### 1982 FG9.2

方程  $x^2 - 15x + B = 0$  的根為 7 及  $C \circ x B$  和 C的值。

The roots of the equation  $x^2 - 15x + B = 0$  are 7 and C. Find the values of B and C.

# 1983 FG6.3

雨數之和為 10,其乘積為 20。若該兩倒數之和為 c,求 c的值。

The sum of two numbers is 10, their product is 20.

The sum of their reciprocal is c. What is the value of c?

### 1984 FI1.3

若 1 為  $x^2 + cx - 5 = 0$  之一根,求 c 的值。

If 1 is a root of  $x^2 + cx - 5 = 0$ , find the value of c.

## 1984 FI2.1

If  $\alpha$ ,  $\beta$  are roots of  $x^2 - 10x + 20 = 0$ , find the value of a, where  $a = \frac{1}{\alpha} + \frac{1}{\beta}$ .

### 1984 FSG.1

某兩數之和為20,其積為10,若該兩數倒數之和為a,求a的值。

The sum of 2 numbers is 20, their product is 10.

If the sum of their reciprocals is a, find the value of a.

### 1985 FSI.1

某兩數之和為40,其積為20。若該兩數倒數之和為 a,求 a 的值。

The sum of two numbers is 40, and their product is 20.

If the sum of their reciprocals is a, find the value of a.

# 1985 FI1.3

若 2 為方程  $x^3 + cx + 10 = 0$  之一根, 求 c 的值。

If one root of the equation  $x^3 + cx + 10 = 0$  is 2, find the value of c.

## 1986 FSG.1

某兩數之和為50,其積為25。若該兩數倒數之和為 a,求 a 的值。

The sum of two numbers is 50, and their product is 25.

If the sum of their reciprocals is a, find the value of a.

## 1987 FG6.1

If  $\alpha$ ,  $\beta$  are the roots of  $x^2 - 10x + 20 = 0$ , and  $p = \alpha^2 + \beta^2$ , find the value of p.

## 1988 FI1.4

若  $3x^2 - ax + 50 = 0$  的其中一根是  $\frac{50}{9}$  ,而另一根是 S ,求 S 的值。

If one root of the equation  $3x^2 - ax + 50 = 0$  is  $\frac{50}{9}$  and the other root is S,

find the value of S.

#### 1989 HI16

已知  $\alpha \cdot \beta$  為  $x^2 - 10x + c = 0$  的雨根,且  $\alpha\beta = -11$ 及 $\alpha > \beta$ ,求 $\alpha - \beta$ 的值。  $\alpha$ ,  $\beta$  are the roots of the equation  $x^2 - 10x + c = 0$ .

If  $\alpha\beta = -11$  and  $\alpha > \beta$ , find the value of  $\alpha - \beta$ .

#### 1989 HG1

 $a \cdot b$  為兩相異實數,且  $a^2 = 5a + 10$  及  $b^2 = 5b + 10$ ,求  $\frac{1}{a^2} + \frac{1}{b^2}$  的值。

Given a and b are distinct real numbers satisfying  $a^2 = 5a + 10$  and  $b^2 = 5b + 10$ .

Find the value of  $\frac{1}{a^2} + \frac{1}{b^2}$ .

### 1989 FI3.

已知 $\alpha + \beta = 11$ , $\alpha\beta = 24$ ,且 $\alpha > \beta$ ,求  $\alpha$  的值。

If  $\alpha + \beta = 11$ ,  $\alpha\beta = 24$  and  $\alpha > \beta$ , find the value of  $\alpha$ .

### 1990 HI8

若α、β是方程  $2x^2+4x-3=0$  的根,且  $\alpha^2$ 、 $\beta^2$  是方程  $x^2+px+q=0$  的根,求 p 的值。

If  $\alpha$ ,  $\beta$  are the roots of the equation  $2x^2 + 4x - 3 = 0$  and  $\alpha^2$ ,  $\beta^2$  are the roots of the equation  $x^2 + px + q = 0$ , find the value of p.

## 1990 FI4.2

若 5 是 方程  $x^2 - bx + 35 = 0$  的 一 個 根 , 求 b 的 值 。

If 5 is one root of the equation  $x^2 - bx + 35 = 0$ , find the value of b.

## 1991 HI12

設方程  $(x^2-11x-10)+k(x+2)=0$  的其中一根為零,求另一根。

If one root of the equation  $(x^2 - 11x - 10) + k(x + 2) = 0$  is zero, find the other root.

## 1991 HI14

 $a \cdot b$  為兩個不同之實數,且 $a^2 = 6a + 8$ 及 $b^2 = 6b + 8$ ,求 $\left(\frac{4}{a}\right)^2 + \left(\frac{4}{b}\right)^2$  的值。

a, b are two different real numbers such that  $a^2 = 6a + 8$  and  $b^2 = 6b + 8$ .

