Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 一個動物園內有 a 頭駱駝，單峯的比雙峯的多 10 頭。若牠們共有 55 個峯，求 a 的值。
There are a camels in a zoo.
The number of one-hump camels exceeds that of two-hump camels by 10.
If there have 55 humps altogether, find the value of a .

$a =$

2. 若 LCM(a, b) = 280 及 HCF(a, b) = 10，求 b 的值。
If LCM(a, b) = 280 and HCF(a, b) = 10, find the value of b .

$b =$

3. 設 C 是一正整數且小於 $\sqrt{b}$ 。若 b 除以 C，餘數是 2。除以(C + 2)，餘數是 C，求 C 的值。
Let C be a positive integer less than $\sqrt{b}$. If b is divided by C, the remainder is 2; when divided by C + 2, the remainder is C, find the value of C .

$C =$

4. 一個正 2C 邊形共有 d 條對角綫，求 d 的值。
A regular 2C-sided polygon has d diagonals, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 陳先生有 8 個兒子和 a 個女兒，他的每個兒子都有 8 個兒子和 a 個女兒。他的每個女兒都有 a 個兒子和 8 個女兒。已知陳先生的男孫比女孫多 1 個及 a 是個質數，求 a 的值。
Mr. Chan has 8 sons and a daughters. Each of his sons has 8 sons and a daughters. Each of his daughters has a sons and 8 daughters. It is known that the number of his grand sons is one more than the number of his grand daughters and a is a prime number, find the value of a .

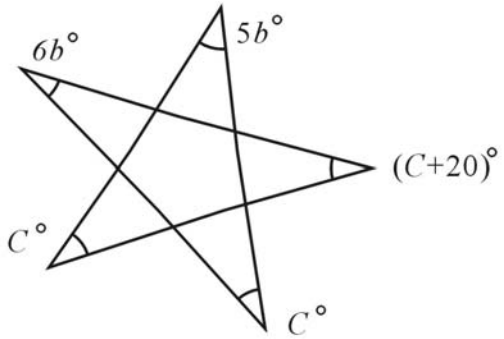
$a =$

2. 設 $\frac{a}{7} = \sqrt[3]{2+\sqrt{b}} + \sqrt[3]{2-\sqrt{b}}$ ，求 b 的值。
Let $\frac{a}{7} = \sqrt[3]{2+\sqrt{b}} + \sqrt[3]{2-\sqrt{b}}$. Find the value of b .

$b =$

3. 如圖一，求 C 的值。
In Figure 1, find the value of C .

$C =$



圖一 Figure 1

4. 已知 P_1、P_2、⋯、P_d 是 d 個連續質數。
若 $P_1 + P_2 + \cdots + P_{d-2} = P_{d-1} + P_d = C + 1$，求 d 的值。
Given that P_1 , P_2 , ⋯ , P_d are d consecutive prime numbers.
If $P_1 + P_2 + \cdots + P_{d-2} = P_{d-1} + P_d = C + 1$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

\+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 a 是方程 $2^{x+1}=8^{\frac{1}{x}-\frac{1}{3}}$ 的正實數解，求 a 的值。

   Given that a is a positive real root of the equation $2^{x+1}=8^{\frac{1}{x}-\frac{1}{3}}$. Find the value of a.

   $a =$

2. 在周界為 a 米的長方形中，最大面積的一個長方形的面積是 b 平方米，求 b 的值。

   The largest area of the rectangle with perimeter a meter is b square meter, find the value of b.

   $b =$

3. 若 $c = [1234^3 - 1232\times(1234^2 + 2472)]\times b$，求 c 的值。

   If $c = [1234^3 - 1232\times(1234^2 + 2472)]\times b$, find the value of c.

   $c =$

4. 若 $\frac{1}{(c+1)(c+2)}+\frac{1}{(c+2)(c+3)}+\cdots+\frac{1}{(c+d)(c+d+1)}=\frac{8}{15}$，求 d 的值。

   If $\frac{1}{(c+1)(c+2)}+\frac{1}{(c+2)(c+3)}+\cdots+\frac{1}{(c+d)(c+d+1)}=\frac{8}{15}$, find the value of d.

   $d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $A^2 + B^2 + C^2 = AB + BC + CA = 3$ 及 $a = A^2$ ，求 a 的值。
If $A^2 + B^2 + C^2 = AB + BC + CA = 3$ and $a = A^2$, find the value of a.

$a =$

2. 已知 n 及 b 是整數，並滿足方程 $29n + 42b = a$，若 $5 < b < 10$，求 b 的值。
Given that n and b are integers satisfying the equation $29n + 42b = a$.
If $5 < b < 10$, find the value b.

$b =$

3. 若 $\dfrac{\sqrt{3}-\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \dfrac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+b}{59}$ ，求 c 的值。
If $\dfrac{\sqrt{3}-\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \dfrac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+b}{59}$, find the value of c.

$c =$

4. 若 c 有 d 個正因數，求 d 的值。
If c has d positive factors, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若在 1 至 200 內能同時被 3 和 7 整除的數有 a 個，求 a 的值。
Suppose there are a numbers between 1 and 200 that can be divisible by 3 and 7, find the value of a .

$a =$

2. 設質數 p 和 q 是方程 $x^2 - 13x + R = 0$ 的兩個不同的根，其中 R 是實數。
若 $b = p^2 + q^2$，求 b 的值。
Let p and q be prime numbers that are the two distinct roots of the equation $x^2 - 13x + R = 0$, where R is a real number. If $b = p^2 + q^2$, find the value of b .

