Individual Events

SI	***A***	20	**I1**	***n***	10	**I2**	***a***	48	**I3**	***a***	2	**I4**	***A***	40	**I5**	***a***	45
	B	4		***a***	25		***b***	144		***b***	–3		***B***	6		***b***	15
	C	5		***z***	205		***c***	4		***c***	12		***C***	198		***c***	12
	D	$\frac{5}{2}$		***S***	1		***d***	572		***d***	140		***D***	7		***d***	2

Group Events

SG		2550	**G6**	***a***	1	**G7**	***a***	–8	**G8**	***A***	2	**G9**	***x***	6	**G10**	***c***	3
		2452		***b***	52		***b***	10		***b***	171		***y***	6		***a***	–2
	P	2501		***c***	13		**area**	116		***c***	3		T_{10}	200		***b***	5
	Q	10001		***d***	3		**tan θ**	2		***d***	27		***n***	19		***d***	5

Sample Individual Event

SI.1 Given $A = (b^m)^n + b^{m+n}$. Find the value of A when $b = 4$, $m = n = 1$.

$A = (4^1)^1 + 4^{1+1} = 4 + 16 = 20$

SI.2 If $2^A = B^{10}$ and $B > 0$, find the value of B .

$2^{20} = 4^{10}$

$\Rightarrow B = 4$

SI.3 Solve for C in the following equation: $\sqrt{\frac{20B+45}{C}} = C$.

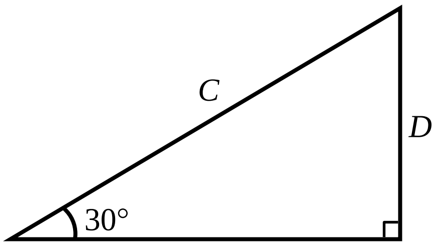
$\sqrt{\frac{20\times 4+45}{C}} = C$

$125 = C^3$

$\Rightarrow C = 5$

SI.4 Find the value of D in the figure.

$D = C \sin 30° = \frac{5}{2}$



Individual Event 1

I1.1 If the sum of the interior angles of an n-sided polygon is 1440°, find the value of n .

$180°\times(n - 2) = 1440°$

$\Rightarrow n = 10$

I1.2 If $x^2 - nx + a = 0$ has 2 equal roots, find the value of a .

$(-10)^2 - 4a = 0$

$\Rightarrow a = 25$

I1.3 In the figure, if $z = p + q$, find the value of z .

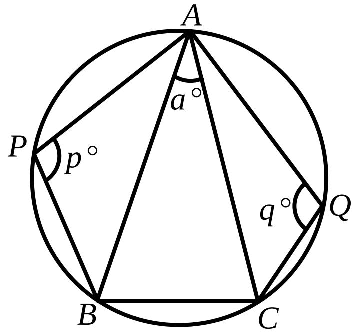
Reference: 1989 HI19

$\angle ACB = 180° - p°$ (opp. $\angle$s cyclic quad.)

$\angle ABC = 180° - q°$ (opp. $\angle$s cyclic quad.)

$180 - p + 180 - q + a = 180$ ($\angle$s sum of Δ)

$z = p + q = 180 + a = 205$



I1.4 If $S = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \dots + z$, find the value of S .

Reference: 1985 FG7.4, 1988 FG6.4, 1990 FG10.1, 1991 FSI.1

$S = 1 + (2 - 3 - 4 + 5) + (6 - 7 - 8 + 9) + \dots + (202 - 203 - 204 + 205) = 1$

Individual Event 2

I2.1 If $ar = 24$ and $ar^4 = 3$, find the value of a .

$$r^3 = \frac{ar^4}{ar} = \frac{3}{24} = \frac{1}{8}$$

$$\Rightarrow r = \frac{1}{2}$$

$ar = 24$

$$\Rightarrow \frac{1}{2}a = 24$$

$\Rightarrow a = 48$

I2.2 If $\left(x+\frac{a}{4}\right)^2 = x^2 + \frac{a}{2}\cdot x + b$, find the value of b .

