Correlated shocks within firms

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December 6, 2017

Abstract

Due to its association with cross-country business cycles, propagation of idiosyncratic shocks, and even financial contagion, firm comovement is an important facet of macroeconomic research. However, we know little about whether pairs of establishments within firms comove more than pairs of establishments from different firms. Using a long panel of Canadian manufacturing establishments, I investigate the correlations and covariances of within-firm pairs of establishments and decompose them into labour inputs, intermediates and profit. I find that within-firm establishment pairs have correlations 0.0477 higher than between-firm establishment pairs (which have an average correlation very close to zero) after controlling for industry and region effects. Covarying intermediate input costs account for 49% of the within-firm comovement effect.

Keywords: Economic Fluctuations; Firm Volatility; Firm Ownership.

JEL: E32; L20; L25.

1 Introduction

This paper explores the determinants of comovement between and within firms. Firm comovement has recently been blamed for things like financial contagion, the propagation of idiosyncratic shocks, and explaining positive cross-country business cycle correlations. However, most of the microeconomic research has focused on individual-level measures and explanations (e.g., size, granularity) and haven't focused on the (even more micro-) economic linkages between and within firms themselves. I use detailed establishment-level panel data to investigate the magnitude and significance of these comovements.

The research on firm comovement is broadly divided into macro and microeconomic areas. In the macroeconomic literature, aggregate GDP comovement is either associated with aggregate measures of integration (the country-country strategy) or individual firms and their integration with other countries (the firm-country strategy).

The country-country studies, such as Frankel and Rose (1998); Kose and Yi (2006), have little information on the actual mechanism driving the comovement—do establishments within multinationals really comove? Is it because of vertical linkages? Or financial dependence? Or common firm-level shocks? Is it because of capital, labour, or intermediate input comovement? Although the qualitative mechanisms seem obvious, we do not seem to understand them quantitatively (Johnson, 2014).

The firm-country studies in the microeconomic literature on firm comovement attempts to address and uncover these mechanisms, and have shown that shocks to parent firms are correlated with aggregate movements in regions where they have affiliate firms (see Kleinert et al., 2015; Cravino and Levchenko, 2016; di Giovanni et al., 2016, 2017). Overall, the firm-country research has aggregate results and calibrated models that strongly suggest firm-comovement can account for a significant amount of cross-country correlations. Previous empirical and theoretical work on firm comovement (or shock transmission and linkages in general) identified a myriad of possible causes: vertical linkages Burstein et al. (2008), internal capital markets Lamont (1997); Stein (1997), technology shocks Atalay et al. (2014), rent sharing Budd et al. (2005) or labour reallocation Giroud and Mueller (2016). I make identifying the within and between firm comovement itself the primary goal, and discover the components of those comovements.

To tackle this problem, I use data from the Annual Survey of Manufacturers, which is a defacto Census of manufacturing activity in Canada.[1] There are approximately 100,000 total plants in the sample, with around 30,000 alive per year. I focus on the period 1973-1999 to take advantage of consistent surveys and industry classifications. The ASM includes establishment sales, value added, labour and intermediates, as well as the firms that own the establishments. One difference between this and other firm studies is that I study all the correlations of all establishments under a common parent, which is slightly different than a headquarter vs. affiliate analysis. Headquarters often house very different economic activities than their affiliates, such as mangerial and financial activities versus production in the establishments. Here, I test whether the production activities in establishments comove, rather than asking how managerial and production activities are correlated.

There are several advantages of using the ASM over other administrative data typically used in firm volatility studies. The main advantage is the long period of the sample required to efficiently estimate covariances between plants.[2] In addition, the ASM has detailed and consistent information on firm ownership, as well as product-level input and output by establishment to differentiate between possible vertical linkages and firm shocks, and also between industry shocks and product-level shocks. Note that industry level input-output measures and industry shocks may not correctly capture the relationships between establishments, due to the substantial diversity of plant input and output within industries.

This paper makes a significant contribution to the estimation of within-firm comovement by establishing it and decomposing it into its input components. The key to identifying within-firm comovement is applying a comprehensive econometric methodology to a long panel of detailed establishment level data. The strategy I employ is to first calculate correlations between the growth of total sales of each establishment pair in the data. To get a sense of the magnitude of the problem: tens of thousands of establishments mean hundreds of millions of establishment-pair correlations. Next, once I have a picture of the dependence within the economy, I decompose the

[1] The data are available from CDER at Statistics Canada; see `http://www.statcan.gc.ca/eng/cder/data`.