$b =$

3. 已知 $\tan\alpha = -\frac{1}{2}$。若 $c = \frac{2\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha}$，求 c 的值。
Given that $\tan\alpha = -\frac{1}{2}$. If $c = \frac{2\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha}$, find the value of c .

$c =$

4. 設 r 和 s 是方程 $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) = 1$ 的兩個不同的實數根。
若 $d = r + s$，求 d 的值。
Let r and s be the two distinct real roots of the equation $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) = 1$. If $d = r + s$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

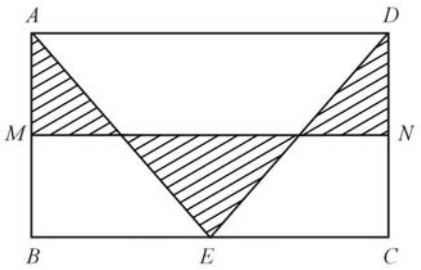
Team No. [ ]

Time [ ] [ ]
Min. Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，在長方形 $ABCD$ 中，$AB = 6\,\text{cm}$，$BC = 10\,\text{cm}$。M 和 N 分別是 AB 和 DC 的中點。若陰影部分的面積是 $a\,\text{cm}^2$，求 a 的值。
In Figure 1, $ABCD$ is a rectangle, $AB = 6\,\text{cm}$ and $BC = 10\,\text{cm}$. M and N are the midpoints of AB and DC respectively. If the area of the shaded region is $a\,\text{cm}^2$, find the value of a.



圖一 Figure 1

$a =$

2. 設 $b = 89 + 899 + 8999 + 89999 + 899999$，求 b 的值。
Let $b = 89 + 899 + 8999 + 89999 + 899999$, find the value of b.

$b =$

3. 已知 $2x + 5y = 3$。若 $c = \sqrt{4^{x+\frac{1}{2}} \times 32^{y}}$，求 c 的值。
Given that $2x + 5y = 3$. If $c = \sqrt{4^{x+\frac{1}{2}} \times 32^{y}}$, find the value of c.

$c =$

4. 設 $d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots + \frac{10}{2^{10}}$，求 d 的值。
Let $d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots + \frac{10}{2^{10}}$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy ____ × Mult. factor for speed ____ = ____ Team No. ____

\+ Bonus score ____ Time ____ ____

Total score ____ Min. Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

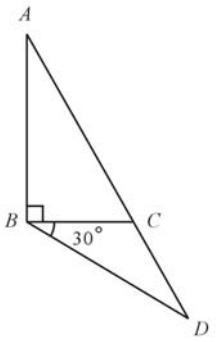
1. 設 $0° < \alpha < 45°$。若 $\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16}$ 及 $A = \sin\alpha$，求 A 的值。

Let $0° < \alpha < 45°$. If $\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16}$ and $A = \sin\alpha$, find the value of A.

$A =$

2. 如圖一，C 在 AD 上且 $AB = BD = 1$ cm，$\angle ABC = 90°$，$\angle CBD = 30°$。若 $CD = b$ cm，求 b 的值。

In figure 1, C lies on AD, $AB = BD = 1$ cm, $\angle ABC = 90°$ and $\angle CBD = 30°$. If $CD = b$ cm, find the value of b.

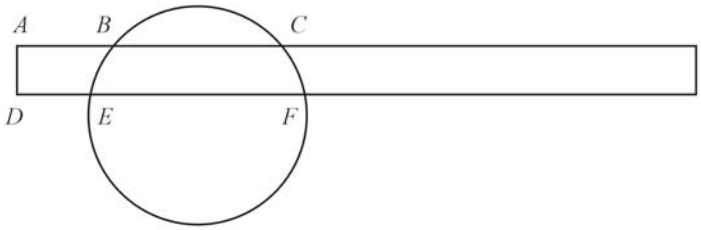


圖一 Figure 1

$b =$

3. 如圖二，一長方形與圓相交於點 B、C、E 及 F。已知 $AB = 4$ cm，$BC = 5$ cm 及 $DE = 3$ cm。若 $EF = c$ cm，求 c 的值。

In Figure 2, a rectangle intersects a circle at points B, C, E and F. Given that $AB = 4$ cm, $BC = 5$ cm and $DE = 3$ cm. If $EF = c$ cm, find the value of c.



圖二 Figure 2

$c =$

4. 假設 x 和 y 都是正數並且成反比。若 x 增加了 10%，則 y 減少了 d %，求 d 的值。

Let x and y be two positive numbers that are inversely proportional to each other. If x is increased by 10 %, y will be decreased by d %, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $a = \log_{\frac{1}{2}} 0.125$ ，求 a 的值。
If $a = \log_{\frac{1}{2}} 0.125$, find the value of a.

$a =$

2. 若方程 $\left| x - \left| 2x + 1 \right| \right| = 3$ 有 b 個不同的解，求 b 的值。
Suppose there are b distinct solutions of the equation $\left| x - \left| 2x + 1 \right| \right| = 3$, find the value of b.

$b =$

3. 若 $c = 2\sqrt{3} \times \sqrt[3]{1.5} \times \sqrt[6]{12}$ ，求 c 的值。
If $c = 2\sqrt{3} \times \sqrt[3]{1.5} \times \sqrt[6]{12}$, find the value of c.

$c =$

4. 已知 $f_1 = 0$ ， $f_2 = 1$ 及對正整數 $n \geq 3$ ， $f_n = f_{n-1} + 2 f_{n-2}$ 。若 $d = f_{10}$ ，求 d 的值。
Given that $f_1 = 0$, $f_2 = 1$ and for any positive integer $n \geq 3$, $f_n = f_{n-1} + 2 f_{n-2}$.
If $d = f_{10}$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.