2. LCBC

The largest interaction distance between the two species

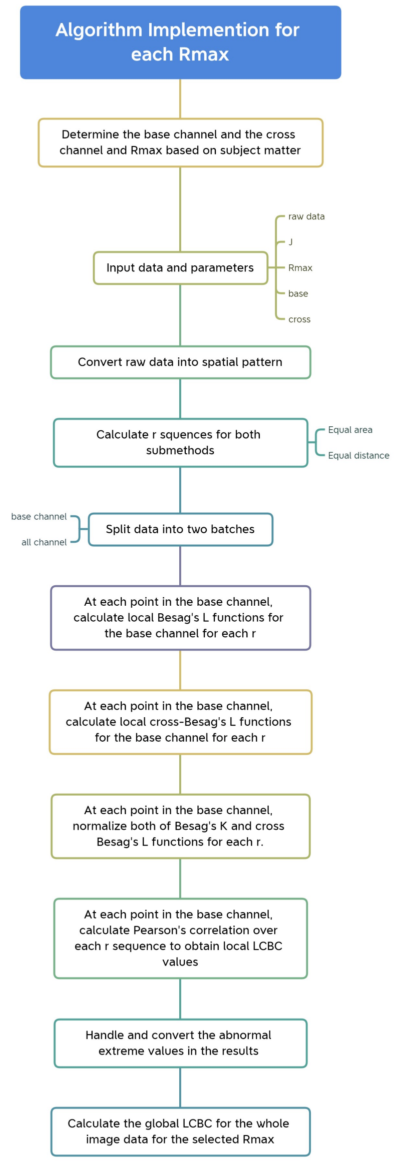
KCBC

Should not exceed a quarter of the lengthof the shortest side of the observation window

$$Rmax \geq \sqrt{\frac{J}{\pi}\left(1 - 0.05^{\frac{1}{n}}\right)}$$

To reduce positive bias due to small sample size in calculating Pearson‘s correlation

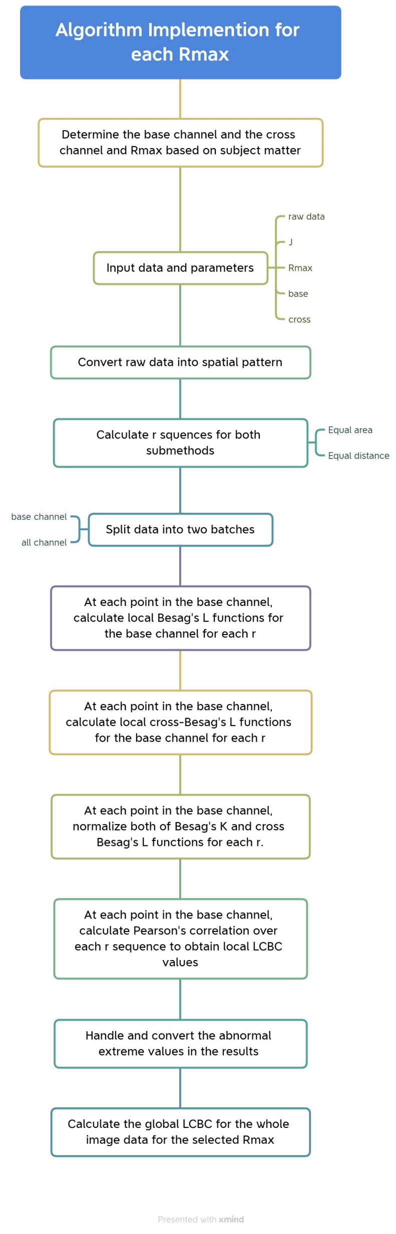
$$J = 10 \text{ and suggest that } J \leq 25$$



2. LCBC

To study the performance of LCBC method on different settings of these rings, we consider two different r sequences: one is for equal area rings, and the other one is for equally spaced rings.

$$\dot{r} = \{r_1, r_2, \ldots, r_{J-1}, r_J\} = \begin{cases} \sqrt{j} * \dfrac{Rmax}{\sqrt{J}} & j \in 1,2,\ldots,J & \text{equal area} \\ j * \dfrac{Rmax}{J} & j \in 1,2,\ldots,J & \text{equal distance} \end{cases}$$



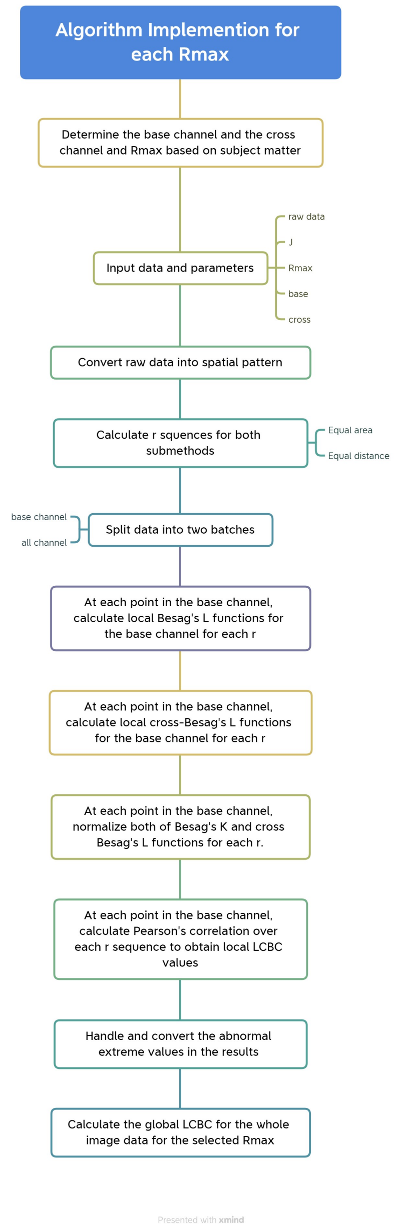
2. LCBC

Local Besag’ s L functions for base channel

$$\hat{L}_{AA}(R_{\max})=\begin{bmatrix} \hat{L}_{AA}(r_1,A_1) & \hat{L}_{AA}(r_1,A_2) & \dots & \hat{L}_{AA}(r_1,A_{N-1}) & \hat{L}_{AA}(r_1,A_N) \\ \hat{L}_{AA}(r_2,A_1) & \hat{L}_{AA}(r_2,A_2) & \dots & \hat{L}_{AA}(r_2,A_{N-1}) & \hat{L}_{AA}(r_2,A_N) \\ \dots & \dots & \dots & \dots & \dots \\ \hat{L}_{AA}(r_J,A_1) & \hat{L}_{AA}(r_J,A_2) & \dots & \hat{L}_{AA}(r_J,A_{N-1}) & \hat{L}_{AA}(r_J,A_N) \end{bmatrix}$$

