

metavariable, x, xs, y, uf, f, d

term, t, u ::=

- | x
- | v
- | $t\ u$
- | $t\ ;\ u$
- | $\text{case } t \text{ of } \{ () \mapsto u \}$
- | $\text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \}$
- | $\text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{roll } x \mapsto u \}$
- | $\text{alloc } d . t$
- | $t \triangleleft_A ()$
- | $t \triangleleft u$
- | $t \triangleleft 1.d . u$
- | $t \triangleleft 2.d . u$
- | $t \triangleleft \langle d_1, d_2 \rangle . u$
- | $t \triangleleft \text{Ur } d . u$
- | $t \triangleleft \text{roll } d . u$
- | $\star l$
- | (t)
- | $t[\text{var_subs}]$

term

- variable
- value
- application
- effect execution
- pattern-matching on unit
- pattern-matching on sum
- pattern-matching on product
- pattern-matching on exponentiated value
- unroll for recursive types
- allocate data
- fill destination with unit
- fill terminal-type destination
- fill sum-type destination with variant 1
- fill sum-type destination with variant 2
- fill product-type destination
- fill destination with exponential
- fill destination with recursive type
- TODO: remove (only there for SemMut)

S
M

var_sub, vs ::=

- | $x := t$

variable substitution

var_subs ::=

- | vs
- | $vs, \text{var_subs}$

variable substitutions

heap_val, h ::=

- | $()$
- | $1.l$
- | $2.l$
- | $\langle l_1, l_2 \rangle$
- | $\text{Ur } l$
- | $\text{roll } l$
- | $C \bar{l}$

M generic for all the cases above

val, v ::=

- | \bullet
- | $[A]$
- | $\lambda x:A . t$
- | h

unreducible value

- no-effect effect. Not part of the user syntax
- allocated destination. Not part of the user syntax
- lambda abstraction
- heap value

label, l ::=

label

label_stmt, s ::=

- | $l \triangleleft v$
- | $l \triangleleft \emptyset$

label statement

	$\bar{l} \triangleleft \bar{v}$	M	generic for multiple occurrences
label_stmts	$::=$ s s, label_stmts		label statements
heap_context, \mathbb{H}	$::=$ \emptyset {label_stmts} $\mathbb{H}_1 \sqcup \mathbb{H}_2$		label statements
type, A, B	$::=$ \perp 1 R $A \otimes B$ $A \oplus B$ $A \multimap B$ $[A]$ $!A$ (A) $W[r := A]$	S M	bottom type unit type recursive type bound to a name product type sum type linear function type destination type exponential
type_with_hole, W	$::=$ \perp r 1 R $W_1 \otimes W_2$ $W_1 \oplus W_2$ $W_1 \multimap W_2$ $[W]$ $!W$ (W)	S	bottom type type hole in recursive definition unit type recursive type bound to a name product type sum type linear function type destination type exponential
rec_type_bound, R	$::=$		recursive type bound to a name
rec_type_def	$::=$ $\mu r. W$		
type_affect, ta	$::=$ $x : A$ $\bar{l} : A$ $\bar{l} : \bar{A}$		type affectation var label generic for multiple occurrences
type_affects	$::=$ ta ta, type_affects		type affectations

typing_context, $\Gamma, \Delta, \mathcal{U}, \Phi$	$::=$ \emptyset $\{\text{type_affects}\}$ $\Gamma \sqcup \Delta$	typing context
types, \bar{A}	$::=$ \cdot A $A \text{ types}$	empty type list
heap_constructor, C	$::=$ $\{()\}$ $\{1.\}$ $\{2.\}$ $\{\langle, \rangle\}$ $\{\text{Ur}\}$ $\{\text{roll } R\}$	
judg	$::=$ $\Phi \vdash \mathbb{H}$ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \mid t : A$ $\Phi ; \mathcal{U} ; \Gamma \vdash t : A$ $C : \bar{A} \xrightarrow{c} A$ $A = B$ $t = u$ $\Gamma = \Delta$ $\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'$ $\text{type_affect} \in \Gamma$ $\text{label_stmt} \in \mathbb{H}$ $\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset$	
terminals	$::=$ $()$ \mapsto \star \otimes \oplus \circ $::=$ \vdash \sqcup $;$ \cap \emptyset \longrightarrow \triangleright \neq \in \notin $\backslash n$	

