第三十四屆香港數學競賽(2016/17)
初賽規則

1. 初賽分個人項目、團體項目和幾何作圖項目三部分，個人項目限時**六十分鐘**，團體項目限時二十分鐘，而幾何作圖項目則限時二十分鐘。
2. 每隊由四至六位**中五**或以下同學組成。其中任何四位可參加個人項目；又其中任何四位可參加團體項目及幾何作圖項目。不足四位同學的隊伍將被撤銷參賽資格。
3. 每隊隊員**必須穿著整齊校服**，**並由負責教師帶領**，於上午9時或以前向會場接待處註冊，同時必須出示**身分證/學生證明文件**，否則**將被撤銷參賽資格**。
4. 指示語言將採用粵語。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英文並列。
5. 每一隊員於個人項目中須解答15條問題（當中**甲部佔10題**、**乙部佔5題**）；而每一隊伍則須於團體項目中解答10條問題；並在幾何作圖項目中解答**所有**問題。
6. 團體項目及幾何作圖項目中，各參賽隊員可進行討論，但必須將聲浪降至最低。
7. 各參賽隊伍須注意：
   (a) 個人項目及團體項目比賽時，**不准使用**計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，
   (b) 幾何作圖項目比賽時，**只准使用**書寫工具(例如: 原子筆、鉛筆等)、圓規及大會提供的直尺，違例隊伍將被撤銷參賽資格或扣分。
8. **除非另有聲明**，**否則所有個人項目及團體項目中問題的答案均為數字**，**並應化至最簡**，**但無須呈交證明及算草**。
9. 參賽者如有攜帶電子通訊器材，應把它關掉(包括響鬧功能)並放入手提包內或座位的椅下。
10. 個人項目中，甲部和乙部的每一正確答案分別可得1分及2分。每隊可得之最高積分為80分。
11. 團體項目中，每一正確答案均可得2分。每隊可得之最高積分為20分。
12. 至於幾何作圖項目，每隊可得之最高積分為20分（必須詳細列出所有步驟，包括作圖步驟）。
13. 初賽中，並不給予快捷分。
14. 參賽者必須自備工具，例如：**原子筆**、**鉛筆及圓規**。
15. 籌委會將根據各參賽隊伍的總成績（個人項目、團體項目及幾何作圖項目的積分總和）選出最高積分的五十隊進入決賽。
16. 初賽獎項：
   (a) 於個人項目比賽中，
      (i) 取得滿分者將獲頒予最佳表現及積分獎狀；
      (ii) 除上述 (i) 中取得最佳表現的參賽者外，
         (1) 成績最佳的首 2% 參賽者將獲頒予一等榮譽獎狀；
         (2) 隨後的 5% 參賽者將獲頒予二等榮譽獎狀；
         (3) 緊接著的 10% 參賽者將獲頒予三等榮譽獎狀。
   (b) 於團體項目中取得滿分的隊伍將獲頒予最佳表現及積分獎狀。
   (c) 於幾何作圖項目中表現優秀的隊伍將獲頒予獎狀。
   (d) 於各分區的比賽中，總成績（個人項目、團體項目及幾何作圖項目的積分總和）最高之首10%的參賽隊伍將獲頒予獎狀。
17. 如有任何疑問，參賽者須於比賽完畢後，立即向會場主任提出。所提出之疑問，將由籌委會作最後裁決。

The Thirty-fourth Hong Kong Mathematics Olympiad (2016/17)
Regulations (Heat Events)

1. The Heat Events consists of three parts: **60 minutes** for the individual event, 20 minutes for the group event and 20 minutes for the geometric construction event.
2. Each team should consist of 4 to 6 members who are students of **Secondary 5** level or below. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event and the geometric construction event. Teams of less than 4 members will be disqualified.
3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception not later than 9:00 a.m. Failing to do so, the team **will be disqualified**.
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.
5. Each member of a team has to solve 15 questions in the individual event (***10 questions in Part A*** and ***5 questions in Part B***) and each team has to solve 10 questions in the group event and **ALL** questions in the geometric construction event.
6. In the group event and geometric construction event, discussions among participating team members are allowed provided that the voice level is kept to a minimum.
7. Please note that
   (a) for the individual and group events, devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will not be allowed to be used; and
   (b) for the geometric construction event, only writing instruments (pens, pencils, etc), **straightedge provided and compasses** will be allowed to be used;

