Destination λ-calculus

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1 Term and value syntax

`tmv`, x, y	Term-level variable name
`hdmv`	Hole or destination static name
`k`	Index for ranges

`hddyn`, `d`	::=		Hole or destination dynamic name
	|	$\star$	Root namespace
	|	`d.1`	Subnamespace 1
	|	`d.2`	Subnamespace 2
	|	`d.3`	Subnamespace 3
`hdnm`, `h`	::=		Hole or destination name
	|	`d`	Dynamic name
	|	`hdmv`	Static name
val, v	::=		Term value
	|	$-\mathtt{h}$	Hole
	|	$+\mathtt{h}$	Destination
	|	$()$	Unit
	|	$\lambda \mathsf{x} \mapsto \mathsf{t}$	Lambda abstraction
	|	$\mathsf{Inl}\ \mathsf{v}$	Left variant for sum
	|	$\mathsf{Inr}\ \mathsf{v}$	Right variant for sum
	|	$\rangle^{m}\ \mathsf{v}$	Exponential
	|	$(\mathsf{v}_1\ ,\ \mathsf{v}_2)$	Product
	|	$\langle \mathsf{v}_1\ ;\ \mathsf{v}_2 \rangle_{\Delta}$	Ampar
term, t, u	::=		Term
	|	v	Value
	|	x	Variable
	|	$\mathsf{t} \succ \mathsf{u}$	Application
	|	$\mathsf{t}\ ;\ \mathsf{u}$	Pattern-match on unit
	|	$\mathsf{t} \succ \mathbf{case}\ \{\ \mathsf{Inl}\ \mathsf{x}_1 \mapsto \mathsf{u}_1\ ,\ \mathsf{Inr}\ \mathsf{x}_2 \mapsto \mathsf{u}_2\ \}$	Pattern-match on sum
	|	$\mathsf{t} \succ \mathbf{case}\ (\mathsf{x}_1\ ,\ \mathsf{x}_2) \mapsto \mathsf{u}$	Pattern-match on product
	|	$\mathsf{t} \succ \mathbf{case}\ \rangle^{m}\ \mathsf{x} \mapsto \mathsf{u}$	Pattern-match on exponential
	|	$\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}$	Map over the right side of the ampar
	|	$\mathbf{to}_{\ltimes}\ \mathsf{t}$	Wrap t into a trivial ampar
	|	$\mathbf{from}_{\ltimes}\ \mathsf{t}$	Extract value from trivial ampar
	|	$\mathbf{alloc}_{\mathsf{T}}$	Return a fresh "identity" ampar object
	|	$\mathsf{t} \triangleleft ()$	Fill destination with unit
	|	$\mathsf{t} \triangleleft \mathsf{Inl}$	Fill destination with left variant
	|	$\mathsf{t} \triangleleft \mathsf{Inr}$	Fill destination with right variant
	|	$\mathsf{t} \triangleleft (,)$	Fill destination with product constructor
	|	$\mathsf{t} \triangleleft \rangle^{m}$	Fill destination with exponential constructor
	|	$\mathsf{t} \triangleleft\!\bullet\ \mathsf{u}$	Fill destination with root of ampar u