	\langle \rangle 1. 2. Ur \triangleleft $ $ \emptyset $\underline{\subseteq}$ $=$ \Downarrow \dots fix \equiv \perp \bullet	
formula	$::=$ $ $ judgement	
Ctx	$::=$ $ $ $\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset$ $ $ $\text{type_affect} \in \Gamma$	Γ and Δ are disjoint typing contexts with no clashing
Heap	$::=$ $ $ $\text{label_stmt} \in \mathbb{H}$	
Eq	$::=$ $ $ $A = B$ $ $ $t = u$ $ $ $\Gamma = \Delta$	
Ty	$::=$ $ $ $R \stackrel{\text{fix}}{=} \text{rec_type_def}$ $ $ $\Phi \vdash \mathbb{H}$ $ $ $\Phi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} t : A$ $ $ $\Phi ; \mathcal{U} ; \Gamma \vdash t : A$ $ $ $C : \bar{A} \subseteq A$	\mathbb{H} is a well-typed heap given heap typing context Φ t is a well-typed term of type A given heap typing context $\Phi ; \mathcal{U} ; \Gamma$ Heap constructor C builds a value of type A given arguments of type \bar{A}
Sem	$::=$ $ $ $\mathbb{H} t \longrightarrow \mathbb{H}' t'$ $ $ $\mathbb{H} t \Downarrow \mathbb{H}' t'$	t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}' t reduces to t' , with heap growing from \mathbb{H} to \mathbb{H}'
judgement	$::=$ $ $ Ctx $ $ Heap $ $ Eq $ $ Ty $ $ Sem	

user_syntax ::=

- metavariable
- term
- var_sub
- var_subs
- heap_val
- val
- label
- label_stmt
- label_stmts
- heap_context
- type
- type_with_hole
- rec_type_bound
- rec_type_def
- type_affect
- type_affects
- typing_context
- types
- heap_constructor
- judg
- terminals

$\text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset$ Γ and Δ are disjoint typing contexts with no clashing variable names or labels

$\text{type_affect} \in \Gamma$

$\text{label_stmt} \in \mathbb{H}$

$A = B$

$t = u$

$\Gamma = \Delta$

$R \stackrel{\text{fix}}{=} \text{rec_type_def}$

$\Phi \vdash \mathbb{H}$ \mathbb{H} is a well-typed heap given heap typing context Φ

$$\frac{\begin{array}{l} \Phi_1 \vdash \mathbb{H}_1 \\ \Phi_2 \vdash \mathbb{H}_2 \\ \text{names}(\Phi_1) \cap \text{names}(\Phi_2) = \emptyset \end{array}}{\Phi_1 \sqcup \Phi_2 \vdash \mathbb{H}_1 \sqcup \mathbb{H}_2} \quad \text{TYHEAP_UNION}$$

$$\frac{\Phi ; \bar{U} ; \Gamma \vdash \lambda x:A. t : A \multimap B}{\Phi \sqcup \{l : A \multimap B\} \vdash \mathbb{H} \sqcup \{l \triangleleft \lambda x:A. t\}} \quad \text{TYHEAP_LAM}$$

$$\frac{\begin{array}{l} C : \bar{A} \sqsubseteq A \\ \Phi \sqcup \{\bar{l} : \bar{A}\} \vdash \mathbb{H} \sqcup \{\bar{l} \triangleleft \bar{v}\} \end{array}}{\Phi \sqcup \{\bar{l} : \bar{A}, l : A\} \vdash \mathbb{H} \sqcup \{l \triangleleft C\bar{l}, \bar{l} \triangleleft \bar{v}\}} \quad \text{TYHEAP_HEAPVAL}$$

$$\overline{\emptyset \vdash \mathbb{H}} \quad \text{TYHEAP_ENLARGE} \quad \text{TODO: remove, needed for empty labels in SemMut}$$

