termvar, x, y, d Term holevar, h Hole	n-level variable	
term_value, v	::= $\langle V_1, \overline{V_2} \rangle_H$ $ \ $	Term value Ampar Destination Unit Left variant for sum Right variant for sum Product Exponential Linear function
extended_value, v	::= v h Inl \overline{v} Inr \overline{v} $(\overline{v_1}, \overline{v_2})$ $)^m \overline{v}$ (\overline{v}) $\overline{v}[e]$	Store value Term value Hole Left variant with val or hole Right variant with val or hole Product with val or hole Exponential with val or hole S M
term, t, u	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Term value Variable Application Pattern-match on unit Pattern-match on sum Pattern-match on product Pattern-match on exponential Map over the left side of the ampar Wrap t into a trivial ampar Extract value from trivial ampar Return a fresh "identity" ampar object Fill destination with unit Fill destination with left variant Fill destination with product constructor Fill destination with exponential constructor Fill destination with root of ampar u S M
sub	::=	Variable substitution
effect, e	$ \begin{array}{cccc} & \varepsilon \\ & \mathbf{h} := \overline{\mathbf{v}} \\ & \mathbf{e}_1 \cdot \mathbf{e}_2 \\ & \mathbf{e} \end{array} $	Effect No effect

type, A, B	::=	$egin{array}{c} 1 & \mathbf{A}_1 \oplus \mathbf{A}_2 & & & & \\ \mathbf{A}_1 \otimes \mathbf{A}_2 & & & & \\ !^m \ \mathbf{A} & & & & & \\ \mathbf{A}_1 ightarrow \mathbf{A}_2 & & & & \\ \mathbf{A}_1 ightarrow \mathbf{A}_2 & & & & \\ m \lfloor \mathbf{A} floor & & & & \\ (\mathbf{A}) & & & & & \end{array}$	S	Type Unit Sum Product Exponential Ampar type (consuming A_1 yields A_2) Linear function Destination
$multiplicity, \ m, \ n$::= 	$ \begin{array}{c} \nu \\ \uparrow \\ \infty \\ m_1 \cdot m_2 \\ (m) \end{array} $	S	Multiplicity (Semiring with product ·) Born now. Identity of the product One scope older Infinitely old / static. Absorbing for product Semiring product
typing_context, Δ	::= 	Γ H $\Gamma \sqcup H$ $m \cdot \Delta$ (Δ)	S	Typing context $ \label{eq:main_main} $
pos_context, Γ	::=	$ \begin{cases} pos_assigns \rbrace \\ \Gamma_1 \sqcup \Gamma_2 \\ m \cdot \Gamma \\ (\Gamma) \\ - \end{cases} $	S M	Positive typing context
pos_assign, pa	::= 	$egin{array}{ll} imes :_m & A \\ @ h :_m & ^n \! ig\lfloor A ig \rfloor \end{array}$		Positive type assignment Variable Destination (m is its own age; n is the age of values it accepts)
pos_assigns	::= 	pa pa, pos_assigns		Positive type assignments
neg_assign, na	::= 	h : ⁿ A		Negative type assignment Hole (n is the age of values it accepts, its own age is undefined)
neg_assigns	::= 	na na, neg_assigns		Negative type assignments
neg_context, H	::=	$ \begin{cases} neg_assigns \rbrace \\ H_1 \sqcup H_2 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$		Negative typing context

```
S
                                          (H)
                                                                                   Μ
eff_app
                                                                                             Effect application
                              ::=
                                          e, \overline{v}_H
                                          apply (eff_app)
                                          e · eff_app
terminals
                                          __0
                                          \bowtie
                                          \mapsto
                                          ()
                                          Inl
                                          Inr
                                          (,)
                                          ◁
                                          ♦
                                         :=
                                          \Box
                                          Ø
                                          \exists
                                          \neq \\ \leq \\ \in
                                          ∉
                                          \subset
                                           \Vdash
                                             \Downarrow
formula
                              ::=
                                          judgement
Ctx
                              ::=
                                          \mathbf{x} \in \mathsf{names}\left(\Delta\right)
                                          \mathbf{h} \, \in \, \mathrm{names} \, (\Delta)
                                          \mathbf{x} \notin \mathsf{names}\left(\Delta\right)
                                          \mathbf{h} \notin \mathsf{names}\left(\Delta\right)
                                          fresh x
                                          fresh\,{\color{red} \underline{h}}
                                          \mathsf{pos}\_\mathsf{assign} \in \Gamma
                                          \mathsf{neg}\_\mathsf{assign}\,\in\,H
                                          \mathbf{onlyPositive}\left(\Delta\right)
                                          \mathbf{onlyNegative}\left(\Delta\right)
Eq
                                          \mathbf{A}_1 = \mathbf{A}_2
                                          \mathbf{A}_1 \neq \mathbf{A}_2
                                          \mathsf{t}=\mathsf{u}
```

