

Hong Kong Mathematics Olympiad 2007-2008
Heat Event (Individual)

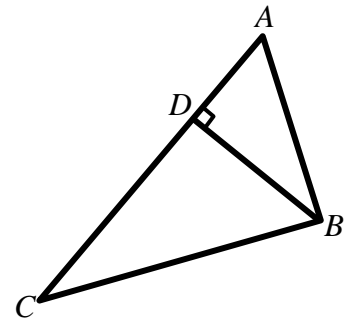
除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 如圖一， ABC 為一個三角形且 $AB = 13$ cm、 $BC = 14$ cm 及 $AC = 15$ cm。 D 為 AC 上的一點使得 $BD \perp AC$ 。
 若 CD 比 AD 長 X cm，求 X 的值。
 In Figure 1, ABC is a triangle, $AB = 13$ cm, $BC = 14$ cm and $AC = 15$ cm. D is a point on AC such that $BD \perp AC$.
 If CD is longer than AD by X cm, find the value of X .



圖一 Figure 1

2. 已知梯形 $PQRS$ 的邊長分別為 $PQ = 6$ cm、 $QR = 15$ cm、 $RS = 8$ cm 及 $SP = 25$ cm，並有 $QR \parallel PS$ 。若 $PQRS$ 的面積為 Y cm²，求 Y 的值。
 Given that a trapezium $PQRS$ with dimensions $PQ = 6$ cm, $QR = 15$ cm, $RS = 8$ cm and $SP = 25$ cm, also $QR \parallel PS$. If the area of $PQRS$ is Y cm², find the value of Y .

3. 已知 x_0 及 y_0 為正整數且滿足方程 $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ 。

若 $35 < y_0 < 50$ 及 $x_0 + y_0 = z_0$ ，求 z_0 的值。

Given that x_0 and y_0 are positive integers satisfying the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$.

If $35 < y_0 < 50$ and $x_0 + y_0 = z_0$, find the value of z_0 .

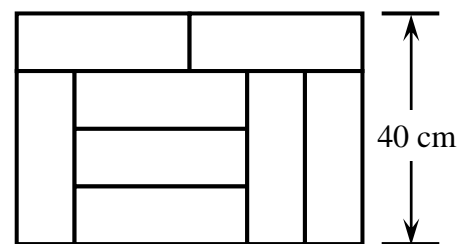
4. 設 a 、 b 、 c 和 d 為實數。若 $\frac{a}{b} = \frac{1}{2}$ ， $\frac{b}{c} = \frac{3}{2}$ ， $\frac{c}{d} = \frac{4}{5}$ 及 $\frac{ac}{b^2 + d^2} = e$ ，求 e 的值。

Let a , b , c and d be real numbers. If $\frac{a}{b} = \frac{1}{2}$, $\frac{b}{c} = \frac{3}{2}$, $\frac{c}{d} = \frac{4}{5}$ and $\frac{ac}{b^2 + d^2} = e$,

find the value of e .

5. 如圖二，利用 8 個相同的小長方形能拼出一個大的長方形。已知在大長方形中較短的邊長為 40 cm。
 若小長方形的面積是 A cm²，求 A 的值。

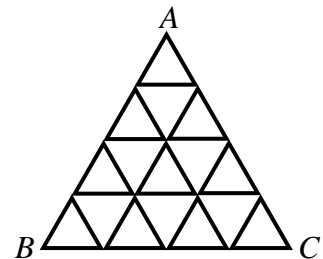
In Figure 2, the large rectangle is formed by eight identical small rectangles. Given that the length of the shorter side of the large rectangle is 40 cm and the area of the small rectangle is A cm², find the value of A .



圖二 Figure 2

6. 在圖三中， $\triangle ABC$ 為等邊三角形。它由多個相同的等邊三角形組成。若圖中共有 N 個等邊三角形，求 N 的值。

In Figure 3, $\triangle ABC$ is an equilateral triangle. It is formed by several identical equilateral triangles. If there are altogether N equilateral triangles in the figure, find the value of N .



圖三 Figure 3

7. 設 r 為方程 $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$ 的較大實根。求 r 的值。

Let r be the larger real root of the equation $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$. Find the value of r .

8. 設 x 為有理數及 $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$ 。求 w 的最小可能值。

Let x be a rational number and $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$. Find the smallest possible value of w .