[2] A rule of thumb to judge significant correlations: using the Fisher transformation, the standard error is approximately $(T-3)^{-1/2}$. With only 9 or 17 periods (in some samples), the variability in the correlation estimates is substantial.

firm component of shocks into capital and profit, labour and inputs.

Here, I delve deeper into the mechanisms and theory behind comovement. There is a considerable amount of research around the relationships between parent firms, their subsidiaries and their operating establishments. Shocks may be transmitted through multiple mechanisms, including technology transfer, operating decisions, legal issues, intermediate contracts, vertical linkages, labour movement within the firm, other types of reallocation, or generic firm demand or productivity shocks. In addition, there are regional and industry shocks that may be attributed to firms, because firms are likely to own establishments with common characteristics.

As a first stab at the problem, one needs to remove industry and regional shocks at least and then ask whether establishments that belong to the same firm move together. To test for vertical linkages within firms, I include establishment level input-output measures. Furthermore, I decompose sales shocks into profit, labour input and intermediate input shocks to expose the sources of the within-firm correlation. The results show within-firm comovement is significantly higher than between-firm comovement, even after accounting for region and industry shocks, distance between establishments, vertical linkages, and common product-level inputs and outputs.

I find an establishment has a correlation 0.0477 higher with an establishment within the same firm relative to an establishment in a different firm. Approximately 49% of the within-firm effect is due to covarying intermediate input costs.

The rest of the paper is organized like so: Section 2 describes the framework I use to analyze the data and the data itself. Section 3 describes the econometric approaches to analyze the problems and the results, and Section 4 concludes. The Appendix follows in Section 5.

2 Empirical framework and data

Here, I outline the framework with which to examine establishment correlations. I'll start with an overview of the sales growth process for each establishment. There are two important ways to think of a sales growth rate: as a combination of industry, province, and idiosyncratic shocks, and also as a combination of the shocks of the components of sales, like payroll, profit and intermediate inputs. Once the different parts of the sales growth process are outlined, I can calculate comovement between establishments after accounting for different parts of the process.

To account for the possible industry and geographic sources of sales shocks, the establishment sales growth process, at a first approximation, follows

$$g^s_{it} = \theta^s_{it} + \mu^s_{pt} + \nu^s_i + u^s_{it} \tag{1}$$

Where $g^s_{it} = \Delta \log s_{it}$, the log-difference growth rate of sales of establishment i at time t, θ^s_{it} is an industry-time shock, μ^s_{pt} is a province-time shock, ν^s_i is a time-invariant establishment effect. $u^s_{it} \equiv \lambda^s_{ft} + e^s_{it}$ is a shock composed of two parts: a firm shock and idiosyncratic establishment shock. The questions: are g^s_{it}s are correlated across establishments within a firm? Are u^s_{it}?s

Of course, g^s_{it} may be correlated within firms because firms tend to own establishments within specific industries and provinces, and so are subject to common shocks. Removing those and examining u^s_{it}, are there common firm shocks that induce correlation within firms?

In addition, sales growth is composed of several components—how do these contribute to within firm correlation? I do so by recognizing that sales can be thought of as a combination of profit, payroll and intermediate inputs:

$$s_{it} = \pi_{it} + pay_{it} + input_{it}, \tag{2}$$

where s_{it} is sales, π_{it} is profit and capital services, pay_{it} is total payroll, $input_{it}$ is total value of intermediate inputs. The growth rate g^s_{it} can then be decomposed into the weighted sum of the growth of those components

$$g^s_{it} = w^{\pi}_{t-1} g^{\pi}_{it} + w^{pay}_{t-1} g^{pay}_{it} + w^{input}_{t-1} g^{input}_{it} \tag{3}$$

