

Regions paper

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ABSTRACT

The aim is to test whether the claims that the polar field at the end of Cycle 23 was weakened by a small number of large, abnormally oriented regions are reasonable, and consequently to investigate what this means for solar cycle prediction. ***WHAT ELSE?***

Keywords: magnetohydrodynamics (MHD) — Sun: activity— Sun: photosphere — Sun: sunspots

1. INTRODUCTION

There is a strong correlation between the strength of the polar field at cycle minimum and the strength of the following cycle (e.g., Schatten et al. 1978; Muñoz-Jaramillo et al. 2013). This relationship calls for accurate predictions of the polar field. Common methods for simulating the evolution of the radial magnetic field at the surface include using dynamo models (for a review, see Charbonneau 2014), but surface flux transport (SFT) models (Wang et al. 1989; Baumann et al. 2004; Sheeley 2005; Jiang et al. 2010; Mackay & Yeates 2012; Upton & Hathaway 2014), introduced in the 1960s (Babcock 1961; Leighton 1964), have risen in popularity over the last decade due to their relative simplicity and accuracy.

Surface flux transport models describe the evolution of magnetic regions on the solar surface, which appear due to the rise of buoyant flux tubes (Fan 2009). Generally they emerge with a leading polarity and an opposing trailing polarity with respect to the east-west direction, and so are known as bipolar magnetic regions (BMRs). There is hemispheric asymmetry in the leading polarities, which are generally the same across a hemisphere, according to Hale’s polarity law (Hale 1924). Helical convective motions in the solar interior impart a tilt to each BMR with respect to the east-west line (the line that connects the centres of the opposing polarities), with the leading polarity located closer to the equator. The effect is stronger at higher latitudes ac-

cording to Joy’s law (Howard 1991), and a sinusoidal fit for the relationship between tilt angle α and latitude λ is $\alpha = 32.1 \sin \lambda$ (Stenflo & Kosovichev 2012), although it should be noted that there is significant variation between different regions. These deviations from Joy’s Law could be the key characteristics in determining polar field strength at cycle minimum, as discussed below.

Leading polarities then diffuse across the surface by being pushed to the edges of convection cells (Leighton 1964). This can lead to cross-equatorial flux cancellation, and any remaining trailing flux is transported polewards by a combination of diffusion and a meridional flow (Howard 1979) of the order $\sim 10\text{--}20 \text{ m s}^{-1}$ (e.g. Komm et al. 1993; Jackiewicz et al. 2015). It is this built-up polar field which provides an early insight into the amplitude of the following cycle.

Of particular interest is the unusually weak and deep-seated polar field (and equivalently weak axial dipole moment) at the end of Cycle 23 (Muñoz-Jaramillo et al. 2012), which in turn is believed to be responsible for the low amplitude of Cycle 24. Jiang et al. (2015) used the BMR data of Li & Ulrich (2012) to investigate the effect of tilt angle on axial dipole moment contribution, using an empirical relation involving tilt angle, latitude and area (Jiang et al. 2014):

$$\Delta = A^{\frac{3}{2}} \sin \alpha \exp \left(-\frac{\lambda^2}{110} \right), \quad (1.1)$$

where A is the area, α is the tilt angle, and λ is the latitude of each region. They found that axial dipole moment contributions from observed tilt angles from Cycle 23 follow those from when Joy’s Law is assumed

at latitudes above $\pm 10^\circ$. Nearer the equator, the regions with observed tilt angles contribute substantially less than would be expected assuming Joy's Law, contrary to the behaviour of Cycles 21 and 22, which follow the Joy's Law contributions more closely at all latitudes. This led to the conclusion that a single large anti-Hale or anti-Joy region emerging at a low latitude, or across the equator (Cameron et al. 2013, 2014), has the ability to significantly alter the dipole moment, and this could have been the catalyst behind the weak polar field at the end of Cycle 23. Therefore the stochasticity behind the properties of emerging regions provides a problem for those attempting to predict the amplitude of future cycles, especially given that the magnetic flux in a large active region is similar to the total polar flux (Wang & Sheeley 1991). With this in mind, it may not be possible to make reliable predictions until the end of the cycle, unless random fluctuations of active region properties are taken into account. Indeed, Nagy et al. (2017) recently demonstrated in a 2×2D dynamo model that large 'rogue' regions can drastically affect the evolution of future solar cycles and introduce hemispheric asymmetries. Such large regions emerging during the early phases of a cycle can even affect the amplitude and duration of the same cycle. In this particular dynamo model, the effect of a single region can persist for multiple cycles. Nagy et al. (2017) found that the effect of a region in their model is dependent on its axial dipole moment at time of emergence, which is in turn approximated by Equation 1.1. So bipolar regions near the equator, and/or with large tilt angle, are particularly strong contributors, although significant effects were found for regions even up to $\pm 20^\circ$ latitude.

In this paper we investigate these claims further by simulating the evolution of real active regions from Cycles 21, 22 and 23 using a 2D SFT model^a with an automated region identification and assimilation process (Yeates et al. 2015). This allows us to identify particular observed properties which could have defined the contribution of each region to the axial dipole moment. In this paper, the emerging regions are determined from NSO line-of-sight magnetograms. In Section 2 we analyse said properties of regions. In Section 3 we discuss the implications of our findings for future cycle predictions, before concluding in Section 4.

