Destination λ-calculus

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1 Term and value syntax

var, x, y		Term-level variable name		
k		Index for ranges		

hdn, h	::=			Hole or destination name ($\mathbb{N}$)
	|	$h + h'$	M	Sum
	|	$\mathrm{max}(H)$	M	Maximum of a set of holes

hdns, H	::=			Set of hole names
	|	$\{h_1, .., h_k\}$		
	|	$H_1 \cup H_2$	M	Union of sets
	|	$H \pm h$	M	Increase all names from H by h.
	|	$\mathrm{hnames}(\Gamma)$	M	Hole names of a context (requires $\mathtt{ctx\_NoVar}(\Gamma)$)
	|	$\mathrm{hnames}(C)$	M	Hole names of an evaluation context

term, t, u	::=			Term
	|	v		Value
	|	x		Variable
	|	$t \succ u$		Application
	|	$t \,;\, u$		Pattern-match on unit
	|	$t \succ \mathbf{case}_m \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}$	bind x_1 in u_1 bind x_2 in u_2	Pattern-match on sum
	|	$t \succ \mathbf{case}_m (x_1 , x_2) \mapsto u$	bind x_1 in u bind x_2 in u	Pattern-match on product
	|	$t \succ \mathbf{case}_m \mathrm{E}^n x \mapsto u$	bind x in u	Pattern-match on exponential
	|	$t \succ \mathbf{map}\ x \mapsto u$	bind x in u	Map over the right side of ampar t
	|	$\mathbf{to}_\ltimes t$		Wrap t into a trivial ampar
	|	$\mathbf{from}_\ltimes t$		Extract value from trivial ampar
	|	$\mathbf{alloc}$		Return a fresh "identity" ampar object
	|	$t \lhd ()$		Fill destination with unit
	|	$t \lhd (\lambda x_m \mapsto u)$	bind x in u	Fill destination with function
	|	$t \lhd \mathsf{Inl}$		Fill destination with left variant
	|	$t \lhd \mathsf{Inr}$		Fill destination with right variant
	|	$t \lhd (,)$		Fill destination with product constructor
	|	$t \lhd \mathrm{E}^m$		Fill destination with exponential constructor
	|	$t \lhd\bullet u$		Fill destination with root of ampar u
	|	$t[x := v]$	M	

val, v	::=			Term value
	|	$-h$		Hole
	|	$+h$		Destination
	|	$()$		Unit
	|	$\lambda^v x_m \mapsto t$	bind x in t	Lambda abstraction
	|	$\mathsf{Inl}\, v$		Left variant for sum
	|	$\mathsf{Inr}\, v$		Right variant for sum
	|	$\mathrm{E}^m v$		Exponential
	|	(v_1 , v_2)		Product
	|	$_H\langle v_1 , v_2 \rangle$		Ampar
	|	$v \pm h$	M	Rename hole names inside v by shifting them by h

eterm, j	::=			Pseudo-term
	|	C[t]		

ectx, C	::=			Evaluation context
	|	$\square$		Identity
	|	$C \succ u$		Application
	|	$v \succ C$		Application
	|	$C \mathbin{;} u$		Pattern-match on unit
	|	$C \succ \mathbf{case}_m \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}$	bind x_1 in u_1	Pattern-match on sum
			bind x_2 in u_2	
	|	$C \succ \mathbf{case}_m\, (x_1 , x_2) \mapsto u$	bind x_1 in u	Pattern-match on product
			bind x_2 in u	
	|	$C \succ \mathbf{case}_m\, \mathrm{E}^n\, x \mapsto u$	bind x in u	Pattern-match on exponential
	|	$C \succ \mathbf{map}\, x \mapsto u$	bind x in u	Map over the right side of ampar
	|	$\mathbf{to}_\ltimes\, C$		Wrap into a trivial ampar
	|	$\mathbf{from}_\ltimes\, C$		Extract value from trivial ampar
	|	$C \triangleleft ()$		Fill destination with unit
	|	$C \triangleleft (\lambda x\,_m \mapsto u)$	bind x in u	Fill destination with function
	|	$C \triangleleft \mathsf{Inl}$		Fill destination with left variant
	|	$C \triangleleft \mathsf{Inr}$		Fill destination with right variant
	|	$C \triangleleft (,)$		Fill destination with product constructor
	|	$C \triangleleft \mathrm{E}^m$		Fill destination with exponential constructor
	|	$C \triangleleft\bullet\, u$		Fill destination with root of ampar
	|	$v \triangleleft\bullet\, C$		Fill destination with root of ampar
	|	$^{\mathrm{OP}}_{\mathtt{H}}\langle v_1 \,,\, C$		Open ampar. Only new addition to term shapes
	|	$C \circ C'$	M	Compose evaluation contexts
	|	$C[\mathtt{h} \coloneqq_{\mathtt{H}} v]$	M	Fill $\mathtt{h}$ with value v (that may contain holes)