In turn, each one could be driven by industry, region, firm or other types of shocks. For instance, a strike in the auto industry could result in a negative labour shock across all establishments in that industry. Or a carbon tax in BC could affect the price of intermediates for all establishments in BC. In that case, we should also be interested in the residual growth of any variable $x \in V = \{\pi, pay, input\}$,

$$g^x_{it} = \theta^x_{it} + \mu^x_{pt} + \nu^x_i + \underbrace{\lambda^x_{ft} + e^x_{it}}_{u^x_{it}} \tag{4}$$

A natural question to ask is then, if u^s_{it} is correlated within firms, does it run through shocks to employment or something else? For instance, Giroud and Mueller (2016) shows firms respond to local shocks by reallocating labour across establishments within the firm (here, that would show up as a negative correlation between payrolls within the firm).

$$\mathrm{cov}(g^s_{it}, g^s_{jt}) = \sum_{x \in V} \sum_{y \in V} \mathrm{cov}(w^x_{it-1} g^x_{it}, w^y_{jt-1} g^y_{jt}) \tag{5}$$

What is the relationship between each component and common firm ownership?

2.1 Data

The data come from the Annual Survey of Manufacturers. The ASM is a long annual panel of manufacturing establishments in Canada, a defacto census of manufacturing activity from 1961-2011 (although I focus on 1973-1999 to get a more consistent sample with respect to establishment and firm identifiers, as well as industry classifications). I have observations on sales, value added, labour (wages and employment), intermediate inputs, and most importantly, firm ownership information. Industries are classified according to 4-digit SIC (1980). In addition, I construct establishment-level input-output linkages using the commodity survey that is included in the ASM for large establishments.

Table 1: Summary statistics.

	All estab.		Multi-estab.		Growth, g^x_{it}	
	(1) Mean	(2) SD	(3) Mean	(4) SD	(5) Mean	(6) SD
Sales	37.3	101.4	96.3	324.6	0.075	0.192
Payroll	5.5	15.1	13.7	46.3	0.067	0.240
Profit & capital	9.7	26.4	28.6	85.8	0.071	0.742
Input cost	22.1	60.2	67.5	212.5	0.074	0.282
N. obs.	4885		1797		125008	

Notes: The ASM data is at an annual frequency. 'Multi-establishment' is the subsample of establishments that are part of multi-establishment firms for at least one year. There are 76,857 plants in the full sample, and 11,092 in the multi-establishment sample. Both samples are restricted to establishments that are alive for at least 5 years in order to calculate reasonably accurate covariances. The static statistics shown [Columns (1–4)] are given for 1990, and are in millions of Canadian dollars. The growth (g^x_{it}) measure is for the multi-establishment subsample only, and calculated over 1974-1999. Profit & capital is everything left over after removing payroll and input cost from total sales. This includes capital costs.

Table 1 shows relevant descriptive statistics for the manufacturing sector. Like many firm or establishment datasets, the distribution of sales and value-added are skewed to the right, with a relatively small number of establishments making the majority of sales in the economy. Before we get started on details, the most important question is whether within-firm comovements matter—is there enough within-firm "mass" for these shocks to matter in the aggregate? Yes; the total value added of multi-establishment firms averages around 90% of total manufacturing value added per year. The most important statistics for this paper are the annual sales growth rates g^s_{it}, with a mean of 7.5% and standard deviation of 19.2%. As a first look at within-firm versus between-firm correlations in sales growth rates, consider Figure 1, the distribution of correlations of within-firms (solid blue line) is pushed significantly to the right of the distribution of the between-firm correlations (dashed black line). The mean within-firm correlation is 0.019, which is significantly higher than the mean between-firm correlation, 0.0037. Establishments are more correlated with other establishments owned by the same firm. Establishments in the same 4-digit industry are only twice as correlated than ones in different industries (0.0064 vs. 0.0037) and ones in the same province are only slight more correlated than ones in different provinces (0.0039 vs. 0.0037). Of course, firmly establishing the importance of within-firm correlations requires addressing the econometric issues raised earlier.