2. CONTRIBUTIONS OF INDIVIDUAL REGIONS TO THE AXIAL DIPOLE MOMENT

We investigate the distribution of various magnetic region properties, namely latitude, magnetic flux, and

initial and final axial dipole moment. The regions and their properties are extracted from NSO Kitt Peak and SOLIS synoptic magnetograms using the 2D SFT model described in Yeates et al. (2015). The radial component of the magnetic field in 2D, $B(\theta, \phi, t)$, evolves according to:

$$\begin{aligned} \frac{\partial B}{\partial t} = & -\omega(\theta) \frac{\partial B}{\partial \phi} + \frac{\eta}{R_\odot^2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial B}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 B}{\partial \phi^2} \right] \\ & - \frac{1}{R_\odot \sin \theta} \frac{\partial}{\partial \theta} \left(v(\theta) \sin \theta B \right) - \frac{1}{\tau} B + S(\theta, \phi, t), \end{aligned} \quad (2.1)$$

where R_\odot is solar radius, $\omega(\theta)$ represents differential rotation, η is turbulent diffusivity, representing the diffusive effect of granular convective motions, τ is an exponential decay term added by Schrijver et al. (2002) to improve regular polar field reversal, and $S(\theta, \phi, t)$ is a source term for newly emerging magnetic regions. The profile $v(\theta)$ describes poleward meridional flow, which we define using the following functional form:

$$v(\theta) = -v_0 \sin^p \theta \cos \theta, \quad (2.2)$$

where p determines the latitude of peak velocity and low-latitude gradient, and the amplitude v_0 is chosen to be the maximum absolute velocity. In order to define the initial conditions, we use the profile of Svalgaard et al. (1978):

$$B(\theta, 0) = B_0 |\cos \theta|^7 \cos \theta, \quad (2.3)$$

where B_0 is the initial field strength. The 2D model also contains a parameter B_{par} which acts as a threshold: only regions with magnetic flux above this value are assimilated into the simulation. All simulations are performed using the optimal values for diffusivity, meridional flow, initial field strength, exponential decay and assimilation threshold, which are obtained systematically using the genetic algorithm PIKAIA^{b,c} (Charbonneau & Knapp 1995; Metcalfe & Charbonneau 2003), as described by Whitbread et al. (2017). These are shown in Table 1, with associated 'acceptable ranges' below each entry. Note that we keep these parameters fixed across the three cycles, and that B_0 is the initial field strength at the start of Cycle 21; each other cycle immediately follows on from the final state of the preceding cycle. We use the optimal value of $B_{\text{par}} = 39.8$ G found in Whitbread et al. (2017). 'Optimal' in this sense refers to the ability to best match the simulated and observed

^a https://github.com/antyeates1983/sft_data

^b <http://www.hao.ucar.edu/modeling/pikaia/pikaia.php>

^c <http://www.hao.ucar.edu/Public/about/Staff/travis/mpikaia/>

Table 1. Optimal parameter set for the simulation shown in Figure 1. Upper and lower bounds for acceptable parameter ranges are given in square brackets below each entry.

η ($\text{km}^2 \text{s}^{-1}$)	v_0 (m s^{-1})	p	τ (yr)	B_0 (G)
466.8	9.2	2.33	10.1	6.7
[325.7, 747.3]	[5.6, 11.9]	[1.12, 3.95]	[3.6, 31.9]	[0.0, 15.0]

butterfly diagrams, and the optimal butterfly diagram for Cycles 21 to 23 is shown in Figure 1. It should be stressed therefore that all conclusions made in this paper are with respect to these parameter values. For differential rotation, the parametrization of [Snodgrass & Ulrich \(1990\)](#) is used:

$$\omega(\theta) = 0.521 - 2.396 \cos^2 \theta - 1.787 \cos^4 \theta \text{ deg day}^{-1}. \quad (2.4)$$

Here we include an exponential decay term of the form $-\frac{1}{\tau}B$. [Baumann et al. \(2006\)](#) offered a physical explanation for the extra term: it is the effect of inward radial diffusion of flux into the convection zone, which is not directly accounted for in the SFT model. Although it was not originally included in this particular 2D model ([Yeates et al. 2015](#)), and [Whitbread et al. \(2017\)](#) showed that it is not necessary in this model for replicating the evolution of the dipole moment for an *individual* cycle, we use it here to ensure regular polar field reversal over multiple cycles and to justify comparison with [Nagy et al. \(2017\)](#), who also utilized such a term. Later we present the case without decay and show that similar conclusions hold in both regimes.

The axial dipole moment of region i is given by:

$$D_i(t) = \frac{3}{2} \int_0^\pi \int_0^{2\pi} B_i(\theta, \phi, t) \cos \theta \sin \theta d\phi d\theta. \quad (2.5)$$

To aid visualization we will also use the relative axial dipole moment D_{rel} , which is defined as a region's axial dipole moment divided by the total change in axial dipole moment over the whole cycle:

$$D_{\text{rel}}(t) = \frac{D_i(t)}{D_{\text{tot}}(T)}, \quad (2.6)$$

where T is the time of the end of the cycle. The simulation is repeated and D_{rel} is calculated for each individual region. The relative axial dipole moment D_{rel} then reflects the proportional contribution of each region. A positive D_{rel} corresponds to a strengthening of the axial dipole moment at the end of the cycle, whilst a negative D_{rel} corresponds to a weakening. ***Explain methodology in more detail***.

Note that most SFT simulations, including [Jiang et al. \(2015\)](#), assume that all regions are BMRs with a simple

