# Destination $\lambda$-calculus

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## 1 Term and value syntax

| | |
|---|---|
| tvar, $x$, $y$, $d$ | Term-level variable |
| hvar, $h$ | Hole |
| $n$ | |

| | | | |
|---|---|---|---|
| $v$ | ::= | | Term value |
| | \| | $\langle v_1 , \overline{v_2} \rangle_\Delta$ | Ampar |
| | \| | $@h$ | Destination |
| | \| | $()$ | Unit |
| | \| | $\mathsf{Inl}\ v$ | Left variant for sum |
| | \| | $\mathsf{Inr}\ v$ | Right variant for sum |
| | \| | $(v_1 , v_2)$ | Product |
| | \| | $\mathsf{Ɛ}^{M}\ v$ | Exponential |
| | \| | $\lambda x . t$ | Linear function |

| | | | |
|---|---|---|---|
| $\overline{v}$ | ::= | | Pseudo-value that may contain holes |
| | \| | $v$ | Term value |
| | \| | $h$ | Hole |
| | \| | $\mathsf{Inl}\ \overline{v}$ | Left variant with val or hole |
| | \| | $\mathsf{Inr}\ \overline{v}$ | Right variant with val or hole |
| | \| | $(\overline{v_1} , \overline{v_2})$ | Product with val or hole |
| | \| | $\mathsf{Ɛ}^{M}\ \overline{v}$ | Exponential with val or hole |

| | | | |
|---|---|---|---|
| $t,\ u$ | ::= | | Term |
| | \| | $v$ | Term value |
| | \| | $x$ | Variable |
| | \| | $t \succ u$ | Application |
| | \| | $t \succ \mathbf{case}\ () \mapsto u$ | Pattern-match on unit |
| | \| | $t \succ \mathbf{case}\ \{ \mathsf{Inl}\ x_1 \mapsto u_1 ,\ \mathsf{Inr}\ x_2 \mapsto u_2 \}$ | Pattern-match on sum |
| | \| | $t \succ \mathbf{case}\ (x_1 , x_2) \mapsto u$ | Pattern-match on product |
| | \| | $t \succ \mathbf{case}\ \mathsf{Ɛ}^{M}\ x \mapsto u$ | Pattern-match on exponential |
| | \| | $t \succ \mathbf{mapL}\ x \mapsto u$ | Map over the left side of the ampar |
| | \| | $\mathbf{to}_{\ltimes}\ t$ | Wrap t into a trivial ampar |
| | \| | $\mathbf{from}_{\ltimes}\ t$ | Extract value from trivial ampar |
| | \| | $\mathbf{alloc}_{T}$ | Return a fresh "identity" ampar object |
| | \| | $t \triangleleft ()$ | Fill destination with unit |
| | \| | $t \triangleleft \mathsf{Inl}$ | Fill destination with left variant |
| | \| | $t \triangleleft \mathsf{Inr}$ | Fill destination with right variant |
| | \| | $t \triangleleft (,)$ | Fill destination with product constructor |
| | \| | $t \triangleleft \mathsf{Ɛ}^{M}$ | Fill destination with exponential constructor |
| | \| | $t \triangleleft\!\bullet\ u$ | Fill destination with root of ampar $u$ |