Local cross-Besag’ s L functions

$$\hat{L}_{AB}(R_{\max})=\begin{bmatrix} \hat{L}_{AB}(r_1,A_1) & \hat{L}_{AB}(r_1,A_2) & \dots & \hat{L}_{AB}(r_1,A_{N-1}) & \hat{L}_{AB}(r_1,A_N) \\ \hat{L}_{AB}(r_2,A_1) & \hat{L}_{AB}(r_2,A_2) & \dots & \hat{L}_{AB}(r_2,A_{N-1}) & \hat{L}_{AB}(r_2,A_N) \\ \dots & \dots & \dots & \dots & \dots \\ \hat{L}_{AB}(r_J,A_1) & \hat{L}_{AA}(r_J,A_2) & \dots & \hat{L}_{AB}(r_J,A_{N-1}) & \hat{L}_{AB}(r_J,A_N) \end{bmatrix}$$



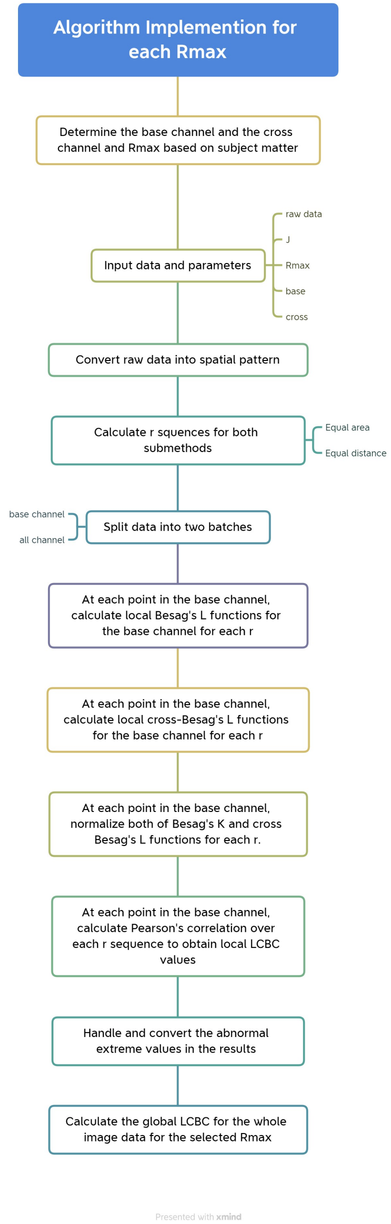
2. LCBC

Conduct subtraction for the L functions for each pair of successive nested circles to get thefunction values for each ring

$$G_{A_iA(r_j)} = \frac{\hat{L}_{AA}(\mathrm{r}_j, A_i) - \hat{L}_{AA}(\mathrm{r}_{j-1}, A_i)}{\mathrm{r}_j - \mathrm{r}_{j-1}} \qquad G_{A_iB(r_j)} = \frac{\hat{L}_{AB}(\mathrm{r}_j, A_i) - \hat{L}_{AB}(\mathrm{r}_{j-1}, A_i)}{\mathrm{r}_j - \mathrm{r}_{j-1}}$$

For the innermost circle (J = 1)

$$G_{A_iA(r_1)} = \frac{\hat{L}_{AA}(\mathrm{r}_1, A_i)}{\mathrm{r}_1} \qquad G_{A_iB(r_1)} = \frac{\hat{L}_{AB}(\mathrm{r}_1, A_i)}{\mathrm{r}_1}$$

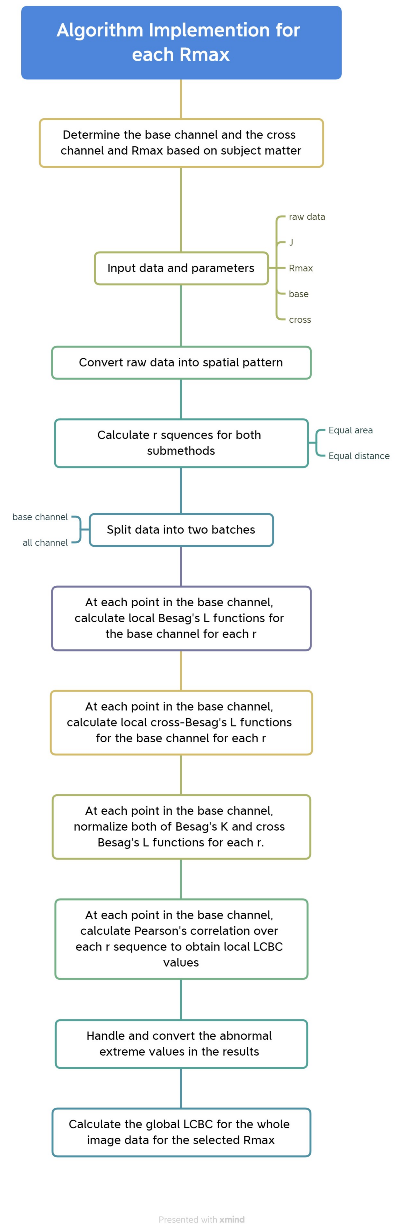


2. LCBC

The above linear transformations can be easily done by matrixmultiplication by a M matrix, which has a dimension of J x J

$$M \cdot \hat{L}_{AA} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \hat{L}_{AA}(\mathrm{r}_1, A_1) & \hat{L}_{AA}(\mathrm{r}_1, A_2) & \dots & \dots & \hat{L}_{AA}(\mathrm{r}_1, A_N) \\ \hat{L}_{AA}(\mathrm{r}_2, A_1) & \hat{L}_{AA}(\mathrm{r}_2, A_2) & \dots & \dots & \hat{L}_{AA}(\mathrm{r}_2, A_N) \\ \dots & \dots & \dots & \dots & \dots \\ \hat{L}_{AA}(\mathrm{r}_j, A_1) & \hat{L}_{AA}(\mathrm{r}_j, A_2) & \dots & \dots & \hat{L}_{AA}(\mathrm{r}_j, A_N) \end{bmatrix}$$

