

Individual Events

I1	P	1	I2	P	12	I3	P	4	I4	P	35
	Q	4		Q	14		Q	7		Q	10
	R	2		R	1		R	14		R	10
	S	32		S	2		S	34		S	222

Group Events

G1	a	4	G2	a	2	G3	a	3	G4	a	3840
	b	1001		b	3		b	20		b	1
	c	8		c	333333		c	14		c	3
	d	3		d	46		d	15		d	1853

Individual Event 1

I1.1 a , b and c are the lengths of the opposite sides $\angle A$, $\angle B$ and $\angle C$ of the $\triangle ABC$ respectively.

If $\angle C = 60^\circ$ and $\frac{a}{b+c} + \frac{b}{a+c} = P$, find the value of P .

$$c^2 = a^2 + b^2 - 2ab \cos 60^\circ = a^2 + b^2 - ab \Rightarrow a^2 + b^2 = c^2 + ab$$

$$P = \frac{a}{b+c} + \frac{b}{a+c} = \frac{a(a+c) + b(b+c)}{(b+c)(a+c)}$$

$$P = \frac{a^2 + ac + b^2 + bc}{ab + ac + bc + c^2} = \frac{ab + ac + bc + c^2}{ab + ac + bc + c^2} = 1$$

I1.2 Given that $f(x) = x^2 + ax + b$ is the common factor of $x^3 + 4x^2 + 5x + 6$ and $2x^3 + 7x^2 + 9x + 10$. If $f(P) = Q$, find the value of Q . **Reference 1992 HI5, 1993 FI5.2, 2011 FI3.2**

Let $g(x) = x^3 + 4x^2 + 5x + 6$; $h(x) = 2x^3 + 7x^2 + 9x + 10$

$g(-3) = -27 + 36 - 15 + 6 = 0$, $(x+3)$ is a factor of $g(x)$; by division, $g(x) = (x+3)(x^2 + x + 2)$

$h(-2.5) = -31.25 + 43.75 - 22.5 + 10 = 0$, $(2x+5)$ is a factor of $h(x)$; by division, $h(x) = (2x+5)(x^2 + x + 2)$

$f(x) = \text{common factor} = (x^2 + x + 2)$

$$Q = f(P) = f(1) = 1 + 1 + 2 = 4$$

I1.3 Given that $\frac{1}{a} + \frac{1}{b} = \frac{Q}{a+b}$ and $\frac{a}{b} + \frac{b}{a} = R$, find the value of R .

$$\frac{1}{a} + \frac{1}{b} = \frac{4}{a+b}$$

$$\Rightarrow (a+b)^2 = 4ab$$

$$\Rightarrow a^2 + 2ab + b^2 = 4ab$$

$$\Rightarrow a^2 - 2ab + b^2 = 0$$

$$\Rightarrow (a-b)^2 = 0$$

$$a = b$$

$$\Rightarrow R = \frac{a}{b} + \frac{b}{a} = 2$$

I1.4 Given that $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ and $a^3+b^3=S$, find the value of S .

$$\begin{cases} a+b=2 \dots\dots(1) \\ a^2+b^2=12 \dots\dots(2) \end{cases}$$

$$(1)^2 - (2): 2ab = -8$$

$$\Rightarrow \begin{cases} ab = -4 \\ a+b=2 \end{cases}$$

$$\Rightarrow \begin{cases} ab = -4 \\ a+b=2 \end{cases}$$

$$S = a^3 + b^3 = (a+b)(a^2 - ab + b^2) = 2(12 + 4) = 32$$

Individual Event 2

I2.1 Suppose P is an integer and $5 < P < 20$. If the roots of the equation

$$x^2 - 2(2P - 3)x + 4P^2 - 14P + 8 = 0 \text{ are integers, find the value of } P.$$

Reference: 2000 FI5.2, 2010 FI2.2, 2011 FI3.1, 2013 HG1

$$\Delta = 4(2P - 3)^2 - 4(4P^2 - 14P + 8) = m^2$$

$$\left(\frac{m}{2}\right)^2 = 4P^2 - 12P + 9 - 4P^2 + 14P - 8 = 2P + 1$$

$$\because 5 < P < 20 \therefore 11 < 2P + 1 < 41$$

The only odd square lying in this interval is 25

$$\Rightarrow 2P + 1 = 25 = 5^2$$

$$\therefore P = 12$$

I2.2 $ABCD$ is a rectangle. $AB = 3P + 4$, $AD = 2P + 6$.

