#### 1987 FI5.3

若 
$$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}=\frac{2\sqrt{21}+3}{c}$$
 , 求  $c$  的值。

If 
$$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+3}{c}$$
, find the value of  $c$ .

#### 1988 FI3.1

若 
$$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+h}{25}$$
 , 求 h 的值。

If 
$$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+h}{25}$$
, find the value of  $h$ .

#### 1989 FG10.1

已知
$$\frac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=3\sqrt{a}+6$$
,求 $a$ 的值。

If 
$$\frac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{a} + 6$$
, find the value of  $a$ .

## 1990 HI1

求下式的值: 
$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$
。

Find the value of  $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ .

## 2000 HG9

設 
$$x = \sqrt{3 + \sqrt{3}}$$
 及  $y = \sqrt{3 - \sqrt{3}}$  , 求  $x^2(1 + y^2) + y^2$  的值。

Let  $x = \sqrt{3+\sqrt{3}}$  and  $y = \sqrt{3-\sqrt{3}}$ , find the value of  $x^2(1+y^2)+y^2$ .

# 2000 FI3.3

已知 
$$\frac{2}{\sqrt{2}+\sqrt{4}} + \frac{2}{\sqrt{4}+\sqrt{6}} + \dots + \frac{2}{\sqrt{1998\times2}+\sqrt{1999\times2}} = \frac{R}{\sqrt{2}+\sqrt{1999\times2}}$$
 ,

Given that 
$$\frac{2}{\sqrt{2}+\sqrt{4}} + \frac{2}{\sqrt{4}+\sqrt{6}} + \dots + \frac{2}{\sqrt{1998\times2}+\sqrt{1999\times2}} = \frac{R}{\sqrt{2}+\sqrt{1999\times2}}, \quad \text{if } c = 2\sqrt{3}\times\sqrt[3]{1.5}\times\sqrt[6]{12}, \text{ find the value of } c.$$

find the value of R.

#### 2000 FG1.2

設 
$$x = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$
 及  $y = \frac{1+\sqrt{3}}{1-\sqrt{3}}$  。如果  $b = 2x^2 - 3xy + 2y^2$ ,求  $b$  的值。

Let  $x = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$  and  $y = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ . If  $b = 2x^2 - 3xy + 2y^2$ , find the value of b.

#### 2001 HG3

設 
$$x = \sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}$$
 及  $y = x^2$ ,求 y 的值。

Let  $x = \sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}$  and  $y = x^2$ , find the value of y.

## 2002 FG3.4

已知 
$$x-y=1+\sqrt{5}$$
 , $y-z=1-\sqrt{5}$  。 若  $x^2+y^2+z^2-xy-yz-zx=d$  , 求  $d$  的 值 。

Given that  $x - y = 1 + \sqrt{5}$ ,  $y - z = 1 - \sqrt{5}$ .

If  $x^2 + v^2 + z^2 - xv - vz - zx = d$ , find the value of d.

#### 2002 FG4.2

設 
$$x > 0$$
,  $y > 0$  且  $\sqrt{x}(\sqrt{x} + \sqrt{y}) = 3\sqrt{y}(\sqrt{x} + 5\sqrt{y})$ 。

若 
$$b = \frac{2x + \sqrt{xy} + 3y}{x + \sqrt{xy} - y}$$
 , 求  $b$  的值。

It is given that x > 0, y > 0 and  $\sqrt{x}(\sqrt{x} + \sqrt{y}) = 3\sqrt{y}(\sqrt{x} + 5\sqrt{y})$ 

If  $b = \frac{2x + \sqrt{xy} + 3y}{x + \sqrt{xy} - y}$ , find the value of b.

## 2005 FI4.3

若 
$$\frac{\sqrt{3}-\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+b}{59}$$
 , 求  $c$  的值。

If 
$$\frac{\sqrt{3} - \sqrt{5} + \sqrt{7}}{\sqrt{3} + \sqrt{5} + \sqrt{7}} = \frac{c\sqrt{21} - 18\sqrt{15} - 2\sqrt{35} + 9}{59}$$
, find the value of  $c$ .