2 Type system

type, **T**, **U**	::=		Type
	|	**1**	Unit
	|	$\mathbf{T}_1 \oplus \mathbf{T}_2$	Sum
	|	$\mathbf{T}_1 \otimes \mathbf{T}_2$	Product
	|	$!^m\, \mathbf{T}$	Exponential
	|	$\mathbf{T}_1 \ltimes \mathbf{T}_2$	Ampar type (consuming $\mathbf{T}_2$ yields $\mathbf{T}_1$)
	|	$\mathbf{T}_{1\ m_1}\!\!\to \mathbf{T}_2$	Function
	|	$\lfloor \mathbf{T} \rfloor^m$	Destination

mode, m, n	::=		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☹	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	M	Semiring product

mul, p	::=		Multiplicity (first component of modality)
	|	*1*	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 \cdot \ldots \cdot p_k$	M	Semiring product

age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	M	Semiring product

bndr, b	::=		Type assignment to either variable, destination or hole
	|	$x :_m \mathbf{T}$	Variable
	|	$+\mathtt{h} :_m \lfloor \mathbf{T} \rfloor^n$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-\mathtt{h} : \mathbf{T}^n$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

ctx, Γ, Δ	::=		Typing context
	$\{b_1, \ldots, b_k\}$		List of bindings
	$m{\cdot}\Gamma$	M	Multiply each binding by m
	$m{\cdot\cdot}\Gamma$	M	TODO remove later
	$\Gamma_1 \uplus \Gamma_2$	M	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mod
	$-\Gamma$	M	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)

$$\boxed{\Gamma \Vdash v : T} \qquad\qquad\qquad\qquad \textit{(Typing of values (raw))}$$

TYR-VAL-H
$$\frac{}{\{-h : T^{1\nu}\} \Vdash -h : T}$$

TYR-VAL-D
$$\frac{\texttt{ctx\_Compatible } \Gamma\ +h :_{1\nu} \lfloor T \rfloor^n}{\Gamma \Vdash +h : \lfloor T \rfloor^n}$$

TYR-VAL-U
$$\frac{}{\{\} \Vdash () : 1}$$

TYR-VAL-F
$$\frac{\texttt{ctx\_DestOnly } \Gamma \qquad \Gamma \uplus \{x :_m T_1\} \vdash t : T_2}{\Gamma \Vdash \lambda^\vee x_m \mapsto t : T_1\ {}_m\!\!\to T_2}$$

TYR-VAL-L
$$\frac{\Gamma \Vdash v : T_1}{\Gamma \Vdash \mathsf{Inl}\, v : T_1 \oplus T_2}$$

TYR-VAL-R
$$\frac{\Gamma \Vdash v : T_2}{\Gamma \Vdash \mathsf{Inr}\, v : T_1 \oplus T_2}$$

TYR-VAL-P
$$\frac{\Gamma_1 \Vdash v_1 : T_1 \qquad \Gamma_2 \Vdash v_2 : T_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (v_1 , v_2) : T_1 \otimes T_2}$$

TYR-VAL-E
$$\frac{\Gamma \Vdash v : T}{n\cdot\Gamma \Vdash \mathrm{E}^n v : !^n T}$$

TYR-VAL-A
$$\frac{\begin{array}{c}\texttt{ctx\_DestOnly } \Gamma_2 \uplus \Gamma_3 \\ \texttt{ctx\_DestOnly } \Gamma_1 \\ \Gamma_1 \uplus (-\Gamma_3) \Vdash v_1 : T_1 \\ \Gamma_2 \uplus \Gamma_3 \Vdash v_2 : T_2\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash {}_{\mathsf{hnames}(-\Gamma_3)}\langle v_1 , v_2 \rangle : T_1 \ltimes T_2}$$

$$\boxed{\Gamma \vdash t : T} \qquad\qquad\qquad\qquad \textit{(Typing of terms)}$$

TY-TERM-VAL
$$\frac{\texttt{ctx\_DestOnly } \Gamma \qquad \Gamma \Vdash v : T}{\Gamma \vdash v : T}$$

