

Destination λ -calculus

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1 Term and value syntax

var, x, y Term-level variable name
 k Index for ranges

hdn, h	$::=$ $ \quad h+h'$ $ \quad \max(H)$	Hole or destination name (\mathbb{N}) M Sum M Maximum of a set of holes
hdns, H	$::=$ $ \quad \{h_1, \dots, h_k\}$ $ \quad H_1 \cup H_2$ $ \quad H \pm h$ $ \quad \text{hnames}(\Gamma)$ $ \quad \text{hnames}(C)$	Set of hole names M Union of sets M Increase all names from H by h . M Hole names of a context (requires $\text{ctx_NoVar}(\Gamma)$) M Hole names of an evaluation context
term, t, u	$::=$ $ \quad v$ $ \quad x$ $ \quad t \succ u$ $ \quad t ; u$ $ \quad t \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$ $ \quad t \succ \text{case}_m (x_1, x_2) \mapsto u$ $ \quad t \succ \text{case}_m E^n x \mapsto u$ $ \quad t \succ \text{map } x \mapsto u$ $ \quad \text{to}_\times t$ $ \quad \text{from}_\times t$ $ \quad \text{alloc}$ $ \quad t \triangleleft ()$ $ \quad t \triangleleft \text{Inl}$ $ \quad t \triangleleft \text{Inr}$ $ \quad t \triangleleft E^m$ $ \quad t \triangleleft (,)$ $ \quad t \triangleleft (\lambda x_m \mapsto u)$ $ \quad t \triangleleft \bullet u$ $ \quad t[x := v]$	Term Value Variable Application Pattern-match on unit Pattern-match on sum Pattern-match on product Pattern-match on exponential Map over the right side of ampar t Wrap t into a trivial ampar Extract value from trivial ampar Return a fresh "identity" ampar object Fill destination with unit Fill destination with left variant Fill destination with right variant Fill destination with exponential constructor Fill destination with product constructor Fill destination with function Fill destination with root of ampar u M
val, v	$::=$ $ \quad -h$ $ \quad +h$ $ \quad ()$ $ \quad \lambda^v x_m \mapsto t$ $ \quad \text{Inl } v$ $ \quad \text{Inr } v$ $ \quad E^m v$ $ \quad (v_1, v_2)$ $ \quad H(v_1, v_2)$ $ \quad v \pm h$	Term value Hole Destination Unit Lambda abstraction Left variant for sum Right variant for sum Exponential Product Ampar M Rename hole names inside v by shifting them by h

ectx, c	::=	$\square \triangleright u$ $v \triangleright \square$ $\square ; u$ $\square \triangleright \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$ $\square \triangleright \text{case}_m (x_1, x_2) \mapsto u$ $\square \triangleright \text{case}_m E^n x \mapsto u$ $\square \triangleright \text{map } x \mapsto u$ $\text{to}_\times \square$ $\text{from}_\times \square$ $\square \triangleleft ()$ $\square \triangleleft \text{Inl}$ $\square \triangleleft \text{Inr}$ $\square \triangleleft E^m$ $\square \triangleleft ()$ $\square \triangleleft (\lambda x_m \mapsto u)$ $\square \triangleleft \bullet u$ $v \triangleleft \bullet \square$ $\overset{\text{op}}{\text{H}} \langle v_1, \square$	Evaluation context component
			Application
			Application
			Pattern-match on unit
			Pattern-match on sum
			Pattern-match on product
			Pattern-match on exponential
			Map over the right side of ampar
			Wrap into a trivial ampar
			Extract value from trivial ampar
			Fill destination with unit
			Fill destination with left variant
			Fill destination with right variant
			Fill destination with exponential constructor
			Fill destination with product constructor
			Fill destination with function
			Fill destination with root of ampar
			Fill destination with root of ampar
			Open ampar. Only new addition to term shapes
ectxs, C	::=	\square $C \circ c$ $C[\overset{\text{h}}{\text{h}} :=_{\text{H}} v]$	Evaluation context stack
			Represent the empty stack / "identity" evaluation context
			Push c on top of C
		M	Fill h in C with value v (that may contain holes)

