Hong Kong Mathematics Olympiad (2003-04)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知有 a 個少於 200 的正整數，它們每個都只有三個正因數，求 a 的值。
Given that there are a positive integers less than 200 and each of them has exactly three positive factors, find the value of a.

$a =$

2. 若 a 個斜邊是 $\sqrt{2}$ cm 的等腰直角三角形能拼成一個周界是 b cm 的梯形，求 b 的最小可能的值。(答案用根號表示)
If a copies of a right-angled isosceles triangle with hypotenuse $\sqrt{2}$ cm can be assembled to form a trapezium with perimeter equal to b cm, find the least possible value of b. (give the answer in surd form).

$b =$

3. 若 $\sin(c^2 - 3c + 17)° = \dfrac{4}{b-2}$，其中 $0 < c^2 - 3c + 17 < 90$ 及 $c > 0$，求 c 的值。
If $\sin(c^2 - 3c + 17)° = \dfrac{4}{b-2}$, where $0 < c^2 - 3c + 17 < 90$ and $c > 0$,
find the value of c.

$c =$

4. 已知兩個三位數 $\overline{xyz}$ 和 $\overline{zyx}$ 的差等於 $700 - c$，其中 $x > z$。若 d 是 $x + z$ 的最大值，求 d 的值。
Given that the difference between two 3-digit numbers $\overline{xyz}$ and $\overline{zyx}$ is $700 - c$, where $x > z$. If d is the greatest value of $x + z$, find the value of d.

$d =$

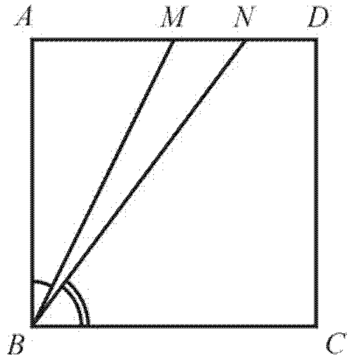
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，$ABCD$ 為一正方形，M 是 AD 的中點及 N 是 MD 的中點及 N 是 MD 的中點。
若 $\angle CBN : \angle MBA = P : 1$，求 P 的值。
In Figure 1, $ABCD$ is a square, M is the mid-point of AD and N is the mid-point of MD.
If $\angle CBN : \angle MBA = P : 1$, find the value of P.



圖一
Figure 1

$P =$

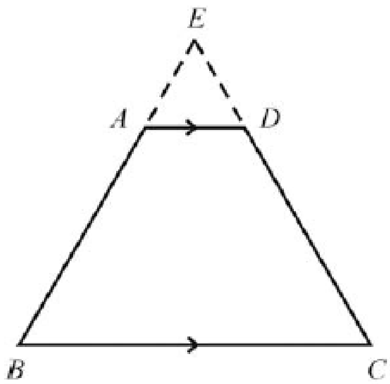
2. 已知 $ABCD$ 為一坐標平面上的菱形，其頂點的座標分別為 $A(0, 0)$，$B(P, 1)$，$C(u, v)$ 及 $D(1, P)$。若 $u + v = Q$，求 Q 的值。
Given that $ABCD$ is a rhombus on a Cartesian plane, and the co-ordinates of its vertices are $A(0, 0)$, $B(P, 1)$, $C(u, v)$ and $D(1, P)$ respectively.
If $u + v = Q$, find the value of Q.

$Q =$

3. 若 $1 + (1 + 2) + (1 + 2 + 3) + \ldots + (1 + 2 + 3 + \ldots + Q) = R$，求 R 的值。
If $1 + (1 + 2) + (1 + 2 + 3) + \ldots + (1 + 2 + 3 + \ldots + Q) = R$, find the value of R.

$R =$

4. 如圖二，EBC 是一等邊三角形，A 和 D 分別在 EB 和 EC 上。已知 $AD \mathbin{//} BC$，$AB = CD = R$，且 $AC \perp BD$。
若梯形 $ABCD$ 的面積是 S，求 S 的值。
In figure 2, EBC is an equilateral triangle, and A, D lie on EB and EC respectively. Given that $AD \mathbin{//} BC$, $AB = CD = R$ and $AC \perp BD$. If the area of the trapezium $ABCD$ is S, find the value of S.



圖二
Figure 2

$S =$

FOR OFFICIAL USE

Score for accuracy $\times$ Mult. factor for speed $=$

Team No.

$+$ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $x \neq \pm 1$ 及 $x \neq -3$。若 a 是方程 $\frac{1}{x-1}+\frac{1}{x+3}=\frac{2}{x^2-1}$ 的實根，求 a 的值。
Let $x \neq \pm 1$ and $x \neq -3$. If a is the real root of the equation $\frac{1}{x-1}+\frac{1}{x+3}=\frac{2}{x^2-1}$, find the value of a.

