TWEAG

Bayesian data analysis with TensorFlow Probability

DataScienceConference Europe 2020

Your hosts

Simeon

...will give the presentation

background in computational structural biology

Data Scientist at Tweag I/O since May 2019

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... will happily answers your questions in the chat

Tweag I/O

TODO

Tweag I/O is a software innovation lab and consultancy based in Paris, but with employees all around the world.

We specialize in

- software engineering, with a focus on functional programming
- DevOps, with a focus on reproducible software systems and builds
- data science

What you're in for

This tutorial consists of alternating blocks of

- theory / example slides
- practical examples on either external websites or Google Colab notebooks. Links are provided at `https://github.com/tweag/tutorial-dsc-2020/`

Requirements:

- a Google account (for the practical exercises)
- elementary knowledge in probability theory and statistics

Bayesian vs frequentist probabilities

Example: fair coin flip with $p(\text{head}) = p(\text{tail}) = \frac{1}{2}$

Frequentist probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

$\frac{\text{\# of heads}}{\text{\# total flips}}$ for ∞ many fair coin flips

Bayesian probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

measure of *belief* in the statement "flip results in head" given single fair coin flip

TODO: this slide looks complicated and is aesthetically offputting

Prior beliefs

Assume: unknown bias *b*

Prior probability

Encodes prior belief in *b* *before* flipping the coin

What is known about *b*?

b is a probability: $0 \leq b \leq 1$

most coins are fair

$\rightarrow$ choose prior distribution defined between 0 and 1, with maximum at and symmetric around $b = \frac{1}{2}$

Example:

$$b \sim \mathsf{Beta}(\alpha = 2, \beta = 2)$$

with

$$\mathsf{Beta}(x; \alpha, \beta) \propto x^{\alpha-1}(1-x)^{\beta-1}$$

Posterior belief

Now: flip coin one time, result: head

Posterior belief

$p(b|\text{head})$**:** updated prior belief after obtaining new data

TODO: some illustration of shifting distribution

Update rule

Bayes' theorem

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

(easily derived from rules for conditional probabilities)

In data analysis:

$$\underbrace{p(x|D, I)}_{\text{posterior}} = \underbrace{p(D|x, I)}_{\text{likelihood}} \times \underbrace{p(x|I)}_{\text{prior}} / \underbrace{p(D|I)}_{\text{evidence}}$$

x**:** model parameter

D**:** data

I**:** prior information (often not made explicit)

Likelihood

$p(D|x)$: probability of the data given fixed model parameters
$\rightarrow$ models data-generating process

In our case:

$$p(k|b) = b^k(1-b)^{k-1} = \mathrm{Ber}(k; b)$$

with

$$k = \begin{cases} 0: & \text{tail} \\ 1: & \text{head} \end{cases}$$

TODO: graph of likelihood function

Evidence

$p(D) = \int \mathrm{d}x \; p(D|x)p(x)$:

normalization constant (long story...)

In our case:

$$
\begin{aligned}
p(k=1) &= \int_0^1 \mathrm{d}b \; L(k=1|b)p(b) \\
&= \int_0^1 \mathrm{d}b \;\; \mathrm{Ber}(k; b) \times \mathrm{Beta}(k; \alpha=2, \beta=2)\big|_{k=1} \\
&= \int_0^1 \mathrm{d}b \;\; b^k (1-b)^{k-1} b(1-b)\Big|_{k=1} \\
&\;\vdots \\
&= \frac{1}{12}
\end{aligned}
$$

Evidence

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In our case:

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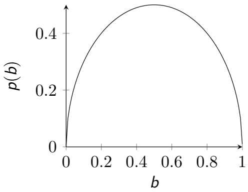
&\;\;\vdots \\
&= \frac{1}{12}
\end{aligned}
$$

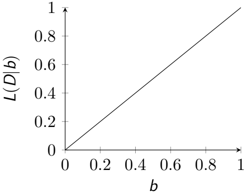
YIKES

Update rule

In our coin flip example:

prior
$$\begin{aligned} p(b) &= \mathrm{Beta}(b; \alpha = 2, \beta = 2) \\ &\propto b^{\frac{1}{2}}(1-b)^{\frac{1}{2}} \end{aligned}$$





likelihood
$$\begin{aligned} p(D|b) &= \mathrm{Bern}(k = 1; b) \\ &= b \end{aligned}$$