An educational game about the problem of climate change

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(Dated: Date: January 30, 2022)

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I. INTRODUCTION

Quantum computers promise to deliver an exponential speed up to various problems that are hard on the classical counterpart. One such class of problems is combinatorial optimization. To solve this type of problems classically, one is generally required to explore all possible configurations, which is in general exponentially hard in the number of degrees of freedom. On a quantum computer, the famous Quantum Approximate Optimization Algorithm (QAOA) promises possible ease of this complexity. During this hackathon, we have created a game that is aimed at helping to understand the challenges of the optimization problems and the benefits that quantum computing can offer. Many popular computer games, from the historical Monopoly to more advanced civilization games like Sid Mayer's Civilization, are based on the difficulty in finding optimal solutions to a resource allocation problem. Generally, the game is aimed to optimize a metric on a graph and has two game regimes: *standard* and *cooperative*. In the former, player is challenged by a *Quantum Artificial Player* which uses the QAOA to tackle the resource allocation problem. In the latter, quantum computer *helps* the player to solve an hard optimization problem.

Many challenges that the humanity is currently facing can be formulated in terms of a resource allocation problem. One of such challenges is the *fight against the climate change*. Even though our game is ran on an arbitrary graph, we specifically tailored one special case that illustrated applicability of QAOA to tackling this pressing issue.

In part, the game would educate quantum novices on possible usages and applications of *variational quantum algorithms*. At the same time, it will make the users aware of non-triviality of resource allocation problems including fighting the climate crisis.

II. GRAPH OPTIMIZATION PROBLEM AND QAOA CIRCUIT

In this project, we employ the cost Hamiltonian

$$\hat{H}_C = \sum_i p_i(1-\hat{Z}_i) + \sum_{i<j} p_{ij}(1-\hat{Z}_i)(1-\hat{Z}_j), \qquad (1)$$

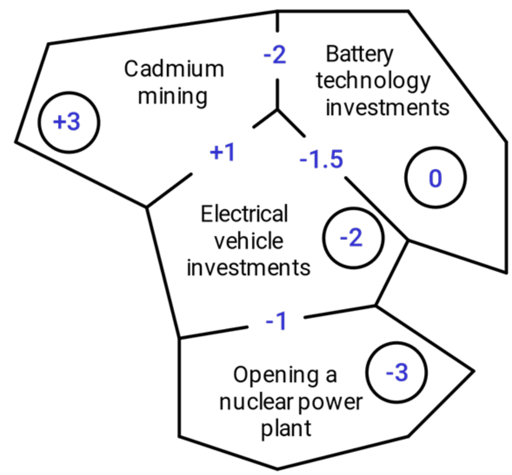

FIG. 1. An example set up of an optimization problem and its weights

where p_i are the *on-site* terms and p_{ij} are the *pair terms*. The meaning of the terms can be understood as follows. Let us assume that the $\hat{H}_C$ is some *goal* one needs to achieve. In our illustration, CO_2 emissions reduction. Some factors have a negative impact on the cost function *per se*, thus the on-site terms. Some factors have a *synergy* between them, which justifies the pair-terms. Here, Fig. 1 shows an example decisions, their "costs" and "synetgies" between them in the climate change setting.

The inspiration for the model Hamiltonian comes from the famous Ising model. Generally, such problem has no efficient solution classically, and finding the most optimal configuration with the "lowest energy" takes an exponential time with respect to the number of "decisions". However, such optimization problems can be tackled efficiently on a quantum device using the QAOA. The circuit performing QAOA is schematically depicted in Fig. 2. The $|0\rangle^{\otimes N}$ state is brought to the ground state of the $H_X = \sum\limits_i X_i$ mixing Hamiltonian by product of Hadamard gates $H^{\otimes N}$. Then p times repeats consecutive application of unitaries $\exp\left(i\alpha_k \hat{H}_X\right)$ and $\exp\left(i\beta_k \hat{H}_C\right)$ with $0 \leqslant k < p$. The resulting bitstring is being measured in the $\hat{Z}$–basis. This algorithm is inspired by the *annealing* approach. Starting the system in ground state of some "easy" Hamiltonian and evolving it adiabatically, we can end in the ground state of the final H_C Hamiltonian.

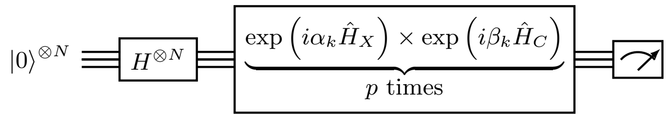

FIG. 2. Schematic representation of the QAOA algorithm. The $|0\rangle^{\otimes N}$ state is brought to the ground state of the H_X mixing Hamiltonian by product of Hadamard gates $H^{\otimes N}$. Then p times repeats consecutive application of unitaries $\exp\left(i\alpha_k \hat{H}_X\right)$ and $\exp\left(i\beta_k \hat{H}_C\right)$ with $0 \leqslant k < p$. The resulting bitstring is being measured in the $\hat{Z}$–basis.

III. THE GAME PROCESS

We have created a game teaching players about the nature of the QAOA and allow them to compete against a quantum computer in optimization. The player is given a set of areas for possible investment. The effectiveness of investment in a particular area is also affected by the development of other areas, negatively of positively. For example, the opening of a new nuclear power plant makes the production of electric vehicles more profitable due to more affordable electricity. The whole picture with some areas "selected" and others "deselected" is referring to some *spin configuration* and the total energy can be recomputed after change of a particular spin. The goal is to find the configuration minimizing energy, i. e. to find the optimal way to develop some areas We will describe two different modes for this game.

A. Standard mode

At the beginning of this game, we run QAOA at a selected *difficulty level* (number of circuit shots used for gradient estimation and final sampling of bit strings). The obtained best energy is shown to the player. The player starts with a random spin configuration and at each turn is allowed to flip one of spins. After each flip, energy of player's configuration string is recomputed ans is displayed. In this regime, the QAOA estimation can be perceived as the lower bound for energy, *however not necessarily the perfect solution.*

B. Cooperative mode

Imagine a system so big that it can not be tacked on a near-term device. To tackle such system, the developed game has the *cooperative mode*. In this mode, at each turn the player specifies the *subgraph* of a size feasible to the quantum hardware. In this setting, QAOA is only allowed to change variables on the selected subgraph $\mathcal{G}$. In this case, interaction with the environment renormalizes on-site terms and results in a slightly modified cost Hamiltonian

$$\hat{H}'_C = \sum_{i,j\in\mathcal{G}} J_{ij}(1-\hat{Z}_i)(1-\hat{Z}_j) + \sum_{i\in\mathcal{G}} () \,, \qquad (2)$$

Here, the user has the opportunity to improve their performance by using quantum computing for a certain part of the map, so he can apply QAOA to only a few areas and thereby get the minimum energy value after several iterations. This approach will help to work with large-sized systems by breaking them into subtasks, in addition, small-sized tasks can be successfully run on real quantum computers.