## 2005 FG4.3

## 2006 FG3.1

已知 
$$r = 2006 \times \frac{\sqrt{8} - \sqrt{2}}{\sqrt{2}}$$
 ,求  $r$  的值。Given that  $r = 2006 \times \frac{\sqrt{8} - \sqrt{2}}{\sqrt{2}}$  , find the

value of r.

#### 2007 FI1.1

設 
$$a$$
 為實數,且  $\sqrt{a} = \sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}}$ ,求  $a$  的值。

Let a be a real number and  $\sqrt{a} = \sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}}$ . Find the value of a. **2008 FI2.1** 

# 已知 $P = \left[\sqrt[3]{6} \times \left(\sqrt[3]{\frac{1}{162}}\right)\right]^{-1}$ ,求 P 的值。

Given that 
$$P = \left[ \sqrt[3]{6} \times \left( \sqrt[3]{\frac{1}{162}} \right) \right]^{-1}$$
, find the value of  $P$ .

#### 2008 FI2.3

設 
$$R = \left(\sqrt{\sqrt{3} + \sqrt{2}}\right)^4 + \left(\sqrt{\sqrt{3} - \sqrt{2}}\right)^4$$
。求  $R$  的值。

Let 
$$R = \left(\sqrt{\sqrt{3} + \sqrt{2}}\right)^4 + \left(\sqrt{\sqrt{3} - \sqrt{2}}\right)^4$$
. Find the value of  $R$ .

## 2009 FI3.1

已知 
$$\frac{4}{\sqrt{6}+\sqrt{2}} - \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{a} - \sqrt{b}$$
 。若  $m = a - b$  ,求  $m$  的值。

Given that  $\frac{4}{\sqrt{6}+\sqrt{2}} - \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{a} - \sqrt{b}$ . If m = a - b, find the value of m.

## 2011 HI3

已知 
$$a+b=\sqrt{2011}+\sqrt{2010}$$
 及  $a-b=\sqrt{2011}-\sqrt{2010}$  ,求  $ab$  的值。  
(答案以根式表示)

Given that  $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$  and  $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$ ,

find the value of ab. (Give your answer in surd form)

# 2012 HG4

求 
$$\frac{1}{\sqrt{2012} + \sqrt{2011}} + \frac{1}{\sqrt{2011} + \sqrt{2010}} + \dots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{1}}$$
 的值。  
(答案可以根式表示。)

Evaluate 
$$\frac{1}{\sqrt{2012} + \sqrt{2011}} + \frac{1}{\sqrt{2011} + \sqrt{2010}} + \dots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{1}}$$
.

(Answer can be expressed in surd form.)

#### 2012 FG3.1

設 
$$x = \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$
 ,  $y = \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$  及  $192z = x^4 + y^4 + (x + y)^4$  , 求  $z$  的 值 。

Let 
$$x = \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$
,  $y = \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$  and  $192z = x^4 + y^4 + (x + y)^4$ , find the value of z.

#### 2014 FI2.3

求正整數 γ 的最小值,以使得方程  $\sqrt{x} - \sqrt{24\gamma} = 4\sqrt{2}$  對 x 有正整數解。

Determine the smallest positive integer  $\gamma$  such that the equation

$$\sqrt{x} - \sqrt{24\gamma} = 4\sqrt{2}$$
 has an integer solution in x.

#### 2014 FI4.1

若 
$$\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=3\sqrt{\alpha}-6$$
,求  $\alpha$  的值。

If 
$$\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 3\sqrt{\alpha} - 6$$
, determine the value of  $\alpha$ .

## 2014 FG4.2

若 
$$x = \frac{\sqrt{5}+1}{2}$$
 及  $y = \frac{\sqrt{5}-1}{2}$  , 求  $x^3y + 2x^2y^2 + xy^3$  的值。

If 
$$x = \frac{\sqrt{5} + 1}{2}$$
 and  $y = \frac{\sqrt{5} - 1}{2}$ , determine the value of  $x^3y + 2x^2y^2 + xy^3$ .

## 2016 FI1.2

若 
$$\sqrt{b} = \sqrt{8 + \sqrt{15}} + \sqrt{8 - \sqrt{15}}$$
 , 求  $b$  的實數值。

If  $\sqrt{b} = \sqrt{8 + \sqrt{15}} + \sqrt{8 - \sqrt{15}}$ , determine the real value of b.