$\Phi ; \bar{U} ; \Gamma \vdash \mathbb{H} \mid t : A$

$$\frac{\begin{array}{l} \Phi \vdash \mathbb{H} \\ \Phi ; \bar{U} ; \Gamma \vdash t : A \end{array}}{\Phi ; \bar{U} ; \Gamma \vdash \mathbb{H} \mid t : A} \quad \text{TYCOMMAND_DEF}$$

$\boxed{\Phi ; \mathcal{U} ; \Gamma \vdash t : A}$ t is a well-typed term of type A given heap typing context Φ , unrestricted typing context

$$\begin{array}{c}
\overline{\Phi ; \mathcal{U} ; \emptyset \vdash \bullet : \perp} \quad \text{TYTERM_NOEFF} \\
\\
\overline{\Phi ; \mathcal{U} ; \emptyset \vdash \textcolor{brown}{l} : \textcolor{blue}{[A]}} \quad \text{TYTERM_LDEST} \quad \text{TODO: we should have either } l : A \text{ in } P \text{ or } l \text{ not in names}(P) \\
\\
\frac{\Phi ; \mathcal{U} ; \Gamma \sqcup \{x : A\} \vdash t : B}{\Phi ; \mathcal{U} ; \Gamma \vdash \lambda x : A . t : A \multimap B} \quad \text{TYTERM_LAM} \\
\\
\frac{C : \bar{A} \xrightarrow{\mathcal{C}} A}{\Phi \sqcup \{\bar{l} : \bar{A}\} ; \mathcal{U} ; \emptyset \vdash C \bar{l} : A} \quad \text{TYTERM_HEAPVAL} \\
\\
\overline{\Phi ; \mathcal{U} ; \{x : A\} \vdash x : A} \quad \text{TYTERM_ID} \\
\\
\overline{\Phi ; \mathcal{U} \sqcup \{x : A\} ; \emptyset \vdash x : A} \quad \text{TYTERM_ID'} \\
\\
\overline{\Phi \sqcup \{l : A\} ; \mathcal{U} ; \emptyset \vdash \star l : A} \quad \text{TYTERM_DEREF} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A \multimap B \\ \Phi ; \mathcal{U} ; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t u : B} \quad \text{TYTERM_APP} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : \perp \\ \Phi ; \mathcal{U} ; \Delta \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash t ; u : B} \quad \text{TYTERM_Witheff} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : 1 \\ \Phi ; \mathcal{U} ; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{() \mapsto u\} : A} \quad \text{TYTERM_PATU} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \oplus A_2 \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1\} \vdash u_1 : B \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_2 : A_2\} \vdash u_2 : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} : B} \quad \text{TYTERM_PATs} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : A_1 \otimes A_2 \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x_1 : A_1, x_2 : A_2\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} : B} \quad \text{TYTERM_PATP} \\
\\
\frac{\begin{array}{c} \Phi ; \mathcal{U} ; \Gamma \vdash t : !A \\ \Phi ; \mathcal{U} \sqcup \{x : A\} ; \Delta \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \} : B} \quad \text{TYTERM_PATE} \\
\\
\frac{\begin{array}{c} R \stackrel{\text{fix}}{=} \mu r . W \\ \Phi ; \mathcal{U} ; \Gamma \vdash t : R \\ \Phi ; \mathcal{U} ; \Delta \sqcup \{x : W[r := R]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi ; \mathcal{U} ; \Gamma \sqcup \Delta \vdash \text{case } t \text{ of } \{ \text{roll } x \mapsto u \} : B} \quad \text{TYTERM_PATR} \\
\end{array}$$

$$\frac{\Phi; \mathcal{U}; \Gamma \sqcup \{d : [A]\} \vdash t : \perp}{\Phi; \mathcal{U}; \Gamma \vdash \text{alloc } d.t : A} \quad \text{TYTERM_ALLOC} \quad \text{TODO: add guard for LDest and NoEff}$$

