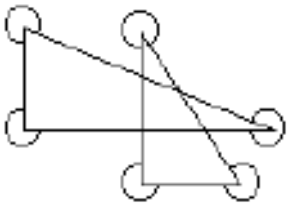
Hong Kong Mathematics Olympiad (1982-83)
Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖，所有有記號的角的總和是 $a°$，求 a 的值。
In the following figure, the sum of the marked angles is $a°$, find the value of a.



$a =$

(ii) 一正 b-邊形的內角和是 $a°$。求 b 的值。
The sum of the interior angles of a regular b-sided polygon is $a°$.
Find the value of b.

$b =$

(iii) 求 c 的值，若 $2^b = c^4$ 及 $c > 0$。
Find the value of c, if $2^b = c^4$ and $c > 0$.

$c =$

(iv) 若 $\frac{b}{c} = k$ 及 $c : d = k : 100$，求 d 的值。
Find the value of d, if $\frac{b}{c} = k$ and $c : d = k : 100$.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

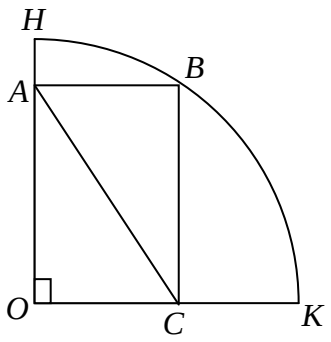
Hong Kong Mathematics Olympiad (1982-83)
Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = 1.8 \times 5.0865 + 1 - 0.0865 \times 1.8$，求 a 的值。
If $a = 1.8 \times 5.0865 + 1 - 0.0865 \times 1.8$, find the value of a.

$a =$

(ii) 如圖，$OH = OK = 10$ 及 $OABC$ 為一個長方形。$AC = b$，問 b 為何值？
In the diagram shown, $OH = OK = a$ units and $OABC$ is a rectangle. $AC = b$ units.
What is the value of b?



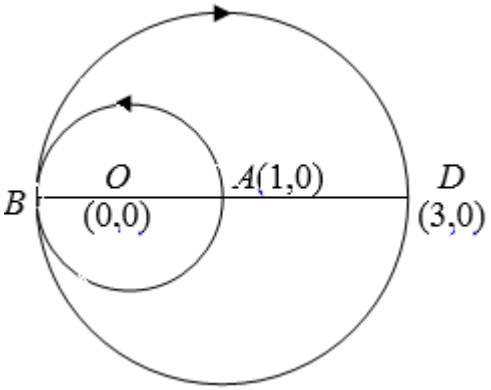
$b =$

(iii) 依下圖之分數，當計算至分子是 x^8 時，c 為何值？
In the expression shown, what is c when it is expanded to the term with $x^{(b-2)}$ as the numerator?

$$\cfrac{x^0}{2+\cfrac{x^1}{6+\cfrac{x^2}{10+\cfrac{x^3}{14+\cfrac{\cdots\cdots\cdots\cdots}{\cfrac{\cdots}{c+\cfrac{x^{(b-2)}}{\ddots}}}}}}}$$

$c =$

(iv) 如圖，一兔子花了 c 分鐘經半圓跑道由 A 去到 B。以相同速度，牠花了 d 分鐘經半圓跑道由 $A \to B \to D$。問 d 為何值？
As shown a rabbit spends c minutes in travelling from A to B along half circle. With the same speed, it spends d minutes in travelling from $A \to B \to D$ along half circles.
What is the value of d?



$d =$

FOR OFFICIAL USE

Score for accuracy ____ × Mult. factor for speed ____ = ____ Team No. ____

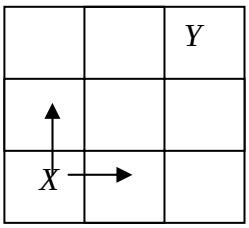
\+ Bonus score ____ Time ____ ____

Total score ____ Min. Sec.

Hong Kong Mathematics Olympiad (1982-83)
Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 右方棋盤為一 3×3 九宮格。一隻棋子放置在 X 的位置上，每次只可向上行一格，或向右行一格。
問：由 X 行到 Y，共有多少種不同的路徑？
The figure shows a board consisting of nine squares. A counter originally on square X can be moved either upwards or to the right one square at a time. By how many different routes may the counter be moved from X to Y?



(ii) 已知 $\sqrt{2a} = -b\tan\frac{\pi}{3}$。求 b 的值。
Given $\sqrt{2a} = -b\tan\frac{\pi}{3}$. Find the value of b.

$b =$

(iii) 已知 $p * q = \frac{p-q}{p}$，求 c 的值，若 $c = (a + b) * (b - a)$。
Given that $p * q = \frac{p-q}{p}$, find the value of c if $c = (a + b) * (b - a)$.

$c =$

(iv) 把一 c cm 的鐵綫屈曲成一半徑為 1 cm 的扇形。問扇形的圓心角為何？
A wire of c cm is bent to form a sector of radius 1 cm. What is the angle of the sector in degrees (correct to the nearest degree)?

angle =

FOR OFFICIAL USE

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Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1982-83)
Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a(x+1) \equiv x^3 + 3x^2 + 3x + 1$，以 x 表示 a。
If $a(x+1) \equiv x^3 + 3x^2 + 3x + 1$, express a in terms of x.