$(x + 12)^2 = x^2 + 24x + 144$

$\Rightarrow b = 144$

I2.3 If $c = \log_2 \frac{b}{9}$, find the value of c .

$$c = \log_2 \frac{144}{9}$$

$= \log_2 16$

$= 4$

I2.4 If $d = 12^c - 142^2$, find the value of d .

$d = 12^4 - 142^2$

$= 144^2 - 142^2$

$= (144 + 142)(144 - 142)$

$= 2(286) = 572$

Individual Event 3

I3.1 If $a=\frac{\sin 15°}{\cos 75°}+\frac{1}{\sin^2 75°}-\tan^2 15°$, find the value of a.

$a=\frac{\sin 15°}{\sin 15°}+\sec^2 15°-\tan^2 15°$

$= 1 + 1 = 2$

I3.2 If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular to each other, find the value of b.

$-\frac{a}{2}\times\left(-\frac{3}{b}\right)=-1$

$\Rightarrow b = -3$

I3.3 The three points $(2, b)$, $(4, -b)$ and $(5, \frac{c}{2})$ are collinear. Find the value of c.

The three points are $(2, -3)$, $(4, 3)$ and $(5, \frac{c}{2})$, so their slopes are equal.

$\frac{3-(-3)}{4-2}=\frac{\frac{c}{2}-3}{5-4}$

$\Rightarrow \frac{c}{2}-3=3$

$\Rightarrow c = 12$

I3.4 If $\frac{1}{x}:\frac{1}{y}:\frac{1}{z}=3:4:5$ and $\frac{1}{x+y}:\frac{1}{y+z}=9c:d$, find the value of d.

$x:y:z=\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$

$=\frac{20}{60}:\frac{15}{60}:\frac{12}{60}$

$= 20 : 15 : 12$

$x = 20k$, $y = 15k$, $z = 12k$

$\frac{1}{x+y}:\frac{1}{y+z}=\frac{1}{20k+15k}:\frac{1}{15k+12k}$

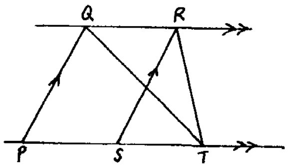
$= 27 : 35$

$= 108 : 140 = 9c : d$

$\Rightarrow d = 140$

Individual Event 4

I4.1 In the figure, the area of *PQRS* is 80 cm^2.



If the area of ΔQRT is A cm^2, find the value of A .

ΔQRT has the same base and same height as the parallelogram *PQRS*.

$A = \frac{1}{2}\cdot 80 = 40$

I4.2 If $B = \log_2\left(\frac{8A}{5}\right)$, find the value of B .

$B = \log_2\left(\frac{8\cdot 40}{5}\right)$

$= \log_2 64$

$= \log_2 2^6$

$= 6$

I4.3 Given $x + \frac{1}{x} = B$. If $C = x^3 + \frac{1}{x^3}$, find the value of C .

$x + \frac{1}{x} = 6$

$x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$

$= 6^2 - 2 = 34$

$C = x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

$= 6(34 - 1) = 198$

I4.4 Let $(p, q) = qD + p$. If $(C, 2) = 212$, find the value of D .

$2D + C = 212$

$\Rightarrow 2D = 212 - 198 = 14$

$\Rightarrow D = 7$

Individual Event 5

I5.1 Let p, q be the roots of the quadratic equation $x^2 – 3x – 2 = 0$ and $a = p^3 + q^3$.
Find the value of a.

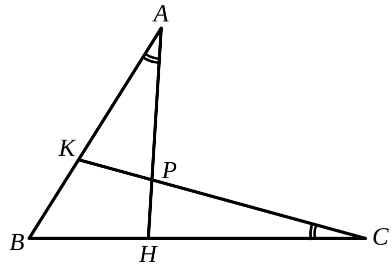
$p + q = 3$, $pq = –2$

$a = (p + q)(p^2 – pq + q^2)$

$= 3[(p + q)^2 – 3pq]$

$= 3[3^2 – 3(–2)] = 45$

I5.2 If $AH = a$, $CK = 36$, $BK = 12$ and $BH = b$, find the value of b.