## 2 Type system

| | | | |
|---|---|---|---|
| $\mathsf{T},\ \mathsf{U}$ | $::=$ | | Type |
| | $\mid$ | $\mathsf{1}$ | Unit |
| | $\mid$ | $\mathsf{T}_1 \oplus \mathsf{T}_2$ | Sum |
| | $\mid$ | $\mathsf{T}_1 \otimes \mathsf{T}_2$ | Product |
| | $\mid$ | $!^{\mathrm{M}}\,\mathsf{T}$ | Exponential |
| | $\mid$ | $\mathsf{T}_1 \rtimes \mathsf{T}_2$ | Ampar type (consuming $\mathsf{T}_1$ yields $\mathsf{T}_2$) |
| | $\mid$ | $\mathsf{T}_1 \;{}_{\mathrm{M}}\!\!\to \mathsf{T}_2$ | Linear function |
| | $\mid$ | ${}^{\mathrm{M}}\lfloor \mathsf{T} \rfloor$ | Destination |
| $\mathrm{M},\ \mathrm{N}$ | $::=$ | | Multiplicity (Semiring with product $\cdot$) |
| | $\mid$ | $\nu$ | Born now. Identity of the product |
| | $\mid$ | $\uparrow$ | One scope older |
| | $\mid$ | $\infty$ | Infinitely old / static. Absorbing for product |
| | $\mid$ | $\mathrm{M}_1 \cdot \ldots \cdot \mathrm{M}_n$ | Semiring product |
| $\mathsf{PC},\ \Gamma$ | $::=$ | | Positive typing context |
| | $\mid$ | $\{\mathsf{PA}_1, .., \mathsf{PA}_n\}$ | |
| | $\mid$ | $\mathrm{M}\cdot\Gamma$ | Increase age of bindings by $\mathrm{M}$ |
| | $\mid$ | $\Gamma_1 \sqcup \Gamma_2$ | |
| $\mathsf{PA}$ | $::=$ | | Positive type assignment |
| | $\mid$ | $x :_{\mathrm{M}} \mathsf{T}$ | Variable |
| | $\mid$ | $@h :_{\mathrm{M}} {}^{\mathrm{N}}\lfloor \mathsf{T} \rfloor$ | Destination ($\mathrm{M}$ is its own age; $\mathrm{N}$ is the age of values it accepts) |
| $\mathsf{NC},\ \Delta$ | $::=$ | | Negative typing context |
| | $\mid$ | $\{\mathsf{NA}_1, .., \mathsf{NA}_n\}$ | |
| | $\mid$ | ${}^{\mathrm{M}\cdot}\Delta$ | Increase age of bindings by $\mathrm{M}$ |
| | $\mid$ | $@^{-1}\Gamma$ | Invert the sign of the context |
| | $\mid$ | $\Delta_1 \sqcup \Delta_2$ | |
| $\mathsf{NA}$ | $::=$ | | Negative type assignment |
| | $\mid$ | $h :^{\mathrm{N}} \mathsf{T}$ | Hole ($\mathrm{N}$ is the age of values it accepts, its own age is undefined) |

$\boxed{\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \overline{v} : \mathsf{T}}$ *(Typing of extended values (require both positive and negative contexts))*

$$\textsc{Ty-w-H}\quad \frac{}{\{\,\} \mathrel{\mathsf{u}} \{h :^{\nu} \mathsf{T}\} \Vdash h : \mathsf{T}}$$

$$\textsc{Ty-w-D}\quad \frac{}{\{@h :_{\nu} {}^{\mathrm{N}}\lfloor \mathsf{T} \rfloor\} \mathrel{\mathsf{u}} \{\,\} \Vdash @h : {}^{\mathrm{N}}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{Ty-w-U}\quad \frac{}{\{\,\} \mathrel{\mathsf{u}} \{\,\} \Vdash () : \mathsf{1}}$$

$$\textsc{Ty-w-L}\quad \frac{\begin{array}{c}\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \overline{v} : \mathsf{T}_1 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \mathsf{Inl}\,\overline{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{Ty-w-P}\quad \frac{\begin{array}{c}\Gamma_1 \mathrel{\mathsf{u}} \Delta_1 \Vdash \overline{v_1} : \mathsf{T}_1 \\ \Gamma_2 \mathrel{\mathsf{u}} \Delta_2 \Vdash \overline{v_2} : \mathsf{T}_2 \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Delta_1) \cap \mathbf{dom}(\Delta_2) = \emptyset\end{array}}{\Gamma_1 \sqcup \Gamma_2 \mathrel{\mathsf{u}} \Delta_1 \sqcup \Delta_2 \Vdash (\overline{v_1}, \overline{v_2}) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{Ty-w-R}\quad \frac{\begin{array}{c}\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \overline{v} : \mathsf{T}_2 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \mathsf{Inr}\,\overline{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{Ty-w-E}\quad \frac{\begin{array}{c}\Gamma \mathrel{\mathsf{u}} \Delta \Vdash \overline{v} : \mathsf{T} \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{\mathrm{M}\cdot\Gamma \mathrel{\mathsf{u}} {}^{\mathrm{M}\cdot}\Delta \Vdash \rrparenthesis^{\mathrm{M}}\,\overline{v} : !^{\mathrm{M}}\,\mathsf{T}}$$