Find the value of  $\left(\frac{4}{a}\right)^2 + \left(\frac{4}{b}\right)^2$ .

Relation between roots and coefficients (HKMO Classified Questions by topics)

### 1991 FG6.3

兩數之和是20,積是5。若該兩數倒數之和是2,求2的值。

The sum of two numbers is 20 and their product is 5.

If the sum of their reciprocals is z, find the value of z.

### 1992 FI5.1

設 $p \cdot q$  為二次方程 $x^2 - 3x - 2 = 0$  的雨根,且 $a = p^3 + q^3$ ,求a的值。

Let p, q be the roots of the quadratic equation  $x^2 - 3x - 2 = 0$  and  $a = p^3 + q^3$ . Find the value of a.

# 1992 FG8.3

方程  $x^3 - 173x^2 + 339x + 513 = 0$  之根為  $-1 \cdot 171$  及  $c \circ 求 c$  的值。

The roots of the equation  $x^3 - 173x^2 + 339x + 513 = 0$  are -1, 171 and c. Find the value of c.

## 1993 HG2

若  $\alpha$ 、β 為方程  $x^2-3x-3=0$  的雨根,求  $\alpha^3+12\beta$  的值。

If  $\alpha$ ,  $\beta$  are the roots of the equation  $x^2 - 3x - 3 = 0$ , find the value of  $\alpha^3 + 12\beta$ .

### 1997 FI5.4

若方程式 $x^2-6x+5=0$  兩根之差為d,求d的值。

If the difference of the two roots of the equation  $x^2 - 6x + 5 = 0$  is d, find the value of d.

### 1998 FI5.1

若方程  $5x^2 + ax - 2 = 0$  的根的和為它的根的積的兩倍,求 a 的值。

If the sum of roots of  $5x^2 + ax - 2 = 0$  is twice the product of roots, find the value of a.

## 1999 FG4.2

設 $\alpha \setminus \beta$ 是  $x^2 + bx - 2 = 0$  的根。若 $\alpha > 1$  及 $\beta < -1$ ,且 b 為一整數,求 b 之值。

Let  $\alpha$ ,  $\beta$  be the roots of  $x^2 + bx - 2 = 0$ . If  $\alpha > 1$  and  $\beta < -1$ , and b is an integer, find the value of b.

# 1999 FG5.1

If the roots of  $x^2 - 2x - P = 0$  differ by 12, find the value of P.

## 2001 FI1.4

已知 
$$\begin{cases} a+b=2 \\ a^2+b^2=12 \end{cases}$$
 及  $a^3+b^3=S$ , 求  $S$  的值。

Given that 
$$\begin{cases} a+b=2 \\ a^2+b^2=12 \end{cases}$$
 and  $a^3+b^3=S$ , find the value of  $S$ .

#### 2003 HG6

設拋物綫  $y = 4x^2 - 5x + c$  與 x-軸相交於  $(\cos \theta, 0)$  及  $(\cos \phi, 0)$ 。

Suppose the parabola  $y = 4x^2 - 5x + c$  intersects the x-axis at  $(\cos \theta, 0)$  and  $(\cos \phi, 0)$  respectively. If  $\theta$  and  $\phi$  are two acute angles of a right-angled triangle, find the value of c.

#### 2004 HI10

若 α 和 β 是二次方程式  $4x^2-10x+3=0$  的根及  $k=\alpha^2+\beta^2$ ,求 k 的值。

If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $4x^2 - 10x + 3 = 0$  and  $k = \alpha^2 + \beta^2$ , find the value of k.

#### 2004 FI3.4

若 -2 和 9 是方程  $px^2 + dx = 1$  的根,求 d 的值。

If -2 and 9 are the roots of the equation  $px^2 + dx = 1$ , find the value of d.

### 2005 HG3

已知  $p \cdot q$  和 r 是方程  $x^3 - x^2 + x - 2 = 0$  的三個不同的根。

若 
$$Q = p^3 + q^3 + r^3$$
, 求  $Q$  的值。

Given that p, q and r are distinct roots of the equation  $x^3 - x^2 + x - 2 = 0$ .

If  $Q = p^3 + q^3 + r^3$ , find the value of Q.

### 2006 HG4

考慮二次方程  $x^2-(a-2)x-a-1=0$  ,其中 a 為實數。設  $\alpha$  和  $\beta$  是方程的根。求 a 的值使得  $\alpha^2+\beta^2$  的值最小。

Consider the quadratic equation  $x^2 - (a-2)x - a - 1 = 0$ , where a is a real number. Let  $\alpha$  and  $\beta$  be the roots of the equation.

Find the value of a such that the value of  $\alpha^2 + \beta^2$  will be the least.