   otherwise the team will be disqualified or risk deduction of marks.
8. **All answers in the individual event and the group event should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**
9. Participants having electronic communication devices should have them turned off (including the alarm function) and put them inside their bags or under their chairs.
10. For the individual event, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The maximum score for a team should be 80.
11. For the group event, 2 marks will be given to each correct answer. The maximum score for a team should be 20.
12. For the geometric construction event, the maximum score for a team should be 20 (all working, including construction work, must be clearly shown).
13. No mark for speed will be awarded in the Heat Event.
14. Participants should bring along their own instruments, e.g. **ball pens**, **pencils and compasses**.
15. The 50 teams with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) will be qualified for the Final Event.
16. Awards of the Heat Event:
   (a) For the individual event,
      (i) candidates obtaining full score will be awarded Best Performance and Score certificates ;
      (ii) apart from the best performer(s) in (i),
         (1) the first 2% of top scoring candidates will be awarded First-class honour certificates ;
         (2) the next 5% of top scoring candidates will be awarded Second-class honour certificates ; and
         (3) the next 10% of top scoring candidates will be awarded Third-class honour certificates;
   (b) For the group event, teams obtaining full marks will be awarded Best Performance and Score certificates.
   (c) For the geometric construction event, teams having outstanding performance will be awarded certificates of merit.
   (d) About 10% of participating schools with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) in each region will be awarded certificates of merit.
17. Should there be any queries, participants should reach the Centre Supervisor immediately after the competition. The decision of the Organising Committee on the queries is final.

Hong Kong Mathematics Olympiad 2016-2017
Heat Event (Individual)
香港數學競賽 2016 – 2017
初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。　　時限：1 小時 Time allowed: 1 hour
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
Q1- Q10 每題 1 分，Q11-Q15 每題 2 分。Q1- Q10 1 mark each, Q11-Q15 2 marks each.
全卷滿分 20 分。The maximum mark for this paper is 20.

1. 已知 $A2017B$ 是一個六位數，且可被 72 整除，求 A 的值。
Given that $A2017B$ is a 6-digit number which is divisible by 72, find the value of A.

2. 已知 $0 \le p \le 1$，求 $Q = 3p^2(1-p) + 6p(1-p)^2 + 3(1-p)^3$ 的最大值。
Given that $0 \le p \le 1$, find the greatest value of $Q = 3p^2(1-p) + 6p(1-p)^2 + 3(1-p)^3$.

3. 已知 ΔABC 的三條邊的長是 a 、b 和 c，其中 $3 \le a \le 5 \le b \le 12 \le c \le 15$，求當 ΔABC 的面積最大時，它的周界是多少？
Given that the three sides of ΔABC are of lengths a, b and c, where $3 \le a \le 5 \le b \le 12 \le c \le 15$, find the perimeter of ΔABC when its area attains the maximum value.

4. 設 B 及 C 為正整數，求 C 的最小值使得 $B^2 = C + 134$。
Let B and C be positive integers. Find the least value of C satisfying $B^2 = C + 134$.

5. 若把一組自然數之和 $1 + 2 + 3 + \cdots + 2015 + 2016 + 2017$ 除以 9，餘數是甚麼？
Determine the remainder when the sum of natural numbers $1 + 2 + 3 +\cdots+ 2015 +2016 +2017$ is divided by 9.

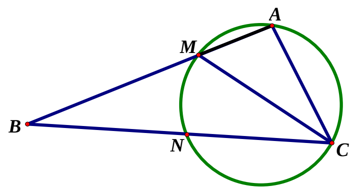
6. 已知 $a_0 = 2$，$a_1 = -1$ 及 $a_{n+1} = 2a_n - a_{n-1}$，其中 $n \ge 1$，求 a_{2017} 的值。
Given that $a_0 = 2$, $a_1 = -1$ and $a_{n+1} = 2a_n - a_{n-1}$, where $n \ge 1$, determine the value of a_{2017}.

