

# Password Authenticated Key Exchange: From Two Party Methods to Group Schemes

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# Introduction

1. Classical Two Party PAKEs
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  - 2.2 Examples of GPAKEs
3. Timings
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# PART 1

## Classical Two Party PAKEs

# Password Authenticated Key Exchange (PAKE)

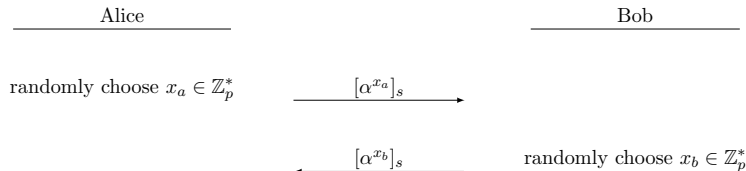
PAKEs allow two parties sharing a *short/weak* password to establish a shared key

Cannot broadcast password directly – would need to be protected (expensive)

Instead, modern PAKEs use *zero-knowledge proof* of password in protocol

# First Protocol: EKE (Bellare and Merritt 1992)

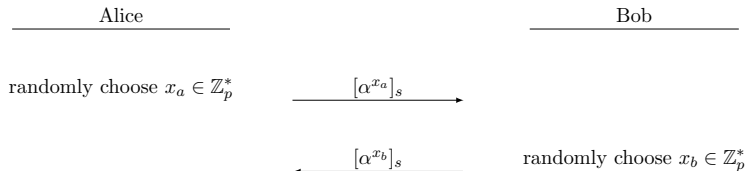
Pick prim. root  $\alpha \in \mathbb{Z}_p$



Alice and Bob share  $K = \alpha^{x_a \cdot x_b}$ .

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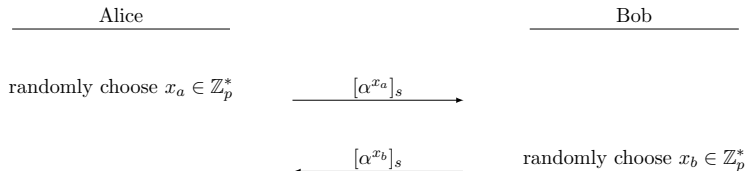


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Ex: Decypher  $[\alpha^{x_a}]_{s'}$  – rule out  $s'$  if output in  $[p, 2^n - 1]$

# Desired Security Properties

## **Offline dictionary attack resistance**

Don't leak info which can be used in brute force search

## **Forward secrecy for established keys**

Past keys secure if password disclosed

Implies passive attacker w/ password cannot compute key

## **Known session security**

All secrets of one session reveals nothing about others

## **Online dictionary attack resistance**

Attacker can only test one password per protocol execution



# J-PAKE

**Setup** Let  $G = \langle g \rangle$  be an order  $q$  subgroup of  $\mathbb{Z}_p^*$

Alice picks  $x_1 \in_R [0, q-1]$  and  $x_2 \in_R [1, q-1]$

Bob picks  $x_3 \in_R [0, q-1]$  and  $x_4 \in_R [1, q-1]$

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**Rd 1** Alice sends  $g^{x_1}, g^{x_2}$  and ZKPs of  $x_1$  and  $x_2$  to Bob

Bob sends  $g^{x_3}, g^{x_4}$  and ZKPs of  $x_3$  and  $x_4$  to Alice

[Alice and Bob verify ZKPs and  $g^{x_2}, g^{x_4} \neq 1$ ]

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**Rd 2** Alice sends  $A = g^{(x_1+x_3+x_4)x_2 \cdot s}$  and ZKP of  $x_2s$

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They share

$$K = \underbrace{(B/g^{x_2x_4s})^{x_2}}_{\text{Computable by Alice}} = g^{(x_1+x_3)x_2x_4s} = \underbrace{(A/g^{x_2x_4s})^{x_4}}_{\text{Computable by Bob}} .$$

# J-PAKE

Satisfies all 4 desired properties  
(under DDH and CDH assumptions)

Only two rounds of communication

No *explicit* key confirmation (only implicit)

Not patented

# Dragonfly

Description of protocol

Security properties

# PAK/PPK

Description of protocol

Security properties

# PART 2

## Extension to Group Setting



# The Fairy-Ring Dance

Description of general procedure

How pairwise + group keys are constructed

# J-PAKE+

Description of J-PAKE+

Uses 3 rounds as J-PAKE does not have explicit key confirmation

# Dragonfly+

Description of Dragonfly+

PPK+

Description of PPK+

# PART 3

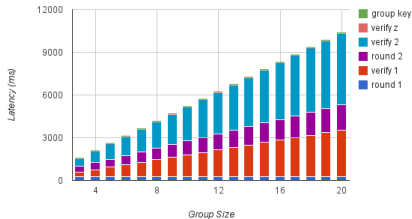
## Timings

# Specifications

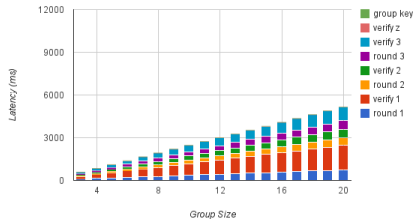
Computer info + how timings done (Java version etc.)

# Results

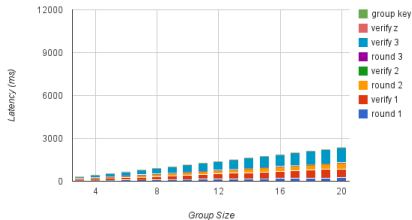
Latency measurement in SPEKE



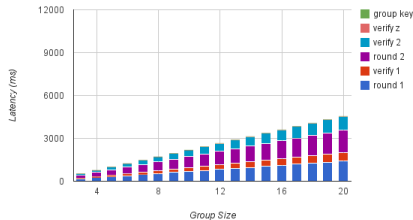
Latency measurement in JPAKE



Latency measurement in Dragonfly



Latency measurement in PPK



## Conclusion



# Conclusion

It is possible to transfer PAKEs into GPAKEs while preserving round efficiency

SPEKE+ is very slow

J-PAKE+ is a bit slow, but proven secure (under CDH)

PPK is faster but weaker security proof

Dragonfly is fastest, but no security proof  
(despite IEEE 802.11-2012 standard)

THANK YOU