$a =$

2. 設 $b > 1$，$\mathrm{f}(b)=\frac{-a}{\log_2 b}$ 及 $\mathrm{g}(b)=1+\frac{1}{\log_3 b}$。
若 b 滿足方程 $|\mathrm{f}(b)-\mathrm{g}(b)|+\mathrm{f}(b)+\mathrm{g}(b)=3$，求 b 的值。
If $b > 1$，$\mathrm{f}(b)=\frac{-a}{\log_2 b}$ and $\mathrm{g}(b)=1+\frac{1}{\log_3 b}$.
If b satisfies the equation $|\mathrm{f}(b)-\mathrm{g}(b)|+\mathrm{f}(b)+\mathrm{g}(b)=3$, find the value of b.

$b =$

3. 已知實數 x_0 滿足方程 $x^2-5x+(b-8)=0$。若 $c=\frac{x_0^2}{x_0^4+x_0^2+1}$，求 c 的值。
Given that x_0 satisfies the equation $x^2-5x+(b-8)=0$.
If $c=\frac{x_0^2}{x_0^4+x_0^2+1}$, find the value of c.

$c =$

4. 若 -2 和 $216c$ 是方程 $px^2+dx=1$ 的根，求 d 的值。
If -2 and $216c$ are the roots of the equation $px^2+dx=1$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

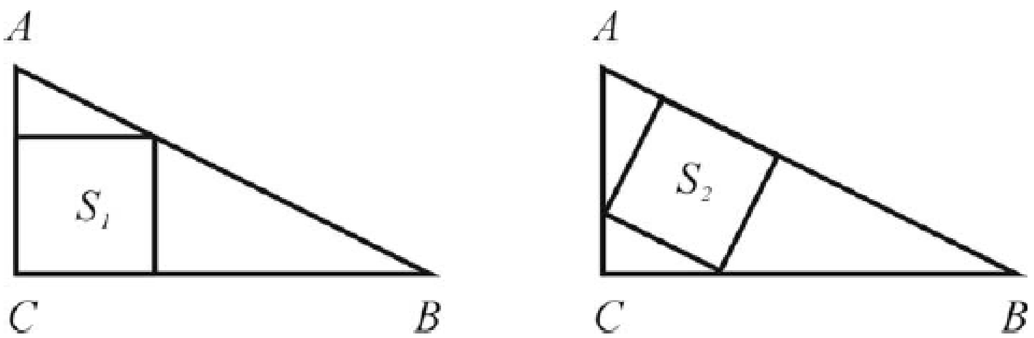
1. 設 a 為實數。若 a 滿足方程 $\log_2(4^x + 4) = x + \log_2(2^{x+1} - 3)$，求 a 的數值。
Let a be a real number.
If a satisfies the equation $\log_2(4^x + 4) = x + \log_2(2^{x+1} - 3)$, find the value of a .

$a =$

2. 已知 n 是自然數。若 $b = n^3 - 4an^2 - 12n + 144$ 是質數，求 b 的數值。
Given that n is a natural number. If $b = n^3 - 4an^2 - 12n + 144$ is a prime number, find the value of b .

$b =$

3.



圖一
Figure 1

如圖一，S_1 和 S_2 都是直角三角形 ABC 的兩個不同的正方形。
若 S_1 的面積是 $40b + 1$， S_2 的面積是 $40b$，及 $AC + CB = c$，求 c 的值。
In Figure 1, S_1 and S_2 are two different inscribed squares of the right-angled triangle ABC.
If the area of S_1 is $40b + 1$, the area of S_2 is $40b$ and $AC + CB = c$, find the value of c .

$c =$

4. 已知 $241c + 214 = d^2$，求 d 的正數值。
Given that $241c + 214 = d^2$, find the positive value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

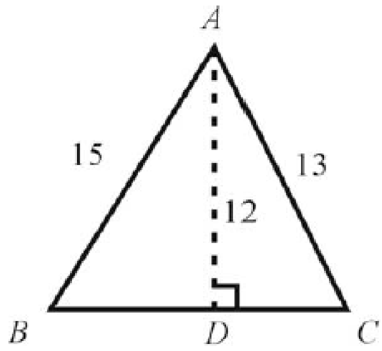
Time [ ] Min. [ ] Sec.

Total score [ ]

Hong Kong Mathematics Olympiad (2003-04)
Final Event Spare (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，ΔABC 為一銳角三角形，$AB = 15$，$AC = 13$，而高 $AD = 12$。若ΔABC 的面積為 P，求 P 的值。
In figure 1, ΔABC is an acute triangle, $AB = 15$, $AC = 13$, and its altitude $AD = 12$.
If the area of the ΔABC is P, find the value of P.



