Hong Kong Mathematics Olympiad 2003-2004
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。   時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 設 $A = 1^2 - 2^2 + 3^2 - 4^2 + ... + 2003^2 - 2004^2$，求 A 的值。

   Let $A = 1^2 - 2^2 + 3^2 - 4^2 + ... + 2003^2 - 2004^2$, find the value of A .

2. 若 $\sqrt[2003]{B} = 2003$，C 是 B 的個位數，求 C 的值。

   If $\sqrt[2003]{B} = 2003$, C is the units digit of B, find the value of C .

3. 若 $x + y + z = 10$，$x^2 + y^2 + z^2 = 10$ 及 $xy + yz + zx = m$，求 m 的值。

   If $x + y + z = 10$, $x^2 + y^2 + z^2 = 10$ and $xy + yz + zx = m$, find the value of m .

4. 把自然數按下列方式排列，其中 9 的位置是第 3 列第 2 行。若 2003 的位置是第 x 列第 y 行，求 xy 的值。

   Arrange the natural numbers in the following order. In this arrangement, 9 is in the row 3 and the column 2. If the number 2003 is in the row x and the column y, find the value of xy .

1	2	4	7	11	16	…
3	5	8	12	17	…	
6	9	13	18	…		
10	14	19	…			
15	20	…				
21	…					
…						

5. 設 $E = \sqrt{12 + 6\sqrt{3}} + \sqrt{12 - 6\sqrt{3}}$，求 E 的值。

   Let $E = \sqrt{12 + 6\sqrt{3}} + \sqrt{12 - 6\sqrt{3}}$, find the value of E .

6. 在圖中，大半圓的圓心是 O，半徑是 $10\,cm$，OB 是小半圓的直徑，C 是弧 OB 的中點且在綫段 OA 上。設陰影部分的面積是 $K\,cm^2$，求 K 的值。(取 $\pi = 3$)

   In the figure, O is the centre of the bigger semicircle with radius 10 cm, OB is the diameter of the smaller semicircle and C is the midpoint of arc OB and it lies on the segment OA. Let the area of the shaded region be $K\,cm^2$, find the value of K. (Take $\pi = 3$)
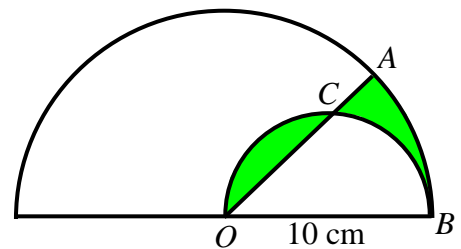
7. 在圖中，設被三條直綫 $y = -x + 3$，$y = x + 1$ 及 $y = -x + 19$ 所圍出的陰影部分的面積是 R，求 R 的值。

   In the figure, let the shaded area formed by the three straight lines
 $y = -x + 3$, $y = x + 1$ and $y = -5x + 19$ be R, find the value of R.
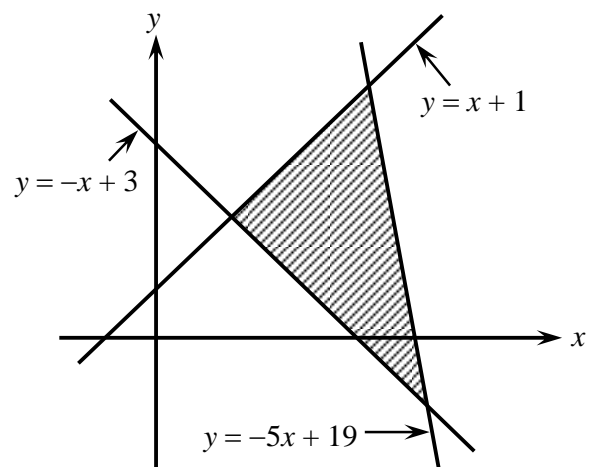
8. 若 $t = \sin^4 \dfrac{\pi}{6} - \cos^2 \dfrac{2\pi}{6}$，求 t 的值。

   If $t = \sin^4 \dfrac{\pi}{6} - \cos^2 \dfrac{2\pi}{6}$, find the value of t.

9.   在圖中，C 在 AE 上，ΔABC 和 ΔCDE 是等邊
      三角形，且 F、G 分別是 BC 和 DE 的中點。
      若 ΔABC 的面積是 24cm^2，ΔCDE 的面積是 60
      cm^2，ΔAFG 的面積是 Q cm^2，求 Q 的值。
      In the figure, C lies on AE, ΔABC and ΔCDE are
      equilateral triangles, F and G are the mid-points
      of BC and DE respectively. If the area of ΔABC is
      24 cm^2, the area of ΔCDE is 60 cm^2, and the area
      of ΔAFG is Q cm^2, find the value of Q .
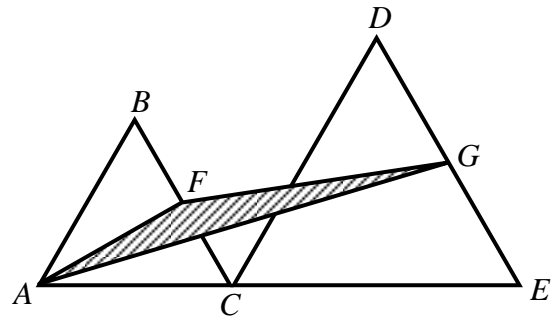
10.   若 α 和 β 是二次方程式 $4x^2 - 10x + 3 = 0$ 的根及 $k = \alpha^2 + \beta^2$，求 k 的值。
      If α and β are the roots of the quadratic equation $4x^2 - 10x + 3 = 0$ and $k = \alpha^2 + \beta^2$,
      find the value of k.