# 2016 FI3.3, 2017 FG3.2

若 
$$0 < x < 1$$
,求  $c = \left(\frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1}\right) \cdot \left(\sqrt{\frac{1}{x^2} - 1} - \frac{1}{x}\right)$ 的值。

If 0 < x < 1, determine the value of

$$c = \left(\frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1}\right) \times \left(\sqrt{\frac{1}{x^2} - 1} - \frac{1}{x}\right).$$

#### 2019 FI2.1

若 
$$\sqrt{A} = \sqrt{11+\sqrt{21}} - \sqrt{11-\sqrt{21}}$$
 ,求  $A$  的值。  
If  $\sqrt{A} = \sqrt{11+\sqrt{21}} - \sqrt{11-\sqrt{21}}$  , determine the value of  $A$  .

#### 2019 FG1.4

設 
$$x = \frac{\sqrt{5} + \sqrt{7}}{\sqrt{5} - \sqrt{7}}$$
 和  $y = \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} + \sqrt{7}}$  。若  $d = 3x^2 - 7xy + 3y^2$ ,求  $d$  的值。

Let  $x = \frac{\sqrt{5} + \sqrt{7}}{\sqrt{5} - \sqrt{7}}$  and  $y = \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} + \sqrt{7}}$ . If  $d = 3x^2 - 7xy + 3y^2$ , determine the value of  $d$ .

## 2019 FG3.1

若 
$$\sqrt{32\times81\times343} = b\sqrt{a}$$
 , 其中  $a$  和  $b$  是正整數 , 求  $a$  的最小值。

If  $\sqrt{32 \times 81 \times 343} = b\sqrt{a}$ , where a and b are positive integers, determine the least value of a.

#### 2023 HG2

對於 
$$0 < x < 2$$
 ,求  $\left(\frac{\sqrt{2+x}}{\sqrt{2+x}-\sqrt{2-x}} + \frac{2-x}{\sqrt{4-x^2}+x-2}\right) \left(\sqrt{\frac{4}{x^2}-1} - \frac{2}{x}\right)$  的值。

For 
$$0 < x < 2$$
, find the value of  $\left(\frac{\sqrt{2+x}}{\sqrt{2+x} - \sqrt{2-x}} + \frac{2-x}{\sqrt{4-x^2} + x - 2}\right) \left(\sqrt{\frac{4}{x^2} - 1} - \frac{2}{x}\right)$ .

## 2023 HG8

已知 
$$x$$
 及  $y$  為正實數且滿足  $x^2-y^2=4$  及  $xy=2$ 。若  $x+y$  可寫成  $a\sqrt{b+\sqrt{c}}$ ,其中  $a$ 、 $b$  及  $c$  均為正整數,求  $100a+10b+c$  的最小值。

Given that x and y are positive real numbers satisfying  $x^2 - y^2 = 4$  and xy = 2.

If the value of x + y can be expressed in the form of  $a\sqrt{b + \sqrt{c}}$ ,

where a, b and c are positive integers, find the least value of 100a + 10b + c.

# **Answers**

1987 FI5.3	1988 FI3.1	1989 FG10.1	1990 HI1	2000 HG9
25	3	6	5	12
2000 FI3.3	2000 FG1.2	2001 HG3	2002 FG3.4	2002 FG4.2
3996	25	10	8	2
2005 FI4.3	2005 FG4.3	2006 FG3.1	2007 FI1.1	2008 FI2.1
20	6	2006	2	3
2008 FI2.3 10	2009 FI3.1 3	$\frac{2011 \text{ HI3}}{\frac{1}{2}\sqrt{2010}}$	$2012 \text{ HG4}$ $2\sqrt{503} - 1$	2012 FG3.1 6
2014 FI2.3	2014 FI4.1	2014 FG4.2	2016 FI1.2	2016 FI3.3, 2017 FG3.2
3	6	5	30	-1
2019 FI2.1	2019 FG1.4	2019 FG3.1	2023 HG2	2023 HG8
2	419	14	-1	172