TY-TERM-VAR
$$\frac{\texttt{ctx\_Compatible } \Gamma\ x :_{1\nu} T}{\Gamma \vdash x : T}$$

TY-TERM-APP
$$\frac{\Gamma_1 \vdash t : T_1 \qquad \Gamma_2 \vdash u : T_1\ {}_m\!\!\to T_2}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ u : T_2}$$

TY-TERM-PATS
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_2\ \{x_1 :_m T_1\} \\ \texttt{ctx\_Disjoint } \Gamma_2\ \{x_2 :_m T_2\} \\ \Gamma_1 \vdash t : T_1 \oplus T_2 \\ \Gamma_2 \uplus \{x_1 :_m T_1\} \vdash u_1 : U \\ \Gamma_2 \uplus \{x_2 :_m T_2\} \vdash u_2 : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_m \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \} : U}$$

TY-TERM-PATU
$$\frac{\Gamma_1 \vdash t : 1 \qquad \Gamma_2 \vdash u : U}{\Gamma_1 \uplus \Gamma_2 \vdash t\,;u : U}$$

TY-TERM-PATP
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_2\ \{x_1 :_m T_1\} \\ \texttt{ctx\_Disjoint } \Gamma_2\ \{x_2 :_m T_2\} \\ \texttt{ctx\_Disjoint } \{x_1 :_m T_1\}\ \{x_2 :_m T_2\} \\ \Gamma_1 \vdash t : T_1 \otimes T_2 \\ \Gamma_2 \uplus \{x_1 :_m T_1, x_2 :_m T_2\} \vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_m\, (x_1 , x_2) \mapsto u : U}$$

TY-TERM-PATE
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_2\ \{x :_{m\cdot n} T\} \\ \Gamma_1 \vdash t : !^n T \\ \Gamma_2 \uplus \{x :_{m\cdot n} T\} \vdash u : U\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{case}_m\, \mathrm{E}^n x \mapsto u : U}$$

TY-TERM-MAP
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_2\ \{x :_{1\nu} T_2\} \\ \Gamma_1 \vdash t : T_1 \ltimes T_2 \\ 1{\uparrow}\cdot\Gamma_2 \uplus \{x :_{1\nu} T_2\} \vdash u : U\end{array}}{\Gamma_1 \uplus \Gamma_2 \vdash t \succ \mathbf{map}\, x \mapsto u : T_1 \ltimes U}$$

TY-TERM-TOA
$$\frac{\Gamma \vdash t : T}{\Gamma \vdash \mathbf{to}_\ltimes t : T \ltimes 1}$$

TY-TERM-FROMA
$$\frac{\Gamma \vdash t : T \ltimes 1}{\Gamma \vdash \mathbf{from}_\ltimes t : T}$$

TY-TERM-ALLOC
$$\frac{}{\{\} \vdash \mathbf{alloc} : T \ltimes \lfloor T \rfloor^{1\nu}}$$

TY-TERM-FILLU
$$\frac{\Gamma \vdash t : \lfloor 1 \rfloor^n}{\Gamma \vdash t \triangleleft () : 1}$$

TY-TERM-FILLF
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_2\ \{x :_m T_1\} \\ \Gamma_1 \vdash t : \lfloor T_1\ {}_m\!\!\to T_2 \rfloor^n \\ \Gamma_2 \uplus \{x :_m T_1\} \vdash u : T_2\end{array}}{\Gamma_1 \uplus (1{\uparrow}\cdot n)\cdot\Gamma_2 \vdash t \triangleleft (\lambda x_m \mapsto u) : 1}$$

TY-TERM-FILLL
$$\frac{\Gamma \vdash t : \lfloor T_1 \oplus T_2 \rfloor^n}{\Gamma \vdash t \triangleleft \mathsf{Inl} : \lfloor T_1 \rfloor^n}$$

TY-TERM-FILLR
$$\frac{\Gamma \vdash t : \lfloor T_1 \oplus T_2 \rfloor^n}{\Gamma \vdash t \triangleleft \mathsf{Inr} : \lfloor T_2 \rfloor^n}$$

TY-TERM-FILLP
$$\frac{\Gamma \vdash t : \lfloor T_1 \otimes T_2 \rfloor^n}{\Gamma \vdash t \triangleleft (,) : \lfloor T_1 \rfloor^n \otimes \lfloor T_2 \rfloor^n}$$

