1991 HI2

某科學家發現某樣本中細菌的數量每小時增加一倍。

於下午四時，他發現細菌的數量為3.2×10^8，

若於同日正午該樣本中細菌的數量為$N\times10^7$，求 N 的值。

A scientist found that the population of a bacteria culture doubled every hour. At 4:00 pm, he found that the number of bacteria was 3.2×10^8. If the number of bacteria in that culture at noon on the same day was $N\times10^7$, find the value of N.

1994 HI1

設 $\log_3 p=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ 至無窮項，求 p 的值。

Suppose $\log_3 p=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ to an infinite number of terms.

Find the value of p.

1997 FG1.3

若 $1+3+3^2+\cdots+3^8=\frac{3^c-1}{2}$，求 c 的值。

If $1+3+3^2+\cdots+3^8=\frac{3^c-1}{2}$, find the value of c.

1998 HI2

已知 8、a、b 形成一等差級數，且 a、b、36 形成一等比級數。

若 a 和 b 皆為正數，求 a、b 的和。

Given that 8, a, b form an A.P. and a, b, 36 form a G.P.

If a and b are both positive numbers, find the sum of a and b.

1998 FG4.3

圖形 S_0，S_1，S_2，⋯ 用以下方法構成：把綫段[0, 1]的中間三分之一取去，得到 S_0，把 S_0 的兩條組成綫段，每段的中間三分之一取去，得到 S_1，把 S_1 的四條組成綫段，每段的中間三分之一取去，得到 S_2，S_3、S_4 …等用類似方法獲得。求在構成 S_5 的過程中取去的綫段的總長度 c(答案以分數表示)。

A sequence of figures S_0, S_1, S_2, ⋯ are constructed as follows. S_0 is obtained by removing the middle third of [0,1] interval; S_1 by removing the middle third of each of the two intervals in S_0; S_2 by removing the middle third of each of the four intervals in S_1; S_3, S_4, … are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 (Give your answer in fraction).

|————( )————| S_0

0 $\frac{1}{3}$ $\frac{2}{3}$ 1

|——( )——( )——( )——| S_1

0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{7}{9}$ $\frac{8}{9}$ 1

|-( )-( )-( )-( )-( )-( )-( )-| S_2

2001 FI3.3

若 $\sin 30^\circ+\sin^2 30^\circ+\cdots+\sin^7 30^\circ=1-\cos^R 45^\circ$，求 R 的值。

If $\sin 30^\circ+\sin^2 30^\circ+\cdots+\sin^7 30^\circ=1-\cos^R 45^\circ$, find the value of R.

2002 FI2.2

已知 $99Q=\frac{99}{100}\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\cdots)$，求 Q 的值。

Given that $99Q=\frac{99}{100}\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\cdots)$, find the value of Q.

2005 FG2.4

設 $d=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\cdots+\frac{10}{2^{10}}$，求 d 的值。

Let $d=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\cdots+\frac{10}{2^{10}}$, find the value of d.

2006 HG3

已知 $0^\circ<\theta<90^\circ$ 及 $1+\sin\theta+\sin^2\theta+\cdots=\frac{3}{2}$。若 $y=\tan\theta$，求 y 的值。

Given that $0^\circ<\theta<90^\circ$ and $1+\sin\theta+\sin^2\theta+\cdots=\frac{3}{2}$.

If $y=\tan\theta$, find the value of y.

2007 FG2.1

若 $R=1\times2+2\times2^2+3\times2^3+\cdots+10\times2^{10}$，求 R 的值。

If $R=1\times2+2\times2^2+3\times2^3+\cdots+10\times2^{10}$, find the value of R.

2009 FI1.3

設 $F=1+2+2^2+2^3+\cdots+2^{120}$ 及 $T=\sqrt{\frac{\log(1+F)}{\log 2}}$，求 T 的值。

Let $F=1+2+2^2+2^3+\cdots+2^{120}$ and $T=\sqrt{\frac{\log(1+F)}{\log 2}}$, find the value of T.

2010 FG2.1

若 $p = 2 - 2^2 - 2^3 - 2^4 - \cdots - 2^9 - 2^{10} + 2^{11}$，求 p 的值。

If $p = 2 - 2^2 - 2^3 - 2^4 - \cdots - 2^9 - 2^{10} + 2^{11}$, find the value of p.

2012 HI5

已知 $\log_4 N = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$，求 N 的值。

Given that $\log_4 N = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$, find the value of N.

2015 FI1.4

若 n 為正整數及 $\mathrm{f}(n) = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 1$，求 $\delta = \mathrm{f}(10)$ 的值。

If n is a positive integer and $\mathrm{f}(n) = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 1$, determine the value of $\delta = \mathrm{f}(10)$.

2017 FI3.4

若 $\mathrm{f}(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$，求 $d = \mathrm{f}(10)$ 的值。

If $\mathrm{f}(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$, determine the value of $d = \mathrm{f}(10)$.

2019 HI4

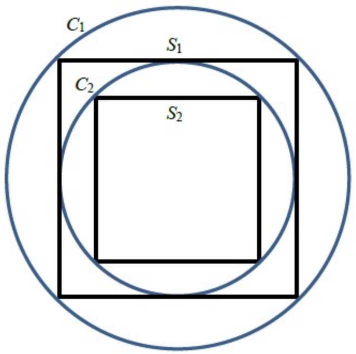
設 n 為正整數。若 $a_n = 1 + 2 + \cdots + 2^n$ 及 $b = a_{10} - a_5 + a_1$，求 b 的值。

Let n be a positive integer. If $a_n = 1 + 2 + \cdots + 2^n$ and $b = a_{10} - a_5 + a_1$, find the value of b.

2023 FI2.2

C_1 是正方形 S_1 的外接圓，它的半徑為 9，C_2 是正方形 S_1 的內切圓；同時也是正方形 S_2 的外接圓，如此類推。求正方形 S_6 的面積 β。

A circle C_1 of radius 9 circumscribes a square S_1 which inscribes a circle C_2. C_2 circumscribes square S_2 and so forth indefinitely.
Find the area β of the square S_6.



Answers

1991 HI2 2	1994 HI1 9	1997 FG1.4 9	1998 HI2 40	1998 FG4.3 $\frac{665}{729}$
2001 FI3.3 14	2002 FI2.2 1	2005 FG2.4 $\frac{509}{256}$	2006 HG3 $\frac{\sqrt{2}}{4}$	2007 FG2.1 18434
2009 FI1.3 11	2010 FG2.1 6	2012 HI5 8	2015 FI1.4 2047	2017 FI3.4 1023
2019 HI4 1987	2023 FI2.2 $\frac{81}{16}=5\frac{1}{16}$			