## 2006 FI1.4

已知 10 是方程  $kx^2 + 2x + 5 = 0$  的一個根,其中 k 為常數。

若 D 是另一個根, 求 D 的值。

Given that 10 is a root of the equation  $kx^2 + 2x + 5 = 0$ , where k is a constant.

If D is another root, find the value of D.

## 2006 FG1.3

已知  $x=2+\sqrt{3}$  是方程  $x^2-(\tan\alpha+\cot\alpha)x+1=0$  的一個根。

若  $C = \sin \alpha \times \cos \alpha$ , 求 C 的值。

Given that  $x = 2 + \sqrt{3}$  is a root of the equation  $x^2 - (\tan \alpha + \cot \alpha)x + 1 = 0$ .

If  $C = \sin \alpha \times \cos \alpha$ , find the value of C.

### Relation between roots and coefficients (HKMO Classified Questions by topics)

#### 2007 HI4

設  $r_1$  和  $r_2$ 是方程 (x-2006)(x-2007)=2007 的兩個實根。 若 r 是方程  $(x-r_1)(x-r_2)=-2007$  較小的實根, 求 r 的值。

Let  $r_1$  and  $r_2$  be the two real roots of the equation (x - 2006)(x - 2007) = 2007.

If r is the smaller real root of the equation  $(x - r_1)(x - r_2) = -2007$ ,

find the value of r.

#### 2007 HI5

已知  $\alpha$  及  $\beta$  是方程  $x^2 - 5^{2007}x + 5^{1000} = 0$  的根。

若 
$$s = \log_{25} \frac{\alpha^2}{\beta} + \log_{25} \frac{\beta^2}{\alpha}$$
 , 求  $s$  的值。

Given that  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 5^{2007}x + 5^{1000} = 0$ .

If 
$$s = \log_{25} \frac{\alpha^2}{\beta} + \log_{25} \frac{\beta^2}{\alpha}$$
, find the value of s.

#### 2008 FI3.1

已知 
$$\frac{1-\sqrt{3}}{2}$$
 滿足方程  $x^2+px+q=0$ , 其中  $p$  和  $q$  是有理數。

若 A = |p| + 2|q|, 求 A 的值。

Given that  $\frac{1-\sqrt{3}}{2}$  satisfies the equation  $x^2+px+q=0$ , where p and q are  $\frac{1}{2}$  1 2  $\frac{1}{2}$  3  $\frac{1}{2}$   $\frac{1$ 

rational numbers. If A = |p| + 2|q|, find the value of A.

## 2008 FIS.3

設α和 β是方程  $x^2 + cx + 2 = 0$  的兩個根,其中 c < 0 及 $\alpha - \beta = 1$ 。求 c 的值。 Let  $\alpha$  and  $\beta$  be the two roots of the equation  $x^2 + cx + 2 = 0$ , where c < 0 and  $\alpha - \beta = 1$ . Find the value of c.

## 2009 FI1.1

設  $a \cdot b \cdot c$  及 d 為方程  $x^4 - 15x^2 + 56 = 0$  相異的根。 若  $R = a^2 + b^2 + c^2 + d^2$  , 求 R 的值。

Let a, b, c and d be the distinct roots of the equation  $x^4 - 15x^2 + 56 = 0$ . If  $R = a^2 + b^2 + c^2 + d^2$ , find the value of R.

### 2010 HI2

若 α 及 β 為二次方程  $x^2-x-1=0$  的兩個實根,求  $\alpha^6+8\beta$  的值。 If  $\alpha$  and  $\beta$  are the two real roots of the quadratic equation  $x^2 - x - 1 = 0$ , find the value of  $\alpha^6 + 8\beta$ .

#### 2011 FG3.2

設 $a \cdot b$ 及c為實數。若 1 為  $x^2 + ax + 2 = 0$ 的根及 a 和 b 為  $x^2 + 5x + c = 0$ 的根,求a+b+c的值。

Let a, b and c be real numbers. If 1 is a root of  $x^2 + ax + 2 = 0$ and a and b be roots of  $x^2 + 5x + c = 0$ , find the value of a + b + c.

#### 2012 FI1.1

若 A 是 多項式  $x^4 + 6x^3 + 12x^2 + 9x + 2$  的所有根的平方之和,求 A 的值。 If A is the sum of the squares of the roots of  $x^4 + 6x^3 + 12x^2 + 9x + 2$ . find the value of A.

#### 2013 HG4

 若α、β是方程  $x^2 + 2013x + 5 = 0$  的根,  $求(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$ 的值。

If  $\alpha$ ,  $\beta$  are roots of  $x^2 + 2013x + 5 = 0$ ,

find the value of  $(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$ .

