**Discriminant** (HKMO Classified Questions by topics)

#### 1986 FI2.1

若方程  $3x^2 - 4x + \frac{h}{3} = 0$  有等根,求 h 的值。

If the equation  $3x^2 - 4x + \frac{h}{3} = 0$  has equal roots, find the value of h.

### 1987 FI4.1

若曲線  $y = 2x^2 - 8x + a$  與 x-軸相切,求 a 的值。

If the curve  $y = 2x^2 - 8x + a$  touches the x-axis, find the value of a.

### 1989 FI4.4

已知曲線 $y = 3x^2 + 12x + c$ 與x-軸相切,求c的值。

If the curve  $y = 3x^2 + 12x + c$  touches the x-axis, find the value of c.

### 1989 FSG.1

已知  $2at^2 + 12t + 9 = 0$  有等根,求 a 的值。

If  $2at^2 + 12t + 9 = 0$  has equal roots, find the value of a.

### 1990 FI3.4

若  $64t^2 + 16t + d$  是完全平方,求 d 的值。

If  $64t^2 + 16t + d$  is a perfect square, find the value of d.

# 1991 FG7.2

某方程 $x^2 + 2x + c = 0$ 無實根,且c為小於3之整數,求c的值。

If the equation  $x^2 + 2x + c = 0$  has no real root and c is an integer less than 3, find the value of c.

### 1992 FI1.2

若  $x^2 - 10x + a = 0$  有兩等根,求 a 的值。

If  $x^2 - 10x + a = 0$  has 2 equal roots, find the value of a.

### 1992 FG10.4

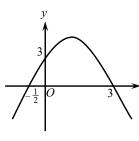
下圖為  $y = -2x^2 + 5x + 3$  的圖形。

若 y = x + d 為  $y = -2x^2 + 5x + 3$ .的切線,求 d 的值。

The following shows the graph of  $y = -2x^2 + 5x + 3$ .

If y = x + d is tangent to  $y = -2x^2 + 5x + 3$ ,

find the value of d.



### 1994 FI1.1

方程式 $x^2 - ax + (a+3) = 0$ 有等根。若a為一正整數,求a的值。

The equation  $x^2 - ax + (a + 3) = 0$  has equal roots.

Find the value of a, if a is a positive integer.

### 1994 FG7.3

若  $c=2-x+2\sqrt{x-1}$  且 x>1, 求 c 之最大值。

Find the largest value of c, if  $c = 2 - x + 2\sqrt{x-1}$  and x > 1.

### 1996 FG9.2

 $\ddot{x} (x^2 + y^2)^2 \le b(x^4 + y^4)$  對任意實數 x 和 y 都成立,求 b 的最小可能整數值。 If the expression  $(x^2 + y^2)^2 \le b(x^4 + y^4)$  holds for all real values of x and y, find the least possible integral value of b.

#### 1997 FI2.3

若對於所有實數 x ,  $x^2 + cx + 36$  不小於 0 , 求 c 的最大值。 If the value of  $x^2 + cx + 36$  is not less than 0 for all real number x, find the maximum value of c.

#### 1999 FI4.3

設 c 為一正實數,若  $x^2 + 2\sqrt{c}x + 2 = 0$  僅有一實數解,求 c 之值。

Let *c* be a positive real number.

If  $x^2 + 2\sqrt{c}x + 2 = 0$  has one real root only, find the value of c.

### 1999 FI5.3

若方程 $x^2-x+1=0$ 有c個實數解,求c之值。

If the number of real roots of the equation  $x^2 - x + 1 = 0$  is c, find the value of c.

## 1999 FG5.2 2013 HG6

已知方程式  $x^2 + ax + 2b = 0$  及  $x^2 + 2bx + a = 0$  的根為實數,且 a, b > 0。 若 a + b 的最小值為 Q,求 Q 之值。

Given that the roots of  $x^2 + ax + 2b = 0$  and  $x^2 + 2bx + a = 0$  are both real and a, b > 0. If the minimum value of a + b is Q, find the value of Q.

### 2000 FI5.2

如果 (x-2)(x-2Q)-1=0 有兩個整數根,求Q的值。

If (x-2)(x-2Q) - 1 = 0 has two integral roots, find the value of Q.

### 2000 FG2.4

設  $f(x) = 41x^2 - 4x + 4$ ,  $g(x) = -2x^2 + x$ 。如果 f(x) + kg(x) = 0 只有一個根, 求 k 的最小值 d。

Let  $f(x) = 41x^2 - 4x + 4$  and  $g(x) = -2x^2 + x$ . If d is the smallest value of k such that f(x) + kg(x) = 0 has a single root, find the value of d.

### 2001 FI2.1

若 P 為整數,及 5 < P < 20。

若方程  $x^2-2(2P-3)x+4P^2-14P+8=0$  的兩個根皆為整數,求 P 的值。 Suppose P is an integer and 5 < P < 20. If the roots of the equation  $x^2-2(2P-3)x+4P^2-14P+8=0$  are integers, find the value of P.

Discriminant (HKMO Classified Questions by topics)

#### 2002 FI2.3

已知
$$x$$
及 $R$ 為實數。若對所有 $x$ , $\frac{2x^2+2Rx+R}{4x^2+6x+3} \le 1$ ,求 $R$ 的最大值。

Given that x and R are real numbers and  $\frac{2x^2 + 2Rx + R}{4x^2 + 6x + 3} \le 1$  for all x,

find the maximum value of R.

