

# Hong Kong Mathematics Olympiad (1983 – 1984)

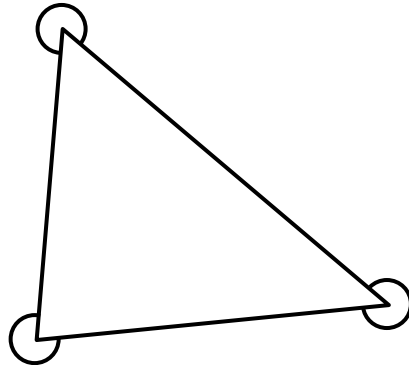
## Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 附圖所示三角之和為  $a^\circ$ ，求  $a$  的值。

In the given diagram, the sum of the three marked angles is  $a^\circ$ .  
Find the value of  $a$ .



$a =$

- (ii) 一正  $b$  邊形之內角和為  $a^\circ$ ，求  $b$  的值。

The sum of the interior angles of a regular  $b$ -sided polygon is  $a^\circ$ .  
Find the value of  $b$ .

$b =$

- (iii) 若  $8^b = c^{21}$ ，求  $c$  的值。

If  $8^b = c^{21}$ , find the value of  $c$ .

$c =$

- (iv) 若  $c = \log_d 81$ ，求  $d$  的值。

If  $c = \log_d 81$ , find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

×

Mult. factor for  
speed

=

Team No.

+ Bonus  
score

Time



Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若  $100a = 35^2 - 15^2$ ，求  $a$  的值。

If  $100a = 35^2 - 15^2$ , find the value of  $a$ .

$a =$

(ii) 若  $(a - 1)^2 = 3^{4b}$ ，求  $b$  的值。

If  $(a - 1)^2 = 3^{4b}$ , find the value of  $b$ .

$b =$

(iii) 若  $b$  為  $x^2 + cx - 5 = 0$  之一根，求  $c$  的值。

If  $b$  is a root of  $x^2 + cx - 5 = 0$ , find the value of  $c$ .

$c =$

(iv) 若  $x + c$  為  $2x^2 + 3x + 4d$  之因式，求  $d$  的值。

If  $x + c$  is a factor of  $2x^2 + 3x + 4d$ , find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $\alpha, \beta$  為  $x^2 - 10x + 20 = 0$  之根，且  $a = \frac{1}{\alpha} + \frac{1}{\beta}$ ，求  $a$  的值。

If  $\alpha, \beta$  are roots of  $x^2 - 10x + 20 = 0$ , find the value of  $a$ , where  $a = \frac{1}{\alpha} + \frac{1}{\beta}$ .

$a =$

- (ii) 若  $\sin \theta = a$  ( $0^\circ < \theta < 90^\circ$ )，且  $10 \cos 2\theta = b$ ，求  $b$  的值。

If  $\sin \theta = a$  ( $0^\circ < \theta < 90^\circ$ ), and  $10 \cos 2\theta = b$ , find the value of  $b$ .

$b =$

- (iii) 點  $A(b, c)$  在直線  $2y = x + 15$  上，求  $c$  的值。

The point  $A(b, c)$  lies on the line  $2y = x + 15$ . Find the value of  $c$ .

$c =$

- (iv) 若  $x^2 - cx + 40 \equiv (x + k)^2 + d$ ，求  $d$  的值。

If  $x^2 - cx + 40 \equiv (x + k)^2 + d$ , find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

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accuracy

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speed

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Team No.

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Bonus  
score

Time



Total score

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $a$  為  $2x^3 - 3x^2 + x - 1$  被  $x + 1$  除所得之餘數，求  $a$  的值。

If  $a$  is the remainder when  $2x^3 - 3x^2 + x - 1$  is divided by  $x + 1$ , find the value of  $a$ .

$a =$

- (ii) If  $b \text{ cm}^2$  is the total surface area of a cube of side  $(8 + a) \text{ cm}$ , find the value of  $b$ .

