Examples on Mathematical Induction: divisibility 7

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1. Prove by mathematical induction $9^n - 2^n$ is divisible by 7 for all non-negative integer n .
   Let $P(n) \equiv$ "$9^n - 2^n$ is divisible by 7 for all non-negative integer n."
   $n = 0$, $9^0 - 2^0 = 0\times 7$, which is divisible by 7
   $P(1)$ is true
   Suppose $P(k)$ is true some some non-negative integer k
   i.e. $9^k - 2^k = 7m$, where m is an integer $\cdots\cdots$ (*)
   When $n = k + 1$,
   $9^{k+1} - 2^{k+1} = 9(9^k) - 2(2^k)$
   $= 9(9^k - 2^k + 2^k) - 2(2^k)$
   $= 9(7m) + 9(2^k) - 2(2^k)$
   $= 9(7m) + 7(2^k)$
   $= 7(9m + 2^k)$
   $\because$ $9m + 2^k$ is an integer
   $\therefore$ $9^{k+1} - 2^{k+1}$ is divisible by 7
   If $P(k)$ is true, then $P(k + 1)$ is also true
   By the principle of mathematical induction, $P(n)$ is true for all positive integer n .
2. Prove by mathematical induction $8^n + 2\cdot 7^n - 1$ is divisible by 7 for all positive integer n .
   Let $P(n) \equiv$ "$8^n + 2\cdot 7^n - 1$ is divisible by 7 for all positive integer n ."
   $n = 1$, $8 + 2\cdot 7 - 1 = 21 = 3\times 7$, which is divisible by 7, $P(1)$ is true.
   Suppose $P(k)$ is true. i.e. $8^k + 2\cdot 7^k - 1 = 7m$, where m is an integer.
   When $n = k + 1$, $8^{k+1} + 2\cdot 7^{k+1} - 1 = 8\cdot 8^k + 2\cdot 7\cdot 7^k - 1$
   $= 8(7m - 2\cdot 7^k + 1) + 2\cdot 7\cdot 7^k - 1$, by induction assumption.
   $= 56m - 16\cdot 7^k + 8 + 2\cdot 7\cdot 7^k - 1$
   $= 56m - 14\cdot 7^k - 7$
   $= 7(8m - 2\cdot 7^k - 1)$, which is divisible by 7.
   If $P(k)$ is true then $P(k + 1)$ is also true.
   By the principle of mathematical induction, $P(n)$ is true for all positive integer n.
3. **1995 Paper 2 Q6**
   Prove, by mathematical induction, that $(8^n - 1)$ is divisible by 7 for all positive integers n .
   Let $P(n) \equiv$ "$(8^n - 1)$ is divisible by 7 for all positive integers n ."
   $n = 1$, $8^1 - 1 = 7$, which is divisible by 7
   $P(1)$ is true
   Suppose $P(k)$ is true for some positive integer k
   i.e. $8^k - 1 = 7m$, where m is an integer $\cdots\cdots$ (*)
   When $n = k + 1$,
   $8^{k+1} - 1 = 8(8^k) - 1$
   $= 8(8^k - 1 + 1) - 1$
   $= 8(7m) + 8 - 1$ by (*)
   $= 7(8m + 1)$
   $\because$ $8m + 1$ is an integer
   $\therefore$ 8^{k+1} is divisible by 7
   If $P(k)$ is true, then $P(k + 1)$ is also true
   By the principle of mathematical induction, $P(n)$ is true for all positive integer n .