 $t \neq u$

```
\Delta_1 = \Delta_2
                                    \mathsf{names}(\Delta_1) \cap \mathsf{names}(\Delta_2) = \emptyset
Ту
                           ::=
                                    \Delta \Vdash \mathsf{e}
                                    \Gamma \vdash \mathsf{v} \mid \mathsf{e} : \mathsf{A}
                                    \Delta \, \Vdash \, \overline{\mathbf{v}} : \mathbf{A}
                                    \Gamma \, \vdash \, \mathsf{t} : \textbf{A}
Sem
                           ::=
                                    H_1 = H_2
                                                                                           "Inverse sign of context" operation
                                    eff_app_1 = eff_app_2
                                                                                           (we assume effect lists are \varepsilon-terminated)
                                    judgement
                           ::=
                                    Ctx
                                    Eq
                                    Ту
                                    Sem
user_syntax
                                    termvar
                                    holevar
                                    term_value
                                    extended_value
                                    term
                                    sub
                                    effect
                                    type
                                    multiplicity
                                    typing_context
                                    pos_context
                                    pos_assign
                                    pos_assigns
                                    neg_assign
                                    neg_assigns
                                    neg_context
                                    eff_app
                                    terminals
x \in \mathsf{names}(\Delta)
\mathbf{h} \in \mathsf{names}\left(\Delta\right)
\mathbf{x} \notin \mathsf{names}\left(\Delta\right)
\mathbf{h} \notin \mathsf{names}\,(\Delta)
fresh x
fresh h
pos\_assign \in \Gamma
\mathsf{neg\_assign} \, \in \, H
onlyPositive (\Delta)
onlyNegative (\Delta)
```