若  $b \text{ cm}^2$  為一邊長  $(8 + a) \text{ cm}$  的立方體之總表面積，求  $b$  的值。

$b =$

- (iii) 一袋內有紅球  $b + 4$  個，白球  $2b - 2$  個。若隨意於袋內取球一個，而該球為白色之機會為  $x$ ，求  $x$  的值。

One ball is taken at random from a bag containing  $b + 4$  red balls and  $2b - 2$  white balls. If  $x$  is the probability that the ball is white, find the value of  $x$ .

$x =$

- (iv) 若  $\sin \theta = x$  ( $90^\circ < \theta < 180^\circ$ ) 及  $\tan(\theta - 15^\circ) = y$ ，求  $y$  的值。

If  $\sin \theta = x$  ( $90^\circ < \theta < 180^\circ$ ) and  $\tan(\theta - 15^\circ) = y$ , find the value of  $y$ .

$y =$

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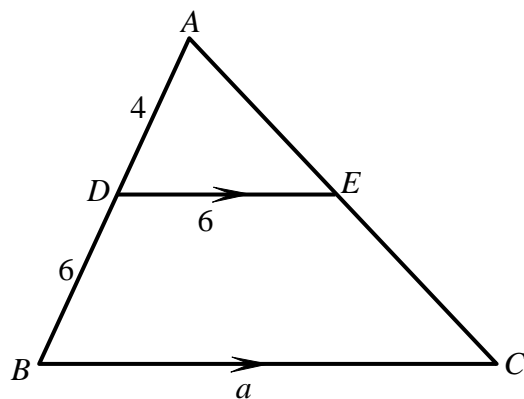
**Hong Kong Mathematics Olympiad (1983 – 1984)**  
**Final Event 4 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.  
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在圖一中， $DE \parallel BC$ ，若  $AD = 4$ ， $DB = 6$ ， $DE = 6$ ，且  $BC = a$ ，求  $a$  的值。

In figure 1,  $DE \parallel BC$ . If  $AD = 4$ ,  $DB = 6$ ,  $DE = 6$  and  $BC = a$ , find the value of  $a$ .

$a =$



圖一 Figure 1

- (ii)  $\theta$  為銳角， $\cos \theta = \frac{a}{17}$ 。若  $\tan \theta = \frac{b}{15}$ ，求  $b$  的值。

$\theta$  is an acute angle such that  $\cos \theta = \frac{a}{17}$ . If  $\tan \theta = \frac{b}{15}$ , find the value of  $b$ .

$b =$

- (iii) 若  $c^3 = b^2$ ，求  $c$  的值。

If  $c^3 = b^2$ , find the value of  $c$ .

$c =$

- (iv) 一等邊三角形之面積為  $c\sqrt{3} \text{ cm}^2$ 。若其周界長  $d \text{ cm}$ ，求  $d$  的值。

The area of an equilateral triangle is  $c\sqrt{3} \text{ cm}^2$ .

If its perimeter is  $d \text{ cm}$ , find the value of  $d$ .

$d =$

**FOR OFFICIAL USE**

Score for  
accuracy

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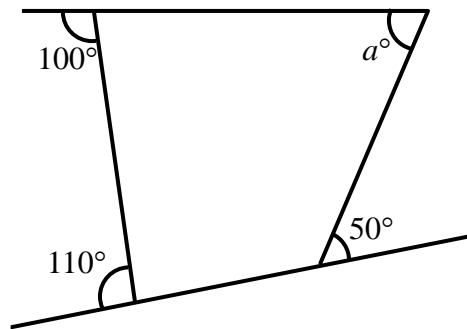
## Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在圖二，求  $a$  的值。

In Figure 2, find the value of  $a$ .



圖二 Figure 2

- (ii) 若  $b = \log_2\left(\frac{a}{5}\right)$ ，求  $b$  的值。

If  $b = \log_2\left(\frac{a}{5}\right)$ , find the value of  $b$ .

