quad \text{fresh } \textcolor{brown}{l}}{S_0 \mid \text{to}_\rtimes t \Downarrow S_1 \sqcup [\textcolor{violet}{l} \mapsto \langle (), v \rangle] \mid \textcolor{brown}{l}} \quad \text{BIGSTEP_TOAMPAR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \sqcup [\textcolor{violet}{l} \mapsto \langle (), v \rangle] \mid \textcolor{brown}{l}}{S_0 \mid \text{from}_\rtimes t \Downarrow S_1 \mid v} \quad \text{BIGSTEP_FROMAMPAR}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h}{S_0 \mid t \triangleleft () \Downarrow S_1[h := ()] \mid ()} \quad \text{BIGSTEP_FILLUNIT}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \quad \text{fresh } h'}{S_0 \mid t \triangleleft \text{Inl} \Downarrow S_1[h := \text{Inl } h'] \mid @h'} \quad \text{BIGSTEP_FILLINL}$$

$$\frac{S_0 \mid t \Downarrow S_1 \mid @h \quad \text{fresh } h'}{S_0 \mid t \triangleleft \text{Inr} \Downarrow S_1[h := \text{Inr } h'] \mid @h'} \quad \text{BIGSTEP_FILLINR}$$

$$\begin{array}{c}
S_0 \mid t \Downarrow S_1 \mid @h \\
\text{fresh } h_1 \\
\text{fresh } h_2 \\
\hline
S_0 \mid t \triangleleft (,) \Downarrow S_1[h := (h_1, h_2)] \mid (@h_1, @h_2) \quad \text{BIGSTEP_FILLPROD}
\end{array}$$

$$\begin{array}{c}
S_0 \mid t \Downarrow S_1 \mid @h \\
S_1 \mid u \Downarrow S_2 \sqcup [l \mapsto \langle v_1, \underline{v}_1 \rangle] \mid l \\
\hline
S_0 \mid t \triangleleft \bullet u \Downarrow S_2[h := \underline{v}_1] \mid v_1 \quad \text{BIGSTEP_FILLCOMP}
\end{array}$$

Definition rules: 42 good 0 bad
 Definition rule clauses: 112 good 0 bad