Hong Kong Mathematics Olympiad 2012-2013
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　　時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark.　　Time allowed: 40 minutes

1. 化簡 $\sqrt{94-2\sqrt{2013}}$ 。

   Simplify $\sqrt{94-2\sqrt{2013}}$.

2. 一個平行四邊形可被分成 178 個邊長為 1 單位的等邊三角形，若該平行四邊形的周界為 P 單位，求 P 的最大值。

   A parallelogram is cut into 178 pieces of equilateral triangles with sides 1 unit. If the perimeter of the parallelogram is P units, find the maximum value of P .

3. 圖一所示為一直角三角形 ACD，其中 B 是 AC 上的點且 $BC = 2AB$。已知 $AB = a$ 及 $\angle ACD = 30°$，求 θ 的值。

   Figure 1 shows a right-angled triangle ACD where B is a point on AC and $BC = 2AB$.
   Given that $AB = a$ and $\angle ACD = 30°$, find the value of θ .

   圖一 Figure 1
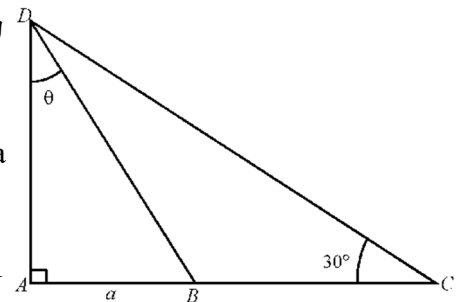
4. 已知 $x^2 + 399 = 2^y$，其中 x、y 為正整數。求 x 的值。

   Given that $x^2 + 399 = 2^y$, where x, y are positive integers. Find the value of x .

5. 已知 $y = (x+1)(x+2)(x+3)(x+4) + 2013$，求 y 的最小值。

   Given that $y = (x + 1)(x + 2)(x + 3)(x + 4) + 2013$, find the minimum value of y.

6. 從一個有 n 條邊的凸多邊形中，選取其中一隻內角。若餘下的 $n - 1$ 隻內角之和是 2013°，求 n 的值。

   In a convex polygon with n sides, one interior angle is selected. If the sum of the remaining $n – 1$ interior angle is 2013°, find the value of n .

7. 圖二所示為一通過 B 點及 C 點的圓，而 A 點則在圓之外。已知 BC 是圓的直徑，AB 及 AC 分別與圓相交於 D 點及 E 點，且 $\angle BAC = 45°$，求 $\dfrac{\Delta ADE\ 的面積}{BCED\ 的面積}$ 。

   Figure 2 shows a circle passes through two points B and C, and a point A is lying outside the circle. Given that BC is a diameter of the circle, AB and AC intersect the circle at D and E respectively and $\angle BAC = 45°$, find $\dfrac{\text{area of } \Delta ADE}{\text{area of } BCED}$.
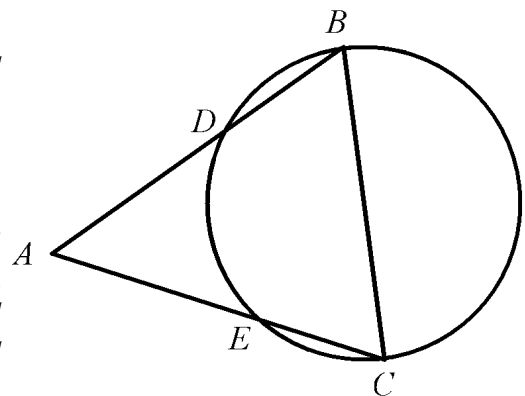
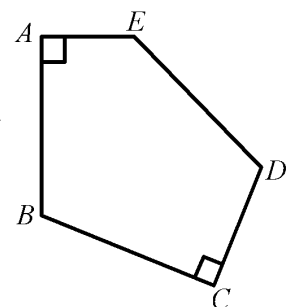   圖二 Figure 2

8. 解 $\sqrt{31-\sqrt{31+x}} = x$ 。

   Solve $\sqrt{31-\sqrt{31+x}} = x$.