3 Econometrics and Results

The economic goal is to understand within versus between firm comovement. The econometric strategy that achieves the economic goal requires the proper analysis of the growth processes in $\{\mathbf{g}^s_t\}_{t=0}^T = \{(g^s_{1t}, \ldots, g^s_{Nt})\}_{t=0}^T$. I proceed in two steps. First, remove the effects of industries and regions and recover the residual growth estimates $u^s_{it} = \lambda^s_{ft} + \epsilon^s_{it}$. Next, calculate the matrix of correlations of the residual growth rates and compare within-firm establishment pairs to between-

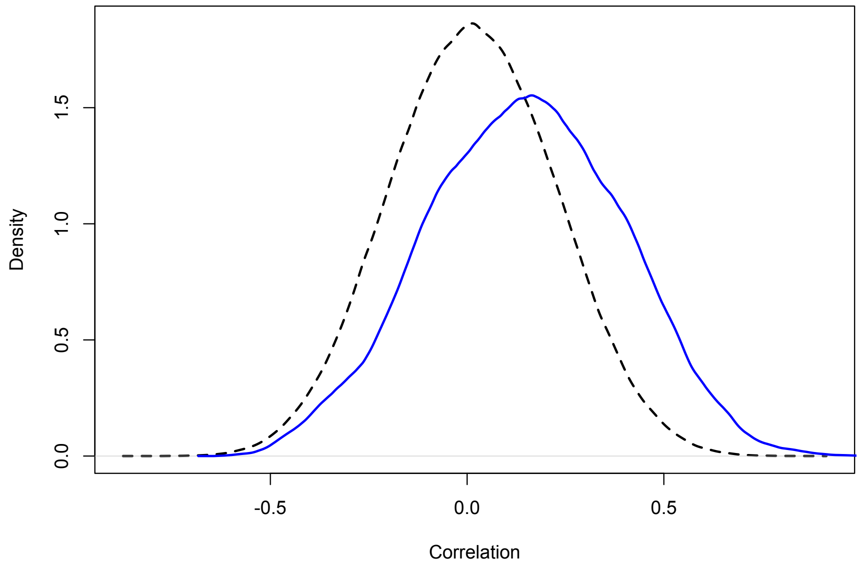

Figure 1: Density of establishment-establishment correlations, within-firm (solid blue line) and across firm (dotted black line).

firm pairs. Then decompose the sales growth rates and see which elements drive the within-firm correlation.

I write the sample correlation of sales growth between two establishments as r_{ij} (dropping the superscript s unless otherwise needed), and use the Fisher transformation to change the sample correlation to a normally distributed variable,

$$z_{ij} = \frac{1}{2} \ln \left(\frac{1 + r_{ij}}{1 - r_{ij}} \right). \tag{6}$$

Note the standard error of each z_{ij} is $(T - 3)^{-1/2}$ depends only on the number of periods T. In addition, since contemporaneous correlations are symmetric (and $r_{ii} = 1$) means we only get $N(N-1)/2$ unique correlations. The typical approach to solving inference issues in these cases are to use multi-way clustering, which I apply at the establishment-pair level.

In order to perform the decomposition, I define

$$c_{ij}^{x,y} \equiv \text{cov}(w_{it-1}^{x} u_{it}^{x}, w_{jt-1}^{y} u_{jt}^{y}), \tag{7}$$

where $w_{it-1}^{x} = x_{it-1}/s_{it-1}$. Note that $w_{it-1}^{s} = 1$, and Equation (5) can be rewritten as $c_{ij}^{s,s} = \sum_x \sum_y c_{ij}^{x,y}$.

Next, consider the economic purpose of this paper. The main effect we want to understand is whether z_{ij} is higher or lower if i, j are both owned by a common firm f. To that end, I define

firm_{ij} as an indicator for common ownership. Similarly, industry_{ij} and region_{ij} are indicators of common industries and provinces. In addition, I use commodity input and output information at the establishment level to examine the effects of vertical integration and other IO measures on correlations. The relevant measures, outlined in Appendix 5, are output_{ij}, input_{ij}, and io_{ij}.

The first thing to do is estimate via OLS the equations

$$z_{ij}^{x} = \mathbf{X}_{ij}\beta + \beta_f^x \text{firm}_{ij} + e_{ij}^x \tag{8}$$

$$c_{ij}^{x,y} = \mathbf{X}_{ij}\beta + \beta_f^{x,y} \text{firm}_{ij} + e_{ij}^{x,y} \tag{9}$$

where $\mathbf{X}$ is a vector of controls that include common industry and region dummies. The common industry and region dummies in these regressions serve to check whether industry and region shocks were removed correctly after estimating Equation (1).