2 Type system

type, T, U	::=		Type
	|	$\mathbf{1}$	Unit
	|	$\mathsf{T}_1 \oplus \mathsf{T}_2$	Sum
	|	$\mathsf{T}_1 \otimes \mathsf{T}_2$	Product
	|	$!^{m}\,\mathsf{T}$	Exponential
	|	$\mathsf{T}_1 \ltimes \mathsf{T}_2$	Ampar type (consuming T_2 yields T_1)
	|	$\mathsf{T}_1 \;{}_{m_1}\!\!\to \mathsf{T}_2$	Function
	|	${}^{m}\lfloor \mathsf{T} \rfloor$	Destination
mode, m, n	::=		Mode (Semiring)
	|	pa	Pair of a multiplicity and age
	|	☠	Error case (incompatible types, multiplicities, or ages)
	|	$m_1 \cdot \ldots \cdot m_k$	Semiring product
mul, p	::=		Multiplicity (first component of modality)
	|	1	Linear. Neutral element of the product
	|	ω	Non-linear. Absorbing for the product
	|	$p_1 . \ldots . p_k$	Semiring product
age, a	::=		Age (second component of modality)
	|	ν	Born now. Neutral element of the product
	|	$\uparrow$	One scope older
	|	∞	Infinitely old / static. Absorbing for the product
	|	$a_1 \cdot \ldots \cdot a_k$	Semiring product
ctx, Γ, Δ	::=		Typing context
	|	$\{\mathsf{b}_1, .., \mathsf{b}_k\}$	List of bindings
	|	$m \cdot \Gamma$	Multiply each binding by m
	|	$\Gamma_1 \uplus \Gamma_2$	Sum contexts Γ_1 and Γ_2. Duplicates/incompatible elements will give bindings with mo
	|	$\Gamma_1 \,{}^{-}\!\!\uplus^{+}\, \Gamma_2$	Sum contexts, but allow linear holes from Γ_1 to be compensated by linear dests from Γ
	|	$-\Gamma$	Transforms every hole binding into a dest binding (requires `ctx_DestOnly` Γ)
bndr, b	::=		Type assignment to either variable, destination or hole
	|	$\mathsf{x} :_{m} \mathsf{T}$	Variable
	|	$+\mathsf{h} :_{m} {}^{n}\lfloor \mathsf{T} \rfloor$	Destination (m is its own modality; n is the modality for values it accepts)
	|	$-\mathsf{h} :^{n} \mathsf{T}$	Hole (n is the modality for values it accepts, it doesn't have a modality on its own)

$\boxed{\Gamma \Vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms (raw))*

$$\textsc{TyR-term-H}\quad \frac{}{\{-\mathtt{h} :^{1\nu} \mathsf{T}\} \Vdash -\mathtt{h} : \mathsf{T}}$$

$$\textsc{TyR-term-D}\quad \frac{}{\{+\mathtt{h} :_{1\nu} {}^{n}\lfloor \mathsf{T} \rfloor\} \Vdash +\mathtt{h} : {}^{n}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{TyR-term-U}\quad \frac{}{\{\,\} \Vdash () : \mathbf{1}}$$

$$\textsc{TyR-term-L}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_1}{\Gamma \Vdash \mathsf{Inl}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-term-R}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}_2}{\Gamma \Vdash \mathsf{Inr}\,\mathsf{v} : \mathsf{T}_1 \oplus \mathsf{T}_2}$$

$$\textsc{TyR-term-P}\quad \frac{\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2}{\Gamma_1 \uplus \Gamma_2 \Vdash (\mathsf{v}_1\,,\,\mathsf{v}_2) : \mathsf{T}_1 \otimes \mathsf{T}_2}$$

$$\textsc{TyR-term-E}\quad \frac{\Gamma \Vdash \mathsf{v} : \mathsf{T}}{m\cdot\Gamma \Vdash \rrparenthesis^{m}\,\mathsf{v} : {!}^{m}\,\mathsf{T}}$$

$$\textsc{TyR-term-A}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash \mathsf{v}_1 : \mathsf{T}_1 \\ \Gamma_2 \Vdash \mathsf{v}_2 : \mathsf{T}_2 \qquad \mathtt{ctx\_DestOnly}\ \Gamma_2 \\ \mathtt{ctx\_SubsetEq}\ \ -\Gamma_2\ \Gamma_1\end{array}}{\Gamma_1 {}^{-}\!\uplus^{+}\, \Gamma_2 \Vdash \langle \mathsf{v}_1\,{\scriptstyle 9}\,\mathsf{v}_2 \rangle_{-\Gamma_1} : \mathsf{T}_1 \ltimes \mathsf{T}_2}$$

$$\textsc{TyR-term-F}\quad \frac{\Gamma \uplus \{\mathsf{x} :_{m} \mathsf{T}_1\} \Vdash \mathsf{t} : \mathsf{T}_2}{\Gamma \Vdash \lambda \mathsf{x} \mapsto \mathsf{t} : \mathsf{T}_1 \,{}_{m}\!\!\to \mathsf{T}_2}$$

$$\textsc{TyR-term-Var}\quad \frac{\mathtt{ctx\_Compatible}\ \Gamma\ \{\mathsf{x} :_{1\nu} \mathsf{T}\}}{\Gamma \Vdash \mathsf{x} : \mathsf{T}}$$