9. 圖三所示為五邊形 $ABCDE$。$AB = BC = DE = AE + CD = 3$，且 $\angle A = \angle C = 90°$，求該五邊形的面積。

   Figure 3 shows a pentagon $ABCDE$. $AB = BC = DE = AE + CD = 3$ and $\angle A = \angle C = 90°$, find the area of the pentagon.

   圖三 Figure 3

10. 若 a 及 b 為實數，且 $a^2 + b^2 = a + b$。求 $a + b$ 的最大值。

    If a and b are real numbers, and $a^2 + b^2 = a + b$. Find the maximum value of $a + b$.

*** 試卷完　End of Paper ***

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每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1.　已知一個直角三角形三邊的長度皆為整數，且其中兩邊的長度為方程 $x^2 - (m + 2)x + 4m = 0$ 的根。求第三邊長度的最大值。

Given that the length of the sides of a right-angled triangle are integers, and two of them are the roots of the equation $x^2 - (m + 2)x + 4m = 0$.
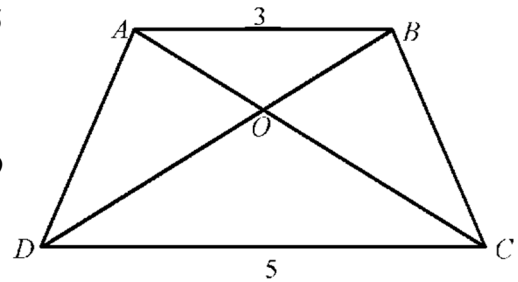
Find the maximum length of the third side of the triangle.

2.　圖一所示為一梯形 $ABCD$，其中 $AB = 3$、$CD = 5$ 及 AC、BD 相交於點 O。若 ΔAOB 的面積是 27，求梯形 $ABCD$ 的面積。

Figure 1 shows a trapezium $ABCD$, where $AB = 3$, $CD = 5$ and the diagonals AC and BD meet at O.

If the area of ΔAOB is 27,

find the area of the trapezium $ABCD$.

圖一 Figure 1

3.　設 x 及 y 為實數使得 $x^2 + xy + y^2 = 2013$。求 $x^2 - xy + y^2$ 的最大值。

Let x and y be real numbers such that $x^2 + xy + y^2 = 2013$.

Find the maximum value of $x^2 - xy + y^2$.

4.　若 α、β 是方程 $x^2 + 2013x + 5 = 0$ 的根，求 $(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$ 的值。

If α, β are roots of $x^2 + 2013x + 5 = 0$, find the value of $(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$.

5.　如圖二所示，$ABCD$ 為一個邊長為 10 單位的正方形，E 及 F 分別為 CD 及 AD 的中點，BE 及 FC 相交於G。求 AG 的長度。

As shown in Figure 2, $ABCD$ is a square of side 10 units, E and F are the mid-points of CD and AD respectively, BE and FC intersect at G. Find the length of AG.
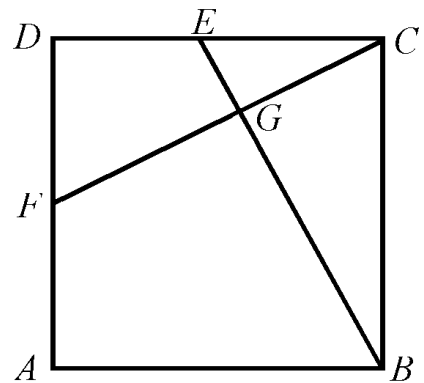
圖二 Figure 2

6.　　若 a 及 b 為正實數，且方程 $x^2 + ax + 2b = 0$ 及 $x^2 + 2bx + a = 0$ 都有實數根。
　　　　求 $a + b$ 的最小值。

Let a and b are positive real numbers, and the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ have real roots. Find the minimum value of $a + b$.

