Aircraft Sizing Project

AE 6343 A - Aircraft Design I

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Honor Code Statement

I certify that I have abided by the Honor Code of the Georgia Institute of Technology and followed the collaboration guidelines as specified in the project description for this assignment.

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1 Introduction

1.1 Abstract

The purpose of this project is to apply the sizing and synthesis theory presented in AE 6343 Aircraft Design class on the conceptual design of a new advanced pilot training aircraft (APTA). A aircraft sizing tool was created using MATLAB. It was first tested and validated using data from a real world aircraft, the North American F-86L Sabre. Then, the tool was used to generate a conceptual design to meet the requirements for APTA listed in the Request for Proposal.

1.2 Design Objectives

The design objective is to develop and use the sizing and synthesis tool to perform conceptual design on an advanced pilot training aircraft that meets all the requirements specified on the Request for Proposal (RFP).

1.3 Benchmark Aircraft

Data from the North America F-86L Sabre aircraft was used for the benchmarking exercise. Inside the project description, the benchmark vehicle mission, miscellaneous requirements, and the engine performance were given and are enumerated below. Results from the aircraft sizing tool were compared against real world data for the benchmark aircraft. Further detail of the results are provided in section 4.5

Benchmark Vehicle Mission Requirements

Details of the benchmark vehicle mission are as follows:

1. Take-off and clear a 50 ft. obstacle in less than 4400 ft. (sea level, 90 degree day) at maximum power (with afterburners). A maximum rate of climb of 90 ft/sec should be assumed.
2. Achieve 1200 ft/sec air speed at sea level using maximum power (with afterburners)
3. Climb to a cruise altitude of 35,400 ft. under full military power (without afterburners)
4. Perform a cruise climb from 35,400 ft to 38,700 ft for 550 nautical miles. Use a cruising speed of 458 knots and normal power. Integrate the fuel burn from first principles (do not use the Breguet range equation).
5. Search (Loiter) at 38,700 ft at normal power for 10 minutes.
6. Climb to 47,550 ft

7. Combat at 47,550 ft at maximum power (with afterburners) for 5 minutes. A combat speed of 536 knots should be achievable at this altitude.
8. Cruise at 37,000 ft for 550 nautical miles at a speed of 458 knots under normal power.
9. Loiter at 35,000 ft for 10 minutes under maximum endurance conditions.
10. Land in less than 5,000ft (sea level, 90 degree day) without high lift devices.

Additional requirements included:

1. Land with a 10 percent fuel reserve
2. Carry 1 crew member (pilot) with gear, totaling 210 lbs
3. Carry 432 lbs of payload

1.4 Advanced Pilot Training Aircraft

Since this advanced pilot training aircraft is supposed to support pilot training in the next 10 years, as a result current technologies can be used. For example, this aircraft will utilize an existing, commercial off-the-shelf engine, and advancements in aerodynamics will allow the aircraft to have a lower drag coefficient.

A pilot training aircraft needs to carry a crew of two. The pilots and their personal equipment add up to 550 lbs. Conformal fuel tanks can be added if necessary.

Pilot Training Mission Requirements

Details of the pilot training mission is shown below:

1. Fuel allowance for start (35 lb/engine), warm-up/taxi (25 lb/min/engine – plan on 30 minutes ground time), mil-power run-up (85 lb/engine)
2. Take-off and acceleration allowance (computed at sea level, 59 deg F). Fuel to accelerate to climb speed at take- off thrust
3. Climb from sea level to optimum cruise altitude
4. Cruise out 150 nm at best cruise Mach and Best cruise Altitude (BCM/BCA)
5. Tanker rendezvous 100 nm at 300 knots indicated airspeed (KIAS) at 20,000 ft MSL
6. Simulated or actual air refueling (full mechanical hookup required but fuel transfer optional, depending on whether aircraft is designed with a full air refueling system or not) 20 minutes at 250 KIAS at 20,000 ft MSL
7. Climb from 20,000 ft MSL to BCM/BCA

8. Cruise to practice area 100 nm at BCM/BCA
9. Descend to 15,000 ft MSL
10. Air combat maneuvering training: Fuel required to maneuver for 20 minutes at 8-9 gs at 15,000 ft
11. Descend / climb to optimum cruise altitude
12. Cruise back 150 nm at BCM/BCA
13. Descend to sea level (distance credit allowed)
14. Reserves: fuel for 30 minutes at 10,000 feet and speed for maximum endurance

Minimum Performance Requirements

There are some additional requirements listed in the RFP. The requirement thresholds and requirement objectives are shown in the table below. Thresholds correspond to the minimum requirements, and objectives are the desired requirements for the aircraft.

Criteria	Requirement Threshold	Requirement Objective
Sustained g at 15,000 ft MSL	8	9
Ceiling	40,000 ft	50,000 ft
Minimum Runway Length	8,000 ft	6,000 ft
Payload (Expendable)	500 lbs	1,000 lbs
Range (Unrefueled)	1,000 nmi	1,500 nmi
Cruise Speed	0.7M	0.8M
Dash Speed	0.95M	1.2M

Table 1: Minimum Performance Requirements for APTA

2 Conceptual Design Methodology

2.1 Assumptions

For conceptual design, the aircraft is treated as a point mass. No higher order or high fidelity analyses were used during this phase of design. Gravitational acceleration is assumed to be constant because it's variation due to the change in altitude is negligible.

2.2 Models

Atmospheric Model

The atmospheric model is obtained from White's textbook[3]. From mean sea level to roughly 36,000 ft is known as the troposphere. In this layer of the atmosphere, the temperature decreases linearly with increasing altitude. From 36,000 ft to 66,000 ft is the stratosphere, where the temperature is roughly constant throughout. However, pressure decreases exponentially as altitude increases. A typical airplane usually operates at an altitude below 60,000 ft. Equations for this atmospheric model provided in White's textbook[3] is shown below:

$$T \approx T_0 - Bh \tag{1}$$

$$p = p_a \left(1 - \frac{Bh}{T_0}\right)^{g/(RB)} \tag{2}$$

$$\rho = \rho_o \left(1 - \frac{Bh}{T_o}\right)^{\frac{g}{RB}-1} \tag{3}$$

where

$$\frac{g}{RB} = 5.26 \quad B = 0.003566°\mathrm{R/ft} \tag{4}$$

and

$$T_0 = 518.69°\mathrm{R} \quad P_a = 2116.8\mathrm{lbf/ft^2} \quad \rho_a = 0.002377\mathrm{slugs/ft^3} \tag{5}$$

Knowing temperature, the speed of sound at a specific altitude can also be found:

$$a = \sqrt{\gamma RT} \tag{6}$$

where

$$\gamma = 1.4 \quad R = 1716.49\mathrm{ft \cdot lbf/(slug \cdot ^\circ R)} \tag{7}$$

Empty Aircraft Weight Model

The empty aircraft weight fraction can be estimated from curve fits based on historic data. According to Mattingly[1], for a fighter aircraft

$$\Gamma = 2.34 W_{TO}^{-0.13} \tag{8}$$

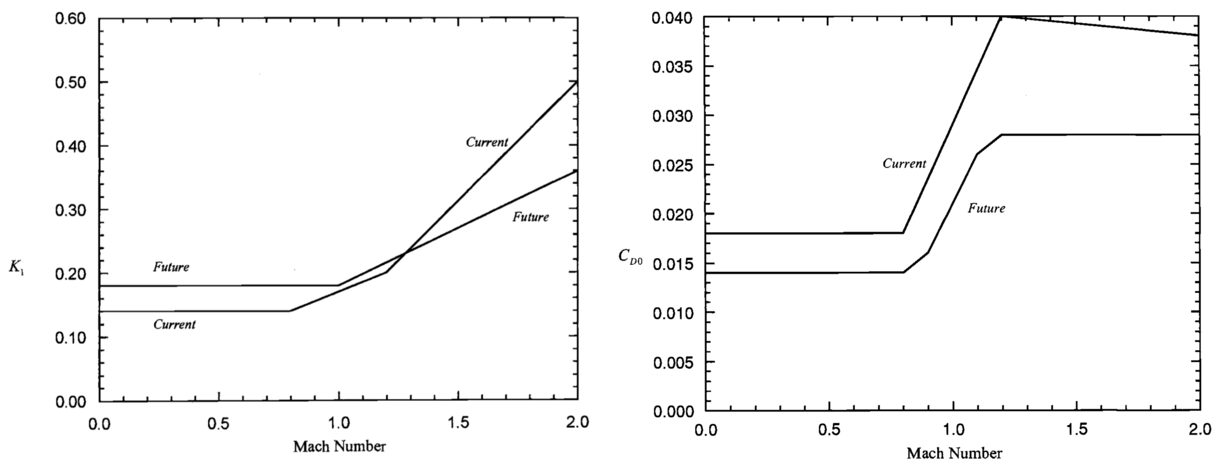
where $\Gamma = {}^{W_E}/_{W_{TO}}$ is the ratio of empty weight W_E to take off weight W_{TO}.