$$\frac{\Phi; \mathcal{U}; \Gamma \vdash t : [1]}{\Phi; \mathcal{U}; \Gamma \vdash t \triangleleft () : \perp} \quad \text{TYTERM_FILLU}$$

$$\frac{\begin{array}{l} \Phi; \mathcal{U}; \Gamma \vdash t : [A] \\ \Phi; \mathcal{U}; \Delta \vdash u : A \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi; \mathcal{U}; \Gamma \sqcup \Delta \vdash t \triangleleft u : \perp} \quad \text{TYTERM_FILLL}$$

$$\frac{\begin{array}{l} \Phi; \mathcal{U}; \Gamma \vdash t : [A_1 \oplus A_2] \\ \Phi; \mathcal{U}; \Delta \sqcup \{d' : [A_1]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi; \mathcal{U}; \Gamma \sqcup \Delta \vdash t \triangleleft 1.d'.u : B} \quad \text{TYTERM_FILLV1}$$

$$\frac{\begin{array}{l} \Phi; \mathcal{U}; \Gamma \vdash t : [A_1 \oplus A_2] \\ \Phi; \mathcal{U}; \Delta \sqcup \{d' : [A_2]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi; \mathcal{U}; \Gamma \vdash t \triangleleft 2.d'.u : B} \quad \text{TYTERM_FILLV2}$$

$$\frac{\begin{array}{l} \Phi; \mathcal{U}; \Gamma \vdash t : [A_1 \otimes A_2] \\ \Phi; \mathcal{U}; \Delta \sqcup \{d_1 : [A_1], d_2 : [A_2]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi; \mathcal{U}; \Gamma \sqcup \Delta \vdash t \triangleleft \langle d_1, d_2 \rangle.u : B} \quad \text{TYTERM_FILLP}$$

$$\frac{\begin{array}{l} \Phi; \mathcal{U}; \Gamma \vdash t : [!A] \\ \Phi; \mathcal{U}; \emptyset \sqcup \{d : [A]\} \vdash u : B \end{array}}{\Phi; \mathcal{U}; \Gamma \vdash t \triangleleft \text{Ur } d.u : B} \quad \text{TYTERM_FILLE}$$

$$\frac{\begin{array}{l} R \stackrel{\text{fix}}{=} \mu r.W \\ \Phi; \mathcal{U}; \Gamma \vdash t : [R] \\ \Phi; \mathcal{U}; \Delta \sqcup \{d : [W[r := R]]\} \vdash u : B \\ \text{names}(\Gamma) \cap \text{names}(\Delta) = \emptyset \end{array}}{\Phi; \mathcal{U}; \Gamma \sqcup \Delta \vdash t \triangleleft_{\text{R}} d.u : B} \quad \text{TYTERM_FILLR}$$

$C : \bar{A} \hookrightarrow A$ Heap constructor C builds a value of type A given arguments of type \bar{A}

$$\frac{}{\{()\} : \cdot \hookrightarrow 1} \quad \text{TYCTOR_U}$$

$$\frac{}{\{1.\} : A \hookrightarrow A \oplus B} \quad \text{TYCTOR_V1}$$

$$\frac{}{\{2.\} : B \hookrightarrow A \oplus B} \quad \text{TYCTOR_V2}$$

$$\frac{}{\{\langle, \rangle\} : A \ B \hookrightarrow A \otimes B} \quad \text{TYCTOR_P}$$

$$\frac{}{\{\text{Ur}\} : A \hookrightarrow !A} \quad \text{TYCTOR_E}$$

$$\frac{R \stackrel{\text{fix}}{=} \mu r.W}{\{\text{roll } R\} : W[r := R] \hookrightarrow R} \quad \text{TYCTOR_R}$$