$$\textsc{Ty-w-A}\quad \frac{\begin{array}{c}\Gamma_1 \mathrel{\mathsf{u}} \{\,\} \Vdash v_1 : \mathsf{T}_1 \\ \Gamma_2 \mathrel{\mathsf{u}} @^{-1}\Gamma_1 \Vdash \overline{v_2} : \mathsf{T}_2 \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset\end{array}}{\Gamma_2 \mathrel{\mathsf{u}} \{\,\} \Vdash \langle v_1 , \overline{v_2} \rangle_{@^{-1}\Gamma_1} : \mathsf{T}_1 \rtimes \mathsf{T}_2}$$

$$\textsc{Ty-w-F}\quad \frac{\begin{array}{c}\Gamma \sqcup \{x :_{\mathrm{M}} \mathsf{T}_1\} \vdash t : \mathsf{T}_2 \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\{x :_{\mathrm{M}} \mathsf{T}_1\}) = \emptyset\end{array}}{\Gamma \mathrel{\mathsf{u}} \{\,\} \Vdash \lambda x\,.\,t : \mathsf{T}_1 \;{}_{\mathrm{M}}\!\!\to \mathsf{T}_2}$$

$\boxed{\Gamma \vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms (only a positive context is needed))*

$$\textsc{Ty-t-V}\quad \frac{\Gamma \mathbin{\mathsf{u}} \{\} \Vdash \mathsf{v} : \mathsf{T}}{\Gamma \vdash \mathsf{v} : \mathsf{T}} \qquad \textsc{Ty-t-X0}\quad \frac{}{\{\mathsf{x} :_{\nu} \mathsf{T}\} \vdash \mathsf{x} : \mathsf{T}} \qquad \textsc{Ty-t-XInf}\quad \frac{}{\{\mathsf{x} :_{\infty} \mathsf{T}\} \vdash \mathsf{x} : \mathsf{T}}$$

$$\textsc{Ty-t-App}\quad \frac{\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 {}_{\mathrm{M}}\!\!\rightarrow \mathsf{T}_2 \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{\mathrm{M}\cdot\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-t-PatU}\quad \frac{\Gamma_1 \vdash \mathsf{t} : \mathbf{1} \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{U} \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\ () \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-t-PatS}\quad \frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Gamma_2 \sqcup \{\mathsf{x}_1 :_{\mathrm{M}} \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U} \\ \Gamma_2 \sqcup \{\mathsf{x}_2 :_{\mathrm{M}} \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U} \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x}_1 :_{\mathrm{M}} \mathsf{T}_1\}) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x}_2 :_{\mathrm{M}} \mathsf{T}_2\}) = \emptyset\end{array}}{\mathrm{M}\cdot\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\ \{\, \mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2 \,\} : \mathsf{U}}$$