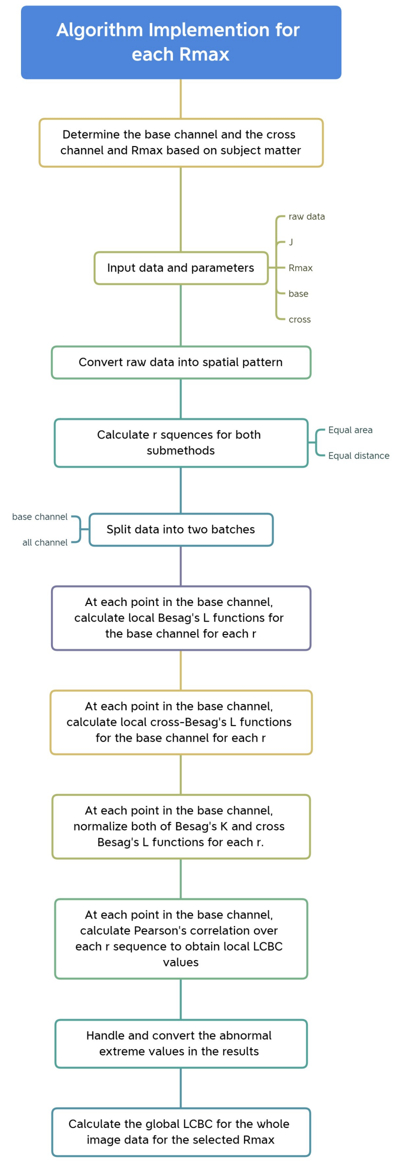
$$M \cdot \hat{L}_{AB} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \hat{L}_{AB}(\mathrm{r}_1, A_1) & \hat{L}_{AB}(\mathrm{r}_1, A_2) & \dots & \dots & \hat{L}_{AB}(\mathrm{r}_1, A_N) \\ \hat{L}_{AB}(\mathrm{r}_2, A_1) & \hat{L}_{AB}(\mathrm{r}_2, A_2) & \dots & \dots & \hat{L}_{AB}(\mathrm{r}_2, A_N) \\ \dots & \dots & \dots & \dots & \dots \\ \hat{L}_{AB}(\mathrm{r}_j, A_1) & \hat{L}_{AB}(\mathrm{r}_j, A_2) & \dots & \dots & \hat{L}_{AB}(\mathrm{r}_j, A_N) \end{bmatrix}$$



2. LCBC

Local LCBC is calculated using the empirical Pearson's correlation for every individual signal in the basechannel A_i

$$LCBC(A_i) = \frac{\sum_{j=1}^{J}[G_{A_iA}(r_j) - \overline{G_{A_iA}}]\cdot[G_{A_iB}(r_j) - \overline{G_{A_iB}}]}{\sqrt{\sum_{j=1}^{J}[G_{A_iA}(r_j) - \overline{G_{A_iA}}]^2}\sqrt{\sum_{j=1}^{J}[G_{A_iB}(r_j) - \overline{G_{A_iB}}]^2}}.$$



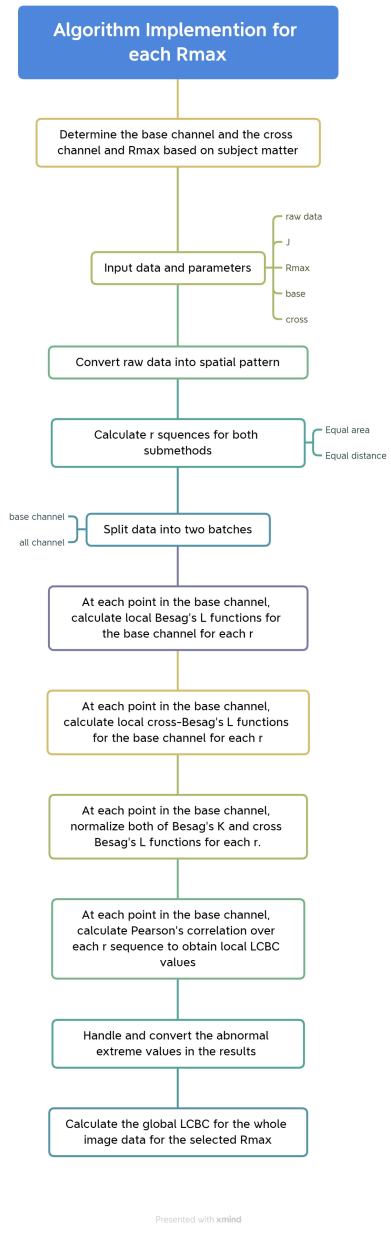
2. LCBC

Extreme cases are likely to occur, such as when R_{max} is too small and no other observations of either channel are found in any rings except for the point at the center

(i) If $G_{A_iA}(r_j)=0$ for all j, and $G_{A_iB}(r_j)\neq 0$ for some *j*, then $LCBC(A_i)=1$

(ii) If $G_{A_iA}(r_j)\neq 0$ for some j, and $G_{A_iB}(r_j)=0$ *for* all *j*, then $LCBC(A_i)=-1$

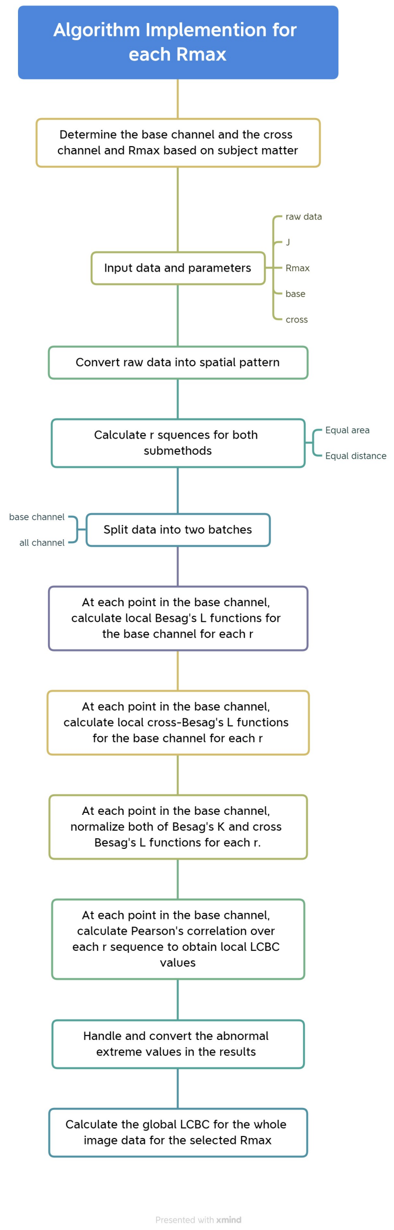
(iii) If $G_{A_iA}(r_j)=G_{A_iB}(r_j)=0$ for all j, then $LCBC(A_i)=0$



2. LCBC

calculating the global LCBC for the whole image, which quantifies theinteraction of channel B towards A in the global scope

$$LCBC = \frac{1}{N}\sum_{i=1}^{N} LCBC(A_i)$$



2. LCBC

Procedure: LCBC

Input

data: three columns which has coordinates in R^2 and the channel label

J: the total number of concentric rings around each point in the base channel

Rmax: a sequence of the largest observation range

base & *cross*: labels set manually

Output

a list of data frames storing **local LCBC** for each Rmax

a data frame containing **global LCBC** for all Rmax

FUNCTION cal.LCBC(*data, J, Rmax, base, cross*):