To perform the decomposition, recover $\beta_f^{x,y}$ for each component $x, y \in \{s, \pi, pay, input\}$ after estimating Equation (9). Then the total firm effect on sales comovement can be decomposed into the firm effects on each component,

$$\hat{\beta}_f^{s,s} = \sum_x \sum_y \hat{\beta}_f^{x,y} \tag{10}$$

In other words, the average increase in covariance resulting from a pair of plants being owned by a common firm can be decomposed into the firm effect on each component. Dividing both sides by $\hat{\beta}_f^{s,s}$ shows the contribution of each component to the total firm effect on comovement.

3.1 Results

The results, shown in Table 2, show within-firm correlations are positive and significant. An establishment has on average a correlation coefficient that is 0.0477 higher with establishments owned by the same firm than with other establishments. In other words, after accounting for industry and region shocks, residual growth rates are correlated within firms. This evidence lends support for the firm comovement theory. The results are stronger for the balanced panel, suggesting longer-lived plants are more likely than short-lived plants to covary within firms.

Next, I decompose the covariance coefficients into the within-firm effects of sales (s), profit and capital (π), total payroll (pay), and total intermediate input costs ($input$). The majority of sales covariances come from input cost comovement, which accounts for 49% of the total effect. The covariances of profit and capital within firms matters slightly more than payroll covariances, but both are much lower than input costs. In fact, the majority of the leftover within-firm effect comes from the covariance of input costs with payroll and profit (i.e., the off-diagonal terms in the decomposition like $\hat{\beta}_f^{input,\pi}$).

Think of the effect of each component on overall volatility in two parts: size and individual volatility. The effect on overall establishment volatility will be greater if the component is a large share of sales, and will be greater if the component is more volatile. In this view, the decomposition

Table 2: Firm correlation and covariance, u_{it}^{s}

	Balanced panel		Full sample	
	Cov.	Corr.	Cov.	Corr.
Firm	0.00130^{***}	0.0839^{***}	0.00142^{***}	0.0477^{***}
	(0.00021)	(0.0130)	(0.00010)	(0.0030)
Industry	4.92×10^{-5}	0.00758	$-9.97\times10^{-5**}$	8.15×10^{-5}
	(9.93×10^{-5})	(0.00517)	(3.87×10^{-5})	(0.00114)
Province	$8.16\times10^{-5**}$	0.00416^{**}	$2.30\times10^{-5**}$	0.00100^{***}
	(3.47×10^{-5})	(0.00176)	(1.10×10^{-5})	(0.00031)
Constant	0.000148^{***}	0.00828^{***}	-2.19×10^{-7}	0.000636^{***}
	(2.37×10^{-5})	(0.00126)	(7.85×10^{-6})	(0.000207)
Observations	245,520	245,520	56,029,880	53,517,864
R-squared	0.000	0.000	0.000	0.000

Notes: Standard errors calculated using multi-way clustering at the establishment level (i and j). *** and ** denote significance at the 1% and 5% level, respectively.

Table 3: Firm covariance decomposition, u_{it}^{s}, u_{it}^{π}, u_{it}^{pay}, u_{it}^{input}

		$\hat{\beta}_f^{x,x}$			$\hat{\beta}_f^{x,x}/\hat{\beta}_f^{s,s}$	
	Corr.	Cov.	S.D.	Corr.	Cov.	S.D.
Sales	0.081	0.0013	0.036	1.000	1.000	1.000
Profit & capital	0.013	0.0001	0.009	0.160	0.070	0.264
Payroll	0.066	0.0000	0.007	0.810	0.034	0.185
Input cost	0.085	0.0006	0.025	1.047	0.490	0.700

Notes: The remainder of the decomposition comes from covariance of the components with input cost (especially profit & capital, less so payroll). Profit and capital is everything left over after removing payroll and input cost from total sales. This includes capital costs. The relative correlation term is only to get a sense of relative magnitudes, it does not actually decompose (its components do not need to add to 1).

results are consistent with the typical view of establishment inputs: labour and capital are tough to adjust in response to shocks, so they have low volatility and contribute less to overall establishment comovement. However, intermediate inputs vary a lot, and have a large share of sales for most plants, so they make up a lot of the overall firm comovement effect. Although recent studies have suggesting firms may reallocate labour across plants within the firm after suffering local shocks (which would induce negative comovement of payroll within firms), this suggests that overall, labour positively comoves. This may be because common firm shocks dominate local labour market shocks, either in magnitude or frequency, or both. This is bolstered by the fact that leaving in firm and regional shocks gives much higher coefficients on the comovement effect (see Figure 1).