Drag Model

A lift-drag polar is typically used to measure the aerodynamic performance of an aircraft. It relates the coefficient of drag, C_D to the coefficient of lift, C_L. The general form of equation is parabolic, given below:

$$C_D = C_{D_0} + K_1 {C_L}^2 + K_2 C_L \tag{9}$$

According to Mattingly[1], the lift drag polar for a fighter type aircraft can be estimated using figure 1 with $K_2 = 0$. K_1 affects lift induced drag because this coefficient is in front of ${C_L}^2$. C_{D_0} is the coefficient of drag at zero lift. Relations for K_1 and C_{D_0} as a function of Mach number are shown in the figures below:



(a) K_1 vs. Mach Number (b) C_{D_0} vs. Mach Number

Figure 1: Drag Polar Constants vs. Mach Number[1]

It should be noted that two different relations exist for both of the constants. One shows the trend based on current technology, and the other one is based on future technology.

Thrust Specific Fuel Consumption (TSFC) Model

TSFC is a parameter that relates the fuel consumption of the jet engine to its thrust. However, other variables can also affect TSFC, such as the temperature ratio θ, which is a function of altitude, and the Mach number, which depends on flight speed.

For a turbojet with afterburner (AB), there are a few different operation modes: First one is max power, where the AB is on. Under this mode, the engine consumes the most fuel and produces the most thrust. The second one is military power. Under military power, the thrust and TSFC drops back significantly from max power. There are two additional models that characterize TSFC under cruise and loiter conditions.

According to Mattingly[1], TSFC for turbojet under military power and maximum power can be modeled using equations 10 and 11. Equations 12 and 13 model the TSFC of turbojet engines during cruise and loiter, respectively. They can be found in Raymer's book[4]. All four equations are shown below:

$$TSFC_{\text{Mil}} = (1.1 + 0.30M)\sqrt{\theta} \tag{10}$$

$$TSFC_{\text{Max}} = (1.5 + 0.23M)\sqrt{\theta} \tag{11}$$

$$TSFC_{\text{Cruise}} = 0.9\sqrt{\theta} \tag{12}$$

$$TSFC_{\text{Loiter}} = 0.8\sqrt{\theta} \tag{13}$$

TSFC is commonly given in the unit of $\frac{\text{lbf/hr}}{\text{lbf}} = \text{hr}^{-1}$.

Thrust Lapse Model

The concept behind thrust lapse is that the thrust output of the engine varies depending on the Mach number of free-stream air as well as air density. The maximum thrust produced by the engine at sea level is used as a reference value.

$$T = \alpha T_{SL} \tag{14}$$

A model for the thrust lapse of afterburning turbojet is provided in the lecture slides[2]:

$$\alpha_{\text{Mil}} = 0.76(0.907 + 0.262(|M - 0.5|)^{1.5})\sigma^{0.7} \tag{15}$$

$$\alpha_{\text{Max}} = (0.952 + 0.3(M - 0.4)^2)\sigma^{0.7} \tag{16}$$

In this model M is the mach number and σ is the density ratio.

Airspeed Model

The pilot training mission requirement specified the indicated air speed (IAS) during rendezvous and simulated air refueling. However, indicated airspeed is not necessarily the same as the true air speed (TAS), which is the actual speed of the airplane relative to incoming air. The true air speed is used for calculating drag and lift.

In order to find the true air speed (TAS), it is assumed that the air speed indicator is calibrated and has no error. Based on this assumption, indicated air speed is the same as calibrated air speed (CAS), IAS = CAS. However, IAS and CAS are both based on air properties at standard sea level pressure and temperature. Since 300 knots at 20000 ft corresponds to a relatively low Mach number, compressibility effects are neglected, which means calibrated air speed equals to equivalent air speed (EAS), CAS = EAS. Finally, there exists a simple relationship that can be used to apply density correction:

$$\text{TAS} = \text{EAS}\sqrt{\frac{\rho_0}{\rho}} = \text{IAS}\sqrt{\frac{\rho_0}{\rho}} \tag{17}$$

2.3 Constraint Analysis

Constraint analysis in preliminary aircraft design refers to the use of main aircraft mission profile to find the minimum thrust-to-weight ratio required for that mission ($^{T_{SL}}/_{W_{TO}}$) and the optimal wing loading ($^{W_{TO}}/_{S}$). The thrust-to-weight ratio is defined as the ratio of the aircrafts sea-level static thrust T_{SL} to its maximum takeoff weight W_{TO}. Wing loading is defined as the ratio of the aircraft's maximum takeoff weight to its wing surface area (S). By employing the functional relationship between $^{T_{SL}}/_{W_{TO}}$ and $^{W_{TO}}/_{S}$, a solution space can be derived among specified performance constraints. For example, we shown in Figure 2, performance constraints for required speed, landing, takeoff, turn are used to converge onto a solution space that satisfies these particular aircraft requirements. Locating the minimum $^{T_{SL}}/_{W_{TO}}$ and corresponding $^{W_{TO}}/_{S}$ relationship within the bounds of the solution space establishes an optimum preliminary sizing of the relationship among aircrafts take off weight, required thrust and wing area.

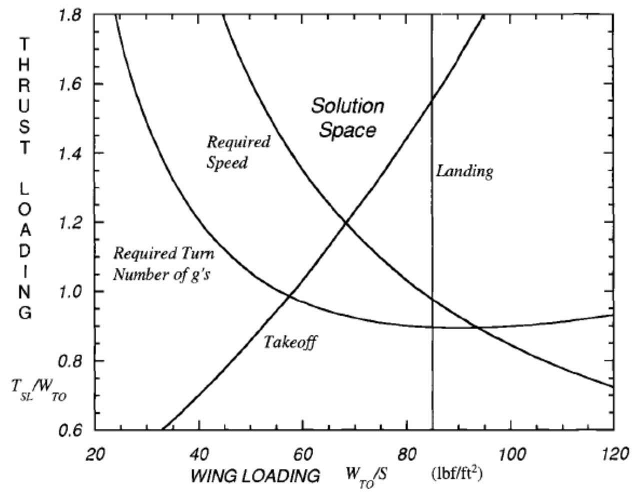


Figure 2: Thrust Loading vs. Wing Loading [1]

Constraint Equation

There exists an equation that describes the relationship between thrust-to-weight ratio and wing loading. Most of the constraints plotted on the constraint analysis plot are derived based on this equation. As a result, it is typically referred to as the "master equation", shown in Eq.19. This equation is ultimately derived based on the energy balance of the aircraft. The derivation involves the four fundamental forces of flight: thrust, drag, lift and weight (as depicted in Figure 3). Furthermore, lift can be related to weight and drag. The relationship between lift coefficient and drag coefficient is known as the drag polar (shown in Eq.9), and the equation that associates lift and weight is follows:

$$L = nW = qSC_L = \frac{1}{2}\rho V^2 SC_L \tag{18}$$

The actual derivation of Eq.19 will not be provided herein for brevity, but can be referred to in Mattingly[1] for further clarity.

