

Destination λ -calculus

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1 Term and value syntax

tvar, **x**, **y** Term-level variable name
hvar, **h** Hole or destination name
k Index for ranges

val , v	::=	$\langle v_1, \overline{v_2} \rangle_\Delta$ $@h$ $()$ $\text{Inl } v$ $\text{Inr } v$ (v_1, v_2) $\mathbin{\text{J}}^m v$ $\lambda x \mapsto t$	Term value Ampar Destination Unit Left variant for sum Right variant for sum Product Exponential Linear function
\overline{xval} , \overline{v}	::=	v h $\text{Inl } \overline{v}$ $\text{Inr } \overline{v}$ $(\overline{v_1}, \overline{v_2})$ $\mathbin{\text{J}}^m \overline{v}$	Pseudo-value that may contain holes Term value Hole Left variant with val or hole Right variant with val or hole Product with val or hole Exponential with val or hole
term , t , u	::=	v x $t \succ u$ $t ; u$ $t \succ \text{case } \{ \text{Inl } x_1 \mapsto u_1, \text{Inr } x_2 \mapsto u_2 \}$ $t \succ \text{case } (x_1, x_2) \mapsto u$ $t \succ \text{case } \mathbin{\text{J}}^m x \mapsto u$ $t \succ \text{map } x \mapsto u$ $\text{to}_\times t$ $\text{from}_\times t$ alloc_T $t \triangleleft ()$ $t \triangleleft \text{Inl}$ $t \triangleleft \text{Inr}$ $t \triangleleft (,)$ $t \triangleleft \mathbin{\text{J}}^m$ $t \triangleleft \bullet u$	Term Term value Variable Application Pattern-match on unit Pattern-match on sum Pattern-match on product Pattern-match on exponential Map over the left side of the ampar Wrap t into a trivial ampar Extract value from trivial ampar Return a fresh "identity" ampar object Fill destination with unit Fill destination with left variant Fill destination with right variant Fill destination with product constructor Fill destination with exponential constructor Fill destination with root of ampar u

2 Type system

typ, T, U	$::=$	1 $T_1 \oplus T_2$ $T_1 \otimes T_2$ $!^m T$ $T_1 \times T_2$ $T_1 \xrightarrow{m_1} T_2$ $m[T]$	Type Unit Sum Product Exponential Ampar type (consuming T_1 yields T_2) Linear function Destination
moda, m, n	$::=$	$p a$ $\underline{\nu}$ \uparrow ∞ $m_1 \dots m_k$	Modality (Semiring) Pair of a multiplicity and age Neutral element of the product. Notation for 1ν . Same multiplicity, but one scope older. Notation for $1 \uparrow$. Linear, infinitely old / static. Notation for 1∞ . Semiring product
mul, p	$::=$	1 ω $p_1 \dots p_k$	Multiplicity (first component of modality) Linear. Neutral element of the product Non-linear. Absorbing for the product Semiring product
age, a	$::=$	ν \uparrow ∞ $a_1 \dots a_k$	Age (second component of modality) Born now. Neutral element of the product One scope older Infinitely old / static. Absorbing for the product Semiring product
pctx, Γ	$::=$	$\{\text{pas}_1, \dots, \text{pas}_k\}$ $m \cdot \Gamma$ $\Gamma_1 \cup \Gamma_2$	Positive typing context Multiply each binding by m
pas	$::=$	$x :_m T$ $@h :_m {}^n[T]$	Positive type assignment Variable Destination (m is its own modality; n is the modality for values it accepts)
nctx, Δ	$::=$	$\{\text{nas}_1, \dots, \text{nas}_k\}$ $m \cdot \Delta$ $@^{-1}\Gamma$ $\Delta_1 \cup \Delta_2$	Negative typing context Multiply each binding by m Maps each destination of Γ to a hole (requires <code>ctx_DestOnly</code> Γ)
nas	$::=$	$h :^n T$	Negative type assignment Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$$\boxed{\Gamma \cup \Delta \Vdash \bar{v} : T}$$

(Typing of extended values (require both positive and negative contexts))