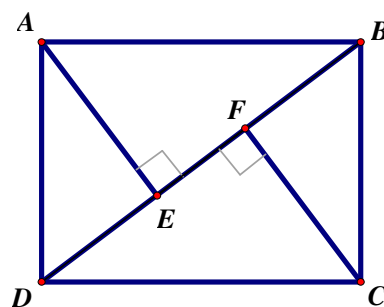
AE and CF are perpendiculars to the diagonal BD .

If $EF = Q$, find the value of Q .

$$AB = 40, AD = 30, BD = 50, \text{ let } \angle ADB = \theta, \cos \theta = \frac{3}{5}$$

$$DE = AD \cos \theta = 30 \times \frac{3}{5} = 18 = BF$$

$$EF = 50 - 18 - 18 = 14$$



I2.3 There are less than $4Q$ students in a class. In a mathematics test, $\frac{1}{3}$ of the students got grade

A, $\frac{1}{7}$ of the students got grade B, half of the students got grade C, and the rest failed. Given

that R students failed in the mathematics test, find the value of R .

$4Q = 56$, let the number of students be x , then x is divisible by 2, 3 and 7.

i.e. x is divisible by 42, as $x < 56$, so $x = 42$

$$R = \text{number of students failed in mathematics} = 42 \times \left(1 - \frac{1}{3} - \frac{1}{7} - \frac{1}{2}\right) = 1; R = 1$$

I2.4 $[a]$ represents the largest integer not greater than a . For example, $\left[2\frac{1}{3}\right] = 2$.

Given that the sum of the roots of the equation $[3x + R] = 2x + \frac{3}{2}$ is S , find the value of S .

Reference: 1994 HG9

$$[3x + 1] = 2x + \frac{3}{2} \Rightarrow 3x + 1 = 2x + \frac{3}{2} + a, \text{ where } 0 \leq a < 1$$

$$a = x - \frac{1}{2} \Rightarrow 0 \leq x - \frac{1}{2} < 1 \Rightarrow 2.5 \leq 2x + \frac{3}{2} < 4.5$$

$$\because 2x + \frac{3}{2} \text{ is an integer } \therefore 2x + \frac{3}{2} = 4 \text{ or } 3$$

$$x = 0.75 \text{ or } 1.25$$

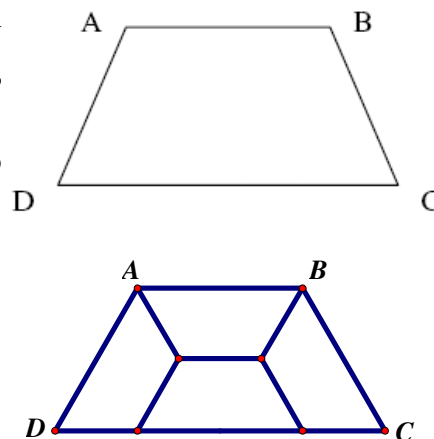
$$S = 0.75 + 1.25 = 2$$

Individual Event 3

I3.1 $ABCD$ is a trapezium such that $\angle ADC = \angle BCD = 60^\circ$ and $AB = BC = AD = \frac{1}{2}CD$. If this trapezium is divided into P

equal portions ($P > 1$) and each portion is similar to trapezium $ABCD$ itself, find the minimum value of P .

From the graph, $P = 4$



I3.2 The sum of tens and units digits of $(P + 1)^{2001}$ is Q . Find the value of Q .

$5^{2001} = 100a + 25$, where a is a positive integer.

$Q = 2 + 5 = 7$.