7. 設 N 為完全立方數，已知 $N = 161x + 23y$，其中 x 和 y 均為正整數。
求 $x + y$ 的最小值。
Let N be a perfect cube number. Given that $N = 161x + 23y$, where x and y are positive integers. Find the minimum value of $x + y$.

8. 已知 ② $= 1\times2\times3\times4$，③ $= 2\times3\times4\times5$，④ $= 3\times4\times5\times6$，… 及 $\frac{1}{⑮} - \frac{1}{⑰} = \frac{1}{⑰} \times A$，求 A 的值。
Given that ② $= 1\times2\times3\times4$, ③ $= 2\times3\times4\times5$, ④ $= 3\times4\times5\times6$, … and $\frac{1}{⑮} - \frac{1}{⑰} = \frac{1}{⑰} \times A$, find the value of A.

9. 已知 $\sin x \cdot \cos x = 0$ 及 $\sin^3 x - \cos^3 x = 1$，其中 $90° \le x < 180°$，求 x 的值。
Given that $\sin x \cdot \cos x = 0$ and $\sin^3 x - \cos^3 x = 1$, where $90° \le x < 180°$, find the value of x .

10. 如圖一，CM 是$\angle ACB$ 的角平分綫，且 $AB = 2AC$。已知ΔAMC 的外接圓與 BC 相交於 N。若 $BN = 10$，求 AM 的長度。
In Figure 1, CM is the angle bisector of $\angle ACB$ and $AB = 2AC$. Given that the circumscribed circle of ΔAMC intersects BC at N. If $BN = 10$, find the length of AM.



圖一 Figure 1

11. 已知 x 為一實數，求 $\sqrt{x(x+3)(x+6)(x+9)+2017}$ 的最小值。

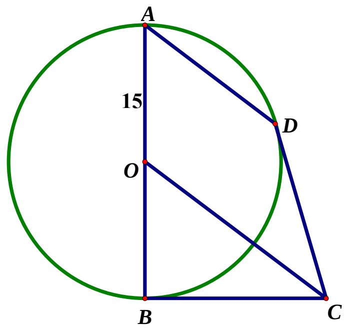
Given that x is a real number, find the least value of $\sqrt{x(x+3)(x+6)(x+9)+2017}$.

12. 已知 $\frac{x}{x^2-5x+1}=\frac{1}{2}$，求 $\frac{x^2}{x^4-5x^2+1}$ 的值。

Given $\frac{x}{x^2-5x+1}=\frac{1}{2}$, find the value of $\frac{x^2}{x^4-5x^2+1}$.

13. 如圖二，O 是圓 ADB 的圓心。BC 及 CD 分別是圓形在點 B 及 D 的切綫。OC // AD，$OA=15$。若 $AD+OC=43$，求 CD 的長。

As shown in Figure 2, O is the centre of the circle ADB. BC and CD are tangents to the circle at points B and D respectively. OC // AD, $OA = 15$. If $AD + OC = 43$, find the length of CD.



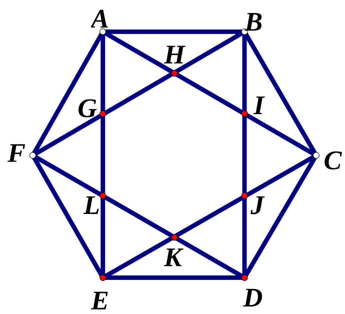
圖二 Figure 2

14. 若 $a+\log_2 b = a^2 + \log_2 b^3 - 10 = 3$，其中 $b > 1$，求 b 的值。

If $a+\log_2 b = a^2 + \log_2 b^3 - 10 = 3$, where $b > 1$, find the value of b.

15. 在圖三中，已知 $ABCDEF$ 為正六邊形，且它的面積是 $90\sqrt{3}$，求 GJ 的值。

In Figure 3, given that $ABCDEF$ is a regular hexagon and its area is $90\sqrt{3}$, find the length of GJ.



