Destination λ-calculus

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1 Term and value syntax

tvar, x, y	Term-level variable name
hvar, h	Hole or destination name
k	Index for ranges

$$
\begin{array}{llll}
\mathsf{val},\ \mathsf{v} & ::= & & \text{Term value} \\
& | & \langle \mathsf{v}_1 \,\mathbin{,}\, \overline{\mathsf{v}_2} \rangle_\Delta & \quad\text{Ampar} \\
& | & @\mathtt{h} & \quad\text{Destination} \\
& | & () & \quad\text{Unit} \\
& | & \mathsf{Inl}\,\mathsf{v} & \quad\text{Left variant for sum} \\
& | & \mathsf{Inr}\,\mathsf{v} & \quad\text{Right variant for sum} \\
& | & (\mathsf{v}_1\,,\,\mathsf{v}_2) & \quad\text{Product} \\
& | & \rrbracket^m\,\mathsf{v} & \quad\text{Exponential} \\
& | & \lambda \mathsf{x} \mapsto \mathsf{t} & \quad\text{Linear function}
\end{array}
$$

$$
\begin{array}{llll}
\overline{\mathsf{xval}},\ \overline{\mathsf{v}} & ::= & & \text{Pseudo-value that may contain holes} \\
& | & \mathsf{v} & \quad\text{Term value} \\
& | & \mathtt{h} & \quad\text{Hole} \\
& | & \mathsf{Inl}\,\overline{\mathsf{v}} & \quad\text{Left variant with val or hole} \\
& | & \mathsf{Inr}\,\overline{\mathsf{v}} & \quad\text{Right variant with val or hole} \\
& | & (\overline{\mathsf{v}_1}\,,\,\overline{\mathsf{v}_2}) & \quad\text{Product with val or hole} \\
& | & \rrbracket^m\,\overline{\mathsf{v}} & \quad\text{Exponential with val or hole}
\end{array}
$$

$$
\begin{array}{llll}
\mathsf{term},\ \mathsf{t},\ \mathsf{u} & ::= & & \text{Term} \\
& | & \mathsf{v} & \quad\text{Term value} \\
& | & \mathsf{x} & \quad\text{Variable} \\
& | & \mathsf{t} \succ \mathsf{u} & \quad\text{Application} \\
& | & \mathsf{t}\ ;\ \mathsf{u} & \quad\text{Pattern-match on unit} \\
& | & \mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} & \quad\text{Pattern-match on sum} \\
& | & \mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u} & \quad\text{Pattern-match on product} \\
& | & \mathsf{t} \succ \mathbf{case}\,\rrbracket^m\,\mathsf{x} \mapsto \mathsf{u} & \quad\text{Pattern-match on exponential} \\
& | & \mathsf{t} \succ \mathbf{map}\,\mathsf{x} \mapsto \mathsf{u} & \quad\text{Map over the left side of the ampar} \\
& | & \mathbf{to}_{\ltimes}\,\mathsf{t} & \quad\text{Wrap } \mathsf{t} \text{ into a trivial ampar} \\
& | & \mathbf{from}_{\ltimes}\,\mathsf{t} & \quad\text{Extract value from trivial ampar} \\
& | & \mathbf{alloc}_{\mathsf{T}} & \quad\text{Return a fresh "identity" ampar object} \\
& | & \mathsf{t} \triangleleft () & \quad\text{Fill destination with unit} \\
& | & \mathsf{t} \triangleleft \mathsf{Inl} & \quad\text{Fill destination with left variant} \\
& | & \mathsf{t} \triangleleft \mathsf{Inr} & \quad\text{Fill destination with right variant} \\
& | & \mathsf{t} \triangleleft (,) & \quad\text{Fill destination with product constructor} \\
& | & \mathsf{t} \triangleleft \rrbracket^m & \quad\text{Fill destination with exponential constructor} \\
& | & \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} & \quad\text{Fill destination with root of ampar } \mathsf{u}
\end{array}
$$