圖一
Figure 1

$P =$

2. 已知 x 和 y 是正整數。若 $x^4 + y^4$ 除以 $x + y$，所得的商是 $P + 13$，餘數是 Q，求 Q 的值。
Given that x and y are positive integers. If $x^4 + y^4$ is divided by $x + y$, the quotient is $P + 13$ and the remainder is Q, find the value of Q.

$Q =$

3. 已知一等邊三角形的周界與一個半徑是 $\frac{12}{Q}$cm 的圓的周界相等。若這三角形的面積是 $R\pi^2$ cm^2，求 R 的值。(答案以根式表示)。
Given that the perimeter of an equilateral triangle equals to that of a circle with radius $\frac{12}{Q}$cm. If the area of the triangle is $R\pi^2$ cm^2, find the value of R.

$R =$

4. 設 $W = \frac{\sqrt{3}}{2R}$，$S = W + \cfrac{1}{W + \cfrac{1}{W + \cfrac{1}{W + \cdots}}}$，求 S 的值。

Let $W = \frac{\sqrt{3}}{2R}$, $S = W + \cfrac{1}{W + \cfrac{1}{W + \cfrac{1}{W + \cdots}}}$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 a 為整數。若 50! 能被 2^a 整除，求 a 的最大可能的值。
Given that a is an integer.
If 50! is divisible by 2^a, find the largest possible value of a.

$a =$

2. 設 $[x]$ 表示不大於 x 的最大整數，例如 $[2.5] = 2$。
若 $b = \left[100 \times \frac{11\times 77 + 12\times 78 + 13\times 79 + 14\times 80}{11\times 76 + 12\times 77 + 13\times 78 + 14\times 79}\right]$，求 b 的值。
Let $[x]$ be the largest integer not greater than x. For example, $[2.5] = 2$.
If $b = \left[100 \times \frac{11\times 77 + 12\times 78 + 13\times 79 + 14\times 80}{11\times 76 + 12\times 77 + 13\times 78 + 14\times 79}\right]$, find the value of b.

$b =$

3. 若在 200 至 500 之間有 c 個數是 7 的倍數，求 c 的值。
If there are c multiples of 7 between 200 and 500, find the value of c.

$c =$

4. 已知 $0 \le x_0 \le \frac{\pi}{2}$ 且 x_0 滿足方程 $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin\frac{x}{2}$。
若 $d = \tan x_0$，求 d 的值。
Given that $0 \le x_0 \le \frac{\pi}{2}$ and x_0 satisfies the equation $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin\frac{x}{2}$.
If $d = \tan x_0$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

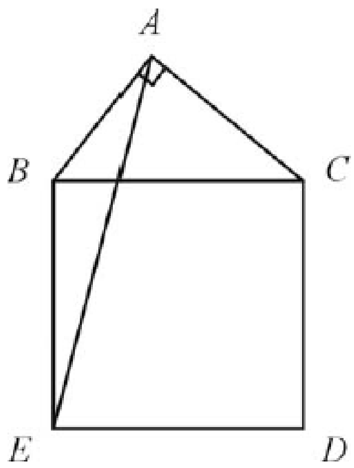
Hong Kong Mathematics Olympiad (2003-04)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 5^{5^5} 的十位數是 a，求 a 的值。
If the tens digit of 5^{5^5} is a, find the value of a.

$a =$

2. 如圖一，ΔABC 是一直角三角形，$AB = 3$ cm，$AC = 4$ cm 及 $BC = 5$ cm。若 $BCDE$ 是一正方形且 ΔABE 的面積是 b cm^2，求 b 的值。
In Figure 1, ΔABC is a right-angled triangle, $AB = 3$ cm, $AC = 4$ cm and $BC = 5$ cm. If $BCDE$ is a square and the area of ΔABE is b cm^2, find the value of b.



圖一 Figure 1

$b =$

3. 已知在 100 以內的質數中，其個位並非平方數的數目有 c 個，求 c 的值。
Given that there are c prime numbers less than 100 such that their unit digits are not square numbers, find the values of c.

$c =$

4. 若直綫 $y = x + d$ 與 $x = -y + d$ 相交於點 $(d - 1, d)$，求 d 的值。
If the lines $y = x + d$ and $x = -y + d$ intersect at the point $(d - 1, d)$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a 是方程 $\sqrt{x(x+1)(x+2)(x+3)+1}=71$ 的最小實數解，求 a 的值。
If a is the smallest real root of the equation $\sqrt{x(x+1)(x+2)(x+3)+1}=71$, find the value of a.