圖三 Figure 3

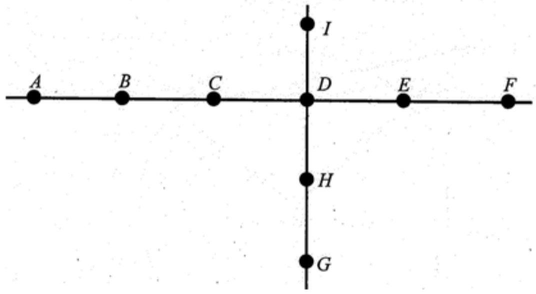
***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2016-2017
Heat Event (Group)
香港數學競賽 2016 – 2017
初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘 Time allowed: 20 minutes
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得兩分。Each correct answer will be awarded 2 marks.
全卷滿分 20 分。The maximum mark for this paper is 20.

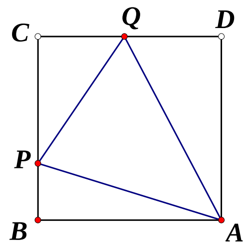
1. 設 ΔABC 為一等腰直角三角形，頂點 A 及 B 的座標分別為 $(-2, 0)$ 及 $(18, 0)$，且 C 的座標是正數。當 ΔABC 的面積為最小時，求 C 的座標。
   Suppose that ΔABC is an isosceles right-angled triangle with the coordinates of the vertices A and B as $(-2, 0)$ and $(18, 0)$, respectively, and the coordinates of C having positive values. Determine the coordinates of C when the area of ΔABC attains its minimum.

2. 如圖一所示，點 A、B、C、D、E 及 F 均在一直綫上。點 G、H、D 及 I 在另一直綫上。揀選三點，可形成多少個三角形？
   As shown in Figure 1, points A, B, C, D, E and F lie on the same straight line, and G, H, D and I lie on another straight line. How many triangles can be made by connecting any three points?



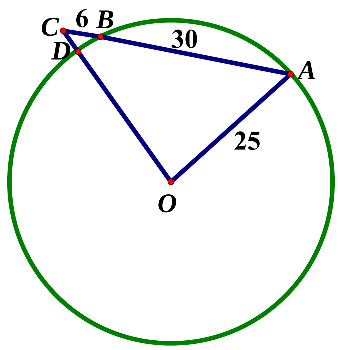
圖一 Figure 1

3. 如圖二所示，P、Q 分別是正方形 $ABCD$ 的邊 BC 及 CD 上的點。已知 ΔPCQ 的周界的長等於正方形 $ABCD$ 的周界的長的 $\frac{1}{2}$，求 $\angle PAQ$ 的值。
   As shown in Figure 2, P, Q are points on the sides BC and CD of a square $ABCD$. Given that the perimeter of ΔPCQ is $\frac{1}{2}$ of that of the square $ABCD$, find the value $\angle PAQ$.



圖二 Figure 2

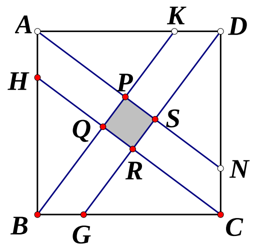
4. 在圖三中，O 是圓心。弦 AB 及半徑 OD 的延綫相交於 C。已知 $OA = 25$、$AB = 30$ 及 $BC = 6$。求 CD 的長。
   In Figure 3, O is the centre of the circle. Chord AB and radius OD are produced to meet at C. Given that $OA = 25$, $AB = 30$ and $BC = 6$, find the length of CD.



圖三 Figure 3

5. 設 Q 為所有能滿足不等式 $\frac{9p^2}{\left(\sqrt{3p+1}-1\right)^2} < 3p+10$ 的整數 p 之和，求 Q 的值。
   Let Q be the sum of all integers p satisfying the inequality $\frac{9p^2}{\left(\sqrt{3p+1}-1\right)^2} < 3p+10$, find the value of Q.

6. 在圖四中，正方形 $ABCD$ 的邊長為 20。已知 $DK : KA = AH : HB = 1 : 3$ 及 $BK \,//\, GD$，$HC \,//\, AN$，求陰影部分 $PQRS$ 的面積。
In Figure 4, square $ABCD$ has sides of length 20 .
Given that $DK : KA = AH : HB = 1 : 3$ and $BK \,//\, GD$, $HC \,//\, AN$,
find the area of shaded region $PQRS$.