2 Type system

typ, T, U	::=		Type
	|	1	Unit
	|	$T_1 \oplus T_2$	Sum
	|	$T_1 \otimes T_2$	Product
	|	$!^m\ T$	Exponential
	|	$T_1 \ltimes T_2$	Ampar type (consuming T_1 yields T_2)
	|	$T_1\ {}_{m_1}\!\!\to T_2$	Linear function
	|	${}^m\lfloor T \rfloor$	Destination
moda, m, n	::=		Modality (Semiring)
	|	$p\|a$	Pair of a multiplicity and age
	|	$\underline{\nu}$	Neutral element of the product. Notation for $1\|\nu$.
	|	$\underline{\uparrow}$	Same multiplicity, but one scope older. Notation for $1\|\uparrow$.
	|	$\underline{\infty}$	Linear, infinitely old / static. Notation for $1\|\infty$.
	|	$m_1 \cdot \ldots \cdot m_k$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	1	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	Semiring product
pctx, Γ	::=		Positive typing context
	|	$\{\mathsf{pas}_1, .., \mathsf{pas}_k\}$	
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \cup \Gamma_2$	
pas	::=		Positive type assignment
	|	$x :_m T$	Variable
	|	$@h :_m {}^n\lfloor T \rfloor$	Destination (m is its own modality; n is the modality for values it accepts)
nctx, Δ	::=		Negative typing context
	|	$\{\mathsf{nas}_1, .., \mathsf{nas}_k\}$	
	|	$m \cdot \Delta$	Multiply each binding by m
	|	$@^{-1}\Gamma$	Maps each destination of Γ to a hole (requires `ctx_DestOnly` Γ)
	|	$\Delta_1 \cup \Delta_2$	
nas	::=		Negative type assignment
	|	$h :^n T$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \cup \Delta \Vdash \overline{v} : T}$ *(Typing of extended values (require both positive and negative contexts))*

$$\textsc{Ty-xval-H}\quad \frac{}{\{\,\} \cup \{h :^{\underline{\nu}} T\} \Vdash h : T}$$

$$\textsc{Ty-xval-D}\quad \frac{}{\{@h :_{\underline{\nu}} {}^n\lfloor T \rfloor\} \cup \{\,\} \Vdash @h : {}^n\lfloor T \rfloor}$$

$$\textsc{Ty-xval-U}\quad \frac{}{\{\,\} \cup \{\,\} \Vdash () : 1}$$

$$\textsc{Ty-xval-L}\quad \frac{\Gamma \cup \Delta \Vdash \overline{v} : T_1 \qquad \texttt{ctx\_Disjoint}\ \Gamma\ \Delta}{\Gamma \cup \Delta \Vdash \mathsf{Inl}\,\overline{v} : T_1 \oplus T_2}$$

$$\textsc{Ty-xval-R}\quad \frac{\begin{array}{c}\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{v} : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \Delta\end{array}}{\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \mathsf{Inr}\,\overline{v} : T_1 \oplus T_2}$$

$$\textsc{Ty-xval-P}\quad \frac{\begin{array}{c}\Gamma_1 \mathbin{\scriptstyle\cup} \Delta_1 \Vdash \overline{v_1} : T_1 \\ \Gamma_2 \mathbin{\scriptstyle\cup} \Delta_2 \Vdash \overline{v_2} : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \Delta_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \Delta_2 \\ \texttt{ctx\_Disjoint}\ \Delta_1\ \ \Delta_2\end{array}}{\Gamma_1 \cup \Gamma_2 \mathbin{\scriptstyle\cup} \Delta_1 \cup \Delta_2 \Vdash (\overline{v_1}\ ,\ \overline{v_2}) : T_1 \otimes T_2}$$

$$\textsc{Ty-xval-E}\quad \frac{\begin{array}{c}\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{v} : T \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \Delta\end{array}}{m\cdot\Gamma \mathbin{\scriptstyle\cup} m\cdot\Delta \Vdash \mathclose{|\!)}^{m}\,\overline{v} : !^{m}\,T}$$

$$\textsc{Ty-xval-A}\quad \frac{\begin{array}{c}\Gamma_1 \mathbin{\scriptstyle\cup} \{\} \Vdash v_1 : T_1 \\ \Gamma_2 \mathbin{\scriptstyle\cup} @^{-1}\Gamma_1 \Vdash \overline{v_2} : T_2 \\ \texttt{pctx\_DestOnly}\ \Gamma_1 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2\end{array}}{\Gamma_2 \mathbin{\scriptstyle\cup} \{\} \Vdash \langle v_1 \,,\, \overline{v_2}\rangle_{@^{-1}\Gamma_1} : T_1 \ltimes T_2}$$

$$\textsc{Ty-xval-F}\quad \frac{\begin{array}{c}\Gamma \cup \{x :_{m} T_1\} \vdash t : T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma\ \ \{x :_{m} T_1\}\end{array}}{\Gamma \mathbin{\scriptstyle\cup} \{\} \Vdash \lambda x \mapsto t : T_1\ {}_{m}\!\!\to T_2}$$