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right) + C_{D_0} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)\right\} \tag{19}$$

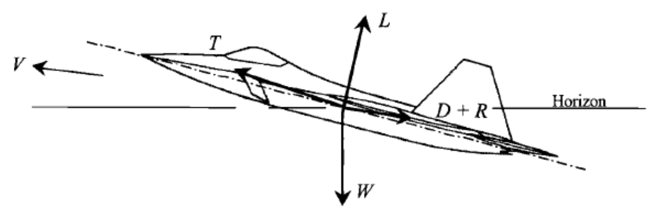


Figure 3: Fundamental Forces of Flight [1]

In the equation above, β represents the ratio of the aircraft weight at some instance during the mission to the take of weight of the aircraft. To compute the thrust-to-weight ratio vs. wing loading relation for a particular mission phase, specific flight requirement and β of that phase needs to be given. The relationships for main flight phases have been outlined below with equations 20 through 26, and they are used to carry out the constraint analysis for both the benchmark and APTA aircraft. It should be noted that for several of the flight segments of interest, i.e. refueling, reserve (max endurance cruise), loiter and cruise, the master equations are identical with the exception of the adjusted α and β values. To avoid redundancies, only a single energy equation for the redundant segments are included.

Note that for each flight phase K_2 and C_{DR} are assumed to be zero, and the respective β values are updated using the numbers computed during mission analysis described in section 2.4.

Takeoff Acceleration

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta^2}{\alpha}\frac{{k_{TO}}^2}{S_G \rho g_0 C_{L\max}}\left(\frac{W_{TO}}{S}\right) \tag{20}$$

where

$$S_G = \frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right)\frac{{V_{TO}}^2}{2g_0} \tag{21}$$

and $k_{TO} = 1.2$ which is the ratio of takeoff velocity and stall velocity (V_{TO}/V_{STALL}).

Accelerating Climb

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\beta}{q}\frac{W_{TO}}{S}\right)^2 + C_{D_0}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)\right\} \tag{22}$$

Cruise

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_0}}{\beta/q\left(W_{TO}/S\right)}\right\} \tag{23}$$

Cruise Climb

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_0}}{\beta/q\left(W_{TO}/S\right)} + \frac{1}{V}\frac{dh}{dt}\right\} \tag{24}$$

Combat

For combat, the load factor value n needs to be specified.

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + C_{D_0}\right] + \frac{1}{g_o}\left(\frac{dV^2}{dt}\right)\right\} \tag{25}$$

Landing

$$\left(\frac{W_{TO}}{S}\right) = \left\{\frac{-b + \sqrt{b^2 + 4ac}}{2a}\right\}^2 \tag{26}$$

where

$$a = \frac{\beta}{\rho g_0 \xi_L}\ln\left\{1 + \xi_L/\left[\left(\mu_B + \frac{(-\alpha)}{\beta}\frac{T_{SL}}{W_{TO}}\right)\frac{C_{L\max}}{k_{TD}^2}\right]\right\} \tag{27}$$

$$\xi_L = C_D + C_{DR} - \mu_B C_L \tag{28}$$

$$b = t_{FR}k_{TD}\sqrt{2\beta/\left(\rho C_{L\max}\right)} \quad c = S_L \tag{29}$$

finally

$$S_L = S_{FR} + S_B \tag{30}$$

The term -α represents reverse thrust. However, this was not applicable to our design so it was set to 0. The term μ_β was set to 0.40 for the coefficient of friction between the runway and aircraft tires during breaking at a nominal 120-160 kts according to FAA AIM [5]. S_L is the total runway length, S_{FR} is the free roll distance after touchdown and S_B is the braking distance.

2.4 Mission Analysis

Equations for mission analysis mainly provide the relationship between the weight after and before a mission segment. The final to initial weight ratio can be calculated by multiplying all the weight fractions:

$$\beta_f = \prod_{i=1}^{n} \beta_i = \frac{W_n}{W_0} = \frac{W_1}{W_0}\frac{W_2}{W_1}\frac{W_3}{W_2}\cdots\frac{W_n}{W_{n-1}} \tag{31}$$

where n is the total number of phases in a given mission. Below are the equations for each type of mission phase.

Prior to Take Off

The prior to take off stage considers fuel consumption for engine start, warm up/taxing and military power run up.

$$\frac{W_f}{W_i} = \frac{W_i - W_{\text{start}} - W_{\text{warmup}} - W_{\text{runup}}}{W_i} \tag{32}$$

Takeoff Acceleration

For take-off acceleration:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{g_o}\left[\frac{V_{TO}}{1-u}\right]\right\} \tag{33}$$

where

$$u = \left[\xi_{TO}\left(\frac{q}{\beta}\right)\left(\frac{S}{W_{TO}}\right) + \mu_{TO}\right]\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right) \tag{34}$$

$$\xi_{TO} = (C_D + C_{DR} - \mu_{TO}C_L) \tag{35}$$

According to Mattingly[1], $C_{DR} = 0$ can be taken as a rough estimate. The value for $\mu_{TO} = 0.05$ was set according to Dr.Mavris' lecture slides[2].

Take-off Rotation

For take-off rotation, the equation is given by:

$$\frac{W_f}{W_i} = 1 - C\sqrt{\theta}\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)t_R \tag{36}$$

where t_R is the rotation time in seconds.

Accelerating Climb

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta\left(h + V^2/(2g_0)\right)}{1 - \left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)}\right]\right\} \tag{37}$$

For this case the excess power P_s is given by

$$P_s = \frac{d}{dt}\left(\frac{h + V^2}{2g_0}\right) \tag{38}$$

The fraction of engine thrust dissipated is: $u = (C_D/C_L)(\beta/\alpha)(W_{TO}/T_{SL})$

Constant Speed Climb

The equation is given by:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta h}{1 - \left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)}\right]\right\} \tag{39}$$

Where the excess power P_s is given by

$$P_s = \frac{dh}{dt} \tag{40}$$

Constant Altitude Acceleration

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta\left(V^2/(2g_0)\right)}{1 - \left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)}\right]\right\} \tag{41}$$

Where the excess power P_s is given by

$$P_s = \frac{d}{dt}\left(\frac{V^2}{2g_0}\right) \tag{42}$$

Cruise

$$\frac{W_f}{W_i} = \exp\left[-\frac{C\sqrt{\theta}}{V}\left(\frac{C_D}{C_L}\right)\Delta s\right] \tag{43}$$

The excess power $P_s = 0$ for this case.

Loiter

During loiter, the maximum $\frac{C_L}{C_D}$ should be achieved for lowest fuel consumption. It can be shown that this optimal lift to drag ratio can be achieved when $\frac{C_D}{C_L} = \sqrt{4C_{D_0}K1} + K2$ As a result, the weight equation for loiter is:

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\sqrt{4C_{D_0}K_1} + K_2\right)\Delta t\right\} \tag{44}$$

The excess power $P_s = 0$ for loiter as well.

Best Cruise Altitude (BCA) and Best Cruise Mach Number (BCM)

The pressure ratio condition for BCA can be found by using the equation shown below:

$$\delta_{\text{BCA}} = \frac{2\beta}{\gamma P_{SL} M^2} \frac{1}{\sqrt{C_{D_0}/K_1}} \left(\frac{W_{TO}}{S}\right) \tag{45}$$

This relation shows that the pressure ratio (δ) changes depending on the Mach number and instantaneous weight of the plane. As a result, by the time the aircraft reaches its optimal cruise altitude based on its weight before the climb, the optimal altitude would have slightly increased. As a result, during mission analysis, another cruise climb is performed to achieve a more optimal altitude.

The best cruise Mach number (BCM) is obtained by observing graph 1b and 1a. It can be seen that once Mach number goes beyond 0.8 into the transonic region, the drag increases significantly. In order to cruise as fast as possible without sacrificing efficiency, BCM is chosen to be 0.8.

Maximum Endurance

The maximum endurance condition for a jet aircraft is achieved by maximizing L/D, or C_L/C_D. It can be shown that

$$C_L = \sqrt{\frac{C_{D_0}}{K_1}} \quad \text{and} \quad C_D = 2C_{D_0} \tag{46}$$

Then, the cruise speed for max endurance is given by:

$$V_{\text{MaxEnduro}} = \sqrt{\frac{2}{\rho}\frac{\beta W_{TO}}{S}\sqrt{\frac{K_1}{C_{D_0}}}} \tag{47}$$

These parameters can be plug into equation 43 to find the weight fraction for the reserve section.