Preprocess data

data.ppp = convert data to point pattern

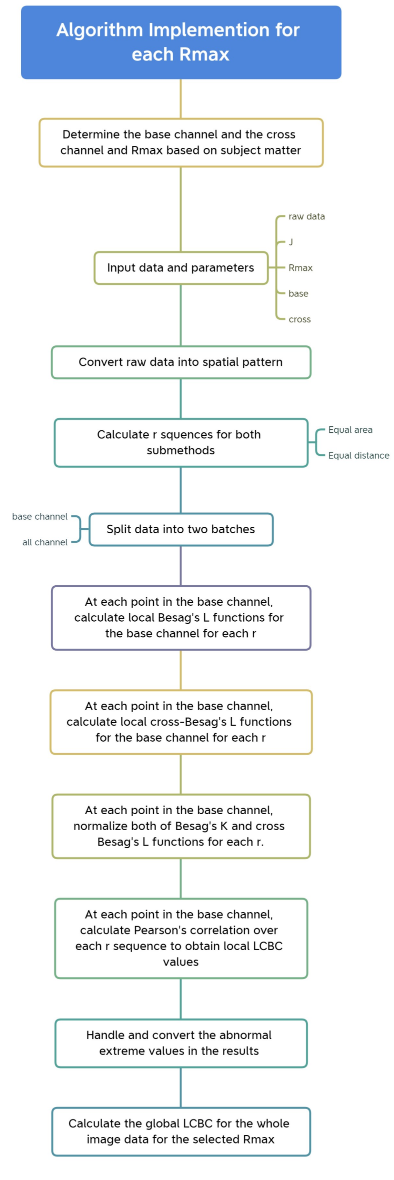
base.ppp = extract data.ppp with the label of base

Define r sequence

r.seq.eq.d = a sequence from 0 to Rmax with increment of $\frac{Rmax}{J}$ *# Equal distance*

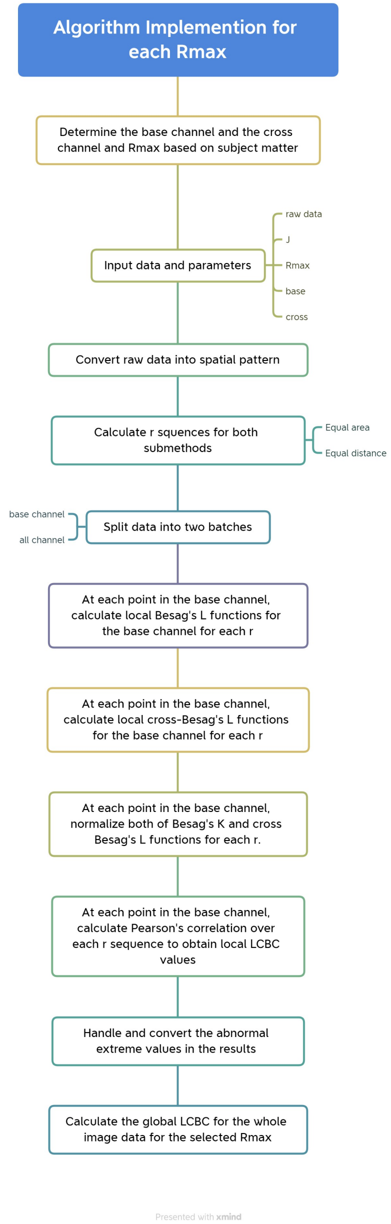
r.seq.eq.a = a sequence of the product between $\sqrt{\frac{j}{J}}$ and Rmax *# Equal area*

Calculate local LCBC



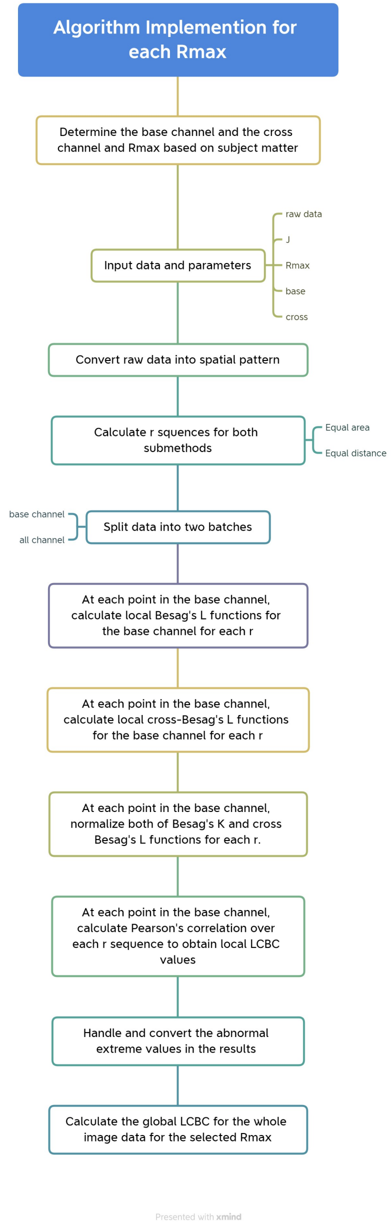
2. LCBC

```
local = FUNCTION(r.seq){
    ### Calculate localL, localLcross
    FOR EACH r IN r.seq:
        Calculate local Besag's L
        Calculate cross-channel local Besag's L
    ### Normalize
    construct the transformation matrix M
    g.self = sub_matrix * self.list / r.vec # Calculate function G for base channel; See formula in 2.1
    g.cross = sub_matrix * cross.list / r.vec # Calculate function G for cross channel; See formula in 2.1
    c.self = get the column sums of g.self
    c.cross = get the column sums of g.cross
    IF c.self == 0 & c.cross == 0
        cor.ex = 0
    ELSE IF c.cross == 0
        cor.ex = -1
    ELSE
        cor.ex = NA
    cor = calculate Pearson correlation of g.self and g.cross
    LocalLCBC = (cor==NA)?cor.ex:cor
}
```

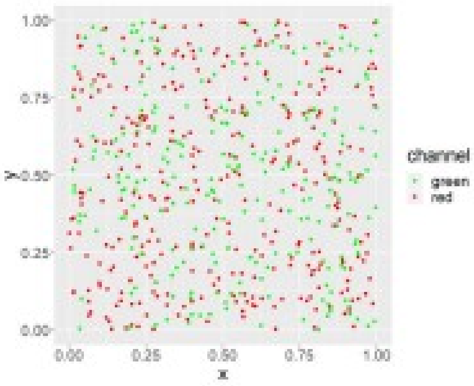


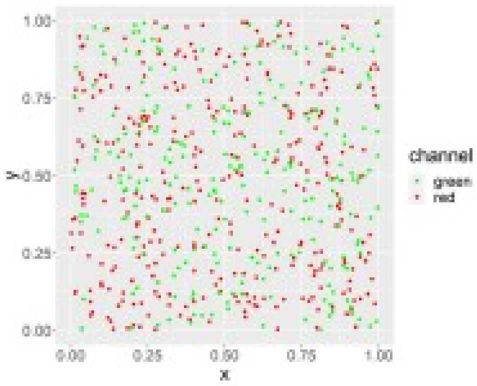
2. LCBC

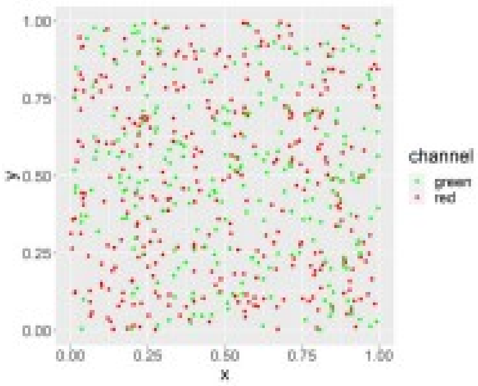
```
### Calculate local LCBC for equal distance and equal area separately
l.d = local(r.seq.eq.d)
l.a = local(r.seq.eq.a)
### Calculate global LCBC for both methods
LCBC.d = mean of l.d
LCBC.a = mean of l.a
}
```



