# Password Authenticated Key Exchange: From Two Party Methods to Group Schemes

Stephen Melczer, Taras Mychaskiw, and Yi Zhang

# Introduction

# PART 1
# Classical Two Party PAKEs

# Password Authenticated Key Exchange (PAKE)

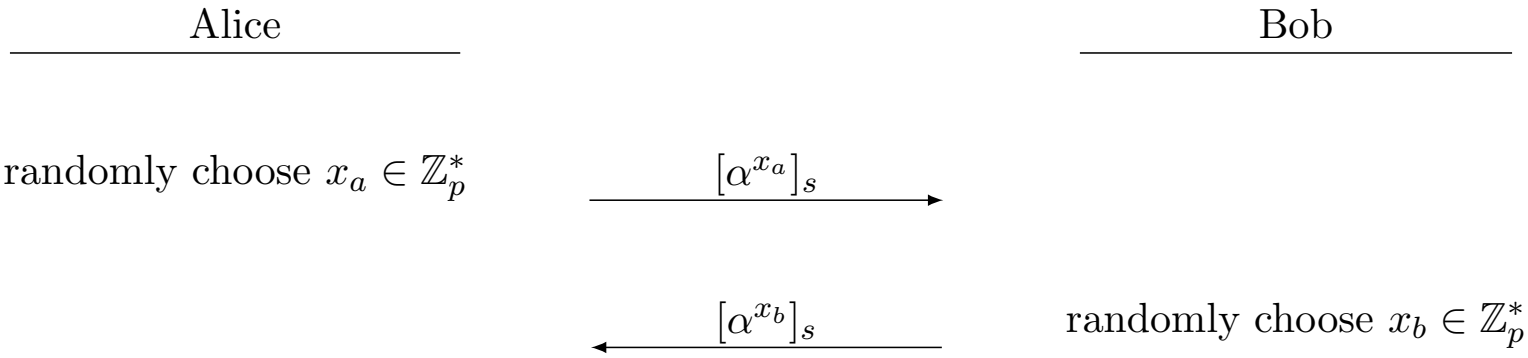
PAKEs allow two parties sharing a *short/weak* password to establish a shared key

Cannot broadcast password directly – would need to be protected (expensive)

Instead, modern PAKEs use *zero-knowledge proof* and/or *hash* of password in protocol

# First Protocol: EKE (Bellovin and Merrit 1992)

Pick prim. root $\alpha \in \mathbb{Z}_p$

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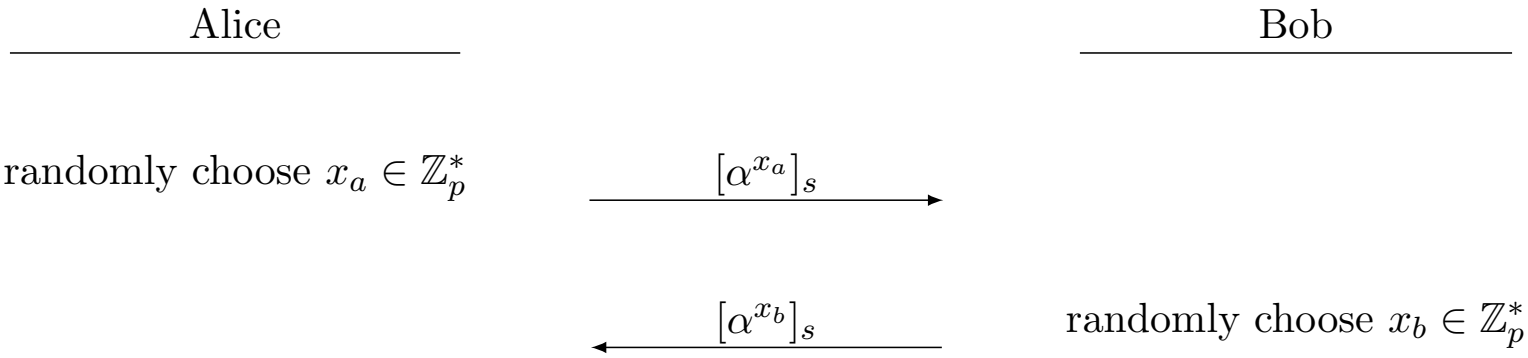