2.5 Other Requirements for APTA

Maximum Range

For turbojet aircraft, the condition for maximizing range is to maximize $\frac{C_L^{\frac{1}{2}}}{C_D}$. Taking its derivative and equate it to zero, it can be shown that

$$C_{D_0} = 3K_1 {C_L}^2 \tag{48}$$

Under this condition, V is give by

$$V_{\text{MaxRange}} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{3K_1}{C_{D_0}}}} \tag{49}$$

Note that this optimal cruise speed depends on ρ, which in turn depends on the altitude. Finally, the range R can be obtained using the following relation:

$$R = \frac{2}{TSFC}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\left(\sqrt{W_{ini}} - \sqrt{W_{fin}}\right) \tag{50}$$

V_{MaxRange} is based on a different criteria than BCM. Since fuel would be needed for take off and accelerating climb, W_{ini} should be the aircraft weight at the end of the accelerating climb, given by the corresponding β value. For example, $W_{ini} = \beta_{ini} W_{TO}$.

Dash Speed

The dash speed of the aircraft is also known as the top speed. To apply this constraint, a top speed is first chosen prior to aircraft sizing. Then, this top speed requirement is imposed during constraint analysis.

Service Ceiling

The service ceiling can be found using the rate of climb (R/C) equation provided in Anderson's book[6].

$$\text{R/C} = V\left[\frac{T}{W} - \frac{1}{2}\rho V^2 \left(\frac{W}{S}\right)^{-1} C_{D_0} - \frac{W}{S}\frac{2K_1}{\rho V^2}\right] \tag{51}$$

Taking thrust lapse and weight fraction into consideration, the equation becomes

$$\text{R/C} = V\left[\frac{\alpha T_{SL}}{\beta W_{TO}} - \frac{1}{2}\rho V^2 \left(\frac{\beta W_{TO}}{S}\right)^{-1} C_{D_0} - \frac{\beta W_{TO}}{S}\frac{2K_1}{\rho V^2}\right] \tag{52}$$

At different altitudes, both the thrust lapse and density changes. So this rate of climb is evaluated for a range of altitudes. The definition used for service ceiling in this project is the altitude when R/C drops below 500 ft/min. β is set to be 0.9 to account for a realistic worst-case scenario.

3 Aircraft Sizing Tool

3.1 Algorithm and Data Flow

The detailed procedure for aircraft sizing is represented by the figure below:

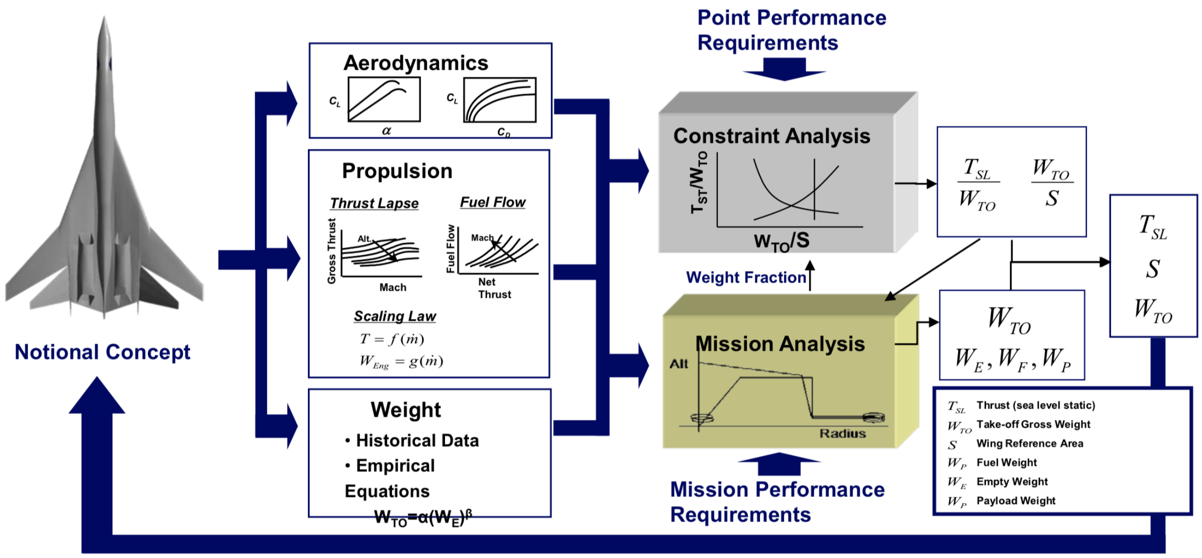


Figure 4: Aircraft Sizing Framework [2]

The first step is to establish the models for atmospheric condition, aerodynamics, thrust lapse, TSFC, and empty weight. Using the models, we can perform constraint analysis and mission analysis.

To start the iteration process, first we need an initial guess for β, which is the weight fraction for each segment of the mission. This array of β values is needed because each constraint should be evaluated based on the weight during that specific segment of the mission. The design point can be chosen from constraint analysis, which gives the thrust to weight ratio T_{SL}/W_{TO} and wing loading W_{TO}/S. In the sizing tool, a 5% margin is added to the thrust to weight ratio to raise the design point slightly above the minimum thrust loading required.

With the design point chosen, mission analysis can then be performed. After going through the mission analysis, the array of β values are updated to give a more accurate representation of fuel usage throughout the mission. These updated β values are then fed back into constraint analysis until the values converge.

The next step is the iteration for take off gross weight (W_{TO}). The procedure for W_{TO} is illustrated more clearly in this diagram shown below:

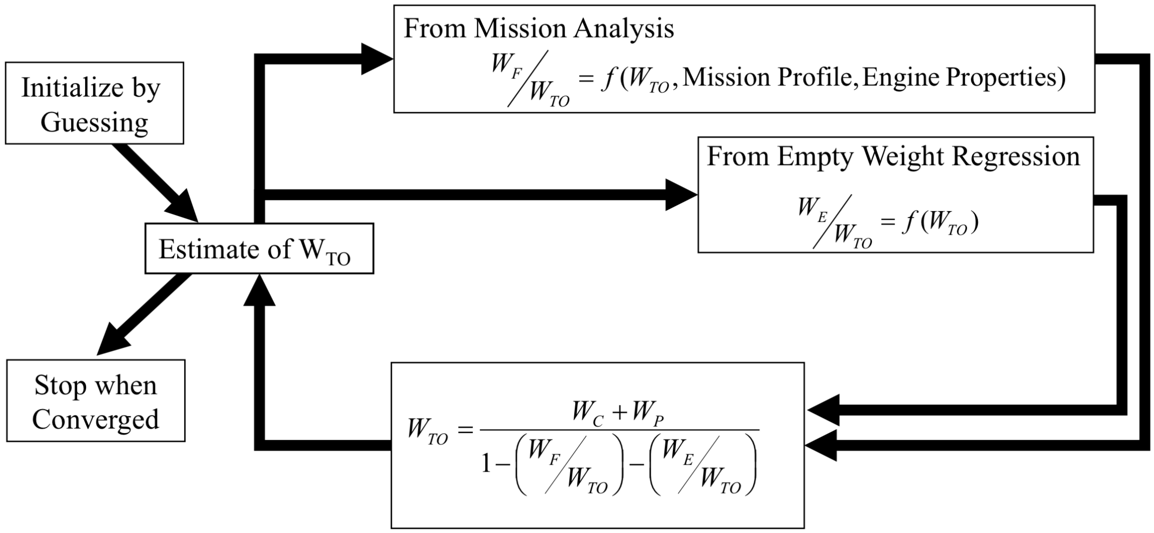


Figure 5: Aircraft Sizing Process [2]

In Figure 5, W_F is the fuel weight, W_E is the empty weight, W_C is crew weight and W_P is payload weight. W_C and W_P are typically specified in the Request for Proposal.