### 2002 FG1.1

假設曲綫 
$$x^2 + 3y^2 = 12$$
 及直綫  $mx + y = 16$  只相交於一點。

Assume that the curve  $x^2 + 3y^2 = 12$  and the straight line mx + y = 16 intersect at only one point. If  $a = m^2$ , find the value of a.

### 2002 FG3.2

已知
$$x$$
和 $y$ 為兩實數且滿足關係  $y = \frac{x}{2x-1}$ 。

若 
$$\frac{1}{x^2} + \frac{1}{y^2}$$
 的最小值為  $b$  ,求  $b$  的值。

It is given that the real numbers x and y satisfy the relation  $y = \frac{x}{2x-1}$ .

If the minimum value of  $\frac{1}{x^2} + \frac{1}{y^2}$  is b, find the value of b.

### 2003 HG5

對任意實數 a ,方程  $x^2 + ax + 3b - a + 2 = 0$  都有實數解 ,求 b 的最大值。 If the equation  $x^2 + ax + 3b - a + 2 = 0$  has real root(s) for any real number a , find the maximum value of b .

# 2009 FI4.4

設 
$$x$$
 及  $y$  為實數並滿足方程  $(x-3)^2 + (y-3)^2 = 1$ 。

Let x and y be real numbers satisfying the equation  $(x-3)^2 + (y-3)^2 = 1$ .

If  $k = \frac{y}{x-3}$  and q is the least possible values of  $k^2$ , find the value of q.

### 2010 HG9

若 
$$\log_4(x+2y) + \log_4(x-2y) = 1$$
, 求  $|x| - |y|$  的最小值。

If 
$$\log_4(x+2y) + \log_4(x-2y) = 1$$
, find the minimum value of  $|x| - |y|$ .

#### 2010 FI2.2

若 b 及 h 為正整數,且滿足 b < h 及  $b^2 + h^2 = b(2+h) + 2h$ ,求 b 的值。

If b and h are positive integers with b < h and  $b^2 + h^2 = b(2 + h) + 2h$ , find the value of b.

### 2010 FIS.3

已知 
$$c$$
 為  $f(x) = \frac{x^2 - 2x - 3}{2x^2 + 2x + 1}$  的最小值。求  $c$  的值。

Given that *c* is the minimum value of  $f(x) = \frac{x^2 - 2x - 3}{2x^2 + 2x + 1}$ . Find the value of *c*.

#### 2011 FI3.1

若 P 為一質數,而且方程  $x^2+2(P+1)x+P^2-P-14=0$  的根為整數,求 P 的最小值。 If P is a prime number and the roots of the equation  $x^2+2(P+1)x+P^2-P-14=0$  are integers, find the least value of P.

#### 2013 HG1

已知一個直角三角形三邊的長度皆為整數,且其中兩邊的長度為方程 $x^2-(m+2)x+4m=0$ 的根。求第三邊長度的最大值。

Given that the length of the sides of a right-angled triangle are integers, and two of them are the roots of the equation  $x^2 - (m+2)x + 4m = 0$ .

Find the maximum length of the third side of the triangle.

#### 2013 FI3.4

若 
$$x$$
 為實數及  $d$  為函數  $y = \frac{3x^2 + 3x + 4}{x^2 + x + 1}$  的最大值,求  $d$  的值。

If x is a real number and d is the maximum value of the function  $y = \frac{3x^2 + 3x + 4}{x^2 + x + 1}$ ,

find the value of d.

# 2013 FG2.3

設 
$$f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$$
,  $x$  為實數且  $f(x)$  的最大值和最小值分別是  $\frac{1}{2}$  和  $-1$ 。

若 
$$t = f(0)$$
, 求  $t$  的值。

Let 
$$f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$$
, where x is a real number and the maximum value of  $f(x)$  is  $\frac{1}{2}$  and the

minimum value of f(x) is -1. If t = f(0), find the value of t.

#### 2016 FG3.4

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  $\stackrel{.}{z}$   $\stackrel{.$ 

If a, b and y are real numbers and satisfy
$$\begin{cases}
a + b + y = 5 \\
ab + by + ay = 3
\end{cases}$$

determine the greatest possible value of y.

#### 2017 FI4.1

**2023 HI3** 設 m 為一個整數常數,其中 4 < m < 40。若方程  $x^2 - 2(2m-3)x + 4m^2 - 14m + 8 = 0$  有兩個整數根,求 x 的最大可能值。 Let m be an integral constant, where 4 < m < 40. If the equation  $x^2 - 2(2m-3)x + 4m^2 - 14m + 8 = 0$  has two integral roots, find the largest possible value of x.

### Answers

Allowels				
1986 FI2.1	1987 FI4.1	1989 FI4.4	1989 FSG.1	1990 FI3.4
4	8	12	2	1
1991 FG7.2	1992 FI1.2	1992 FG10.4	1994 FI1.1	1994 FG7.3
2	25	5	6	2
1996 FG9.2	1997 FI2.3	1999 FI4.3	1999 FI5.3	1999 FG5.2 2013 HG6
2	12	2	0	6
2000 FI5.2	2000 FG2.4	2001 FI2.1	2002 FI2.3	2002 FG1.1
1	<del>-4</del> 0	12	3	21
2002 FG3.2	2003 HG5	2009 FI4.4	2010 HG9	2010 FI2.2
2	-1	8	$\sqrt{3}$	2
2010 EIG 2	2011 EI2 1	2012 HC1	2013 FI3.4	2013 FG2.3
2010 FIS.3	2011 FI3.1	2013 HG1	13	1
<del>-4</del>	/	13	3	$-{2}$
2016 FG3.4	2017 FI4.1	2023 HI3		
13	201 / F14.1			
3	2	52		
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