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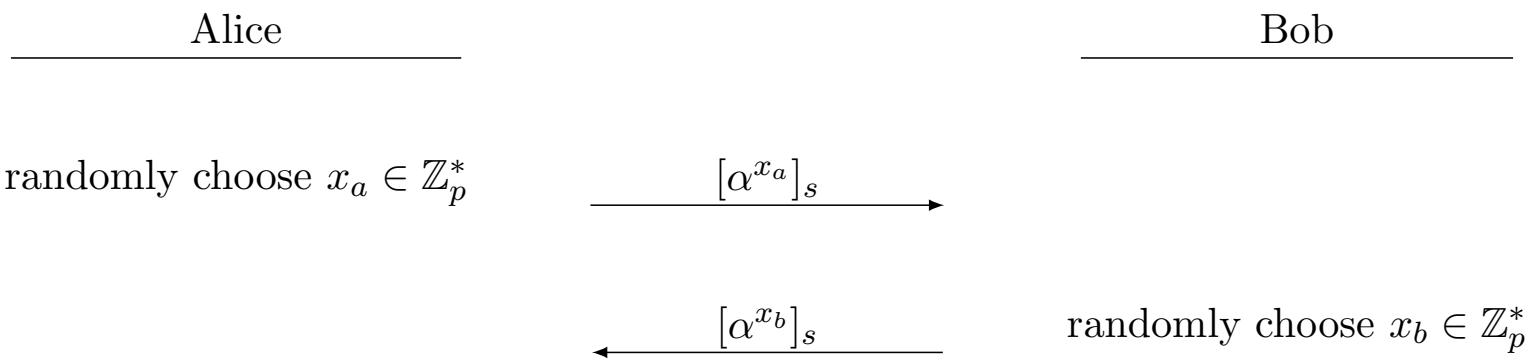


Alice and Bob share $K = \alpha^{x_a \cdot x_b}$.

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Ex: Decypher $[\alpha^{x_a}]_{s'}$ – rule out $s'$ if output in $[p, 2^n - 1]$

# Desired Security Properties

**Offline dictionary attack resistance**

Don't leak info which can be used in brute force search

**Forward secrecy for established keys**

Past keys secure if password disclosed

Implies passive attacker w/ password cannot compute key

**Known session security**

All secrets of one session reveals nothing about others

**Online dictionary attack resistance**

Attacker can only test one password per protocol execution

# J-PAKE

**Setup** Let $G = <g>$ be an order $q$ subgroup of $\mathbb{Z}_p^*$

Alice picks $x_1 \in_R [0, q-1]$ and $x_2 \in_R [1, q-1]$

Bob picks $x_3 \in_R [0, q-1]$ and $x_4 \in_R [1, q-1]$

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**Rd 1** Alice sends $g^{x_1}, g^{x_2}$ and ZKPs of $x_1$ and $x_2$ to Bob
Bob sends $g^{x_3}, g^{x_4}$ and ZKPs of $x_3$ and $x_4$ to Alice
[Alice and Bob verify ZKPs and $g^{x_2}, g^{x_4} \neq 1$]

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**Rd 2** Alice sends $A = g^{(x_1+x_3+x_4)x_2 \cdot s}$ and ZKP of $x_2 s$
Bob sends $B = g^{(x_1+x_2+x_3)x_4 \cdot s}$ and ZKP of $x_4 s$

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Bob sends $B = g^{(x_1+x_2+x_3)x_4 \cdot s}$ and ZKP of $x_4 s$