$\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'$ t reduces to t' , with heap changing from \mathbb{H} to \mathbb{H}'

$$\begin{array}{c}
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \text{ u} \longrightarrow \mathbb{H}' \mid t' \text{ u}} \quad \text{SEMMUT_UAPP} \\
\\
\frac{}{\mathbb{H} \mid (\lambda x:A. t) \text{ u} \longrightarrow \mathbb{H} \mid t[x := u]} \quad \text{SEMMUT_APP} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \lambda x:A. t\} \mid \star l \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \lambda x:A. t\} \mid \lambda x:A. t} \quad \text{SEMMUT_DLAM} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid \star l \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid C\bar{l}} \quad \text{SEMMUT_DHEAPVAL} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{() \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{() \mapsto u\}} \quad \text{SEMMUT_UPATU} \\
\\
\frac{}{\mathbb{H} \mid \text{case } () \text{ of } \{() \mapsto u\} \longrightarrow \mathbb{H} \mid u} \quad \text{SEMMUT_PATU} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}} \quad \text{SEMMUT_UPATV12} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \mid \text{case } 1.l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \mid u_1[x_1 := v]} \quad \text{SEMMUT_PATV1} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \mid \text{case } 2.l \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \longrightarrow \mathbb{H} \mid u_2[x_2 := v]} \quad \text{SEMMUT_PATV2} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\}} \quad \text{SEMMUT_UPATP} \\
\\
\frac{}{\mathbb{H} \sqcup \{l_1 \triangleleft v_1, l_2 \triangleleft v_2\} \mid \text{case } \langle l_1, l_2 \rangle \text{ of } \{\langle x_1, x_2 \rangle \mapsto u\} \longrightarrow \mathbb{H} \mid u[x_1 := v_1, x_2 := v_2]} \quad \text{SEMMUT_PATP} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{Ur } x \mapsto u\}} \quad \text{SEMMUT_UPATE} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \mid \text{case Ur } l \text{ of } \{\text{Ur } x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := v]} \quad \text{SEMMUT_PATE} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \text{case } t \text{ of } \{\text{roll}_R x \mapsto u\} \longrightarrow \mathbb{H}' \mid \text{case } t' \text{ of } \{\text{roll}_R x \mapsto u\}} \quad \text{SEMMUT_UPATR} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft v\} \mid \text{case roll}_R l \text{ of } \{\text{roll}_R x \mapsto u\} \longrightarrow \mathbb{H} \mid u[x := v]} \quad \text{SEMMUT_PATR} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t ; u \longrightarrow \mathbb{H}' \mid t' ; u} \quad \text{SEMMUT_UWitheff} \\
\\
\frac{}{\mathbb{H} \mid \bullet ; u \longrightarrow \mathbb{H} \mid u} \quad \text{SEMMUT_Witheff} \\
\\
\frac{}{\mathbb{H} \mid \text{alloc}_A d. t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid t[d := \overset{l}{[A]}] ; \star l} \quad \text{SEMMUT_ALLOC} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft () \longrightarrow \mathbb{H}' \mid t' \triangleleft ()} \quad \text{SEMMUT_UFILLU} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \emptyset\} \mid \overset{l}{[1]} \triangleleft () \longrightarrow \mathbb{H} \sqcup \{l \triangleleft ()\} \mid \bullet} \quad \text{SEMMUT_FILLU} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft u \longrightarrow \mathbb{H}' \mid t' \triangleleft u} \quad \text{SEMMUT_UFILLL} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid \overset{l}{[A]} \triangleleft t \longrightarrow \mathbb{H}' \mid \overset{l}{[A]} \triangleleft t'} \quad \text{SEMMUT_UFILLL}'
\end{array}$$

