

# The Kolmogorov Theorem

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## 1 Introduction

### Abstract

This paper gives a proof of the Kolmogorov Theorem on the conservation of invariant tori. We follow the approach given by Hubbard and Ilyashenko in .

**Theorem 1.1** (The Kolmogorov Theorem). *Let  $\rho, \gamma > 0$  be given, and let  $h(\mathbf{q}, \mathbf{p}) = h_0(\mathbf{p}) + h_1(\mathbf{q}, \mathbf{p})$  be a Hamiltonian, with  $h_0, h_1 \in \mathcal{A}_\rho$  and  $\|h\|_\rho \leq 1$ . Suppose the Taylor polynomial of  $h_0$  is*

$$h_0(\mathbf{p}) = a + \omega \mathbf{p} + \frac{1}{2} \mathbf{p} \cdot C \mathbf{p} + o(|\mathbf{p}|^2),$$

*with  $\omega \in \Omega_\gamma$  and  $C$  is symmetric and invertible. Then for any  $\rho_* \leq \rho$ , there exists  $\epsilon > 0$ , which depends on  $C$  and  $\gamma$ , but not on the remainder term in  $o(|\mathbf{p}|^2)$ , such that if  $\|h_1\|_\rho \leq \epsilon$ , there exists a symplectic mapping  $\Phi : A_{\rho_*} \rightarrow A_\rho$  such that if we set  $(\mathbf{q}, \mathbf{p}) = \Phi(\mathbf{Q}, \mathbf{P})$  and  $H = h \circ \Phi$ , we have*

$$H(\mathbf{Q}, \mathbf{P}) = A + \omega \mathbf{P} + R(\mathbf{Q}, \mathbf{P}),$$

*with  $R(\mathbf{Q}, \mathbf{P}) \in O(|\mathbf{P}|^2)$ .*

*Proof.* The proof is left as an exercise for the reader.

QED