They share

$$K = \underbrace{(B/g^{x_2 x_4 s})^{x_2}}_{\text{Computable by Alice}} = g^{(x_1+x_3)x_2 x_4 s} = \underbrace{(A/g^{x_2 x_4 s})^{x_4}}_{\text{Computable by Bob}}.$$

# J-PAKE

Satisfies all 4 desired properties
(under DDH and CDH assumptions)

Only two rounds of communication

No *explicit* key confirmation (only implicit)

Not patented

# Dragonfly

**Setup** Let $Q$ be a cyclic subgroup of $\mathbb{Z}_p^*$ with prime order $q$.

Alice picks $r_A, m_A \in_R \mathbb{Z}_q^*$

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**Rd 1** Alice sends $s_A = r_A + m_A \mod q$ and $E_A = \pi^{-m_A} \mod p$

Bob sends $s_B = r_B + m_B \mod q$ and $E_B = \pi^{-m_B} \mod p$

[Alice and Bob check that $s_A \neq s_B$ or $E_A \neq E_B$]

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[Alice and Bob check that $s_A \neq s_B$ or $E_A \neq E_B$]

**Rn 2** Alice computes $ss = (\pi^{s_B} E_B)^{r_A} = \pi^{r_A r_B}$

Bob computes $ss = (\pi^{s_A} E_A)^{r_B} = \pi^{r_A r_B}$

Alice sends $H(ss|E_A|s_A|E_B|s_B)$

Bob sends $H(ss|E_B|s_B|E_A|s_A)$

Each member confirms the hashes.

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Each member confirms the hashes.

They share:

$$K = H(ss|E_A \times E_B|(s_A + s_B) \mod q)$$

# Dragonfly

No formal security proofs, but claims resistance to offline dictionary attacks

Only two rounds of communication

Very fast compared to other protocols

# PAK/PPK – PAK

**Setup** Let $G = <g>$ be an order $q$ subgroup of $\mathbb{Z}_p^*$

Let $p = rq + 1$ where $r, q$ relatively prime.

Let $\pi$ be the password and $H_1, H_{2a}, H_{2b}, H_3$ be random, independent hash functions.

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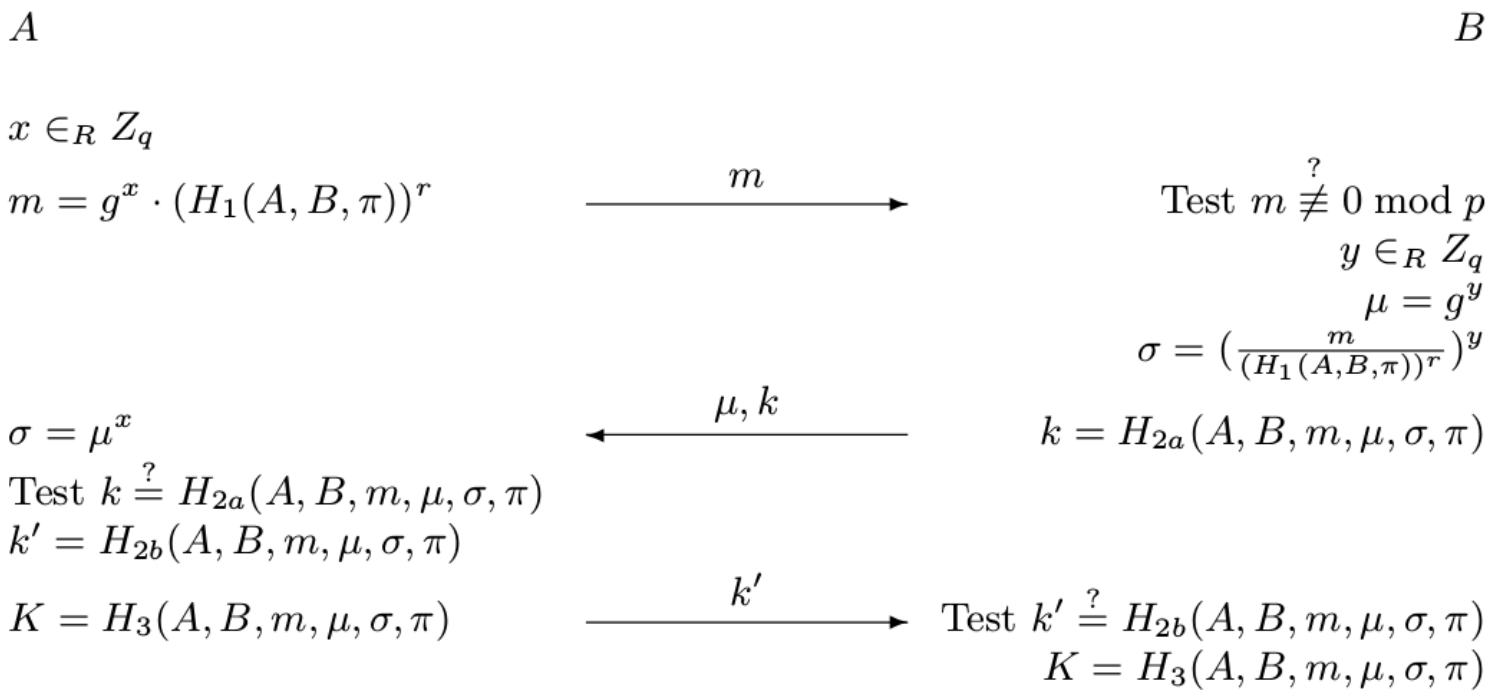
## PAK