$\boxed{\Gamma \vdash t : T}$ *(Typing of terms (only a positive context is needed))*

$$\textsc{Ty-term-V}\quad \frac{\Gamma \mathbin{\scriptstyle\cup} \{\} \Vdash v : T}{\Gamma \vdash v : T}$$

$$\textsc{Ty-term-X0}\quad \frac{}{\{x :_{\underline{\nu}} T\} \vdash x : T}$$

$$\textsc{Ty-term-XInf}\quad \frac{}{\{x :_{\underline{\infty}} T\} \vdash x : T}$$

$$\textsc{Ty-term-App}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \qquad \Gamma_2 \vdash u : T_1\ {}_{m}\!\!\to T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2\end{array}}{m\cdot\Gamma_1 \cup \Gamma_2 \vdash t \succ u : T_2}$$

$$\textsc{Ty-term-PatS}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \oplus T_2 \\ \Gamma_2 \cup \{x_1 :_{m} T_1\} \vdash u_1 : U \\ \Gamma_2 \cup \{x_2 :_{m} T_2\} \vdash u_2 : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} T_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} T_2\}\end{array}}{m\cdot\Gamma_1 \cup \Gamma_2 \vdash t \succ \mathsf{case}\,\{\mathsf{Inl}\,x_1 \mapsto u_1\ ,\ \mathsf{Inr}\,x_2 \mapsto u_2\} : U}$$

$$\textsc{Ty-term-PatU}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : 1 \qquad \Gamma_2 \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2\end{array}}{\Gamma_1 \cup \Gamma_2 \vdash t\ ;\ u : U}$$

$$\textsc{Ty-term-PatP}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \otimes T_2 \\ \Gamma_2 \cup \{x_1 :_{m} T_1, x_2 :_{m} T_2\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_1 :_{m} T_1\} \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x_2 :_{m} T_2\} \\ \texttt{ctx\_Disjoint}\ \{x_1 :_{m} T_1\}\ \ \{x_2 :_{m} T_2\}\end{array}}{m\cdot\Gamma_1 \cup \Gamma_2 \vdash t \succ \mathsf{case}\,(x_1\ ,\ x_2) \mapsto u : U}$$

$$\textsc{Ty-term-PatE}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : !^{n}\,T \\ \Gamma_2 \cup \{x :_{m\cdot n} T\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{m\cdot n} T\}\end{array}}{m\cdot\Gamma_1 \cup \Gamma_2 \vdash t \succ \mathsf{case}\ \mathclose{|\!)}^{n}\,x \mapsto u : U}$$

$$\textsc{Ty-term-Map}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : T_1 \ltimes T_2 \\ \uparrow\cdot\Gamma_2 \cup \{x :_{\underline{\nu}} T_1\} \vdash u : U \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_2\ \ \{x :_{\underline{\nu}} T_1\}\end{array}}{\Gamma_1 \cup \Gamma_2 \vdash t \succ \mathsf{map}\ x \mapsto u : U \ltimes T_2}$$

$$\textsc{Ty-term-FillC}\quad \frac{\begin{array}{c}\Gamma_1 \vdash t : {}^{n}\lfloor T_2\rfloor \qquad \Gamma_2 \vdash u : T_1 \ltimes T_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_1\ \ \Gamma_2\end{array}}{\Gamma_1 \cup (\uparrow\cdot n)\cdot\Gamma_2 \vdash t \triangleleft\!\bullet\, u : T_1}$$

$$\textsc{Ty-term-FillU}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor 1\rfloor}{\Gamma \vdash t \triangleleft () : 1}$$

$$\textsc{Ty-term-FillL}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \oplus T_2\rfloor}{\Gamma \vdash t \triangleleft \mathsf{Inl} : {}^{n}\lfloor T_1\rfloor}$$

$$\textsc{Ty-term-FillR}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \oplus T_2\rfloor}{\Gamma \vdash t \triangleleft \mathsf{Inr} : {}^{n}\lfloor T_2\rfloor}$$

$$\textsc{Ty-term-FillP}\quad \frac{\Gamma \vdash t : {}^{n}\lfloor T_1 \otimes T_2\rfloor}{\Gamma \vdash t \triangleleft (,) : {}^{n}\lfloor T_1\rfloor \otimes {}^{n}\lfloor T_2\rfloor}$$