4 Conclusion

In this note, I've shown that growth rates of establishments within firms are significantly more positive correlated than establishments that are not owned by the same firm. This comovement survives, but is reduced, after accounting for the fact that firms are likely to own establishments in the same industries and same geographic regions. On another note, about 49% of the covariance in growth rates within firms is due to covarying input costs, and not labour or profit and capital movements. This is consistent with the fact that labour and capital are tough to adjust in response to shocks, while intermediate inputs are more volatile.

This result is significant for the measurement of establishment and firm growth, and the application of those measures to understanding how volatility is transferred across borders within firms. Although this study uses domestic plants, it is suggestive that similar mechanisms operate across international borders, and contributes to the understanding of how MNCs contribute to global economic fluctuations.

References

Atalay, E., A. Hortaçsu, and C. Syverson (2014). Vertical Integration and Input Flows. *American Economic Review 104*(4), 1120–1148.

Budd, J. W., J. Konings, and M. J. Slaughter (2005). Wages and International Rent Sharing in Multinational Firms. *Review of Economics & Statistics 87*(1), 73–84.

Burstein, A., C. Kurz, and L. Tesar (2008). Trade, production sharing, and the international transmission of business cycles. *Journal of Monetary Economics 55*(4), 775–795.

Cravino, J. and A. A. Levchenko (2016). Multinational Firms and Business Cycle Transmission. *The Quarterly Journal of Economics 132*(2), 921–962.

Davis, S. J. and J. Haltiwanger (1992). Gross Job Creation, Gross Job Destruction, and Employment Reallocation. *The Quarterly Journal of Economics 107*(3), 819–863.

di Giovanni, J., A. A. Levchenko, and I. Mejean (2016). The Micro Origins of International Business Cycle Comovement. *NBER Working Paper* (21885).

di Giovanni, J., A. A. Levchenko, and I. Mejean (2017). Large Firms and International Business Cycle Comovement. *American Economic Review: Papers and Proceedings*, Forthcoming.

Frankel, J. A. and A. K. Rose (1998). The endogenity of the optimum currency area criteria. *The Economic Journal 108*(449), 1009–1025.

Giroud, X. and H. M. Mueller (2016). Redistribution of Local Labor Market Shocks Through Firms' Internal Networks. *NBER Working Paper Series.* (July).

Johnson, R. C. (2014). Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics 6*(4), 39–83.

Kleinert, J., J. Martin, and F. Toubal (2015). The few leading the many: Foreign affiliates and business cycle comovement. *American Economic Journal: Macroeconomics 7*(4), 134–159.

Kose, M. A. and K.-M. Yi (2006). Can the standard international business cycle model explain the relation between trade and comovement? *Journal of International Economics 68*(2), 267–295.

Lamont, O. (1997). Cash Flow and Investment: Evidence from Internal Capital Markets. *The Journal of Finance 52*(1), 83–109.

Stein, J. C. (1997). Internal Capital Markets and the Competition for Corporate Resources. *The Journal of Finance 52*(1), 111.

5 Appendix

5.1 Data

Data available from CDER in Statistics Canada. The data need some processing in order to be analyzed in this paper. First, there are two choices for the meaning of "firm", either the "parent" or the "ultimate parent". An establishment may be owned and directly controlled by firm A (the parent), which is in turn owned by a firm, which is owned by a firm, which is owned by a much larger firm B (the ultimate parent). I use the ultimate parent to be faithful to work on MNCs, although an argument could be made that direct parents have much more control and are more likely to transmit shocks among their establishments, both more often and more strongly. In that sense, the results may be viewed as an lower bound. On the other hand, there would be fewer establishment pairs that have common ownership, which would reduce the overall effect. In any case, the definition of firm in this case does not seem to affect the results. In addition, firm identifiers may change over time for reasons unrelated to business activities. The firm may undergo an organizational change that results in a new statistical identifier; suppose firm A owns three establishments in the sample, and then all three establishments have a new firm identifier B at some year t; it's likely the firm identifier has changed, or a new firm acquired all of the former direct parent's establishments—in any case, the effect is likely to attenuate the effect of firm ownership if anything, but the proportion of these establishments is very low.

