Tackling high-dimentional optimization problems within Quantum Approximate Optimization Algorithm in a gamified setting

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High-dimensional optimization problems with exponentially large search space lie in the foundation of many yet unresolved scientific problems or pressing issues that the world society is currently facing. During the MIT iQuHACK 2022 hackaton, we developed the *Opti-maze* game aimed at increasing awareness of complexity of such problems. In this game, a player either competes with or is being assisted by the Quantum Approximate Optimization Algorithm which can run on a near-term noisy quantum device. The game showcases (1) how quantum advantage can be beneficial to researchers and the general public and (2) how even a near-term noisy quantum device can be used to tackle large-scale optimization problems.

I. INTRODUCTION

Quantum computers promise to deliver an exponential speed up to various problems that are hard on the classical counterpart. One such class of problems is combinatorial optimization. To solve this type of problems classically, one is generally required to explore all possible configurations, which is in general exponentially hard in the number of degrees of freedom. On a quantum computer, the famous Quantum Approximate Optimization Algorithm (QAOA) promises possible ease of this complexity. During the MIT iQuHACK 2022 hackathon, we have created a game that is aimed at helping to understand the challenges of the optimization problems and the benefits that quantum computing can offer. Many popular computer games, from the historical Monopoly to more advanced civilization games like Sid Mayer's Civilization, are based on the difficulty in finding optimal solutions to a resource allocation problem. Generally, the game is aimed to optimize a metric on a graph and has two game regimes: *standard* and *cooperative*. In the former, player is challenged by a *Quantum Artificial Player* which uses the QAOA to tackle the resource allocation problem. In the latter, quantum computer *helps* the player to solve an hard optimization problem.

Many challenges that the humanity is currently facing can be formulated in terms of a resource allocation problem. One of such challenges is the *fight against the climate change*. Even though our game is ran on an arbitrary graph, we specifically tailored one special case that illustrated applicability of QAOA to tackling this pressing issue.

In part, the game would educate quantum novices on possible usages and applications of *variational quantum algorithms*. At the same time, it will make the users aware of non-triviality of resource allocation problems including fighting the climate crisis.

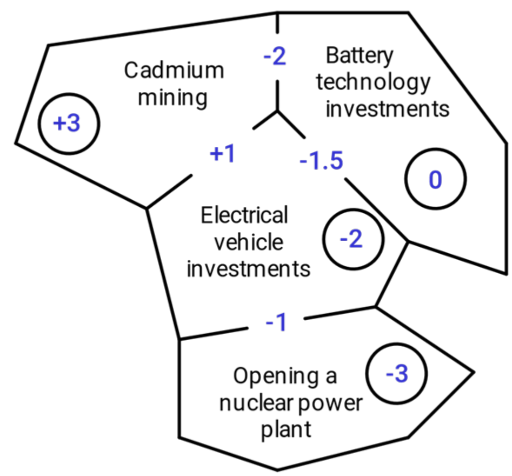

FIG. 1. An example set up of an optimization problem and its weights

II. GRAPH OPTIMIZATION PROBLEM AND QAOA CIRCUIT

In this project, we employ the cost Hamiltonian

$$\hat{H}_C = \sum_i p_i(1 - \hat{Z}_i) + \sum_{i<j} p_{ij}(1 - \hat{Z}_i)(1 - \hat{Z}_j), \quad (1)$$

where p_i are the *on-site* terms and p_{ij} are the *pair terms*. The meaning of the terms can be understood as follows. Let us assume that the $\hat{H}_C$ is some *goal* one needs to achieve. In our illustration, CO_2 emissions reduction. Some factors have a negative impact on the cost function *per se*, thus the on-site terms. Some factors have a *synergy* between them, which justifies the pair-terms. Here, Fig. 1 shows an example decisions, their "costs" and "synetgies" between them in the climate change setting.