$$\textsc{Ty-term-FillE}\quad \frac{\Gamma \vdash t : {}^{m}\lfloor !^{n}\,T\rfloor}{\Gamma \vdash t \triangleleft \mathclose{|\!)}^{n} : {}^{m\cdot n}\lfloor T\rfloor}$$

$$\textsc{Ty-term-Alloc}\quad \frac{}{\{\} \vdash \mathsf{alloc}_{T} : {}^{\underline{\nu}}\lfloor T\rfloor \ltimes T}$$

$$\textsc{Ty-term-ToA}\quad \frac{\Gamma \vdash t : T}{\Gamma \vdash \mathsf{to}_{\ltimes}\ t : 1 \ltimes T}$$

$$\textsc{Ty-term-FromA}\quad \frac{\Gamma \vdash t : 1 \ltimes T}{\Gamma \vdash \mathsf{from}_{\ltimes}\ t : T}$$

$\boxed{\Gamma \vdash v \diamond e : T}$ *(Typing of commands (only a positive context is needed))*

$$\textsc{Ty-cmd-C}\quad \frac{\begin{array}{c}\Gamma_{11} \cup \Gamma_{12} \vdash v : T \\ \Gamma_2 \mathbin{\scriptstyle\cup} @^{-1}\Gamma_{12} \Vdash e \\ \texttt{pctx\_DestOnly}\ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \ \Gamma_{12} \\ \texttt{ctx\_Disjoint}\ \Gamma_{11}\ \ \Gamma_2 \\ \texttt{ctx\_Disjoint}\ \Gamma_{12}\ \ \Gamma_2\end{array}}{\Gamma_{11} \cup \Gamma_2 \vdash v \diamond e : T}$$

3 Effects and big-step semantics

$$
\begin{array}{llll}
\mathit{eff},\ e & ::= & & \text{Effect} \\
 & | & \varepsilon & \\
 & | & \mathtt{h} \coloneqq \overline{\mathsf{v}} & \\
 & | & e_1, \ldots, e_{\mathsf{k}} & \text{Chain effects}
\end{array}
$$

$\boxed{\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash e}$ *(Typing of effects (require both positive and negative contexts))*

$$
\textsc{Ty-eff-N}\quad \dfrac{}{\{\,\} \mathbin{\scriptstyle\cup} \{\,\} \Vdash \varepsilon}
$$

$$
\textsc{Ty-eff-A}\quad \dfrac{\begin{array}{c}
\Gamma \mathbin{\scriptstyle\cup} \Delta \Vdash \overline{\mathsf{v}} : \mathsf{T} \\
\mathtt{ctx\_Disjoint}\ \Gamma\ \{@\mathtt{h} :_{m}{}^{n}\lfloor \mathsf{T} \rfloor\} \\
\mathtt{ctx\_Disjoint}\ \Gamma\ \Delta \\
\mathtt{ctx\_Disjoint}\ \{@\mathtt{h} :_{m}{}^{n}\lfloor \mathsf{T} \rfloor\}\ \Delta
\end{array}}{(\underline{\uparrow}\cdot m\cdot n)\cdot\Gamma \cup \{@\mathtt{h} :_{m}{}^{n}\lfloor \mathsf{T} \rfloor\} \mathbin{\scriptstyle\cup} (m\cdot n)\cdot\Delta \Vdash \mathtt{h} \coloneqq \overline{\mathsf{v}}}
$$

$$
\textsc{Ty-eff-P}\quad \dfrac{\begin{array}{c}
\Gamma_1 \mathbin{\scriptstyle\cup} \Delta_1 \cup @^{-1}\Gamma_{22} \Vdash e_1 \\
\Gamma_{21} \cup \Gamma_{22} \mathbin{\scriptstyle\cup} \Delta_2 \Vdash e_2 \\
\mathtt{pctx\_DestOnly}\ \Gamma_{22} \\
\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{21} \\
\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_{22} \\
\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Delta_1 \\
\mathtt{ctx\_Disjoint}\ \Gamma_1\ \Delta_2 \\
\mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Gamma_{22} \\
\mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_1 \\
\mathtt{ctx\_Disjoint}\ \Gamma_{21}\ \Delta_2 \\
\mathtt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_1 \\
\mathtt{ctx\_Disjoint}\ \Gamma_{22}\ \Delta_2 \\
\mathtt{ctx\_Disjoint}\ \Delta_1\ \Delta_2
\end{array}}{\Gamma_1 \cup \Gamma_{21} \mathbin{\scriptstyle\cup} \Delta_1 \cup \Delta_2 \Vdash e_1, e_2}
$$