2 Type system

type, T, U	::=	1 $T_1 \oplus T_2$ $T_1 \otimes T_2$ $!^m T$ $T_1 \times T_2$ $T_1 \xrightarrow{m_1} T_2$ $[T]^m$	Type
			Unit
			Sum
			Product
			Exponential
			Ampar type (consuming T_2 yields T_1)
			Function
			Destination
mode, m, n	::=	pa ω $m_1 \cdot \dots \cdot m_k$	Mode (Semiring)
			Pair of a multiplicity and age
			Error case (incompatible types, multiplicities, or ages)
		M	Semiring product
mul, p	::=	1 ω $p_1 \cdot \dots \cdot p_k$	Multiplicity (first component of modality)
			Linear. Neutral element of the product
			Non-linear. Absorbing for the product
		M	Semiring product
age, a	::=	ν \uparrow ∞ $a_1 \cdot \dots \cdot a_k$	Age (second component of modality)
			Born now. Neutral element of the product
			One scope older
			Infinitely old / static. Absorbing for the product
		M	Semiring product
bndr_var	::=	$x :_m T$	Variable binding
bndr_dest	::=	$+\text{h} :_m [T]^n$	Dest binding (m is its own modality; n is the modality for values it accepts)
bndr_hole	::=	$-\text{h} : T^n$	Hole binding (n is the modality for values it accepts, it doesn't have a modality on i

ctx, Γ , Δ , Π	::=	Typing context
	(mvar mdest mhole)	Actual representation of contexts for Coq proofs (cannot hide)
	$m\cdot\Gamma$	M Multiply each binding by m
	$\Gamma_1 \uplus \Gamma_2$	M Sum contexts Γ_1 and Γ_2 . Duplicate keys with incompatible values will be
	$-\Gamma$	M Transforms dest bindings into a hole bindings (requires <code>ctx_DestOnly</code> Γ a

$\boxed{\Gamma \Vdash v : \mathbf{T}}$

(Typing of values (raw))

$\frac{\text{TYR-VAL-H}}{\{-h : \mathbf{T}^{\text{lv}}\} \Vdash -h : \mathbf{T}}$	$\frac{\text{TYR-VAL-D} \quad \text{ctx_Compatible } \Gamma \text{ } +h : \text{lv } [\mathbf{T}]^n}{\Gamma \Vdash +h : [\mathbf{T}]^n}$	$\frac{\text{TYR-VAL-U}}{\{\} \Vdash () : \mathbf{1}}$	$\frac{\text{TYR-VAL-F} \quad \text{ctx_DestOnly } \Delta \quad \Delta \uplus \{x : m \mathbf{T}_1\} \vdash t : \mathbf{T}_2}{\Delta \Vdash \lambda^v x_m \mapsto t : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2}$
$\frac{\text{TYR-VAL-L} \quad \Gamma \Vdash v : \mathbf{T}_1}{\Gamma \Vdash \text{Inl } v : \mathbf{T}_1 \oplus \mathbf{T}_2}$	$\frac{\text{TYR-VAL-R} \quad \Gamma \Vdash v : \mathbf{T}_2}{\Gamma \Vdash \text{Inr } v : \mathbf{T}_1 \oplus \mathbf{T}_2}$	$\frac{\text{TYR-VAL-P} \quad \Gamma_1 \Vdash v_1 : \mathbf{T}_1 \quad \Gamma_2 \Vdash v_2 : \mathbf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (v_1, v_2) : \mathbf{T}_1 \otimes \mathbf{T}_2}$	$\frac{\text{TYR-VAL-E} \quad \Gamma \Vdash v : \mathbf{T}}{n\cdot\Gamma \Vdash \mathbf{E}^n v : !^n \mathbf{T}}$
$\frac{\text{TYR-VAL-A} \quad \begin{array}{l} \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_DestOnly } \Delta_3 \\ \text{ctx_LinOnly } \Delta_3 \\ \text{ctx_IsValid } \Delta_3 \\ \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_Disjoint } \Delta_1 \Delta_3 \\ \text{ctx_Disjoint } \Delta_2 \Delta_3 \\ \Delta_1 \uplus (-\Delta_3) \Vdash v_1 : \mathbf{T}_1 \\ \Delta_2 \uplus \Delta_3 \Vdash v_2 : \mathbf{T}_2 \end{array}}{\Delta_1 \uplus \Delta_2 \Vdash \text{hnames}(-\Delta_3)(v_1, v_2) : \mathbf{T}_1 \ltimes \mathbf{T}_2}$			

$\boxed{\Pi \vdash t : \mathbf{T}}$

(Typing of terms)