*** 試卷完 **End of Paper** ***

除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　　　時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark.　　Time allowed: 20 minutes

1. 若 $x = \dfrac{1}{2} + \left(\dfrac{1}{3} + \dfrac{2}{3}\right) + \left(\dfrac{1}{4} + \dfrac{2}{4} + \dfrac{3}{4}\right) + \left(\dfrac{1}{5} + \dfrac{2}{5} + \dfrac{3}{5} + \dfrac{4}{5}\right) + \cdots + \left(\dfrac{1}{100} + \dfrac{2}{100} + \cdots + \dfrac{99}{100}\right)$，求 x 的值。

   If $x = \dfrac{1}{2} + \left(\dfrac{1}{3} + \dfrac{2}{3}\right) + \left(\dfrac{1}{4} + \dfrac{2}{4} + \dfrac{3}{4}\right) + \left(\dfrac{1}{5} + \dfrac{2}{5} + \dfrac{3}{5} + \dfrac{4}{5}\right) + \cdots + \left(\dfrac{1}{100} + \dfrac{2}{100} + \cdots + \dfrac{99}{100}\right)$,

   find the value of x.

2. 若 z 是方程 $6 \times 4^x - 13 \times 6^x + 6 \times 9^x = 0$ 的正數根，求 z 的值。

   If z is the positive root of the equation $6 \times 4^x - 13 \times 6^x + 6 \times 9^x = 0$, find the value of z.

3. 若最多有 k 個互不全等的等腰三角形，其周界為 25 cm 及其三邊的長度以 cm 表示時均為正整數，求 k 的值。

   If there are at most k mutually non-congruent isosceles triangles whose perimeter is 25cm and the lengths of the three sides are positive integers when expressed in cm, find the value of k.

4. 已知 a、b 為實數並且滿足 $a^3 = 2004$ 及 $b^2 = 2004$。若滿足不等式 $a < x < b$ 的整數 x 有 h 個，求 h 的值。

   Given that a, b are positive real numbers satisfying $a^3 = 2004$ and $b^2 = 2004$.
   If the number of integers x that satisfy the inequality $a < x < b$ is h, find the value of h.

5. 若 R 個連續正整數之和是 1000 (其中 $R > 1$)，求 R 的最小值。

   If the sum of R consecutive positive integers is 1000 (where $R > 1$), find the least value of R.

6. 若 a、b 及 c 是正整數且 $abc + ab + bc + ac + a + b + c = 2003$，求 abc 的最小值。

   If a, b and c are positive integers such that $abc + ab + bc + ac + a + b + c = 2003$,
   find the least value of abc.

7. 在圖中，$ABCD$ 是梯形，AB、CD 垂直於 BC，對角綫 AC 和 BD 相交於 X。若 $AB = 9$ cm，$BC = 12$ cm，$CD = 16$ cm，ΔBXC 的面積為 W cm^2，求 W 的值。

   In the figure, $ABCD$ is a trapezium, the segments AB and CD are both perpendicular to BC and the diagonals AC and BD intersect at X. If $AB = 9$ cm, $BC = 12$ cm and $CD = 16$ cm, and the area of ΔBXC is W cm^2, find the value of W.
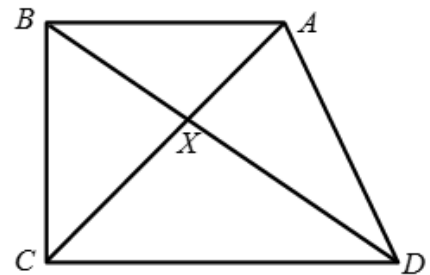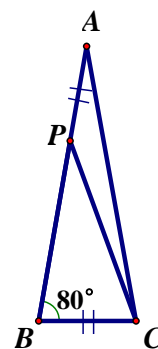
8. 設 $y = \log_{1400} \sqrt{2} + \log_{1400} \sqrt[3]{5} + \log_{1400} \sqrt[6]{7}$，求 y 的值。

   Let $y = \log_{1400} \sqrt{2} + \log_{1400} \sqrt[3]{5} + \log_{1400} \sqrt[6]{7}$, find the value of y.

9. 在圖中，ΔABC 是等腰三角形，$AB = AC$ 及 $\angle ABC = 80°$。
    若 P 是 AB 上一點使得 $AP = BC$，$\angle ACP = k°$，求 k 的值。
    In the figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 80°$.
    If P is a point on the AB such that $AP = BC$, $\angle ACP = k°$, find the value of k.

10. 若點 $P(a, b)$ 在直綫 $x - y + 1 = 0$ 上使得點 P 與點 $A(1, 0)$ 之間的距離和點 P 與點 $B(3, 0)$ 之間的距離之和為最小，求 $a + b$ 的值。
    Suppose $P(a, b)$ is a point on the straight line $x - y + 1 = 0$ such that the sum of the distance between P and the point $A(1,0)$ and the distance between P and the point $B(3,0)$ is the least, find the value of $a + b$.

*** 試卷完 End of Paper ***