$$\textsc{TyR-term-App}\quad \frac{\Gamma_1 \Vdash \mathsf{t} : \mathsf{T}_1 \qquad \Gamma_2 \Vdash \mathsf{u} : \mathsf{T}_1 \,{}_{m}\!\!\to \mathsf{T}_2}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{TyR-term-PatU}\quad \frac{\Gamma_1 \Vdash \mathsf{t} : \mathbf{1} \qquad \Gamma_2 \Vdash \mathsf{u} : \mathsf{U}}{\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t}\ ;\ \mathsf{u} : \mathsf{U}}$$

$$\textsc{TyR-term-PatS}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash \mathsf{t} : \mathsf{T}_1 \oplus \mathsf{T}_2 \\ \Gamma_2 \uplus \{\mathsf{x}_1 :_{m} \mathsf{T}_1\} \Vdash \mathsf{u}_1 : \mathsf{U} \\ \Gamma_2 \uplus \{\mathsf{x}_2 :_{m} \mathsf{T}_2\} \Vdash \mathsf{u}_2 : \mathsf{U}\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\} : \mathsf{U}}$$

$$\textsc{TyR-term-PatP}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash \mathsf{t} : \mathsf{T}_1 \otimes \mathsf{T}_2 \\ \Gamma_2 \uplus \{\mathsf{x}_1 :_{m} \mathsf{T}_1, \mathsf{x}_2 :_{m} \mathsf{T}_2\} \Vdash \mathsf{u} : \mathsf{U}\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{TyR-term-PatE}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash \mathsf{t} : {!}^{n}\,\mathsf{T} \\ \Gamma_2 \uplus \{\mathsf{x} :_{m\cdot n} \mathsf{T}\} \Vdash \mathsf{u} : \mathsf{U}\end{array}}{m\cdot\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t} \succ \mathbf{case}\ \rrparenthesis^{n}\,\mathsf{x} \mapsto \mathsf{u} : \mathsf{U}}$$

$$\textsc{TyR-term-Map}\quad \frac{\begin{array}{c}\Gamma_1 \Vdash \mathsf{t} : \mathsf{T}_1 \ltimes \mathsf{T}_2 \\ 1{\uparrow}\cdot\Gamma_2 \uplus \{\mathsf{x} :_{1\nu} \mathsf{T}_2\} \Vdash \mathsf{u} : \mathsf{U}\end{array}}{\Gamma_1 \uplus \Gamma_2 \Vdash \mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{U}}$$

$$\textsc{TyR-term-FillC}\quad \frac{\Gamma_1 \Vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \rfloor \qquad \Gamma_2 \Vdash \mathsf{u} : \mathsf{T}_1 \ltimes \mathsf{T}_2}{\Gamma_1 \uplus (1{\uparrow}\cdot n)\cdot\Gamma_2 \Vdash \mathsf{t} \triangleleft\!\bullet\, \mathsf{u} : \mathsf{T}_2}$$

$$\textsc{TyR-term-FillU}\quad \frac{\Gamma \Vdash \mathsf{t} : {}^{n}\lfloor \mathbf{1} \rfloor}{\Gamma \Vdash \mathsf{t} \triangleleft () : \mathbf{1}}$$

$$\textsc{TyR-term-FillL}\quad \frac{\Gamma \Vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \Vdash \mathsf{t} \triangleleft \mathsf{Inl} : {}^{n}\lfloor \mathsf{T}_1 \rfloor}$$

$$\textsc{TyR-term-FillR}\quad \frac{\Gamma \Vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor}{\Gamma \Vdash \mathsf{t} \triangleleft \mathsf{Inr} : {}^{n}\lfloor \mathsf{T}_2 \rfloor}$$

$$\textsc{TyR-term-FillP}\quad \frac{\Gamma \Vdash \mathsf{t} : {}^{n}\lfloor \mathsf{T}_1 \otimes \mathsf{T}_2 \rfloor}{\Gamma \Vdash \mathsf{t} \triangleleft (,) : {}^{n}\lfloor \mathsf{T}_1 \rfloor \otimes {}^{n}\lfloor \mathsf{T}_2 \rfloor}$$

$$\textsc{TyR-term-FillE}\quad \frac{\Gamma \Vdash \mathsf{t} : {}^{m}\lfloor {!}^{n}\,\mathsf{T} \rfloor}{\Gamma \Vdash \mathsf{t} \triangleleft \rrparenthesis^{n} : {}^{m\cdot n}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{TyR-term-Alloc}\quad \frac{}{\{\,\} \Vdash \mathbf{alloc}_{\mathsf{T}} : \mathsf{T} \ltimes {}^{1\nu}\lfloor \mathsf{T} \rfloor}$$

$$\textsc{TyR-term-ToA}\quad \frac{\Gamma \Vdash \mathsf{t} : \mathsf{T}}{\Gamma \Vdash \mathbf{to}_{\ltimes}\,\mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}$$

$$\textsc{TyR-term-FromA}\quad \frac{\Gamma \Vdash \mathsf{t} : \mathsf{T} \ltimes \mathbf{1}}{\Gamma \Vdash \mathbf{from}_{\ltimes}\,\mathsf{t} : \mathsf{T}}$$