I3.3 If $\sin 30^\circ + \sin^2 30^\circ + \dots + \sin^Q 30^\circ = 1 - \cos^R 45^\circ$, find the value of R .

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^7} = 1 - \frac{1}{\sqrt{2}^R}$$

$$1 - \frac{1}{2^7} = 1 - \frac{1}{2^{\frac{R}{2}}}$$

$$R = 14$$

I3.4 Let α and β be the roots of the equation $x^2 - 8x + (R + 1) = 0$. If $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ are the roots of the equation $225x^2 - Sx + 1 = 0$, find the value of S .

Reference: 1996 FI2.2

$$x^2 - 8x + 15 = 0, \alpha = 3, \beta = 5$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{9} + \frac{1}{25} = \frac{34}{225} = \frac{S}{225}$$

$$S = 34$$

Individual Event 4

I4.1 Let $a^{\frac{2}{3}} + b^{\frac{2}{3}} = 17\frac{1}{2}$, $x = a + 3a^{\frac{1}{3}}b^{\frac{2}{3}}$ and $y = b + 3a^{\frac{2}{3}}b^{\frac{1}{3}}$. If $P = (x + y)^{\frac{2}{3}} + (x - y)^{\frac{2}{3}}$, find the value of P .

$$\begin{aligned} P &= \left(a + 3a^{\frac{2}{3}}b^{\frac{1}{3}} + 3a^{\frac{1}{3}}b^{\frac{2}{3}} + b\right)^{\frac{2}{3}} + \left(a - 3a^{\frac{2}{3}}b^{\frac{1}{3}} + 3a^{\frac{1}{3}}b^{\frac{2}{3}} - b\right)^{\frac{2}{3}} \\ &= \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^{3 \times \frac{2}{3}} + \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)^{3 \times \frac{2}{3}} \\ P &= \left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^2 + \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)^2 \\ &= a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + a^{\frac{2}{3}} + a^{\frac{2}{3}} - 2a^{\frac{1}{3}}b^{\frac{1}{3}} + a^{\frac{2}{3}} \\ &= 2\left(a^{\frac{2}{3}} + a^{\frac{2}{3}}\right) \\ &= 2 \times 17.5 = 35 \end{aligned}$$

I4.2 If a regular Q -sided polygon has P diagonals, find the value of Q .

Reference: 1984 FG10.3, 1985 FG8.3, 1988 FG6.2, 1989 FG6.1, 1991 FI2.3, 2005 FI1.4

The number of diagonals = $C_2^Q - Q = 35$

$$\frac{Q(Q-1)}{2} - Q = 35$$

$$Q^2 - 3Q - 70 = 0$$

$$Q = 10$$

I4.3 Let $x = \sqrt{\frac{Q}{2}} + \sqrt{\frac{Q}{2}}$ and $y = \sqrt{\frac{Q}{2}} - \sqrt{\frac{Q}{2}}$. If $R = \frac{x^6 + y^6}{40}$, find the value of R .

$$\begin{aligned} R &= \frac{(x^2 + y^2)(x^4 + y^4 - x^2y^2)}{40} \\ &= \frac{\left(\frac{Q}{2} + \sqrt{\frac{Q}{2}} + \frac{Q}{2} - \sqrt{\frac{Q}{2}}\right) \left[\left(\frac{Q}{2} + \sqrt{\frac{Q}{2}}\right)^2 + \left(\frac{Q}{2} - \sqrt{\frac{Q}{2}}\right)^2 - \left(\frac{Q}{2} + \sqrt{\frac{Q}{2}}\right)\left(\frac{Q}{2} - \sqrt{\frac{Q}{2}}\right)\right]}{40} \\ &= \frac{Q \left[2\left(\frac{Q}{2}\right)^2 + 2\left(\sqrt{\frac{Q}{2}}\right)^2 - \left(\frac{Q}{2}\right)^2 + \left(\sqrt{\frac{Q}{2}}\right)^2\right]}{40} \\ &= \frac{10(5^2 + 3 \times 5)}{40} = 10 \end{aligned}$$

I4.4 $[a]$ represents the largest integer not greater than a . For example, $[2.5] = 2$.