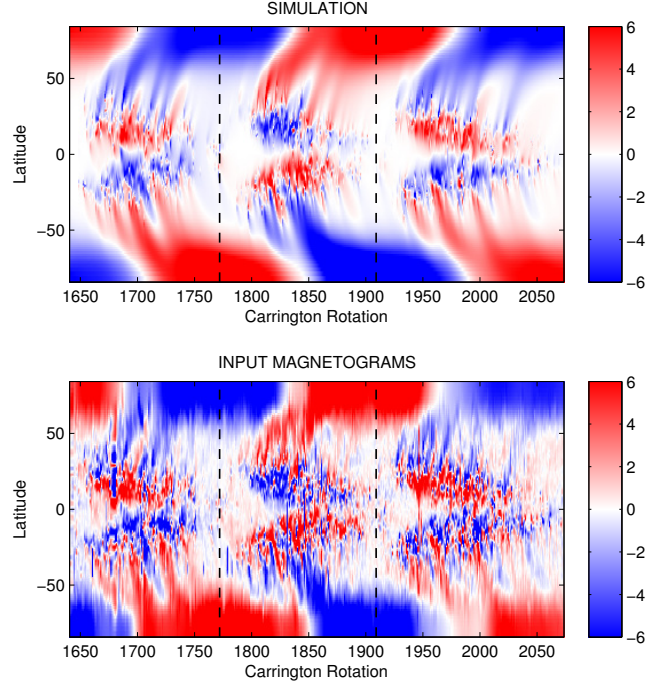


Figure 1. Top: Optimal butterfly diagram for Cycle 21 through to Cycle 23, simulated using the parameters from Table 1. Bottom: ‘Ground truth’ data for the same period. Vertical dashed lines indicate start/end points of cycles as used in this paper. ***CHANGE X-AXIS***

bipolar structure. However in our 2D model is this not always the case. The model inserts specific shapes of active regions, meaning that complex multipolar configurations are often assimilated. Figure 2 shows a couple of examples of such configurations. Whilst some regions are clearly bipolar, some are less clear and are harder to separate into BMRs even manually, let alone using an automated algorithm. Because of this, a ‘tilt angle’ is no longer a sensible measure, and so instead we opt to use the initial (relative) axial dipole moment which still takes into account orientation and polarity. Similarly, we also do not consider polarity separation distance. Another consequence of this is that the model considers a cluster of active regions to be one single large region, whilst human observers would usually separate a sunspot group into multiple BMRs. Hence, for the optimal threshold B_{par} , we tend to extract fewer regions per cycle than other studies. Despite this, the insertion of actual configurations of active regions combined with the optimization procedure means that the overall evolution of the observed axial dipole moment D_{tot} is well reproduced by the simulation, as shown in Figure 11 of [Whitbread et al. \(2017\)](#), even though the axial dipole moment is not considered directly in the fitness function.

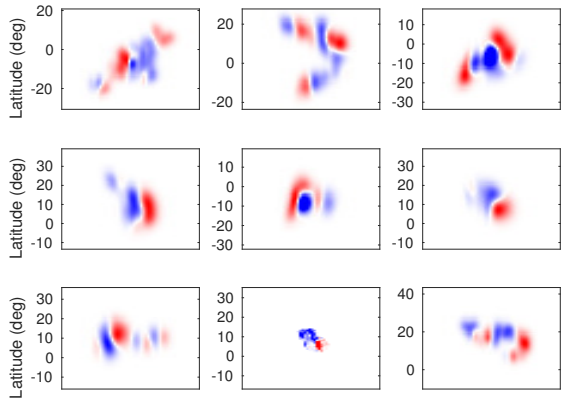


Figure 2. 9 strong regions from Cycle 23.

2.1. Location and size

Initially we consider the effects of emergence latitude, flux and initial D_{rel} on the axial dipole moment contribution of each region. The top panels of Figure 4 show the relationships between final D_{rel} and these three quantities respectively for the regions from Cycle 21 (1st May 1976–10th March 1986). We find that most significant contributors to the axial dipole moment emerge below $\pm 20^\circ$, the very largest of which emerge below $\pm 10^\circ$. We also find that these regions do not necessarily have strong levels of magnetic flux; very few of the biggest contributors are stronger than 1.5×10^{22} Mx.

Furthermore, we find that the relationship between initial and final D_{rel} is largely latitude-dependent: regions emerging at mid-latitudes (dark purple) tend to contribute little to the final axial dipole moment, regardless of their initial values. Conversely, regions emerging at low latitudes (yellow and orange) can undergo an increase in axial dipole moment contribution as cross-equatorial flux cancellation occurs and flux is transported polewards by the meridional flow.

The central line of panels of Figure 4 shows the same relationships as discussed above but for Cycle 22 (10th March 1986–1st June 1996). The left and middle panels tell a different story to that of Cycle 21. There are fewer big contributions (i.e. contributions of more than 2.5%) to the axial dipole moment, and the largest is a strengthening rather than a weakening as in Cycle 21. This provides an explanation as to why the axial dipole moment increased in amplitude during Cycle 22. This region is also the only significant contributor to lie below $\pm 10^\circ$, although the others still emerge below $\pm 20^\circ$ as in Cycle 21. The most striking difference between the two cycles is the effect of strong-flux regions. ***In Cycle 22 some of the most significant contributions to

the axial dipole moment come from regions with fluxes above 3×10^{22} Mx, which is not the case in Cycle 21***.

The same latitudinal dependence of the initial to final D_{rel} relationship is found as in Cycle 21, supporting the idea that latitude of emergence plays an important role in determining whether a region will contribute significantly to the polar field. ***Furthermore, the incidence of more regions emerging at higher latitudes (i.e. at cycle maximum) suggests a stronger cycle. This can suppress any tilt imparted by the Coriolis force during emergence, which could lead to fewer large axial dipole moment contributions, as we find in Figure 4. This is in agreement with the anti-correlation between cycle strength and tilt angle found by Dasi-Espuig et al. (2010).***

The bottom three panels of Figure 4 show the same three distributions as before but for Cycle 23 (1st June 1996–3rd August 2008). We return to a similar regime to Cycle 21; of the most significant contributors, we observe more regions which weaken the axial dipole moment, and the biggest contributors have fluxes smaller than 2×10^{22} Mx. Again, most of these regions emerge below $\pm 20^\circ$. If we consider the sign, we find that the most significant regions in Cycle 23 induce a weakening of the overall axial dipole moment. These low-latitude regions could indeed be the cause of the weak polar field at the end of Cycle 23, and hence the low amplitude of Cycle 24, as suggested by Jiang et al. (2015). The latitude-dependent relationship between initial and final D_{rel} still also holds in Cycle 23. In fact, we can separate the regions into bins of 5° and calculate the gradient of the lines in the right-hand panels of Figure 4 for each bin. We then see in Figure 5 that down to $\pm 20^\circ$ the relationship between initial and final D_{rel} is practically identical across the three cycles, and even down to $\pm 5^\circ$ the relationships over the three cycles are equivalent within error bars. However, for the 0 – 5° bin, the gradient is much steeper for Cycle 23 ***WHY??***. What should be noted is that the standard errors for these fits are very small, indicating that there is clearly a strong correlation between the overall amplification in D_{rel} and latitude of emergence. ***Talk about Jiang’s functional form and Anthony’s similar findings, i.e. exponential latitudinal dependence?***