$\boxed{\Gamma \vdash \mathsf{t} : \mathsf{T}}$ *(Typing of terms (valid ones only))*

$$\textsc{Ty-term-T}\quad \frac{\begin{array}{c}\Gamma \Vdash \mathsf{t} : \mathsf{T} \\ \mathtt{ctx\_Valid}\ \Gamma \qquad \mathtt{ctx\_NoHole}\ \Gamma\end{array}}{\Gamma \vdash \mathsf{t} : \mathsf{T}}$$

3 Effects and big-step semantics

$$\begin{array}{llll} eff,\ e & ::= & & \text{Effect}\\ & | & \varepsilon & \quad\text{No-op effect}\\ & | & \mathsf{f} & \quad\text{Single hole assignment}\\ & | & e_1 \gg .. \gg e_{\mathsf{k}} & \quad\text{Chain effects}\end{array}$$

$$\begin{array}{llll} \mathsf{hf},\ \mathsf{f} & ::= & & \text{Hole filling}\\ & | & \mathsf{h} \mathrel{:=} \mathsf{v} & \quad\text{Fill } \mathsf{h} \text{ with value } \mathsf{v} \text{ (that may contain holes)}\end{array}$$

$\boxed{\Gamma \Vdash e}$ *(Typing of effects (raw))*

TyR-eff-N
$$\frac{}{\{\,\} \Vdash \varepsilon}$$

TyR-eff-A
$$\frac{\Gamma \uplus \Delta \Vdash \mathsf{v} : \mathsf{T} \qquad \texttt{ctx\_DestOnly}\ \Gamma \qquad \texttt{ctx\_HoleOnly}\ \Delta}{(1\!\uparrow\!\cdot m\cdot n)\cdot\Gamma \uplus \{+\mathsf{h} :_m {}^{n}\lfloor \mathsf{T}\rfloor\} \uplus (m\cdot n)\cdot\Delta \Vdash \mathsf{h} \mathrel{:=} \mathsf{v}}$$

TyR-eff-C
$$\frac{\Gamma_1 \Vdash e_1 \qquad \Gamma_2 \Vdash e_2}{\Gamma_1 {}^{-}\!\uplus^{+} \Gamma_2 \Vdash e_1 \gg e_2}$$

$\boxed{\Gamma \vdash e}$ *(Typing of effects (valid ones only))*

Ty-eff-T
$$\frac{\Gamma \Vdash e \qquad \texttt{ctx\_Valid}\ \Gamma}{\Gamma \vdash e}$$

$\boxed{\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}}$ *(Typing of commands (valid ones only))*

Ty-cmd-C
$$\frac{\Gamma_1 \vdash e \qquad \Gamma_2 \vdash \mathsf{v}:\mathsf{T} \qquad \texttt{ctx\_DestOnly}\ \Gamma_1 {}^{-}\!\uplus^{+}\Gamma_2}{\Gamma_1{}^{-}\!\uplus^{+}\Gamma_2 \vdash \mathsf{v} \diamond e : \mathsf{T}}$$

$\boxed{\mathsf{v}_1\ {\scriptstyle\Gamma_1}\,|\ e_1 \Downarrow \mathsf{v}_2\ {\scriptstyle\Gamma_2}\,|\ e_2}$ *(Big-step evaluation of effects on values (with potential holes))*

Sem-eff-N
$$\frac{}{\mathsf{v}_1\ {\scriptstyle\Gamma_1}\,|\ \varepsilon \Downarrow \mathsf{v}_1\ {\scriptstyle\Gamma_1}\,|\ \varepsilon}$$

