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CSE 340 Fall 2015
HOMEWORK 5
Assigned 11/30/2015
Due 12/5/2015 by 11:59:59 pm on Blackboard
Remember that late submissions are not accepted for homework.
For all answers, show your work for partial credit.

All submissions should be typed. Exception can only be made for drawing parse trees, which can be handwritten and scanned in the submitted document.

Problem 1. Underline the free variables in each of the following lambda expressions:

1.1 λ z . (λ x . z x) (x y z)   Y is free across the whole expression, X is free in the second set of parenthesis.
1.2 λ z . λ x . (z x) (x y z)   only Y is free
1.3 λ z . (λ x . (z x) (x y)) z   Only Y is free

Problem 2. Fully parenthesize the following lambda expressions:

2.1 λ x . x y (λ y . w x (u p))   (λx.(λy.(λy. w x u p)))
2.2 (λ z . (λ y . z y z) z z) (λ x . λ y . x)   (λz. λy.z y z)(z z) (λx. Λy.x)
2.3 a b c λ x . x a b   (a b c) (λx.x a b)

Problem 3. Fully β-reduce the following expressions:

3.1 (λ x . x λ y . λ z . z y x) (y z)
(x λ.y. λz.zyx)(z) {w/y} [x-> y] (change all the y's of the original expression to w to account for the new y variable.)
(yλ.w. λz.zwy) {v/z} [w->z] (change the original z's to account for the free var z being substituted in.)
(y λv.vzy)

3.2 (λ n . λ f . λ x . f (n f x)) (λ x . λ f . x x x x f)
(λf. λ x.f(nfx)) {w/x} {g/f} [n-> λx. λf.xxxxf)]
(λg. λw.g((λx. λf.xxxxf)(gw))) [x->f], [f->w]
(λg. λ w.g(ggggw)) = λg. λw. Gggggw = 5

3.3 (λ x . x (λ x . λ y . y) (λ y . λ x . y)) (λ x . λ y . y)
F T F
(x (λ x . λ y . y) (λ y . λ x . y)) [x->(λ x . λ y . y)]
(λ x . λ y . y) (λ x . λ y . y) (λ y . λ x . y) [x-> λ x . λ y . y]
(λ y . y) (λ y . λ x . y)[y-> λ y . λ x . y]
λ y . λ x . y = T

Problem 4. Using the definitions of T and F given in class, write an "or" lambda function and prove that it is correct (fully β-reduce all four possible inputs, as we did for "and" in class).

T = λx.λy.x
F = λx. λy.y
and = (λx. λy.xyF)

or = (λx. λy.xTy)
Tests: (T F), (F T), (T T), (F F). All except the (F F) should be equal to T. These four are the only inputs that are possible.

or T F: (λx. λy.xTy) T F
    (λy.xTy) (F) [x ->T]
    (TTy) [y->F]
    TTF = T
or F T : (λx. λy.xTy) F T
    (λy.xTy) (T) [x->F]
    (FTy) [y->T]
    (FTT) = T
or T T: (λx. λy.xTy) T T
    (λy.xTy) (T) [x->T]
    (TTy) [y->T]
    TTT = T
or F F: (λx. λy.xTy) F F
    (λy.xTy) (F) [x->F]
    (FTy) [y->F]
    FTF = F
With these 4 possible outputs being correct, or = (λx. λy.xTy) is correct.