圖四 Figure 4

7. 已知對於實數 x_1、x_2、x_3、⋯、x_{2017}，

$$\sqrt{x_1 - 1} + \sqrt{x_2 - 1} + \sqrt{x_3 - 1} + \cdots + \sqrt{x_{2017} - 1} = \frac{1}{2}(x_1 + x_2 + x_3 + \cdots + x_{2017})，$$

求 $x_1 + x_2 + x_3 + \cdots + x_{2017}$ 的值。

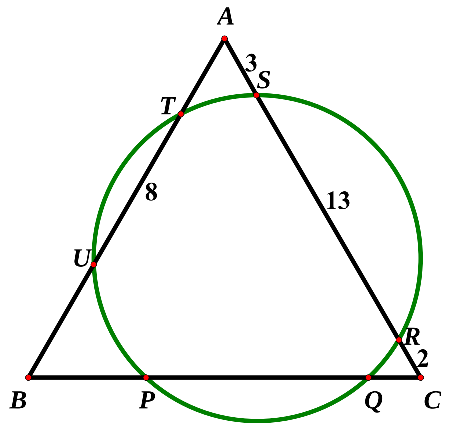
It is given that for real numbers $x_1, x_2, x_3, \cdots, x_{2017}$,

$$\sqrt{x_1 - 1} + \sqrt{x_2 - 1} + \sqrt{x_3 - 1} + \cdots + \sqrt{x_{2017} - 1} = \frac{1}{2}(x_1 + x_2 + x_3 + \cdots + x_{2017})，$$

Find the value of $x_1 + x_2 + x_3 + \cdots + x_{2017}$.

8. 設正整數 T 能滿足條件：T 的數字的積是 $T^2 - 11T - 23$。求該等正整數之和，S 的值。
Let positive integers, T, satisfy the condition: the product of the digits of T is $T^2 - 11T - 23$.
Find the sum S, of all such positive integers .

9. 在圖五中，ABC 是一個等邊三角形且與一圓相交於六點：P、Q、R、S、T 及 U。若 $AS = 3$，$SR = 13$，$RC = 2$ 及 $UT = 8$，求 $BP - QC$ 的值。
In Figure 5, ABC is an equilateral triangle intersecting the circle at six points P, Q, R, S, T and U. If $AS = 3$, $SR = 13$, $RC = 2$ and $UT = 8$, find the value of $BP - QC$.



圖五 Figure 5

10. 已知方程 $a^2x^2 - (4a - 3a^2)x + 2a^2 - a - 21 = 0$ (其中 $a > 0$) 最少有一個整數根，求所有 a 的可能整數值之和。
It is given that the equation $a^2x^2 - (4a - 3a^2)x + 2a^2 - a - 21 = 0$ (where $a > 0$) has at least one integral root. Find the sum of all possible integral values of a .

