Factor (HKMO Classified Questions by topics)

#### 1985 FG7.2

正整數 N 分別被 6、5、4、3 及 2 除時,其餘數依次為 5、4、3、2 及 1。求 給出整數 a、b、c,使得  $a^2 = b^3 = c$ 。若 c > 1,求 c 的最小值。 N之最小值。

When the positive integer N is divided by 6, 5, 4, 3 and 2,

the remainders are 5, 4, 3, 2 and 1 respectively. Find the least value of N.

## 1985 FG8.1-2

 $M \cdot N$  均為小於 10 之正整數,且 258024M8 × 9 = 2111110N × 11。 求M及N的值。

M, N are positive integers less than 10 and  $258024M8 \times 9 = 2111110N \times 11$ . Find the value of M and N.

#### 1988 FG7.1-2

 $M \cdot N$  是小於 10 的正整數,且  $8M420852 \times 9 = N9889788 \times 11$ 。 求M及N的值。

M, N are positive integers less than 10 and  $8M420852 \times 9 = N9889788 \times 11$ . Find the value of M and N.

### 1990 HI13 2013FG4.3

正整數 N 被 10、9、8、7、6、5、4、3 及 2 除所得的餘數依次是 <math>9、8、 7、6、5、4、3、2及1, 求 N 的最小值。

A positive integer N, when divided by 10, 9, 8, 7, 6, 5, 4, 3 and 2, leaves 如果 a 是一個三位數, 駁在 504 之後, 新組成的六位數可被 7、9、11 整除, remainders 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively. Find the least value of N.

# 1990 FG7.3-4

 $A \cdot B$  均為小於 10 的正整數,且  $21A104 \times 11 = 2B8016 \times 9$ 。求  $A \otimes B$  的 值。

A, B are positive integers less than 10 such that  $21A104 \times 11 = 2B8016 \times 9$ . Find the value of A and B.

## 1995 FG6.1-2

 $2^{a} \cdot 9^{b}$  為一四位數,其千位數是 2,百位數是 a,十位數是 9,個位數是 b, 求a及b的值。

 $2^a \cdot 9^b$  is a four digit number and its thousands digit is 2, its hundreds digit is a, its tens digit is 9 and its units digit is b, find the value of a and b.

# 1996 FG6.4

若 r, s, t, u 是正整數及  $r^5 = s^4, t^3 = u^2, t - r = 19$  及 d = u - s , 求 d 的值。 If r, s, t, u are positive integers and  $r^5 = s^4$ ,  $t^3 = u^2$ , t - r = 19 and d = u - s, find the value of d.

#### 1998 HG4

Let a, b, c be integers such that  $a^2 = b^3 = c$ . If c > 1, find the smallest value of c.

#### 1999 FI4.1

李先生今年a歲,a<100。若把李先生的出生月份與a相乘,其結果是 253。 求a的值。

Mr. Lee is a years old, a < 100.

If the product of a and his month of birth is 253, find the value of a.

#### 1999 FIS.3

W和F為兩大於20的整數。

若W與F之積為770,W與F之和為c,求c之值。

W and F are two integers which are greater than 20. If the product of W and F is 770 and the sum of W and F is c, find the value of c.

### 1999 FG3.1

設小於 100 的正整數,同時又是完全平方及完全立方的數目共有 a 個, 求a之值。

Let a be the number of positive integers less than 100 such that they are both square and cubic numbers, find the value of a.

### 2000 FG4.1

求a的值。

Let a be a 3-digit number. If the 6-digit number formed by putting a at the end of the number 504 is divisible by 7, 9, and 11, find the value of a.

# 2000 FG5.1

如果a是可被810整除的最小立方數,求a的值。

If a is the smallest cubic number divisible by 810, find the value of a.

## 2001 FI2.3

某班學生的人數少於 56 人。在一次數學測驗中有  $\frac{1}{3}$  學生得甲等, $\frac{1}{7}$  學 生得乙等,一半學生得丙等,餘下的學生都不及格。已知不及格的學生人數

是 R, 求 R 的值。

There are less than 56 students in a class. In a mathematics test,  $\frac{1}{2}$  of the students got grade A,  $\frac{1}{7}$  of the students got grade B, half of the students got grade C, and the rest failed. Given that R students failed in the mathematics test, find the value of R.

Factor (HKMO Classified Questions by topics)

#### 2002 HI2

已知  $a^3 = 150b$ ,且 a 和 b 都是正整數。求 b 的最小值。

Given  $a^3 = 150b$  and a, b are positive integers, find the least value of b.

### 2002 HG2

已知  $a \cdot b \cdot c$  為三個連續奇數且  $b^3 = 3375$  , 求 ac 的數值。

Given that a, b, c are three consecutive odd numbers and  $b^3 = 3375$ , find the value of ac.

#### 2003 HI9

已知六位數  $N=\overline{x1527y}$  是 4 的倍數,且 N 被 11 除餘 5。求 x+y 的值。

It is given that the 6-digit number  $N = x\overline{1527y}$  is a multiple of 4, and the remainder is 5 when N is divided by 11. Find the value of x + y.