$\frac{\text{TY-TERM-VAL} \quad \text{ctx_DestOnly } \Delta \quad \Delta \vdash v : \mathbf{T}}{\Delta \vdash v : \mathbf{T}}$	$\frac{\text{TY-TERM-VAR} \quad \text{ctx_Compatible } \Pi \quad x : \textcolor{teal}{lv} \mathbf{T}}{\Pi \vdash x : \mathbf{T}}$	$\frac{\text{TY-TERM-APP} \quad \Pi_1 \vdash t : \mathbf{T}_1 \quad \Pi_2 \vdash u : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2}{\textcolor{teal}{m} \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ u : \mathbf{T}_2}$		
$\frac{\text{TY-TERM-PATU} \quad \Pi_1 \vdash t : \mathbf{1} \quad \Pi_2 \vdash u : \mathbf{U}}{\Pi_1 \uplus \Pi_2 \vdash t ; u : \mathbf{U}}$	$\frac{\text{TY-TERM-PATS} \quad \begin{array}{l} \text{ctx_Disjoint } \Pi_2 \{x_1 : \textcolor{teal}{m} \mathbf{T}_1\} \\ \text{ctx_Disjoint } \Pi_2 \{x_2 : \textcolor{teal}{m} \mathbf{T}_2\} \\ \Pi_1 \vdash t : \mathbf{T}_1 \oplus \mathbf{T}_2 \\ \Pi_2 \uplus \{x_1 : \textcolor{teal}{m} \mathbf{T}_1\} \vdash u_1 : \mathbf{U} \\ \Pi_2 \uplus \{x_2 : \textcolor{teal}{m} \mathbf{T}_2\} \vdash u_2 : \mathbf{U} \end{array}}{\textcolor{teal}{m} \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_{\textcolor{teal}{m}} \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \} : \mathbf{U}}$			
$\frac{\text{TY-TERM-PATP} \quad \begin{array}{l} \text{ctx_Disjoint } \Pi_2 \{x_1 : \textcolor{teal}{m} \mathbf{T}_1\} \\ \text{ctx_Disjoint } \Pi_2 \{x_2 : \textcolor{teal}{m} \mathbf{T}_2\} \\ \text{ctx_Disjoint } \{x_1 : \textcolor{teal}{m} \mathbf{T}_1\} \{x_2 : \textcolor{teal}{m} \mathbf{T}_2\} \\ \Pi_1 \vdash t : \mathbf{T}_1 \otimes \mathbf{T}_2 \\ \Pi_2 \uplus \{x_1 : \textcolor{teal}{m} \mathbf{T}_1\} \uplus \{x_2 : \textcolor{teal}{m} \mathbf{T}_2\} \vdash u : \mathbf{U} \end{array}}{\textcolor{teal}{m} \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_{\textcolor{teal}{m}} (x_1, x_2) \mapsto u : \mathbf{U}}$	$\frac{\text{TY-TERM-PATE} \quad \begin{array}{l} \text{ctx_Disjoint } \Pi_2 \{x : \textcolor{teal}{m} \cdot \textcolor{teal}{n} \mathbf{T}\} \\ \Pi_1 \vdash t : !^{\textcolor{teal}{n}} \mathbf{T} \\ \Pi_2 \uplus \{x : \textcolor{teal}{m} \cdot \textcolor{teal}{n} \mathbf{T}\} \vdash u : \mathbf{U} \end{array}}{\textcolor{teal}{m} \cdot \Pi_1 \uplus \Pi_2 \vdash t \succ \text{case}_{\textcolor{teal}{m}} \textcolor{teal}{E}^{\textcolor{teal}{n}} x \mapsto u : \mathbf{U}}$	$\frac{\text{TY-TERM-MAP} \quad \begin{array}{l} \text{ctx_Disjoint } \Pi_2 \{x : \textcolor{teal}{lv} \mathbf{T}_2\} \\ \Pi_1 \vdash t : \mathbf{T}_1 \ltimes \mathbf{T}_2 \\ \textcolor{teal}{l} \uparrow \cdot \Pi_2 \uplus \{x : \textcolor{teal}{lv} \mathbf{T}_2\} \vdash u : \mathbf{U} \end{array}}{\Pi_1 \uplus \Pi_2 \vdash t \succ \text{map } x \mapsto u : \mathbf{T}_1 \ltimes \mathbf{U}}$		