$\boxed{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid e_1 \quad \downdownarrows \quad \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2}$ *(Big-step evaluation of effects on extended values)*

$$
\textsc{Sem-eff-N}\quad \dfrac{}{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \varepsilon \quad \downdownarrows \quad \overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \varepsilon}
$$

$$
\textsc{Sem-eff-S}\quad \dfrac{\begin{array}{c}
\mathtt{ctx\_HvarNotMem}\ \mathtt{h}\ \Delta_1 \\
\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid e_1 \quad \downdownarrows \quad \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2
\end{array}}{\overline{\mathsf{v}_1}\ {}_{\Delta_1} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}'}, e_1 \quad \downdownarrows \quad \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}'}, e_2}
$$

$$
\textsc{Sem-eff-F}\quad \dfrac{\begin{array}{c}
\Gamma_0 \mathbin{\scriptstyle\cup} \Delta_0 \Vdash \overline{\mathsf{v}_0} : \mathsf{T} \\
\mathtt{ctx\_Disjoint}\ \Gamma_0\ \Delta_0 \\
\mathtt{ctx\_Disjoint}\ \Delta_1\ \{\mathtt{h} :^{n} \mathsf{T}\} \\
\mathtt{ctx\_Disjoint}\ \Delta_1\ \Delta_0 \\
\overline{\mathsf{v}_1}[\mathtt{h} \coloneqq \overline{\mathsf{v}_0}]\ {}_{(\Delta_1 \cup n\cdot\Delta_0)} \mid e_1 \quad \downdownarrows \quad \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2
\end{array}}{\overline{\mathsf{v}_1}\ {}_{\Delta_1 \cup \{\mathtt{h}:^{n}\mathsf{T}\}} \mid \mathtt{h} \coloneqq \overline{\mathsf{v}_0}, e_1 \quad \downdownarrows \quad \overline{\mathsf{v}_2}\ {}_{\Delta_2} \mid e_2}
$$

$\boxed{\mathsf{t} \Downarrow \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

$$
\textsc{Sem-term-V}\quad \dfrac{}{\mathsf{v} \Downarrow \mathsf{v} \diamond \varepsilon}
$$

$$
\textsc{Sem-term-App}\quad \dfrac{\begin{array}{c}
\mathsf{t}_1 \Downarrow \mathsf{v}_1 \diamond e_1 \\
\mathsf{t}_2 \Downarrow \lambda \mathsf{x} \mapsto \mathsf{u} \diamond e_2 \\
\mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_3
\end{array}}{\mathsf{t}_1 \succ \mathsf{t}_2 \Downarrow \mathsf{v}_3 \diamond e_1, e_2, e_3}
$$

$$
\textsc{Sem-term-PatU}\quad \dfrac{\mathsf{t}_1 \Downarrow () \diamond e_1 \qquad \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1 \mathbin{;} \mathsf{t}_2 \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-term-PatL}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow \mathsf{Inl}\, \mathsf{v}_1 \diamond e_1 \\
\mathsf{u}_1[\mathsf{x}_1 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2
\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\mathsf{Inl}\, \mathsf{x}_1 \mapsto \mathsf{u}_1,\ \mathsf{Inr}\, \mathsf{x}_2 \mapsto \mathsf{u}_2\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-term-PatR}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow \mathsf{Inr}\, \mathsf{v}_1 \diamond e_1 \\
\mathsf{u}_2[\mathsf{x}_2 \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_2 \diamond e_2
\end{array}}{\mathsf{t} \succ \mathbf{case}\,\{\mathsf{Inl}\, \mathsf{x}_1 \mapsto \mathsf{u}_1,\ \mathsf{Inr}\, \mathsf{x}_2 \mapsto \mathsf{u}_2\} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-term-PatP}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow (\mathsf{v}_1, \mathsf{v}_2) \diamond e_1 \\
\mathsf{u}[\mathsf{x}_1 \coloneqq \mathsf{v}_1][\mathsf{x}_2 \coloneqq \mathsf{v}_2] \Downarrow \mathsf{v}_2 \diamond e_2
\end{array}}{\mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1, \mathsf{x}_2) \mapsto \mathsf{u} \Downarrow \mathsf{v}_2 \diamond e_1, e_2}
$$

