Kolmogorov's Theorem

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Our goal is to understand Kolmogorov's theorem:

Theorem 0.1 (Kolmogorov's Theorem)**.** *Let $\rho, \gamma > 0$ be given, and let $h(\boldsymbol{q}, \boldsymbol{p}) = h_0(\boldsymbol{p}) + h_1(\boldsymbol{q}, \boldsymbol{p})$ be a Hamiltonian, with $h_0, h_1 \in \mathcal{A}_\rho$ and $\|h\|_\rho \leq 1$. Suppose the Taylor polynomial of h_0 is*

$$h_0(\boldsymbol{p}) = a + \omega \boldsymbol{p} + \frac{1}{2}\boldsymbol{p} \cdot C\boldsymbol{p} + o(|\boldsymbol{p}|^2),$$

with $\omega \in \Omega_\gamma$ and C is symmetric and invertible. Then for any $\rho_* \leq \rho$, there exists $\epsilon > 0$, which depends on C and γ, but not on the remainder term in $o(|\boldsymbol{p}|^2)$, such that if $\|h_1\|_\rho \leq \epsilon$, there exists a symplectic mapping $\Phi : A_{\rho_*} \to A_\rho$ such that if we set $(\boldsymbol{q}, \boldsymbol{p}) = \Phi(\boldsymbol{Q}, \boldsymbol{P})$ and $H = h \circ \Phi$, we have

$$H(\boldsymbol{Q}, \boldsymbol{P}) = A + \omega \boldsymbol{P} + R(\boldsymbol{Q}, \boldsymbol{P}),$$

with $R(\boldsymbol{Q}, \boldsymbol{P}) \in O(|\boldsymbol{P}|^2)$.

To read this theorem, we require some notation.