Bayesian data analysis with TensorFlow Probability

Simeon Carstens & Dorran Howell, Tweag I/O

DataScienceConference Europe 2020

Your hosts

Simeon

...will give the presentation

background in computational structural biology

Data Scientist at Tweag I/O since May 2019

Dorran

... will happily answers your questions in the chat

background and previous positions in geophysics

Data Scientist at Tweag I/O since August 2019

Tweag I/O

TODO

Tweag I/O is a software innovation lab and consultancy based in Paris, but with employees all around the world.

We specialize in

- software engineering, with a focus on functional programming
- DevOps, with a focus on reproducible software systems and builds
- data science

What you're in for

This tutorial consists of alternating blocks of

- theory / example slides
- practical examples on either external websites or Google Colab notebooks. Links are provided at `https://github.com/tweag/tutorial-dsc-2020/`

Requirements:

- a Google account (for the practical exercises)
- elementary knowledge in probability theory and statistics

Bayesian vs frequentist probabilities

Example: fair coin flip with $p(\mathrm{head}) = p(\mathrm{tail}) = \frac{1}{2}$

Frequentist probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

$\frac{\#\text{ of heads}}{\#\text{ total flips}}$ for ∞ many fair coin flips

Bayesian probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

measure of *belief* in the statement "flip results in head" given single fair coin flip

TODO: this slide looks complicated and is aesthetically offputting

Prior beliefs

Assume: unknown bias b

Prior probability

Encodes prior belief in b *before* flipping the coin

What is known about b?

- b is a probability: $0 \leq b \leq 1$
- most coins are fair

$\rightarrow$ choose prior distribution defined between 0 and 1, with maximum at and symmetric around $b = \frac{1}{2}$

Example:

$$b \sim \mathrm{Beta}(\alpha = 2, \beta = 2)$$

with

$$\mathrm{Beta}(x; \alpha, \beta) \propto x^{\alpha - 1}(1 - x)^{\beta - 1}$$

Posterior belief

Now: flip coin one time, result: head

Posterior belief

$p(b|\mathrm{head})$: updated prior belief after obtaining new data

TODO: some illustration of shifting distribution

Update rule

Bayes' theorem

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

(easily derived from rules for conditional probabilities)

In data analysis:

$$\underbrace{p(x|D, I)}_{\text{posterior}} = \underbrace{p(D|x, I)}_{\text{likelihood}} \times \underbrace{p(x|I)}_{\text{prior}} \,/\, \underbrace{p(D|I)}_{\text{evidence}}$$

x: model parameter

D: data

I: prior information (often not made explicit)

Likelihood

$p(D|x)$: probability of the data given fixed model parameters
$\rightarrow$ models data-generating process

In our case:

$$p(k|b) = b^k(1-b)^{k-1} = \mathrm{Ber}(k; b)$$

with

$$k = \begin{cases} 0: & \text{tail} \\ 1: & \text{head} \end{cases}$$

TODO: graph of likelihood function

Evidence

$p(D) = \int \mathrm{d}x \;\, p(D|x)p(x)$:

normalization constant (long story...)

In our case:

$$
\begin{aligned}
p(k=1) &= \int_0^1 \mathrm{d}b \; L(k=1|b)p(b) \\
&= \int_0^1 \mathrm{d}b \;\; \mathrm{Ber}(k;b) \times \mathrm{Beta}(k;\alpha=2,\beta=2)|_{k=1} \\
&= \int_0^1 \mathrm{d}b \;\; b^k(1-b)^{k-1}b(1-b)\Big|_{k=1} \\
&\;\;\vdots \\
&= \frac{1}{12}
\end{aligned}
$$

Evidence

$p(D) = \int \mathrm{d}x \; p(D|x)p(x)$:

normalization constant (long story...)

In our case:

$$
\begin{aligned}
p(k=1) &= \int_0^1 \mathrm{d}b \; L(k=1|b)p(b) \\
&= \int_0^1 \mathrm{d}b \;\; \mathrm{Ber}(k;b) \times \mathrm{Beta}(b;\alpha=2,\beta=2)\big|_{k=1} \\
&= \int_0^1 \mathrm{d}b \;\; b^k(1-b)^{k-1}b(1-b)\Big|_{k=1} \\
&\;\vdots \\
&= \frac{1}{12}
\end{aligned}
$$

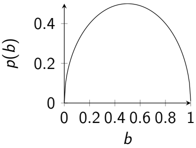
YIKES

Update rule

In our coin flip example:

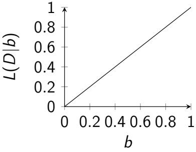
prior

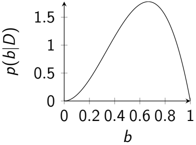
$$p(b) = \mathrm{Beta}(b; \alpha = 2, \beta = 2) \propto b^{\frac{1}{2}}(1-b)^{\frac{1}{2}}$$



likelihood

$$p(D|b) = \mathrm{Bern}(k = 1; b) = b$$





posterior

$$p(b|D) \propto p(D|b) \times p(b)/p(D) = \mathrm{Beta}(b; \alpha = 3, \beta = 2)$$