 $\mathbf{A}_1 = \mathbf{A}_2$

```
\mathbf{A}_1 \neq \mathbf{A}_2
t = u
t \neq u
\Delta_1 = \Delta_2
\mathsf{names}(\Delta_1) \cap \mathsf{names}(\Delta_2) = \emptyset
\Delta \Vdash \mathsf{e}
                                                                                                                                                                                 \frac{1}{\emptyset \cup \emptyset \Vdash \varepsilon} TyEff_NoEff
                                                                                                                                                                       \Gamma \,{\scriptstyle \sqcup}\, H \,\Vdash\, \overline{\vee}: \mathsf{A}
                                                                                                                                                                       \mathbf{h} \notin \mathsf{names}(\Gamma)
                                                                                                                                                                                                                                                                                             TyEff_Single
                                                                                                                  \overline{m \cdot ((n \cdot \uparrow) \cdot \Gamma \sqcup \{ \bigcirc \mathbf{h} :_{\nu} | \mathbf{A} | \} \sqcup n \cdot \mathbf{H}) \Vdash \mathbf{h} := \overline{\mathbf{v}}}
                                                                                                                         \Gamma_1 \sqcup H_1 \sqcup \bigcirc^{-1}\Gamma_{22} \Vdash e_1
                                                                                                                         \Gamma_{21} \sqcup \Gamma_{22} \sqcup H_2 \Vdash e_2
                                                                                                                         \frac{\mathsf{names}(\Gamma_1 \mathrel{\sqcup} H_1) \cap \mathsf{names}(\Gamma_{21} \mathrel{\sqcup} H_2) = \emptyset}{\Gamma_1 \mathrel{\sqcup} \Gamma_{21} \mathrel{\sqcup} H_1 \mathrel{\sqcup} H_2 \mathrel{\Vdash} \mathsf{e}_1 \cdot \mathsf{e}_2} \quad \mathsf{TYEFF\_UNION}
   \Gamma \vdash \mathsf{v} \mid \mathsf{e} : \mathsf{A}
                                                                                                                                               \Gamma_{11} \sqcup \Gamma_{12} \vdash \mathsf{v} : \mathsf{A}
                                                                                                                                               \Gamma_2 \mathrel{\sqcup} @^{\text{-1}}\Gamma_{12} \Vdash \mathsf{e}
                                                                                                                                               \frac{\mathsf{names}(\Gamma_{11}) \cap \mathsf{names}(\Gamma_2) = \emptyset}{\Gamma_{11} \sqcup \Gamma_2 \vdash \mathsf{v} \mid \mathsf{e} : \mathbf{A}} \quad \mathsf{TYCMD\_CMD}
   \Delta \Vdash \overline{\mathsf{v}} : \mathsf{A}

\sqrt[b]{\|\mathbf{h}\|^{\nu} \|\mathbf{A}\|^{\nu} \|\mathbf{h}\|^{2}} \quad \text{TyValExt\_Hole}

                                                                                                                                  \overline{\{ @\mathbf{h} :_{\nu} {}^{n} \hspace{-0.5mm} \lfloor \mathbf{A} \rfloor \} \sqcup \emptyset \hspace{0.1cm} \Vdash \hspace{0.1cm} @\mathbf{h} : \hspace{0.1cm} {}^{n} \hspace{-0.5mm} \lfloor \mathbf{A} \rfloor} \hspace{0.3cm} \text{TyValext\_Dest}

\sqrt{\emptyset \cup \emptyset \Vdash (): 1}
 TyValExt_Unit
                                                                                                                                                       \frac{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \overline{\nu} : \textbf{A}_1}{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \textbf{Inl} \, \overline{\nu} : \textbf{A}_1 \! \oplus \! \textbf{A}_2} \quad \text{TyValext\_Inl}
                                                                                                                                                     \frac{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \overline{\nu} : \mathsf{A}_2}{\Gamma \, {\scriptstyle \sqcup} \, H \, \Vdash \, \mathsf{Inr} \, \overline{\nu} : \mathsf{A}_1 \oplus \mathsf{A}_2} \quad \mathsf{TYVALEXT\_INR}