2.2. Number of regions required

Figure 6 shows the effect of removing the smallest contributions to the axial dipole moment from the simulation of Cycle 21. Again, incorporating 750 regions makes only a little difference (a decrease of 3%), but using 500 regions corresponds to a reduction of 10% of the axial dipole moment. However, this is still a less extreme re-

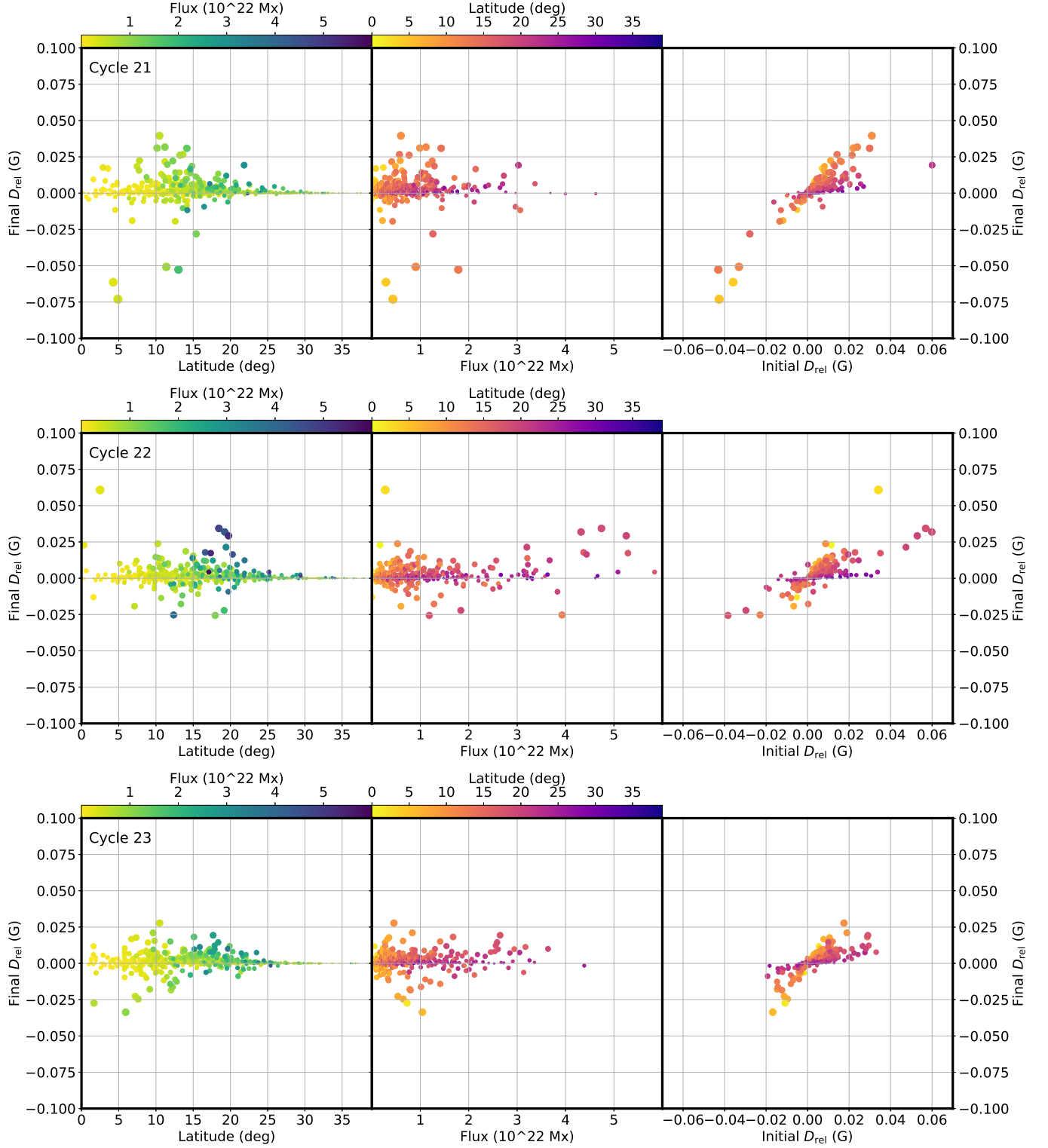


Figure 3. Final D_{rel} for each region against latitude (left panels), flux (middle panels) and initial D_{rel} (right panels). Markers are sized by final D_{rel} , and coloured by flux (left panels) and latitude (middle and right panels).

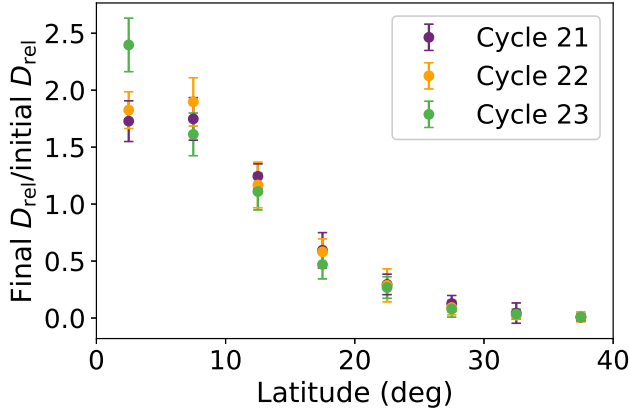


Figure 4. Ratio between final D_{rel} and initial D_{rel} for 5° latitudinal bins for Cycles 21 (purple), 22 (orange) and 23 (green). Error bars show squared standard error. Markers are plotted at the midpoint of each bin.

duction compared to Cycle 23. For the same cycle, 21, Wang & Sheeley (1991) found that about 54% of the axial dipole moment came from about 10.7% of regions, and here we find a similar result (blue curve). In fact, we find the same outcome for Cycle 22 but not for Cycle 23.