- (iii) 一繩長 20 m，依  $b - 2 : b : b + 2$  之比例分成三段。

若最長一段為  $N$  m，求  $N$  的值。

A piece of string, 20 m long, is divided into 3 parts in the ratio of  $b - 2 : b : b + 2$ .

If  $N$  m is the length of the longest portion, find the value of  $N$ .

- (iv) 正  $N$  邊形之每一內角為  $x^\circ$ 。求  $x$  的值。

Each interior angle of an  $N$ -sided regular polygon is  $x^\circ$ . Find the value of  $x$ .

### FOR OFFICIAL USE

Score for  
accuracy

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Mult. factor for  
speed

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Team No.

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score

Time

Total score

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 某兩數之和為 20，其積為 10，若該兩數倒數之和為  $a$ ，求  $a$  的值。

The sum of 2 numbers is 20, their product is 10.

If the sum of their reciprocals is  $a$ , find the value of  $a$ .

$a =$

- (ii)  $1^2 - 1 = 0 \times 2$ ,  $2^2 - 1 = 1 \times 3$ ,  $3^2 - 1 = 2 \times 4$ , ...,  $b^2 - 1 = 135 \times 137$ .

若  $b > 0$ ，求  $b$  的值。

$1^2 - 1 = 0 \times 2$ ,  $2^2 - 1 = 1 \times 3$ ,  $3^2 - 1 = 2 \times 4$ , ...,  $b^2 - 1 = 135 \times 137$ .

If  $b > 0$ , find the value of  $b$ .

$b =$

- (iii) 若兩直線  $x + 2y + 1 = 0$  及  $cx + 3y + 1 = 0$  互相垂直，求  $c$  的值。

If the lines  $x + 2y + 1 = 0$  and  $cx + 3y + 1 = 0$  are perpendicular, find the value of  $c$ .

$c =$

- (iv)  $(2, -1)$ 、 $(0, 1)$ 、 $(c, d)$  三點共線。求  $d$  的值。

The points  $(2, -1)$ ,  $(0, 1)$ ,  $(c, d)$  are collinear. Find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

Score for  
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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若  $p = \frac{21^3 - 11^3}{21^2 + 21 \times 11 + 11^2}$ ，求  $p$  的值。

If  $p = \frac{21^3 - 11^3}{21^2 + 21 \times 11 + 11^2}$ , find the value of  $p$ .

$p =$

(ii) 若  $p$  人可在 6 日完成某一工程，且 4 人可在  $q$  日完成同一工程，求  $q$  的值。

If  $p$  men can do a job in 6 days and 4 men can do the same job in  $q$  days, find the value of  $q$ .

$q =$

(iii) 某年三月第  $q$  日為星期三，而同年三月第  $r$  日為星期五，且  $18 < r < 26$ ，求  $r$  的值。

If the  $q^{\text{th}}$  day of March in a year is Wednesday and the  $r^{\text{th}}$  day of March in the same year is Friday, where  $18 < r < 26$ , find the value of  $r$ .

$r =$

(iv) 若  $a * b = ab + 1$ ，且  $s = (3 * 4) * 2$ ，求  $s$  的值。

If  $a * b = ab + 1$ , and  $s = (3 * 4) * 2$ , find the value of  $s$ .

$s =$

### FOR OFFICIAL USE

Score for accuracy

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Mult. factor for speed

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Bonus score

Time



Total score

Min.

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 凌晨三點卅分，時鐘兩針間之銳角為  $p^\circ$ ，求  $p$  的值。

The acute angle between the 2 hands of a clock at 3:30 a.m. is  $p^\circ$ .

Find the value of  $p$ .

$p =$

- (ii) 在  $\triangle ABC$  中， $\angle B = \angle C = p^\circ$ 。若  $q = \sin A$ ，求  $q$  的值。

In  $\triangle ABC$ ,  $\angle B = \angle C = p^\circ$ . If  $q = \sin A$ , find the value of  $q$ .