Sem-eff-S
$$\frac{\texttt{ctx\_HdnmNotMem}\ \mathsf{h}\ \Gamma_1 \qquad \mathsf{v}_1\ {\scriptstyle\Gamma_1}\,|\ e_1 \Downarrow \mathsf{v}_2\ {\scriptstyle\Gamma_2}\,|\ e_2}{\mathsf{v}_1\ {\scriptstyle\Gamma_1}\,|\ \mathsf{h}\mathrel{:=}\mathsf{v}' \gg e_1 \Downarrow \mathsf{v}_2\ {\scriptstyle\Gamma_2}\,|\ \mathsf{h}\mathrel{:=}\mathsf{v}'\gg e_2}$$

Sem-eff-F
$$\frac{\Gamma_0 \Vdash \mathsf{v}_0 : \mathsf{T} \qquad \texttt{ctx\_Valid}\ \Gamma_0 \qquad \mathsf{v}_1[\mathsf{h}\mathrel{:=}\mathsf{v}_0]\ {\scriptstyle(\Gamma_1 \uplus n\cdot\Gamma_0)}\,|\ e_1 \Downarrow \mathsf{v}_2\ {\scriptstyle\Gamma_2}\,|\ e_2}{\mathsf{v}_1\ {\scriptstyle\Gamma_1 \uplus \{-\mathsf{h}:{}^{n}\mathsf{T}\}}\,|\ \mathsf{h}\mathrel{:=}\mathsf{v}_0 \gg e_1 \Downarrow \mathsf{v}_2\ {\scriptstyle\Gamma_2}\,|\ e_2}$$

$\boxed{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}$ *(Big-step evaluation into commands)*

Sem-term-V
$$\frac{}{\mathsf{v}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}\diamond\varepsilon}$$

Sem-term-App
$$\frac{\mathsf{t}_1\ {}_{\mathsf{d.1}}\!\Downarrow\ \mathsf{v}_1 \diamond e_1 \qquad \mathsf{t}_2\ {}_{\mathsf{d.2}}\!\Downarrow\ \lambda \mathsf{x} \mapsto \mathsf{u} \diamond e_2 \qquad \mathsf{u}[\mathsf{x}\mathrel{:=}\mathsf{v}_1]\ {}_{\mathsf{d.3}}\!\Downarrow\ \mathsf{v}_3 \diamond e_3}{\mathsf{t}_1 \succ \mathsf{t}_2\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_3 \diamond e_1 \gg e_2 \gg e_3}$$

Sem-term-PatU
$$\frac{\mathsf{t}_1\ {}_{\mathsf{d.1}}\!\Downarrow\ () \diamond e_1 \qquad \mathsf{t}_2\ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_2 \diamond e_2}{\mathsf{t}_1\ ;\ \mathsf{t}_2\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}$$

Sem-term-PatL
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ \mathsf{Inl}\,\mathsf{v}_1 \diamond e_1 \qquad \mathsf{u}_1[\mathsf{x}_1\mathrel{:=}\mathsf{v}_1]\ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}$$

Sem-term-PatR
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ \mathsf{Inr}\,\mathsf{v}_1 \diamond e_1 \qquad \mathsf{u}_2[\mathsf{x}_2\mathrel{:=}\mathsf{v}_1]\ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathbf{case}\,\{\,\mathsf{Inl}\,\mathsf{x}_1 \mapsto \mathsf{u}_1\,,\ \mathsf{Inr}\,\mathsf{x}_2 \mapsto \mathsf{u}_2\,\}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}$$

Sem-term-PatP
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ (\mathsf{v}_1\,,\,\mathsf{v}_2) \diamond e_1 \qquad \mathsf{u}[\mathsf{x}_1\mathrel{:=}\mathsf{v}_1][\mathsf{x}_2\mathrel{:=}\mathsf{v}_2]\ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_2 \diamond e_2}{\mathsf{t} \succ \mathbf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_2 \diamond e_1 \gg e_2}$$

Sem-term-Map
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ \langle \mathsf{v}_1 \,⨾\, \mathsf{v}_2\rangle_{\Delta} \diamond e_1 \qquad \mathsf{u}[\mathsf{x}\mathrel{:=}\mathsf{v}_1]\ {}_{\mathsf{d.2}}\!\Downarrow\ \mathsf{v}_3 \diamond e_2 \qquad \mathsf{v}_2\ {\scriptstyle\Delta}\,|\ e_2 \Downarrow \mathsf{v}_4\ {\scriptstyle\Delta'}\,|\ e_3}{\mathsf{t} \succ \mathbf{map}\ \mathsf{x} \mapsto \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \langle \mathsf{v}_3 \,⨾\, \mathsf{v}_4\rangle_{\Delta'} \diamond e_1 \gg e_3}$$