If $S = \left[\frac{2001}{R}\right] + \left[\frac{2001}{R^2}\right] + \left[\frac{2001}{R^3}\right] + \dots$, find the value of S .

$$\begin{aligned} S &= \left[\frac{2001}{10}\right] + \left[\frac{2001}{100}\right] + \left[\frac{2001}{1000}\right] + \dots \\ &= 200 + 20 + 2 + 0 + \dots = 222 \end{aligned}$$

Group Event 1**G1.1** Given that $(a + b + c)^2 = 3(a^2 + b^2 + c^2)$ and $a + b + c = 12$, find the value of a .Sub. (2) into (1), $12^2 = 3(a^2 + b^2 + c^2)$

$$\Rightarrow a^2 + b^2 + c^2 = 48 \dots\dots(3)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\Rightarrow 12^2 = 48 + 2(ab + bc + ca)$$

$$\Rightarrow ab + bc + ca = 48$$

$$2[a^2 + b^2 + c^2 - (ab + bc + ca)] = (a - b)^2 + (b - c)^2 + (c - a)^2$$

$$2[48 - 48] = 0 = (a - b)^2 + (b - c)^2 + (c - a)^2$$

$$\Rightarrow a = b = c$$

$$a + b + c = 3a = 12$$

$$\Rightarrow a = 4$$

G1.2 Given that $b \left[\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{1999 \times 2001} \right] = 2 \times \left[\frac{1^2}{1 \times 3} + \frac{2^2}{3 \times 5} + \dots + \frac{1000^2}{1999 \times 2001} \right]$, find the value of b .

Note that $\frac{1}{(2r-1) \times (2r+1)} = \frac{1}{2} \left(\frac{1}{2r-1} - \frac{1}{2r+1} \right)$ and $\frac{r^2}{(2r-1) \times (2r+1)} = \frac{1}{4} + \frac{1}{8} \left(\frac{1}{2r-1} - \frac{1}{2r+1} \right)$

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{1999 \times 2001} = \frac{1}{2} \left[1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{1999} - \frac{1}{2001} \right] = \frac{1}{2} \left(1 - \frac{1}{2001} \right) = \frac{1000}{2001}$$

$$\frac{1^2}{1 \times 3} + \frac{2^2}{3 \times 5} + \dots + \frac{1000^2}{1999 \times 2001} = \frac{1}{4} + \frac{1}{8} \left(1 - \frac{1}{3} \right) + \frac{1}{4} + \frac{1}{8} \left(\frac{1}{3} - \frac{1}{5} \right) + \dots + \frac{1}{4} + \frac{1}{8} \left(\frac{1}{1999} - \frac{1}{2001} \right) \quad (1000 \text{ terms})$$

$$= \frac{1000}{4} + \frac{1}{8} \left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{1999} - \frac{1}{2001} \right) = \frac{1000}{4} + \frac{1}{8} \left(1 - \frac{1}{2001} \right)$$

$$= \frac{1000}{4} + \frac{1}{8} \cdot \frac{2000}{2001} = 250 + \frac{250}{2001} = 250 \left(1 + \frac{1}{2001} \right) = \frac{250 \cdot 2002}{2001}$$

The given equation becomes: $b \cdot \frac{1000}{2001} = 2 \cdot \frac{250 \cdot 2002}{2001} \Rightarrow b = 1001$