TY-TERM-FILLE
$$\frac{\Gamma \vdash t : \lfloor !^{n'} T \rfloor^n}{\Gamma \vdash t \triangleleft \mathrm{E}^{n'} : \lfloor T \rfloor^{n'\cdot n}}$$

TY-TERM-FILLC
$$\frac{\Gamma_1 \vdash t : \lfloor T_1 \rfloor^n \qquad \Gamma_2 \vdash u : T_1 \ltimes T_2}{\Gamma_1 \uplus (1{\uparrow}\cdot n)\cdot\Gamma_2 \vdash t \triangleleft\bullet u : T_2}$$

$$\boxed{\Gamma \Vdash C : T_1 \rightarrowtail T_2} \qquad\qquad\qquad\qquad \textit{(Typing of evaluation contexts)}$$

TYR-ECTX-ID
$$\frac{}{\{\} \Vdash \square : U_0 \rightarrowtail U_0}$$

TYR-ECTX-APP1
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_0\ \Gamma \\ \texttt{ctx\_DestOnly } \Gamma_0 \\ \texttt{ctx\_HoleOnly } \Gamma \\ -\Gamma_0 \uplus \Gamma \Vdash C : T_2 \rightarrowtail U_0 \\ \Gamma \neq \{\} \implies m = 1\nu \\ \Gamma_0 \vdash u : T_1\ {}_m\!\!\to T_2\end{array}}{\Gamma \Vdash C \circ (\square \succ u) : T_1 \rightarrowtail U_0}$$

TYR-ECTX-APP2
$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint } \Gamma_0\ \Gamma \\ \texttt{ctx\_DestOnly } \Gamma_0 \\ \texttt{ctx\_HoleOnly } \Gamma \\ -\Gamma_0 \uplus \Gamma \Vdash C : T_2 \rightarrowtail U_0 \\ \Gamma_0 \vdash v : T_1\end{array}}{\Gamma \Vdash C \circ (v \succ \square) : (T_1\ {}_m\!\!\to T_2) \rightarrowtail U_0}$$

$\textsc{TyR-ectx-PatS}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \{x_1 :_m T_1\}$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \{x_2 :_m T_2\}$$
$$\Gamma \neq \{\} \implies m = 1\nu$$
$$-\Gamma_0 \uplus \Gamma \Vdash C : U \rightarrowtail U_0$$
$$\Gamma_0 \uplus \{x_1 :_m T_1\} \vdash u_1 : U$$
$$\Gamma_0 \uplus \{x_2 :_m T_2\} \vdash u_2 : U$$
$$\overline{\Gamma \Vdash C \circ (\square \succ \mathsf{case}_m \{\mathsf{Inl}\, x_1 \mapsto u_1 \, , \ \mathsf{Inr}\, x_2 \mapsto u_2 \}) : (T_1 \oplus T_2) \rightarrowtail U_0}$$

$\textsc{TyR-ectx-PatU}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\frac{-\Gamma_0 \uplus \Gamma \Vdash C : U \rightarrowtail U_0 \qquad \Gamma_0 \vdash u : U}{\Gamma \Vdash C \circ (\square \, ; u) : 1 \rightarrowtail U_0}$$

$\textsc{TyR-ectx-PatP}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \{x_1 :_m T_1\}$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \{x_2 :_m T_2\}$$
$$\text{ctx\_Disjoint } \{x_1 :_m T_1\} \ \{x_2 :_m T_2\}$$
$$\Gamma \neq \{\} \implies m = 1\nu$$
$$-\Gamma_0 \uplus \Gamma \Vdash C : U \rightarrowtail U_0$$
$$\Gamma_0 \uplus \{x_1 :_m T_1, x_2 :_m T_2\} \vdash u : U$$
$$\overline{\Gamma \Vdash C \circ (\square \succ \mathsf{case}_m \, (x_1 \, , \, x_2) \mapsto u) : (T_1 \otimes T_2) \rightarrowtail U_0}$$

$\textsc{TyR-ectx-PatE}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \{x :_{m \cdot m'} T\}$$
$$\Gamma \neq \{\} \implies m = 1\nu$$
$$-\Gamma_0 \uplus \Gamma \Vdash C : U \rightarrowtail U_0$$
$$\Gamma_0 \uplus \{x :_{m \cdot m'} T\} \vdash u : U$$
$$\overline{\Gamma \Vdash C \circ (\square \succ \mathsf{case}_m \, E^{m'} \, x \mapsto u) : \, !^{m'} T \rightarrowtail U_0}$$