$$\textsc{Ty-t-PatP}\quad \frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Gamma_2 \sqcup \{\mathsf{x}_1 :_{\mathrm{M}} \mathsf{T}_1, \mathsf{x}_2 :_{\mathrm{M}} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U} \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x}_1 :_{\mathrm{M}} \mathsf{T}_1\}) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x}_2 :_{\mathrm{M}} \mathsf{T}_2\}) = \emptyset \\ \mathbf{dom}(\{\mathsf{x}_1 :_{\mathrm{M}} \mathsf{T}_1\}) \cap \mathbf{dom}(\{\mathsf{x}_2 :_{\mathrm{M}} \mathsf{T}_2\}) = \emptyset\end{array}}{\mathrm{M}\cdot\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\ (\mathsf{x}_1\,,\ \mathsf{x}_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-t-PatE}\quad \frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : !^{\mathrm{N}}\,\mathsf{T} \\ \Gamma_2 \sqcup \{\mathsf{x} :_{\mathrm{M}\cdot\mathrm{N}} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U} \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x} :_{\mathrm{M}\cdot\mathrm{N}} \mathsf{T}\}) = \emptyset\end{array}}{\mathrm{M}\cdot\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{case}\ \rrparenthesis^{\mathrm{N}}\,\mathsf{x} \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-t-Map}\quad \frac{\begin{array}{c}\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1 \rtimes \mathsf{T}_2 \\ \uparrow\cdot\Gamma_2 \sqcup \{\mathsf{x} :_{\nu} \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{U} \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_2) \cap \mathbf{dom}(\{\mathsf{x} :_{\nu} \mathsf{T}_1\}) = \emptyset\end{array}}{\Gamma_1 \sqcup \Gamma_2 \vdash \mathsf{t} \succ \mathbf{mapL}\ \mathsf{x} \mapsto \mathsf{u} : \mathsf{U} \rtimes \mathsf{T}_2}$$

$$\textsc{Ty-t-FillC}\quad \frac{\Gamma_1 \vdash \mathsf{t} : {}^{\mathrm{N}}\lfloor \mathsf{T}_2 \rfloor \qquad \Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \rtimes \mathsf{T}_2 \qquad \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_2) = \emptyset}{\Gamma_1 \sqcup (\uparrow\cdot\mathrm{N})\cdot\Gamma_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_1}$$

$$\textsc{Ty-t-FillU}\quad \frac{\Gamma \vdash \mathsf{t} : {}^{\mathrm{N}}\lfloor \mathbf{1} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft () : \mathbf{1}} \qquad \textsc{Ty-t-FillL}\quad \frac{\Gamma \vdash \mathsf{t} : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inl} : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \rfloor} \qquad \textsc{Ty-t-FillR}\quad \frac{\Gamma \vdash \mathsf{t} : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \mathsf{Inr} : {}^{\mathrm{N}}\lfloor \mathsf{T}_2 \rfloor}$$

$$\textsc{Ty-t-FillP}\quad \frac{\Gamma \vdash \mathsf{t} : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft (,) : {}^{\mathrm{N}}\lfloor \mathsf{T}_1 \rfloor \otimes {}^{\mathrm{N}}\lfloor \mathsf{T}_2 \rfloor} \qquad \textsc{Ty-t-FillE}\quad \frac{\Gamma \vdash \mathsf{t} : {}^{\mathrm{M}}\lfloor !^{\mathrm{N}}\,\mathsf{T} \rfloor}{\Gamma \vdash \mathsf{t} \triangleleft \rrparenthesis^{\mathrm{N}} : {}^{\mathrm{M}\cdot\mathrm{N}}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{Ty-t-Alloc}\quad \frac{}{\{\} \vdash \mathbf{alloc}_{\mathsf{T}} : {}^{\nu}\lfloor \mathsf{T} \rfloor \rtimes \mathsf{T}} \qquad \textsc{Ty-t-ToA}\quad \frac{\Gamma \vdash \mathsf{t} : \mathsf{T}}{\Gamma \vdash \mathbf{to}_{\rtimes}\ \mathsf{t} : \mathbf{1} \rtimes \mathsf{T}} \qquad \textsc{Ty-t-FromA}\quad \frac{\Gamma \vdash \mathsf{t} : \mathbf{1} \rtimes \mathsf{T}}{\Gamma \vdash \mathbf{from}_{\rtimes}\ \mathsf{t} : \mathsf{T}}$$