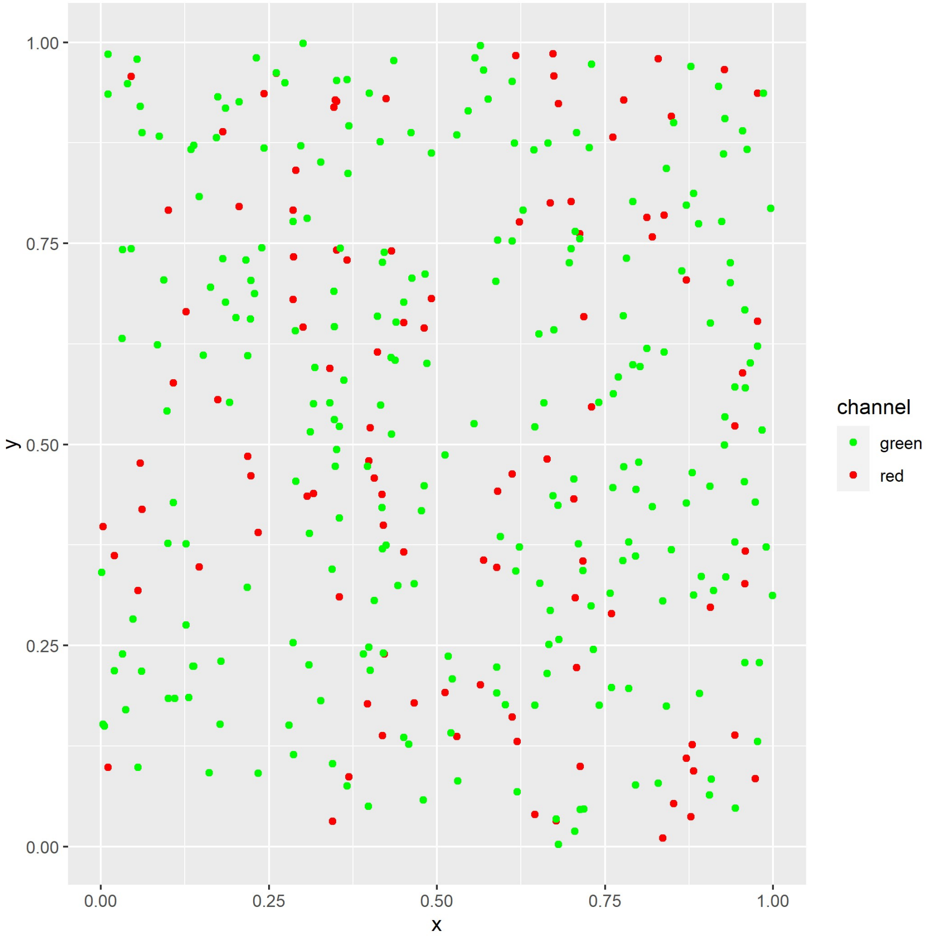
3. Simulation Study







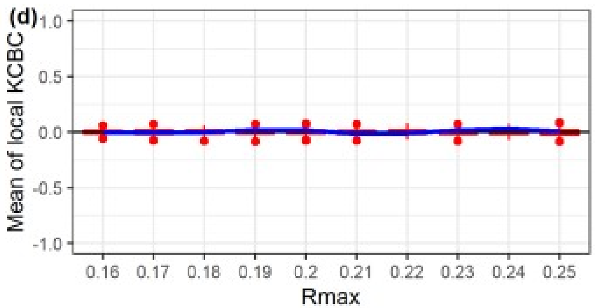
3. Simulation Study

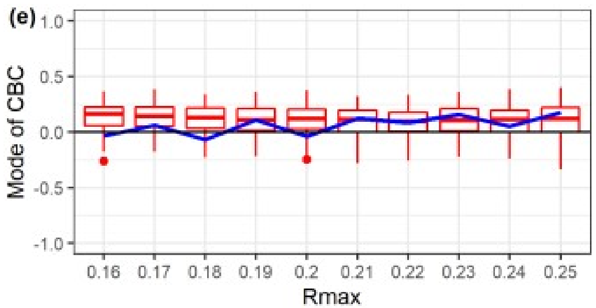


- Null Hypothesis:
  - If the two channels are independent CSR, thentheglobal LCBC is expected to be zero
- Window: [0,1]X[0,1].
- Base channel: red
  - Intensity: 328
- Cross channel: green
  - Intensity: 270

3. Simulation Study (KCBC)

The box plots of mean of local KCBC and mode of CBC of all simulations



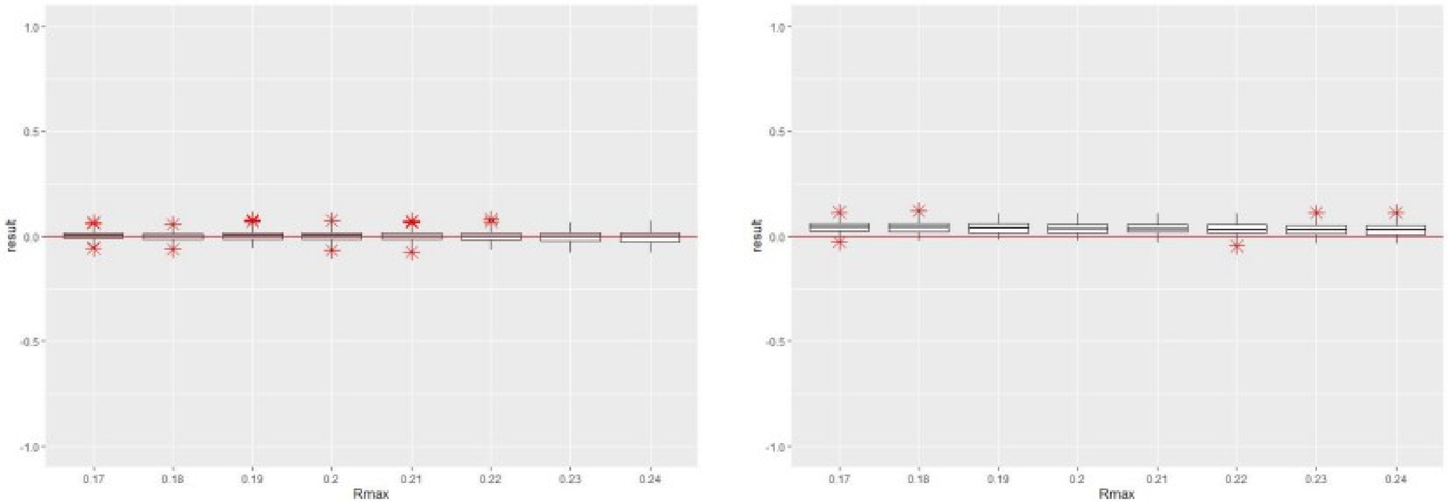


the mean of local KCBC is unbiased with very small variance

the mode of CBC is slightly upward biased but with roughly consistent bias and variance

