Q6. In ΔABC, D, E and F are points on BC, CA and AB such that $\frac{BD}{DC}=\frac{CE}{EA}=\frac{AF}{FB}$. Prove that AD, BE and CF can form a triangle.

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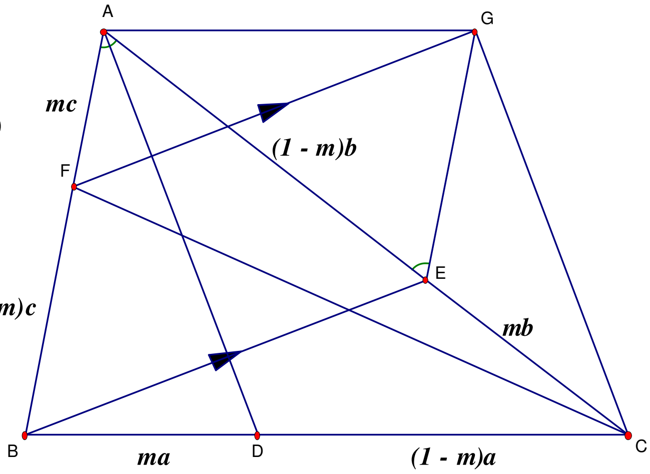
Case 1 D, E and F are not concurrent.

Translate BE along the direction BF to FG.

Then $BEGF$ is a parallelogram. (opp. sides are eq. and //)

Join EG, CG, AG.



$EG = (1 - m)c$ (opp. sides, //-gram)

$BF \parallel EG \Rightarrow BA \parallel EG$ (property of //-gram)

$\angle BAC = \angle AEG$ (alt. $\angle$s $BA \parallel EG$)

$\frac{AE}{EG}=\frac{(1-m)b}{(1-m)c}=\frac{b}{c}=\frac{AC}{AB}$

$\therefore \Delta AEG \sim \Delta CAB$ (SAS)

$\angle EAG = \angle ACB$ (corr. $\angle$s. $\sim\Delta$s)

$AG \parallel DC$ (alt. $\angle$s eq.)

$\frac{AG}{BC}=\frac{AE}{AC}$ (corr. sides, $\sim\Delta$s)

$\frac{AG}{a}=\frac{(1-m)b}{b}=(1-m)$

$AG = (1 - m)a = DC$

$ADCG$ is a //-gram (opp. sides are eq. and //)

$AD = CG$ (opp. sides //-gram)

$\therefore \Delta CFG$ is the required triangle.

Case 2 D, E and F are concurrent.

Note that the proof is exactly the same as the case 1.

You may also prove that $m=\frac{1}{2}$.