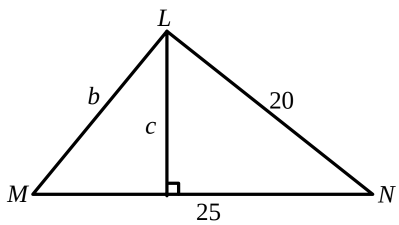


$\Delta ABH \sim \Delta CBK$ (equiangular)

$\frac{b}{12} = \frac{45}{36}$ (ratio of sides, ~Δs)

$b = 15$

I5.3 Find the value of c.



Reference: 1985 FG6.4

$15^2 + 20^2 = 25^2$

$\Rightarrow ML \perp LN$ (converse, Pythagoras' theorem)

Area of $\Delta MNL = \frac{1}{2}\cdot 15\cdot 20 = \frac{1}{2}\cdot 25c$

$c = 12$

I5.4 Let $\sqrt{2x+23}+\sqrt{2x-1}=c$ and $d=\sqrt{2x+23}-\sqrt{2x-1}$. Find the value of d.

Reference: 2014 HG1

$cd = \left(\sqrt{2x+23}+\sqrt{2x-1}\right)\left(\sqrt{2x+23}-\sqrt{2x-1}\right)$

$12d = (2x + 23) – (2x – 1) = 24$

$\Rightarrow d = 2$

Sample Group Event Reference HKCEE Mathematics 1990 Paper 1 Q14

Consider the following groups of numbers:

(2)

(4, 6)

(8, 10, 12)

(14, 16, 18, 20)

(22, 24, 26, 28, 30)

……………………………

SG.1 Find the last number of the 50th group.

$2 = 2\times 1$

$6 = 2(1 + 2)$

$12 = 2(1 + 2 + 3)$

$20 = 2(1 + 2 + 3 + 4)$

$30 = 2(1 + 2 + 3 + 4 + 5)$

The last number of the 50th group

$= 2(1 + 2 + \ldots + 50)$

$= 2\cdot\frac{1}{2}\cdot 50\cdot(1+50) = 2550$

SG.2 Find the first number of the 50th group.

There are 50 numbers in the 50th group.

The first number of the 50th group $= 2550 - 2(50 - 1) = 2452$

SG.3 Find the value of P if the sum of the numbers in the 50th group is $50P$.

$2452 + 2454 + \ldots + 2550 = 50P$

$\frac{1}{2}\cdot 50\cdot(2452+2550) = 50P$

$P = 2501$

SG.4 Find the value of Q if the sum of the numbers in the 100th group is $100Q$.

The last number in the 100th group $= 2(1 + 2 + \ldots + 100) = 2\cdot\frac{1}{2}\cdot 100\cdot(1+100) = 10100$

The first number of the 100th group $= 10100 - 2(100 - 1) = 9902$

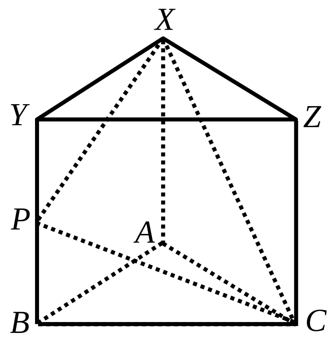
$9902 + 9904 + \ldots + 10100 = 100P$

$\frac{1}{2}\cdot 100\cdot(9902+10100) = 100P$

$P = 10001$

Group Event 6

As shown in the figure, ΔABC and ΔXYZ are equilateral triangles and are ends of a right prism. P is the mid-point of BY and BP = 3 cm, XY = 4 cm.



G6.1 If $a = \dfrac{CP}{PX}$, find the value of a.

$CP = \sqrt{3^2 + 4^2}$ cm = 5 cm = PX (Pythagoras' theorem)

a = 1

G6.2 If $CX = \sqrt{b}$ cm, find the value of b.

$CX = \sqrt{6^2 + 4^2}$ cm $= \sqrt{52}$ cm (Pythagoras' theorem)

b = 52

G6.3 If $\cos\angle PCX = \dfrac{\sqrt{c}}{5}$, find the value of c.

