Q2 In ΔABC, D is the mid point of AB, E and F are points on AC and BC respectively. Prove that $S_{\Delta DEF} \le S_{\Delta ADE} + S_{\Delta BDF}$.

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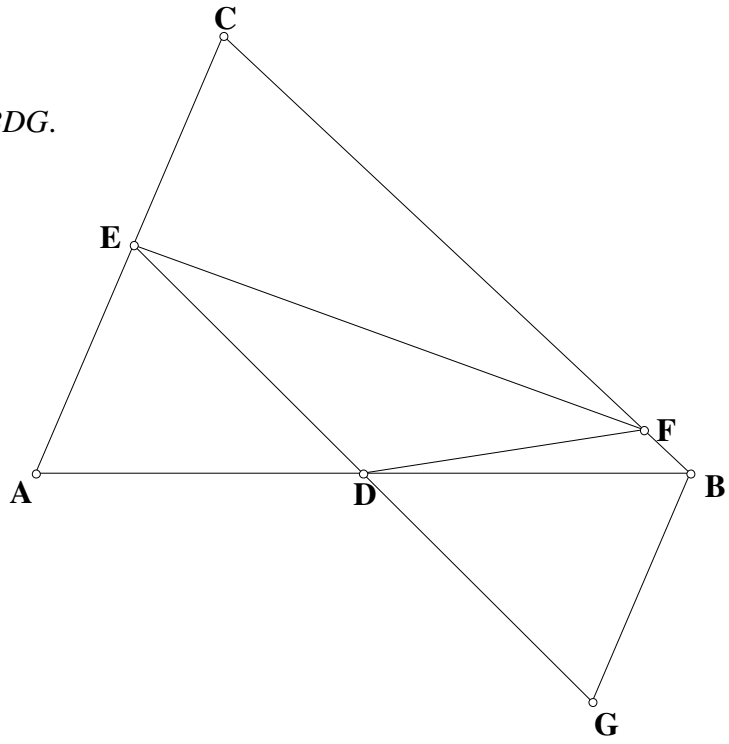

Rotate ΔADE about D by 180° so that $\Delta ADE \cong \Delta BDG$.

Then $\angle ADE = \angle BDG$ (corr. $\angle \cong \Delta$s)

EDG is a straight line. (converse, vert. opp. $\angle$s)

$ED = DG$ (corr. sides $\cong \Delta$s)

$$S_{\Delta ADE} + S_{\Delta BDF} = S_{\Delta BDG} + S_{\Delta BDF}$$
$$= S_{BGDF}$$
$$\ge S_{\Delta DGF}$$
$$= S_{\Delta DEF} \text{ (equal base, same height)}$$