A FULL REDUCTION RULES FOR λ_d

$$\frac{\texttt{NotVal}\ t}{E\left[\,t'\ t\,\right] \longrightarrow (E \circ t'\ [])\left[\,t\,\right]}\ \textsc{App-Focus1} \qquad \frac{}{(E \circ t'\ [])\left[\,v\,\right] \longrightarrow E\left[\,t'\ v\,\right]}\ \textsc{App-Unfocus1}$$

$$\frac{\texttt{NotVal}\ t'}{E\left[\,t'\ v\,\right] \longrightarrow (E \circ []\ v)\left[\,t'\,\right]}\ \textsc{App-Focus2} \qquad \frac{}{(E \circ []\ v)\left[\,v'\,\right] \longrightarrow E\left[\,v'\ v\,\right]}\ \textsc{App-Unfocus2}$$

$$\frac{}{E\left[\,(\lambda x_{\,m} \mapsto u)\ v\,\right] \longrightarrow E\left[\,u[x := v]\,\right]}\ \textsc{App-Red} \qquad \frac{\texttt{NotVal}\ t}{E\left[\,t \fatsemi u\,\right] \longrightarrow (E \circ [] \fatsemi u)\left[\,t\,\right]}\ \textsc{PatU-Focus}$$

$$\frac{}{(E \circ [] \fatsemi u)\left[\,v\,\right] \longrightarrow E\left[\,v \fatsemi u\,\right]}\ \textsc{PatU-Unfocus} \qquad \frac{}{E\left[\,() \fatsemi u\,\right] \longrightarrow E\left[\,u\,\right]}\ \textsc{PatU-Red}$$

$$\frac{\texttt{NotVal}\ t}{E\left[\,\mathbf{case}_m\ t\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\}\,\right] \longrightarrow (E \circ \mathbf{case}_m\ []\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\})\left[\,t\,\right]}\ \textsc{PatS-Focus}$$

$$\frac{}{(E \circ \mathbf{case}_m\ []\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\})\left[\,v\,\right] \longrightarrow E\left[\,\mathbf{case}_m\ v\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\}\,\right]}\ \textsc{PatS-Unfocus}$$

$$\frac{}{E\left[\,\mathbf{case}_m\ (\mathsf{Inl}\ v_1)\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\}\,\right] \longrightarrow E\left[\,u_1[x_1 := v_1]\,\right]}\ \textsc{PatL-Red}$$

$$\frac{}{E\left[\,\mathbf{case}_m\ (\mathsf{Inr}\ v_2)\ \mathbf{of}\ \{\mathsf{Inl}\ x_1 \mapsto u_1,\ \mathsf{Inr}\ x_2 \mapsto u_2\}\,\right] \longrightarrow E\left[\,u_2[x_2 := v_2]\,\right]}\ \textsc{PatR-Red}$$

$$\frac{\texttt{NotVal}\ t}{E\left[\,\mathbf{case}_m\ t\ \mathbf{of}\ (x_1, x_2) \mapsto u\,\right] \longrightarrow (E \circ \mathbf{case}_m\ []\ \mathbf{of}\ (x_1, x_2) \mapsto u)\left[\,t\,\right]}\ \textsc{PatP-Focus}$$

$$\frac{}{(E \circ \mathbf{case}_m\ []\ \mathbf{of}\ (x_1, x_2) \mapsto u)\left[\,v\,\right] \longrightarrow E\left[\,\mathbf{case}_m\ v\ \mathbf{of}\ (x_1, x_2) \mapsto u\,\right]}\ \textsc{PatP-Unfocus}$$

$$\frac{}{E\left[\,\mathbf{case}_m\ (v_1, v_2)\ \mathbf{of}\ (x_1, x_2) \mapsto u\,\right] \longrightarrow E\left[\,u[x_1 := v_1][x_2 := v_2]\,\right]}\ \textsc{PatP-Red}$$

