

metavariable, x, xs, y, uf, f, d

term, t, u ::=

- | x
- | v
- | $t\ u$
- | $t\ ;\ u$
- | $\text{case } t \text{ of } \{ () \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}$
- | $\text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \}$
- | $\text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \}$
- | $\text{case } t \text{ of } \{ \text{roll } x \mapsto u \}$
 $\textcolor{blue}{R}$
- | $\text{alloc } d . t$
 $\textcolor{blue}{A}$
- | $t \triangleleft ()$
- | $t \triangleleft u$
- | $t \triangleleft \text{Ur } u$
- | $t \triangleleft 1.d . u$
- | $t \triangleleft 2.d . u$
- | $t \triangleleft \langle d_1, d_2 \rangle . u$
- | $t \triangleleft \text{roll } d . u$
 $\textcolor{blue}{R}$
- | (t)
- | $t[\text{var_subs}]$

term

- variable
- value
- application
- effect execution
- pattern-matching on unit
- pattern-matching on exponentiated value
- pattern-matching on sum
- pattern-matching on product
- unroll for recursive types
- allocate data
- fill destination with unit
- fill terminal-type destination
- fill destination with exponential
- fill sum-type destination with variant 1
- fill sum-type destination with variant 2
- fill product-type destination
- fill destination with recursive type

S
M

var_sub, vs ::=

- | $x := t$

variable substitution

var_subs ::=

- | vs
- | $vs, \text{var_subs}$

variable substitutions

heap_val, h ::=

- | $()$
- | $\text{Ur } l$
- | $1.l$
- | $2.l$
- | $\langle l_1, l_2 \rangle$
- | $\text{roll } l$
 $\textcolor{blue}{R}$
- | $C\bar{l}$

M generic for all the cases above

val, v ::=

- | \bullet
- | $[l]$
- | $\lambda x:A . t$
- | h

unreducible value

- no-effect effect
- address of an allocated memory area
- lambda abstraction
- heap value

$label, l$::=

memory address

labels ::=

- | l
- | $l \text{ bar}$
- | l, labels

		$\bar{l}bar$, labels	
$label_set, L$	$::=$		set of used labels
		\emptyset	
		$\{labels\}$	
		$L_1 \sqcup L_2$	
heap_affect, ha	$::=$		heap cell
		$l \triangleleft v$	
		$\bar{l} \triangleleft \bar{v}$	M generic for multiple occurrences
heap_affects	$::=$		heap cells
		ha	
		ha, heap_affects	
heap_context, \mathbb{H}	$::=$		heap contents
		\emptyset	
		$\{heap_affects\}$	
		$\mathbb{H}_1 \sqcup \mathbb{H}_2$	
type, A, B	$::=$		
		\perp	bottom type
		1	unit type
		R	recursive type bound to a name
		$A \otimes B$	product type
		$A \oplus B$	sum type
		$A \multimap B$	linear function type
		$[A]$	destination type
		$!A$	exponential
		(A)	S
		$W[r := A]$	M
type_with_hole, W	$::=$		
		\perp	bottom type
		r	type hole in recursive definition
		1	unit type
		R	recursive type bound to a name
		$W_1 \otimes W_2$	product type
		$W_1 \oplus W_2$	sum type
		$W_1 \multimap W_2$	linear function type
		$[W]$	destination type
		$!W$	exponential
		(W)	S
rec_type_bound, R	$::=$		recursive type bound to a name
rec_type_def	$::=$		
		$\mu r. W$	

terminals

::=

()
→
★
⊕
⊕
—○
:=
⊢
⊔
;
∩
∅
→
▷
≠
∈
∉
↵
⟨
⟩
1.
2.
Ur
◁
|
⊙
⊔
=
⇓
...
fix
≡
⊥
•

formula

::=

| judgement

Ctx

::=

| $l \in \text{names}(\Phi)$
| $l \notin \text{names}(\Phi)$
| $\text{type_affect} \in \Gamma$
| $\text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset$