$q =$

- (iii) 三點  $(1, 3)$ 、 $(a, 5)$ 、 $(4, 9)$  共綫，求  $a$  的值。

The 3 points  $(1, 3)$ ,  $(a, 5)$ ,  $(4, 9)$  are collinear. Find the value of  $a$ .

$a =$

- (iv)  $7$ 、 $9$ 、 $x$ 、 $y$ 、 $17$  之平均數為  $10$ 。若  $m$  為  $x+3$ 、 $x+5$ 、 $y+2$ 、 $8$ 、 $y+18$  之平均數，求  $m$  的值。

The average of  $7, 9, x, y, 17$  is  $10$ .

If  $m$  is the average of  $x+3, x+5, y+2, 8, y+18$ , find the value of  $m$ .

$m =$

### FOR OFFICIAL USE

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Total score

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

如圖所示加法中，每字母代表由零至九之不同整數。

已知  $S = 9$ ， $O = \text{零}$ ， $E = 5$ 。

求下列字母所代表之數字：

(i)  $M$

(ii)  $N$

(iii)  $R$

(iv)  $Y$

In the addition shown, each letter represents a different digit ranging from zero to nine. It is already known that  $S = 9$ ,  $O = \text{zero}$ ,  $E = 5$ .

Find the numbers represented by

(i)  $M$

(ii)  $N$

(iii)  $R$

(iv)  $Y$

$$\begin{array}{r} S \quad E \quad N \quad D \\ + \quad M \quad O \quad R \quad E \\ \hline M \quad O \quad N \quad E \quad Y \end{array}$$

$$\begin{array}{r} S \quad E \quad N \quad D \\ + \quad M \quad O \quad R \quad E \\ \hline M \quad O \quad N \quad E \quad Y \end{array}$$

$M =$

$N =$

$R =$

$Y =$

### FOR OFFICIAL USE

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Time

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $x = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{100}\right)$ ，試以最簡單的分數表  $x$ 。

If  $x = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{100}\right)$ , find  $x$  in the simplest fractional form.

$x =$

- (ii) 一長方體之長、闊、高依次為 2、3 及 4。若其總面積為  $A$ ，求  $A$  的值。  
The length, width and height of a rectangular block are 2, 3 and 4 respectively.  
Its total surface area is  $A$ , find the value of  $A$ .

$A =$

- (iii) 若  $m$  為 1、2、3、...、1001 之平均數，求  $m$  的值。

The average of the integers 1, 2, 3, ..., 1001 is  $m$ . Find the value of  $m$ .

$m =$

- (iv) 一面積為  $12\pi$  之圓，內接於一周界為  $P$  之等邊三角形，求  $P$  的值。

The area of a circle inscribed in an equilateral triangle is  $12\pi$ .

If  $P$  is the perimeter of this triangle, find the value of  $P$ .

$P =$

### FOR OFFICIAL USE

Score for  
accuracy

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Mult. factor for  
speed

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Time



Total score

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# Hong Kong Mathematics Olympiad (1983 – 1984)

## Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 一正方形內接於一直徑為 10 之圓。若  $A$  為正方形的面積，求  $A$  的值。

If  $A$  is the area of a square inscribed in a circle of diameter 10,  
find the value of  $A$ .

$A =$

- (ii) 若  $a + \frac{1}{a} = 2$ ，及  $S = a^3 + \frac{1}{a^3}$ ，求  $S$  的值。

If  $a + \frac{1}{a} = 2$ , and  $S = a^3 + \frac{1}{a^3}$ , find the value of  $S$ .

$S =$

- (iii) 一凸  $n$  邊形有 14 條對角線，求  $n$  的值。

An  $n$ -sided convex polygon has 14 diagonals. Find the value of  $n$ .

$n =$

- (iv) 若  $d$  為兩點  $(2, 3)$  及  $(-1, 7)$  間之距離，求  $d$  的值。

If  $d$  is the distance between the 2 points  $(2, 3)$  and  $(-1, 7)$ , find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
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