$$\begin{array}{c}
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A} \rrbracket \triangleleft \lambda x:A.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \lambda x:A.t\} \mid \bullet} \text{SEMMUT_FILLLLAM} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A} \rrbracket \triangleleft C\bar{l} \longrightarrow \mathbb{H} \sqcup \{l \triangleleft C\bar{l}\} \mid \bullet} \text{SEMMUT_FILLHEAPVAL} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 1.d.u \longrightarrow \mathbb{H}' \mid t' \triangleleft 1.d.u} \text{SEMMUT_UFILLV1} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A_1 \oplus A_2} \rrbracket \triangleleft 1.d.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 1.l', l' \triangleleft \odot\} \mid t[d := \llbracket \underline{A_1} \rrbracket]} \text{SEMMUT_FILLV1} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft 2.d.u \longrightarrow \mathbb{H}' \mid t' \triangleleft 2.d.u} \text{SEMMUT_UFILLV2} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A_1 \oplus A_2} \rrbracket \triangleleft 2.d.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft 2.l', l' \triangleleft \odot\} \mid t[d := \llbracket \underline{A_2} \rrbracket]} \text{SEMMUT_FILLV2} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \langle d_1, d_2 \rangle . u \longrightarrow \mathbb{H}' \mid t' \triangleleft \langle d_1, d_2 \rangle . u} \text{SEMMUT_UFILLP} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A_1 \otimes A_2} \rrbracket \triangleleft \langle d_1, d_2 \rangle . t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft \odot, l_2 \triangleleft \odot\} \mid t[d_1 := \llbracket \underline{A_1} \rrbracket, d_2 := \llbracket \underline{A_2} \rrbracket]} \text{SEMMUT_FILLP} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \text{Ur } d.u \longrightarrow \mathbb{H}' \mid t' \triangleleft \text{Ur } d.u} \text{SEMMUT_UFILLE} \\
\\
\frac{}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{A} \rrbracket \triangleleft \text{Ur } d.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \text{Ur } l', l' \triangleleft \odot\} \mid t[d := \llbracket \underline{A} \rrbracket]} \text{SEMMUT_FILLE} \\
\\
\frac{\mathbb{H} \mid t \longrightarrow \mathbb{H}' \mid t'}{\mathbb{H} \mid t \triangleleft \text{roll}_{\underline{R}} d.u \longrightarrow \mathbb{H}' \mid t' \triangleleft \text{roll}_{\underline{R}} d.u} \text{SEMMUT_UFILLR} \\
\\
\frac{R \stackrel{\text{fix}}{=} \mu r.W}{\mathbb{H} \sqcup \{l \triangleleft \odot\} \mid \llbracket \underline{R} \rrbracket \triangleleft \text{roll}_{\underline{R}} d.t \longrightarrow \mathbb{H} \sqcup \{l \triangleleft \text{roll}_{\underline{R}} l', l' \triangleleft \odot\} \mid t[d := \llbracket \underline{W[r := R]} \rrbracket]} \text{SEMMUT_FILLR} \\
\\
\boxed{\mathbb{H} \mid t \Downarrow \mathbb{H}' \mid t'} \quad t \text{ reduces to } t', \text{ with heap growing from } \mathbb{H} \text{ to } \mathbb{H}' \\
\\
\frac{}{\mathbb{H} \mid \bullet \Downarrow \mathbb{H} \mid \bullet} \text{SEMMUT_NOEFF (value)} \\
\\
\frac{}{\mathbb{H} \mid \llbracket \underline{A} \rrbracket \Downarrow \mathbb{H} \mid \llbracket \underline{A} \rrbracket} \text{SEMMUT_LDEST (value)} \\
\\
\frac{}{\mathbb{H} \mid \lambda x:A.t \Downarrow \mathbb{H} \mid \lambda x:A.t} \text{SEMMUT_LAM (value)} \\
\\
\frac{}{\mathbb{H} \mid C\bar{l} \Downarrow \mathbb{H} \mid C\bar{l}} \text{SEMMUT_HEAPVAL (value)} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid \lambda x:A.t' \quad \mathbb{H}_1 \mid t'[x := u] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid t u \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMMUT_APP} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid () \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{() \mapsto u\} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMMUT_PATU} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} \mid 1.l \quad \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} \mid u_1[x_1 := v] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMMUT_PATV1}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | 2.l \quad \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | u_2[x_2 := v] \Downarrow \mathbb{H}_2 | v_2}{\mathbb{H}_0 | \text{case t of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\} \Downarrow \mathbb{H}_2 | v_2} \text{SEMIMM_PATV2} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 \sqcup \{l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} | \langle l_1, l_2 \rangle \quad \mathbb{H}_1 \sqcup \{l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} | u[x_1 := v_1, x_2 := v_2] \Downarrow \mathbb{H}_2 | v_2}{\mathbb{H}_0 | \text{case t of } \{\langle x_1, x_2 \rangle \mapsto u\} \Downarrow \mathbb{H}_2 | v_2} \text{SEMIMM_PATP} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | \text{Ur } l \quad \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | u[x := v_1] \Downarrow \mathbb{H}_2 | v_2}{\mathbb{H}_0 | \text{case t of } \{\text{Ur } x \mapsto u\} \Downarrow \mathbb{H}_2 | v_2} \text{SEMIMM_PATE} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | \text{roll } l \quad \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | u[x := v_1] \Downarrow \mathbb{H}_2 | v_2}{\mathbb{H}_0 | \text{case t of } \{\text{roll } x \mapsto u\} \Downarrow \mathbb{H}_2 | v_2} \text{SEMIMM_PATR} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \bullet \quad \mathbb{H}_1 | u \Downarrow \mathbb{H}_2 | v_2}{\mathbb{H}_0 | t ; u \Downarrow \mathbb{H}_2 | v_2} \text{SEMIMM_WITHEFF} \\
\frac{\mathbb{H}_0 | t[d := \underline{A}] \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | \bullet}{\mathbb{H}_0 | \text{alloc } d . t \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft v_1\} | v_1} \text{SEMIMM_ALLOC} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{1}}{\mathbb{H}_0 | t \triangleleft () \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft ()\} | \bullet} \text{SEMIMM_FILLU} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A \rightarrow B} \quad \mathbb{H}_1 | u \Downarrow \mathbb{H}_2 | \lambda x:A. t}{\mathbb{H}_0 | t \triangleleft u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \lambda x:A. t\} | \bullet} \text{SEMIMM_FILLLLAM } \textit{block on NoEff/LDest} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A} \quad \mathbb{H}_1 | u \Downarrow \mathbb{H}_2 | \underline{C \bar{l}}}{\mathbb{H}_0 | t \triangleleft u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft C \bar{l}\} | \bullet} \text{SEMIMM_FILLLHEAPVAL } \textit{block on NoEff/LDest} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A \oplus B} \quad \mathbb{H}_1 | u[d := \underline{A'}] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} | v_2}{\mathbb{H}_0 | t \triangleleft 1.d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 1.l', l' \triangleleft v_1\} | v_2} \text{SEMIMM_FILLV1} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A \oplus B} \quad \mathbb{H}_1 | u[d := \underline{B'}] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} | v_2}{\mathbb{H}_0 | t \triangleleft 2.d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 2.l', l' \triangleleft v_1\} | v_2} \text{SEMIMM_FILLV2} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A \otimes B} \quad \mathbb{H}_1 | u[d_1 := \underline{A}, d_2 := \underline{B}] \Downarrow \mathbb{H}_2 \sqcup \{l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} | v_2}{\mathbb{H}_0 | t \triangleleft \langle d_1, d_2 \rangle . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} | v_2} \text{SEMIMM_FILLP} \\
\frac{\mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{A} \quad \mathbb{H}_1 | u[d := \underline{A'}] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} | v_2}{\mathbb{H}_0 | t \triangleleft \text{Ur } d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{Ur } l', l' \triangleleft v_1\} | v_2} \text{SEMIMM_FILLE} \\
\frac{\text{R} \stackrel{\text{fix}}{=} \mu r. W \quad \mathbb{H}_0 | t \Downarrow \mathbb{H}_1 | \underline{R} \quad \mathbb{H}_1 | u[d := \underline{W[r := R]}] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} | v_2}{\mathbb{H}_0 | t \triangleleft \text{roll } d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{roll } l', l' \triangleleft v_1\} | v_2} \text{SEMIMM_FILLR}
\end{array}$$

Definition rules: 88 good 0 bad

Definition rule clauses: 191 good 0 bad