Γ_1 and Γ_2 are disjoint typing contexts with no clashing

LabelSet

::=

| $\text{names}(\Phi) \sqcup \text{names}(\Psi) = L$
| $l \in L$
| $l \notin L$

Heap

::=

		$\text{heap_affect} \in \mathbb{H}$	
Eq	::=	$A = B$ $t = u$ $\Gamma = D$	
Ty	::=	$R \stackrel{\text{fix}}{=} \text{rec_type_def}$ $C : \bar{A} \xrightarrow{c} A$ $\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{command} : A$ $\Phi \vdash \mathbb{H}$ $\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A$	<p>Heap constructor C builds a value of type A given a</p> <p>\mathbb{H} is a well-typed heap given heap typing context Φ t is a well-typed term of type A given heap typing c</p>
Sem	::=	$\text{command} \Downarrow \text{command}'$	t reduces to t' , with heap growing from H to H'
judgement	::=	Ctx LabelSet Heap Eq Ty Sem	
user_syntax	::=	metavariable term var_sub var_subs heap_val val <i>label</i> labels <i>label_set</i> heap_affect heap_affects heap_context type type_with_hole rec_type_bound rec_type_def type_affect type_affects typing_context types command heap_constructor judg terminals	

$$l \in \text{names}(\Phi)$$

$$l \notin \text{names}(\Phi)$$

$$\text{type_affect} \in \Gamma$$

$$\text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset$$

Γ_1 and Γ_2 are disjoint typing contexts with no clashing variable names or labels

$$\text{names}(\Phi) \sqcup \text{names}(\Psi) = L$$

$$l \in L$$

$$l \notin L$$

$$\text{heap_affect} \in \mathbb{H}$$

$$A = B$$

$$t = u$$

$$\Gamma = D$$

$$R \stackrel{\text{fix}}{=} \text{rec_type_def}$$

$$C : \bar{A} \hookrightarrow A$$

Heap constructor C builds a value of type A given arguments of type \bar{A}

$$\frac{}{\{()\} : \cdot \hookrightarrow 1} \text{TyCTOR_U}$$

$$\frac{}{\{1.\} : A \hookrightarrow A \oplus B} \text{TyCTOR_V1}$$

$$\frac{}{\{2.\} : B \hookrightarrow A \oplus B} \text{TyCTOR_V2}$$

$$\frac{}{\{\langle, \rangle\} : A \ B \hookrightarrow A \otimes B} \text{TyCTOR_P}$$

$$\frac{}{\{U_r\} : A \hookrightarrow !A} \text{TyCTOR_E}$$

$$\frac{R \stackrel{\text{fix}}{=} \mu r. W}{\{\text{roll } R\} : W[r := R] \hookrightarrow R} \text{TyCTOR_R}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{command} : A$$

$$\text{names}(\Phi) \sqcup \text{names}(\Psi) = L$$

$$\Phi \vdash \mathbb{H}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A$$

$$\frac{}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \mathbb{H} \sqcup t : A} \text{TyCOMMAND_DEF}$$

$$\Phi \vdash \mathbb{H}$$

\mathbb{H} is a well-typed heap given heap typing context Φ

$$\frac{}{\emptyset \vdash \emptyset} \text{TyHEAP_EMPTY}$$

$$\Phi \vdash \mathbb{H}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash v : A$$

$$\frac{}{\Phi \sqcup \{l : A\} \vdash \mathbb{H} \sqcup \{l \triangleleft v\}} \text{TyHEAP_VAL}$$

$$\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : A$$

t is a well-typed term of type A given heap typing context Φ , unrestricted typing context Ψ , and heap typing context \mathcal{U}

$$\frac{}{\Phi ; \emptyset ; \mathcal{U} ; \emptyset \vdash \bullet : \perp} \text{TyTERM_NOEFF}$$