$\boxed{\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}}$ *(Typing of commands (only a positive context is needed))*

$$\textsc{Ty-c}\quad \frac{\begin{array}{c}\Gamma_{11} \sqcup \Gamma_{12} \vdash \mathsf{v} : \mathsf{T} \\ \Gamma_2 \mathbin{\mathsf{u}} @^{-1}\Gamma_{12} \Vdash e \\ \mathbf{dom}(\Gamma_{11}) \cap \mathbf{dom}(\Gamma_{12}) = \emptyset \\ \mathbf{dom}(\Gamma_{11}) \cap \mathbf{dom}(\Gamma_2) = \emptyset \\ \mathbf{dom}(\Gamma_{12}) \cap \mathbf{dom}(\Gamma_2) = \emptyset\end{array}}{\Gamma_{11} \sqcup \Gamma_2 \vdash \mathsf{v} \diamond e : \mathsf{T}}$$

# 3 Effects and big-step semantics

$$
\begin{array}{llll}
e & ::= & & \text{Effect} \\
& | & \varepsilon & \\
& | & \mathsf{h} \coloneqq \overline{\mathsf{v}} & \\
& | & e_1, \ldots, e_{\mathsf{n}} & \text{Chain effects}
\end{array}
$$

$\boxed{\Gamma \,\mathsf{u}\, \Delta \Vdash e}$ *(Typing of effects (require both positive and negative contexts))*

$$
\textsc{Ty-E-N}\quad \frac{}{\{\,\} \,\mathsf{u}\, \{\,\} \Vdash \varepsilon}
$$

$$
\textsc{Ty-E-A}\quad \frac{\begin{array}{c}\Gamma \,\mathsf{u}\, \Delta \Vdash \overline{\mathsf{v}} : \mathsf{T} \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\{@\mathsf{h} :_{\nu} {}^{\mathsf{N}}\lfloor \mathsf{T} \rfloor\}) = \emptyset \\ \mathbf{dom}(\Gamma) \cap \mathbf{dom}(\Delta) = \emptyset \\ \mathbf{dom}(\{@\mathsf{h} :_{\nu} {}^{\mathsf{N}}\lfloor \mathsf{T} \rfloor\}) \cap \mathbf{dom}(\Delta) = \emptyset\end{array}}{(\uparrow\cdot\mathsf{M}\cdot\mathsf{N})\cdot\Gamma \sqcup \mathsf{M}\cdot\{@\mathsf{h} :_{\nu} {}^{\mathsf{N}}\lfloor \mathsf{T} \rfloor\} \,\mathsf{u}\, {}^{(\mathsf{M}\cdot\mathsf{N})\cdot}\Delta \Vdash \mathsf{h} \coloneqq \overline{\mathsf{v}}}
$$

$$
\textsc{Ty-E-X}\quad \frac{\begin{array}{c}\Gamma_1 \,\mathsf{u}\, \Delta_1 \sqcup @^{-1}\Gamma_{22} \Vdash e_1 \\ \Gamma_{21} \sqcup \Gamma_{22} \,\mathsf{u}\, \Delta_2 \Vdash e_2 \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_{21}) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Gamma_{22}) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_1) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Gamma_{21}) \cap \mathbf{dom}(\Gamma_{22}) = \emptyset \\ \mathbf{dom}(\Gamma_{21}) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_{21}) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Gamma_{22}) \cap \mathbf{dom}(\Delta_1) = \emptyset \\ \mathbf{dom}(\Gamma_{22}) \cap \mathbf{dom}(\Delta_2) = \emptyset \\ \mathbf{dom}(\Delta_1) \cap \mathbf{dom}(\Delta_2) = \emptyset\end{array}}{\Gamma_1 \sqcup \Gamma_{21} \,\mathsf{u}\, \Delta_1 \sqcup \Delta_2 \Vdash e_1, e_2}
$$