$$
\textsc{Sem-term-Map}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow \langle \mathsf{v}_1 \mathbin{,} \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_1 \\
\mathsf{u}[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_2 \\
\overline{\mathsf{v}_2}\ {}_{\Delta} \mid e_2 \quad \downdownarrows \quad \overline{\mathsf{v}_4}\ {}_{\Delta'} \mid e_3
\end{array}}{\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u} \Downarrow \langle \mathsf{v}_3 \mathbin{,} \overline{\mathsf{v}_4} \rangle_{\Delta'} \diamond e_1, e_3}
$$

$$
\textsc{Sem-term-Alloc}\quad \dfrac{\mathtt{hvar\_Fresh}\ \mathtt{h}}{\mathbf{alloc}_{\mathsf{T}} \Downarrow \langle @\mathtt{h} \mathbin{,} \mathtt{h} \rangle_{\{\mathtt{h}:^{\underline{\nu}}\mathsf{T}\}} \diamond \varepsilon}
$$

$$
\textsc{Sem-term-ToA}\quad \dfrac{\mathsf{t} \Downarrow \mathsf{v} \diamond e}{\mathbf{to}_{\ltimes}\, \mathsf{t} \Downarrow \langle () \mathbin{,} \mathsf{v} \rangle_{\{\,\}} \diamond e}
$$

$$
\textsc{Sem-term-FromA}\quad \dfrac{\mathsf{t} \Downarrow \langle () \mathbin{,} \mathsf{v} \rangle_{\{\,\}} \diamond e}{\mathbf{from}_{\ltimes}\, \mathsf{t} \Downarrow \mathsf{v} \diamond e}
$$

$$
\textsc{Sem-term-FillU}\quad \dfrac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e}{\mathsf{t} \triangleleft () \Downarrow () \diamond e, \mathtt{h} \coloneqq ()}
$$

$$
\textsc{Sem-term-FillL}\quad \dfrac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e \qquad \mathtt{hvar\_Fresh}\ \mathtt{h}'}{\mathsf{t} \triangleleft \mathsf{Inl} \Downarrow @\mathtt{h}' \diamond e, \mathtt{h} \coloneqq \mathsf{Inl}\, \mathtt{h}'}
$$

$$
\textsc{Sem-term-FillR}\quad \dfrac{\mathsf{t} \Downarrow @\mathtt{h} \diamond e}{\mathsf{t} \triangleleft \mathsf{Inr} \Downarrow @\mathtt{h}' \diamond e, \mathtt{h} \coloneqq \mathsf{Inr}\, \mathtt{h}'}
$$

$$
\textsc{Sem-term-FillP}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow @\mathtt{h} \diamond e \qquad \mathtt{hvar\_Fresh}\ \mathtt{h}_1 \\
\mathtt{hvar\_Fresh}\ \mathtt{h}_2
\end{array}}{\mathsf{t} \triangleleft (,) \Downarrow (@\mathtt{h}_1, @\mathtt{h}_2) \diamond e, \mathtt{h} \coloneqq (\mathtt{h}_1, \mathtt{h}_2)}
$$

$$
\textsc{Sem-term-FillC}\quad \dfrac{\begin{array}{c}
\mathsf{t} \Downarrow @\mathtt{h} \diamond e_1 \\
\mathsf{u} \Downarrow \langle \mathsf{v}_1 \mathbin{,} \overline{\mathsf{v}_2} \rangle_{\Delta} \diamond e_2
\end{array}}{\mathsf{t} \triangleleft\!\bullet\, \mathsf{u} \Downarrow \mathsf{v}_1 \diamond e_1, e_2, \mathtt{h} \coloneqq \overline{\mathsf{v}_2}}
$$

4 Type safety

Theorem 1 (Type safety)**.** *If* $\texttt{pctx\_DestOnly}\ \Gamma$ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$*.*