                                                                                                                    \Gamma_1 \sqcup H_1 \Vdash \overline{\mathsf{v}_1} : \mathsf{A}_1
                                                                                                                    \Gamma_2 \sqcup H_2 \Vdash \overline{\mathsf{v}_2} : \mathsf{A}_2
                                                                                                                \frac{\mathsf{names}(\Gamma_1 \, {\scriptstyle \sqcup} \, H_1) \cap \mathsf{names}(\Gamma_2 \, {\scriptstyle \sqcup} \, H_2) = \emptyset}{\Gamma_1 \, {\scriptstyle \sqcup} \, \Gamma_2 \, {\scriptstyle \sqcup} \, H_1 \, {\scriptstyle \sqcup} \, H_2 \, \Vdash \, (\overline{\mathsf{v}_1} \, , \, \overline{\mathsf{v}_2}) : \mathsf{A}_1 \otimes \mathsf{A}_2} \quad \mathrm{TyValExt\_Prod}
                                                                                                                                               \frac{\Gamma \sqcup \mathbf{H} \ \Vdash \overline{\mathbf{v}} : \mathbf{A}}{m \cdot \Gamma \sqcup m \cdot \mathbf{H} \ \Vdash \ \mathbb{D}^m \overline{\mathbf{v}} : \mathbb{P}^m \mathbf{A}} \quad \text{TyValext\_Exp}
                                                                                                                                                 \Gamma_1 \cup \emptyset \Vdash \mathsf{v}_1 : \mathsf{A}_1
                                                                                                                                 \frac{\Gamma_2 \, \sqcup \, \textcircled{0}^{\text{-}1}\Gamma_1 \, \Vdash \, \overline{v_2} : \mathsf{A}_2}{\Gamma_2 \, \sqcup \, \emptyset \, \Vdash \, \langle \mathsf{v}_1 \, , \, \overline{\mathsf{v}_2} \rangle_H : \mathsf{A}_1 \rtimes \mathsf{A}_2} \quad \text{TyValext\_Ampar}
                                                                                                                                      \frac{\Gamma \sqcup \{ \mathsf{x} :_m \mathsf{A}_1 \} \vdash \mathsf{t} : \mathsf{A}_2}{\Gamma \sqcup \emptyset \Vdash \lambda \mathsf{x} \cdot \mathsf{t} : \mathsf{A}_1 \longrightarrow \mathsf{A}_2} \quad \mathsf{TYVALEXT\_LAMBDA}
  \Gamma \vdash \mathsf{t} : \mathsf{A}
```

```
\overline{\{\times :_{\nu} \mathbf{A}\} \vdash \times : \mathbf{A}} TYTERM_VARNOW
                                                             \frac{}{\{x :_{\infty} A\} \vdash x : A} TYTERM_VARINF
                                                  \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1
                                                  \Gamma_2 \vdash \mathsf{u} : \mathbf{A}_1 \xrightarrow{m} \mathbf{A}_2
                                                  \frac{\mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \; \succ \; \mathsf{u} : \mathsf{A}_2} \quad \mathsf{TYTERM\_APP}
                                         \Gamma_1 \vdash t: \mathbf{1}
                                         \Gamma_2 \vdash \mathsf{u} : \mathsf{B}
                                     \frac{\mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \; \succ \! \mathsf{case} \, () \mapsto \; \mathsf{u} : \mathsf{B}} \quad \mathsf{TYTERM\_PATUNIT}
                                           \Gamma_1 \vdash t : \mathbf{A}_1 \oplus \mathbf{A}_2
                                           \Gamma_2 \sqcup \{ \mathsf{x}_1 :_m \mathsf{A}_1 \} \vdash \mathsf{u}_1 : \mathsf{B}
                                           \Gamma_2 \sqcup \{\mathsf{x}_2 :_m \mathsf{A}_2\} \vdash \mathsf{u}_2 : \mathsf{B}
                                           \mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset
                                                                                                                                                                                          TyTerm_PatSum
\frac{1}{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case} \{ \mathsf{Inl} \times_1 \mapsto \mathsf{u}_1, \; \mathsf{Inr} \times_2 \mapsto \mathsf{u}_2 \} : \mathsf{B}}
                              \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1 \otimes \mathsf{A}_2
                              \Gamma_2 \sqcup \{ \mathsf{x}_1 :_m \mathsf{A}_1, \mathsf{x}_2 :_m \mathsf{A}_2 \} \vdash \mathsf{u} : \mathsf{B}