$\boxed{\overline{\mathsf{v}_1}\ \Delta_1 \mid e_1 \rightsquigarrow \overline{\mathsf{v}_2}\ \Delta_2 \mid e_2}$ *(Big-step evaluation of effects on extended values)*

$$
\textsc{Sem-E-S}\quad \frac{\begin{array}{c}\mathsf{h} \notin \mathbf{dom}(\Delta_1) \\ \overline{\mathsf{v}_1}\ \Delta_1 \mid e_1 \rightsquigarrow \overline{\mathsf{v}_2}\ \Delta_2 \mid e_2\end{array}}{\overline{\mathsf{v}_1}\ \Delta_1 \mid \mathsf{h} \coloneqq \overline{\mathsf{v}'}, e_1 \rightsquigarrow \overline{\mathsf{v}_2}\ \Delta_2 \mid \mathsf{h} \coloneqq \overline{\mathsf{v}'}, e_2}
$$

$$
\textsc{Sem-E-F}\quad \frac{\begin{array}{c}\Gamma_1' \,\mathsf{u}\, \Delta_1' \Vdash \overline{\mathsf{v}'} : \mathsf{T} \\ \mathbf{dom}(\Gamma_1') \cap \mathbf{dom}(\Delta_1') = \emptyset \\ \mathbf{dom}(\Delta_1) \cap \mathbf{dom}(\{\mathsf{h} :^{\mathsf{N}} \mathsf{T}\}) = \emptyset \\ \mathbf{dom}(\Delta_1) \cap \mathbf{dom}(\Delta_1') = \emptyset \\ \overline{\mathsf{v}_1}[\mathsf{h} \coloneqq \overline{\mathsf{v}'}]\ (\Delta_1 \sqcup {}^{\mathsf{N}\cdot}\Delta_1') \mid e_1 \rightsquigarrow \overline{\mathsf{v}_2}\ \Delta_2 \mid e_2\end{array}}{\overline{\mathsf{v}_1}\ \Delta_1 \sqcup \{\mathsf{h} :^{\mathsf{N}} \mathsf{T}\} \mid \mathsf{h} \coloneqq \overline{\mathsf{v}'}, e_1 \rightsquigarrow \overline{\mathsf{v}_2}\ \Delta_2 \mid e_2}
$$

$\boxed{\mathsf{t} \Downarrow \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

$$
\textsc{Sem-T-V}\quad \frac{}{\mathsf{v} \Downarrow \mathsf{v} \diamond \varepsilon}
$$

$$
\textsc{Sem-T-App}\quad \frac{\begin{array}{c}\mathsf{t}_1 \Downarrow \mathsf{v}_1 \diamond e_1 \qquad \mathsf{t}_2 \Downarrow \lambda \mathsf{x} \,.\, \mathsf{u} \diamond e_2 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_3\end{array}}{\mathsf{t}_1 \succ \mathsf{t}_2 \Downarrow \mathsf{v}_3 \diamond e_1, e_2, e_3}
$$

$$
\textsc{Sem-T-PatU}\quad \frac{\mathsf{t}_1 \Downarrow () \diamond e_1 \qquad \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1 \succ \mathsf{case}\,() \mapsto \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-T-PatL}\quad \frac{\begin{array}{c}\mathsf{t} \Downarrow \mathsf{Inl}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_1[\mathsf{x}_1 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathsf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-T-PatR}\quad \frac{\begin{array}{c}\mathsf{t} \Downarrow \mathsf{Inr}\,\mathsf{v}_1 \diamond e_1 \\ \mathsf{u}_2[\mathsf{x}_2 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathsf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-T-PatP}\quad \frac{\begin{array}{c}\mathsf{t} \Downarrow (\mathsf{v}_1\,,\,\mathsf{v}_2) \diamond e_1 \\ \mathsf{u}[\mathsf{x}_1 \coloneqq \mathsf{v}_1][\mathsf{x}_2 \coloneqq \mathsf{v}_2] \Downarrow \mathsf{v}_2 \diamond e_2\end{array}}{\mathsf{t} \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-T-Map}\quad \frac{\begin{array}{c}\mathsf{t} \Downarrow \langle \mathsf{v}_1 \,{\scriptstyle\mathbf{9}}\, \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_1 \\ \mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_2 \\ \overline{\mathsf{v}_2}\ \Delta \mid e_2 \rightsquigarrow \overline{\mathsf{v}_4}\ \Delta' \mid e_3\end{array}}{\mathsf{t} \succ \mathsf{mapL}\,\mathsf{x} \mapsto \mathsf{u} \Downarrow \langle \mathsf{v}_3 \,{\scriptstyle\mathbf{9}}\, \overline{\mathsf{v}_4} \rangle_{\Delta'} \diamond e_1, e_3}
$$