Sem-term-Alloc
$$\frac{}{\mathbf{alloc}_{\mathsf{T}}\ {}_{\mathsf{d}}\!\Downarrow\ \langle +\mathsf{d} \,⨾\, -\mathsf{d}\rangle_{\{-\mathsf{d}:{}^{1\nu}\mathsf{T}\}} \diamond \varepsilon}$$

Sem-term-ToA
$$\frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}{\mathbf{to}_{⋉}\ \mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \langle () \,⨾\, \mathsf{v}\rangle_{\{\}} \diamond e}$$

Sem-term-FromA
$$\frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \langle () \,⨾\, \mathsf{v}\rangle_{\{\}} \diamond e}{\mathbf{from}_{⋉}\ \mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v} \diamond e}$$

Sem-term-FillU
$$\frac{\mathsf{t}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd ()\ {}_{\mathsf{d}}\!\Downarrow\ () \diamond e \gg \mathsf{h}\mathrel{:=}()}$$

Sem-term-FillL
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd \mathsf{Inl}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{d.2} \diamond e \gg \mathsf{h}\mathrel{:=}\mathsf{Inl}\ -\mathsf{d.2}}$$

Sem-term-FillR
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd \mathsf{Inr}\ {}_{\mathsf{d}}\!\Downarrow\ +\mathsf{d.2} \diamond e \gg \mathsf{h}\mathrel{:=}\mathsf{Inr}\ -\mathsf{d.2}}$$

Sem-term-FillP
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ +\mathsf{h} \diamond e}{\mathsf{t} \lhd (,)\ {}_{\mathsf{d}}\!\Downarrow\ (+\mathsf{d.2}\,,\,+\mathsf{d.3}) \diamond e \gg \mathsf{h}\mathrel{:=}(-\mathsf{d.2}\,,\,-\mathsf{d.3})}$$

Sem-term-FillC
$$\frac{\mathsf{t}\ {}_{\mathsf{d.1}}\!\Downarrow\ +\mathsf{h} \diamond e_1 \qquad \mathsf{u}\ {}_{\mathsf{d.2}}\!\Downarrow\ \langle \mathsf{v}_1 \,⨾\, \mathsf{v}_2\rangle_{\Delta} \diamond e_2}{\mathsf{t} \lhd\!\bullet\ \mathsf{u}\ {}_{\mathsf{d}}\!\Downarrow\ \mathsf{v}_1 \diamond e_1 \gg e_2 \gg \mathsf{h}\mathrel{:=}\mathsf{v}_2}$$

4 Type safety

Theorem 1 (Type safety). *If* `ctx_DestOnly` Γ *and* $\Gamma \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t}\ {}_{\mathsf{d}} \Downarrow\ \mathsf{v} \diamond e$ *and* $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

Theorem 2 (Type safety for complete programs). *If* $\{\} \vdash \mathsf{t} : \mathsf{T}$ *then* $\mathsf{t}\ {}_{\star} \Downarrow\ \mathsf{v} \diamond \varepsilon$ *and* $\{\} \vdash \mathsf{v} : \mathsf{T}$

Proof. By induction on the typing derivation.

- TyTerm_Val: (0) $\Gamma \vdash \mathsf{v} : \mathsf{T}$
  (0) gives (1) $\mathsf{v}\ {}_{\mathsf{d}} \Downarrow\ \mathsf{v} \diamond \varepsilon$ immediately. From TyEff_NoEff and TyCmd_Cmd we conclude (2) $\Gamma \vdash \mathsf{v} \diamond e : \mathsf{T}$.