$\textsc{TyR-ectx-Map}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\text{ctx\_Disjoint } 1\!\uparrow\!\Gamma_0 \ \{x :_{1\nu} T_2\}$$
$$-\Gamma_0 \uplus \Gamma \Vdash C : U \rightarrowtail U_0$$
$$1\!\uparrow\!\Gamma_0 \uplus \{x :_{1\nu} T_2\} \vdash u : U$$
$$\overline{1\!\uparrow\!\cdot\Gamma \Vdash C \circ (\square \succ \mathsf{map} \ x \mapsto u) : (T_1 \ltimes T_2) \rightarrowtail U_0}$$

$\textsc{TyR-ectx-ToA}$
$$\frac{\Gamma \Vdash C : (T \ltimes 1) \rightarrowtail U_0}{\Gamma \Vdash C \circ (\mathsf{to}_\ltimes \square) : T \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FromA}$
$$\frac{\Gamma \Vdash C : T \rightarrowtail U_0}{\Gamma \Vdash C \circ (\mathsf{from}_\ltimes \square) : (T \ltimes 1) \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillF}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$\text{ctx\_Disjoint } \Gamma_2 \ \{x :_m T_1\}$$
$$1\!\uparrow\!\cdot - \Gamma_0 \uplus \Gamma \Vdash C : 1 \rightarrowtail U_0$$
$$\Gamma_0 \uplus \{x :_m T_1\} \vdash u : T_2$$
$$\Gamma_0 \neq \{\} \implies n = 1\nu$$
$$\overline{\Gamma \Vdash C \circ (\square \triangleleft (\lambda x_m \mapsto u)) : \lfloor T_{1 \ m} {\rightarrow} T_2 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillU}$
$$\frac{\Gamma \Vdash C : 1 \rightarrowtail U_0}{\Gamma \Vdash C \circ (\square \triangleleft ()) : \lfloor 1 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillL}$
$$\frac{\Gamma \Vdash C : \lfloor T_1 \rfloor^n \rightarrowtail U_0}{\Gamma \Vdash C \circ (\square \triangleleft \mathsf{Inl}) : \lfloor T_1 \oplus T_2 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillR}$
$$\frac{\Gamma \Vdash C : \lfloor T_2 \rfloor^n \rightarrowtail U_0}{\Gamma \Vdash C \circ (\square \triangleleft \mathsf{Inr}) : \lfloor T_1 \oplus T_2 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillP}$
$$\frac{\Gamma \Vdash C : (\lfloor T_1 \rfloor^n \otimes \lfloor T_2 \rfloor^n) \rightarrowtail U_0}{\Gamma \Vdash C \circ (\square \triangleleft (,)) : \lfloor T_1 \otimes T_2 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillE}$
$$\frac{\Gamma \Vdash C : \lfloor T \rfloor^{m \cdot n} \rightarrowtail U_0}{\Gamma \Vdash C \circ (\square \triangleleft E^m) : \lfloor !^m T \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillC1}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$1\!\uparrow\!\cdot - \Gamma_0 \uplus \Gamma \Vdash C : T_2 \rightarrowtail U_0$$
$$\Gamma_0 \vdash u : T_1 \ltimes T_2$$
$$\Gamma_0 \neq \{\} \implies n = 1\nu$$
$$\overline{\Gamma \Vdash C \circ (\square \triangleleft \bullet u) : \lfloor T_1 \rfloor^n \rightarrowtail U_0}$$

$\textsc{TyR-ectx-FillC2}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma$$
$$-\Gamma_0 \uplus 1\!\uparrow\!\cdot\Gamma \Vdash C : T_2 \rightarrowtail U_0$$
$$\Gamma_0 \vdash v : \lfloor T_1 \rfloor^n$$
$$\Gamma \neq \{\} \implies n = 1\nu$$
$$\overline{\Gamma \Vdash C \circ (v \triangleleft \bullet \square) : T_1 \ltimes T_2 \rightarrowtail U_0}$$