The inspiration for the model Hamiltonian comes from the famous Ising model. Generally, such problem has no efficient solution classically, and finding the most optimal configuration with the "lowest energy" takes an exponential time with respect to the number of "decisions". However, such optimization problems can be

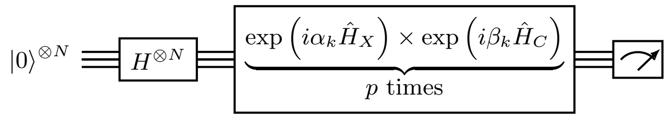

FIG. 2. Schematic representation of the QAOA algorithm. The $|0\rangle^{\otimes N}$ state is brought to the ground state of the H_X mixing Hamiltonian by product of Hadamard gates $H^{\otimes N}$. Then p times repeats consecutive application of unitaries $\exp\left(i\alpha_k \hat{H}_X\right)$ and $\exp\left(i\beta_k \hat{H}_C\right)$ with $0 \leqslant k < p$. The resulting bitstring is being measured in the $\hat{Z}$–basis.

tackled efficiently on a quantum device using the QAOA. The circuit performing QAOA is schematically depicted in Fig. 2. The $|0\rangle^{\otimes N}$ state is brought to the ground state of the $H_X = \sum_i X_i$ mixing Hamiltonian by product of Hadamard gates $H^{\otimes N}$. Then p times repeats consecutive application of unitaries $\exp\left(i\alpha_k \hat{H}_X\right)$ and $\exp\left(i\beta_k \hat{H}_C\right)$ with $0 \leqslant k < p$. The resulting bitstring is being measured in the $\hat{Z}$–basis. This algorithm is inspired by the *annealing* approach. Starting the system in ground state of some "easy" Hamiltonian and evolving it adiabatically, we can end in the ground state of the final H_C Hamiltonian.

III. THE GAME PROCESS

We have created a game teaching players about the nature of the QAOA and allow them to compete against a quantum computer in optimization. The player is given a set of areas for possible investment. The effectiveness of investment in a particular area is also affected by the development of other areas, negatively of positively. For example, the opening of a new nuclear power plant makes the production of electric vehicles more profitable due to more affordable electricity. The whole picture with some areas "selected" and others "deselected" is referring to some *spin configuration* and the total energy can be recomputed after change of a particular spin. The goal is to find the configuration minimizing energy, i. e. to find the optimal way to develop some areas. The games comes with two different possible modes.

A. Standard mode

At the beginning of this game, we run QAOA at a selected *difficulty level* (number of circuit shots used for gradient estimation and final sampling of optimal bit string). The obtained best energy is shown to the player. The player starts with a random spin configuration and at each turn is allowed to flip one of spins. After each flip, energy of player's configuration string is recomputed ans is displayed. In this regime, the QAOA estimation can be perceived as the lower bound for energy, *however not necessarily the perfect solution.* Indeed, due to the approximation nature of QAOA and depending on the difficulty selected by the user there will be a certain probability the optimal value is just a local minima[1], and players can actually *outsmart* the given solution to beat the Quantum Player.

B. Cooperative mode

Imagine a system so big that it can not be tacked on a near-term device. To tackle such system, the developed game has the *cooperative mode*. In this mode, at each turn the player specifies the *subgraph* of a size feasible to the quantum hardware. In this setting, QAOA is only allowed to change variables on the selected subgraph $\mathcal{G}$. In this case, interaction with the environment renormalizes on-site terms and results in a slightly modified cost Hamiltonian

$$\hat{H}'_C = \sum_{i,j\in\mathcal{G}} p_{ij}\hat{Z}_i\hat{Z}_j + \sum_{i\in\mathcal{G}} \underbrace{\left(p_i + \sum_{j\notin\mathcal{G}} p_{ij}\hat{Z}_j\right)}_{\text{renormalization}} \hat{Z}_i. \quad (2)$$

The QAOA finds solution of the subgraph and updates spins. By such iterative user-guided subproblem solving, one aims at reaching the minimum energy. This mode showcases that, even in case of a problem too big to being tackled at available resources, quantum computer can even though be utilized on a subproblem.

IV. DISCUSSION

The game delivered could be a prototype of a more ambitious project. Our idea behind the *fights against climate change* scenario is to have realistic binary variables, with plausible values for on-site and pair terms. These values would be retrieved with the help of some climate and policy-maker expert, in order to deliver a final experience to the user as close as possible to effective reality. In this context, the standard mode would pose a much harder challenge where players can train themselves, while the cooperative mode would have an online *multiplayer leader-board* where player, aided by their quantum QAOA assistant, can compete each other whilst actually submitting innovative solution to a challenging real-world problem.

Few technicalities still have to be addressed for this further stage, with the main one relative to the *online* usage of quantum devices. We however hope this problem will be naturally solved in next years. We also hope this kind of initiatives will spread interest and knowledge toward recent quantum technologies, while exploiting them for common good.

[1] Notice that the most frequent eigenstates in the resulting wave function will still be close the the global minimum of the energy.