V Reciprocal roots.

Theory If α is a non-zero root of a polynomial equation $f(x) = 0$, then $\frac{1}{\alpha}$ is a root of a polynomial equation $f(\frac{1}{y}) = 0$.

That is to say, change $x \to \frac{1}{y}$.

Class Work V.1

If α, β and γ be the roots of $x^3 + px^2 + qx + r = 0$.

If $\alpha, \beta, \gamma \neq 1$, find the value of $\frac{1}{\alpha-1}+\frac{1}{\beta-1}+\frac{1}{\gamma-1}$.

Transform $x \to \frac{1}{y} + 1$.

Class Work V.2

(a) Show that the equation $x^3 - 16x + 16 = 0$ has 2 positive roots and one negative root.

(b) If α, β and γ be the roots, prove that $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=1$.

(Solution on the next page.)

Class Work V.2 solution

Let $f(x) = x^3 - 16x + 16$

$f(-10) = -1000 + 160 + 16 < 0$

$f(0) = 16 > 0$

$f(2) = 8 - 32 + 16 < 0$

$f(10) = 1000 - 160 + 16 > 0$

$\because$ f(x) is continuous

$\therefore$ There is one negative root and two positive roots.

Transform $x \to \frac{1}{\sqrt{y}}$

Then $\frac{1}{\left(\sqrt{y}\right)^3} - \frac{16}{\sqrt{y}} + 16 = 0$ has roots $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$, $\frac{1}{\gamma^2}$.

$1 - 16y + 16y\sqrt{y} = 0$

$16y\sqrt{y} = 16y - 1$

$(16y\sqrt{y})^2 = (16y - 1)^2$

$256y^3 = 256y^2 - 32y + 1$

$256y^3 - 256y^2 + 32y - 1 = 0$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = 1$