

1 Electronic Band Structure of Solids

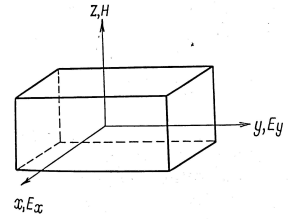
1.1 Free electron model

1.1.1 Drude model

- Electrons are free: no electron-electron and no electron-phonon interactions.
- Electrons collide with ionic cores with probability per unit time τ ; such collisions are instantaneous and inelastic (all of electron's momentum is transferred to the lattice).

In this approximation, the change in the momentum \mathbf{p} within dt is

$$d\mathbf{p} = \overbrace{\left(1 - \frac{dt}{\tau}\right) \mathbf{F} dt}^{\text{no collision: Newton's second law}} + \underbrace{\frac{dt}{\tau} (-\mathbf{p})}_{\text{collision: all momentum transferred to the lattice}}$$

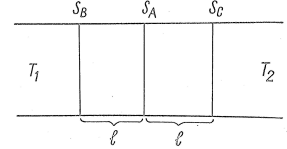


Hence, the equation of motion is $d\mathbf{p}/dt = -\mathbf{p}/\tau + \mathbf{F}$.

Assume that a field \mathbf{E} is applied along a metallic wire. The field acts on the electron with a force $\mathbf{F} = -e\mathbf{E}$. In the steady-state case ($d\mathbf{p}/dt = 0$), we get $\mathbf{v} = -e\mathbf{E}\tau/2m$. The current density is thus $\mathbf{j} = -ne\mathbf{v} = ne^2\tau\mathbf{E}/2m$, and the specific electric conductivity is $\sigma = ne^2\tau/2m$.

Suppose now that our wire (aligned along x -axis) is placed in a magnetic field \mathbf{H} oriented along the z -axis. An electron is then subject to the sum of Coulomb and Lorentz forces $\mathbf{F} = -e\mathbf{E} - e/c\mathbf{v} \times \mathbf{H}$. Hall voltage is developed in the y -direction and Hall coefficient $R_H \equiv E_y/j_x H = -1/ne$.

The Drude model achieved complete success in the derivation of the Wiedemann-Franz law, according to which the ratio of thermal and electrical conductivities is a universal linear function of temperature for many metals: $\kappa/\sigma = LT$ (L here is called Lorentz number). Say, T depends on x only. Then the heat transfer $W = \kappa \partial T / \partial x$. The number of electrons passing through unit cross section S_A per unit time is $n|v_x|/2$. S_B and S_C are cross sections on which the electrons, on average, have suffered the last collisions. $l = 2\tau\bar{v}/3$ is the mean-free path¹. From the equipartition of energy, we get $1/2 m \bar{v}^2 = 3/2 k_B T(x)$.² Hence,



$$W = \frac{1}{2} n |v_x| \left(\left. \frac{mv^2}{2} \right|_{x-l} - \left. \frac{mv^2}{2} \right|_{x+l} \right) \approx \frac{1}{2} n k_B \bar{v}^2 \tau \frac{\partial T}{\partial x}$$

and we get that $\kappa = 3k_B^2 n \tau T / 2m$. Finally, $\kappa/\sigma = 3(k_B/e)^2 T$.

¹This is known from the kinetic theory of gases.

² \bar{v} is the average velocity and we assume that $|v_x|^2 \approx \bar{v}^2/3$

1.1.2 Sommerfeld model

1.2 Nearly-Free electron model

1.3 Tight-binding model

2 References