TY-XVAL-H

$$\frac{}{\{\} \cup \{h : \underline{\nu} T\} \Vdash h : T}$$

TY-XVAL-D

$$\frac{}{\{ @h : \underline{\nu} {}^n[T] \} \cup \{\} \Vdash @h : {}^n[T]}$$

TY-XVAL-U

$$\frac{}{\{\} \cup \{\} \Vdash () : 1}$$

TY-XVAL-L

$$\frac{\Gamma \cup \Delta \Vdash \bar{v} : T_1 \quad \text{ctx_Disjoint } \Gamma \Delta}{\Gamma \cup \Delta \Vdash \text{Inl } \bar{v} : T_1 \oplus T_2}$$

TY-XVAL-P

$$\begin{array}{c} \Gamma_1 \cup \Delta_1 \Vdash \bar{v}_1 : \mathbf{T}_1 \\ \Gamma_2 \cup \Delta_2 \Vdash \bar{v}_2 : \mathbf{T}_2 \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_1 \Delta_1 \\ \text{ctx_Disjoint } \Gamma_1 \Delta_2 \\ \text{ctx_Disjoint } \Gamma_2 \Delta_1 \\ \text{ctx_Disjoint } \Gamma_2 \Delta_2 \\ \text{ctx_Disjoint } \Delta_1 \Delta_2 \end{array}$$

TY-XVAL-R

$$\frac{\Gamma \cup \Delta \Vdash \bar{v} : \mathbf{T}_2 \quad \text{ctx_Disjoint } \Gamma \Delta}{\Gamma \cup \Delta \Vdash \text{Inr } \bar{v} : \mathbf{T}_1 \oplus \mathbf{T}_2}$$

TY-XVAL-E

$$\frac{\Gamma \cup \Delta \Vdash \bar{v} : \mathbf{T} \quad \text{ctx_Disjoint } \Gamma \Delta}{m \cdot \Gamma \cup m \cdot \Delta \Vdash \bar{v} : !^m \mathbf{T}}$$

TY-XVAL-A

$$\frac{\begin{array}{c} \Gamma_1 \cup \{ \} \Vdash v_1 : \mathbf{T}_1 \\ \Gamma_2 \cup @^{-1} \Gamma_1 \Vdash \bar{v}_2 : \mathbf{T}_2 \\ \text{pctx_DestOnly } \Gamma_1 \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \end{array}}{\Gamma_2 \cup \{ \} \Vdash \langle v_1, \bar{v}_2 \rangle @^{-1} \Gamma_1 : \mathbf{T}_1 \rtimes \mathbf{T}_2}$$

TY-XVAL-F

$$\frac{\Gamma \cup \{ \mathbf{x} :_m \mathbf{T}_1 \} \vdash t : \mathbf{T}_2 \quad \text{ctx_Disjoint } \Gamma \{ \mathbf{x} :_m \mathbf{T}_1 \}}{\Gamma \cup \{ \} \Vdash \lambda \mathbf{x} \mapsto t : \mathbf{T}_1 \multimap \mathbf{T}_2}$$

$\boxed{\Gamma \vdash t : \mathbf{T}}$

(Typing of terms (only a positive context is needed))

TY-TERM-V

$$\frac{\Gamma \cup \{ \} \Vdash v : \mathbf{T}}{\Gamma \vdash v : \mathbf{T}}$$

TY-TERM-X0

$$\frac{}{\{ \mathbf{x} :_{\underline{v}} \mathbf{T} \} \vdash \mathbf{x} : \mathbf{T}}$$

TY-TERM-XINF

$$\frac{}{\{ \mathbf{x} :_{\underline{\infty}} \mathbf{T} \} \vdash \mathbf{x} : \mathbf{T}}$$

TY-TERM-APP

$$\frac{\Gamma_1 \vdash t : \mathbf{T}_1 \quad \Gamma_2 \vdash u : \mathbf{T}_1 \multimap \mathbf{T}_2 \quad \text{ctx_Disjoint } \Gamma_1 \Gamma_2}{m \cdot \Gamma_1 \cup \Gamma_2 \vdash t \succ u : \mathbf{T}_2}$$

TY-TERM-PATS

$$\frac{\begin{array}{c} \Gamma_1 \vdash t : \mathbf{T}_1 \oplus \mathbf{T}_2 \\ \Gamma_2 \cup \{ \mathbf{x}_1 :_m \mathbf{T}_1 \} \vdash u_1 : \mathbf{U} \\ \Gamma_2 \cup \{ \mathbf{x}_2 :_m \mathbf{T}_2 \} \vdash u_2 : \mathbf{U} \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x}_1 :_m \mathbf{T}_1 \} \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x}_2 :_m \mathbf{T}_2 \} \end{array}}{m \cdot \Gamma_1 \cup \Gamma_2 \vdash t \succ \text{case } \{ \text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2 \} : \mathbf{U}}$$