$\textsc{TyR-ectx-AOpen}$

$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma_1$$
$$\text{ctx\_DestOnly } \Gamma_0$$
$$\text{ctx\_HoleOnly } \Gamma_1$$
$$\text{ctx\_Disjoint } \Gamma_0 \ \Gamma_2$$
$$\text{ctx\_DestOnly } \Gamma_2$$
$$\text{hdns\_Disjoint } \mathsf{hnames}(C) \ \mathsf{hnames}(-\Gamma_2)$$
$$-\Gamma_0 \uplus \Gamma_1 \Vdash C : (T_1 \ltimes U) \rightarrowtail U_0$$
$$\Gamma_0 \uplus -\Gamma_2 \vdash v_1 : T_1$$
$$\overline{\Gamma_1 \uplus -\Gamma_2 \Vdash C \circ (^{\mathrm{op}}_{\mathsf{hnames}(-\Gamma_2)} \langle v_1 \, , \, \square \rangle : U \rightarrowtail U_0}$$

$\boxed{\vdash j : T}$ *(Typing of extended terms (pair of evaluation context and term))*

$\textsc{Ty-eterm-ClosedEterm}$
$$\frac{-\Gamma \Vdash C : T \rightarrowtail U_0 \qquad \Gamma \vdash t : T}{\vdash C[t] : U_0}$$

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3 Small-step semantics

$$\boxed{j \longrightarrow j'}$$
<div align="right">(Small-step evaluation of terms using evaluation contexts)</div>

Sem-eterm-App
$$C[v \succ (\lambda^{\mathsf{v}} x_m \mapsto t)] \longrightarrow C[t[x := v]]$$

Sem-eterm-PatU
$$C[() \,;\, t_2] \longrightarrow C[t_2]$$

Sem-eterm-PatL
$$C[(\mathsf{Inl}\, v) \succ \mathbf{case}_m \{\, \mathsf{Inl}\, x_1 \mapsto u_1 \,,\, \mathsf{Inr}\, x_2 \mapsto u_2 \,\}] \longrightarrow C[u_1[x := v]]$$

Sem-eterm-PatR
$$C[(\mathsf{Inr}\, v) \succ \mathbf{case}_m \{\, \mathsf{Inl}\, x_1 \mapsto u_1 \,,\, \mathsf{Inr}\, x_2 \mapsto u_2 \,\}] \longrightarrow C[u_2[x := v]]$$

Sem-eterm-PatP
$$C[(v_1 \,,\, v_2) \succ \mathbf{case}_m\, (x_1 \,,\, x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]$$

Sem-eterm-PatE
$$C[E^n\, v \succ \mathbf{case}_m\, E^n\, x \mapsto u] \longrightarrow C[u[x := v]]$$

Sem-eterm-MapOpen
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C))}{C[_H\langle v_1 \,,\, v_2\rangle \succ \mathbf{map}\, x \mapsto u] \longrightarrow (C \circ (^{\mathrm{op}}_{H \pm h'}\langle v_1 \pm h' \,,\, \square\rangle))[u[x := v_2 \pm h']]}$$

Sem-eterm-MapClose
$$(C \circ {}^{\mathrm{op}}_H\langle v_1 \,,\, \square\rangle)[v_2] \longrightarrow C[_H\langle v_1 \,,\, v_2\rangle]$$

Sem-eterm-Alloc
$$\mathbf{alloc} \longrightarrow {}_{\{1\}}\langle +1 \,,\, -1\rangle$$

Sem-eterm-ToA
$$C[\mathbf{to}_\ltimes v] \longrightarrow C[_{\{\,\}}\langle v \,,\, ()\rangle]$$

Sem-eterm-FromA
$$C[\mathbf{from}_\ltimes {}_{\{\}}\langle v \,,\, ()\rangle] \longrightarrow v$$

Sem-eterm-FillU
$$C[+h \triangleleft ()] \longrightarrow C[h :=_{\{\,\}} ()][()]$$

Sem-eterm-FillF
$$C[+h \triangleleft (\lambda x_m \mapsto u)] \longrightarrow C[h :=_{\{\,\}} \lambda^{\mathsf{v}} x_m \mapsto u][()]$$

Sem-eterm-FillL
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inl}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inl} -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillR
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inr}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inr} -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillE
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft E^m] \longrightarrow C[h :=_{\{h'+1\}} E^m -(h'+1)][+(h'+1)]}$$

Sem-eterm-FillP
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft (,)] \longrightarrow C[h :=_{\{h'+1, h'+2\}} (-(h'+1) \,,\, -(h'+2))][(+(h'+1) \,,\, +(h'+2))]}$$

Sem-eterm-FillC
$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \bullet_H\langle v_1 \,,\, v_2\rangle] \longrightarrow C[h :=_{(H \pm h')} v_1 \pm h'][v_2 \pm h']}$$