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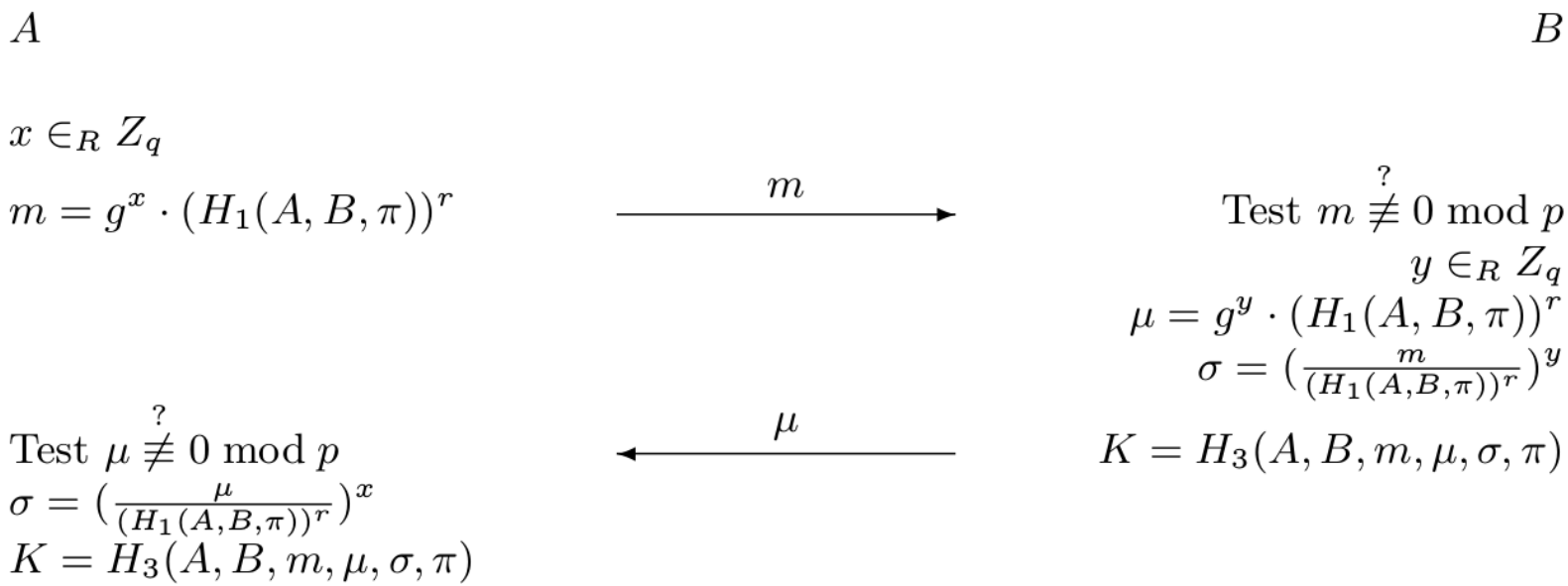