TY-TERM-PATU

$$\frac{\Gamma_1 \vdash t : \mathbf{1} \quad \Gamma_2 \vdash u : \mathbf{U} \quad \text{ctx_Disjoint } \Gamma_1 \Gamma_2}{\Gamma_1 \cup \Gamma_2 \vdash t ; u : \mathbf{U}}$$

TY-TERM-PATP

$$\frac{\begin{array}{c} \Gamma_1 \vdash t : \mathbf{T}_1 \otimes \mathbf{T}_2 \\ \Gamma_2 \cup \{ \mathbf{x}_1 :_m \mathbf{T}_1, \mathbf{x}_2 :_m \mathbf{T}_2 \} \vdash u : \mathbf{U} \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x}_1 :_m \mathbf{T}_1 \} \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x}_2 :_m \mathbf{T}_2 \} \\ \text{ctx_Disjoint } \{ \mathbf{x}_1 :_m \mathbf{T}_1 \} \{ \mathbf{x}_2 :_m \mathbf{T}_2 \} \end{array}}{m \cdot \Gamma_1 \cup \Gamma_2 \vdash t \succ \text{case } (\mathbf{x}_1, \mathbf{x}_2) \mapsto u : \mathbf{U}}$$

TY-TERM-PATE

$$\frac{\begin{array}{c} \Gamma_1 \vdash t : !^n \mathbf{T} \\ \Gamma_2 \cup \{ \mathbf{x} :_{m \cdot n} \mathbf{T} \} \vdash u : \mathbf{U} \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x} :_{m \cdot n} \mathbf{T} \} \end{array}}{m \cdot \Gamma_1 \cup \Gamma_2 \vdash t \succ \text{case } !^n \mathbf{x} \mapsto u : \mathbf{U}}$$

TY-TERM-MAP

$$\frac{\begin{array}{c} \Gamma_1 \vdash t : \mathbf{T}_1 \rtimes \mathbf{T}_2 \\ \uparrow \Gamma_2 \cup \{ \mathbf{x} :_{\underline{v}} \mathbf{T}_1 \} \vdash u : \mathbf{U} \\ \text{ctx_Disjoint } \Gamma_1 \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_2 \{ \mathbf{x} :_{\underline{v}} \mathbf{T}_1 \} \end{array}}{\Gamma_1 \cup \Gamma_2 \vdash t \succ \text{map } \mathbf{x} \mapsto u : \mathbf{U} \rtimes \mathbf{T}_2}$$

TY-TERM-FILLC

$$\frac{\Gamma_1 \vdash t : {}^n[\mathbf{T}_2] \quad \Gamma_2 \vdash u : \mathbf{T}_1 \rtimes \mathbf{T}_2 \quad \text{ctx_Disjoint } \Gamma_1 \Gamma_2}{\Gamma_1 \cup (\uparrow \cdot n) \cdot \Gamma_2 \vdash t \blacktriangleleft u : \mathbf{T}_1}$$

TY-TERM-FILLU

$$\frac{}{\Gamma \vdash t : {}^n[\mathbf{1}]}$$

TY-TERM-FILLL

$$\frac{}{\Gamma \vdash t : {}^n[\mathbf{T}_1 \oplus \mathbf{T}_2]}$$

TY-TERM-FILLR

$$\frac{}{\Gamma \vdash t : {}^n[\mathbf{T}_1 \oplus \mathbf{T}_2]}$$

TY-TERM-FILLP

$$\frac{\Gamma \vdash t : {}^n[\mathbf{T}_1 \otimes \mathbf{T}_2]}{\Gamma \vdash t \blacktriangleleft (,) : {}^n[\mathbf{T}_1] \otimes {}^n[\mathbf{T}_2]}$$

TY-TERM-FILLE

$$\frac{\Gamma \vdash t : {}^m[!^n \mathbf{T}]}{\Gamma \vdash t \blacktriangleright^n : {}^{m \cdot n}[\mathbf{T}]}$$

TY-TERM-ALLOC

$$\frac{}{\{ \} \vdash \text{alloc}_{\mathbf{T}} : \underline{v}[\mathbf{T}] \rtimes \mathbf{T}}$$