Figure 6 shows the effect of removing the smallest contributions to the axial dipole moment from the simulation of Cycle 22. The effect of removing small contributions is not as strong here; as few as 500 regions can be used with a shortfall of just 3.4%. In contrast to Cycle 23, using 750 regions makes very little difference to the evolution of the axial dipole moment. ***One reason for this is that 750 makes up a greater proportion of the regions from Cycle 22 compared to Cycle 23***, but ultimately it comes down to the polarity distribution of regions with a small contribution to the axial dipole moment.

Figure 6 shows the effect of removing the smallest contributions to the axial dipole moment from the simulation of Cycle 23. Even when the largest 750 of the 951 regions are assimilated, there is a large discrepancy between the resulting axial dipole moment and the axial dipole moment obtained using every region. One explanation of this is that most of the least significant regions from Cycle 23 strengthen the axial dipole moment, and the cumulative effect of removing these causes it to weaken. So although a small number of regions have a disproportionate effect, the cumulative contribution of the many regions with weaker dipole moment cannot be ignored.

2.3. Location and time

Figure 7 shows the corresponding butterfly diagrams of Cycle 21 for the cases shown in the first section of Figure 6, where border colours match profile colours. We find fewer strong regions that have emerged across the equator, but from Figure 4 we see that there are a couple of regions with strong initial D_{rel} , suggesting that large contributors from Cycle 21 are likely to be because of orientation reasons rather than being cross-equatorial. There is a cluster of negatively contributing regions in the northern hemisphere around 1983 which is not succeeded by many significant regions during the remainder of the cycle; this cluster could be responsible for a lower axial dipole moment in Cycle 21 (compared to Cycle 22), and is certainly why the polar field fails to reverse when only 10 regions are used in Cycle 21, as seen in Figure 6.

Figure 8 shows the corresponding butterfly diagrams of Cycle 22 for the cases shown in the second section of Figure 6, where border colours match profile colours. As inferred from Figure 4, the majority of large contributions to the axial dipole moment in Cycle 22 enhance the dipole moment, but are clustered around -20° . However, there are two regions at low latitudes, possibly cross-equatorial, which would support the claim of Cameron et al. (2013): that regions emerging across the equator can change the amount of net flux in each hemisphere, in turn weakening or strengthening the axial dipole moment, meaning future cycle predictions will be less reliable.

Figure 9 shows the corresponding butterfly diagrams of Cycle 23 for the cases shown in the third section of Figure 6, where border colours match profile colours. Significant negatively-contributing regions include a cluster across the equator around 2002, and a group regions in the southern hemisphere towards the end of the cycle, visible as blue patches in all but the bottom-right frame. While the cross-equatorial group is important for reasons discussed above, the majority of regions in the late-emerging cluster might not have as significant an effect on the current cycle as if they had instead emerged earlier in the cycle, as discussed by Nagy et al. (2017). Nevertheless, by analysing Cycles 21 and 23 we see that a lack of disruption from a major cross-equatorial region in Cycle 21 led to a stronger axial dipole moment compared to Cycle 23.

3. HOW THOROUGH DO WE NEED TO BE?/WHAT ARE THE IMPLICATIONS FOR MAKING PREDICTIONS?

Figure 10 displays the change in D_{rel} as more active regions are included in the simulation (ordered by flux as opposed to D_{rel} above) for Cycles 21 (green), 22 (orange)

