Q4 E is a point inside the square $ABCD$ such that $EA + EB + EC$ attains its minimum value $\sqrt{2}+\sqrt{6}$, find the length of the square.

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Rotate ΔABE anti-clockwise by 60° about B to ΔFBG.
Join EG.
$BE = BG$ and $\angle EBG = 60°$
ΔBEG is an equilateral triangle.
$\angle ABF = 60°$, $\angle CBF = 150°$
$EA + EB + EC = FG + EG + EC$
which is a minimum when C, E, G, F are collinear.
Minimum distance = $CF = \sqrt{2}+\sqrt{6}$
Let $AB = BC = x$
$CF^2 = x^2 + x^2 - 2(x)(x)\cos 150°$
$(\sqrt{2}+\sqrt{6})^2 = x^2(2+\sqrt{3})$
$x^2 = \dfrac{2+6+4\sqrt{3}}{2+\sqrt{3}}$
$x^2 = 4$
$x = 2$