### 2013 HG7

已知 $\triangle ABC$ 的三邊的長度組成一個等差數列,

且為方程  $x^3 - 12x^2 + 47x - 60 = 0$  的根,求  $\triangle ABC$  的面積。

Given that the length of the three sides of  $\triangle ABC$  form an arithmetic sequence, and are the roots of the equation  $x^3 - 12x^2 + 47x - 60 = 0$ , find the area of  $\triangle ABC$ .

### 2013 FI4.3

If 1, 2 and 3 are three roots of the equation  $x^4 + rx^2 + sx + t = 0$ , find the value of c = r + t.

## 2014 HI4

設  $\alpha$  及  $\beta$  為二次方程  $x^2-14x+1=0$  的根。求  $\frac{\alpha^2}{\beta^2+1}+\frac{\beta^2}{\alpha^2+1}$  的值。

Let  $\alpha$  and  $\beta$  be the roots of the quadratic equation  $x^2 - 14x + 1 = 0$ .

Find the value of  $\frac{\alpha^2}{\beta^2+1} + \frac{\beta^2}{\alpha^2+1}$ .

## 2015 FI2.4

若方程  $x^4 + ax^2 + bx + \delta = 0$  有四實根,且已知其中三個為  $1 \cdot 2$  及 4, 求δ的值。

If the equation  $x^4 + ax^2 + bx + \delta = 0$  has four real roots with three of them being 1, 2 and 4, determine the value of  $\delta$ .

#### 2016 HI11

已知方程  $100[\log(63x)][\log(32x)] + 1 = 0$  有兩個相異的實數根  $\alpha$  及  $\beta$ ,求  $\alpha\beta$  的值。

It is known that the equation  $100[\log(63x)][\log(32x)] + 1 = 0$  has two distinct real roots  $\alpha$  and  $\beta$ . Find the value of  $\alpha\beta$ .

### 2016 FI1.3

若方程  $x^2 - cx + 30 = 0$  有兩個實數根及兩根之差為 1 ,求兩根之和的最大可能值 c 。

If the equation  $x^2 - cx + 30 = 0$  has two distinct real roots and their difference is 1, determine the greatest possible value of the sum of the roots, c.

#### 2019 HI9

已知  $\alpha$  及  $\beta$  為方程  $x^2 + 32x - 1 = 0$  的兩個根。

若 
$$P = (\alpha^2 + 31\alpha - 2)(\beta^2 + 33\beta)$$
, 求  $P$  的值。

Given that  $\alpha$  and  $\beta$  are the two roots of the equation  $x^2 + 32x - 1 = 0$ .

If  $P = (\alpha^2 + 31\alpha - 2)(\beta^2 + 33\beta)$ , find the value of P.

### 2021 P1Q3

α 及 β 為方程  $x^2 - 7x + 4 = 0$  的根。求  $\alpha^3 + \beta^3$  的值。

 $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 7x + 4 = 0$ . Find the value of  $\alpha^3 + \beta^3$ .

## 2022 P1Q1

α 及 β 是方程  $x^2 - 100x + k = 0$  的實根。若  $\alpha - 7 = 30\beta$ ,求 k 的值。

 $\alpha$  and  $\beta$  are the real roots of the equation  $x^2 - 100x + k = 0$ .

If  $\alpha - 7 = 30\beta$ , find the value of k.

## **Answers**

Allsweis	1	1	Ī	1
1982 FG9.2 B = 56, C = 8	1983 FG6.3 $\frac{1}{2}$	1984 FI1.3 4	1984 FI2.1 $\frac{1}{2}$	1984 FSG.1 2
1985 FSI.1	1985 FI1.3	1986 FSG.1	1987 FG6.1	1988 FI1.4
2	-9	2	60	3
1989 HI16	1989 HG1	1989FI3.1	1990 HI8	1990 FI4.2
12	9/20	8	-7	12
1991 HI12	1991 HI14	1991 FG6.3	1992 FI5.1	1992 FG8.3
6	13	4	45	3
1993 HG2	1997 FI5.4	1998 FI5.1	1999 FG4.2	1999 FG5.1
45	4	4	0	35
2001 FI1.4 32	2003 HG6 9/8	2004 HI10 19 4	$ \begin{array}{r} 2004 \text{ FI3.4} \\ -\frac{7}{18} \end{array} $	2005 HG3 4
2006 HG4	2006 FI1.4	2006 FG1.3 $\frac{1}{4}$	2007 HI4	2007 HI5
1	-2		2006	500
2008 FI3.1	2008 FIS.3	2009 FI1.1	2010 HI2	2011 FG3.2
2	-3	30	13	1
2012 FI1.1	2013 HG4	2013 HG7	2013 FI4.3	2014 HI4
12	8028	6	-61	193
2015 FI2.4 -56	2016 HI11 1 2016	2016 FI1.3 11	2019 HI9 32	2021 P1Q3 259
2022 P1Q1 291				