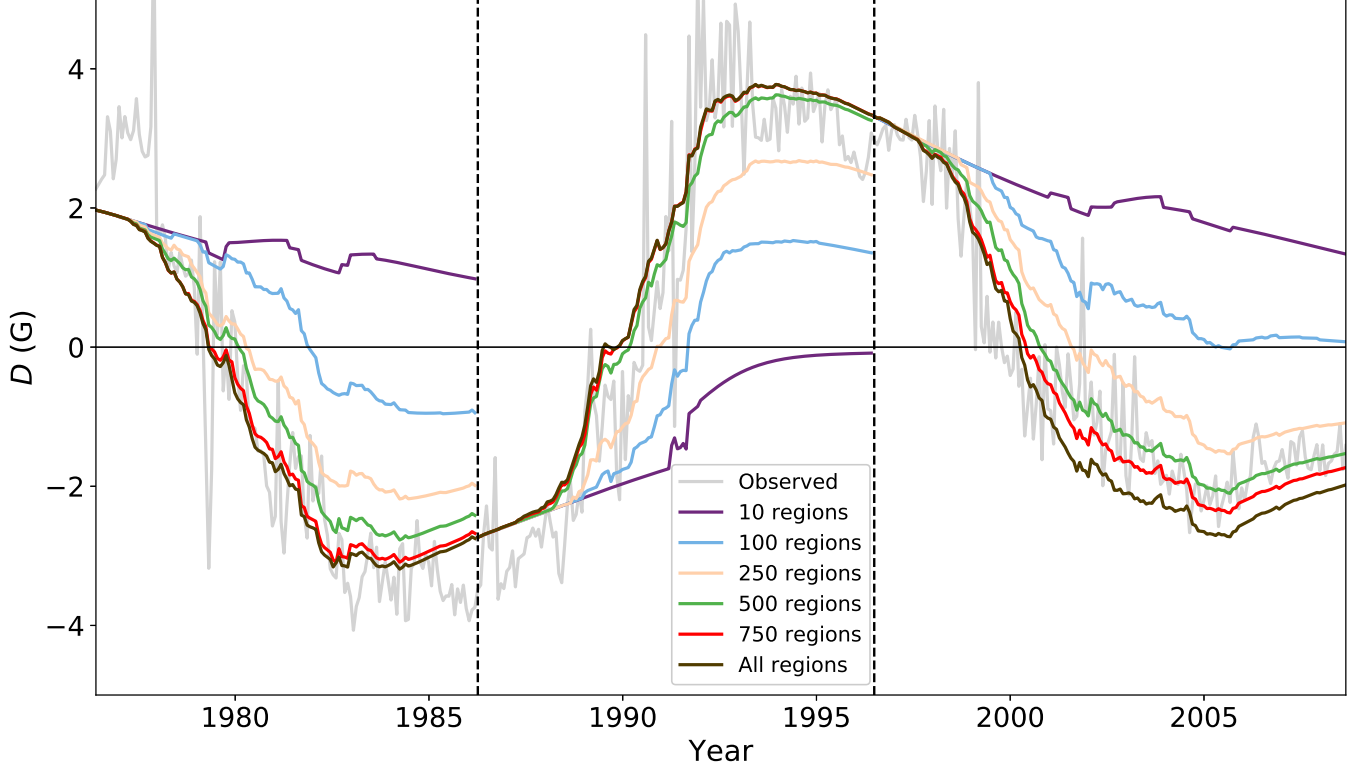


Figure 5. Evolution of the axial dipole moment for Cycles 21 to 23. Each profile is obtained by only using a certain number of the biggest contributors to the axial dipole moment. The legend shows the number of regions from each cycle per profile. The light grey curve shows the observed axial dipole moment. Vertical dashed lines indicate start/end points of cycles as used in this paper.

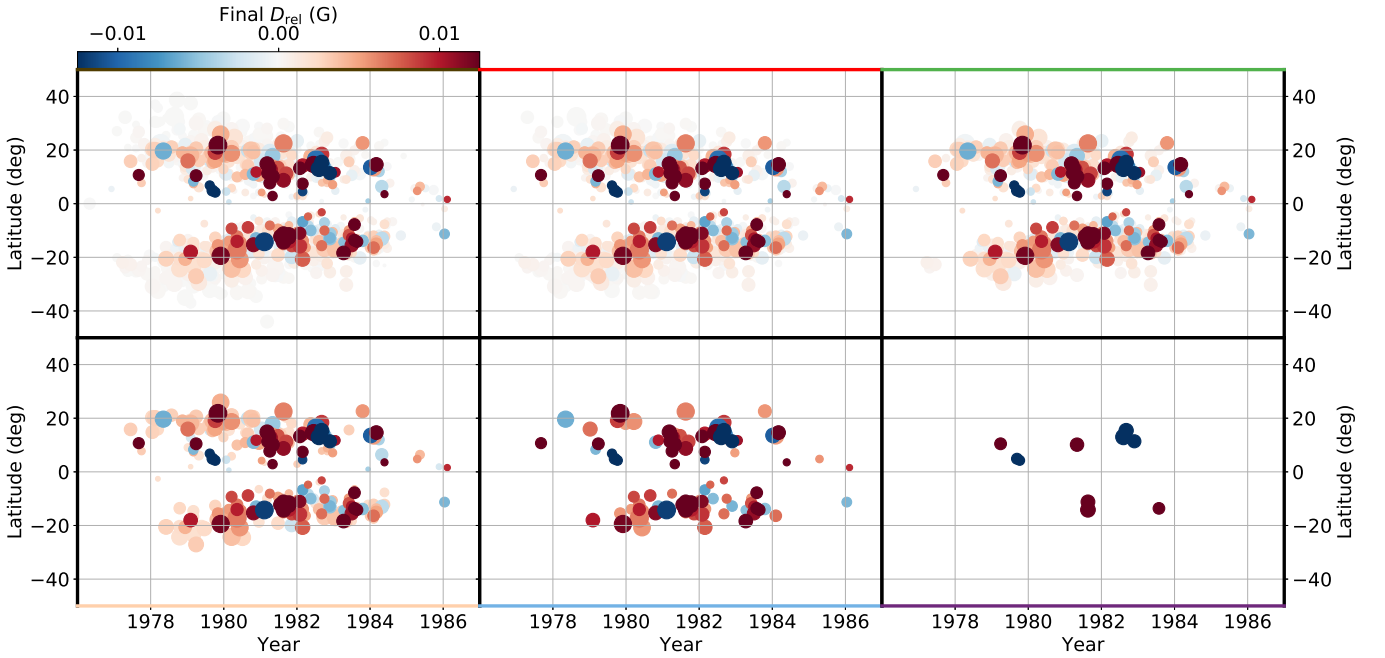


Figure 6. Time-latitude distributions of regions used to obtain the profiles in the first section of Figure 6 (profile colours match border colours). Markers are sized by flux and coloured by final D_{rel} .