$\cos\angle PCX = \dfrac{\sqrt{52} \div 2}{5} = \dfrac{\sqrt{13}}{5}$

$\Rightarrow c = 13$

G6.4 If $\sin\angle PCX = \dfrac{2\sqrt{d}}{5}$, find the value of d.

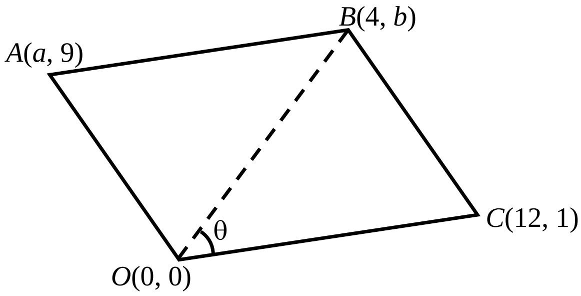
$\sin^2\angle PCX = 1 - \cos^2\angle PCX = \dfrac{12}{25}$

$\sin\angle PCX = \dfrac{2\sqrt{3}}{5}$

$\Rightarrow d = 3$

Group Event 7

In the figure, *OABC* is a parallelogram.



G7.1 Find the value of a .

$a - 0 = 4 - 12$

$\Rightarrow a = -8$

G7.2 Find the value of b .

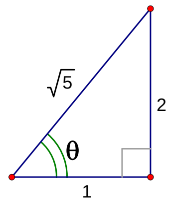
$b - 1 = 9 - 0$

$\Rightarrow b = 10$

G7.3 Find the area of *OABC*.

$$\text{Area} = 2 \cdot \frac{1}{2} \begin{vmatrix} 0 & 0 \\ 12 & 1 \\ 4 & 10 \\ 0 & 0 \end{vmatrix} = 116$$

G7.4 Find the value of $\tan \theta$.



$OC = \sqrt{145}$

$OB = \sqrt{116}$

$BC = \sqrt{(12-4)^2 + (1-10)^2} = \sqrt{145}$

$$\cos \theta = \frac{\sqrt{145}^2 + \sqrt{116}^2 - \sqrt{145}^2}{2\left(\sqrt{145}\right)\left(\sqrt{116}\right)} = \frac{1}{\sqrt{5}}$$

$\tan \theta = 2$

Method 2

$$m_{OC} = \frac{1-0}{12-0} = \frac{1}{12}$$

$$m_{OB} = \frac{10-0}{4-0} = \frac{5}{2}$$

$$\tan \theta = \frac{\frac{5}{2} - \frac{1}{12}}{1 + \frac{5}{2} \cdot \frac{1}{12}} = 2$$

Group Event 8

G8.1 The area of an equilateral triangle of side A cm is $\sqrt{3}$ cm^2. Find the value of A .

$\frac{1}{2}\cdot A^2\sin 60^\circ=\sqrt{3}$

$\Rightarrow A = 2$

G8.2 If $19\times 243^{\frac{A}{5}}=b$, find the value of b .

$b=19\times\left(3^5\right)^{\frac{2}{5}}=171$

G8.3 The roots of the equation $x^3 - 173x^2 + 339x + 513 = 0$ are -1, b and c. Find the value of c .

$-1 + 171 + c$ = sum of roots = 173

$\Rightarrow c = 3$

G8.4 The base of a triangular pyramid is an equilateral triangle of side $2c$ cm.

If the height of the pyramid is $\sqrt{27}$ cm, and its volume is d cm^3, find the value of d .

$d=\frac{1}{3}\cdot\frac{1}{2}\cdot\left(6^2\cdot\sin 60^\circ\right)\cdot\sqrt{27}=27$

Group Event 9

If the area of a regular hexagon *ABCDEF* is $54\sqrt{3}\text{ cm}^2$ and $AB = x$ cm, $AC = y\sqrt{3}$ cm,

G9.1 find the value of x .

The hexagon can be cut into 6 identical equilateral triangles

$$6\cdot\frac{1}{2}\cdot\left(x^2\cdot\sin 60^\circ\right)=54\sqrt{3}$$

$\Rightarrow x = 6$

G9.2 find the value of y .

