**Hong Kong Mathematics Olympiad (1995– 96)**
**Heat Event (Individual)**

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 已知 $4^{x-3} = 8^{x-2}$，求 $x$ 的值。
Find the value of $x$ if $4^{x-3} = 8^{x-2}$.

2. 已知 $f\left(\frac{1+x}{x}\right) = \frac{x^2+1}{x^2} + \frac{1}{x}$，求 $f(x^3)$ 的值。
If $f\left(\frac{1+x}{x}\right) = \frac{x^2+1}{x^2} + \frac{1}{x}$, find $f(x^3)$.

3. 考慮 $n! = n\times(n-1)\times(n-2)\times\ldots\times 3\times 2\times 1$，求 100! 的末尾 0 的數目。
By considering $n! = n\times(n-1)\times(n-2)\times\ldots\times 3\times 2\times 1$, find the number of trailing zeros of 100!.

4. 能滿足不等式 $n^{200} < 5^{300}$ 的最大整數 $n$ 是多少？
What is the largest integral value $n$ that satisfies the inequality $n^{200} < 5^{300}$?

5. 三個面值 \$0.1、\$3 和 \$5 和郵票共有 110 個，這組郵票共值 \$100。求 \$3 郵票的數目。
A set of 110 stamps of the denominations of \$0.1, \$3, \$5 worth \$100 in total.
Find the number of \$3 stamps in the set of stamps.

6. 對於任何數值 $m$，直線 $y = mx + 2m + 2$ 必經一定點 $P$。求 $P$ 之座標。
For any value of $m$, a straight line $y = mx + 2m + 2$ passes through a fixed point $P$. Find the coordinates of $P$.

7. 若各數字可重複選用，從數字 4, 5, 6, 7, 8, 9 中，可組成多少個 3 位數？
How many 3-digit numbers can be made from the figures 4, 5, 6, 7, 8, 9 when repetitions are allowed?

8. 將 $2.3\dot{1}$ 以分數表示。
Express $2.3\dot{1}$ as a fraction.

9. 若 $x$ 及 $y$ 為正整數及 $x - y = 5$，求 $x^2 - y^2 + 5$ 之最小值。
If $x$ and $y$ are positive integers and $x - y = 5$, find the least value of $x^2 - y^2 + 5$.

10. 因式分解 $x^2(y-z) + y^2(z-x) + z^2(x-y)$。
Factorize $x^2(y-z) + y^2(z-x) + z^2(x-y)$.

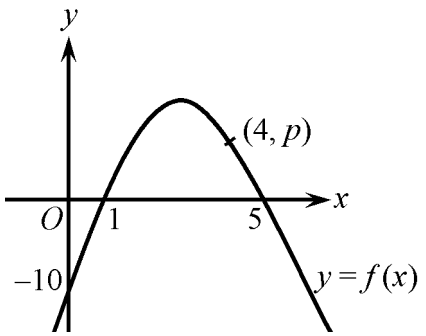
***** 試卷完 End of Paper *****

**Hong Kong Mathematics Olympiad (1995 – 96)**
**Heat Event (Group)**

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

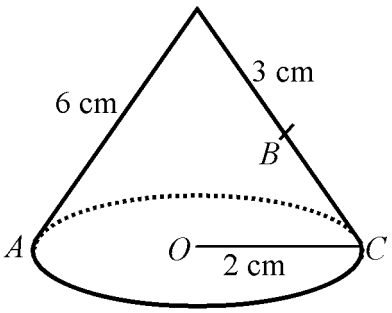
1. 圖中二次函數曲線 $y = f(x)$ 切 $x$-軸於點 (1, 0) 和 (5, 0)，及 $y$-軸於點 (0, –10)。求 $p$ 的值。

In the figure, the quadratic curve $y = f(x)$ cuts the $x$-axis at the two points (1, 0) and (5, 0) and the $y$-axis at the point (0, –10). Find the value of $p$ .




