Hong Kong Mathematics Olympiad (1988 – 89)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 已知 $x+\frac{1}{x}=3$，求 $x^2+\frac{1}{x^2}$ 的值。
   Given that $x+\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$.

2. 設 $x\#y=xy-2x$，求 $2\#3$ 的值。
   If $x\#y=xy-2x$, find the value of $2\#3$.

3. 若一正多邊形的某內角較其外角大的 150°，求該正多邊形邊的數目。
   Find the number of sides of a regular polygon if an interior angle exceeds an exterior angle by 150°.

4. 已知 $10^{\log_{10}9}=8b+5$，求 b 的值。
   Find the value of b such that $10^{\log_{10}9}=8b+5$.

5. 某人以 15 km/h 速率乘單車由 P 至 Q，然後以 10 km/h 速率由 Q 返回 P。
   求該人來回全程的平均速率。
   A man cycles from P to Q with a uniform speed of 15 km/h and then back from Q to P with a uniform speed of 10 km/h. Find the average speed for the whole journey.

6. $[x]$ 是小於或等於 x 的最大整數。例如，$[3]=3$，$[5.7]=5$。
   若 $\left[\sqrt[5]{1}\right]+\left[\sqrt[5]{2}\right]+\cdots+\left[\sqrt[5]{n}\right]=n+14$，求 n 的值。
   $[x]$ denotes the greatest integer less than or equal to x. For example, $[3]=3$, $[5.7]=5$.
   If $\left[\sqrt[5]{1}\right]+\left[\sqrt[5]{2}\right]+\cdots+\left[\sqrt[5]{n}\right]=n+14$, find the value of n.

7. 某小孩以平行四邊形的兩條相鄰邊長的乘積當作該圖形的面積，他計算的答案是正確面積的 $\sqrt{2}$ 倍。若該平行四邊形的銳角是 $x°$，求 x 的值。
   A boy tries to find the area of a parallelogram by multiplying together the lengths of two adjacent sides. His answer is $\sqrt{2}$ times the correct area. If the acute angle of the parallelogram is $x°$, find the value of x.

8. 已知三點 $A(-8, 6)$、$B(-2, 1)$ 及 $C(4, c)$ 共綫，求 c 的值。
   If the points $A(-8, 6)$, $B(-2, 1)$ and $C(4, c)$ are collinear, find the value of c.

9. 曲綫 $x^2+y=8$ 與直綫 $x+y=8$ 相交於兩點。若該兩點的距離是 $\sqrt{d}$，求 d 的值。
   The graphs of $x^2+y=8$ and $x+y=8$ meet at two points. If the distance between these two points is $\sqrt{d}$, find the value of d.

10. 在某三角形中，各內角正弦的比是 3 : 4 : 5。若 A 是這個三角形的最小內角，且 $\tan A=\frac{x}{16}$，求 x 的值。
    The sines of the three angles of a triangle are in the ratio 3 : 4 : 5.
    If A is the smallest interior angle of the triangle and $\tan A=\frac{x}{16}$, find the value of x.

11. 兩骰同擲，求兩數的和大於 7 的概率。
    Two dice are thrown. Find the probability that the sum of the two numbers shown is greater than 7.

12. 函數 F 定義為 $F(x)=\begin{cases}2x+1, & \text{if } x\le 3\\ 3x^2, & \text{if } x>3\end{cases}$。求 $F\big(F(3)\big)$ 的值。
    F is a function defined by $F(x)=\begin{cases}2x+1, & \text{if } x\le 3\\ 3x^2, & \text{if } x>3\end{cases}$. Find the value of $F\big(F(3)\big)$.

13. 設 $\begin{pmatrix}a & b & c\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=ax+by+cz$，且 $\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}14\\y\\2\end{pmatrix}=26$，求 y 的值。

If $\begin{pmatrix}a & b & c\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=ax+by+cz$ and $\begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}14\\y\\2\end{pmatrix}=26$, find the value of y.

14. 設 $\frac{1}{B}=\frac{\sin 37°\sin 45°\cos 60°\sin 60°}{\cos 30°\cos 45°\cos 53°}$，求 B 的值。

If $\frac{1}{B}=\frac{\sin 37°\sin 45°\cos 60°\sin 60°}{\cos 30°\cos 45°\cos 53°}$, find the value of B.

15. 已知 $x+y=-4$、$y+z=5$ 及 $z+x=7$，求 xyz 的值。

If $x+y=-4$, $y+z=5$ and $z+x=7$, find the value of xyz.

16. 已知 α、β 為 $x^2-10x+c=0$ 的兩根，且 $\alpha\beta=-11$ 及 $\alpha>\beta$，求 $\alpha-\beta$ 的值。

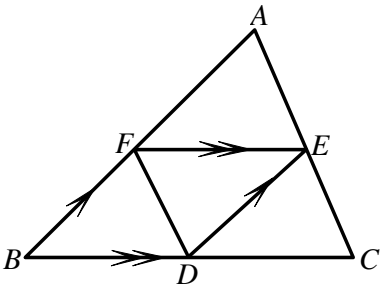
α, β are the roots of the equation $x^2-10x+c=0$.

If $\alpha\beta=-11$ and $\alpha>\beta$, find the value of $\alpha-\beta$.

17. 如圖一，FE // BC 及 ED // AB。

若 $AF:FB=3:2$，求 ΔDEF 的面積 : ΔABC 的面積。

In figure 1, FE // BC and ED // AB. If $AF:FB=3:2$, find the ratio area of ΔDEF : area of ΔABC.