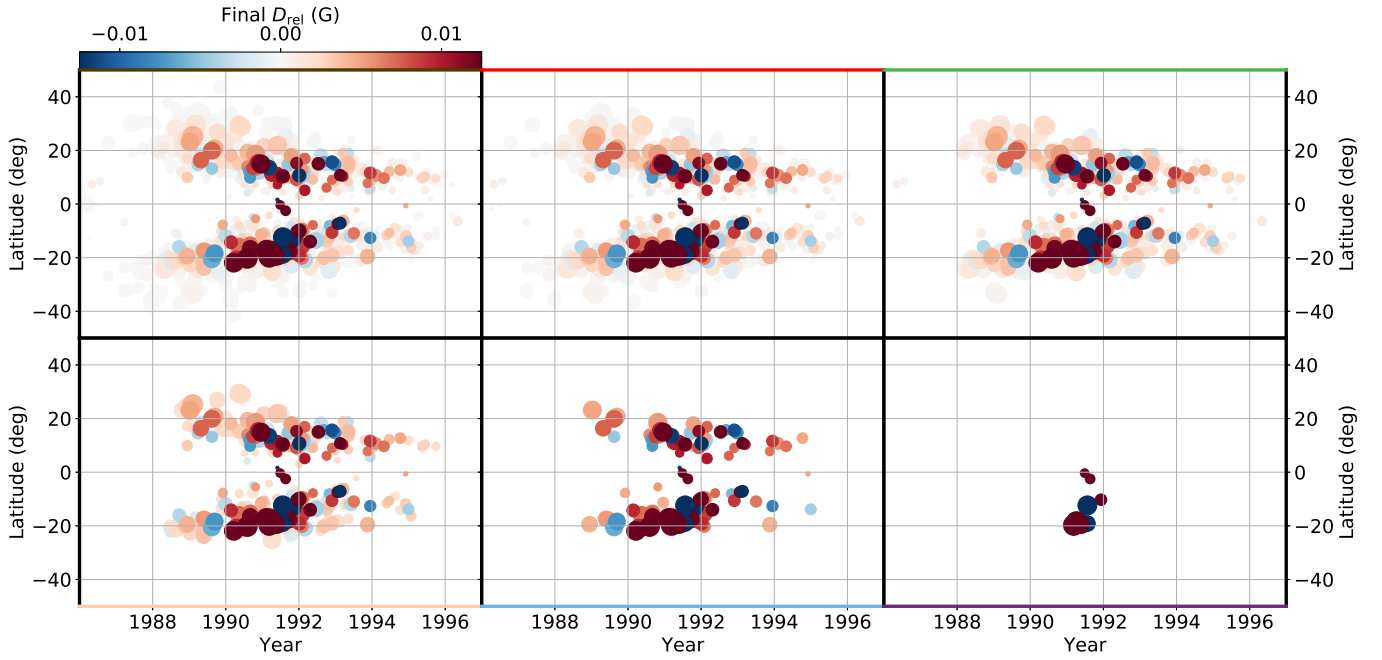


Figure 7. Time-latitude distributions of regions used to obtain the profiles in the second section of Figure 6 (profile colours match border colours). Markers are sized by flux and coloured by final D_{rel} .

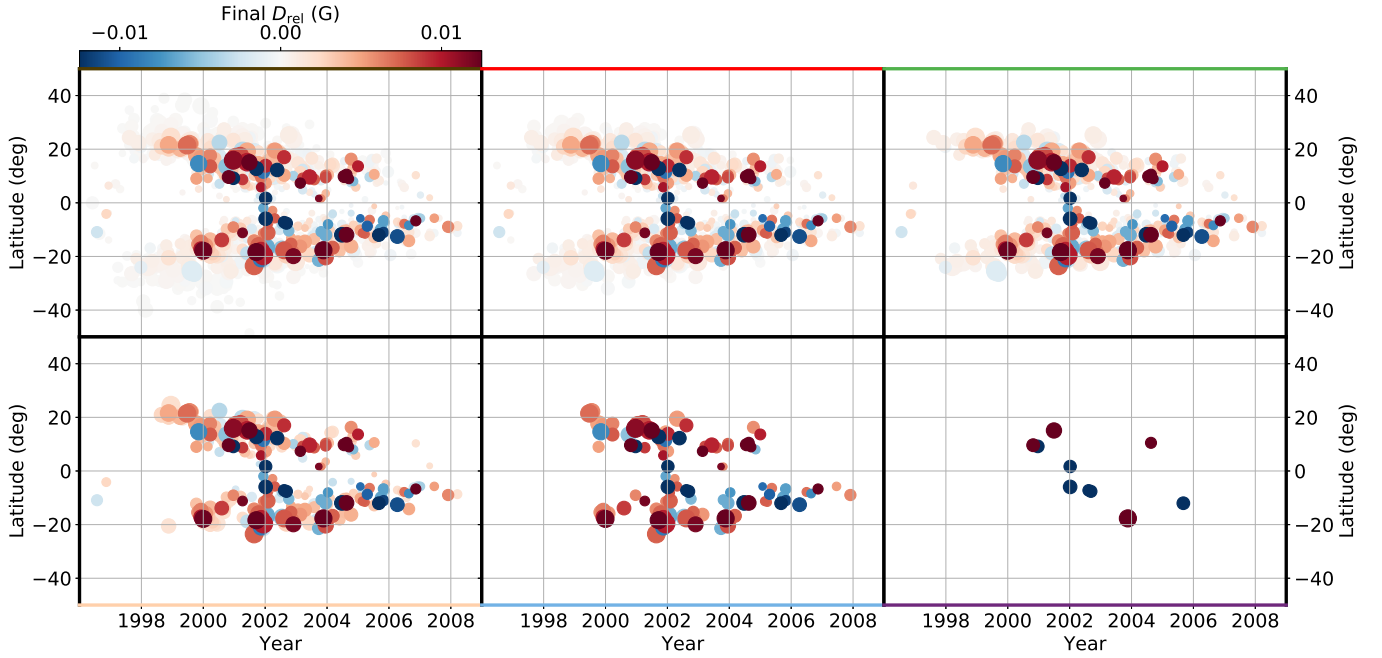


Figure 8. Time-latitude distributions of regions used to obtain the profiles in the third section of Figure 6 (profile colours match border colours). Markers are sized by flux and coloured by final D_{rel} .

and 23 (purple). This further highlights the distinctions between the three cycles.

There are many more regions with large flux that contribute positively to the axial dipole moment during Cycle 21. Because of this, 80% of the axial dipole moment is attained when less than 40% of regions are considered (bearing in mind the threshold for the top 40% is $\sim 4\text{--}4.5 \times 10^{21}$ Mx depending on the cycle). There is then a sharp decrease when the two biggest contributions are included, before the 80% mark is reached again, corresponding to half the number of regions being used.