G1.3 A six-digit number 1234xy is divisible by both 8 and 9. Given that $x + y = c$, find the value of c .**Reference: 2003 FI4.1, 2017 HI1**The number formed by last 3 digits must be divisible by 8 and the sum of digits must be divisible by 9. i.e. $400 + 10x + y$ is divisible by 8 and $1 + 2 + 3 + 4 + x + y = 9m$

$$10x + y = 8n \dots\dots(1); \quad x + y = 9m - 10 \dots\dots(2)$$

$$(1) - (2): 9x = 8n - 9m + 9 + 1$$

$$\Rightarrow n = 1 \text{ or } 10$$

When $n = 1$, (1) has no solution; when $n = 10$, $x = 8$, $y = 0$; $c = x + y = 8$ **G1.4** Suppose $\log_x t = 6$, $\log_y t = 10$ and $\log_z t = 15$. If $\log_{xyz} t = d$, find the value of d .**Reference: 1999 FG1.4, 2015 HI7**

$$\frac{\log t}{\log x} = 6, \quad \frac{\log t}{\log y} = 10, \quad \frac{\log t}{\log z} = 15$$

$$\Rightarrow \frac{\log x}{\log t} = \frac{1}{6}, \quad \frac{\log y}{\log t} = \frac{1}{10}, \quad \frac{\log z}{\log t} = \frac{1}{15}$$

$$\frac{\log x}{\log t} + \frac{\log y}{\log t} + \frac{\log z}{\log t} = \frac{1}{6} + \frac{1}{10} + \frac{1}{15} = \frac{10}{30} = \frac{1}{3}$$

$$\frac{\log x + \log y + \log z}{\log t} = \frac{1}{3}$$

$$\frac{\log xyz}{\log t} = \frac{1}{3}$$

$$d = \frac{\log t}{\log xyz} = 3$$

Group Event 2

G2.1 Given that $x = \sqrt{7 - 4\sqrt{3}}$ and $\frac{x^2 - 4x + 5}{x^2 - 4x + 3} = a$, find the value of a .

Reference: 1993 FI1.4, 1999 HG3, 2011 HI7, 2015 FI4.2, 2015 FG3.1

Reference: 1993 HI9, 2000HG1, 2007 HG3, 2009HG2

$$\begin{aligned} x &= \sqrt{7 - 4\sqrt{3}} = \sqrt{4 - 2\sqrt{12} + 3} \\ &= \sqrt{\sqrt{4}^2 - 2\sqrt{4}\sqrt{3} + \sqrt{3}^2} \\ &= \sqrt{(\sqrt{4} - \sqrt{3})^2} = \sqrt{4} - \sqrt{3} = 2 - \sqrt{3} \\ \sqrt{3} &= 2 - x \\ \Rightarrow 3 &= (2 - x)^2 \\ \Rightarrow x^2 - 4x + 1 &= 0 \\ a &= \frac{x^2 - 4x + 5}{x^2 - 4x + 3} = \frac{x^2 - 4x + 1 + 4}{x^2 - 4x + 1 + 2} = 2 \end{aligned}$$

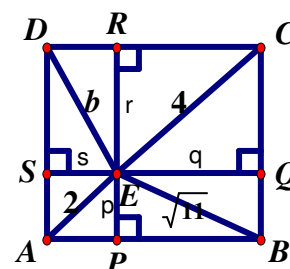
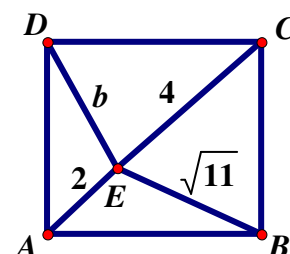
G2.2 E is an interior point of the rectangle $ABCD$. Given that the lengths of EA , EB , EC and ED are 2, $\sqrt{11}$, 4 and b respectively, find the value of b .

Reference: 1994 FG10.1-2, 2003 FI3.4, 2018 HI7

Let P , Q , R and S be the foot of perpendiculars drawn from E onto AB , BC , CD and DA respectively. $PE = p$, $QE = q$, $RE = r$, $SE = s$.

Using Pythagoras' Theorem, it can be proved that

$$\begin{aligned} p^2 + s^2 &= 4 \dots\dots\dots(1) \\ p^2 + q^2 &= 11 \dots\dots\dots(2) \\ q^2 + r^2 &= 16 \dots\dots\dots(3) \\ r^2 + s^2 &= b^2 \dots\dots\dots(4) \\ (1) + (3) - (2) - (4): 0 &= 4 + 16 - 11 - b^2 \\ b &= 3 \end{aligned}$$



G2.3 Given that $111111222222 = c \times (c + 1)$, find the value of c .