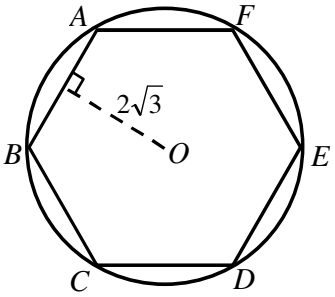


(Figure 1) (圖一)

18. 如圖二，$ABCDEF$ 為一正六邊形內接於圓形上，O 為圓心。

若 O 至 AB 的距離為 $2\sqrt{3}$，且 p 為該正六邊形的周界，求 p 的值。

In figure 2, a regular hexagon $ABCDEF$ is inscribed in a circle centred at O. If the distance of O from AB is $2\sqrt{3}$ and p is the perimeter of the hexagon, find the value of p.

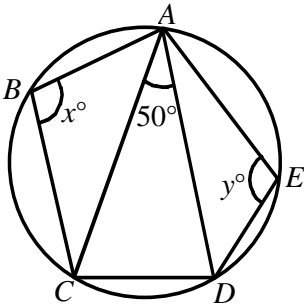


(Figure 2) (圖二)

19. 在圖三，$ABCD$ 及 $ACDE$ 是圓內接四邊形，求 $x+y$ 的值。

In figure 3, $ABCD$ and $ACDE$ are cyclic quadrilaterals.

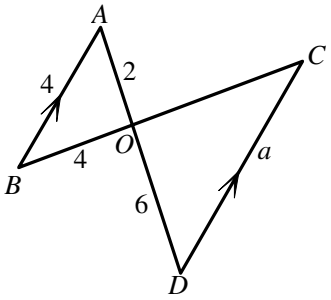
Find the value of $x+y$.



(Figure 3) (圖三)

20. 如圖四，求 a 的值。

Find the value of a in figure 4.



(Figure 4) (圖四)

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Heat Event (Group)

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1. a、b 為兩相異實數，且 $a^2 = 5a + 10$ 及 $b^2 = 5b + 10$，求 $\frac{1}{a^2}+\frac{1}{b^2}$ 的值。
Given a and b are distinct real numbers satisfying $a^2 = 5a + 10$ and $b^2 = 5b + 10$.
Find the value of $\frac{1}{a^2}+\frac{1}{b^2}$.

2. 一凸 n 邊形的一個內角是 $x°$，其他內角的和是 800°，求 n 的值。
An interior angle of an n-sided convex polygon is $x°$ while the sum of other interior angles is 800°. Find the value of n.

3. 已知對所有正整數 n，$1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$，求 $21^2 + 22^2 + \cdots + 30^2$ 的值。
It is known that $1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$ for all positive integers n.
Find the value of $21^2 + 22^2 + \cdots + 30^2$.

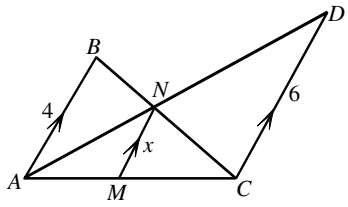
4. 方程 $19x + 88y = 1988$ 的其中一組正整數解是 (100, 1)，求另一組正整數解。
One of the positive integral solutions of the equation $19x + 88y = 1988$ is given by (100, 1) .
Find another positive integral solution.

5. A(2, 3) 與 B(17, 23) 的連綫交 $2x – y = 7$ 於 P，求 $\frac{AP}{PB}$ 的值。
The line joining $A(2, 3)$ and $B(17, 23)$ meets the line $2x – y = 7$ at P. Find the value of $\frac{AP}{PB}$.

6. 求 7^{2047} 被 100 除所得的餘數。
Find the remainder when 7^{2047} is divided by 100 .

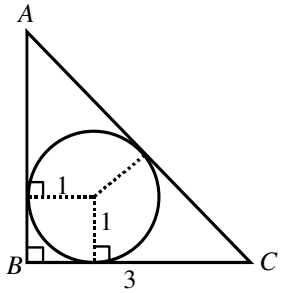
7. 若 $\log_2[\log_3(\log_7 x)]=\log_3[\log_7(\log_2 y)]=\log_7[\log_2(\log_3 z)]=0$，求 $x + y + z$ 的值。
If $\log_2[\log_3(\log_7 x)]=\log_3[\log_7(\log_2 y)]=\log_7[\log_2(\log_3 z)]=0$, find the value of $x + y + z$.

8. 在圖一中，AB // MN // CD。若 $AB = 4$、$CD = 6$ 及 $MN = x$，求 x 的值。
In figure 1, AB // MN // CD. If $AB = 4$, $CD = 6$ and $MN = x$, find the value of x .



(Figure 1)(圖一)

9. 在圖二中，$\angle B = 90°$、$BC = 3$，且 ΔABC 的內切圓半徑長 1 單位，求 AC 的長度。
In figure 2, $\angle B = 90°$, $BC = 3$ and the radius of the inscribed circle of ΔABC is 1. Find the length of AC.



(Figure 2)(圖二)

10. 在所附除法算式中(見圖三)，(a)列的被除數可被(b) 列的除數整除。求(a)列的被除數。
(每一星號*為由 0 至 9 的整數。)
In the attached division (see figure 3), the dividend in (a) is divisible by the divisor in line (b). Find the dividend in line (a). (Each asterisk * is an integer from 0 to 9.)

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          * 8 *
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(b) ...* * * ) * * * * * * .. (a)
        * * * *
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          * * *
          * * *
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(Figure 3)(圖三)

***** 試卷完 End of Paper *****