Examples on Mathematical Induction: divisibility 64 & 512

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1. Prove by mathematical induction $3^{2n+2} - 8n - 9$ is divisible by 64 for all non-negative integers n.
2. Prove that $3^{2n} - 32n^2 + 24n - 1$ is divisible by 512 for all non-negative integers n.

We first prove that $3^{2n} - 8n - 1$ is divisible by 64 for all non-negative integers n.

$3^0 - 8\times 0 - 1 = 0$, which is divisible by 64.

Suppose $3^{2k} - 8k - 1 = 64p$, where p is an integer.

$3^{2k+2} - 8(k + 1) - 1 = 3^{2k+2} - 8k - 9$ which is divisible by 64, (by Q1)

By MI, $3^{2n} - 8n - 1$ is divisible by 64. $\cdots\cdots$ (*)

$n = 0$, $3^0 - 0 + 0 - 1 = 0$, which is divisible by 512.

Suppose $3^{2k} - 32k^2 + 24k - 1 = 512m$, where m is an integer, for some integer k.

$$\begin{aligned}
&\ \ 3^{2(k+1)} - 32(k + 1)^2 + 24(k + 1) - 1 - (3^{2k} - 32k^2 + 24k - 1)\\
&= (9\times 3^{2k} - 3^{2k}) - 32[(k + 1)^2 - k^2] + 24(k + 1 - k)\\
&= 8\times 3^{2k} - 32(2k + 1) + 24\\
&= 8\times 3^{2k} - 64k - 8\\
&= 8(3^{2k} - 8k - 1)\\
&= 8\times 64p, \text{ by (*)}\\
&= 512p
\end{aligned}$$

$\therefore$ $3^{2(k+1)} - 32(k + 1)^2 + 24(k + 1) - 1 = 512m + 512p$, which is divisible by 512.

$\therefore$ If $P(k)$ is true then $P(k+1)$ is also true.

By the principle of mathematical induction, $P(n)$ is true for all positive integers n.