Destination λ-calculus

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1 Term and value syntax

var, x, y	Term-level variable name
k	Index for ranges

hdn, h	::=			Hole or destination name ($\mathbb{N}$)
	|	$\mathtt{h}+\mathtt{h}'$	M	Sum
	|	$\mathtt{max}(\mathtt{H})$	M	Maximum of a set of holes
hdns, H	::=			Set of hole names
	|	$\{\mathtt{h}_1, .., \mathtt{h}_k\}$		
	|	$\mathtt{H}_1 \cup \mathtt{H}_2$	M	Union of sets
	|	$\mathtt{H} \underline{\pm} \mathtt{h}$	M	Increase all names from H by h.
	|	$\mathtt{hnames}(\Gamma)$	M	Hole names of a context (requires `ctx_NoVar`(Γ))
	|	$\mathtt{hnames}(\mathsf{C})$	M	Hole names of an evaluation context
term, t, u	::=			Term
	|	v		Value
	|	x		Variable
	|	$\mathsf{t} \succ \mathsf{u}$		Application
	|	$\mathsf{t} \,;\, \mathsf{u}$		Pattern-match on unit
	|	$\mathsf{t} \succ \mathbf{case}_m \{ \mathsf{Inl}\, \mathsf{x}_1 \mapsto \mathsf{u}_1 ,\ \mathsf{Inr}\, \mathsf{x}_2 \mapsto \mathsf{u}_2 \}$		Pattern-match on sum
	|	$\mathsf{t} \succ \mathbf{case}_m (\mathsf{x}_1 , \mathsf{x}_2) \mapsto \mathsf{u}$		Pattern-match on product
	|	$\mathsf{t} \succ \mathbf{case}_m\, \mathrm{E}^n\, \mathsf{x} \mapsto \mathsf{u}$		Pattern-match on exponential
	|	$\mathsf{t} \succ \mathbf{map}\, \mathsf{x} \mapsto \mathsf{u}$		Map over the right side of ampar t
	|	$\mathbf{to}_{\ltimes}\, \mathsf{t}$		Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\, \mathsf{t}$		Extract value from trivial ampar
	|	$\mathbf{alloc}$		Return a fresh "identity" ampar object
	|	$\mathsf{t} \triangleleft ()$		Fill destination with unit
	|	$\mathsf{t} \triangleleft (\lambda \mathsf{x}\, {}_m \mapsto \mathsf{u})$		Fill destination with function
	|	$\mathsf{t} \triangleleft \mathsf{Inl}$		Fill destination with left variant
	|	$\mathsf{t} \triangleleft \mathsf{Inr}$		Fill destination with right variant
	|	$\mathsf{t} \triangleleft (,)$		Fill destination with product constructor
	|	$\mathsf{t} \triangleleft \mathrm{E}^m$		Fill destination with exponential constructor
	|	$\mathsf{t} \triangleleft\!\bullet\, \mathsf{u}$		Fill destination with root of ampar u
	|	$\mathsf{t}[\mathsf{x} \coloneqq \mathsf{v}]$	M	
val, v	::=			Term value
	|	$-\mathtt{h}$		Hole
	|	$+\mathtt{h}$		Destination
	|	$()$		Unit
	|	$\lambda^{\mathsf{v}} \mathsf{x}\, {}_m \mapsto \mathsf{t}$		Lambda abstraction
	|	$\mathsf{Inl}\, \mathsf{v}$		Left variant for sum
	|	$\mathsf{Inr}\, \mathsf{v}$		Right variant for sum
	|	$\mathrm{E}^m\, \mathsf{v}$		Exponential
	|	$(\mathsf{v}_1 , \mathsf{v}_2)$		Product
	|	${}_{\mathtt{H}}\langle \mathsf{v}_1 \,,\, \mathsf{v}_2 \rangle$		Ampar
	|	$\mathsf{v} \underline{\pm} \mathtt{h}$	M	Rename hole names inside v by shifting them by h
ectx, C	::=			Evaluation context

