

Hong Kong Mathematics Olympiad (1996-97)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知 $\frac{3}{a} + \frac{1}{u} = \frac{7}{2}$ 及 $\frac{2}{a} - \frac{3}{u} = 6$ 為 a 與 u 的聯立方程。求 a 的解。

Given that $\frac{3}{a} + \frac{1}{u} = \frac{7}{2}$ and $\frac{2}{a} - \frac{3}{u} = 6$ are simultaneous equations in a and u .

Solve for a .

$a =$

- (ii) 方程 $px + qy + bz = 1$ 的根分別為 $(0, 3a, 1)$ 、 $(9a, -1, 2)$ 和 $(0, 3a, 0)$ 。
求係數 b 的值。

Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b .

$b =$

- (iii) 若 $y = mx + c$ 的圖像經過 $(b + 4, 5)$ 及 $(-2, 2)$ 兩點。求 c 的值。

Find the value of c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(-2, 2)$.

$c =$

- (iv) 不等式 $x^2 + 5x - 2c \leq 0$ 的解為 $d \leq x \leq 1$ 。求 d 的值。

The solution of the inequality $x^2 + 5x - 2c \leq 0$ is $d \leq x \leq 1$. Find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

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Mult. factor for
speed

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Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 考慮： $\frac{1^2}{1}=1$ ， $\frac{1^2+2^2}{1+2}=\frac{5}{3}$ ， $\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$ ， $\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$ ，

求 a 的值使得 $\frac{1^2+2^2+\cdots+a^2}{1+2+\cdots+a}=\frac{25}{3}$ 。

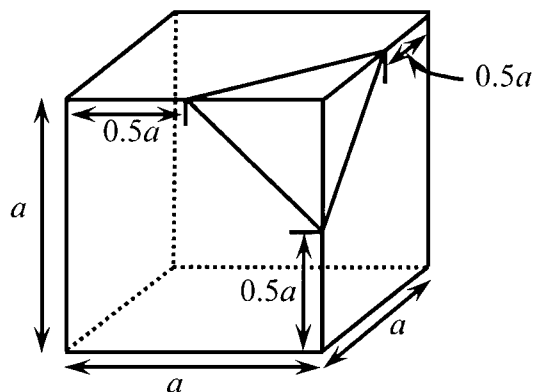
By considering: $\frac{1^2}{1}=1$ ， $\frac{1^2+2^2}{1+2}=\frac{5}{3}$ ， $\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$ ， $\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$ ，

find the value of a such that $\frac{1^2+2^2+\cdots+a^2}{1+2+\cdots+a}=\frac{25}{3}$.

$a =$

(ii) 如圖所示，從邊長為 a cm 的正立方體的一角割出一個三角錐體。
若三角錐體的體積為 b cm³，求 b 的值。

A triangular pyramid is cut from a corner of a cube with side length a cm as the figure shown. If the volume of the pyramid is b cm³, find the value of b .



$b =$

(iii) 若對於所有實數 x ， x^2+cx+b 不小於 0，求 c 的最大值。

If the value of x^2+cx+b is not less than 0 for all real number x , find the maximum value of c .

$c =$

(iv) 若 1997^{1997} 的個位數為 $c-d$ ，求 d 的值。

If the units digit of 1997^{1997} is $c-d$, find d .

$d =$

FOR OFFICIAL USE

Score for accuracy

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Mult. factor for speed

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Team No.

+ Bonus score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) a 、 b 、 c 和 d 的平均值為 8。若 a 、 b 、 c 、 d 和 P 的平均值為 P ，求 P 的值。
 The average of a , b , c and d is 8. If the average of a , b , c , d and P is P , find the value of P .

$P =$

- (ii) 若直線 $2x + 3y + 2 = 0$ 和 $Px + Qy + 3 = 0$ 互相平行，求 Q 的值。
 If the lines $2x + 3y + 2 = 0$ and $Px + Qy + 3 = 0$ are parallel, find the value of Q .

$Q =$

- (iii) 若等邊三角形的周界和面積分別為 Q cm 和 $\sqrt{3}R$ cm²。求 R 的值。
 The perimeter and the area of an equilateral triangle are Q cm and $\sqrt{3}R$ cm² respectively. Find the value of R .

$R =$

- (iv) 若 $(1 + 2 + \dots + R)^2 = 1^2 + 2^2 + \dots + R^2 + S$ ，求 S 的值。
 If $(1 + 2 + \dots + R)^2 = 1^2 + 2^2 + \dots + R^2 + S$, find the value of S .

$S =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (1996-97)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若正 n 邊形的內角為 140° ，求 n 的值。

If each interior angle of a n -sided regular polygon is 140° , find the value of n .

$n =$

- (ii) 若不等式 $2x^2 - nx + 9 < 0$ 的解為 $k < x < b$ ，求 b 的值。

If the solution of the inequality $2x^2 - nx + 9 < 0$ is $k < x < b$, find the value of b .

$b =$

- (iii) 若 $cx^3 - bx + x - 1$ 除以 $x + 1$ ，餘數為 -7 ，求 c 的值。

If $cx^3 - bx + x - 1$ is divided by $x + 1$, the remainder is -7 , find the value of c .