The D_{rel} of Cycle 22 rises at a steady rate as more regions are added, but there are two clear phases with a large jump in between. One can attribute this jump to the inclusion of the largest contributor of Cycle 22, which is weak in terms of flux (see Figure 4). Because of this significant addition to the dipole moment, using 55% of regions is enough to ensure that 80% of the axial dipole moment is reached. In this sense Cycles 21 and 22 behave in a similar manner.

However, the profile for Cycle 23 initially struggles to contribute much to the final D_{rel} , presumably due to the most significant regions which are weakening the polar fields, and requires many smaller regions to enhance the axial dipole moment. In fact, 70% of regions are needed to accumulate 80% of the axial dipole moment. This mimics the problem found in Figure 6; Cycle 23 is largely dominated by negative D_{rel} active regions, meaning simulations of Cycle 23 require more input regions than others to fully capture its behaviour.

We hope this will give insight to those wishing to model full solar cycles, in particular giving an indication as to what resolution is sufficient (or equivalently how small regions are allowed to be in order to be ignored), to give an acceptable representation of the axial dipole moment. It may be noteworthy that when 90% of regions are used, all three cycles reach a similar relative level close to $D_{\text{tot}}(T)$.

4. CONCLUSIONS

Our aim was to extract active region properties from magnetograms using a 2D model with an automated region assimilation technique, and analyse the relationships between these properties and the evolution of the axial dipole moment. We examined how the final relative axial dipole moment D_{rel} is affected by a region's latitude, flux and initial D_{rel} , and compared these relationships across Cycle 21, 22 and 23.

We found some consistent results across all three cycles. Generally all large contributions to the axial dipole moment emerge below $\pm 20^\circ$, with the largest emerging below $\pm 10^\circ$. This supports the idea that regions emerg-

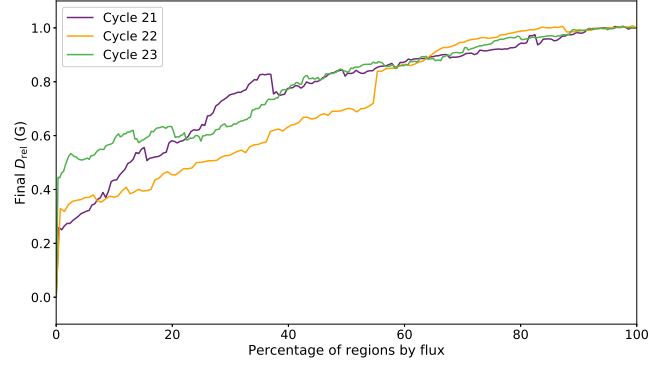


Figure 9. D_{rel} against percentage of regions included for Cycles 21 (purple), 22 (yellow) and 23 (green). Regions are ordered by flux and the top $x\%$ of the strongest regions are incorporated.

ing at low-latitude can have a large effect on the evolution of the axial dipole moment, given by Cameron et al. (2013) and Jiang et al. (2015). However, this idea also combined the latitudinal dependence with the effect of random scatter of tilt angles. For our more realistically shaped multipolar regions, we cannot measure the conventional tilt angle, so instead we calculated the more meaningful parameter of initial D_{rel} which takes into account orientation as well as latitude. We found a general positive correlation between initial and final D_{rel} within all latitudinal bins in all cycles, but that the constant of proportionality depended on latitude with regions at low latitudes contributing most, whence we conclude that latitude is the most dominant parameter in this respect.

We found that the distribution of dipole moment was not consistent across the three cycles. In particular, Cycle 22 contained multiple strong-flux regions which were also some of the largest contributors to the axial dipole moment. This was not the case in Cycles 21 and 23; most large contributors had fluxes of less than 1.5×10^{22} Mx, reinforcing that flux alone is not an appropriate measure of contribution. We conclude that while latitude appears to be the most important property in determining a region's contribution to the axial dipole moment, there is also an important dependence on size and tilt/orientation. This reinforces the expression of the kind given in Equation 1.1. Nagy et al. (2017) showed that changing BMR tilt and emergence latitude had more immediate consequences than changing flux, unless a very large amount of flux was included ***again, mention Anthony's work on latitude dependence?***. Consequently, if a very large, anti-Joy, anti-Hale region was to emerge close to the equator, it could have a significant detrimental impact on the polar field and hence the amplitude of the next cycle. Following the results

of Nagy et al. (2017) it could even be speculated that, in the most extreme case, such an event could lead to a grand minimum.

We also looked at the effect of removing regions with the smallest final D_{rel} in increments. Using the 500 biggest contributors produced an acceptable axial dipole moment in Cycles 21 and 22, but the removal was more damaging in Cycle 23, where at least 750 regions are required to produce an acceptable match. Similarly, when regions are removed in order of flux, there are differences between cycles; ‘acceptable’ matches are achieved when different percentages of regions are included in simulations.

As we approach the minimum at the end of Cycle 24, predictions of Cycle 25 will become more reliable, since it becomes less likely that any more large regions which can significantly alter the polar field will emerge. Indeed, from our analysis of the previous three cycles, we only found regions emerging in the early stages of the descending phase, although that isn’t to say such an event is not possible. For completeness we should

go back and repeat this analysis once we reach cycle minimum in a few years’ time, using the results to assess any current predictions of Cycle 25.

Some predictions of Cycle 25 have already been made, for example by Hathaway & Upton (2016) and Cameron et al. (2016), who used two distinct models but came to a similar conclusion: that Cycle 25 will be another weak cycle. However, by incorporating uncertainty in tilt angles and performing multiple simulations, a wider range of cycle amplitudes was found, suggesting that the behaviour of our Sun really does hinge on the random fluctuations in active region properties, and perhaps making *early* predictions of the next cycle, let alone future cycles, is futile.

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