|	$\square$		Identity
|	$C \succ u$		Application
|	$v \succ C$		Application
|	$C \, ; u$		Pattern-match on unit
|	$C \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}$		Pattern-match on sum
|	$C \succ \mathbf{case}_{m} (x_1 , x_2) \mapsto u$		Pattern-match on product
|	$C \succ \mathbf{case}_{m} \mathrm{E}^{n}\, x \mapsto u$		Pattern-match on exponential
|	$C \succ \mathbf{map}\ x \mapsto u$		Map over the right side of ampar
|	$\mathbf{to}_{\ltimes}\ C$		Wrap into a trivial ampar
|	$\mathbf{from}_{\ltimes}\ C$		Extract value from trivial ampar
|	$C \triangleleft ()$		Fill destination with unit
|	$C \triangleleft (\lambda x_{m} \mapsto u)$		Fill destination with function
|	$C \triangleleft \mathsf{Inl}$		Fill destination with left variant
|	$C \triangleleft \mathsf{Inr}$		Fill destination with right variant
|	$C \triangleleft (,)$		Fill destination with product constructor
|	$C \triangleleft \mathrm{E}^{m}$		Fill destination with exponential constructor
|	$C \triangleleft\!\bullet\ u$		Fill destination with root of ampar
|	$v \triangleleft\!\bullet\ C$		Fill destination with root of ampar
|	${}^{\mathrm{op}}_{\mathrm{H}}\langle v_1 ,\, C$		Open ampar. Only new addition to term shapes
|	$C \circ C'$	M	Compose evaluation contexts
|	$C[h :=_{\mathrm{H}} v]$	M	Fill h with value v (that may contain holes)

2 Type system

$\mathsf{type},\ \mathsf{T},\ \mathsf{U}$	::=			Type
	|	$\mathbf{1}$		Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$		Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$		Product
	|	$!^{m}\, \mathsf{T}$		Exponential
	|	$\mathsf{T}_1 \ltimes \mathsf{T}_2$		Ampar type (consuming T_2 yields T_1)
	|	$\mathsf{T}_1\ {}_{m_1}\!\!\to \mathsf{T}_2$		Function
	|	$\lfloor \mathsf{T} \rfloor^{m}$		Destination
$\mathit{mode},\ m,\ n$	::=			Mode (Semiring)
	|	pa		Pair of a multiplicity and age
	|	☠		Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	M	Semiring product
$\mathit{mul},\ p$	::=			Multiplicity (first component of modality)
	|	1		Linear. Neutral element of the product
	|	ω		Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	M	Semiring product
$\mathit{age},\ a$	::=			Age (second component of modality)
	|	ν		Born now. Neutral element of the product
	|	$\uparrow$		One scope older
	|	∞		Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	M	Semiring product
bndr, b	::=			Type assignment to either variable, destination or hole
	|	$x :_{m} \mathsf{T}$		Variable
	|	$+h :_{m} \lfloor \mathsf{T} \rfloor^{n}$		Destination (m is its own modality; n is the modality for values it accepts)
	|	$-h : \mathsf{T}^{n}$		Hole (n is the modality for values it accepts, it doesn't have a modality on its own)
ctx, Γ, Δ, Π	::=			Typing context
	|	$\{b_1, .., b_k\}$		List of bindings
	|	$m \cdot \Gamma$	M	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	M	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with
	|	$-\Gamma$	M	Transforms every dest binding into a hole binding (requires `ctx_DestOnly` Γ)

$\boxed{\Gamma \Vdash \mathsf{v} : \mathsf{T}}$ *(Typing of values (raw))*

$$\textsc{TyR-val-H}\quad \frac{}{\{-h : \mathsf{T}^{1\nu}\} \Vdash -h : \mathsf{T}}$$

$$\textsc{TyR-val-D}\quad \frac{\texttt{ctx\_Compatible}\ \Gamma\ \ {+h} :_{1\nu} \lfloor \mathsf{T} \rfloor^{n}}{\Gamma \Vdash {+h} : \lfloor \mathsf{T} \rfloor^{n}}$$

$$\textsc{TyR-val-U}\quad \frac{}{\{\} \Vdash () : \mathbf{1}}$$

$$\textsc{TyR-val-F}\quad \frac{\texttt{ctx\_DestOnly}\ \ \Delta \qquad \Delta \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{t} : \mathsf{T}_2}{\Delta \Vdash \lambda^{\mathsf{v}} x\ {}_{m} \mapsto \mathsf{t} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}$$

$$\textsc{TyR-val-L}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-R}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-val-P}\quad \frac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-val-E}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{n{\cdot}\Gamma \Vdash \mathrm{E}^{n}\,\mathsf{v} : !^{n}\,\mathsf{T}}$$