2. 在圖中 $O$ 是圓錐體底部的圓心；$A$、$B$、$C$ 及 $O$ 躺於同一平面上。若螞蟻在圓錐曲面上由 $A$ 走到 $B$，找出由 $A$ 到 $B$ 的最短路綫的長度。

In the figure, $O$ is the centre of the base circle of a cone and the points $A$, $B$, $C$ and $O$ lie in the same plane. An ant walks from $A$ to $B$ on the surface of the cone. Find the length of the shortest path from $A$ to $B$ .




3. 當一疊面值 7020 元的十元紙幣被等分給 $x$ 人時，餘下 650 元。把剩下的 650 元換成 5 元硬幣再等分給 $x$ 人時，餘下 195 元。求 $x$ 的值。

When a sum of \$7020, in the form of ten-dollar notes, is divided equally among $x$ persons, \$650 remains. When this sum \$650 is changed to five-dollar coins and then divided equally among the $x$ persons, \$195 remains. Find the value of $x$ .

4. 射擊比賽中，根據統計紀錄，$A$ 每 5 發射擊，有一發落空，$B$ 每 4 發射擊則有一發落空，而 $C$ 每 3 發射擊則有一發落空。

若 $A$、$B$、$C$ 同時各發一槍，求 $A$、$B$ 命中而 $C$ 落空的概率。

In a shooting competition, according to statistics, $A$ misses one in every 5 shoots, $B$ misses one in every 4 shoots and $C$ misses one in every 3 shoots. Find the probability of obtaining successful shoots by $A$, $B$ but not $C$ .

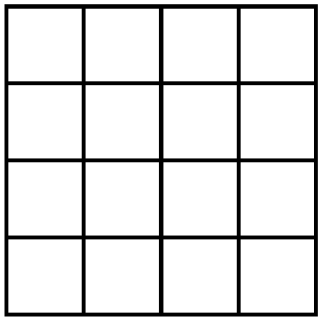
5. Given that $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$, find the value of $\dfrac{1}{2\times 3} + \dfrac{1}{3\times 4} + \cdots + \dfrac{1}{99\times 100}$.

已知 $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$，求 $\dfrac{1}{2\times 3} + \dfrac{1}{3\times 4} + \cdots + \dfrac{1}{99\times 100}$ 之值。

6. 若三位數 $A$ 加上 3，新數的三個數字之和為原先 $A$ 的三位數之和的三分之一。求所有這些可能的數目 $A$ 的總和。

If 3 is added to a 3-digit number $A$, the sum of the digits of the new number is $\frac{1}{3}$ of the value of the sum of digits of the original number $A$. Find the sum of all such possible numbers $A$.

7. 圖中每個小正方形的邊長為 1 單位。求圖中所有可能組成之長方形(包括正方形)的面積之和。



In the figure, the side of each smaller square is 1 unit long. Find the sum of the area of all possible rectangles (squares included) that can be formed in the figure.

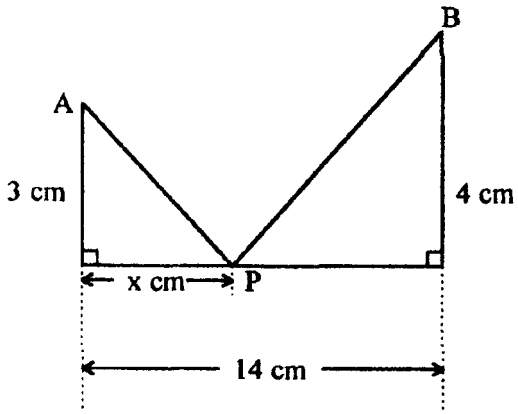
8. 若質數 $a$、$b$ 為二次方程 $x^2 - 21x + t = 0$ 的根，求 $\left(\frac{b}{a}+\frac{a}{b}\right)$ 的值。

If prime numbers $a$, $b$ are the roots of the quadratic equation $x^2 - 21x + t = 0$,

find the value of $\left(\frac{b}{a}+\frac{a}{b}\right)$.

9. 求 $x$ 的值使得圖中路綫 $APB$ 的長度最小。

Find the value of $x$ such that the length of the path $APB$ in the figure is the smallest .




10. 求總數 $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 123456789^2$ 的個位數。

Find the units digit of the sum $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 123456789^2$ .

***** 試卷完 End of Paper *****