Growth rates have always their own issues with outliers. To deal with this, I experiment with winsorizing or trimming the log growth rates at different percentiles by year, (0.05,0.95), (0.025,0.975), (0.01,0.99), and (0.001,0.999). The results do not change. However, the payroll, profit and intermediate numbers are less reliable than sales, and so calculating the growth rates for the decomposition can result in a smaller sample size. For this reason, I use the balanced panel sample for the decomposition. I find little difference if I trim the growth rates to a greater degree (0.1,0.9) and run the decomposition on the full sample.

The input-output measures are defined in a similar way to industry IO measures, but using establishment commodity data instead of industry commodity data. The relevant measures are output_{ij}, input_{ij}, and io_{ij}. Output (output_{ij}) is a measure of how similar two establishments commodity outputs are,

$$\text{output}_{ij} = \frac{\sum_c s_{ci}s_{cj}}{\sqrt{\sum_c s_{ci}^2}\sqrt{\sum_c s_{cj}^2}}, \tag{11}$$

where s_{ci} is sales of commodity c by establishment i. This measure is called "cosine similarity". The Input and IO measures are defined as

$$\text{input}_{ij} = \frac{\sum_c inp_{ci}inp_{cj}}{\sqrt{\sum_c inp_{ci}^2}\sqrt{\sum_c s_{inpj}^2}}, \tag{12}$$

$$\text{io}_{ij} = \frac{1}{2}\left(\frac{\sum_c inp_{ci}s_{cj}}{\sqrt{\sum_c inp_{ci}^2}\sqrt{\sum_c s_{cj}^2}} + \frac{\sum_c s_{ci}inp_{cj}}{\sqrt{\sum_c s_{ci}^2}\sqrt{\sum_c inp_{cj}^2}}\right). \tag{13}$$

$$\tag{14}$$

5.2 Robustness

To check the robustness of the conclusions to different specifications, I perform the following checks. In all checks, I continue the practice of using both a balanced panel and the much larger unbalanced panel. First, I include measures of vertical integration and input and output competition to see if the intra-firm correlation is due to similar input-output structures that aren't accounted for by industry specific effects. The results are shown in Table 4. Although some of the input-output measures are significant, the intafirm correlation coefficient does not change much, suggesting the product-level measures of establishment relationships are not causing the observed intra-firm comovement coefficient. In addition, there is no consistent finding of vertical linkages within firms associated with comovement. However, in the one case it is significant (covariances in the full sample), it is of the same magnitude of the within-firm effect itself. The balanced and full sample panels cannot reject the null of no within-firm vertical linkage effect.

Next, I calculate use a different growth rate measure for each sample to see if the log growth rate approximation is affecting the true relationship. The Davis-Haltiwanger-Schuh (DHS, Davis and Haltiwanger, 1992) growth rates are defined,

$$g_{it}^s = \frac{1}{2}\left(\frac{s_{it} - s_{it-1}}{s_{it} + s_{it-1}}\right) \tag{15}$$

the results are similar for both measures, in both the overall relationship and the decomposition.

Table 4: Firm correlation and covariance, u^s_{it}, including extra covariates

	Balanced panel		Full sample	
	Cov.	Corr.	Cov.	Corr.
Firm	0.00135***	0.0797***	0.00134***	0.0456***
	(0.00023)	(0.0137)	(0.00010)	(0.0031)
Industry	0.000184	0.00765	−0.000199***	−0.00428***
	(0.000131)	(0.00638)	(3.96×10^{-5})	(0.00107)
Province	8.16×10^{-5}**	0.00417**	2.32×10^{-5}**	0.00101***
	(3.48×10^{-5})	(0.00177)	(1.10×10^{-5})	(0.00031)
Output	-3.24×10^{-5}	0.0132	0.000629***	0.0268***
	(0.000213)	(0.0127)	(0.000141)	(0.0046)
Input	−0.000432**	−0.0185*	0.000311***	0.0136***
	(0.000183)	(0.0097)	(9.65×10^{-5})	(0.0032)
IO	1.27×10^{-5}	0.00372	-2.75×10^{-5}	0.000247
	(6.95×10^{-5})	(0.00400)	(3.93×10^{-5})	(0.001310)
IO × Firm	3.78×10^{-5}	0.0396	0.00108**	0.0167*
	(0.000473)	(0.0382)	(0.00045)	(0.0101)
Constant	0.000149***	0.00829***	-6.51×10^{-7}	0.000613***
	(2.38×10^{-5})	(0.00127)	(7.87×10^{-6})	(0.000207)
Observations	245,520	245,520	56,029,880	53,517,864
R-squared	0.000	0.000	0.000	0.000