$$\textsc{TyR-val-A}\quad \frac{\begin{array}{c}\texttt{ctx\_DestOnly}\ \ \Delta_1\\ \texttt{ctx\_DestOnly}\ \ \Delta_2\\ \texttt{ctx\_DestOnly}\ \ \Delta_3\\ \texttt{ctx\_Disjoint}\ \ \Delta_1\ \ \Delta_2\\ \texttt{ctx\_Disjoint}\ \ \Delta_1\ \ \Delta_3\\ \texttt{ctx\_Disjoint}\ \ \Delta_2\ \ \Delta_3\\ \Delta_1 \uplus (-\Delta_3) \Vdash \mathsf{v}_1 : \mathsf{T}_1\\ \Delta_2 \uplus \Delta_3 \Vdash \mathsf{v}_2 : \mathsf{T}_2\end{array}}{\Delta_1 \uplus \Delta_2 \Vdash_{\texttt{hnames}(-\Delta_3)} \langle \mathsf{v}_1\,{,}\,\mathsf{v}_2 \rangle : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$\boxed{\Pi \vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms)*

$$\textsc{Ty-term-Val}\quad \frac{\texttt{ctx\_DestOnly}\ \ \Delta \qquad \Delta \Vdash \mathsf{v} : \mathsf{T}}{\Delta \vdash \mathsf{v} : \mathsf{T}}$$

$$\textsc{Ty-term-Var}\quad \frac{\texttt{ctx\_Compatible}\ \Pi\ \ x :_{1\nu} \mathsf{T}}{\Pi \vdash x : \mathsf{T}}$$

$$\textsc{Ty-term-App}\quad \frac{\Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{Ty-term-PatU}\quad \frac{\Pi_1 \vdash \mathsf{t} : \mathbf{1} \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{U}}{\Pi_1 \uplus \Pi_2 \vdash \mathsf{t}\,;\,\mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x_1 :_{m} \mathsf{T}_1\}\\ \texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x_2 :_{m} \mathsf{T}_2\}\\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2\\ \Pi_2 \uplus \{x_1 :_{m} \mathsf{T}_1\} \vdash \mathsf{u}_1 : \mathsf{U}\\ \Pi_2 \uplus \{x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u}_2 : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,\{\,\mathsf{Inl}\,x_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,x_2 \mapsto \mathsf{u}_2\,\} : \mathsf{U}}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x_1 :_{m} \mathsf{T}_1\}\\ \texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x_2 :_{m} \mathsf{T}_2\}\\ \texttt{ctx\_Disjoint}\ \ \{x_1 :_{m} \mathsf{T}_1\}\ \ \{x_2 :_{m} \mathsf{T}_2\}\\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2\\ \Pi_2 \uplus \{x_1 :_{m} \mathsf{T}_1, x_2 :_{m} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,(x_1\,,\,x_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x :_{m\cdot n} \mathsf{T}\}\\ \Pi_1 \vdash \mathsf{t} : !^{n}\,\mathsf{T}\\ \Pi_2 \uplus \{x :_{m\cdot n} \mathsf{T}\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{m{\cdot}\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{case}_{m}\,\mathrm{E}^{n}\,x \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x :_{1\nu} \mathsf{T}_2\}\\ \Pi_1 \vdash \mathsf{t} : \mathsf{T}_1 \ltimes \mathsf{T}_2\\ 1{\uparrow}{\cdot}\Pi_2 \uplus \{x :_{1\nu} \mathsf{T}_2\} \vdash \mathsf{u} : \mathsf{U}\end{array}}{\Pi_1 \uplus \Pi_2 \vdash \mathsf{t} \succ \mathsf{map}\ x \mapsto \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Pi \vdash \mathsf{t} : \mathsf{T}}{\Pi \vdash \mathsf{to}_{\ltimes}\,\mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Pi \vdash \mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}{\Pi \vdash \mathsf{from}_{\ltimes}\,\mathsf{t} : \mathsf{T}}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathsf{alloc} : \mathsf{T} \ltimes \lfloor \mathsf{T} \rfloor^{1\nu}}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathbf{1} \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft () : \mathbf{1}}$$