$\angle ABC = 120°$

$AC^2 = (x^2 + x^2 - 2x^2 \cos 120°)\text{ cm}^2$

$= [6^2 + 6^2 - 2(6)^2\cdot\left(-\frac{1}{2}\right)]\text{ cm}^2$

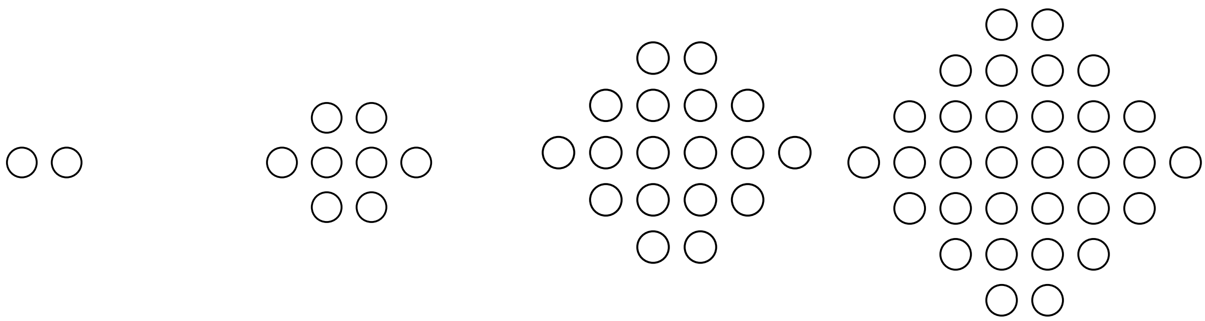
$= 3\times 6^2\text{ cm}^2$

$y\sqrt{3}=6\sqrt{3}$

$\Rightarrow y = 6$

G9.3 - G9.4 (Reference: 1991 FG8.1-2)

Consider the following number pattern:



$T_1 = 2$ $T_2 = 8$ $T_3 = 18$ $T_4 = 32$

G9.3 Find the value of T_{10} .

$8 - 2 = 6$, $18 - 8 = 10$, $32 - 18 = 14$

$\Rightarrow T_1 = 2$, $T_2 = 2 + 6$, $T_3 = 2 + 6 + 10$, $T_4 = 2 + 6 + 10 + 14$

$$T_{10}=\frac{10}{2}\cdot[2(2)+(10-1)\cdot 4]=200$$

G9.4 If $T_n = 722$, find the value of n .

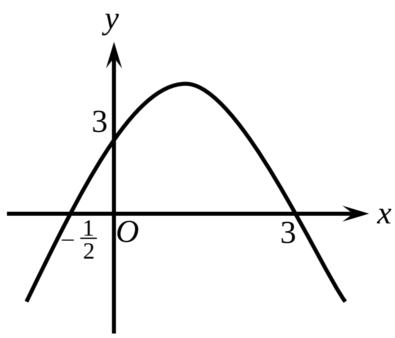
$$\frac{n}{2}\cdot[2(2)+(n-1)\cdot 4]=722$$

$n^2 = 361$

$n = 19$

Group Event 10

The following shows the graph of $y = ax^2 + bx + c$.



G10.1 Find the value of c .

$x = 0$, $y = c = 3$

G10.2 Find the value of a .

$y = a(x + \frac{1}{2})(x - 3)$

Sub. $x = 0$, $y = 3$

$\Rightarrow -\frac{3}{2}a = 3$

$a = -2$

G10.3 Find the value of b .

$3 - \frac{1}{2} = \text{sum of roots} = -\frac{b}{(-2)}$

$b = 5$

G10.4 If $y = x + d$ is tangent to $y = ax^2 + bx + c$, find the value of d .

Sub. $y = x + d$ into $y = ax^2 + bx + c$

$-2x^2 + 5x + 3 = x + d$

$2x^2 - 4x + d - 3 = 0$

$\Delta = (-4)^2 - 4(2)(d - 3) = 0$

$4 - 2d + 6 = 0$

$d = 5$