$\frac{\text{TY-TERM-TOA} \quad \Pi \vdash t : \mathbf{T}}{\Pi \vdash \text{to}_{\ltimes} t : \mathbf{T} \ltimes \mathbf{1}}$	$\frac{\text{TY-TERM-FROMA} \quad \Pi \vdash t : \mathbf{T} \ltimes \mathbf{1}}{\Pi \vdash \text{from}_{\ltimes} t : \mathbf{T}}$	$\frac{\text{TY-TERM-ALLOC} \quad \{\} \vdash \text{alloc} : \mathbf{T} \ltimes [\mathbf{T}]^{\textcolor{teal}{lv}}}{\{\} \vdash \text{alloc} : \mathbf{T} \ltimes [\mathbf{T}]^{\textcolor{teal}{lv}}}$	$\frac{\text{TY-TERM-FILLU} \quad \Pi \vdash t : [\mathbf{1}]^{\textcolor{teal}{n}}}{\Pi \vdash t \triangleleft () : \mathbf{1}}$	$\frac{\text{TY-TERM-FILLL} \quad \Pi \vdash t : [\mathbf{T}_1 \oplus \mathbf{T}_2]^{\textcolor{teal}{n}}}{\Pi \vdash t \triangleleft \text{Inl} : [\mathbf{T}_1]^{\textcolor{teal}{n}}}$
$\frac{\text{TY-TERM-FILLR} \quad \Pi \vdash t : [\mathbf{T}_1 \oplus \mathbf{T}_2]^{\textcolor{teal}{n}}}{\Pi \vdash t \triangleleft \text{Inr} : [\mathbf{T}_2]^{\textcolor{teal}{n}}}$	$\frac{\text{TY-TERM-FILLP} \quad \Pi \vdash t : [\mathbf{T}_1 \otimes \mathbf{T}_2]^{\textcolor{teal}{n}}}{\Pi \vdash t \triangleleft (,) : [\mathbf{T}_1]^{\textcolor{teal}{n}} \otimes [\mathbf{T}_2]^{\textcolor{teal}{n}}}$	$\frac{\text{TY-TERM-FILLE} \quad \Pi \vdash t : [!^{\textcolor{teal}{n}'} \mathbf{T}]^{\textcolor{teal}{n}}}{\Pi \vdash t \triangleleft \textcolor{teal}{E}^{\textcolor{teal}{n}'} : [\mathbf{T}]^{\textcolor{teal}{n}' \cdot \textcolor{teal}{n}}}$	$\frac{\text{TY-TERM-FILLF} \quad \begin{array}{l} \text{ctx_Disjoint } \Pi_2 \{x : \textcolor{teal}{m} \mathbf{T}_1\} \\ \Pi_1 \vdash t : [\mathbf{T}_1 \xrightarrow{\textcolor{teal}{m}} \mathbf{T}_2]^{\textcolor{teal}{n}} \\ \Pi_2 \uplus \{x : \textcolor{teal}{m} \mathbf{T}_1\} \vdash u : \mathbf{T}_2 \end{array}}{\Pi_1 \uplus (\textcolor{teal}{l} \uparrow \cdot \textcolor{teal}{n}) \cdot \Pi_2 \vdash t \triangleleft (\lambda x_{\textcolor{teal}{m}} \mapsto u) : \mathbf{1}}$	
$\frac{\text{TY-TERM-FILLC} \quad \Pi_1 \vdash t : [\mathbf{T}_1]^{\textcolor{teal}{n}} \quad \Pi_2 \vdash u : \mathbf{T}_1 \ltimes \mathbf{T}_2}{\Pi_1 \uplus (\textcolor{teal}{l} \uparrow \cdot \textcolor{teal}{n}) \cdot \Pi_2 \vdash t \triangleleft \bullet u : \mathbf{T}_2}$				