- TyTerm_App: (0) $m{\cdot}\Gamma_1 \uplus \Gamma_2 \vdash \mathsf{t} \succ \mathsf{u} : \mathsf{T}_2$
  We have
  (1) $\Gamma_1 \vdash \mathsf{t} : \mathsf{T}_1$
  (2) $\Gamma_2 \vdash \mathsf{u} : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$
  (3) `ctx_Disjoint` $\Gamma_1\ \Gamma_2$
  Using recursion hypothesis on (1) we get (4) $\mathsf{t}\ {}_{\mathsf{d.1}} \Downarrow\ \mathsf{v}_1 \diamond e_1$ where (5) $\Gamma_1 \vdash \mathsf{v}_1 \diamond e_1 : \mathsf{T}_1$.
  Inverting TyCmd_Cmd we get (5) $\Gamma_{11} \uplus \Gamma_{13} \vdash \mathsf{v}_1 : \mathsf{T}_1$ and (6) $\Gamma_{12} \uplus -\Gamma_{13} \Vdash e_1$ where (7) $\Gamma_1 = \Gamma_{11} \uplus \Gamma_{12}$.
  Using recursion hypothesis on (2) we get (8) $\mathsf{u}\ {}_{\mathsf{d.2}} \Downarrow\ \mathsf{v}_2 \diamond e_2$ where (9) $\Gamma_2 \vdash \mathsf{v}_2 \diamond e_2 : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$.
  Inverting TyCmd_Cmd we get (10) $\Gamma_{21} \uplus \Gamma_{23} \vdash \mathsf{v}_2 : \mathsf{T}_1\ {}_{m}\!\!\to \mathsf{T}_2$ and (11) $\Gamma_{22} \uplus -\Gamma_{23} \Vdash e_2$ where (12) $\Gamma_2 = \Gamma_{21} \uplus \Gamma_{22}$.
  Using Lemma **??** on (9) we get (13) $\mathsf{v}_2 = \lambda \mathsf{x} \mapsto \mathsf{t}'$ and (14) $\Gamma_{21} \uplus \Gamma_{23} \uplus \{\mathsf{x} :_m \mathsf{T}_1\} \vdash \mathsf{t}' : \mathsf{T}_2$.

  Typing value part of the result

  Using Lemma **??** on (14) and (5) we get (15) $m{\cdot}(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1] : \mathsf{T}_2$.
  Using recursion hypothesis on (15) we get (16) $\mathsf{t}'[\mathsf{x} \coloneqq \mathsf{v}_1]\ {}_{\mathsf{d.3}} \Downarrow\ \mathsf{v}_3 \diamond e_3$ where (17) $m{\cdot}(\Gamma_{11} \uplus \Gamma_{13}) \uplus (\Gamma_{21} \uplus \Gamma_{23}) \vdash \mathsf{v}_3 \diamond e_3 : \mathsf{T}_2$.

  Typing effect part of the result

  We have
  (6) $\Gamma_{12} \uplus -\Gamma_{13} \Vdash e_1$
  (11) $\Gamma_{22} \uplus -\Gamma_{23} \Vdash e_2$

  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{22}$ comes naturally from (3), (7) and (12).

  We must show:
  `ctx_Disjoint` $\Gamma_{12}\ \Gamma_{23}$: holes in e_2 (associated to u) are fresh so they cannot match a destination name from t as they don't exist yet when t is evaluated.
  `ctx_Disjoint` $\Gamma_{22}\ \Gamma_{13}$: slightly harder. Holes in e_1 (associated to t) are fresh too, so I don't see a way for u to create a term that could mention them, but sequentially, at least, they exist during u evaluation. In fact, Γ_{22} might have intersection with Γ_{13} (see TyEff_Union) as long as they share the same modalities (it's even harder to prove I think).
  `ctx_Disjoint` $\Gamma_{13}\ \Gamma_{23}$: freshness of holes in both effects, executed sequentially, should be enough.

  Let say this is solved by Lemma 1, with no holes of e_1 negative context appearing as dests in e_2 positive context.

  By TyEff_Union we get (18) $\Gamma_{12} \uplus \Gamma_{22} \uplus -\Gamma_{13} \uplus -\Gamma_{23} \Vdash e_1 \gg e_2$.
  Inverting TyCmd_Cmd on (17) we get (19) $m{\cdot}(\Gamma_{111} \uplus \Gamma_{131}) \uplus \Gamma_{211} \uplus \Gamma_{231} \uplus \Gamma_3 \vdash \mathsf{v}_3 : \mathsf{T}_2$ and (20) $m{\cdot}(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \uplus$
  $\Gamma_3 \Vdash e_3$ where (21) $\Gamma_{k\,1} \uplus \Gamma_{k\,2} = \Gamma_{k}$

  We have
  (18) $\Gamma_{12} \uplus \Gamma_{22} \uplus -\Gamma_{13} \uplus -\Gamma_{23} \Vdash e_1 \gg e_2$
  (20) $m{\cdot}(\Gamma_{112} \uplus \Gamma_{132}) \uplus \Gamma_{212} \uplus \Gamma_{232} \uplus -\Gamma_3 \Vdash e_3$

  Using (21) on (18) to decompose $-\Gamma_{23}$, we get (22) $\Gamma_{12} \uplus \Gamma_{22} \uplus -(\Gamma_{131} \uplus \Gamma_{231}) \uplus -(\Gamma_{132} \uplus \Gamma_{232}) \Vdash e_1 \gg e_2$

  We want Γ_{132} from (22) to cancel $m{\cdot}\Gamma_{132}$ from (20), but the multiplicity doesn't match apparently.