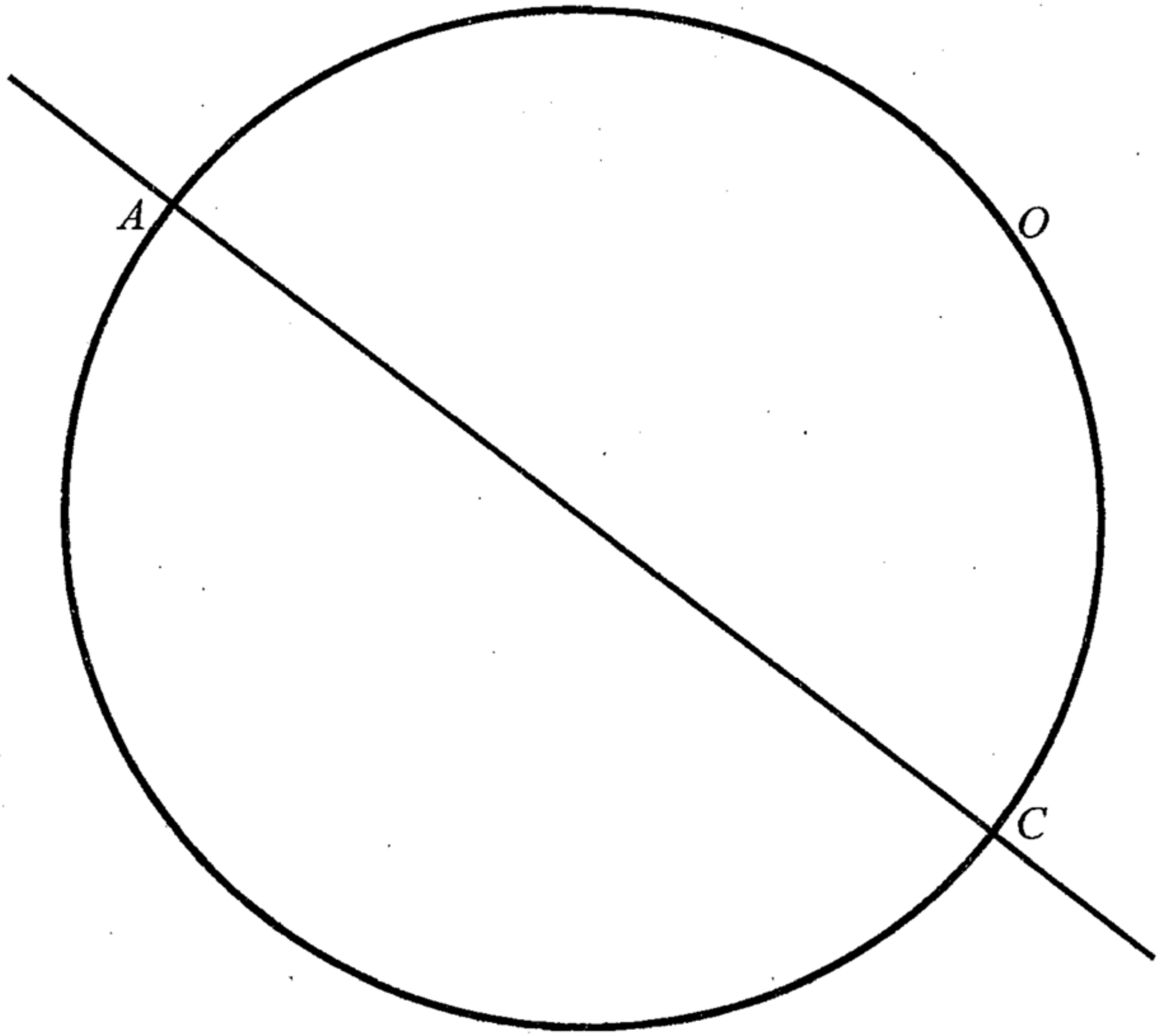
***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2016 – 2017
Heat Event (Geometric Construction)
香港數學競賽 2016 – 2017
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為二十分。The full marks of this part is 20 marks. Time allowed: 20 minutes

School ID: ____
School Name: ____

第一題 Question No. 1
如下圖，已知一圓 O 的其中一條直徑為 AC。
求作圓上兩點 B、D 使得 $ABCD$ 成為一個正方形。
As shown in the figure below, given that O is a circle with a diameter AC.
Construct two points B, D on the circle such that $ABCD$ form a square.



Hong Kong Mathematics Olympiad 2016 – 2017
Heat Event (Geometric Construction)
香港數學競賽 2016 – 2017
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為二十分。The full marks of this part is 20 marks. Time allowed: 20 minutes