$$\textsc{Ty-term-FillF}\quad \frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Pi_2\ \ \{x :_{m} \mathsf{T}_1\}\\ \Pi_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2 \rfloor^{n}\\ \Pi_2 \uplus \{x :_{m} \mathsf{T}_1\} \vdash \mathsf{u} : \mathsf{T}_2\end{array}}{\Pi_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Pi_2 \vdash \mathsf{t} \triangleleft (\lambda x\ {}_{m} \mapsto \mathsf{u}) : \mathbf{1}}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathsf{Inl} : \lfloor \mathsf{T}_1 \rfloor^{n}}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathsf{Inr} : \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft (,) : \lfloor \mathsf{T}_1 \rfloor^{n} \otimes \lfloor \mathsf{T}_2 \rfloor^{n}}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Pi \vdash \mathsf{t} : \lfloor !^{n'}\,\mathsf{T} \rfloor^{n}}{\Pi \vdash \mathsf{t} \triangleleft \mathrm{E}^{n'} : \lfloor \mathsf{T} \rfloor^{n'\cdot n}}$$

$$\textsc{Ty-term-FillC}\quad \frac{\Pi_1 \vdash \mathsf{t} : \lfloor \mathsf{T}_1 \rfloor^{n} \qquad \Pi_2 \vdash \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Pi_1 \uplus (1{\uparrow}{\cdot}n){\cdot}\Pi_2 \vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_2}$$

$\boxed{\Delta \dashv C : T_1 \rightarrowtail T_2}$ *(Typing of evaluation contexts)*

TyR-ectx-Id

$$\frac{}{\{\} \dashv \square : U_0 \rightarrowtail U_0}$$

TyR-ectx-App1

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ m\cdot\Delta_1 \uplus \Delta_2 \dashv C : T_2 \rightarrowtail U_0 \\ \Delta_2 \vdash u : T_1 \ {}_{m}\!\!\to T_2\end{array}}{\Delta_1 \dashv C \circ (\square \succ u) : T_1 \rightarrowtail U_0}$$

TyR-ectx-App2

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ m\cdot\Delta_1 \uplus \Delta_2 \dashv C : T_2 \rightarrowtail U_0 \\ \Delta_1 \vdash v : T_1\end{array}}{\Delta_2 \dashv C \circ (v \succ \square) : (T_1 \ {}_{m}\!\!\to T_2) \rightarrowtail U_0}$$

TyR-ectx-PatU

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0 \\ \Delta_2 \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \ ;\ u) : 1 \rightarrowtail U_0}$$

TyR-ectx-PatS

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x_1 :_m T_1\} \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x_2 :_m T_2\} \\ m\cdot\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0 \\ \Delta_2 \uplus \{x_1 :_m T_1\} \vdash u_1 : U \\ \Delta_2 \uplus \{x_2 :_m T_2\} \vdash u_2 : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m \{\mathsf{Inl}\, x_1 \mapsto u_1,\ \mathsf{Inr}\, x_2 \mapsto u_2\}) : (T_1 \oplus T_2) \rightarrowtail U_0}$$

TyR-ectx-PatP

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x_1 :_m T_1\} \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x_2 :_m T_2\} \\ \texttt{ctx\_Disjoint}\ \{x_1 :_m T_1\}\ \{x_2 :_m T_2\} \\ m\cdot\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0 \\ \Delta_2 \uplus \{x_1 :_m T_1, x_2 :_m T_2\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m (x_1 , x_2) \mapsto u) : (T_1 \otimes T_2) \rightarrowtail U_0}$$

TyR-ectx-PatE

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x :_{m\cdot m'} T\} \\ m\cdot\Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0 \\ \Delta_2 \uplus \{x :_{m\cdot m'} T\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{case}_m\ \mathrm{E}^{m'}\ x \mapsto u) : !^{m'}\, T \rightarrowtail U_0}$$

TyR-ectx-Map

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ 1\!\uparrow\cdot\Delta_2\ \{x :_{1\nu} T_2\} \\ \Delta_1 \uplus \Delta_2 \dashv C : U \rightarrowtail U_0 \\ 1\!\uparrow\cdot\Delta_2 \uplus \{x :_{1\nu} T_2\} \vdash u : U\end{array}}{\Delta_1 \dashv C \circ (\square \succ \mathbf{map}\ x \mapsto u) : (T_1 \ltimes T_2) \rightarrowtail U_0}$$