$$\Delta \vdash C : \mathbf{T}_1 \multimap \mathbf{T}_2$$

(Typing of evaluation contexts)

TYR-ECTX-ID $\frac{\{\} \vdash \square : \mathbf{U}_0 \multimap \mathbf{U}_0}{\{\} \vdash \square : \mathbf{U}_0 \multimap \mathbf{U}_0}$	TYR-ECTX-APPFOC1 $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{mode_IsValid } m \\ \text{ctx_IsValid } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{T}_2 \multimap \mathbf{U}_0 \\ \Delta_2 \vdash u : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2 \end{array}}{\Delta_1 \vdash C \circ (\square \succ u) : \mathbf{T}_1 \multimap \mathbf{U}_0}$	TYR-ECTX-APPFOC2 $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } m \cdot \Delta_1 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{T}_2 \multimap \mathbf{U}_0 \\ \Delta_1 \vdash v : \mathbf{T}_1 \end{array}}{\Delta_2 \vdash C \circ (v \succ \square) : (\mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2) \multimap \mathbf{U}_0}$
	TYR-ECTX-PATUFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } \Delta_2 \\ \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square ; u) : \mathbf{1} \multimap \mathbf{U}_0}$	
TYR-ECTX-PATSFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \wedge \text{ctx_DestOnly } \Delta_2 \\ \text{mode_IsValid } m \\ \text{ctx_IsValid } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x_1 : m \mathbf{T}_1\} \vdash u_1 : \mathbf{U} \\ \Delta_2 \uplus \{x_2 : m \mathbf{T}_2\} \vdash u_2 : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}) : (\mathbf{T}_1 \oplus \mathbf{T}_2) \multimap \mathbf{U}_0}$	TYR-ECTX-PATPFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \wedge \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_Disjoint } \{x_1 : m \mathbf{T}_1\} \{x_2 : m \mathbf{T}_2\} \\ \text{mode_IsValid } m \\ \text{ctx_IsValid } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x_1 : m \mathbf{T}_1\} \uplus \{x_2 : m \mathbf{T}_2\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m (x_1, x_2) \mapsto u) : (\mathbf{T}_1 \otimes \mathbf{T}_2) \multimap \mathbf{U}_0}$	
TYR-ECTX-PATEFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{mode_IsValid } m \\ \text{ctx_IsValid } \Delta_2 \\ m \cdot \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x : m \cdot m' \mathbf{T}\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{case}_m E^{m'} x \mapsto u) : !^{m'} \mathbf{T} \multimap \mathbf{U}_0}$	TYR-ECTX-MAPFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } \Delta_2 \\ \Delta_1 \uplus \Delta_2 \vdash C : \mathbf{U} \multimap \mathbf{U}_0 \\ l \uparrow \cdot \Delta_2 \uplus \{x : l \nu \mathbf{T}_2\} \vdash u : \mathbf{U} \end{array}}{\Delta_1 \vdash C \circ (\square \succ \text{map } x \mapsto u) : (\mathbf{T}_1 \ltimes \mathbf{T}_2) \multimap \mathbf{U}_0}$	
TYR-ECTX-TOAFOC $\frac{\Delta \vdash C : (\mathbf{T} \ltimes \mathbf{1}) \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\text{to}_\ltimes \square) : \mathbf{T} \multimap \mathbf{U}_0}$	TYR-ECTX-FROMAFOC $\frac{\Delta \vdash C : \mathbf{T} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\text{from}_\ltimes \square) : (\mathbf{T} \ltimes \mathbf{1}) \multimap \mathbf{U}_0}$	TYR-ECTX-FILLUFOC $\frac{\Delta \vdash C : \mathbf{1} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft ()) : [\mathbf{1}]^n \multimap \mathbf{U}_0}$