# PAK/PPK

Proposes a new formal security model
based on the random oracle model

Satisfies all 4 desired properties
(under DDH assumptions)

Only two/three rounds of communication

PAK has explicit key confirmation
PPK has implicit key confirmation

# PART 2
# Extension to Group Setting

# The Fairy-Ring Dance

Desciption of general procedure

How pairwise + group keys are constructed

# J-PAKE+

Description of J-PAKE+

Uses 3 rounds as J-PAKE does not have explicit key confirmation

# Dragonfly+

Description of Dragonfly+

# PPK+

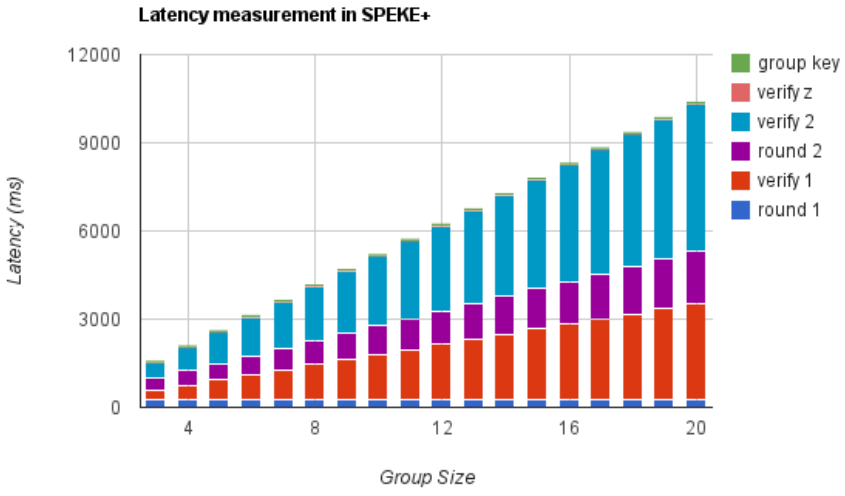
Description of PPK+

# PART 3
# Timings

# Specifications

- All protocol benchmarks were implemented in Java 1.6 and run on a server (3GHz AMD processor, 6GB of RAM) running Ubuntu 12.04.
- Benchmarks measured latency, the amount of work each device would have to do in the group excluding communication.