$$\frac{\texttt{NotVal}\ t}{E\left[\,\mathbf{case}_m\ t\ \mathbf{of}\ \mathsf{Mod}_n\ x \mapsto u\,\right] \longrightarrow (E \circ \mathbf{case}_m\ []\ \mathbf{of}\ \mathsf{Mod}_n\ x \mapsto u)\left[\,t\,\right]}\ \textsc{PatE-Focus}$$

$$\frac{}{(E \circ \mathbf{case}_m\ []\ \mathbf{of}\ \mathsf{Mod}_n\ x \mapsto u)\left[\,v\,\right] \longrightarrow E\left[\,\mathbf{case}_m\ v\ \mathbf{of}\ \mathsf{Mod}_n\ x \mapsto u\,\right]}\ \textsc{PatE-Unfocus}$$

$$\frac{}{E\left[\,\mathbf{case}_m\ \mathsf{Mod}_n\ v'\ \mathbf{of}\ \mathsf{Mod}_n\ x \mapsto u\,\right] \longrightarrow E\left[\,u[x := v']\,\right]}\ \textsc{PatE-Red}$$

$$\frac{\texttt{NotVal}\ t}{E\left[\,\mathbf{upd}_{\mathsf{K}}\ t\ \mathbf{with}\ x \mapsto t'\,\right] \longrightarrow (E \circ \mathbf{upd}_{\mathsf{K}}\ []\ \mathbf{with}\ x \mapsto t')\left[\,t\,\right]}\ \textsc{UpdA-Focus}$$

$$\frac{}{(E \circ \mathbf{upd}_{\mathsf{K}}\ []\ \mathbf{with}\ x \mapsto t')\left[\,v\,\right] \longrightarrow E\left[\,\mathbf{upd}_{\mathsf{K}}\ v\ \mathbf{with}\ x \mapsto t'\,\right]}\ \textsc{UpdA-Unfocus}$$

$$\frac{h''' = \mathbf{max}(H \cup \mathbf{hnames}(E))+1}{E\left[\,\mathbf{upd}_{\mathsf{K}}\ {}_H\langle v_2 \wedge v_1\rangle\ \mathbf{with}\ x \mapsto t'\,\right] \longrightarrow (E \circ {}^{\mathsf{op}}_{H \pm h'''}\langle v_2[H \pm h''']\wedge []\rangle)\left[\,t'[x := v_1[H \pm h''']]\,\right]}\ \textsc{AmpAr-Open}$$

$$\frac{}{(E \circ {}^{\mathsf{op}}_{H}\langle v_2 \wedge []\rangle)\left[\,v_1\,\right] \longrightarrow E\left[\,{}_H\langle v_2 \wedge v_1\rangle\,\right]}\ \textsc{AmpAr-Close} \qquad \frac{\texttt{NotVal}\ u}{E\left[\,\mathbf{to}_{\mathsf{K}}\ u\,\right] \longrightarrow (E \circ \mathbf{to}_{\mathsf{K}}\ [])\left[\,u\,\right]}\ \textsc{ToA-Focus}$$

$$\frac{}{(E \circ \mathbf{to}_{\mathsf{K}}\ [])\left[\,v_2\,\right] \longrightarrow E\left[\,\mathbf{to}_{\mathsf{K}}\ v_2\,\right]}\ \textsc{ToA-Unfocus} \qquad \frac{}{E\left[\,\mathbf{to}_{\mathsf{K}}\ v_2\,\right] \longrightarrow E\left[\,\{\,\}\langle v_2 \wedge ()\rangle\,\right]}\ \textsc{ToA-Red}$$