With converged values for β, we can use the β value at the end of the mission to estimate the fuel weight fraction (W_F/W_{TO}) required to complete the mission. Along with the empty weight model based on historical data, an estimate for a new W_{TO} can be made. To make sure that the new value for W_{TO} does not overshoot, it is updated as follows:

$$W_{TO}{}^{(i+1)} = \frac{W_{TO}{}^{(i)} + W_{TO_{\text{est}}}{}^{(i)}}{2} \tag{53}$$

where the superscript i represents the iteration count.

The fuel weight fraction is fairly independent of the actual take off weight, unless there are requirements in the mission analysis that are given based on actual weight values (such as the phase before take off for the pilot training aircraft).

3.2 Details Regarding Constraint and Mission Analyses

Constraint Analysis

During each phase of the mission, the values for parameters such as thrust lapse, weight fraction, and velocity can all vary. This is especially true for the accelerating climb phase right after take off. The general trend for aircraft such as the F-86L is that its performance parameters (such as lift to drag ratio, thrust level) drops as it flies faster and higher. This trend is most likely not true for aircraft optimized for supersonic flight. Because of this reason, for accelerating climb, the constraint curve is plotted based on the last 10% change in altitude and speed of that phase.

Mission Analysis

Again, due to the fact that some phases of the mission involve large change in operational condition such as speed and altitude, or long flight distance, the mission analysis for each phase is broken down into smaller segments. This default number of segments is set to 10. This will result in more accurate weight estimate.

3.3 Graphic User Interface (GUI) and Output File

A basic graphical user interface (GUI) was developed per the assignment requirements enabling users to select between the benchmark aircraft and the APTA by toggling between the options.

Executing the Matlab tool simply requires the user to open **AST.mlapp** from Matlab to activate the GUI. Once the GUI is open, depressing of the "Compute" button will run the program. A tabulated text file with pertinent performance is generated along with graphical outputs of the mission constraint analysis and the optimal thrust-to-weight ratio and wing loading loci. Both the APTA and F86L output text files can be found in Appendix A. The converged graphical constraint analysis plots are provided within the Results sections below.

One can open the **designParametersAPTA.m** to change the mission profile or aircraft parameters. A **ReadMe.txt** is included in the folder to provide some basic instructions.

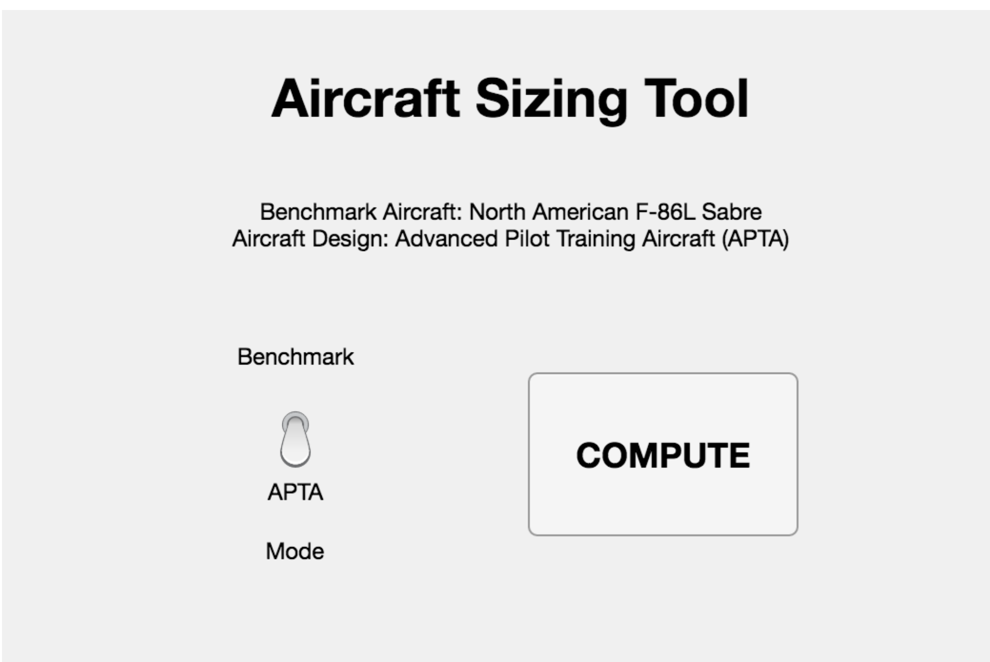


Figure 6: Aircraft Sizing Tool GUI

4 Benchmark Aircraft

4.1 Constraint Analysis

Aerodynamic Constants

The lift coefficient during take off is estimated based on the aircraft's aerodynamic design and historic data. F-86L's wing has leading edge flap and single trailing edge flap. According to the range of values given in the slides [2], $C_{L_{\text{Max}}}$ was chosen to be 1.1.

During landing, a maximum runway length limit is imposed for landing without high lift devices. It is really difficult to give an estimate for the maximum lift coefficient achievable without high lift devices using formulas. Luckily, aerodynamic data for F-86E (another variant for the F-86 series) was found on the Defense Technical Information Center's website [7]. According to the data, $C_{L_{\text{MaxNoHL}}} \approx 0.85$.

Load Factor for Combat

During phase 7, combat takes place at 47550 ft for 5 minutes. The load factor is set to be 1.4 based on the figure on the last page of the Standard Aircraft Characteristics manual[8] for the F-86L.

Top Speed

Even though the maximum speed at sea level phase was removed in mission analysis, this constraint stays in constraint analysis. It was found that even a top speed of 602 knots was constraining the design space too much. This is most likely due to an over-estimate of drag at the transonic region. In order to achieve a more reasonable design point, the top speed requirement is lowered to 550 knots.

The top speed is evaluated at an altitude of 200 ft with a weight fraction $\beta = 0.98$ for the worst case scenario (minimal fuel consumed before take off, and then accelerate to the top speed right after take off).

Landing Distance

Since the constraint requires landing to be achieved in less than 5000 ft, a runway length of 3000 ft was imposed while calculating the landing distance constraint.

4.2 Mission Analysis

Mission Phase Change

While performing the missions analysis for F-86L, an adjustment was made to the second phase (to achieve 1200 ft/s air speed at sea level using maximum power). Looking at actual aircraft data for the F-86L, it can only achieve a maximum speed of 602 knots at sea level, which equates to 1016 ft/s. Since this phase is a relatively unreasonable mission requirement, it was removed from the mission analysis. Instead, a maximum speed constraint was imposed during constraint analysis. An accelerating climb phase replaces this requirement. The airplane climbs from 50 ft to 35,400 ft, while accelerating from V_{TO} (around 250 ft/s) to $V_{\text{cruise}} = 458\,\text{knots} = 773\,\text{ft/s}$. From a mission analysis standpoint, the total fuel consumed to first horizontally accelerate then perform constant speed climb should be relatively similar to an accelerating climb.

Cruise Climb with Range Requirement

In phase 4, the requirement asked for cruise climb from 35,400 ft to 38,700 ft for 550 nautical miles. Because equation (39) for cruise climb (constant speed climb) only consider the fuel required to perform the climb, it does not take the distance that aircraft needs to travel into consideration. To perform mission analysis for this step, the procedure is broken into two steps. First, the weight fraction for constant speed climb from 35,400 ft to 38,700 ft was calculated, then a second weight fraction for cruise distance of 550 nautical miles was found. Multiplying the two weight fractions, the result would give the amount of fuel needed to perform this phase of the mission.

4.3 Other Considerations

Reserve Fuel

One of the additional requirements states the aircraft must land with 10% reserve fuel. This requires a small change to the weight estimate equation. With reserve fuel, the take off weight should be calculated as follows:

$$W_{TO} = \frac{W_C + W_P + 0.1W_F}{1 - (W_F/W_{TO}) - (W_E/W_{TO})} \tag{54}$$

where the 0.1 represents the percentage of reserve fuel.

