# Examples on Mathematical Induction: divisibility 24

Created by Mr. Francis Hung Last updated: September 1, 2021

1. Prove that $n(n + 1)(n + 2)(n + 3)$ is divisible by 24 for all positive integers $n$ .
2. Prove by mathematical induction that $n(n + 1)(n + 2)(3n + 5)$ is divisible by 24 for all positive integers $n$.

   $n = 1$, $1(2)(3)(8) = 48$ which is divisible by 24.

   Suppose $k(k + 1)(k + 2)(3k + 5) = 24N$, where $N$ is a positive integer, for some $k \geq 1$.

   $(k + 1)(k + 2)\underline{(k + 3)}(3k + 8)$

   $= k(k + 1)(k + 2)(3k + 8) + 3(k + 1)(k + 2)(3k + 8)$ (expand throughout the underlying term)

   $= k(k + 1)(k + 2)(\underline{3k + 5 + 3}) + 3(k + 1)(k + 2)(3k + 8)$

   $= k(k + 1)(k + 2)(3k + 5) + \underline{3k(k + 1)(k + 2) + 3(k + 1)(k + 2})(3k + 8)$

   $= 24N + 3(k + 1)(k + 2)(k + 3k + 8)$ (factorise over the underlying terms)

   $= 24N + 3(k + 1)(k + 2)(4k + 8)$

   $= 24N + 12(k + 1)(k + 2)(k + 2)$

   $(k + 1)(k + 2)$ is a product of two consecutive integers, one of which must be even.

   $\therefore$ $12(k + 1)(k + 2)(k + 2)$ must be divisible by 24

   $24N + 12(k + 1)(k + 2)(k + 2)$ is divisible by 24

   If it is true for $n = k$, then it is also true for $n = k + 1$

   By the principle of mathematical induction, $n(n + 1)(n + 2)(3n + 5)$ is divisible by 24 for all positive integers $n$ .