Fig. 10. Full reduction rules for λ_d (part 1)

$$\frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[\mathbf{from}_{\mathsf{K}}\ t\right] \longrightarrow (\mathsf{E} \circ \mathbf{from}_{\mathsf{K}}\ [\,])\left[t\right]}\ \textsc{FromA-Focus} \qquad \frac{}{(\mathsf{E} \circ \mathbf{from}_{\mathsf{K}}\ [\,])\left[v\right] \longrightarrow \mathsf{E}\left[\mathbf{from}_{\mathsf{K}}\ v\right]}\ \textsc{FromA-Unfocus}$$

$$\frac{}{\mathsf{E}\left[\mathbf{from}_{\mathsf{K}}\ \{\,\}\langle v_2 \wedge \mathsf{Mod}_{1\infty}\ v_1\rangle\right] \longrightarrow \mathsf{E}\left[(v_2, \mathsf{Mod}_{1\infty}\ v_1)\right]}\ \textsc{FromA-Red} \qquad \frac{}{\mathsf{E}\left[\mathbf{new}_{\mathsf{K}}\right] \longrightarrow \mathsf{E}\left[\{1\}\langle \boxed{1} \wedge \to 1\rangle\right]}\ \textsc{NewA-Red}$$

$$\frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft ()\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft ())\left[t\right]}\ \textsc{FillU-Focus} \qquad \frac{}{(\mathsf{E} \circ [\,] \triangleleft ())\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft ()\right]}\ \textsc{FillU-Unfocus}$$

$$\frac{}{\mathsf{E}\left[\to h \triangleleft ()\right] \longrightarrow \mathsf{E}(\!|h := \{\,\}\ ()|\!)\left[()\right]}\ \textsc{FillU-Red} \qquad \frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft \mathsf{Inl}\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft \mathsf{Inl})\left[t\right]}\ \textsc{FillL-Focus}$$

$$\frac{}{(\mathsf{E} \circ [\,] \triangleleft \mathsf{Inl})\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft \mathsf{Inl}\right]}\ \textsc{FillL-Unfocus} \qquad \frac{h' = \mathbf{max}(\mathbf{hnames}(\mathsf{E}) \cup \{h\}) + 1}{\mathsf{E}\left[\to h \triangleleft \mathsf{Inl}\right] \longrightarrow \mathsf{E}(\!|h := \{h'+1\}\ \mathsf{Inl}\ \boxed{h'+1}|\!)\left[\to h'+1\right]}\ \textsc{FillL-Red}$$

$$\frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft \mathsf{Inr}\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft \mathsf{Inr})\left[t\right]}\ \textsc{FillR-Focus} \qquad \frac{}{(\mathsf{E} \circ [\,] \triangleleft \mathsf{Inr})\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft \mathsf{Inr}\right]}\ \textsc{FillR-Unfocus}$$

$$\frac{h' = \mathbf{max}(\mathbf{hnames}(\mathsf{E}) \cup \{h\}) + 1}{\mathsf{E}\left[\to h \triangleleft \mathsf{Inr}\right] \longrightarrow \mathsf{E}(\!|h := \{h'+1\}\ \mathsf{Inr}\ \boxed{h'+1}|\!)\left[\to h'+1\right]}\ \textsc{FillR-Red} \qquad \frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft \mathsf{Mod}_{\mathsf{m}}\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft \mathsf{Mod}_{\mathsf{m}})\left[t\right]}\ \textsc{FillE-Focus}$$

$$\frac{}{(\mathsf{E} \circ [\,] \triangleleft \mathsf{Mod}_{\mathsf{m}})\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft \mathsf{Mod}_{\mathsf{m}}\right]}\ \textsc{FillE-Unfocus}$$

$$\frac{h' = \mathbf{max}(\mathbf{hnames}(\mathsf{E}) \cup \{h\}) + 1}{\mathsf{E}\left[\to h \triangleleft \mathsf{Mod}_{\mathsf{m}}\right] \longrightarrow \mathsf{E}(\!|h := \{h'+1\}\ \mathsf{Mod}_{\mathsf{m}}\ \boxed{h'+1}|\!)\left[\to h'+1\right]}\ \textsc{FillE-Red} \qquad \frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft (,)\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft (,))\left[t\right]}\ \textsc{FillP-Focus}$$

$$\frac{}{(\mathsf{E} \circ [\,] \triangleleft (,))\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft (,)\right]}\ \textsc{FillP-Unfocus}$$