TY-TERM-TOA

$$\frac{}{\Gamma \vdash \text{to}_{\rtimes} t : \mathbf{1} \rtimes \mathbf{T}}$$

TY-TERM-FROMA

$$\frac{}{\Gamma \vdash \text{from}_{\rtimes} t : \mathbf{T}}$$

$\boxed{\Gamma \vdash v \diamond e : \mathbf{T}}$

(Typing of commands (only a positive context is needed))

TY-CMD-C

$$\frac{\begin{array}{c} \Gamma_{11} \cup \Gamma_{12} \vdash v : \mathbf{T} \\ \Gamma_2 \cup @^{-1} \Gamma_{12} \Vdash e \\ \text{pctx_DestOnly } \Gamma_{12} \\ \text{ctx_Disjoint } \Gamma_{11} \Gamma_{12} \\ \text{ctx_Disjoint } \Gamma_{11} \Gamma_2 \\ \text{ctx_Disjoint } \Gamma_{12} \Gamma_2 \end{array}}{\Gamma_{11} \cup \Gamma_2 \vdash v \diamond e : \mathbf{T}}$$

3 Effects and big-step semantics

$eff, e ::=$ Effect

ε

has

$e_1 \gg \dots \gg e_k$ Chain effects

$\Gamma \cup \Delta \Vdash e$

(Typing of effects (require both positive and negative contexts))

TY-EFF-N

$\frac{}{\{\} \cup \{\} \Vdash \varepsilon}$

TY-EFF-A

$\frac{\Gamma \cup \Delta \Vdash \bar{v} : \mathbf{T} \quad \text{ctx_Disjoint } \Gamma \{ \text{@h} :_m^{\textcolor{blue}{n}}[\mathbf{T}] \} \quad \text{ctx_Disjoint } \Gamma \Delta \quad \text{ctx_Disjoint } \{ \text{@h} :_m^{\textcolor{blue}{n}}[\mathbf{T}] \} \Delta}{(\uparrow \cdot m \cdot n) \cdot \Gamma \cup \{ \text{@h} :_m^{\textcolor{blue}{n}}[\mathbf{T}] \} \cup (m \cdot n) \cdot \Delta \Vdash \mathbf{h} := \bar{v}}$

TY-EFF-P

$\frac{\Gamma_1 \cup \Delta_1 \cup \text{@}^+ \Gamma_{22} \Vdash e_1 \quad \Gamma_{21} \cup \Gamma_{22} \cup \Delta_2 \Vdash e_2 \quad \text{pctx_DestOnly } \Gamma_{22} \quad \text{ctx_Disjoint } \Gamma_1 \Gamma_{21} \quad \text{ctx_Disjoint } \Gamma_1 \Gamma_{22} \quad \text{ctx_Disjoint } \Gamma_1 \Delta_1 \quad \text{ctx_Disjoint } \Gamma_1 \Delta_2 \quad \text{ctx_Disjoint } \Gamma_{21} \Gamma_{22} \quad \text{ctx_Disjoint } \Gamma_{21} \Delta_1 \quad \text{ctx_Disjoint } \Gamma_{21} \Delta_2 \quad \text{ctx_Disjoint } \Gamma_{22} \Delta_1 \quad \text{ctx_Disjoint } \Gamma_{22} \Delta_2 \quad \text{ctx_Disjoint } \Delta_1 \Delta_2}{\Gamma_1 \cup \Gamma_{21} \cup \Delta_1 \cup \Delta_2 \Vdash e_1 \gg e_2}$

$\overline{v_1} \Delta_1 \mid e_1 \Downarrow \overline{v_2} \Delta_2 \mid e_2$

(Big-step evaluation of effects on extended values)

SEM-EFF-N

$\frac{}{\overline{v_1} \Delta_1 \mid \varepsilon \Downarrow \overline{v_1} \Delta_1 \mid \varepsilon}$

SEM-EFF-S

$\frac{\text{ctx_HvarNotMem } \mathbf{h} \Delta_1 \quad \overline{v_1} \Delta_1 \mid e_1 \Downarrow \overline{v_2} \Delta_2 \mid e_2}{\overline{v_1} \Delta_1 \mid \mathbf{h} := \bar{v}' \gg e_1 \Downarrow \overline{v_2} \Delta_2 \mid \mathbf{h} := \bar{v}' \gg e_2}$