TYR-ECTX-FILLFOC $\frac{\Delta \vdash C : [\mathbf{T}_1]^n \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft \text{Inl}) : [\mathbf{T}_1 \oplus \mathbf{T}_2]^n \multimap \mathbf{U}_0}$	TYR-ECTX-FILLRFOC $\frac{\Delta \vdash C : [\mathbf{T}_2]^n \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft \text{Inr}) : [\mathbf{T}_1 \oplus \mathbf{T}_2]^n \multimap \mathbf{U}_0}$	TYR-ECTX-FILLPFOC $\frac{\Delta \vdash C : ([\mathbf{T}_1]^n \otimes [\mathbf{T}_2]^n) \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft (,)) : [\mathbf{T}_1 \otimes \mathbf{T}_2]^n \multimap \mathbf{U}_0}$
TYR-ECTX-FILLEFOC $\frac{\Delta \vdash C : [\mathbf{T}]^{m \cdot n} \multimap \mathbf{U}_0}{\Delta \vdash C \circ (\square \triangleleft E^m) : [!^m \mathbf{T}]^n \multimap \mathbf{U}_0}$	TYR-ECTX-FILLFFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } (l \uparrow \cdot n) \cdot \Delta_2 \\ \Delta_1 \uplus (l \uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{1} \multimap \mathbf{U}_0 \\ \Delta_2 \uplus \{x : m \mathbf{T}_1\} \vdash u : \mathbf{T}_2 \end{array}}{\Delta_1 \vdash C \circ (\square \triangleleft (\lambda x_m \mapsto u)) : [\mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2]^n \multimap \mathbf{U}_0}$	$\text{TYR-ECTX-FILLCFOC1}$ $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } (l \uparrow \cdot n) \cdot \Delta_2 \\ \Delta_1 \uplus (l \uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{T}_2 \multimap \mathbf{U}_0 \\ \Delta_2 \vdash u : \mathbf{T}_1 \ltimes \mathbf{T}_2 \end{array}}{\Delta_1 \vdash C \circ (\square \triangleleft \bullet u) : [\mathbf{T}_1]^n \multimap \mathbf{U}_0}$
$\text{TYR-ECTX-FILLCFOC2}$ $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \\ \text{ctx_DestOnly } \Delta_1 \\ \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_IsValid } \Delta_1 \\ \text{mode_IsValid } (l \uparrow \cdot n) \\ \Delta_1 \uplus (l \uparrow \cdot n) \cdot \Delta_2 \vdash C : \mathbf{T}_2 \multimap \mathbf{U}_0 \\ \Delta_1 \vdash v : [\mathbf{T}_1]^n \end{array}}{\Delta_2 \vdash C \circ (v \triangleleft \bullet \square) : \mathbf{T}_1 \ltimes \mathbf{T}_2 \multimap \mathbf{U}_0}$	TYR-ECTX-AOPENFOC $\frac{\begin{array}{l} \text{ctx_Disjoint } \Delta_1 \Delta_2 \wedge \text{ctx_Disjoint } \Delta_1 \Delta_3 \\ \text{hdns_Disjoint } \text{hnames}(C) \text{ hnames}(-\Delta_3) \\ \text{ctx_DestOnly } \Delta_1 \wedge \text{ctx_DestOnly } \Delta_2 \\ \text{ctx_DestOnly } \Delta_3 \wedge \text{ctx_LinOnly } \Delta_3 \\ \text{ctx_IsValid } \Delta_1 \\ \Delta_1 \uplus \Delta_2 \vdash C : (\mathbf{T}_1 \ltimes \mathbf{U}) \multimap \mathbf{U}_0 \\ \Delta_1 \uplus -\Delta_3 \Vdash v_1 : \mathbf{T}_1 \end{array}}{l \uparrow \cdot \Delta_2 \uplus \Delta_3 \vdash C \circ (\text{hnames}^{op}(-\Delta_3) \langle v_1, \square \rangle) : \mathbf{U} \multimap \mathbf{U}_0}$	

