TWEAG

Bayesian data analysis with TensorFlow Probability

www.tweag.io

Your hosts

Simeon will give the presentation

- background in computational biology
- Data Scientist at Tweag since 2019

Dorran will happily answer questions

- previous positions in geophysics
- Data Scientist at Tweag since 2019

Tweag I/O

TODO

Tweag I/O is a software innovation lab and consultancy based in Paris with employees all around the world.

We specialize in

- software engineering, with a focus on functional programming
- DevOps, with a focus on reproducible software systems and builds
- data science

What you're in for

This tutorial consists of alternating blocks of

- theory / example slides
- practical examples on either external websites or Google Colab notebooks. Links are provided at `https://github.com/tweag/tutorial-dsc-2020/`

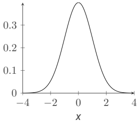
Requirements:

- a Google account (for the practical exercises)
- elementary knowledge in probability theory and statistics

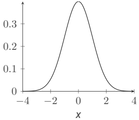
Reminder: Probabilities

Probability distributions can be...

discrete: $\mathrm{Ber}(\mathrm{k};\mathrm{b}) = b^k(1-b)^{1-k}$



continuous: $\mathcal{N}(\mathrm{x};\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}}\,\mathrm{exp}(-\frac{1}{2\sigma^2}(x-\mu)^2)$



Important concepts:

Conditional probability

$p(A|B)$: probability that A is true, given B is true

Joint probability

$p(A,B)$: probability that both A and B are true

Conditional joint probability

$p(A,B|C)$: probability that both A and B are true, given C is true

p(x)

p(x)

Bayesian vs frequentist probabilities

Example: fair coin flip with $p(\text{head}) = p(\text{tail}) = \frac{1}{2}$

Frequentist probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

$\frac{\text{\# of heads}}{\text{\# total flips}}$ for ∞ many fair coin flips

Bayesian probability

$p(\text{"flip results in head"}|b = \frac{1}{2}) = \frac{1}{2}$:

measure of *belief* in the statement "flip results in head" given single fair coin flip

TODO: this slide looks complicated and is aesthetically offputting

Prior beliefs

Assume: unknown bias *b*

> Prior probability Encodes prior belief in *b* *before* flipping the coin

What is known about *b*?

- *b* is a probability: $0 \leq b \leq 1$
- most coins are fair

$\rightarrow$ choose prior distribution defined between 0 and 1, with maximum at and symmetric around $b = \frac{1}{2}$

Example:

$$b \sim \text{Beta}(\alpha = 2, \beta = 2)$$

with

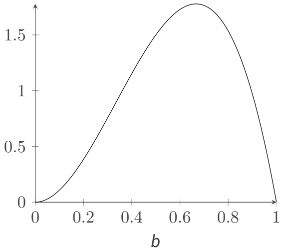
$$\text{Beta}(x; \alpha, \beta) \propto x^{\alpha-1}(1-x)^{\beta-1}$$

Posterior belief

Now: flip coin one time, result: head

Posterior belief

$p(b|\text{head})$: updated prior belief after obtaining new data



Update rule

Bayes' theorem

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

(easily derived from rules for conditional probabilities)

In data analysis:

$$\underbrace{p(x|D,I)}_{\text{posterior}} = \underbrace{p(D|x,I)}_{\text{likelihood}} \times \underbrace{p(x|I)}_{\text{prior}} \,/\, \underbrace{p(D|I)}_{\text{evidence}}$$

x: model parameter

D: data

I: prior information (often not made explicit)

Likelihood

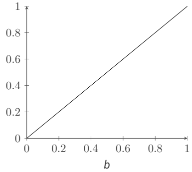
$p(D|x)$: probability of the data given fixed model parameters
$\rightarrow$ models data-generating process

In our case:

$$p(k|b) = b^k(1-b)^{k-1} = \text{Ber}(k; b)$$

with

$$k = \begin{cases} 0: & \text{tail} \\ 1: & \text{head} \end{cases}$$



Evidence

$p(D) = \int \mathrm{d}x \, p(D|x)p(x)$:
normalization constant (long story...)

In our case:

$$
\begin{aligned}
p(k=1) &= \int_0^1 \mathrm{d}b \, L(k=1|b)p(b) \\
&= \int_0^1 \mathrm{d}b \;\; \mathrm{Ber}(k;b) \times \mathrm{Beta}(k;\alpha=2,\beta=2)|_{k=1} \\
&= \int_0^1 \mathrm{d}b \;\; b^k(1-b)^{k-1}b(1-b)\Big|_{k=1} \\
&\vdots \\
&= \frac{1}{12}
\end{aligned}
$$

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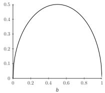
$$

YIKES

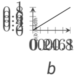
Update rule

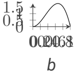
In our coin flip example:

prior: $$p(b) = \text{Beta}(b; \alpha = 2, \beta = 2) \propto b(1-b)$$



likelihood: $$p(D|b) = \text{Ber}(k = 1; b) = b$$





posterior: $$p(b|D) \propto p(D|b) \times p(b)/p(D) = \text{Beta}(b; \alpha = 3, \beta = 2) \propto b^2(1-b)$$

$L(D|b)$

– no slides for interactive stuff –

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YIKES

Intractable distributions

In most cases: $p(D) = ???$

$\rightarrow$