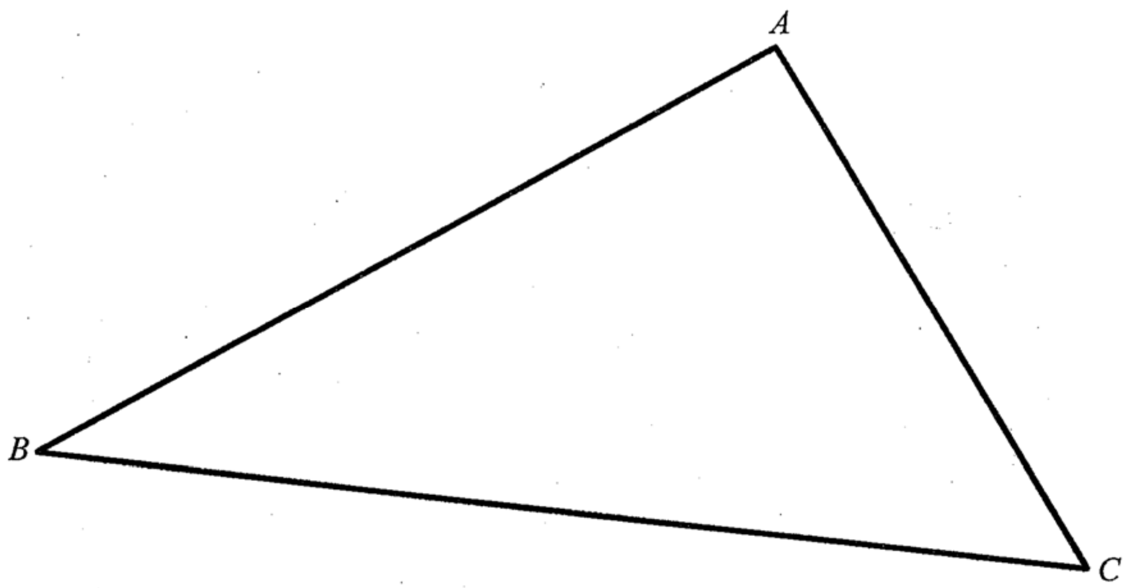
School ID: ____
School Name: ____

第二題 Question No. 2

已知ΔABC，如下圖所示。

求作一點 M，使得 MA、MB 及 MC 三條綫段將ΔABC 的面積三等分。

Given ΔABC as shown in the figure below. Construct a point M such that the line segments MA, MB, MC will divide the area of ΔABC into 3 equal parts.



每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為二十分。The full marks of this part is 20 marks. Time allowed: 20 minutes

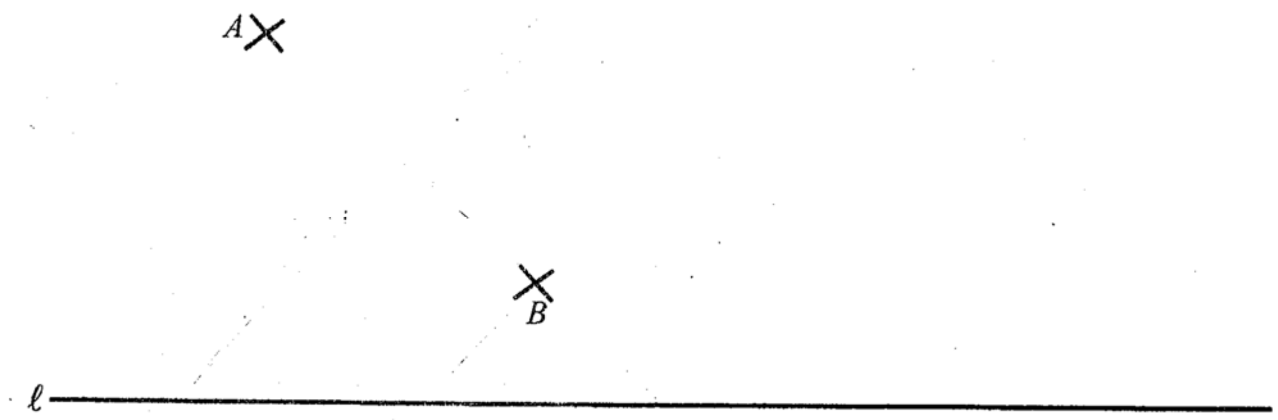
School ID: ____________________

School Name: ____________________

第三題 Question No. 3

已知 A、B 兩點和直綫 ℓ，如下圖所示。求作一圓過 A、B 兩點且與 ℓ 相切。

Given two points A, B and a straight line ℓ as shown in the figure below. Construct a circle which passes through A and B, and is tangent to the straight line ℓ.