$$\vdash C[t] : \mathbf{T}$$

(Typing of extended terms (pair of evaluation context and term))

$$\text{TY-ETERM-CLOSED ETERM}$$

$$\frac{\begin{array}{l} \text{ctx_IsValid } \Gamma \\ \Gamma \vdash C : \mathbf{T} \multimap \mathbf{U}_0 \quad \Gamma \vdash t : \mathbf{T} \end{array}}{\vdash C[t] : \mathbf{U}_0}$$

3 Small-step semantics

$$\boxed{C[t] \longrightarrow C'[t']}$$

(Small-step evaluation of terms using evaluation contexts)

$$\begin{array}{c}
\text{SEM-ETERM-APPFOC1} \\
\frac{\text{term_NotVal } t}{C[t \succ u] \longrightarrow (C \circ (\Box \succ u))[t]} \\
\\
\text{SEM-ETERM-APPUNFOC1} \\
\frac{}{(C \circ (\Box \succ u))[v] \longrightarrow C[v \succ u]} \\
\\
\text{SEM-ETERM-APPFOC2} \\
\frac{\text{term_NotVal } u}{C[v \succ u] \longrightarrow (C \circ (v \succ \Box))[u]} \\
\\
\text{SEM-ETERM-APPUNFOC2} \\
\frac{}{(C \circ (v \succ \Box))[v'] \longrightarrow C[v \succ v']} \\
\\
\text{SEM-ETERM-APPREd} \\
\frac{}{C[v \succ (\lambda^v x_m \mapsto t)] \longrightarrow C[t[x := v]]} \\
\\
\text{SEM-ETERM-PATUFOC} \\
\frac{\text{term_NotVal } t}{C[t; u] \longrightarrow (C \circ (\Box; u))[t]} \\
\\
\text{SEM-ETERM-PATUUNFOC} \\
\frac{}{(C \circ (\Box; u))[v] \longrightarrow C[v; u]} \\
\\
\text{SEM-ETERM-PATUReD} \\
\frac{}{C[(\Box; u)] \longrightarrow C[u]} \\
\\
\text{SEM-ETERM-PATLFOC} \\
\frac{\text{term_NotVal } t}{C[t \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow (C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[t]} \\
\\
\text{SEM-ETERM-PATLUNFOC} \\
\frac{}{(C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[v] \longrightarrow C[v \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}]} \\
\\
\text{SEM-ETERM-PATLReD} \\
\frac{}{C[(\text{Inl } v) \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow C[u_1[x := v]]} \\
\\
\text{SEM-ETERM-PATRFOC} \\
\frac{\text{term_NotVal } t}{C[t \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow (C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[t]} \\
\\
\text{SEM-ETERM-PATRUNFOC} \\
\frac{}{(C \circ (\Box \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}))[v] \longrightarrow C[v \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}]} \\
\\
\text{SEM-ETERM-PATRReD} \\
\frac{}{C[(\text{Inr } v) \succ \text{case}_m \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}] \longrightarrow C[u_2[x := v]]} \\
\\
\text{SEM-ETERM-PATPFOC} \\
\frac{\text{term_NotVal } t}{C[t \succ \text{case}_m (x_1, x_2) \mapsto u] \longrightarrow (C \circ (\Box \succ \text{case}_m (x_1, x_2) \mapsto u))[t]} \\
\\
\text{SEM-ETERM-PATPUNFOC} \\
\frac{}{(C \circ (\Box \succ \text{case}_m (x_1, x_2) \mapsto u))[v] \longrightarrow C[v \succ \text{case}_m (x_1, x_2) \mapsto u]} \\
\\
\text{SEM-ETERM-PATPRed} \\
\frac{}{C[(v_1, v_2) \succ \text{case}_m (x_1, x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]} \\
\\
\text{SEM-ETERM-PATEFOC} \\
\frac{\text{term_NotVal } t}{C[t \succ \text{case}_m E^n x \mapsto u] \longrightarrow (C \circ (\Box \succ \text{case}_m E^n x \mapsto u))[t]} \\
\\
\text{SEM-ETERM-PATEUNFOC} \\
\frac{}{(C \circ (\Box \succ \text{case}_m E^n x \mapsto u))[v] \longrightarrow C[v \succ \text{case}_m E^n x \mapsto u]} \\
\\
\text{SEM-ETERM-PATERed} \\
\frac{}{C[E^n v \succ \text{case}_m E^n x \mapsto u] \longrightarrow C[u[x := v]]} \\
\\
\text{SEM-ETERM-MAPFOC} \\
\frac{\text{term_NotVal } t}{C[t \succ \text{map } x \mapsto u] \longrightarrow (C \circ (\Box \succ \text{map } x \mapsto u))[t]} \\
\\
\text{SEM-ETERM-MAPUNFOC} \\
\frac{}{(C \circ (\Box \succ \text{map } x \mapsto u))[v] \longrightarrow C[v \succ \text{map } x \mapsto u]} \\
\\
\text{SEM-ETERM-MAPREDaOPENFOC} \\
\frac{h' = \max(\text{hnames}(C))}{C[\text{H}\langle v_1, v_2 \rangle \succ \text{map } x \mapsto u] \longrightarrow (C \circ (\overset{\text{OP}}{\text{H} \pm h'} \langle v_1 \pm h', \Box \rangle))[u[x := v_2 \pm h']]} \\
\\
\text{SEM-ETERM-AOPENUNFOC} \\
\frac{}{(C \circ \overset{\text{OP}}{\text{H}} \langle v_1, \Box \rangle)[v_2] \longrightarrow C[\text{H}\langle v_1, v_2 \rangle]} \\
\\
\text{SEM-ETERM-ALLOCRed} \\
\frac{}{C[\text{alloc}] \longrightarrow C[\{1\} \langle +1, -1 \rangle]} \\
\\
\text{SEM-ETERM-TOAFOC} \\
\frac{\text{term_NotVal } t}{C[\text{to}_\times t] \longrightarrow (C \circ (\text{to}_\times \Box))[t]} \\
\\
\text{SEM-ETERM-TOAUNFOC} \\
\frac{}{(C \circ (\text{to}_\times \Box))[v] \longrightarrow C[\text{to}_\times v]}
\end{array}$$