TyR-ectx-ToA

$$\frac{\Delta \dashv C : (T \ltimes 1) \rightarrowtail U_0}{\Delta \dashv C \circ (\mathbf{to}_{\ltimes}\ \square) : T \rightarrowtail U_0}$$

TyR-ectx-FromA

$$\frac{\Delta \dashv C : T \rightarrowtail U_0}{\Delta \dashv C \circ (\mathbf{from}_{\ltimes}\ \square) : (T \ltimes 1) \rightarrowtail U_0}$$

TyR-ectx-FillU

$$\frac{\Delta \dashv C : 1 \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft ()) : \lfloor 1 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillF

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_2\ \{x :_m T_1\} \\ \Delta_1 \uplus (1\!\uparrow\cdot n)\cdot\Delta_2 \dashv C : 1 \rightarrowtail U_0 \\ \Delta_2 \uplus \{x :_m T_1\} \vdash u : T_2\end{array}}{\Delta_1 \dashv C \circ (\square \triangleleft (\lambda x\ {}_m \mapsto u)) : \lfloor T_1 \ {}_{m}\!\!\to T_2 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillL

$$\frac{\Delta \dashv C : \lfloor T_1 \rfloor^{n} \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathsf{Inl}) : \lfloor T_1 \oplus T_2 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillR

$$\frac{\Delta \dashv C : \lfloor T_2 \rfloor^{n} \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathsf{Inr}) : \lfloor T_1 \oplus T_2 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillP

$$\frac{\Delta \dashv C : (\lfloor T_1 \rfloor^{n} \otimes \lfloor T_2 \rfloor^{n}) \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft (,)) : \lfloor T_1 \otimes T_2 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillE

$$\frac{\Delta \dashv C : \lfloor T \rfloor^{m\cdot n} \rightarrowtail U_0}{\Delta \dashv C \circ (\square \triangleleft \mathrm{E}^{m}) : \lfloor !^{m}\, T \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillC1

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \Delta_1 \uplus (1\!\uparrow\cdot n)\cdot\Delta_2 \dashv C : T_2 \rightarrowtail U_0 \\ \Delta_2 \vdash u : T_1 \ltimes T_2\end{array}}{\Delta_1 \dashv C \circ (\square \triangleleft\!\bullet\, u) : \lfloor T_1 \rfloor^{n} \rightarrowtail U_0}$$

TyR-ectx-FillC2

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \Delta_1 \uplus (1\!\uparrow\cdot n)\cdot\Delta_2 \dashv C : T_2 \rightarrowtail U_0 \\ \Delta_1 \vdash v : \lfloor T_1 \rfloor^{n}\end{array}}{\Delta_2 \dashv C \circ (v \triangleleft\!\bullet\, \square) : T_1 \ltimes T_2 \rightarrowtail U_0}$$

TyR-ectx-AOpen

$$\frac{\begin{array}{c}\texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \Delta_3 \\ \texttt{hdns\_Disjoint}\ \texttt{hnames}(C)\ \texttt{hnames}(-\Delta_3) \\ \texttt{ctx\_DestOnly}\ \Delta_1 \\ \texttt{ctx\_DestOnly}\ \Delta_2 \\ \texttt{ctx\_DestOnly}\ \Delta_3 \\ \Delta_1 \uplus \Delta_2 \dashv C : (T_1 \ltimes U) \rightarrowtail U_0 \\ \Delta_1 \uplus -\Delta_3 \Vdash v_1 : T_1\end{array}}{1\!\uparrow\cdot\Delta_2 \uplus \Delta_3 \dashv C \circ ({}^{\mathrm{op}}_{\texttt{hnames}(-\Delta_3)}\langle v_1 ,\, \square) : U \rightarrowtail U_0}$$

$\boxed{\vdash \mathsf{C}[\mathsf{t}] : \mathbf{T}}$ *(Typing of extended terms (pair of evaluation context and term))*

$$\frac{\Gamma \dashv \mathsf{C} : \mathbf{T} \rightarrowtail \mathbf{U}_0 \qquad \Gamma \vdash \mathsf{t} : \mathbf{T}}{\vdash \mathsf{C}[\mathsf{t}] : \mathbf{U}_0} \; \textsc{Ty-eterm-ClosedEterm}$$