9. 設 m 和 n 為正整數。已知表達式 $\left(\left(\left((2)^2 \right)^2 \right)^{\cdots} \right)^2 = \left(\left(\left((4)^4 \right)^4 \right)^{\cdots} \right)^4$ 含有 m 個 2 及 n 個 4。
若 $k = \frac{m}{n}$ ，求 k 的值。

Let m and n be positive integers. Given that the number 2 appears m times and the number 4 appears n times in the expansion $\left(\left(\left((2)^2 \right)^2 \right)^{\cdots} \right)^2 = \left(\left(\left((4)^4 \right)^4 \right)^{\cdots} \right)^4$. If $k = \frac{m}{n}$, find the value of k .

10. 求 $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$ 的值。

Find the value of $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$.

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Heat Event (Group)

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時限：20 分鐘

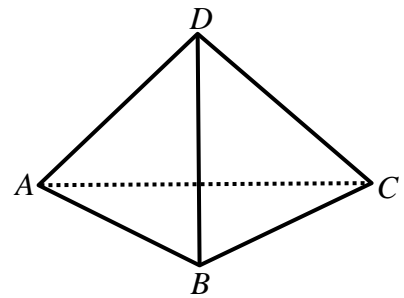
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1. 已知 $5 + \sqrt{11}$ 的小數部分為 A 及 $5 - \sqrt{11}$ 的小數部分為 B 。設 $C = A + B$ ，求 C 的值。
Given that the decimal part of $5 + \sqrt{11}$ is A and the decimal part of $5 - \sqrt{11}$ is B .
Let $C = A + B$, find the value of C .

2. 有一批糖共 x 粒， x 為正整數，這批糖能分別為 851 人及 943 人所均分。
求 x 的最小可能值。
A total number of x candies, x is a positive integer, can be evenly distributed to 851 people as well as 943 people. Find the least possible value of x .

3. 如圖一，正四面體 $ABCD$ 的邊長為 2 cm。
若該正四面體的體積是 $\sqrt{R} \text{ cm}^3$ ，求 R 的值。
In Figure 1, $ABCD$ is a **regular** tetrahedron with side length of 2 cm. If the volume of the tetrahedron is $\sqrt{R} \text{ cm}^3$, find the value of R .



圖一 Figure 1

4. 已知 x 為正整數及 $x < 60$ 。若 x 恰有 10 個正因子，求 x 的值。
Given that x is a positive integer and $x < 60$. If x has exactly 10 positive factors, find the value of x .

5. 已知 $90^\circ < \theta < 180^\circ$ 及 $\sin \theta = \frac{\sqrt{3}}{2}$ 。若 $A = \cos(180^\circ - \theta)$ ，求 A 的值。
Given that $90^\circ < \theta < 180^\circ$ and $\sin \theta = \frac{\sqrt{3}}{2}$. If $A = \cos(180^\circ - \theta)$, find the value of A .

6. 設 x 為正實數，求 $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$ 的最小值。

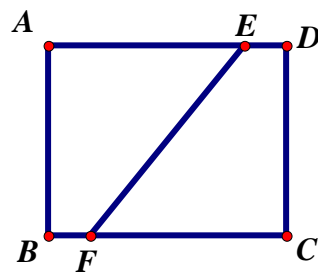
Let x be a positive real number. Find the minimum value of $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$.

7. 設 x 及 y 為實數，且滿足 $\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5, \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5. \end{cases}$ 若 $z = x + y$ ，求 z 的值。

Let x and y be real numbers satisfying $\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5 \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5 \end{cases}$.

If $z = x + y$, find the value of z .

8. 設 R 為 $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ 被 4 除後的餘數，求 R 的值。
 Let R be the remainder of $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ divided by 4. Find the value of R .
9. 已知 k 、 x_1 及 x_2 為正整數且 $x_1 < x_2$ 。設 A 、 B 及 C 為曲線 $y = kx^2$ 上的三點，其 x 座標分別為 $-x_1$ 、 x_1 及 x_2 。若 $\triangle ABC$ 的面積是 15 平方單位，求所有可能 k 值的總和。
 Given that k , x_1 and x_2 are positive integers with $x_1 < x_2$. Let A , B and C be three points on the curve $y = kx^2$, with x -coordinates being $-x_1$, x_1 and x_2 respectively.
 If the area of $\triangle ABC$ is 15 square units, find the sum of all possible values of k .
10. 如圖二， $ABCD$ 是長方形紙張並有 $AB = 4$ cm 及 $BC = 5$ cm。將該紙張對摺，使 C 點與 A 點重合，得摺痕 EF 。若 $EF = r$ cm，求 r 的值。
 In Figure 2, $ABCD$ is rectangular piece of paper with $AB = 4$ cm and $BC = 5$ cm. Fold the paper by putting point C onto A to create a crease EF . If $EF = r$ cm, find the value of r .



圖二 Figure 2