SEM-EFF-F

$\frac{\Gamma_0 \cup \Delta_0 \Vdash \bar{v}_0 : \mathbf{T} \quad \text{ctx_Disjoint } \Gamma_0 \Delta_0 \quad \text{ctx_Disjoint } \Delta_1 \{ \mathbf{h} :^{\textcolor{blue}{n}} \mathbf{T} \} \quad \text{ctx_Disjoint } \Delta_1 \Delta_0 \quad \overline{v_1}[\mathbf{h} := \bar{v}_0] (\Delta_1 \cup \textcolor{blue}{n} \Delta_0) \mid e_1 \Downarrow \overline{v_2} \Delta_2 \mid e_2}{\overline{v_1} \Delta_1 \cup \{ \mathbf{h} :^{\textcolor{blue}{n}} \mathbf{T} \} \mid \mathbf{h} := \bar{v}_0 \gg e_1 \Downarrow \overline{v_2} \Delta_2 \mid e_2}$

$t \Downarrow v \diamond e$

(Big-step evaluation into commands)

SEM-TERM-V

$\frac{}{v \Downarrow v \diamond \varepsilon}$

SEM-TERM-APP

$\frac{t_1 \Downarrow v_1 \diamond e_1 \quad t_2 \Downarrow \lambda \mathbf{x} \mapsto u \diamond e_2 \quad u[\mathbf{x} := v_1] \Downarrow v_3 \diamond e_3}{t_1 \succ t_2 \Downarrow v_3 \diamond e_1 \gg e_2 \gg e_3}$

SEM-TERM-PATU

$\frac{t_1 \Downarrow () \diamond e_1 \quad t_2 \Downarrow v_2 \diamond e_2}{t_1 ; t_2 \Downarrow v_2 \diamond e_1 \gg e_2}$

SEM-TERM-PATL

$\frac{t \Downarrow \text{Inl } v_1 \diamond e_1 \quad u_1[\mathbf{x}_1 := v_1] \Downarrow v_2 \diamond e_2}{t \succ \text{case } \{ \text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2 \} \Downarrow v_2 \diamond e_1 \gg e_2}$

SEM-TERM-PATR

$\frac{t \Downarrow \text{Inr } v_1 \diamond e_1 \quad u_2[\mathbf{x}_2 := v_1] \Downarrow v_2 \diamond e_2}{t \succ \text{case } \{ \text{Inl } \mathbf{x}_1 \mapsto u_1, \text{Inr } \mathbf{x}_2 \mapsto u_2 \} \Downarrow v_2 \diamond e_1 \gg e_2}$

SEM-TERM-PATP

$\frac{t \Downarrow (v_1, v_2) \diamond e_1 \quad u[\mathbf{x}_1 := v_1][\mathbf{x}_2 := v_2] \Downarrow v_2 \diamond e_2}{t \succ \text{case } (\mathbf{x}_1, \mathbf{x}_2) \mapsto u \Downarrow v_2 \diamond e_1 \gg e_2}$

SEM-TERM-MAP

$\frac{t \Downarrow \langle v_1, \bar{v}_2 \rangle_{\Delta} \diamond e_1 \quad u[\mathbf{x} := v_1] \Downarrow v_3 \diamond e_2 \quad \bar{v}_2 \Delta \mid e_2 \Downarrow \bar{v}_4 \Delta' \mid e_3}{t \succ \text{map } \mathbf{x} \mapsto u \Downarrow \langle v_3, \bar{v}_4 \rangle_{\Delta'} \diamond e_1 \gg e_3}$

SEM-TERM-ALLOC

$\frac{\text{hvar_Fresh } \mathbf{h}}{\text{alloc}_{\mathbf{T}} \Downarrow \langle \text{@h}, \mathbf{h} \rangle_{\{ \mathbf{h} :^{\textcolor{blue}{n}} \mathbf{T} \}} \diamond \varepsilon}$

SEM-TERM-TOA

$\frac{t \Downarrow v \diamond e}{\text{to}_{\mathbf{x}} t \Downarrow \langle (), v \rangle_{\{ \}} \diamond e}$

SEM-TERM-FROMA

$\frac{t \Downarrow \langle (), v \rangle_{\{ \}} \diamond e}{\text{from}_{\mathbf{x}} t \Downarrow v \diamond e}$