#### 2004 FI1.1

已知有 a 個少於 200 的正整數,它們每個都只有三個正因數,求 a 的值。

Given that there are a positive integers less than 200 and each of them has exactly three positive factors, find the value of a.

#### 2005 FG1.1

若在1至200內能同時被3和7整除的數有a個,求a的值。

Suppose there are a numbers between 1 and 200 that can be divisible by 3 and 7, find the value of a.

### 2006 HI7

已知在數列 1001,1001001,1001001001,..., $1\underline{001001}$ ... $1\underline{001}$ ,... 中有 R 個質數,

求 R 的值。

Given that the number of prime numbers in the sequence 1001, 1001001, 1001001001,  $\cdots$ ,  $1\underbrace{00}_{2}1\underbrace{00}_{2}1\cdots1\underbrace{00}_{2}1$ ,  $\cdots$  is R, find the value of R.

### 2007 HG1

若由  $1 \le 50$  內與 50 互質的整數有 N 個,求 N 的值。

If there are N integers from 1 to 50 that are relatively prime to 50, find the value of N.

## 2007 FI3.3

若有 c 個正整數 n 使得  $\frac{n+17}{n-7}$  也是正整數, 求 c 的值。

If there is (are) c positive integer(s) n such that  $\frac{n+17}{n-7}$  is also a positive integer,

find the value of c.

## 2008 HG2

有一批糖共x 粒,x 為正整數,這批糖能分別為 851 人及 943 人所均分。 求x 的最小可能值。

A total number of x candies, x is a positive integer, can be evenly distributed to 851 people as well as 943 people. Find the least possible value of x.

### 2008 HG4

已知 x 為正整數及 x < 60。若 x 恰有 10 個正因子,求 x 的值。

Given that x is a positive integer and x < 60. If x has exactly 10 positive factors, find the value of x.

### 2008 FIS.1

若 28 的所有正因子是  $d_1, d_2, \ldots, d_n$  及  $a = \frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_n}$  , 求 a 的值。

If all the positive factors of 28 are  $d_1, d_2, \ldots, d_n$  and  $a = \frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_n}$ ,

find the value of a.

### 2010 HG1

已知六位數 503xyz 可以被 7,9,11 整除。求三位數 xyz 的最小值。 Given that the six-digit number 503xyz is divisible by 7,9,11.

Find the minimum value of the three-digit number xyz.

### 2010 FI3.1

若 a 為 15147 的相異質因數的數目。求 a 的值。

If a is the number of distinct prime factors of 15147, find the value of a.

## 2011 HG1

 $若(1000-a)(1000-b)(1000-c)(1000-d)(1000-e) = 24^2$ ,其中  $a \cdot b \cdot c \cdot d$  及 e 為偶數,且 a > b > c > d > e,求  $a \cdot b \cdot c \cdot d$  及 e 的值。

If  $(1000 - a)(1000 - b)(1000 - c)(1000 - d)(1000 - e) = 24^2$ , where a, b, c, d and e are even numbers and a > b > c > d > e, find the values of a, b, c, d and e.

## 2011 FG3.1

若 a 為一正整數及  $a^2 + 100a$  為一質數,求 a 的最大值。

If a is a positive integer and  $a^2 + 100a$  is a prime number, find the maximum value of a.

## 2011 FG3.4

把數字 1, 2, ..., 10 分成兩組並設  $P_1$  及  $P_2$  分別為該兩組的乘積。

$$\stackrel{}{ ext{$ z$}} P_1 \stackrel{}{ ext{$ a$}} P_2$$
的倍數,求 $\frac{P_1}{P_2}$ 的最小值。

Spilt the numbers 1, 2, ..., 10 into two groups and let  $P_1$  be the product of the first group and  $P_2$  the product of the second group.

If  $P_1$  is a multiple of  $P_2$ , find the minimum value of  $\frac{P_1}{P_2}$ .

#### 2013 HI2

一個平行四邊形可被分成 178 個邊長為 1 單位的等邊三角形,若該平行四邊形的周界為 P 單位,求 P 的最大值。

A parallelogram is cut into 178 pieces of equilateral triangles with sides 1 unit. If the perimeter of the parallelogram is P units, find the maximum value of P.

## 2013 FG2.1

### 2014 HI6

設 n 為正整數,且 n < 1000。若  $(n-1)^2$  整除  $(n^{2014}-1)$ ,求 n 的最大值。 Let n be a positive integer and n < 1000.

If  $(n^{2014} - 1)$  is divisible by  $(n - 1)^2$ , find the maximum value of n.

## **2014 HI8**

設  $\overline{xy} = 10x + y$ 。若  $\overline{xy} + yx$  為一個平方數,這樣的數有多少個? Let  $\overline{xy} = 10x + y$ .

If  $\overline{xy} + \overline{yx}$  is a square number, how many numbers of this kind exist?