$a =$

2. 已知質數 p 和 q 滿足方程 $18p+30q=186$。若 $\log_8 \frac{p}{3q+1}=b\geq 0$，求 b 的值。
Given that p and q are prime numbers satisfying the equation $18p + 30q = 186$.
If $\log_8 \frac{p}{3q+1}=b\geq 0$, find the value of b.

$b =$

3. 已知對任意實數 x、y 及 z，運算 $\oplus$ 滿足
(i) $x\oplus 0=1$；及
(ii) $(x\oplus y)\oplus z=(z\oplus xy)+z$。
若 $1\oplus 2004=c$，求 c 的值。
Given that for any real numbers x, y and z, $\oplus$ is an operation satisfying
(i) $x\oplus 0=1$, and
(ii) $(x\oplus y)\oplus z=(z\oplus xy)+z$.
If $1\oplus 2004=c$, find the value of c.

$c =$

4. 已知 $f(x)=(x^4+2x^3+4x-5)^{2004}+2004$，若 $f\left(\sqrt{3}-1\right)=d$，求 d 的值。
Given that $f(x)=(x^4+2x^3+4x-5)^{2004}+2004$. If $f\left(\sqrt{3}-1\right)=d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $f(x)=\dfrac{4^x}{4^x+2}$ 及 $P=f\left(\dfrac{1}{1001}\right)+f\left(\dfrac{2}{1001}\right)+\cdots+f\left(\dfrac{1000}{1001}\right)$，求 P 的數值。

If $f(x)=\dfrac{4^x}{4^x+2}$ and $P=f\left(\dfrac{1}{1001}\right)+f\left(\dfrac{2}{1001}\right)+\cdots+f\left(\dfrac{1000}{1001}\right)$, find the value of P.

$P =$

2. 設 $f(x) = |x-a| + |x-15| + |x-a-15|$，其中 $a \le x \le 15$ 及 $0 < a < 15$。
若 Q 是 $f(x)$ 的最小值，求 Q 的值。
Let $f(x) = |x-a| + |x-15| + |x-a-15|$, where $a \le x \le 15$ and $0 < a < 15$.
If Q is the smallest value of $f(x)$, find the value of Q.

$Q =$

3. 若 $2^m = 3^n = 36$ 及 $R=\dfrac{1}{m}+\dfrac{1}{n}$，求 R 的值。

If $2^m = 3^n = 36$ and $R=\dfrac{1}{m}+\dfrac{1}{n}$, find the value of R.

$R =$

4. 設 $[x]$ 表示不大於 x 的最大整數，例如 $[2.5]=2$。
若 $a=1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\cdots+\dfrac{1}{2004^2}$ 及 $S=[a]$，求 S 的值。
Let $[x]$ be the largest integer not greater than x, for example, $[2.5] = 2$.
If $a=1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\cdots+\dfrac{1}{2004^2}$ and $S = [a]$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=		Team No.	
		+ Bonus score				Time	
		Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2003-04)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

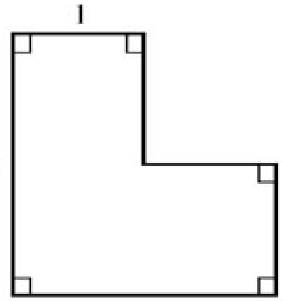
1. 對任意整數 n，F_n 的定義如下：$F_n = F_{n-1} + F_{n-2}$，$F_0 = 0$ 及 $F_1 = 1$。
若 $a = F_{-5} + F_{-4} + \ldots + F_4 + F_5$，求 a 的值。
For all integers n, F_n is defined by $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$ and $F_1 = 1$.
If $a = F_{-5} + F_{-4} + \ldots + F_4 + F_5$, find the value of a.

$a =$

2. 已知 x_0 滿足方程 $x^2 + x + 2 = 0$。若 $b = x_0^4 + 2x_0^3 + 3x_0^2 + 2x_0 + 1$，求 b 的值。
Given that x_0 satisfies the equation $x^2 + x + 2 = 0$.
If $b = x_0^4 + 2x_0^3 + 3x_0^2 + 2x_0 + 1$, find the value of b.

$b =$

3. 圖一所示為一瓷磚圖形。若最少可用 C 塊該類瓷磚便能鋪滿一正方形，求 C 的值。
Figure 1 shows a tile. If C is the minimum number of tiles required to tile a square, find the value of C.



圖一 Figure 1

$C =$

4. 若直綫 $5x + 2y - 100 = 0$ 上有 d 個點，其 x 及 y 坐標的值都是正整數，求 d 的值。
If the line $5x + 2y - 100 = 0$ has d points whose x and y coordinates are both positive integers, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.