  Γ_{13} contains dests associated to holes that may have been created when evaluating t into $\mathsf{v}_1 \diamond e_1$. If v_1 is used with delay (result of multiplying its context by m), then should we also delay the RHS of its associated effect? In other terms, if we have $\{+\mathsf{h} :_{1\nu}\ {}^{n}\lfloor \mathsf{T}_1 \oplus \mathsf{T}_2 \rfloor\} \vdash +\mathsf{h}' \diamond \mathsf{h} \coloneqq \mathsf{Inl}\ -\mathsf{h}' : {}^{n}\lfloor \mathsf{T}_1 \rfloor$, and use h' with delay m (e.g stored inside another dest in the body of the function), should we also type the RHS of $\mathsf{h} \coloneqq \mathsf{Inl}\ -\mathsf{h}'$ with delay ? I think so, if we want to keep the property that age of dests and age of the associated holes are the same. Which means a more refined substitution lemma.

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2) \succ (\lambda \mathsf{x}_2 \mapsto +\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))$

  $+\mathsf{h}_0 \triangleleft (,)\ {}_{\mathsf{d}} \Downarrow\ (+\mathsf{d.2}\,,\,+\mathsf{d.3}) \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3})$

  $(\mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2)[\mathsf{x}_1 \coloneqq +\mathsf{d.2}][\mathsf{x}_2 \coloneqq +\mathsf{d.3}]\ {}_{\mathsf{d}'} \Downarrow\ +\mathsf{d.3} \diamond \mathsf{d.2} \coloneqq +\mathsf{h}_1$

  $(+\mathsf{h}_0 \triangleleft (,) \succ \mathsf{case}\,(\mathsf{x}_1\,,\,\mathsf{x}_2) \mapsto \mathsf{x}_1 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ +\mathsf{h}_1)\ ;\ \mathsf{x}_2)\ {}_{\mathsf{d}''} \Downarrow\ +\mathsf{d.3} \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3}) \gg \mathsf{d.2} \coloneqq +\mathsf{h}_1$

  $(+\mathsf{h}_3 \triangleleft\!\bullet (\mathsf{to}_{\rtimes}\ \mathsf{x}_2))[\mathsf{x}_2 \coloneqq +\mathsf{d.3}]\ {}_{\mathsf{d}'''} \Downarrow\ () \diamond \mathsf{h}_3 \coloneqq +\mathsf{d.3}$

  $\mathsf{t}\ {}_{\mathsf{d}''''} \Downarrow\ () \diamond \mathsf{h}_0 \coloneqq (-\mathsf{d.2}\,,\,-\mathsf{d.3}) \gg \mathsf{d.2} \coloneqq +\mathsf{h}_1 \gg \mathsf{h}_3 \coloneqq +\mathsf{d.3}$

Lemma 1 (Freshness of holes)**.** *Let* t *be a program with no pre-existing ampar sharing hole names.*

During the reduction of t*, the only other place where the names of the holes on the RHS of an effect can appear is in the accompanying value of the command, as destinations.*

Proof. Names of the holes on the RHS of a new effect:

- either are **fresh** (in all BigStep_Fill⟨*Ctor*⟩ rules), which means the only other place where those names are known and can show up is as destinations on the accompanying value of the command (Γ_{12} in TyCmd_Cmd), but not in positive or negative contexts of the command given by the evaluation of a sibling subterm;

- or are those of pre-existing holes coming from the extended value v_2 of an ampar, when BigStep_FillComp is evaluated. Because they come from an ampar, they must be neutralized by this ampar, so the left value v_1 of the ampar is the only place where those names can show up, as destinations, if we disallow pre-existing ampar with shared hole names in the body of the initial program · And v_1 is exactly the accompanying value returned by the evaluation of BigStep_FillComp

TODO: prove that this property is preserved by typing rules □