## 2015 HI3

設正 n 邊形的內角為  $x^{\circ}$ ,其中 x 為整數。問 n 有多少個可能值? Let  $x^{\circ}$  be the measure of an interior angle of an n-sided regular polygon, where x is an integer, how many possible values of n are there?

## 2016 HI13

設 $A \cdot B$  和 C 為三個數字。利用這三 Let A, B and C be three digits. The 個數字組成的三位數有以下性質: number formed by these three digits has

- (a) ACB 可以被 3 整除;
- (b) BAC 可以被 4 整除;
- (c) BCA可以被5整除;及
- (d) CBA 的因數數目為單數。 求三位數 ABC。
- the following properties:
- (a) ACB is divisible by 3;
- (b) BAC is divisible by 4;
- (c) BCA is divisible by 5;
- (d) *CBA* has an odd number of factors.

Find the 3-digit number *ABC*.

# 2017 HI7

設 N 為完全立方數,已知 N=161x+23y,其中 x 和 y 均為正整數。 求 x+y 的最小值。

Let N be a perfect cube number. Given that N = 161x + 23y, where x and y are positive integers. Find the minimum value of x + y.

### 2018 HI15

設  $N^2 = \overline{abcdefabc}$  為一個 9 位整數,其中 N 是 4 個相異質數的積及 a、b、c、d、e、f均為非零數字且滿足  $\overline{def} = 2 \times \overline{abc}$ 。求  $N^2$  的最小值。

Let  $N^2 = \overline{abcdefabc}$  be a nine-digit positive integer, where N is the product of four distinct primes and a, b, c, d, e, f are non-zero digits that satisfy  $\overline{def} = 2 \times \overline{abc}$ . Find the least value of  $N^2$ .

#### 2018 FI2.4

求整數 d,使得對於任何實數 x, $x^{13}+x+90$  可被  $x^2-x+d$  整除。 Determine the integral value of d, so that  $x^{13}+x+90$  is divisible by  $x^2-x+d$  for any real number x.

#### 2018 FI4.2

若  $x \cdot y$  為正整數及 b 為  $x \cdot y$  組合的數量使得它們的乘積  $x \times y = 100$ , 求 b 的值。

If x, y are positive integers numbers and b is the number of groups of x, y such that the product  $x \times y = 100$ , determine the value of b.

### 2019 HI13

已知正整數 $a \cdot b$  及c 满足下列條件:

- (i) a > b > c,
- (ii) (a-b)(b-c)(a-c) = 84,
- (iii)  $abc < 100 \circ$

設 M為 a 的最大值。求M的值。

Given that a, b and c are positive integers satisfying the following conditions:

- (i) a > b > c,
- (ii) (a-b)(b-c)(a-c) = 84,
- (iii) abc < 100.

Let M be the maximum value of a. Find the value of M.

# 2021 P2O4

已知  $a \cdot b \cdot c \cdot d$  及 e 是連續正整數,其中 a < b < c < d < e。若 a + b + c + d + e 是一個立方數及 b + c + d 是一個平方數,求 c 的最小可能值。

Given that a, b, c, d and e are consecutive positive integers, where a < b < c < d < e. If a+b+c+d+e is a perfect cube and b+c+d is a perfect square, find the smallest possible value of c.

# 2022 P1Q5

設 N = 24x + 216y, 其中x及y均為正整數。

若N為完全立方數,求x+y的最小值。

Let N = 24x + 216y, where both x and y are positive integers.

If N is a cube number, find the minimum value of x + y.

### 2023 HI11

已知 n 是一個少於 2023 正整數。

若 n 只有三個不同的因數, 求 n 的可能性的總數。

Given that n is a positive integer less than 2023.

If n has only 3 distinct factors, find the number of possible values of n.

# Answers

Allsweis				
1985 FG7.2	1985 FG8.1-2	1988 FG7.1-2	1990HI13 2013FG4.3	1990 FG7.3-4
59	M = 5, N = 2	M = 5, N = 6	2519	A = 1, B = 5
1995 FG6.1-2	1996 FG6.4	1998 HG4	1999 FI4.1	1999 FIS.3
a = 5, b = 2	757	64	23	57
1999 FG3.1	2000 FG4.1	2000 FG5.1	2001 FI2.3	2002 HI2
2	504	729000	1	180
2002 HG2	2003 HI9	2004 FI1.1	2005 FG1.1	2006 HI7
221	9	6	9	0
2007 HG1	2007 FI3.3	2008 HG2	2008 HG4	2008 FIS.1
20	8	34891	48	2
2010 HG1	2010 FI3.1	2011 HG1	2011 FG3.1	2011 FG3.4
118	3	a = 1006, b = 1002, c = 998, d = 996, e = 994	1	7
2013 HI2	2013 FG2.1	2014 HI6	2014 HI8	2015 HI3
180	1	107	8	22
2016 HI13	2017 HI7	2018 HI15	2018 FI2.4	2018 FI4.2
522	79	289578289	2	9
2019 HI13	2021 P2Q4	2022 P1Q5	2023 HI11	
9	75	16	14	