7.　　已知 ΔABC 的三邊的長度組成一個等差數列，且為方程 $x^3 - 12x^2 + 47x - 60 = 0$ 的根，
　　　　求 ΔABC 的面積。

Given that the length of the three sides of ΔABC form an arithmetic sequence, and are the roots of the equation $x^3 - 12x^2 + 47x - 60 = 0$, find the area of ΔABC.

8.　　圖三中，ΔABC 為一等腰三角形，其中 AB
　　　　$= AC$，$BC = 240$。已知 ΔABC 的內接圓
　　　　的半徑是 24，求 AB 的長度。

　　　　In Figure 3, ΔABC is an isosceles triangle
　　　　with $AB = AC$, $BC = 240$. The radius of the
　　　　inscribed circle of ΔABC is 24.
　　　　Find the length of AB.
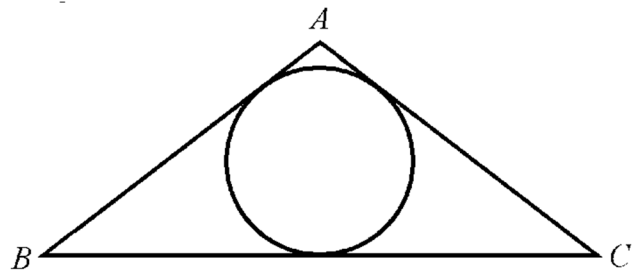
圖三 Figure 3

9.　　從 1、2、3、$\cdots$、2012、2013 中最多可取出多少個數，使得在取出的數中任意兩數之和
　　　　都不是這兩個數之差的倍數？

At most how many numbers can be taken from the set of integers: 1, 2, 3, $\cdots$, 2012, 2013 such that the sum of any two numbers taken out from the set is not a multiple of the difference between the two numbers?

10.　　對所有正整數 n，定義函數 f 為
　　　　(i)　　f(1) = 2012,

　　　　(ii)　　f(1) + f(2) + $\cdots$ + f(n – 1) + f(n) = n^2 f(n)，$n > 1$

　　　　求 f(2012) 的值。

For all positive integers n, define a function f as
(i)　　f(1) = 2012,

(ii)　　f(1) + f(2) + $\cdots$ + f(n – 1) + f(n) = n^2 f(n)，$n > 1$.

Find the value of f(2012) .

*** 試卷完 End of Paper ***

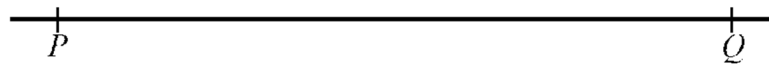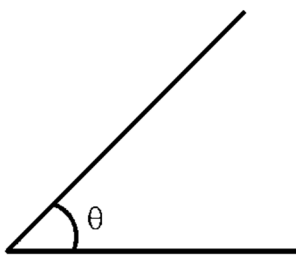| 每隊必須列出詳細所有步驟(包括作圖步驟)。	時限:20 分鐘
All working (including geometric drawing) must be clearly shown.	
此部份滿分為十分。The full marks of this part is 10 marks.	Time allowed: 20 minutes

School Code: _____

School Name: _____

第一題 Question No. 1

下圖所示為綫段 *PQ* 及角 θ。試構作一個等腰三角形 *PQR*,其中 *PQ* = *PR* 及 ∠*QPR* = θ。
Line segment *PQ* and an angle of size θ are given below. Construct the isosceles triangle *PQR* with *PQ* = *PR* and ∠*QPR* = θ.

每隊必須列出詳細所有步驟(包括作圖步驟)。　　　　　　　　　　時限：20 分鐘

All working (including geometric drawing) must be clearly shown.