3 Small-step semantics

$\boxed{C[t] \longrightarrow C'[t']}$ *(Small-step evaluation of terms using evaluation contexts)*

$$\frac{}{C[v \succ (\lambda^{v} x_{m} \mapsto t)] \longrightarrow C[t[x := v]]}\ \textsc{Sem-eterm-App}$$

$$\frac{}{C[() \ ; t_2] \longrightarrow C[t_2]}\ \textsc{Sem-eterm-PatU}$$

$$\frac{}{C[(\mathsf{Inl}\, v) \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}] \longrightarrow C[u_1[x := v]]}\ \textsc{Sem-eterm-PatL}$$

$$\frac{}{C[(\mathsf{Inr}\, v) \succ \mathbf{case}_{m} \{ \mathsf{Inl}\, x_1 \mapsto u_1 ,\ \mathsf{Inr}\, x_2 \mapsto u_2 \}] \longrightarrow C[u_2[x := v]]}\ \textsc{Sem-eterm-PatR}$$

$$\frac{}{C[(v_1 , v_2) \succ \mathbf{case}_{m} (x_1 , x_2) \mapsto u] \longrightarrow C[u[x_1 := v_1][x_2 := v_2]]}\ \textsc{Sem-eterm-PatP}$$

$$\frac{}{C[\mathrm{E}^{n}\, v \succ \mathbf{case}_{m} \mathrm{E}^{n}\, x \mapsto u] \longrightarrow C[u[x := v]]}\ \textsc{Sem-eterm-PatE}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C))}{C[{}_{H}\langle v_1 \, , v_2 \rangle \succ \mathbf{map}\ x \mapsto u] \longrightarrow (C \circ ({}^{\mathrm{op}}_{H \pm h'}\langle v_1 \pm h' , \square \rangle))[u[x := v_2 \pm h']]}\ \textsc{Sem-eterm-MapOpen}$$

$$\frac{}{(C \circ {}^{\mathrm{op}}_{H}\langle v_1 , \square \rangle)[v_2] \longrightarrow C[{}_{H}\langle v_1 , v_2 \rangle]}\ \textsc{Sem-eterm-MapClose}$$

$$\frac{}{C[\mathbf{alloc}] \longrightarrow C[{}_{\{1\}}\langle +1 , -1 \rangle]}\ \textsc{Sem-eterm-Alloc}$$

$$\frac{}{C[\mathbf{to}_{\ltimes}\, v] \longrightarrow C[{}_{\{\,\}}\langle v , () \rangle]}\ \textsc{Sem-eterm-ToA}$$

$$\frac{}{C[\mathbf{from}_{\ltimes}\, {}_{\{\,\}}\langle v , () \rangle] \longrightarrow C[v]}\ \textsc{Sem-eterm-FromA}$$

$$\frac{}{C[+h \triangleleft ()] \longrightarrow C[h :=_{\{\,\}} ()][()]}\ \textsc{Sem-eterm-FillU}$$

$$\frac{}{C[+h \triangleleft (\lambda x_{m} \mapsto u)] \longrightarrow C[h :=_{\{\,\}} \lambda^{v} x_{m} \mapsto u][()]}\ \textsc{Sem-eterm-FillF}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inl}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inl}\, -(h'+1)][+(h'+1)]}\ \textsc{Sem-eterm-FillL}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathsf{Inr}] \longrightarrow C[h :=_{\{h'+1\}} \mathsf{Inr}\, -(h'+1)][+(h'+1)]}\ \textsc{Sem-eterm-FillR}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft \mathrm{E}^{m}] \longrightarrow C[h :=_{\{h'+1\}} \mathrm{E}^{m} -(h'+1)][+(h'+1)]}\ \textsc{Sem-eterm-FillE}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft (,)] \longrightarrow C[h :=_{\{h'+1, h'+2\}} (-(h'+1) , -(h'+2))][(+(h'+1) , +(h'+2))]}\ \textsc{Sem-eterm-FillP}$$

$$\frac{h' = \mathtt{max}(\mathtt{hnames}(C) \cup \{h\})}{C[+h \triangleleft\!\bullet {}_{H}\langle v_1 , v_2 \rangle] \longrightarrow C[h :=_{(H \pm h')} v_1 \pm h'][v_2 \pm h']}\ \textsc{Sem-eterm-FillC}$$