$a =$

(ii) 若 $a - 1 = 0$，則 x 的解為 0 或 b，求 b 的值。
If $a - 1 = 0$, then the value of x is 0 or b, what is the value of b?

$b =$

(iii) 若 $pc^4 = 32$，$pc = b^2$ 及 c 為正數，c 的值為何？
If $pc^4 = 32$, $pc = b^2$ and c is positive, what is the value of c?

$c =$

(iv) P 為一運算子使得 $P(A \cdot B) = P(A) + P(B)$。
$P(A) = y$ 的意思是 $A = 10^y$。若 $d = A \cdot B$，$P(A) = 1$ 及 $P(B) = c$，求 d 的值。
P is an operation such that $P(A \cdot B) = P(A) + P(B)$.
$P(A) = y$ means $A = 10^y$.
If $d = A \cdot B$, $P(A) = 1$ and $P(B) = c$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1982-83)
Event 6 (Group)

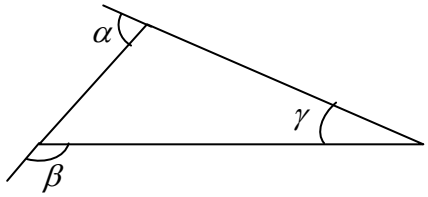
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 右表顯示二元運算子*定義於 P、Q、R、S 時的結果。假設 a 為 P 的反元素。求 a 的值。
The table shows the results of the operation * on P, Q, R, S taken two at a time.
Let a be the inverse of P. Find the value of a .

*	P	Q	R	S
P	Q	R	S	P
Q	R	S	P	Q
R	S	P	Q	R
S	P	Q	R	S

$a =$

(ii) α 與 β 的平均值是 105°，α、β 與 γ 的平均值是 b°。求 b 的值。
The average of α and β is 105°, the average of α, β and γ is b°. Find the value of b .



$b =$

(iii) 兩數之和為 10，其乘積為 20。若該兩倒數之和為 c，求 c 的值。
The sum of two numbers is 10, their product is 20. The sum of their reciprocal is c. What is the value of c ?

$c =$

(iv) 已知 $\sqrt{90} = 9.49$ (準至兩位小數)
若 $d < 7\sqrt{0.9} < d + 1$，其中 d 為整數，求 d 的值。
It is given that $\sqrt{90} = 9.49$, to 2 decimal places.
If $d < 7\sqrt{0.9} < d + 1$, where d is an integer, what is the value of d ?

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

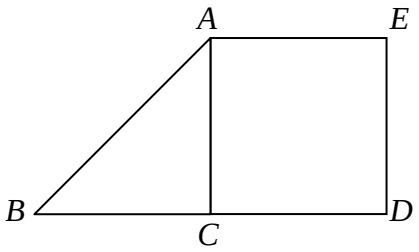
Hong Kong Mathematics Olympiad (1982-83)
Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 求 $3 + 6 + 9 + \ldots + 45$ 的值。
Find the value of $3 + 6 + 9 + \ldots + 45$.

sum =

(ii) 圖中，*ACDE* 為一正方形，*AC* = *BC* 及 $\angle ACB = 90°$。若 *ACDE* 的面積為 10 cm^2，求 ΔABC 的面積。
In the figure shown, *ACDE* is a square and *AC* = *BC*, $\angle ACB = 90°$. Find the area of ΔABC if the area of *ACDE* is 10 cm^2.



Area =

(iii) 若 $a+\frac{1}{a}=3$，求 $a^3+\frac{1}{a^3}$ 的值。
Given that $a+\frac{1}{a}=3$. Evaluate $a^3+\frac{1}{a^3}$.

(iv) 已知 $\sum_{y=1}^{n}\frac{1}{y}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$。
求 $\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ 的值。(答案以份數表示。)
Given that $\sum_{y=1}^{n}\frac{1}{y}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$.
Find the value of $\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$. (Express your answer in fraction.)

FOR OFFICIAL USE

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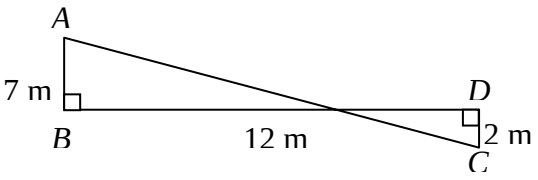
Time [ ] [ ]
Min. Sec.

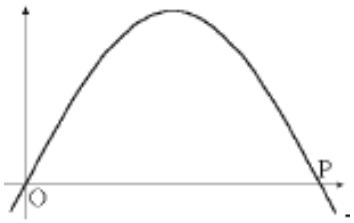
Hong Kong Mathematics Olympiad (1982-83)
Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖，彼得站 A 點而約翰站在 C 點，BD 的距離 12 m。問彼得和約翰之間的最短距離為何？
Peter is standing at *A* and John is at *C*. The distance between *B* and *D* is 12 m. What is the shortest distance between John and Peter?