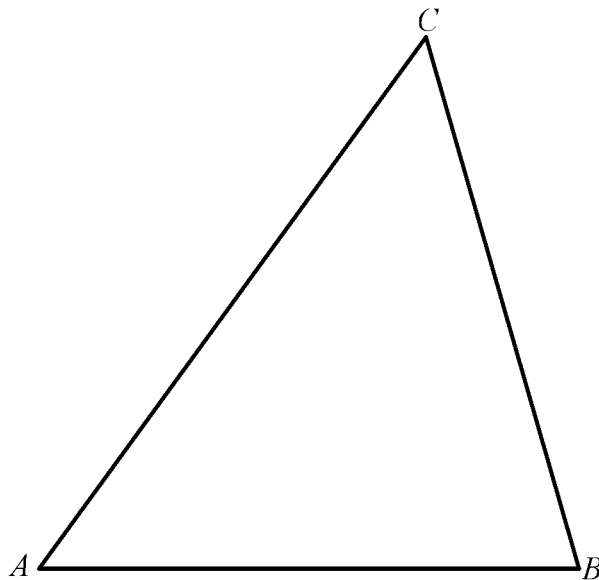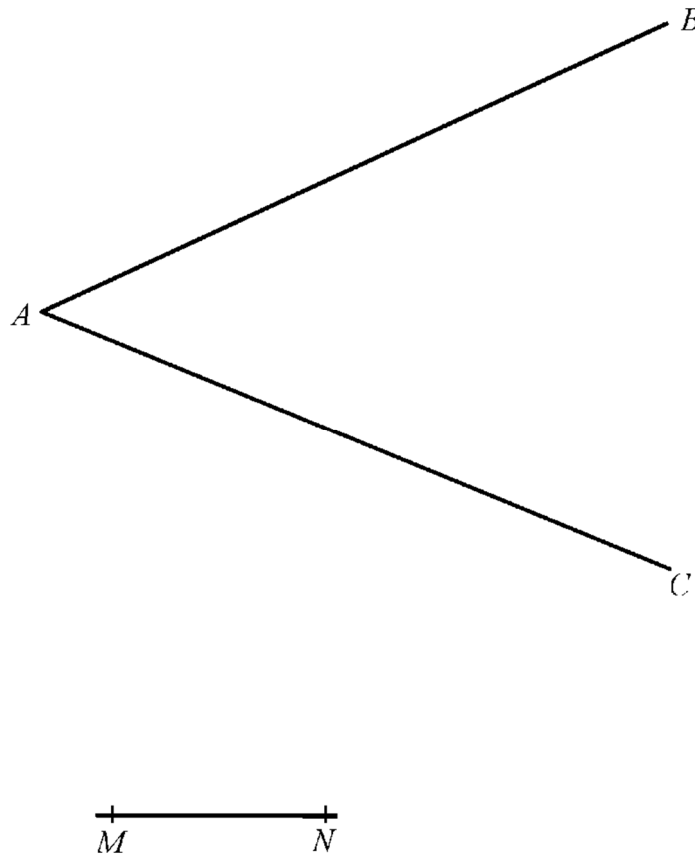此部份滿分為十分。The full marks of this part is 10 marks.　　　　Time allowed: 20 minutes

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第二題　Question No. 2

試構作一個面積與下圖所示的 ΔABC 面積相等的長方形，長方形其中一邊為 AB。

Construct a rectangle with AB as one of its sides and with area equal to that of ΔABC below.

每隊必須列出詳細所有步驟(包括作圖步驟)。	時限：20 分鐘
All working (including geometric drawing) must be clearly shown.	
此部份滿分為十分。The full marks of this part is 10 marks.	Time allowed: 20 minutes

School Code: _____

School Name: _____

第三題 Question No. 3

下圖所示為兩相交於 *A* 點的綫段 *AB* 及 *AC*。試構作一半徑等於綫段 *MN* 的圓使得 *AB* 及 *AC* 均為該圓的切綫。

The figure below shows two straight lines *AB* and *AC* intersecting at the point *A*. Construct a circle with radius equal to the line segment *MN* so that *AB* and *AC* are tangents to the circle.

B

A

C

|―――――――――|
M　　　　　*N*

*** 試卷完 End of Paper ***