SEM-ETERM-TOARED	SEM-ETERM-FROMAFOC $\text{term_NotVal } t$	SEM-ETERM-FROMAUNFOC
$\overline{C[\text{to}_{\times} v] \longrightarrow C[\{\}_{\langle v, () \rangle}]}$	$\overline{C[\text{from}_{\times} t] \longrightarrow (C \circ (\text{from}_{\times} \square))[t]}$	$\overline{(C \circ (\text{from}_{\times} \square))[v] \longrightarrow C[\text{from}_{\times} v]}$
SEM-ETERM-FROMARED	SEM-ETERM-FILLUFOC $\text{term_NotVal } t$	SEM-ETERM-FILLUUNFOC
$\overline{C[\text{from}_{\times} \{\}_{\langle v, () \rangle}] \longrightarrow C[v]}$	$\overline{C[t \triangleleft ()] \longrightarrow (C \circ (\square \triangleleft ()))[t]}$	$\overline{(C \circ (\square \triangleleft ()))[v] \longrightarrow C[v \triangleleft ()]}$
SEM-ETERM-FILLURED	SEM-ETERM-FILLLFOC $\text{term_NotVal } t$	SEM-ETERM-FILLLUNFOC
$\overline{C[+h \triangleleft ()] \longrightarrow C[h := \{\} ()][()]}$	$\overline{C[t \triangleleft \text{Inl}] \longrightarrow (C \circ (\square \triangleleft \text{Inl}))[t]}$	$\overline{(C \circ (\square \triangleleft \text{Inl}))[v] \longrightarrow C[v \triangleleft \text{Inl}]}$
SEM-ETERM-FILLLRED $h' = \max(\text{hnames}(C) \cup \{h\})$	SEM-ETERM-FILLRFOC $\text{term_NotVal } t$	
$\overline{C[+h \triangleleft \text{Inl}] \longrightarrow C[h := \{h'+1\} \text{Inl} - (h'+1)][+(h'+1)]}$	$\overline{C[t \triangleleft \text{Inr}] \longrightarrow (C \circ (\square \triangleleft \text{Inr}))[t]}$	
SEM-ETERM-FILLRUNFOC	SEM-ETERM-FILLRRED $h' = \max(\text{hnames}(C) \cup \{h\})$	
$\overline{(C \circ (\square \triangleleft \text{Inr}))[v] \longrightarrow C[v \triangleleft \text{Inr}]}$	$\overline{C[+h \triangleleft \text{Inr}] \longrightarrow C[h := \{h'+1\} \text{Inr} - (h'+1)][+(h'+1)]}$	
SEM-ETERM-FILLEFOC $\text{term_NotVal } t$	SEM-ETERM-FILLEUNFOC	
$\overline{C[t \triangleleft E^m] \longrightarrow (C \circ (\square \triangleleft E^m))[t]}$	$\overline{(C \circ (\square \triangleleft E^m))[v] \longrightarrow C[v \triangleleft E^m]}$	
SEM-ETERM-FILLERED $h' = \max(\text{hnames}(C) \cup \{h\})$	SEM-ETERM-FILLPFOC $\text{term_NotVal } t$	SEM-ETERM-FILLPUNFOC
$\overline{C[+h \triangleleft E^m] \longrightarrow C[h := \{h'+1\} E^m - (h'+1)][+(h'+1)]}$	$\overline{C[t \triangleleft (,)] \longrightarrow (C \circ (\square \triangleleft (,)))[t]}$	$\overline{(C \circ (\square \triangleleft (,)))[v] \longrightarrow C[v \triangleleft (,)]}$
SEM-ETERM-FILLPRED $h' = \max(\text{hnames}(C) \cup \{h\})$		
$\overline{C[+h \triangleleft (,)] \longrightarrow C[h := \{h'+1, h'+2\} (- (h'+1), - (h'+2))][+(h'+1), +(h'+2)]}$		
SEM-ETERM-FILLFFOC $\text{term_NotVal } t$	SEM-ETERM-FILLFUNFOC	
$\overline{C[t \triangleleft (\lambda x_m \mapsto u)] \longrightarrow (C \circ (\square \triangleleft (\lambda x_m \mapsto u)))[t]}$	$\overline{(C \circ (\square \triangleleft (\lambda x_m \mapsto u)))[v] \longrightarrow C[v \triangleleft (\lambda x_m \mapsto u)]}$	
SEM-ETERM-FILLFRED	SEM-ETERM-FILLCFOC1 $\text{term_NotVal } t$	SEM-ETERM-FILLCUNFOC1
$\overline{C[+h \triangleleft (\lambda x_m \mapsto u)] \longrightarrow C[h := \{\} \lambda^v x_m \mapsto u][()]}$	$\overline{C[t \triangleleft \bullet u] \longrightarrow (C \circ (\square \triangleleft \bullet u))[t]}$	$\overline{(C \circ (\square \triangleleft \bullet u))[v] \longrightarrow C[v \triangleleft \bullet u]}$
SEM-ETERM-FILLCFOC2	SEM-ETERM-FILLCUNFOC2	SEM-ETERM-FILLCRED $h' = \max(\text{hnames}(C) \cup \{h\})$
$\overline{C[v \triangleleft \bullet u] \longrightarrow (C \circ (v \triangleleft \bullet \square))[u]}$	$\overline{(C \circ (v \triangleleft \bullet \square))[v'] \longrightarrow C[v \triangleleft \bullet v']}$	$\overline{C[+h \triangleleft \bullet_h \langle v_1, v_2 \rangle] \longrightarrow C[h := (\text{H} \pm h') v_1 \pm h'][v_2 \pm h']}$