$$\begin{array}{c}
\frac{l \notin \text{names}(\Phi)}{\Phi; \{l:A\}; \mathcal{U}; \emptyset \vdash [l]: [A]} \text{TYTERM_LDEST} \\
\\
\frac{\Phi; \Psi; \mathcal{U}; \Gamma \sqcup \{x:A\} \vdash t:B}{\Phi; \Psi; \mathcal{U}; \Gamma \vdash \lambda x:A. t: A \multimap B} \text{TYTERM_LAM} \\
\\
\frac{C: \bar{A} \xrightarrow{c} A}{\Phi \sqcup \{\bar{l}:\bar{A}\}; \emptyset; \mathcal{U}; \emptyset \vdash C\bar{l}: A} \text{TYTERM_HEAPVAL} \\
\\
\frac{}{\Phi; \emptyset; \mathcal{U}; \{x:A\} \vdash x:A} \text{TYTERM_ID} \\
\\
\frac{}{\Phi; \emptyset; \mathcal{U} \sqcup \{x:A\}; \emptyset \vdash x:A} \text{TYTERM_ID'} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: A \multimap B \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \vdash u:A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash tu: B} \text{TYTERM_APP} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: \perp \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \vdash u: B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash t; u: B} \text{TYTERM_Witheff} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: 1 \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \vdash u: A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{() \mapsto u\}: A} \text{TYTERM_PATU} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: !A \\ \Phi; \Psi_2; \mathcal{U} \sqcup \{x:A\}; \Gamma_2 \vdash u: B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \}: B} \text{TYTERM_PATE} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: A_1 \oplus A_2 \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \sqcup \{x_1:A_1\} \vdash u_1: B \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \sqcup \{x_2:A_2\} \vdash u_2: B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2\}: B} \text{TYTERM_PATs} \\
\\
\frac{\begin{array}{l} \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: A_1 \otimes A_2 \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \sqcup \{x_1:A_1, x_2:A_2\} \vdash u: B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \langle x_1, x_2 \rangle \mapsto u \}: B} \text{TYTERM_PATP} \\
\\
\frac{\begin{array}{l} R \stackrel{\text{fix}}{=} \mu r. W \\ \Phi; \Psi_1; \mathcal{U}; \Gamma_1 \vdash t: R \\ \Phi; \Psi_2; \mathcal{U}; \Gamma_2 \sqcup \{x: W[r:=R]\} \vdash u: B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi; \Psi_1 \sqcup \Psi_2; \mathcal{U}; \Gamma_1 \sqcup \Gamma_2 \vdash \text{case } t \text{ of } \{ \text{roll } x \mapsto u \}: B} \text{TYTERM_PATR}
\end{array}$$

$$\begin{array}{c}
\frac{\Phi ; \Psi ; \mathcal{U} ; \Gamma \sqcup \{d : [A]\} \vdash t : \perp}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash \text{alloc } d . t : A} \quad \text{TYTERM_ALLOC} \\
\\
\frac{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t : [1]}{\Phi ; \Psi ; \mathcal{U} ; \Gamma \vdash t \triangleleft () : \perp} \quad \text{TYTERM_FILLU} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \vdash u : A \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft u : \perp} \quad \text{TYTERM_FILLL} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma \vdash t : [!A] \\ \Phi ; \emptyset ; \mathcal{U} ; \emptyset \vdash u : A \end{array}}{\Phi ; \Psi_1 ; \mathcal{U} ; \Gamma \vdash t \triangleleft \text{Ur } u : \perp} \quad \text{TYTERM_FILLE} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \oplus A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d' : [A_1]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft 1.d'.u : B} \quad \text{TYTERM_FILLV1} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \oplus A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d' : [A_2]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma \vdash t \triangleleft 2.d'.u : B} \quad \text{TYTERM_FILLV2} \\
\\
\frac{\begin{array}{l} \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [A_1 \otimes A_2] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d_1 : [A_1], d_2 : [A_2]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft \langle d_1, d_2 \rangle . u : B} \quad \text{TYTERM_FILLP} \\
\\
\frac{\begin{array}{l} R \stackrel{\text{fix}}{=} \mu r . W \\ \Phi ; \Psi_1 ; \mathcal{U} ; \Gamma_1 \vdash t : [R] \\ \Phi ; \Psi_2 ; \mathcal{U} ; \Gamma_2 \sqcup \{d : [W[r := R]]\} \vdash u : B \\ \text{names}(\Gamma_1) \cap \text{names}(\Gamma_2) = \emptyset \\ \text{names}(\Psi_1) \cap \text{names}(\Psi_2) = \emptyset \end{array}}{\Phi ; \Psi_1 \sqcup \Psi_2 ; \mathcal{U} ; \Gamma_1 \sqcup \Gamma_2 \vdash t \triangleleft \text{roll } d . u : B} \quad \text{TYTERM_FILLR}
\end{array}$$