$$
\textsc{Sem-T-Alloc}\quad \frac{\mathsf{fresh}\ \mathsf{h}}{\mathsf{alloc}_{\mathsf{T}} \Downarrow \langle @\mathsf{h} \,{\scriptstyle\mathbf{9}}\, \mathsf{h} \rangle_{\{\mathsf{h} :^{\nu} \mathsf{T}\}} \diamond \varepsilon}
$$

$$
\textsc{Sem-T-ToA}\quad \frac{\mathsf{t} \Downarrow \mathsf{v} \diamond e}{\mathsf{to}_{\ltimes}\,\mathsf{t} \Downarrow \langle () \,{\scriptstyle\mathbf{9}}\, \mathsf{v} \rangle_{\{\,\}} \diamond e}
$$

$$
\textsc{Sem-T-FromA}\quad \frac{\mathsf{t} \Downarrow \langle () \,{\scriptstyle\mathbf{9}}\, \mathsf{v} \rangle_{\{\,\}} \diamond e}{\mathsf{from}_{\ltimes}\,\mathsf{t} \Downarrow \mathsf{v} \diamond e}
$$

$$
\textsc{Sem-T-FillU}\quad \frac{\mathsf{t} \Downarrow @\mathsf{h} \diamond e}{\mathsf{t} \triangleleft () \Downarrow () \diamond e, \mathsf{h} \coloneqq ()}
$$

$$
\textsc{Sem-T-FillL}\quad \frac{\mathsf{t} \Downarrow @\mathsf{h} \diamond e \qquad \mathsf{fresh}\ \mathsf{h}'}{\mathsf{t} \triangleleft \mathsf{Inl} \Downarrow @\mathsf{h}' \diamond e, \mathsf{h} \coloneqq \mathsf{Inl}\,\mathsf{h}'}
$$

$$
\textsc{Sem-T-FillR}\quad \frac{\mathsf{t} \Downarrow @\mathsf{h} \diamond e}{\mathsf{t} \triangleleft \mathsf{Inr} \Downarrow @\mathsf{h}' \diamond e, \mathsf{h} \coloneqq \mathsf{Inr}\,\mathsf{h}'}
$$

$$
\textsc{Sem-T-FillP}\quad \frac{\mathsf{t} \Downarrow @\mathsf{h} \diamond e \qquad \mathsf{fresh}\ \mathsf{h}_1 \qquad \mathsf{fresh}\ \mathsf{h}_2}{\mathsf{t} \triangleleft (,) \Downarrow (@\mathsf{h}_1\,,\,@\mathsf{h}_2) \diamond e, \mathsf{h} \coloneqq (\mathsf{h}_1\,,\,\mathsf{h}_2)}
$$

$$
\textsc{Sem-T-FillC}\quad \frac{\mathsf{t} \Downarrow @\mathsf{h} \diamond e_1 \qquad \mathsf{u} \Downarrow \langle \mathsf{v}_1 \,{\scriptstyle\mathbf{9}}\, \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_2}{\mathsf{t} \triangleleft\!\bullet\, \mathsf{u} \Downarrow \mathsf{v}_1 \diamond e_1, e_2, \mathsf{h} \coloneqq \overline{\mathsf{v}_2}}
$$

## 4 Type safety

**Theorem 1** (Type safety)**.** *If* $\textsf{destOnly}\,\Gamma$ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$*.*

**Theorem 2** (Type safety for complete programs)**.** *If* $\{\,\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond \varepsilon$ *and* $\{\,\} \vdash \mathsf{v} : \mathsf{T}$