Theorem 2 (Type safety for complete programs)**.** *If* $\{\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t} \Downarrow \mathsf{v} \diamond \varepsilon$ *and* $\{\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v} \Downarrow \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $m{\cdot}\Gamma_1 \cup \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1 \,{}_{m}\!\!\to \mathsf{T}_2$
  (3) $\texttt{ctx\_Disjoint}\ \Gamma_1\ \Gamma_2$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t} \Downarrow \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \cup \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12} \uplus @^{-1}\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \cup \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u} \Downarrow \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1 \,{}_{m}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \cup \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1 \,{}_{m}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22} \uplus @^{-1}\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \cup \Gamma_{22}$.
  Using Lemma **??** on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x} \mapsto \mathsf{t}'$ and (14) $\Gamma_{21} \cup \Gamma_{23} \cup \{\mathsf{x} :_m \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma **??** on (14) and (5) we get (15) $m{\cdot}(\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] \Downarrow \mathsf{v}_3 \diamond e_3$ where (17) $m{\cdot}(\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12} \uplus @^{-1}\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22} \uplus @^{-1}\Gamma_{23} \Vdash e_2$

  $\texttt{ctx\_Disjoint}\ \Gamma_{12}\ \Gamma_{22}$ comes naturally from (3), (7) and (12).

  We must show:
  $\texttt{ctx\_Disjoint}\ \Gamma_{12}\ \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  $\texttt{ctx\_Disjoint}\ \Gamma_{22}\ \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  $\texttt{ctx\_Disjoint}\ \Gamma_{13}\ \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \cup \Gamma_{22} \uplus @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1, e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $m{\cdot}(\Gamma_{111} \cup \Gamma_{131}) \cup \Gamma_{211} \cup \Gamma_{231} \cup \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $m{\cdot}(\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \uplus @^{-1}\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{k\,1} \cup \Gamma_{k\,2} = \Gamma_k$

  We have
  (18) $\Gamma_{12} \cup \Gamma_{22} \uplus @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1, e_2$
  (20) $m{\cdot}(\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \uplus @^{-1}\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $@^{-1}\Gamma_{23}$, we get (22) $\Gamma_{12} \cup \Gamma_{22} \uplus @^{-1}(\Gamma_{131} \cup \Gamma_{231}) \cup @^{-1}(\Gamma_{132} \cup \Gamma_{232}) \Vdash e_1, e_2$

  We want Γ_{132} from (22) to cancel $m{\cdot}\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{@\mathsf{h} :_{\underline{\nu}} {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash @\mathsf{h}' \diamond \mathsf{h} \coloneqq \mathsf{Inl}\,\mathsf{h}' : {}^{n}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} \coloneqq \mathsf{Inl}\,\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(@\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ @\mathsf{h}_1)\ ;\ \mathsf{x}_2) \succ (\lambda \mathsf{x}_2 \mapsto @\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))$

  $@\mathsf{h}_0 \triangleleft (,) \Downarrow (@\mathsf{h}_{01}\,,\,@\mathsf{h}_{02}) \diamond \mathsf{h}_0 \coloneqq (\mathsf{h}_{01}\,,\,\mathsf{h}_{02})$

  $(\mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ @\mathsf{h}_1)\ ;\ \mathsf{x}_2)[\mathsf{x}_1 \coloneqq @\mathsf{h}_{01}][\mathsf{x}_2 \coloneqq @\mathsf{h}_{02}] \Downarrow @\mathsf{h}_{02} \diamond \mathsf{h}_{01} \coloneqq @\mathsf{h}_1$

  $(@\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ @\mathsf{h}_1)\ ;\ \mathsf{x}_2) \Downarrow @\mathsf{h}_{02} \diamond \mathsf{h}_0 \coloneqq (\mathsf{h}_{01}\,,\,\mathsf{h}_{02}), \mathsf{h}_{01} \coloneqq @\mathsf{h}_1$

  $(@\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))[\mathsf{x}_2 \coloneqq @\mathsf{h}_{02}] \Downarrow () \diamond \mathsf{h}_3 \coloneqq @\mathsf{h}_{02}$

  $\mathsf{t} \Downarrow () \diamond \mathsf{h}_0 \coloneqq (\mathsf{h}_{01}\,,\,\mathsf{h}_{02}), \mathsf{h}_{01} \coloneqq @\mathsf{h}_1, \mathsf{h}_3 \coloneqq @\mathsf{h}_{02}$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all BigStep_Fill$\langle Ctor\rangle$ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TyCmd_Cmd), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;

- or are those of pre-existing holes coming from the extended value $\overline{\mathsf{v}_2}$ of an ampar, when BigStep_FillComp is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program · And v_1 is exactly the accompanying value returned by the evaluation of BigStep_FillComp

TODO: prove that this property is preserved by typing rules □