                              \mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset
                                                                                                                                                                TyTerm_PatProd
                      \overline{m \cdot \Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{case}(\mathsf{x}_1, \mathsf{x}_2) \mapsto \mathsf{u} : \mathsf{B}}
                                           \Gamma_1 \vdash \mathsf{t} : !^{m'} \mathsf{A}
                                          \Gamma_2 \sqcup \{ \mathsf{x} :_{m \cdot m'} \mathsf{A}_1 \} \vdash \mathsf{u} : \mathsf{B}
                                           \mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset
                                                                                                                                                             TyTerm_Patexp
                             \overline{m \cdot \Gamma_1 \sqcup \Gamma_2 \, \vdash \, \mathsf{t} \, \succ \! \mathsf{case} \, |\!\!|^{m'} \, \mathsf{x} \mapsto \, \mathsf{u} : \mathsf{B}}
                                    \Gamma_1 \vdash \mathsf{t} : \mathsf{A}_1 \rtimes \mathsf{A}_2
                                    \uparrow \cdot \Gamma_2 \sqcup \{ \mathbf{x} :_{\nu} \mathbf{A}_1 \} \vdash \mathbf{u} : \mathbf{B}
                      \frac{\mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \, \vdash \, t \ \succ \! \mathsf{mapL} \times \mapsto \, \mathsf{u} : \mathsf{B} \rtimes \mathsf{A}_2} \quad \mathrm{TYTERM\_MAPAMPAR}
                                       \Gamma_1 \vdash \mathsf{t} : {}^n | \mathbf{A}_2 |
                                       \Gamma_2 \vdash \mathsf{u} : \mathsf{A}_1 \rtimes \mathsf{A}_2
                                      \frac{\mathsf{names}(\Gamma_1) \cap \mathsf{names}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup (n \cdot \uparrow) \cdot \Gamma_2 \vdash \mathsf{t} \triangleleft \bullet \mathsf{u} : \mathsf{A}_1} \quad \mathsf{TYTERM\_FILLCOMP}
                                                              \frac{\Gamma \vdash \mathsf{t} : \sqrt[n]{1}}{\Gamma \vdash \mathsf{t} \triangleleft () : 1} \quad \mathsf{TYTERM\_FILLUNIT}
                                                        \frac{\Gamma \vdash \mathsf{t} : {}^{n} \lfloor \mathsf{A}_{1} \oplus \mathsf{A}_{2} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : {}^{n} \lfloor \mathsf{A}_{1} \rfloor} \quad \mathsf{TYTERM\_FILLINL}
                                                       \frac{\Gamma \vdash \mathsf{t} : \, {}^{n} \lfloor \mathsf{A}_{1} \oplus \mathsf{A}_{2} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : \, {}^{n} | \mathsf{A}_{2} |} \quad \mathsf{TYTERM\_FILLINR}
                                         \frac{\Gamma \vdash \mathsf{t} : \sqrt[n]{\mathsf{A}_1 \otimes \mathsf{A}_2}}{\Gamma \vdash \mathsf{t} \triangleleft (,) : \sqrt[n]{\mathsf{A}_1} \triangleleft \sqrt[n]{\mathsf{A}_2}} \quad \mathsf{TYTERM\_FILLPROD}
                                                     \frac{\Gamma \vdash \mathsf{t} : {}^{n} \lfloor !^{n'} \mathsf{A} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft | \mathsf{n}' : {}^{n \cdot n'} | \mathsf{A} |} \quad \mathsf{TYTERM\_FILLEXP}
                                                        \overline{\emptyset \vdash \mathsf{alloc}_\mathsf{A} : {}^{\nu} [\mathsf{A}] \rtimes \mathsf{A}} \quad \mathrm{TyTerm}\_\mathrm{Alloc}
                                                        \frac{\Gamma \vdash \mathsf{t} : \mathsf{A}}{\Gamma \vdash \mathsf{to}_{\bowtie} \mathsf{t} : 1 \rtimes \mathsf{A}} \quad \mathsf{TYTERM\_ToAMPAR}
```

$$\frac{\Gamma \vdash t: \mathbf{1} \rtimes \mathbf{A}}{\Gamma \vdash \textbf{from}_{\rtimes} \, t: \mathbf{A}} \quad \text{TyTerm_FromAmpar}$$

 $H_1 = H_2$ "Inverse sign of context" operation

$$\begin{array}{c} \overline{\mathbb{Q}^{\text{--1}}\emptyset = \emptyset} & \operatorname{ATAPP_EMPTY} \\ \\ \overline{\mathbb{Q}^{\text{--1}}(\{\mathbb{Q}\mathbf{h}:_{m} \ ^{n}[\mathbf{A}]\} \sqcup \Gamma) = \{\mathbf{h}:^{m\cdot n} \ \mathbf{A}\} \sqcup \mathbb{Q}^{\text{--1}}\Gamma} & \operatorname{ATAPP_REC} \end{array}$$

 $eff_app_1 = eff_app_2$

(we assume effect lists are ε -terminated)

Definition rules: 51 good 0 bad Definition rule clauses: 127 good 0 bad