III Roots increased by a given constant k .

Theory Let α be a root of a polynomial equation $f(x) = 0$, then $\alpha + h$ is a root of a polynomial equation $f(y - h) = 0$.

That is to say, change $x \to y - h$

Class Work III.1

Given $x^3 + 6x^2 + 2x + 1 = 0$. Transform it into a new equation in y for which the coefficient of y^2 is 0 .

Change $x \to y - h$, then

$(\qquad\qquad)^3 + 6(\qquad\qquad)^2 + 2(\qquad\qquad) + 1 = 0$

$y^3 - (\qquad\qquad)y^2 + (3h^2 - 12h + 2)y - h^3 + 6h^2 - 2h + 1 = 0$

Let ____________ $= 0 \Rightarrow h = 2$

The new equation is: ____________________ $+ 13 = 0$

Class Work III.2

If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$ ……(*)

(a) Prove that $\alpha^2 - \beta\gamma = -(p\alpha + q)$

Proof: If $\alpha \neq 0$, $\alpha^2 - \beta\gamma = \dfrac{1}{\alpha}(\alpha^3 - \alpha\beta\gamma) \cdots \cdots (1)$

$\alpha\beta\gamma$ = product of roots = ______

Put $x = \alpha$ in (*) $\Rightarrow \alpha^3 =$ _______

Sub. into (1) $\Rightarrow \alpha^2 - \beta\gamma =$ ________ = __________

If $\alpha = 0$, $\alpha\beta + \beta\gamma + \alpha\gamma = q$

$\beta\gamma = q$

$\alpha^2 - \beta\gamma = -\beta\gamma = -q$

$\therefore \alpha^2 - \beta\gamma = -(p\alpha + q)$

(b) If $p \neq 0$, construct a cubic equation whose roots are $\alpha^2 - \beta\gamma$, $\beta^2 - \gamma\alpha$, $\gamma^2 - \alpha\beta$.

The new roots are $-(p\alpha + q)$, $-(p\beta + q)$, $-(p\gamma + q)$

New equation is $(x + p\alpha + q)(\qquad\qquad\qquad)(\qquad\qquad\qquad) = 0$

Change old equation by $x \to \dfrac{-y-q}{p}$ (Note: $x = -\dfrac{y}{p} - q$ is wrong!)

Exercise: Substitute $x = y + 1$ into $x^4 - 4x^3 - 2x^2 + 12x + 8 = 0$ and hence solve it.

[Ans. $1 \pm \sqrt{3}$, $1 \pm \sqrt{5}$]