$c =$

- (iv) 若 $x + \frac{1}{x} = c$ 和 $x^2 + \frac{1}{x^2} = d$ ，求 d 的值。

If $x + \frac{1}{x} = c$ and $x^2 + \frac{1}{x^2} = d$, find the value of d .

$d =$

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Hong Kong Mathematics Olympiad (1996-97)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 一直徑為 a 的半球體的體積為 $18\pi \text{ cm}^3$ ，求 a 的值。

The volume of a hemisphere with diameter $a \text{ cm}$ is $18\pi \text{ cm}^3$, find the value of a .

$a =$

- (ii) 若 $\sin 10a^\circ = \cos(360^\circ - b^\circ)$ 和 $0 < b < 90$ ，求 b 的值。

If $\sin 10a^\circ = \cos(360^\circ - b^\circ)$ and $0 < b < 90$, find the value of b .

$b =$

- (iii) 一三角形是由 x -軸、 y -軸和直綫 $bx + 2by = 120$ 所組成。

若所包圍之三角形的面積為 c ，求 c 的值。

The triangle is formed by the x -axis and y -axis and the line $bx + 2by = 120$.

If the bounded area of the triangle is c , find the value of c .

$c =$

- (iv) 若方程式 $x^2 - (c + 2)x + (c + 1) = 0$ 兩根之差為 d ，求 d 的值。

If the difference of the two roots of the equation $x^2 - (c + 2)x + (c + 1) = 0$ is d , find the value of d .

$d =$

FOR OFFICIAL USE

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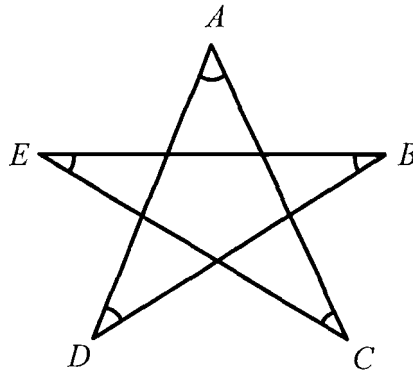
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Hong Kong Mathematics Olympiad (1996-97)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 圖中， $\angle A + \angle B + \angle C + \angle D + \angle E = a^\circ$ 。求 a 的值。

In the diagram, $\angle A + \angle B + \angle C + \angle D + \angle E = a^\circ$. Find the value of a .



$a =$

- (ii) 代數式 $x^6 + x^6 + x^6 + \dots + x^6$ 有 x 項及其總和為 x^b 。求 b 的值。

There are x terms in the algebraic expression $x^6 + x^6 + x^6 + \dots + x^6$ and its sum is x^b . Find the value of b .

- (iii) 若 $1 + 3 + 3^2 + 3^3 + \dots + 3^8 = \frac{3^c - 1}{2}$ ，求 c 的值。

If $1 + 3 + 3^2 + 3^3 + \dots + 3^8 = \frac{3^c - 1}{2}$, find the value of c .

- (iv) 從 16 張寫上 1 至 16 的咭紙中隨意抽出一張，若果抽出的號碼是一個完全平方數的概率為 $\frac{1}{d}$ ，求 d 之值。

16 cards are marked from 1 to 16 and one is drawn at random.

If the chance of it being a perfect square number is $\frac{1}{d}$, find the value of d .

FOR OFFICIAL USE

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Mult. factor for
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Time

Total score

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Hong Kong Mathematics Olympiad (1996-97)

Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若數列 $1, 6 + 2a, 10 + 5a, \dots$ 是一算術級數，求 a 的值。

If the sequence $1, 6 + 2a, 10 + 5a, \dots$ forms an A.P., find the value of a .

$a =$

- (ii) 若 $(0.0025 \times 40)^b = \frac{1}{100}$ ，求 b 的值。

If $(0.0025 \times 40)^b = \frac{1}{100}$, find the value of b .

$b =$

- (iii) 若 c 為正整數及 $c^3 + 3c + \frac{3}{c} + \frac{1}{c^3} = 8$ ，求 c 的值。

If c is an integer and $c^3 + 3c + \frac{3}{c} + \frac{1}{c^3} = 8$, find the value of c .

$c =$

- (iv) 若將 5 個女孩排成一列，共有 d 個不同方法。求 d 的值。

There are d different ways for arranging 5 girls in a row. Find the value of d .

$d =$

FOR OFFICIAL USE

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score

Time

Total score

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Hong Kong Mathematics Olympiad (1996-97)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 設 m 為滿足不等式 $14x - 7(3x - 8) < 4(25 + x)$ 的整數。求 m 的最小值。

Let m be an integer satisfying the inequality: $14x - 7(3x - 8) < 4(25 + x)$.

Find the least value of m .

$m =$

- (ii) 已知 $f(x) = \frac{1}{3}x^3 - 2x^2 + \frac{2}{3}x^3 + 3x^2 + 5x + 7 - 4x$ 。若 $f(-2) = b$ ，求 b 的值。

It is given that $f(x) = \frac{1}{3}x^3 - 2x^2 + \frac{2}{3}x^3 + 3x^2 + 5x + 7 - 4x$.