4.4 Real World Aircraft Engine and Performance Parameters

The F86L used one General Electric J47-GE-33 engine. The sea level thrust values are given in the table below:

Power Setting	**Sea Level Static Thrust (lbf)**
Maximum (with afterburner)	7650
Military (without afterburner)	5550
Normal	5100

Table 2: GE J47-GE-33 Engine Parameters

These engine parameters will be used for the comparison of computed thrust to weight ratio vs. the final engine selection to be employed on the APTA design discussed in section 5. Other characteristic parameters for the F-86L are listed below:

Parameters	**Value**	**Unit**
Take Off Weight (W_{TO})	18484	lbf
Empty Weight (W_E)	13822	lbf
Aspect Ratio (AR)	4.883	
Wing Area (S)	313.4	ft^2
Max Speed (V_{Max})	1016.1	ft/s
Take Off Field Length (TOFL)	3650	ft

Table 3: F-86L Design Parameters

The actual wing loading of the aircraft can be found by dividing the take off weight by the wing area. This will be compared to the converged design point value in the results section.

4.5 Results

Constraint Analysis Plot

The final constraint analysis plot for the F-86L is shown in Figure 7 below with the computed design point and F-86L performance denoted accordingly. The deltas between the actual F-86L performance data and the converged sizing and synthesis tool are tabulated in Table 4.

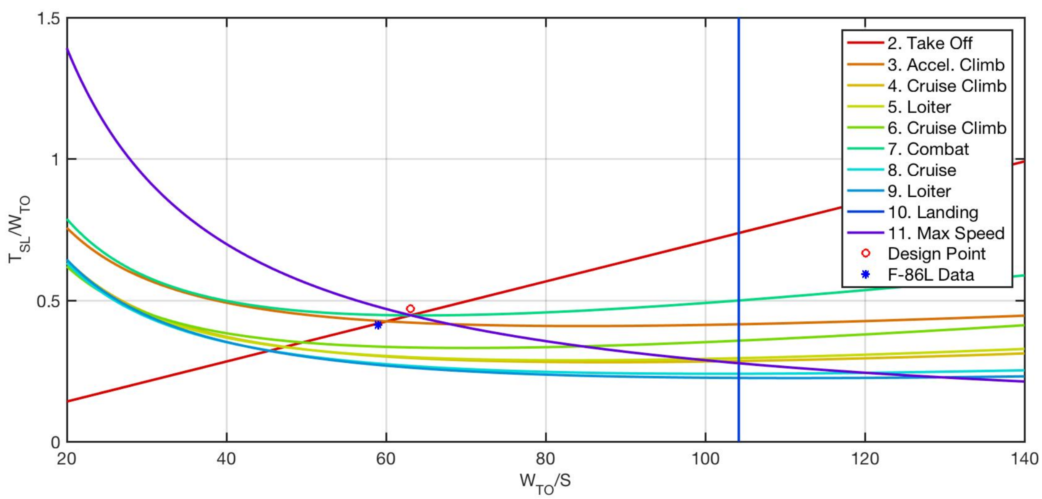


Figure 7: F-86L Constraint Analysis Plot

Mission Analysis Fuel Fractions

The final β values after each mission phase is shown in the bar graph below to aid in visualizing the decrease in the fuel weight fraction throughout the mission.

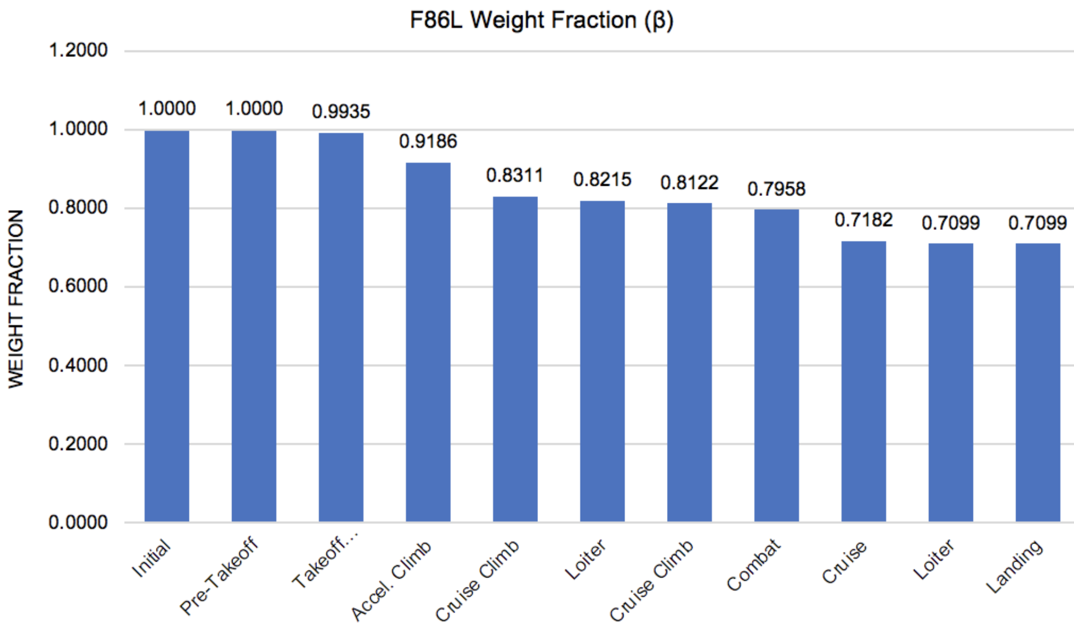


Figure 8: F-86L Fuel Weight Fractions

Take Off Weight and Design Point

The total take off weight differs by a conservative 5.4% and wing loading of 6.9%. The largest difference lies with the thrust-to-weight ratio delta of 10.4%. The differences here can be attributed to several factors including empirical rounding, various differences in initial assumptions, overly optimistic performance requirements for the F-86L as was mentioned previously in sections 4.1 and 4.2, etc. In addition, the F-86L does not exactly follow the empty weight equation based on historic data regression (Eq.8). For the scope of this report however, the fidelity of the aircraft sizing tool agrees well with baseline expectations with the two dominant constraints being take off and max speed.

Parameter	Actual Data	Converged Result	% Difference
W_{TO}	18484 lb	19489.2 lb	5.4%
T_{SL}/W_{TO}	0.4247	0.4689	10.4%
W_{TO}/S	59.0 $^{\text{lb}}/_{\text{ft}^2}$	63.1 $^{\text{lb}}/_{\text{ft}^2}$	6.9%

Table 4: F-86L vs Sizing Tool Results

5 Advanced Pilot Training Aircraft (APTA)

The pilot training mission assumes that fuel transfer does not take place, and conformal fuel tanks are not necessary. So the weight of the aircraft does not increase mid-flight, and there will be no C_{D_R} component due to conformal fuel tank.

5.1 Constraint Analysis

Load Factor for Combat

The requirement for combat asked for a minimum of sustained 8 g at 15000 ft above main sea level. However, this is a unreasonable constraint, since it is almost impossible to sustain such a high load factor due to thrust constraint. A load factor this high can possibly be achieved during a dive or some other un-sustained maneuver with the dominant constraint being imposed by the structural limit instead of thrust limit. To support this argument, it can be seen that for a fighter known for its high maneuverability such as the F-16, it can only achieve sustained loading of around 4 at 30000 ft using maximum power[9]. Hence for this advanced pilot training aircraft, the max load factor during combat is set to 3.

Top Speed

The top Mach number is set to 1.1 due to the dash speed requirement listed in the request for proposal. This constraint is based on the same condition as those used when evaluating the top speed constraint for the benchmark aircraft.

Combat Speed

Since combat speed was not specified in the RFP, a reasonable combat speed of 300 knots was assumed for the purpose of constraint and mission analysis.

5.2 Mission Analysis

For rendezvous and simulated air refueling, IAS is converted to TAS using equation 17. The fuel required for combat maneuver will be based on a load factor of 3 instead of 8 or 9.

The final phase (fuel reserve) is calculated by assuming that the aircraft decreases altitude from phase 12 to 10000 ft. The aircraft does not land and then take off to climb to 10000 ft and accelerate to $V_{\text{MaxEnduro}}$.