# Results


Latency measurement in SPEKE+
12000
9000
6000
3000
0
Latency (ms)
4
8
12
16
20
Group Size
group key
verify z
verify 2
round 2
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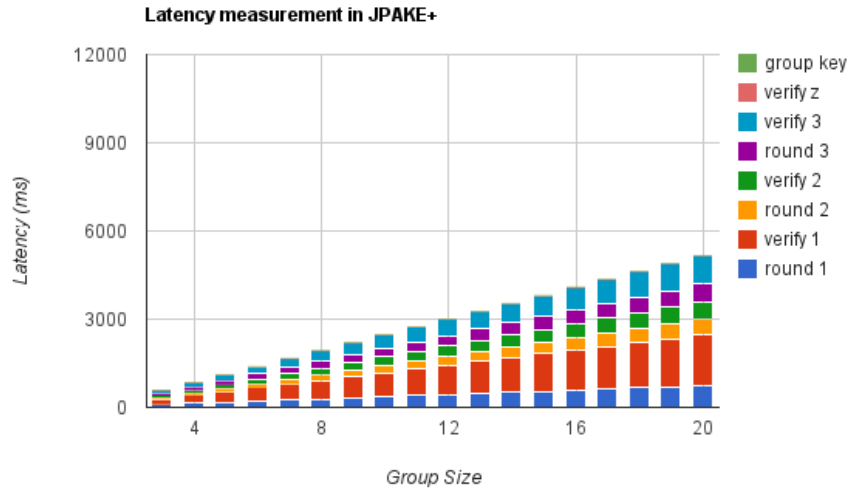
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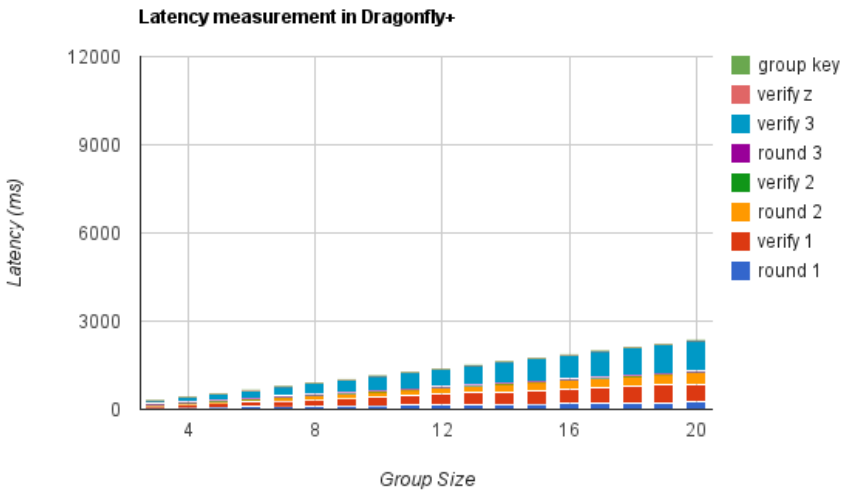
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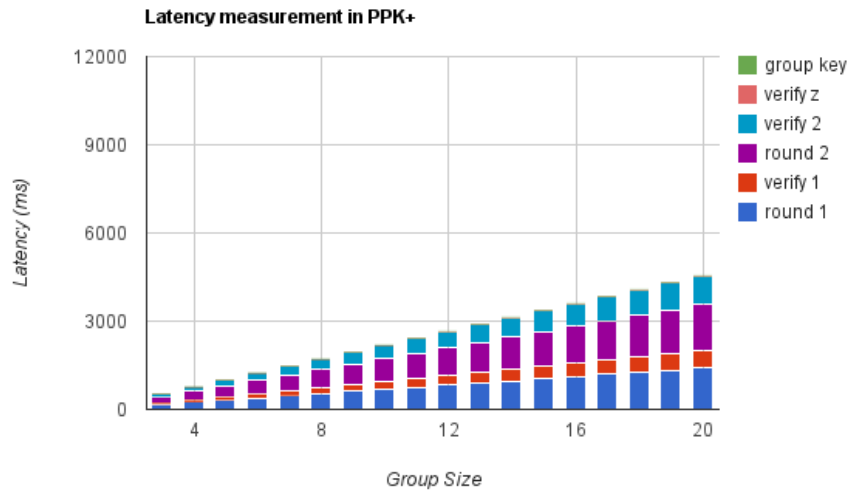
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# Conclusion

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It is possible to transfer PAKEs into GPAKEs while preserving round efficiency

SPEKE+ is very slow

J-PAKE+ is a bit slow, but proven secure (under CDH)

PPK is faster but weaker security proof

Dragonfly is fastest, but no security proof
(despite IEEE 802.11-2012 standard)

THANK YOU