3. Simulation Study

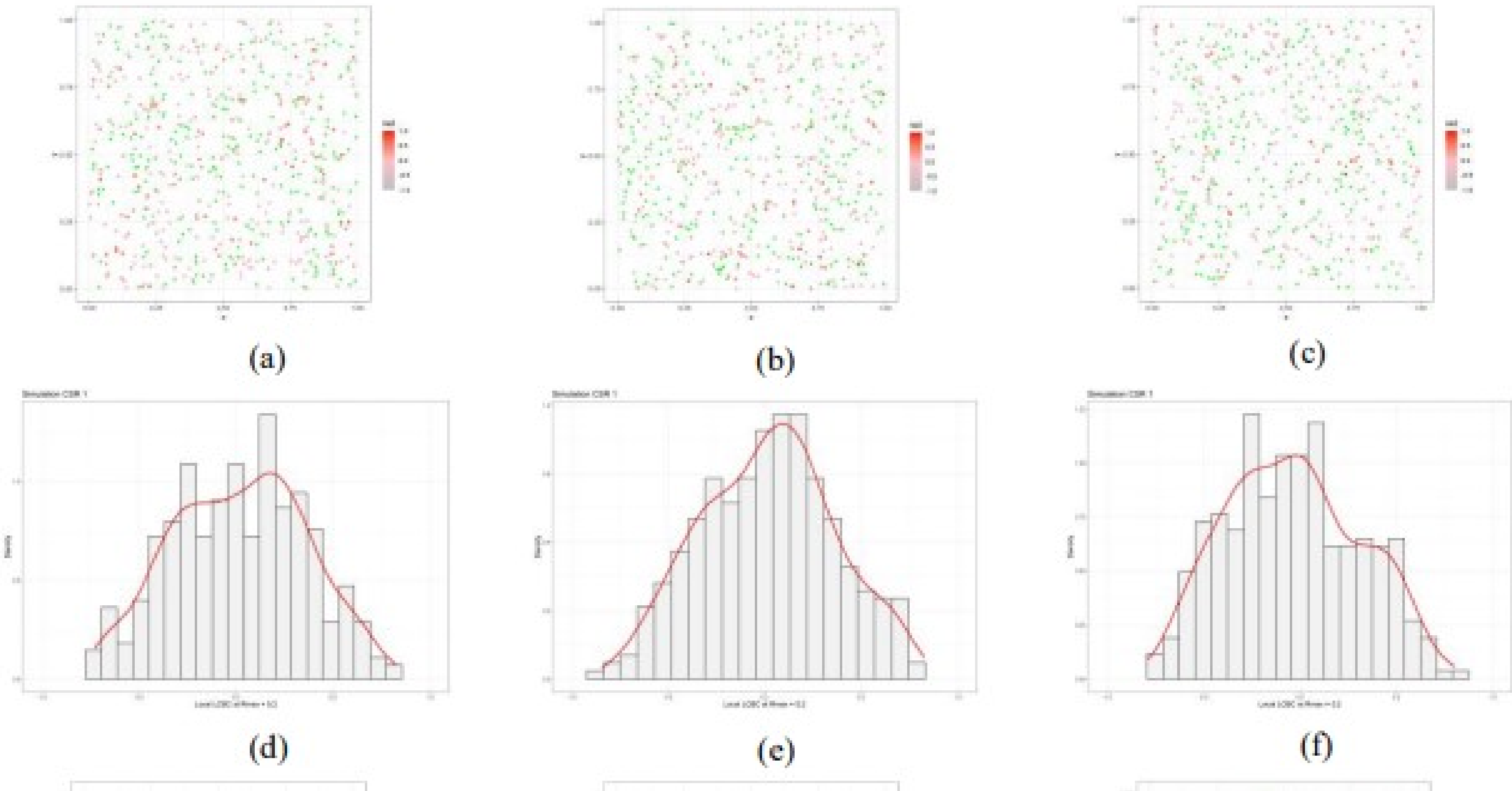
As Rmax values increase the expectations remain around zero. This illustratesthat there’ s no positive or negative correlation between the spatial distributions of red and green points aroundlocations of red points. That fact validates the unbiasedness of LCBC for the equal area case



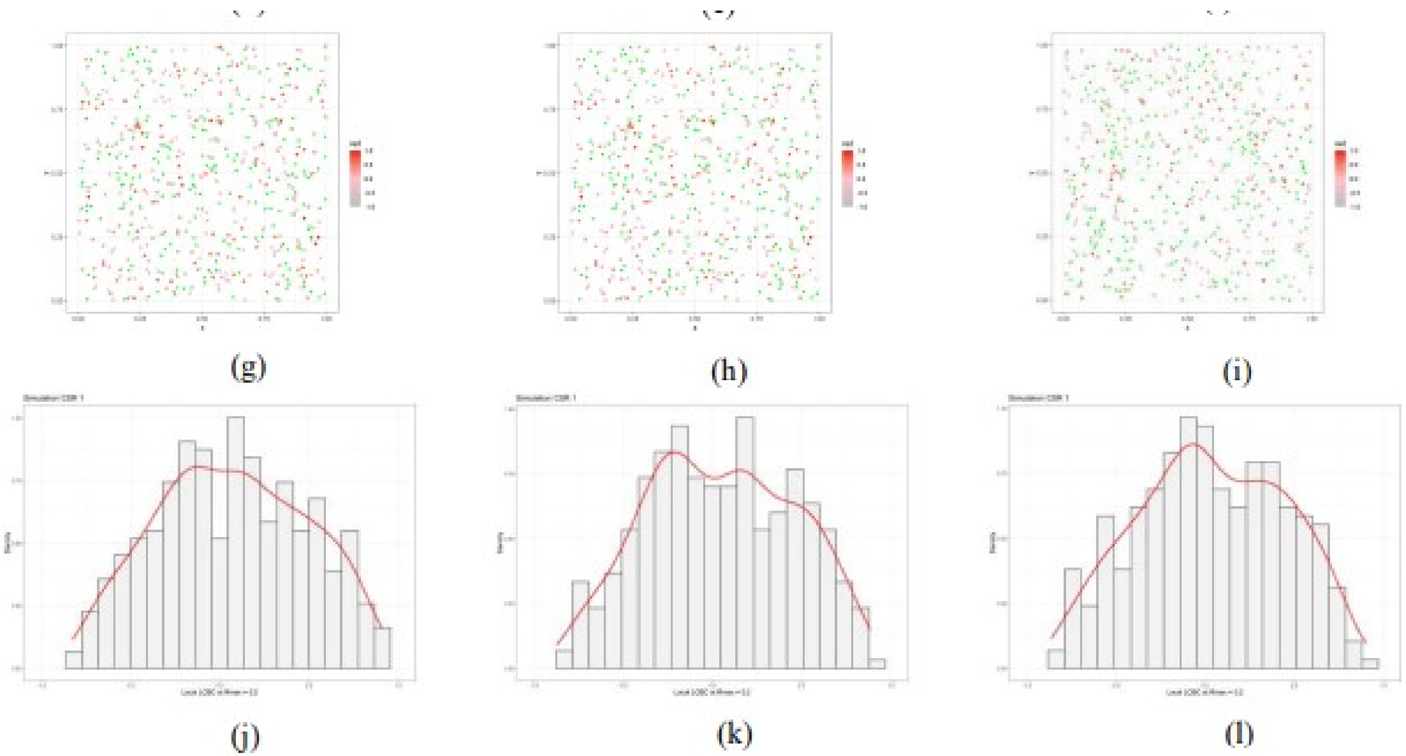
(a) (b)