5.3 Maximum Range

The maximum range of the advanced pilot training aircraft is calculated based on the following mission:

1. Fuel allowance for start (35 lb/engine), warm-up/taxi (25 lb/min/engine – plan on 30 minutes ground time), mil- power run-up (85 lb/engine)

2. Take-off acceleration
3. Accelerating climb to BCA and cruise speed for best range using max power
4. Cruise climb to new BCA based on new aircraft weight using normal power
5. Fly at BCA and cruise speed for best range until fuel runs out

5.4 Results

Constraint Analysis Plot

The final constraint analysis plot for the APTA is shown in Figure 9 below with the computed design point constrained by max speed and the combat mission segments of the mission profile. As noted in section 5.2 above, the load factor was limited to 3 vs. the 8-9 specified in the requirements. This resolves the thrust-to-weight ratio of 0.674 at a wing loading of 88.90 lb/ft^2. These are reasonable values that agree well with engine selection discussed in section 5.5 and the realistic performance for a trainer aircraft with our specific RFP requirements.

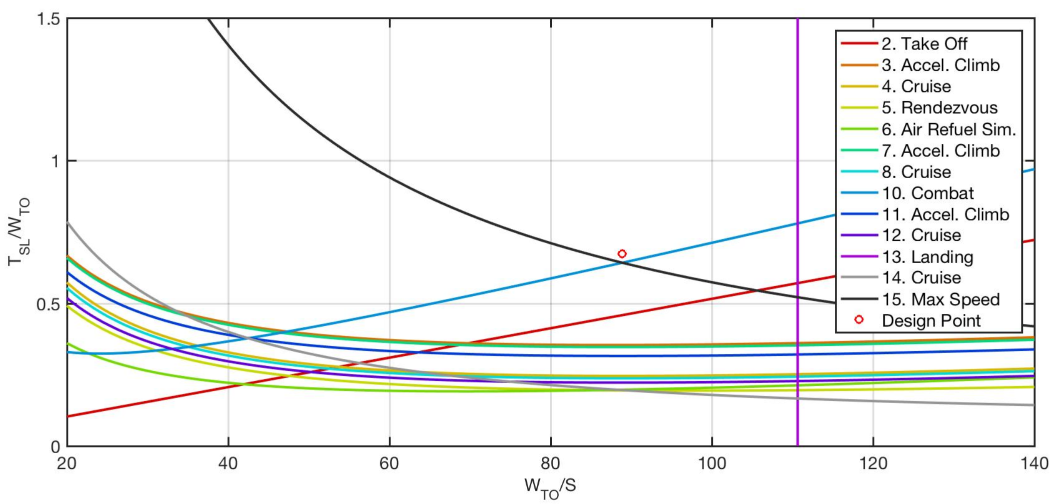


Figure 9: APTA Constraint Analysis Plot

Mission Analysis Fuel Fractions

As with the F-86L, the final β values after each mission phase are illustrated by the bar graph in Figure 10 to aid in visualizing the decrease in the fuel weight fraction throughout the mission. The weight fraction after reserve cruise is 0.6920. This value is used to find the fuel required based on mission analysis.

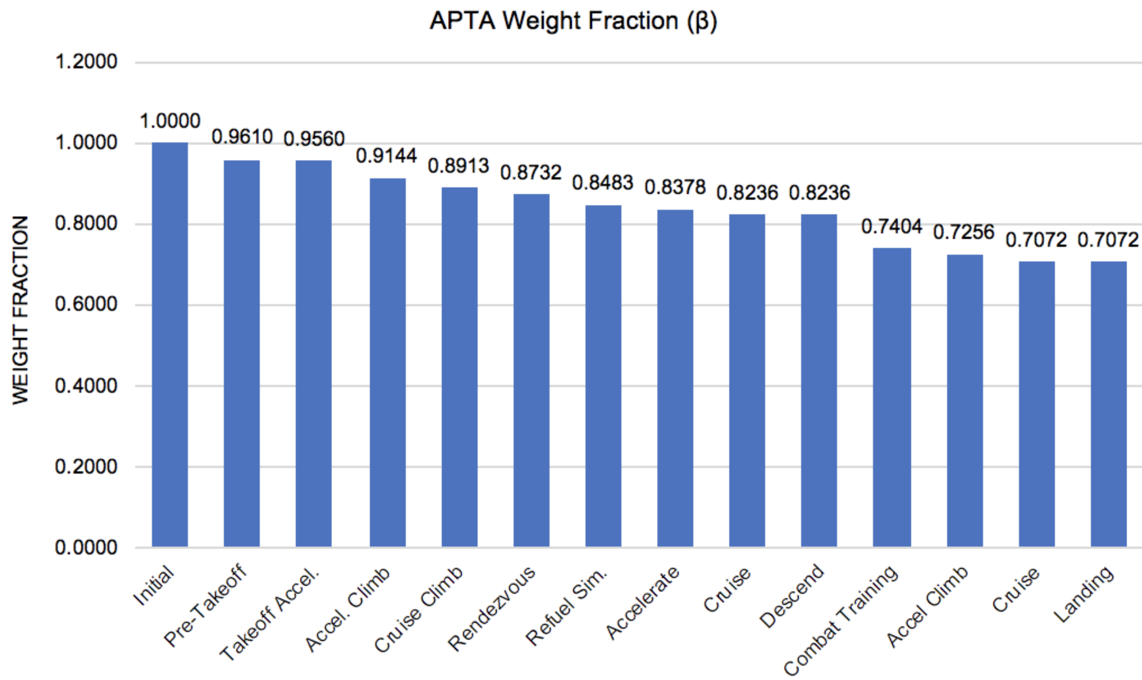


Figure 10: APTA Fuel Weight Fractions

Figure 11 has been provided to present the fuel burn percentages for each segment of the mission profile. We can clearly see that the largest expense (28%) of fuel takes place during the combat training segment followed by accelerated climb (14%) which intuitively makes sense. With the assumption that the descent segment and landing segment (without reverse thrusters) consumes no fuel they have not been included.

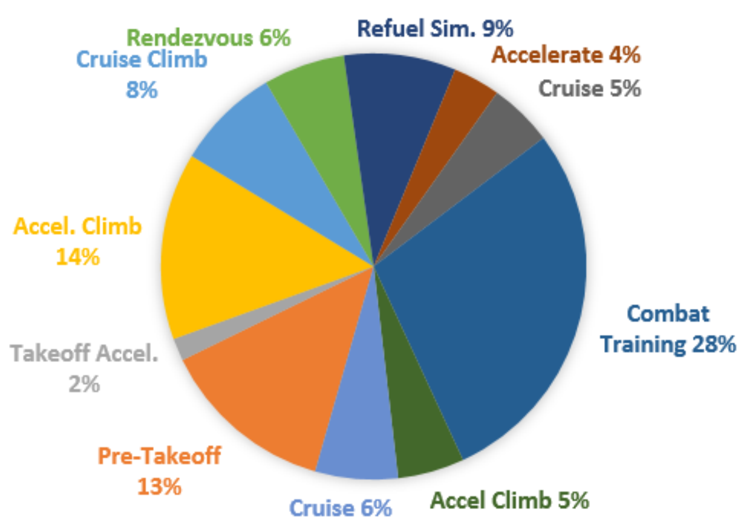


Figure 11: APTA Fuel Burn Percentages of TOGW

Weight Breakdown and Design Point

The take off weight, sea level max thrust, and wing area for the converged result are summarized in the table below:

Parameter	Value
W_{TO}	22785 lb
$T_{SL_{\text{Max}}}$	15357 lbf
S	256.3 ft^2

The weight breakdown is also represented with a pie graph, shown below:

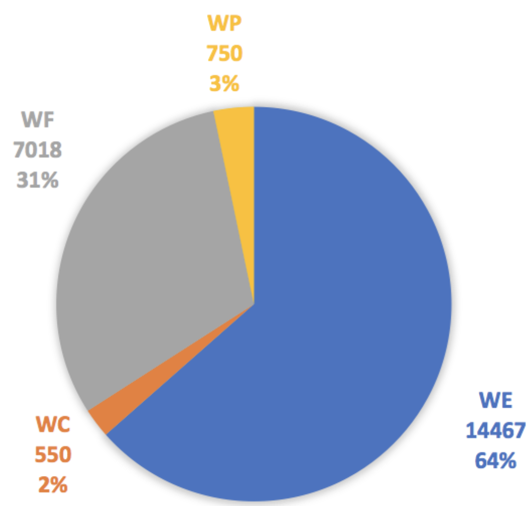


Figure 12: APTA Weight Breakdown (unit in lb)

Minimum Performance Requirements

In Table 5 we find the RFP requirement thresholds, objectives and the final results of the synthesis and sizing of the APTA. Discussed thus far in section 5.1 is the rationale for reducing the sustained loading from 8.0-9.0 down to 3.0 for structural and physiological limits of the aircraft and aircrew. With all of the remaining comparisons under consideration, it can be seen that the proposed APTA design either meets or exceeds performance expectations with greater range, ceiling and payload capacity.