SEM-TERM-FILLU

$\frac{t \Downarrow \text{@h} \diamond e}{t \triangleleft () \Downarrow () \diamond e \gg \mathbf{h} := ()}$

SEM-TERM-FILLL

$\frac{t \Downarrow \text{@h} \diamond e \quad \text{hvar_Fresh } \mathbf{h}'}{t \triangleleft \text{Inl} \Downarrow \text{@h}' \diamond e \gg \mathbf{h} := \text{Inl } \mathbf{h}'}$

SEM-TERM-FILLR

$\frac{t \Downarrow \text{@h} \diamond e}{t \triangleleft \text{Inr} \Downarrow \text{@h}' \diamond e \gg \mathbf{h} := \text{Inr } \mathbf{h}'}$

SEM-TERM-FILLP

$\frac{t \Downarrow \text{@h} \diamond e \quad \text{hvar_Fresh } \mathbf{h}_1 \quad \text{hvar_Fresh } \mathbf{h}_2}{t \triangleleft (,) \Downarrow (\text{@h}_1, \text{@h}_2) \diamond e \gg \mathbf{h} := (\mathbf{h}_1, \mathbf{h}_2)}$

SEM-TERM-FILLC

$\frac{t \Downarrow \text{@h} \diamond e_1 \quad u \Downarrow \langle v_1, \bar{v}_2 \rangle_{\Delta} \diamond e_2}{t \triangleleft \bullet u \Downarrow v_1 \diamond e_1 \gg e_2 \gg \mathbf{h} := \bar{v}_2}$

4 Type safety

Theorem 1 (Type safety). *If $pctx_DestOnly \Gamma$ and $\Gamma \vdash t : \mathbf{T}$ then $t \Downarrow v \diamond e$ and $\Gamma \vdash v \diamond e : \mathbf{T}$.*

Theorem 2 (Type safety for complete programs). *If $\{\} \vdash t : \mathbf{T}$ then $t \Downarrow v \diamond \varepsilon$ and $\{\} \vdash v : \mathbf{T}$*

Proof. By induction on the typing derivation.

- **TYTERM_VAL:** (0) $\Gamma \vdash v : \mathbf{T}$
(0) gives (1) $v \Downarrow v \diamond \varepsilon$ immediately. From **TYEFF_NOEFF** and **TYCMD_CMD** we conclude (2) $\Gamma \vdash v \diamond e : \mathbf{T}$.

- **TYTERM_APP:** (0) $m \cdot \Gamma_1 \cup \Gamma_2 \vdash t \succ u : \mathbf{T}_2$

We have

- (1) $\Gamma_1 \vdash t : \mathbf{T}_1$
- (2) $\Gamma_2 \vdash u : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2$
- (3) **ctx_Disjoint** $\Gamma_1 \Gamma_2$

Using recursion hypothesis on (1) we get (4) $t \Downarrow v_1 \diamond e_1$ where (5) $\Gamma_1 \vdash v_1 \diamond e_1 : \mathbf{T}_1$.

Inverting **TYCMD_CMD** we get (5) $\Gamma_{11} \cup \Gamma_{13} \vdash v_1 : \mathbf{T}_1$ and (6) $\Gamma_{12} \cup @^{-1}\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \cup \Gamma_{12}$.

Using recursion hypothesis on (2) we get (8) $u \Downarrow v_2 \diamond e_2$ where (9) $\Gamma_2 \vdash v_2 \diamond e_2 : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2$.

Inverting **TYCMD_CMD** we get (10) $\Gamma_{21} \cup \Gamma_{23} \vdash v_2 : \mathbf{T}_1 \xrightarrow{m} \mathbf{T}_2$ and (11) $\Gamma_{22} \cup @^{-1}\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \cup \Gamma_{22}$.

Using Lemma ?? on (9) we get (13) $v_2 = \lambda x \mapsto t'$ and (14) $\Gamma_{21} \cup \Gamma_{23} \cup \{x : m \mathbf{T}_1\} \vdash t' : \mathbf{T}_2$.

Typing value part of the result

Using Lemma ?? on (14) and (5) we get (15) $m \cdot (\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash t'[x := v_1] : \mathbf{T}_2$.

Using recursion hypothesis on (15) we get (16) $t'[x := v_1] \Downarrow v_3 \diamond e_3$ where (17) $m \cdot (\Gamma_{11} \cup \Gamma_{13}) \cup (\Gamma_{21} \cup \Gamma_{23}) \vdash v_3 \diamond e_3 : \mathbf{T}_2$.

