Construct a regular octagon inscribed in a circle.

HKMO 2018 heat events contruction Q3

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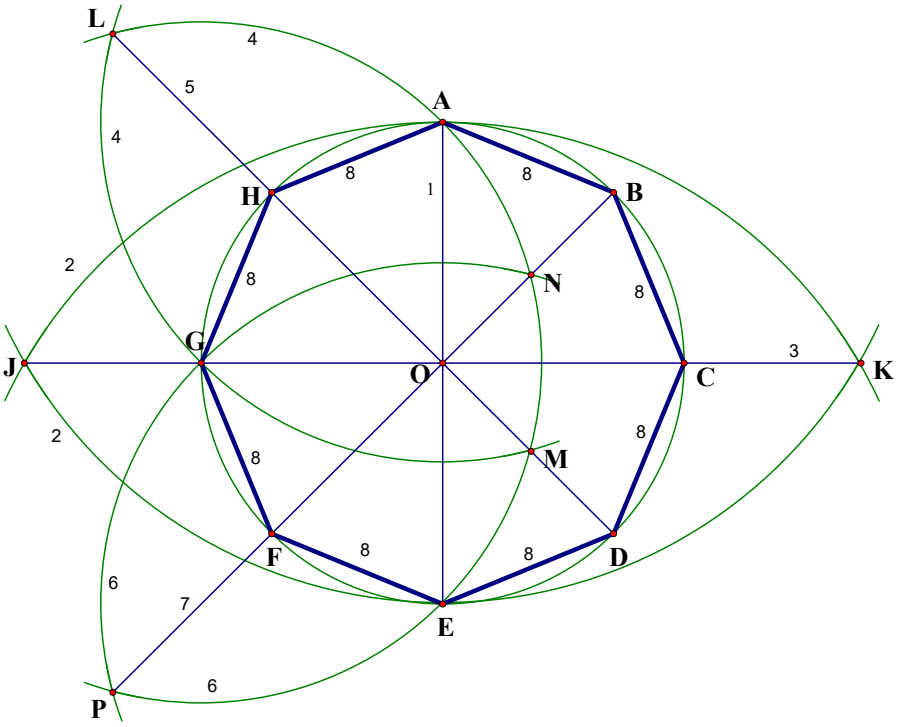
3. 作一個以 O 為圓心的圓上的正八邊形 $ABCDEFGH$。

Construct a regular octagon $ABCDEFGH$ on a circle with centre O .



作圖方法如下： (1) 作直徑 AOE。 (2) 以 A 為圓心，AE 為半徑作一弧；以 E 為圓心，EA 為半徑作一弧；兩弧相交於 J 及 K。 (3) 連接 JK，交圓於 G 及 C。 (4) 以 A 為圓心，AG 為半徑作一弧；以 G 為圓心，GA 為半徑作一弧；兩弧相交於 L 及 M。 (5) 連接並延長 LM，交圓於 H 及 D。 (6) 以 G 為圓心，GE 為半徑作一弧；以 E 為圓心，EG 為半徑作一弧；兩弧相交於 P 及 N。 (7) 連接並延長 PN，交圓於 F 及 B。 (8) 連接 AB、BC、CD、DE、EF、FG、GH 及 HA。 $ABCDEFGH$ 便是所須的正八邊形，證明從略。 利用以上方法，我們可以作圓內接正 16 邊形、圓內接正 32 邊形、…、圓內接正 2^n 邊形 ($n \geq 2$)。	Construction steps: (1) Construct a diameter AOE. (2) Use A as centre, AE as radius to draw an arc; use E as centre, EA as radius to draw another arc; the two arcs intersect at J and K. (3) Join JK, intersecting the circle at G and C. (4) Use A as centre, AG as radius to draw an arc; use G as centre, GA as radius to draw another arc; the two arcs intersect at L and M. (5) Join and produce LM, intersecting the circle at H and D. (6) Use G as centre, GE as radius to draw an arc; use E as centre, EG as radius to draw another arc; the two arcs intersect at P and N. (7) Join and produce PN, intersecting the circle at F and B. (8) Join AB, BC, CD, DE, EF, FG, GH and HA. $ABCDEFGH$ is the required regular octagon, proof omitted. Using this method, we can construct a regular 16-sided polygon, regular 32-sided polygon,…, regular 2^n-gon ($n \geq 2$) inscribed in a circle.