Criteria	Threshold	Objective	Result
Sustained g at 15,000 ft MSL	8.0	9.0	3.0
Ceiling	40000ft	50000ft	50100ft
Minimum Runway Length	8000ft	6000ft	3000ft
Payload (Expendable)	500lb	1000lb	750lb
Range (Unrefueled)	1000nm	1500nm	1734nm
Cruise Mach Number	0.7	0.8	0.8
Dash Mach Number	0.95	1.20	1.10

Table 5: Performance Requirements and Computed Results

Engine Selection

The aircraft sizing tool gave a take off weight of 22,785 lb for the advanced pilot training aircraft. With a thrust-to-weight ratio of 0.674, maximum thrust (with afterburners) of 15,357 lbf at sea level is required to meet the mission profile defined in the RFP. With the T_{SL} now known, several dozen engines of various manufactures were evaluated. Finally, the decision was made and the General Electric F404-GE-400[10] was chosen. This engine was selected due to its particular 16,000 lbf of thrust with afterburners and 10,600 lbf of thrust without. The GE 404 is a relatively small compact turbojet at 2,282 lbs dry weight used on the F-18 Hornet derivatives. There is an excess of approximately 400 lbf thrust with the afterburner, which means that this engine is appropriate for the APTA based on the mission requirements. In addition, due to the use of modern technologies, the TSFC for this engine is also very reasonable.

6 Conclusion

6.1 Review

In review, the performance of the F-86L Sabre aircraft was examined as a benchmark for validating an aircraft sizing tool that employs both mission analysis and constraint analysis. The tool was developed in MATLAB, and proved a degree of fidelity commensurate with that of a conceptual design utility within the scope of this project.

6.2 Tool Performance

The aircraft sizing tool did have small deltas between the benchmark aircraft's known performance, but said deltas were of a small enough magnitude to be within expectations. As noted above and tabulated in Table 5 the tool sized a conceptual design of the APTA in line with the requirements of the customer RFP. The capability of the proposed APTA sizing satisfied all of the performance requirements with positive margins suggesting an aircraft that can perform to the expectations of the requirements and better.

Specifically, it can be seen that the proposed sizing provides a positive margin for the aircraft's max ceiling, payload, range and required runway length. As already discussed, we reduced the sustained loading from 8.0-9.0 down to 3.0 for structural and physiological limits of the aircraft and aircrew to meet a more realistic design space.

Overall, the development of the aircraft sizing tool produced a means to quickly and fairly accurately generate key parameters needed in the conceptual design phase of aircraft maturation.

6.3 Recommended Improvements

A couple improvements that would be prudent to consider, would include the expansion of the GUI's capability. At the moment its intended purpose is to only simplify the execution of the tool and generate graphical and numerical outputs for ease of user interfacing. Additional capability of the GUI to enable the user to select multiple airframe benchmarks or provide discrete inputs for certain sizing parameters of the proposed aircraft,etc are features that could prove useful with future versions.

Although in the opinion of this paper's authors, the aircraft sizing tool demonstrated dependable and accurate performance, a second improvement to consider for future use would of course be a degree of increased fidelity to reduce the magnitude of the deltas between benchmark and the conceptual vehicle sizing.

Bibliography

[1] Jack D. Mattingly, William H. Heiser, and David T. Pratt. *Aircraft Engine Design, Second Edition*. American Institute of Aeronautics and Astronautics, Inc., 2003.

[2] Professor Dimitri. M. Mavris. *An Energy Based Approach to Constraint Analysis*. Aerospace Systems Design Laboratory, 2018.

[3] Frank White. *Fluid Mechanics (McGraw-hill Series in Mechanical Engineering)*. McGraw-Hill Science/Engineering/Math, 2010.

[4] Daniel P. Raymer. *Aircraft Design: A Conceptual Approach (AIAA Education Series)*. American Institute of Aeronautics and Astronautics, Inc., 1992.

[5] FAA. Aeronautical information manual official guide to basic flight information and atc procedures. Technical report, Federal Aviation Administration, 2017.

[6] Anderson. *Aircraft Performance and Design*. McGraw Hill, 2010.

[7] Aerodynamics Group. Estimated aerodynamic characteristics for design of the f-86e airplane. Technical report, North American Aviation Inc., 1950.

[8] North American Aviation Inc. F-86l sabre standard aircraft characterisitics. Technical report, North American Aviation Inc., 1958.

[9] General Dynamics. F-16a standard aircraft characterisitics. Technical report, General Dynamics, 1987.

[10] Denis Frith. The general electric f404 - engine of the raaf's new fighter. Technical report, Australia Department of Defense, 1985.

Appendices

A Aircraft Sizing Tool Output .txt Files

```
Converged Results for Benchmark Aircraft (F-86L Sabre)

Design Point

TSL/WTO  : 0.469
WTO/S    : 63.10 lb/ft^2

Key Parameters

TOGW(WTO): 19489.2 lb
TSL      : 9138.8 lbf
S        : 308.9 ft^2

Mission Analysis

   Mission Phase     |   beta
---------------------+----------
 0. Initial          |  1.0000
 1. Pre Take Off     |  1.0000
 2. Take Off Accel.  |  0.9935
 3. Accel. Climb     |  0.9186
 4. Cruise Climb     |  0.8311
 5. Loiter           |  0.8215
 6. Cruise Climb     |  0.8122
 7. Combat           |  0.7958
 8. Cruise           |  0.7182
 9. Loiter           |  0.7099
10. Landing          |  0.7099

Benchmark Aircraft Comparison

   Parameter    |  Actual Data   | Converged Result  |  % Diff.
----------------+----------------+-------------------+------------
      WTO       |   18484lb      |    19489.2lb      |    5.4%
    TSL/WTO     |    0.4247      |     0.4689        |   10.4%
     WTO/S      |  59.0lb/ft^2   |   63.1lb/ft^2     |    6.9%
```

Figure 13: F86L Converged Results Output

```
Converged Results for Advanced Pilot Training Aircraft

Design Point

TSL/WTO  : 0.674
WTO/S    : 88.90 lb/ft^2

Key Parameters

TOGW(WTO): 22784.7 lb
TSL      : 15361.2 lbf
S        : 256.3 ft^2

Mission Analysis

   Mission Phase     |   beta
---------------------+----------
 0. Initial          |  1.0000
 1. Pre Take Off     |  0.9610
 2. Take Off Accel.  |  0.9560
 3. Accel. Climb     |  0.9144
 4. Cruise Climb     |  0.8913
 5. Rendezvous       |  0.8732
 6. Refueling Sim.   |  0.8483
 7. Accel. Climb     |  0.8378
 8. Cruise           |  0.8236
 9. Descend          |  0.8236
10. Combat Training  |  0.7404
11. Accel. Climb     |  0.7256
12. Cruise           |  0.7072
13. Landing          |  0.7072
14. Reserve          |  0.6920

Minimum Performance Requirements

           Criteria            | Threshold | Objective |   Result
-------------------------------+-----------+-----------+-----------
Sustained g at 15,000 ft MSL   |     8     |     9     |    3.0
Ceiling                        |  40000ft  |  50000ft  |  50100ft
Minimum Runway Length          |   8000ft  |   6000ft  |   3000ft
Payload (Expendable)           |    500lb  |   1000lb  |    750lb
Range (Unrefueled)             |   1000nm  |   1500nm  |   1734nm
Cruise Mach Number             |    0.7    |    0.8    |    0.8
Dash Mach Number               |    0.95   |    1.2    |    1.1
```

Figure 14: APTA Converged Results Output