Figure 3.2: Boxplots for global LCBC over Rmax values for the 200 CSR simulations. Panel (a) shows the outputs for the method of equal area while (b) reveals the results of equal distance method.

3. Simulation Study - Equal Area



3. Simulation Study - Equal Distances

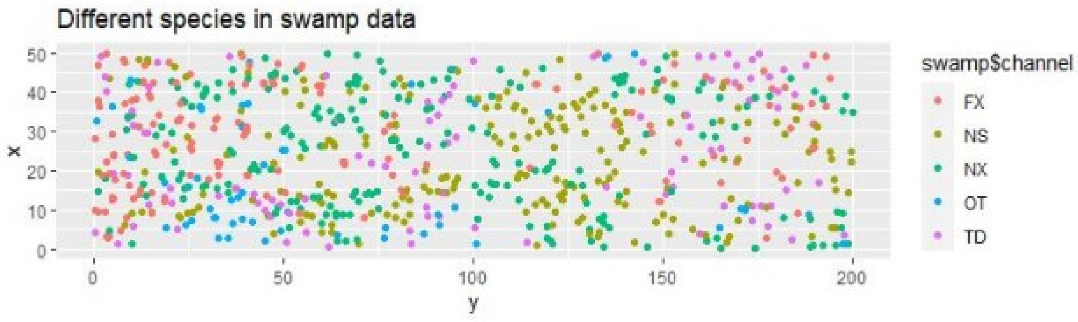


(g) (h) (i)

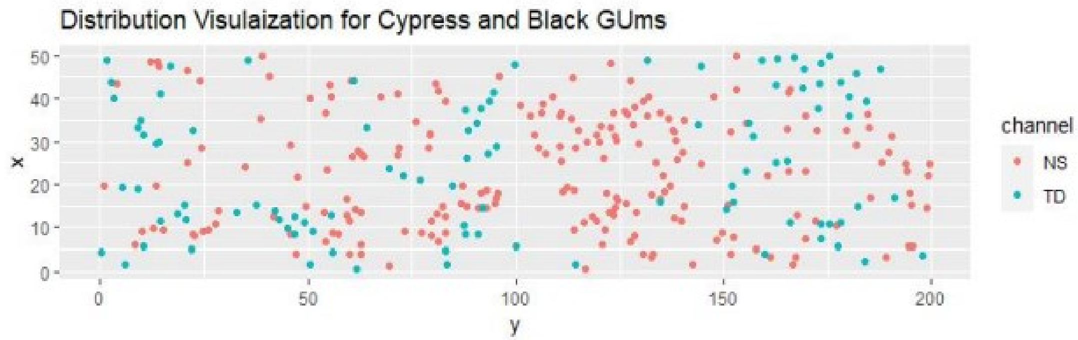
(j) (k) (l)

4. Real Applications

This dataset includes the locations of 13 different types of trees in a 200m x 50m plot of swamp hardwood forest in the Savannah River Site, South California, USA



(a)