Reference 1996 FG7.2 $111aaa = b \times (b + 1) \dots\dots$

$$\begin{aligned} 111111222222 &= 111111000000 + 222222 \\ &= 111111 \times 1000000 + 2 \times 111111 \\ &= 111111 \times 1000002 \\ 111111222222 &= 111111 \times 3 \times 333334 = 333333 \times 333334 \\ c &= 333333 \end{aligned}$$

G2.4 Given that $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and $0 < d < 90$, find the value of d .

$$\begin{aligned} \sin d^\circ &= \cos 16^\circ - \sin 14^\circ \\ \sin d^\circ &= \sin 74^\circ - \sin 14^\circ \\ \sin d^\circ &= 2 \cos \frac{74^\circ + 14^\circ}{2} \sin \frac{74^\circ - 14^\circ}{2} \\ \sin d^\circ &= \cos 44^\circ = \sin 46^\circ \\ d &= 46 \end{aligned}$$

Group Event 3

G3.1 Given that the solution of the equation $\sqrt{3x+1} + \sqrt{3x+6} = \sqrt{4x-2} + \sqrt{4x+3}$ is a , find the value of a .

$$\sqrt{3x+6} - \sqrt{4x-2} = \sqrt{4x+3} - \sqrt{3x+1}$$

$$(\sqrt{3x+6} - \sqrt{4x-2})^2 = (\sqrt{4x+3} - \sqrt{3x+1})^2$$

$$3x+6+4x-2-2\sqrt{12x^2+18x-12} = 4x+3+3x+1-2\sqrt{12x^2+13x+3}$$

$$\sqrt{12x^2+18x-12} = \sqrt{12x^2+13x+3}$$

$$12x^2+18x-12 = 12x^2+13x+3$$

$$x = 3$$

G3.2 Suppose the equation $x^2y - x^2 - 3y - 14 = 0$ has only one positive integral solution (x_0, y_0) . If $x_0 + y_0 = b$, find the value of b .

$$(y-1)x^2 = 3y+14$$

$$x^2 = \frac{3y+14}{y-1} = \frac{3y-3+17}{y-1} = 3 + \frac{17}{y-1} = 3 + 1$$

$$y = 18, x = 2$$

$$b = 20$$

G3.3 $ABCD$ is a cyclic quadrilateral. AC and BD intersect at G . Suppose $AC = 16$ cm, $BC = CD = 8$ cm, $BG = x$ cm and $GD = y$ cm. If x and y are integers and $x + y = c$, find the value of c .

As shown in the figure, let $CG = t$, $AG = 16 - t$.

Let $\angle CBG = \theta$, $\angle ACB = \alpha$.

Then $\angle CAB = \theta$ (eq. chords eq. \angle s)

Then $\triangle BCG \sim \triangle ACB$ (equiangular)

$$t : 8 = 8 : 16 \text{ (ratio of sides, } \sim \Delta \text{s)}$$

$$t = 4$$

It is easy to see that $\triangle ADG \sim \triangle BCG$ (equiangular)

$$(16 - t) : y = x : t \text{ (ratio of sides, } \sim \Delta \text{s)}$$

$$(16 - 4) \times 4 = xy$$

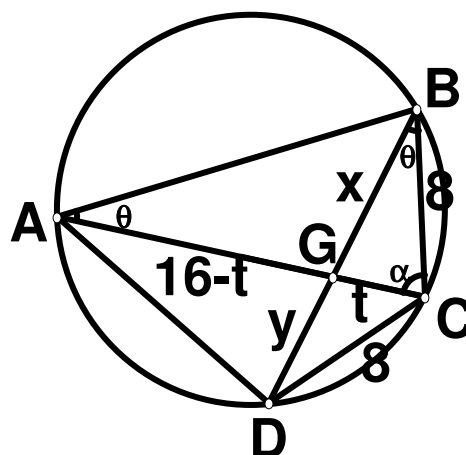
$$xy = 48$$

Assume that x and y are integers, then possible pairs of (x, y) are $(1, 48)$, $(2, 24)$, ..., $(6, 8)$, ..., $(48, 1)$.