Notes: Standard errors calculated using multi-way clustering at the establishment level (i and j). *** and ** denotes significance at the 1% and 5% level, respectively.

Table 5: DHS growth rates; firm correlation and covariance, u^s_{it}

	Balanced panel		Full sample	
	Cov.	Corr.	Cov.	Corr.
Firm	7.91×10^{-5}***	0.0659***	9.91×10^{-5}***	0.0452***
	(1.13×10^{-5})	(0.0087)	(7.11×10^{-6})	(0.0027)
Industry	2.11×10^{-5}***	0.0161***	-1.39×10^{-6}	0.00149
	(7.87×10^{-6})	(0.0049)	(2.58×10^{-6})	(0.00107)
Province	8.98×10^{-6}***	0.00734***	1.54×10^{-6}**	0.00122***
	(2.11×10^{-6})	(0.00161)	(7.17×10^{-7})	(0.00031)
Constant	1.41×10^{-5}***	0.0111***	9.38×10^{-7}	0.000898***
	(1.58×10^{-6})	(0.0012)	(5.57×10^{-7})	(0.000215)
Observations	624,890	624,890	63,024,530	60,777,540
R-squared	0.000	0.001	0.000	0.000

Notes: Standard errors calculated using multi-way clustering at the establishment level (i and j). *** and ** denotes significance at the 1% and 5% level, respectively.

Table 6: DHS growth rates; firm correlation and covariance, u_{it}^{s}, including extra covariates

	Balanced panel		Full sample	
	Cov.	Corr.	Cov.	Corr.
Firm	$7.78\times10^{-5***}$	0.0646^{***}	$9.42\times10^{-5***}$	0.0433^{***}
	(1.22×10^{-5})	(0.0093)	(7.11×10^{-6})	(0.0028)
Industry	2.34×10^{-6}	0.00329	$-7.25\times10^{-6***}$	-0.00192
	(6.54×10^{-6})	(0.00450)	(2.60×10^{-6})	(0.00103)
Province	$8.98\times10^{-6***}$	0.00734^{***}	$1.55\times10^{-6**}$	0.00123^{***}
	(2.11×10^{-6})	(0.00161)	(7.17×10^{-7})	(0.00031)
Output	2.97×10^{-5}	0.0201	$3.22\times10^{-5***}$	0.0216^{***}
	(1.65×10^{-5})	(0.0107)	(8.38×10^{-6})	(0.0041)
Input	1.94×10^{-5}	0.0126	$2.52\times10^{-5***}$	0.0117^{***}
	(1.13×10^{-5})	(0.0074)	(6.20×10^{-6})	(0.0028)
IO	5.00×10^{-6}	0.00598^{**}	-1.71×10^{-6}	0.000243
	(3.75×10^{-6})	(0.00289)	(2.60×10^{-6})	(0.001241)
IO × Firm	-2.96×10^{-5}	-0.0161	$7.90\times10^{-5**}$	0.0183
	(2.78×10^{-5})	(0.0223)	(3.19×10^{-5})	(0.0099)
Constant	$1.39\times10^{-5***}$	0.0110^{***}	9.07×10^{-7}	0.000879^{***}
	(1.58×10^{-6})	(0.0012)	(5.58×10^{-7})	(0.000216)
Observations	624,890	624,890	63,024,530	60,777,540
R-squared	0.001	0.001	0.000	0.000

Notes: Standard errors calculated using multi-way clustering at the establishment level (i and j). *** and ** denotes significance at the 1% and 5% level, respectively.