4. Real Applications (Dixon’s)

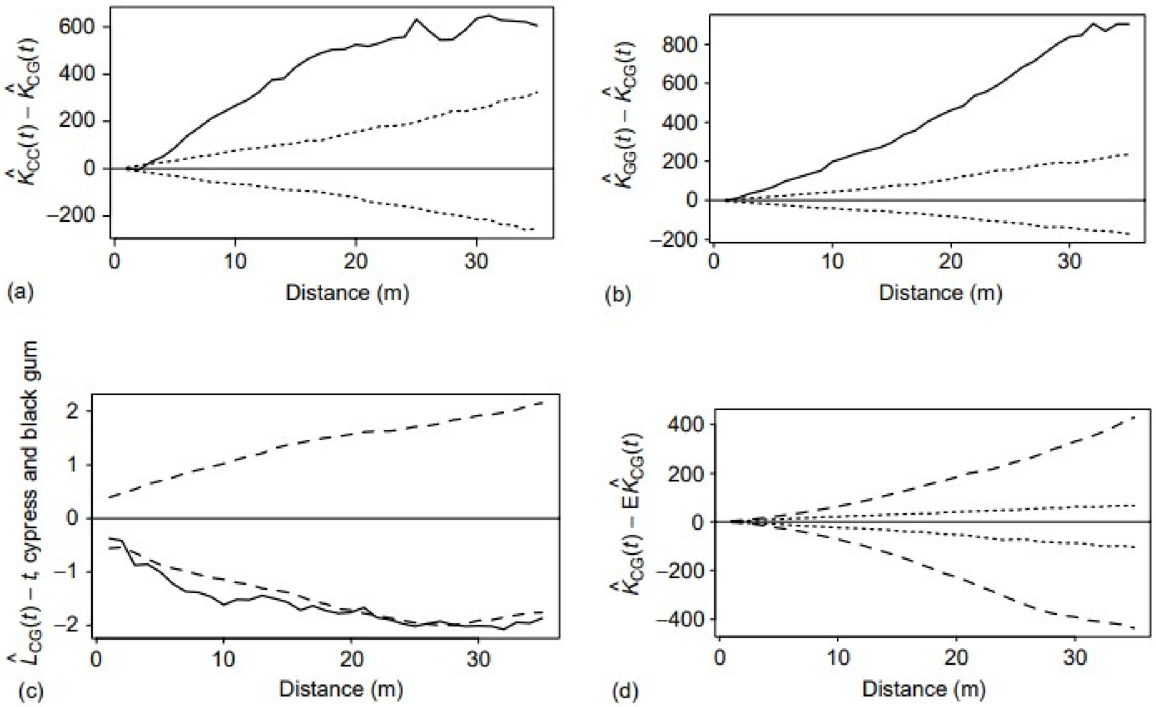


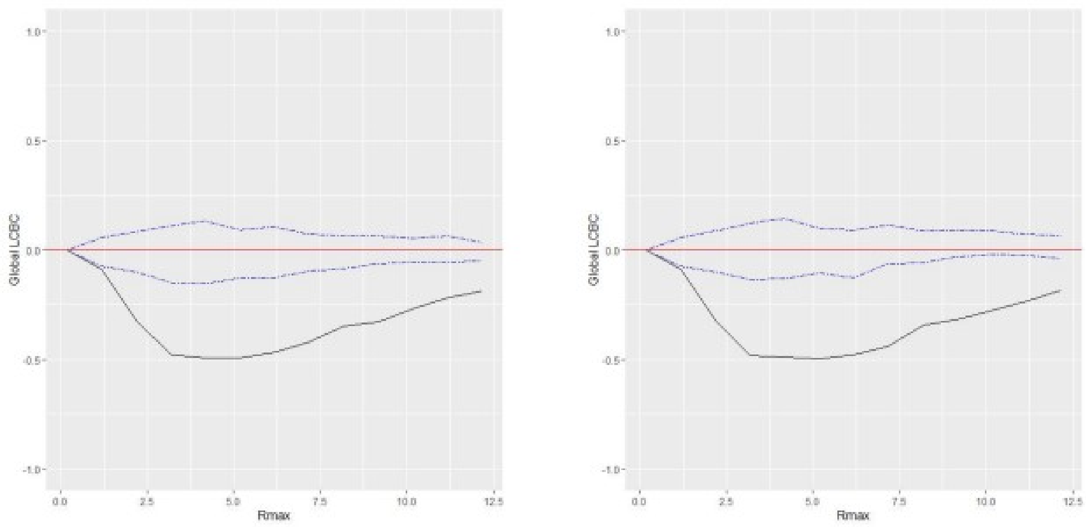
Figure 2 Bivariate $K(t)$ plots to evaluate the spatial relationship between cypress and black gum trees. (a) Plot of $\widehat{K}_{CC}(t) - \widehat{K}_{CG}(t)$ for cypress trees. Solid horizontal line at 0 provides a reference for random labeling. Dotted lines are 0.025 and 0.975 quantiles of $L(t) - t$ estimated from 999 random relabelings. (b) Plot of $\widehat{K}_{GG}(t) - \widehat{K}_{CG}(t)$ for black gum trees. Solid horizontal line at 0 provides a reference for random labeling. Dotted lines are 0.025 and 0.975 quantiles of $L(t) - t$ estimated from 999 random relabelings. (c) Plot of $\widehat{L}_{CG}(t) - t$ for cypress and black gum trees. Solid horizontal line at 0 provides a reference for independence of the two spatial processes. Dotted lines are 0.025 and 0.975 quantiles of $L(t) - t$ estimated from 999 random toroidal shifts. (d) Comparison of 0.025 and 0.975 quantiles computed by random labeling (dotted lines) and random toroidal shifts (dashed lines)

4. Real Applications

The red solid horizontal line at 0 provides a reference value

Two dashed blue lines represent the 0.05 and 0.95 quantiles of global LCBC estimated from 99 random labelings.

The solid black line shows the global LCBC value for Cypress and Black Gum from the original swamp data.



(a) (b)

Figure 4.2: Global LCBC over different Rmax values while (a) for the equal area and (b) for the equal distance method.

4. Real Applications

The prevalence of gray points over red ones suggests that spatially segregated patterns are morefrequently observed.

Moreover, the histogram plots demonstrate a frequency distribution of the local LCBC, which predominantly features extreme negative values.

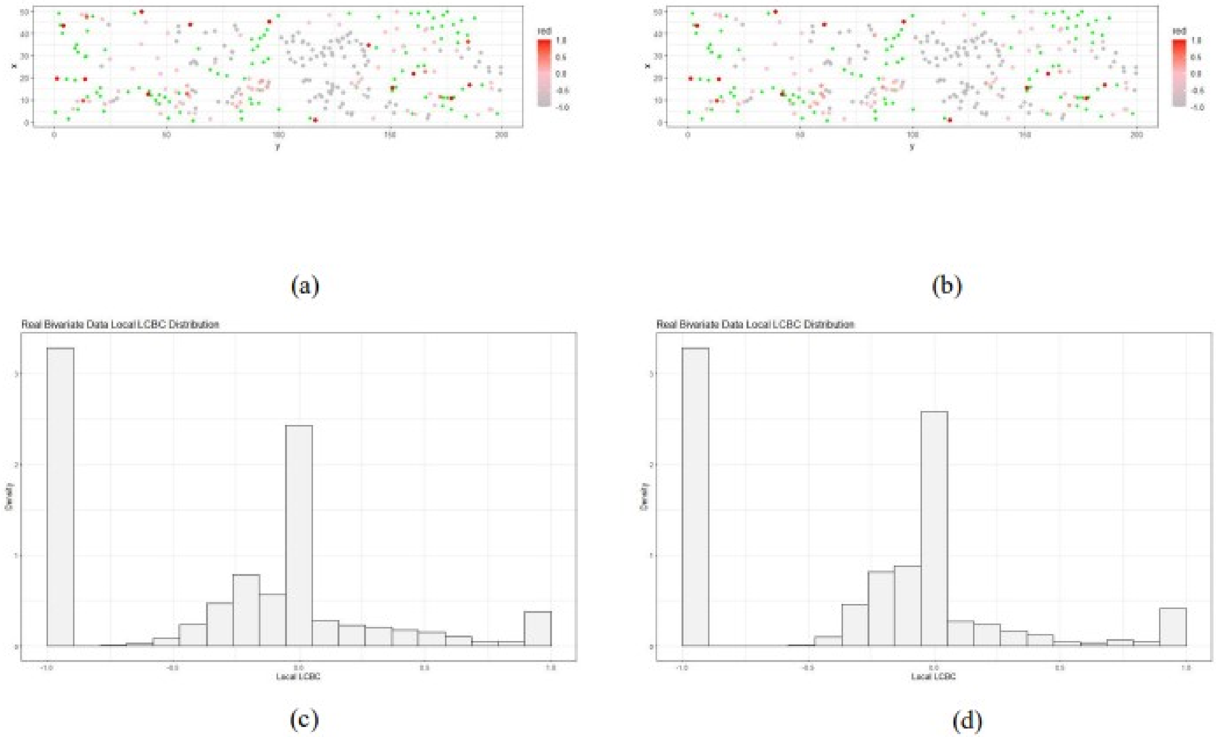


Figure 4.3: (a)(b) are line plots for the mean of local LCBC; (c)(d) are heat maps and (e)(f) are histogram plots for local LCBC. (a)(c) using the equal area method while (b)(d) are same types of analysis visualizations with the equal distance method.

5. Conclusion

- Future Scope:

  - investigate the performance of LCBC algorithmbyapplying it to more extensive simulations, such as spatially segregated patterns and clustering pattern.

  - Comparison with KCBC will be conducted in detail

  - Mathematical proofs for the unbiasedness of LCBC over equal-area rings will be studied as well.