(ii) 右圖顯示 $y = \sin 3x°$ 的圖像，求 P 點的 x 座標。
The following figure shows a part of the graph $y = \sin 3x°$. What is the x-coordinate of P ?



$x =$

(iii) 若 $f(x) = x^2$，以 x 表示 $f(x) - f(x-1)$。
If $f(x) = x^2$, then express $f(x) - f(x-1)$ in terms of x.

(iv) 若果 *mnp*、*nmp*、*mmp* 及 *nnp* 為十進制數字，其位值是由 *m*、*n* 及 *p* 組成，且 $mnp - nmp = 180$ 及 $mmp - nnp = d$。求 d 的值。
If *mnp*, *nmp*, *mmp* and *nnp* are numbers in base 10 composed of the digits *m*, *n* and *p*, such that: $mnp - nmp = 180$ and $mmp - nnp = d$. Find the value of d .

$d =$

FOR OFFICIAL USE

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Min. Sec.

Hong Kong Mathematics Olympiad (1982-83)
Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\sin\theta = \frac{3}{5}$，$a = \sqrt{\tan^2\theta + 1}$，求 a 的值。

If $\sin\theta = \frac{3}{5}$, $a = \sqrt{\tan^2\theta + 1}$, find the value of a.

$a =$

(ii) 考慮以下步驟，用以證明 $\frac{1}{8} > \frac{1}{4}$。

Examine the following proof carefully: To prove $\frac{1}{8} > \frac{1}{4}$.

step _____

步驟 Steps

1	$3 > 2$	$3 > 2$
2	兩邊乘以 $\log\left(\frac{1}{2}\right)$， 使得 $3\log\left(\frac{1}{2}\right) > 2\log\left(\frac{1}{2}\right)$	Multiply both sides by $\log\left(\frac{1}{2}\right)$, then $3\log\left(\frac{1}{2}\right) > 2\log\left(\frac{1}{2}\right)$
3	$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$	$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$
4	$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$

$\therefore \frac{1}{8} > \frac{1}{4}$

Which step is incorrect? 以上哪一步是錯的？

(iii) 若兩直綫 $2y + x + 3 = 0$ 及 $3y + cx + 2 = 0$ 互相垂直，求 c 的值。
If the lines $2y + x + 3 = 0$ and $3y + cx + 2 = 0$ are perpendicular, find the value of c.

$c =$

(iv) 在箱子內有 4 個紅球和 3 個黑球。若從中一個接一個抽出 3 個球，每次抽完之後皆將抽到的球放回原位。求抽到 2 個紅球和 1 個黑球的概率。
There are 4 red balls and 3 black balls in a box. If 3 balls are chosen one by one with replacement, what is the probability of choosing 2 red balls and 1 black ball?

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Hong Kong Mathematics Olympiad (1982-83)
Event 10 (Group)

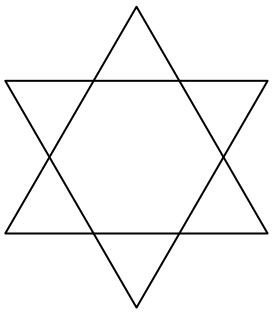
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) $1^2 - 1 = 0 \times 2$
$2^2 - 1 = 1 \times 3$
$3^2 - 1 = 2 \times 4$
$4^2 - 1 = 3 \times 5$
……
$A^2 - 1 = 3577 \times 3579$
若 $A > 0$，求 A 的值。

$1^2 - 1 = 0 \times 2$
$2^2 - 1 = 1 \times 3$
$3^2 - 1 = 2 \times 4$
$4^2 - 1 = 3 \times 5$
……
$A^2 - 1 = 3577 \times 3579$
If $A > 0$, find the value of A.

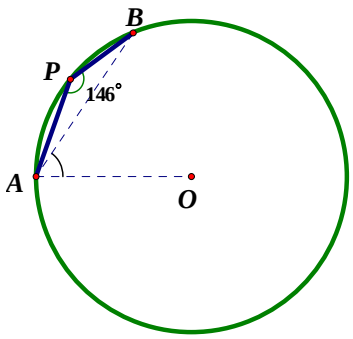
$A =$

(ii) 一正 N-邊形的邊向外延長形成一個"星形"。如果該星形的每一隻角均為 108°，求 N 的值。(例如，由正 6 邊形形成的 6 角星如右圖所示。)
The sides of an N-sided regular polygon are produced to form a "star". If the angle at each point of that "star" is 108°, find the value of N.
(For example, the "star" of a six-sided polygon is given as shown in the diagram.)



6-sided regular polygon.

(iii) A、P 及 B 三點均在圓周上，圓心為 O。
若 $\angle APB = 146°$，求 $\angle OAB$ 的值。
A, P, B are three points on a circle with centre O.
If $\angle APB = 146°$, find the value of $\angle OAB$.



$\angle OAB =$

(iv) 一兩位數 X 的個位與十位相乘等於 24，若將個位與十位對掉，新的兩位數比原來的兩位數大了 18，求 X 的值。
A number X consists of 2 digits whose product is 24. By reversing the digits, the new number formed is 18 greater than the original one. What is the value of X?

$X =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

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