$\boxed{\text{command} \Downarrow \text{command}'}$ t reduces to t' , with heap growing from H to H'

$$\begin{array}{c}
\frac{}{\frac{\mathbb{H}| \bullet}{L} \Downarrow \frac{\mathbb{H}| \bullet}{L}} \quad \text{SEMOP_NOEFF (value)} \\
\\
\frac{}{\frac{\mathbb{H}| [l]}{L} \Downarrow \frac{\mathbb{H}| [l]}{L}} \quad \text{SEMOP_LDEST (value)} \\
\\
\frac{}{\frac{\mathbb{H}| \lambda x:A . t}{L} \Downarrow \frac{\mathbb{H}| \lambda x:A . t}{L}} \quad \text{SEMOP_LAM (value)} \\
\\
\frac{}{\frac{\mathbb{H}| C\bar{l}}{L} \Downarrow \frac{\mathbb{H}| C\bar{l}}{L}} \quad \text{SEMOP_HEAPVAL (value)}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid \lambda x:A. t' \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2 \quad \mathbb{H}_2 \mid t'[x := v_2] \Downarrow \mathbb{H}_3 \mid v_3}{\mathbb{H}_0 \mid t u \Downarrow \mathbb{H}_3 \mid v_3} \text{SEMOP_APP} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid () \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ () \mapsto u \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATU} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid \text{Ur } l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid u[x := v_1] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ \text{Ur } x \mapsto u \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATE} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid 1.l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid u_1[x_1 := v] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATV1} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid 2.l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid u_2[x_2 := v] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ 1.x_1 \mapsto u_1, 2.x_2 \mapsto u_2 \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATV2} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{ l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12} \} \mid \langle l_1, l_2 \rangle \quad \mathbb{H}_1 \sqcup \{ l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12} \} \mid u[x_1 := v_1, x_2 := v_2] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ (x_1, x_2) \mapsto u \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATP} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid \text{roll } l \quad \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid u[x := v_1] \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid \text{case } t \text{ of } \{ \text{roll } x \mapsto u \} \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_PATR} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid \bullet \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid t ; u \Downarrow \mathbb{H}_2 \mid v_2} \text{SEMOP_WITHEFF} \\
\\
\frac{l \notin L_0 \quad \mathbb{H}_0 \mid t[d := l] \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid \bullet}{\mathbb{H}_0 \mid \text{alloc } d . t \Downarrow \mathbb{H}_1 \sqcup \{ l \triangleleft v_1 \} \mid v_1} \text{SEMOP_ALLOC}
\end{array}$$

$$\begin{array}{c}
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l]}{\mathbb{H}_0 \mid t \triangleleft () \Downarrow \mathbb{H}_1 \sqcup \{l \triangleleft ()\} \mid \bullet} \text{SEMOP_FILLU} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid t \triangleleft u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft v_2\} \mid \bullet} \text{SEMOP_FILLL} \\
\\
\frac{\mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u \Downarrow \mathbb{H}_2 \mid v_2}{\mathbb{H}_0 \mid t \triangleleft \text{Ur } u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{Ur } l', l' \triangleleft v_2\} \mid \bullet} \text{SEMOP_FILLE} \\
\\
\frac{l' \notin L_1 \quad \mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \mid v_2}{\mathbb{H}_0 \mid t \triangleleft 1.d.u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 1.l', l' \triangleleft v_1\} \mid v_2} \text{SEMOP_FILLV1} \\
\\
\frac{l' \notin L_1 \quad \mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \mid v_2}{\mathbb{H}_0 \mid t \triangleleft 2.d.u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft 2.l', l' \triangleleft v_1\} \mid v_2} \text{SEMOP_FILLV2} \\
\\
\frac{l_1 \notin L_1 \quad l_2 \notin L_1 \quad \mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u[d_1 := [l_1], d_2 := [l_2]] \Downarrow \mathbb{H}_2 \sqcup \{l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} \mid v_2}{\mathbb{H}_0 \mid t \triangleleft \langle d_1, d_2 \rangle . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \langle l_1, l_2 \rangle, l_1 \triangleleft v_{11}, l_2 \triangleleft v_{12}\} \mid v_2} \text{SEMOP_FILLP} \\
\\
\frac{l' \notin L_1 \quad \mathbb{H}_0 \mid t \Downarrow \mathbb{H}_1 \mid [l] \quad \mathbb{H}_1 \mid u[d := [l']] \Downarrow \mathbb{H}_2 \sqcup \{l' \triangleleft v_1\} \mid v_2}{\mathbb{H}_0 \mid t \triangleleft \text{roll } d . u \Downarrow \mathbb{H}_2 \sqcup \{l \triangleleft \text{roll } l', l' \triangleleft v_1\} \mid v_2} \text{SEMOP_FILLR}
\end{array}$$

Definition rules: 50 good 0 bad
 Definition rule clauses: 151 good 0 bad