Aufgabe 5.1)

Annahme o.B.d.A.: $\mu_B \geq \mu_A$

Gegeben: zwei Normalverteilungen (μ_A, σ_A^2) und (μ_B, σ_B^2)

Gesucht: Position der Schnittpunkte der beiden NV.

$$\frac{1}{\sqrt{2\pi}\,\sigma_A} \exp\left(-\frac{(x-\mu_A)^2}{2\sigma_A^2}\right) = \frac{1}{\sqrt{2\pi}\,\sigma_B} \exp\left(-\frac{(x-\mu_B)^2}{2\sigma_B^2}\right) \quad \Big| \cdot 2\pi\sigma_A\sigma_B$$

$$\sigma_B \cdot \exp\left(-\frac{(x-\mu_A)^2}{2\sigma_A^2}\right) = \sigma_A \cdot \exp\left(-\frac{(x-\mu_B)^2}{2\sigma_B^2}\right) \quad \Big| \ln$$

$$\ln(\sigma_B) - \frac{(x-\mu_A)^2}{2\sigma_A^2} = \ln(\sigma_A) - \frac{(x-\mu_B)^2}{2\sigma_B^2}$$

$$\ln\left(\frac{\sigma_B}{\sigma_A}\right) - \frac{(x-\mu_A)^2}{2\sigma_A^2} + \frac{(x-\mu_B)^2}{2\sigma_B^2} = 0 \quad \Big| \cdot 2\sigma_A^2\sigma_B^2$$

$$2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right) - (x^2 - 2x\mu_A + \mu_A^2)\sigma_B^2 + (x^2 - 2x\mu_B + \mu_B^2)\sigma_A^2 = 0$$

$$2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right) - x^2\sigma_B^2 + 2x\mu_A\sigma_B^2 - \mu_A^2\sigma_B^2 + x^2\sigma_A^2 - 2x\mu_B\sigma_A^2 + \mu_B^2\sigma_A^2 = 0$$

$$(\sigma_A^2 - \sigma_B^2)x^2 + 2(\mu_A\sigma_B^2 - \mu_B\sigma_A^2)x + 2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right) - \mu_A^2\sigma_B^2 + \mu_B^2\sigma_A^2 = 0$$

Fall 1:

$\sigma_A^2 = \sigma_B^2$

$$0x^2 + 2(\mu_A - \mu_B)\sigma_A^2 x + 2\sigma_A^2\sigma_B^2 \cdot \ln(1) + (\mu_B^2 - \mu_A^2)\sigma_A^2 = 0 \quad \Big| : \sigma_A^2$$

$$2(\mu_A - \mu_B)x = -(\mu_B^2 - \mu_A^2)$$

$$2(\mu_A - \mu_B)x = (\mu_A + \mu_B)(\mu_A - \mu_B)$$

$$x = \frac{(\mu_A + \mu_B)(\mu_A - \mu_B)}{2(\mu_A - \mu_B)} = \frac{\mu_A + \mu_B}{2}$$

Fall 2:

$\sigma_A^2 \neq \sigma_B^2$

$$x^2 + \frac{2(\mu_A\sigma_B^2 - \mu_B\sigma_A^2)x}{\sigma_A^2 - \sigma_B^2} + \frac{2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right) - \mu_A^2\sigma_B^2 + \mu_B^2\sigma_A^2}{\sigma_A^2 - \sigma_B^2} = 0$$

$$x_{1,2} = -\frac{\mu_A\sigma_B^2 - \mu_B\sigma_A^2}{\sigma_A^2 - \sigma_B^2} \pm \sqrt{\frac{(\mu_A\sigma_B^2 - \mu_B\sigma_A^2)^2}{(\sigma_A^2 - \sigma_B^2)^2} - \frac{2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right)}{\sigma_A^2 - \sigma_B^2} + \frac{(\mu_A^2\sigma_B^2 - \mu_B^2\sigma_A^2)(\sigma_A^2 - \sigma_B^2)}{(\sigma_A^2 - \sigma_B^2)^2}}$$

$$\ldots \pm \sqrt{\frac{\mu_A^2\sigma_B^4 - 2\mu_A\sigma_B^2\mu_B\sigma_A^2 + \mu_B^2\sigma_A^4 + \mu_A^2\sigma_B^2\sigma_A^2 - \mu_A^2\sigma_A^4 - \mu_B^2\sigma_A^4 + \mu_B^2\sigma_A^2\sigma_B^2}{(\sigma_A^2 - \sigma_B^2)^2} - \frac{2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right)}{\sigma_A^2 - \sigma_B^2}}$$

$$\ldots \pm \sqrt{\frac{\sigma_A^2\sigma_B^2(\mu_A^2 - 2\mu_A\mu_B + \mu_B^2)}{(\sigma_A^2 - \sigma_B^2)^2} - \frac{2\sigma_A^2\sigma_B^2 \ln\left(\frac{\sigma_B}{\sigma_A}\right)(\sigma_A^2 - \sigma_B^2)}{(\sigma_A^2 - \sigma_B^2)^2}}$$

$$\ldots \pm \sqrt{\frac{\sigma_A^2\sigma_B^2}{(\sigma_A^2 - \sigma_B^2)^2} \cdot \left((\mu_A - \mu_B)^2 + 2\ln\left(\frac{\sigma_B}{\sigma_A}\right)(\sigma_B^2 - \sigma_A^2)\right)}$$

$$x_{1,2} = \frac{\mu_A\sigma_B^2 - \mu_B\sigma_A^2}{\sigma_B^2 - \sigma_A^2} \pm \sigma_A \cdot \sigma_B \sqrt{\frac{(\mu_A - \mu_B)^2 + 2\ln\left(\frac{\sigma_B}{\sigma_A}\right)(\sigma_B^2 - \sigma_A^2)}{(\sigma_B^2 - \sigma_A^2)^2}}$$