Using triangle inequality $x + t > 8$ and $8 + t > x$ in $\triangle BCG$, the only possible combinations are:

$$(x, y) = (6, 8) \text{ or } (8, 6)$$

$$c = x + y = 14$$



G3.4 Given that $5^{\log 30} \times \left(\frac{1}{3}\right)^{\log 0.5} = d$, find the value of d .

$$\log 30 \log 5 + \log 0.5 \log \frac{1}{3} = \log d$$

$$\log (3 \times 10) \log \frac{10}{2} + (-\log 2)(-\log 3) = \log d$$

$$(\log 3 + 1)(1 - \log 2) + \log 2 \log 3 = \log d$$

$$\log 3 + 1 - \log 3 \log 2 - \log 2 + \log 2 \log 3 = \log d$$

$$\log d = \log 3 + 1 - \log 2 = \log \frac{3 \times 10}{2}$$

$$d = 15$$

Group Event 4

G4.1 $x_1 = 2001$. When $n > 1$, $x_n = \frac{n}{x_{n-1}}$. Given that $x_1 x_2 x_3 \dots x_{10} = a$, find the value of a .

$$x_2 = \frac{2}{x_1} \Rightarrow x_1 x_2 = 2$$

$$x_4 = \frac{4}{x_3} \Rightarrow x_3 x_4 = 4$$

$$x_6 = \frac{6}{x_5} \Rightarrow x_5 x_6 = 6$$

$$x_8 = \frac{8}{x_7} \Rightarrow x_7 x_8 = 8$$

$$x_{10} = \frac{10}{x_9} \Rightarrow x_9 x_{10} = 10$$

Multiply these equations gives $a = x_1 x_2 x_3 \dots x_{10} = 2 \times 4 \times 6 \times 8 \times 10 = 32 \times 120 = 3840$

G4.2 Given that the units digit of $1^3 + 2^3 + 3^3 + \dots + 2001^3$ is b , find the value of b .

Arrange the numbers in groups of 10 in ascending order, the units digit of sum each group is the same (except the last number, 2001^3).

$$1^3 + 2^3 + \dots + 10^3 \equiv 1 + 8 + 27 + 64 + 125 + 216 + 343 + 512 + 729 + 1000 \pmod{10}$$

$$\equiv 5 \pmod{10}$$

$$1^3 + 2^3 + \dots + 2000^3 + 2001^3 \equiv 200(5) + 1 \pmod{10}$$

So $b = 1$

G4.3 A and B ran around a circular path with constant speeds. They started from the same place and at the same time in opposite directions. After their first meeting, B took 1 minute to go back to the starting place. If A and B need 6 minutes and c minutes respectively to complete one round of the path, find the value of c .

In one minute, A and B ran $\frac{1}{6} + \frac{1}{c} = \frac{c+6}{6c}$ of the total distance.

They will meet at the first time after $\frac{6c}{c+6}$ minutes.

After 1 more minute, (i.e. total time elapsed $= \frac{6c}{c+6} + 1$ minutes), B returned to the starting

point. So $\left(\frac{6c}{c+6} + 1\right) \times \frac{1}{c} = 1$

$$6c + c + 6 = c^2 + 6c$$

$$c^2 - c - 6 = 0$$

$$(c-3)(c+2) = 0$$

$$c = 3$$

G4.4 The roots of the equation $x^2 - 45x + m = 0$ are prime numbers. Given that the sum of the squares of the roots is d , find the value of d .

Reference: 1996 HG8, 1996FG7.1, 2005 FG1.2, 2012 HI6

Let the roots be α, β . $\alpha + \beta = 45$, $\alpha\beta = m$

The sum of two prime numbers $\alpha + \beta = 45$

$\alpha = 2$, $\beta = 43$ (2 is the only even prime number)

$$d = \alpha^2 + \beta^2 = 4 + 43^2 = 1853$$