$$\frac{h' = \mathbf{max}(\mathbf{hnames}(\mathsf{E}) \cup \{h\}) + 1}{\mathsf{E}\left[\to h \triangleleft (,)\right] \longrightarrow \mathsf{E}(\!|h := \{h'+1, h'+2\}\ (\boxed{h'+1}, \boxed{h'+2})|\!)\left[(\to h'+1, \to h'+2)\right]}\ \textsc{FillP-Red}$$

$$\frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \triangleleft (\boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u)\right] \longrightarrow (\mathsf{E} \circ [\,] \triangleleft (\boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u))\left[t\right]}\ \textsc{FillF-Focus}$$

$$\frac{}{(\mathsf{E} \circ [\,] \triangleleft (\boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u))\left[v\right] \longrightarrow \mathsf{E}\left[v \triangleleft (\boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u)\right]}\ \textsc{FillF-Unfocus}$$

$$\frac{}{\mathsf{E}\left[\to h \triangleleft (\boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u)\right] \longrightarrow \mathsf{E}(\!|h := \{\,\}\ \boldsymbol{\lambda} x_{\mathsf{m}} \mapsto u|\!)\left[()\right]}\ \textsc{FillF-Red} \qquad \frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \lhd\!\circ\ t'\right] \longrightarrow (\mathsf{E} \circ [\,] \lhd\!\circ\ t')\left[t\right]}\ \textsc{FillComp-Focus1}$$

$$\frac{}{(\mathsf{E} \circ [\,] \lhd\!\circ\ t')\left[v\right] \longrightarrow \mathsf{E}\left[v \lhd\!\circ\ t'\right]}\ \textsc{FillComp-Unfocus1} \qquad \frac{\mathsf{NotVal}\ t'}{\mathsf{E}\left[v \lhd\!\circ\ t'\right] \longrightarrow (\mathsf{E} \circ v \lhd\!\circ [\,])\left[t'\right]}\ \textsc{FillComp-Focus2}$$

$$\frac{}{(\mathsf{E} \circ v \lhd\!\circ [\,])\left[v'\right] \longrightarrow \mathsf{E}\left[v \lhd\!\circ v'\right]}\ \textsc{FillComp-Unfocus2}$$

$$\frac{h'' = \mathbf{max}(H \cup (\mathbf{hnames}(\mathsf{E}) \cup \{h\})) + 1}{\mathsf{E}\left[\to h \underset{H}{\lhd\!\circ} \langle v_2 \wedge v_1\rangle\right] \longrightarrow \mathsf{E}(\!|h := (H \pm h'')\ v_2[H \pm h'']|\!)\left[v_1[H \pm h'']\right]}\ \textsc{FillComp-Red}$$

$$\frac{\mathsf{NotVal}\ t}{\mathsf{E}\left[t \blacktriangleleft t'\right] \longrightarrow (\mathsf{E} \circ [\,] \blacktriangleleft t')\left[t\right]}\ \textsc{FillLeaf-Focus1} \qquad \frac{}{(\mathsf{E} \circ [\,] \blacktriangleleft t')\left[v\right] \longrightarrow \mathsf{E}\left[v \blacktriangleleft t'\right]}\ \textsc{FillLeaf-Unfocus1}$$

$$\frac{\mathsf{NotVal}\ t'}{\mathsf{E}\left[v \blacktriangleleft t'\right] \longrightarrow (\mathsf{E} \circ v \blacktriangleleft [\,])\left[t'\right]}\ \textsc{FillLeaf-Focus2} \qquad \frac{}{(\mathsf{E} \circ v \blacktriangleleft [\,])\left[v'\right] \longrightarrow \mathsf{E}\left[v \blacktriangleleft v'\right]}\ \textsc{FillLeaf-Unfocus2}$$

$$\frac{}{\mathsf{E}\left[\to h \blacktriangleleft v\right] \longrightarrow \mathsf{E}(\!|h := \{\,\}\ v|\!)\left[()\right]}\ \textsc{FillLeaf-Red}$$

Fig. 11. Full reduction rules for λ_d (part 2)