Typing effect part of the result

We have

- (6) $\Gamma_{12} \cup @^{-1}\Gamma_{13} \Vdash e_1$
- (11) $\Gamma_{22} \cup @^{-1}\Gamma_{23} \Vdash e_2$

ctx_Disjoint $\Gamma_{12} \Gamma_{22}$ comes naturally from (3), (7) and (12).

We must show:

ctx_Disjoint $\Gamma_{12} \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.

ctx_Disjoint $\Gamma_{22} \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see **TYEFF_UNION**) as long as they share the same modalities (it's even harder to prove I think).

ctx_Disjoint $\Gamma_{13} \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

By **TYEFF_UNION** we get (18) $\Gamma_{12} \cup \Gamma_{22} \cup @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1 \gg e_2$.

Inverting **TYCMD_CMD** on (17) we get (19) $m \cdot (\Gamma_{111} \cup \Gamma_{131}) \cup \Gamma_{211} \cup \Gamma_{231} \cup \Gamma_3 \vdash v_3 : \mathbf{T}_2$ and (20) $m \cdot (\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \cup @^{-1}\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{k1} \cup \Gamma_{k2} = \Gamma_k$

We have

- (18) $\Gamma_{12} \cup \Gamma_{22} \cup @^{-1}\Gamma_{13} \cup @^{-1}\Gamma_{23} \Vdash e_1 \gg e_2$
- (20) $m \cdot (\Gamma_{112} \cup \Gamma_{132}) \cup \Gamma_{212} \cup \Gamma_{232} \cup @^{-1}\Gamma_3 \Vdash e_3$

Using (21) on (18) to decompose $@^{-1}\Gamma_{23}$, we get (22) $\Gamma_{12} \cup \Gamma_{22} \cup @^{-1}(\Gamma_{131} \cup \Gamma_{231}) \cup @^{-1}(\Gamma_{132} \cup \Gamma_{232}) \Vdash e_1 \gg e_2$

We want Γ_{132} from (22) to cancel $m \cdot \Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

Γ_{13} contains dests associated to holes that may have been created when evaluating t into $v_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{ @h : \underline{v} \text{ } ^n[\mathbf{T}_1 \oplus \mathbf{T}_2] \} \vdash @h' \diamond h := \text{Inl } h' : \text{ } ^n[\mathbf{T}_1]$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $h := \text{Inl } h'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

$$(@h_0 \triangleleft (,)) \succ \text{case } (x_1, x_2) \mapsto x_1 \triangleleft \bullet (to_{\times} @h_1) ; x_2 \succ (\lambda x_2 \mapsto @h_3 \triangleleft \bullet (to_{\times} x_2))$$

$$@h_0 \triangleleft (,) \Downarrow (@h_{01}, @h_{02}) \diamond h_0 := (h_{01}, h_{02})$$

$$(x_1 \triangleleft \bullet (to_{\times} @h_1) ; x_2)[x_1 := @h_{01}][x_2 := @h_{02}] \Downarrow @h_{02} \diamond h_{01} := @h_1$$

$$(@h_0 \triangleleft (,) \succ \text{case } (x_1, x_2) \mapsto x_1 \triangleleft \bullet (to_{\times} @h_1) ; x_2) \Downarrow @h_{02} \diamond h_0 := (h_{01}, h_{02}) \gg h_{01} := @h_1$$

$$(@h_3 \triangleleft \bullet (to_{\times} x_2))[x_2 := @h_{02}] \Downarrow () \diamond h_3 := @h_{02}$$

$$t \Downarrow () \diamond h_0 := (h_{01}, h_{02}) \gg h_{01} := @h_1 \gg h_3 := @h_{02}$$

Lemma 1 (Freshness of holes). *Let t be a program with no pre-existing ampar sharing hole names.*

During the reduction of t , the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all $\text{BIGSTEP_FILL}\langle Ctor \rangle$ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TYCMD_CMD), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;
- or are those of pre-existing holes coming from the extended value \overline{v}_2 of an ampar, when BIGSTEP_FILLCOMP is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program \cdot . And v_1 is exactly the accompanying value returned by the evaluation of BIGSTEP_FILLCOMP .

TODO: prove that this property is preserved by typing rules

□