*** 試卷完 End of Paper ***

參加學校數目：214
初賽日期：2017 年 2 月 18 日星期六
試場一(HK1)：中華傳道會劉永生中學 (39 隊)
試場二(KLN1)：香海正覺蓮社佛教正覺中學 (52 隊)
試場三(KLN2)：佛教善德英文中學(49 隊)
試場四(NT1)：保良局朱敬文中學(37 隊)
試場四(NT2)：順德聯誼總會翁祐中學 (37 隊)

Regional winners of the Heat Event

Hong Kong Island Region

Position	**School Name**
Winner	Queen's College
1st Runner-up	St Paul's Co-Educational College
2nd Runner-up	Hong Kong Chinese Women's Club College
4th Place	Wah Yan College, Hong Kong

Kowloon Region 1

Position	**School Name**
Winner	La Salle College
1st Runner-up	Sing Ying Secondary School
2nd Runner-up	Diocesan Boys' School
4th Place	G.T. (Ellen Yeung) College
5th Place	PLK Celine Ho Yam Tong College

Kowloon Region 2

Position	**School Name**
Winner	Pui Ching Middle School
1st Runner-up	STFA Lee Shau Kee College
2nd Runner-up	Ying Wa College
4th Place	Queen Elizabeth School
5th Place	Tsuen Wan Government Secondary School

New Territories Region 1

Position	**School Name**
Winner	Baptist Lui Ming Choi Secondary School
1st Runner-up	Sha Tin Government Secondary School
2nd Runner-up	HKTA Tang Hin Memorial Secondary School
4th Place	SKH Tsang Shiu Tim Secondary School

New Territories Region 2

Position	**School Name**
Winner	Chiu Lut Sau Memorial Secondary College
1st Runner-up	Christian Alliance S.C. Chan Memorial College
2nd Runner-up	NTHYK Yuen Long District Secondary School
4th Place	PLK Centenary Li Shiu Chung Memorial College

School ID	Name of School school = new school entering final event this year
FE-01	Baptist Lui Ming Choi Secondary School
FE-02	Buddhist Sin Tak College
FE-03	Cheung Chuk Shan College
FE-04	Chinese Foundation Secondary School
FE-05	Chiu Lut Sau Memorial Secondary School
FE-06	Christian Alliance S.C. Chan Memorial College
FE-07	Diocesan Boys' School
FE-08	Fukien Secondary School
FE-09	G.T. (Ellen Yeung) College
FE-10	Good Hope School
FE-11	HKTA Tang Hin Memorial Secondary School
FE-12	Hoi Ping Chamber of Commerce Secondary School
FE-13	Hong Kong Chinese Women's Club College
FE-14	Hong Kong International School
FE-15	Kiangsu-Chekiang College (Kwai Chung)
FE-16	Kiangsu-Chekiang College (Shatin)
FE-17	Kwun Tong Government Secondary School
FE-18	La Salle College
FE-19	Maryknoll Convent School (Secondary Section)
FE-20	Munsang College
FE-21	Munsang College (Hong Kong Island)
FE-22	NTHYK Yuen Long District Secondary School
FE-23	PLK Celine Ho Yam Tong College
FE-24	PLK Mrs Ma Kam Tong College
FE-25	PLK No. 1 WH Cheung College
FE-26	PLK Tang Yuk Tien College
FE-27	Po Leung Kuk Centenary Li Shiu Chung Memorial College
FE-28	Pui Ching Middle School
FE-29	Pui Kiu College
FE-30	Queen Elizabeth School
FE-31	Queen's College
FE-32	Sha Tin Government Secondary School
FE-33	Sha Tin Methodist College
FE-34	Shatin Tsung Tsin Secondary School
FE-35	Shung Tak Catholic English College
FE-36	Sing Yin Secondary School
FE-37	SKH Bishop Mok Sau Tseng Secondary School
FE-38	SKH Lam Woo Memorial Secondary School
FE-39	SKH Tsang Shiu Tim Secondary School
FE-40	St Paul's Co-Educational College
FE-41	STFA Lee Shau Kee College
FE-42	The ELCHK Yuen Long Lutheran Secondary School
FE-43	Tsuen Wan Government Secondary School
FE-44	Tuen Mun Catholic Secondary School
FE-45	TWGH Chen Zao Men College
FE-46	TWGH Mrs Wu York Yu Memorial College
FE-47	TWGH Kap Yan Directors' College
FE-48	Wah Yan College, Hong Kong
FE-49	Wah Yan College, Kowloon
FE-50	Ying Wa College