If $f(-2) = b$, find the value of b .

$b =$

- (iii) 已知 $\log \frac{x}{2} = 0.5$ 及 $\log \frac{y}{5} = 0.1$ 。若 $\log xy = c$ ，求 c 的值。

It is given that $\log \frac{x}{2} = 0.5$ and $\log \frac{y}{5} = 0.1$. If $\log xy = c$, find the value of c .

$c =$

- (iv) d 、 e 及 f 為三個小於 10 之質數且滿足兩個條件 $d + e = f$ 及 $d < e$ 。求 d 的值。

Three prime numbers d , e and f which are all less than 10, satisfy the two conditions $d + e = f$ and $d < e$. Find the value of d .

$d =$

FOR OFFICIAL USE

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Mult. factor for
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Team No.

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Bonus
score

Time

Total score

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Hong Kong Mathematics Olympiad (1996-97)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知 $a = 103 \times 97 \times 10009$ ，求 a 的值。

It is given that $a = 103 \times 97 \times 10009$, find the value of a .

$a =$

- (ii) 已知 $1 + x + x^2 + x^3 + x^4 = 0$ 。若 $b = 2 + x + x^2 + x^3 + x^4 + \dots + x^{1989}$ ，求 b 的值。

It is given that $1 + x + x^2 + x^3 + x^4 = 0$.

If $b = 2 + x + x^2 + x^3 + x^4 + \dots + x^{1989}$, find the value of b .

$b =$

- (iii) 已知 m 及 n 為兩個不大於 10 的自然數。

若 c 為 m 及 n 滿足方程 $mx = n$ 之組數，其中 $\frac{1}{4} < x < \frac{1}{3}$ 。求 c 的值。

It is given that m and n are two natural numbers and both are not greater than 10. If c is the number of pairs of m and n satisfying the equation $mx = n$, where $\frac{1}{4} < x < \frac{1}{3}$, find the value of c .

$c =$

- (iv) 設 x 及 y 為實數且定義運算 $*$ 為 $x*y = px^y + q + 1$ 。已知 $1*2 = 869$ 及 $2*3 = 883$ 。若 $2*9 = d$ ，求 d 的值。

Let x and y be real numbers and define the operation $*$ as $x*y = px^y + q + 1$.

It is given that $1*2 = 869$ and $2*3 = 883$. If $2*9 = d$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
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Mult. factor for
speed

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Team No.

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score

Time

Total score

Min.

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Hong Kong Mathematics Olympiad (1996-97)
Final Event 5 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 a 是 5 的正倍數，且被 3 除時餘 1，求 a 之最小可能數值。

If a is a positive multiple of 5, which gives remainder 1 when divided by 3, find the smallest possible value of a .

$a =$

- (ii) 若 $x^3 + 6x^2 + 12x + 17 \equiv (x + 2)^3 + b$ ，求 b 的值。

If $x^3 + 6x^2 + 12x + 17 \equiv (x + 2)^3 + b$, find the value of b .

$b =$

- (iii) 若 c 是一兩位正整數，其兩位之和是 10 而兩位之積是 25。求 c 的值。

If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c .

$c =$

- (iv) 設 S_1, S_2, \dots, S_{10} 是一個由正整數組成的 A.P. 之首 10 項。

若 $S_1 + S_2 + \dots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$ 。求 d 的值。

Let S_1, S_2, \dots, S_{10} be the first ten terms of an A.P., which consists of positive integers.

If $S_1 + S_2 + \dots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

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Mult. factor for
speed

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Team No.

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Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event (Spare Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) E 是平行四邊形 $ABCD$ 其中一條邊 CD 的中點。若三角形 ADE 與平行四邊形 $ABCD$ 面積的比等於 $1 : a$ ，求 a 的值。

$ABCD$ is a parallelogram and E is the midpoint of CD . If the ratio of the area of the triangle ADE to the area of the parallelogram $ABCD$ is $1 : a$, find the value of a .

$a =$

- (ii) E 是平行四邊形 $ABCD$ 其中一條邊 CD 的中點，且 AE 和 BD 相交於 M ；
 若 $DM : MB = 1 : k$ ，求 k 的值。

$ABCD$ is a parallelogram and E is the midpoint of CD . AE and BD meet at M .
 If $DM : MB = 1 : k$, find the value of k .

$k =$

- (iii) 若 5 的平方根是 2.236，以同一準確度，80 的平方根是 d 。求 d 的值。

If the square root of 5 is approximately 2.236, the square root of 80 with the same precision is d . Find the value of d .

$d =$

- (iv) 將一個正方形的長增加 20%，同時又將它的闊減少 20%，則我們可得一個長方形。若長方形與正方形面積的比為 $1 : r$ ，求 r 的值。

A square is changed into a rectangle by increasing its length by 20% and decreasing its width by 20%. If the ratio of the area of the rectangle to the area of the square is $1 : r$, find the value of r .

$r =$

FOR OFFICIAL USE

Score for
accuracy

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Mult. factor for
speed

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Team No.

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Bonus
score

Time

Total score

Min.

Sec.