Discriminant (HKMO Classified Questions by topics)

### 1986 FI2.1

若方程  $3x^2 - 4x + \frac{h}{3} = 0$  有等根,求 h 的值。

If the equation  $3x^2 - 4x + \frac{h}{3} = 0$  has equal roots, find the value of h.

### 1987 FI4.1

若曲線  $y = 2x^2 - 8x + a$  與 x-軸相切,求 a 的值。

If the curve  $y = 2x^2 - 8x + a$  touches the x-axis, find the value of a.

### 1989 FI4.4

已知曲線  $y = 3x^2 + 12x + c$  與 x-軸相切,求 c 的值。

If the curve  $y = 3x^2 + 12x + c$  touches the x-axis, find the value of c.

# 1989 FSG.1

已知  $2at^2 + 12t + 9 = 0$  有等根,求 a 的值。

If  $2at^2 + 12t + 9 = 0$  has equal roots, find the value of a.

## 1990 FI3.4

若  $64t^2 + 16t + d$  是完全平方,求 d 的值。

If  $64t^2 + 16t + d$  is a perfect square, find the value of d.

## 1991 FG7.2

某方程 $x^2 + 2x + c = 0$ 無實根,且c為小於3之整數,求c的值。

If the equation  $x^2 + 2x + c = 0$  has no real root and c is an integer less than 3, find the value of c.

## 1992 FI1.2

If  $x^2 - 10x + a = 0$  has 2 equal roots, find the value of a.

# 1992 FG10.4

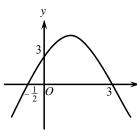
下圖為  $y = -2x^2 + 5x + 3$  的圖形。

若 y = x + d 為  $y = -2x^2 + 5x + 3$ .的切線,求 d 的值。

The following shows the graph of  $y = -2x^2 + 5x + 3$ .

If y = x + d is tangent to  $y = -2x^2 + 5x + 3$ ,

find the value of d.



# 1994 FI1.1

方程式 $x^2 - ax + (a+3) = 0$ 有等根。若a為一正整數,求a的值。

The equation  $x^2 - ax + (a + 3) = 0$  has equal roots.

Find the value of a, if a is a positive integer.

### 1994 FG7.3

若  $c=2-x+2\sqrt{x-1}$  且 x>1, 求 c 之最大值。

Find the largest value of c, if  $c = 2 - x + 2\sqrt{x-1}$  and x > 1.

## 1996 FG9.2

#### 1997 FI2.3

若對於所有實數 x,  $x^2 + cx + 36$  不小於 0, 求 c 的最大值。 If the value of  $x^2 + cx + 36$  is not less than 0 for all real number x,

find the maximum value of c.

### 1999 FI4.3

設 c 為一正實數,若  $x^2 + 2\sqrt{c}x + 2 = 0$  僅有一實數解,求 c 之值。

Let *c* be a positive real number.

If  $x^2 + 2\sqrt{c}x + 2 = 0$  has one real root only, find the value of c.

### 1999 FI5.3

若方程 $x^2-x+1=0$ 有c個實數解,求c之值。

If the number of real roots of the equation  $x^2 - x + 1 = 0$  is c, find the value of c.

### 1999 FG5.2 2013 HG6

已知方程式  $x^2 + ax + 2b = 0$  及  $x^2 + 2bx + a = 0$  的根為實數,且 a, b > 0。 若 a + b 的最小值為 O,求 O 之值。

Given that the roots of  $x^2 + ax + 2b = 0$  and  $x^2 + 2bx + a = 0$  are both real and a, b > 0. If the minimum value of a + b is Q, find the value of Q.

# 2000 FI5.2

如果 (x-2)(x-2Q)-1=0 有兩個整數根,求 Q 的值。

If (x-2)(x-2Q)-1=0 has two integral roots, find the value of Q.

# 2000 FG2.4

設  $f(x) = 41x^2 - 4x + 4$ ,  $g(x) = -2x^2 + x$ 。 如果 f(x) + kg(x) = 0 只有一個根,求 k 的最小值 d。

Let  $f(x) = 41x^2 - 4x + 4$  and  $g(x) = -2x^2 + x$ . If d is the smallest value of k such that f(x) + kg(x) = 0 has a single root, find the value of d.

## 2001 FI2.1

若 P 為整數,及 5 < P < 20。

若方程  $x^2 - 2(2P - 3)x + 4P^2 - 14P + 8 = 0$  的兩個根皆為整數,求 P 的值。 Suppose P is an integer and 5 < P < 20. If the roots of the equation  $x^2 - 2(2P - 3)x + 4P^2 - 14P + 8 = 0$  are integers, find the value of P.

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### 2002 FI2.3

已知
$$x$$
及 $R$ 為實數。若對所有 $x$ , $\frac{2x^2+2Rx+R}{4x^2+6x+3} \le 1$ ,求 $R$ 的最大值。

Given that x and R are real numbers and  $\frac{2x^2 + 2Rx + R}{4x^2 + 6x + 3} \le 1$  for all x,

find the maximum value of R.

### 2002 FG1.1

假設曲綫 
$$x^2 + 3y^2 = 12$$
 及直綫  $mx + y = 16$  只相交於一點。

$$若 a = m^2$$
, 求  $a$  的值。

only one point. If  $a = m^2$ , find the value of a.

### 2002 FG3.2

已知
$$x$$
和 $y$ 為兩實數且滿足關係  $y = \frac{x}{2x-1}$ 。

若 
$$\frac{1}{x^2} + \frac{1}{y^2}$$
 的最小值為  $b$  ,求  $b$  的值。

It is given that the real numbers x and y satisfy the relation  $y = \frac{x}{2x-1}$ .

If the minimum value of  $\frac{1}{x^2} + \frac{1}{y^2}$  is b, find the value of b.

### 2003 HG5

對任意實數 a ,方程  $x^2 + ax + 3b - a + 2 = 0$  都有實數解 ,求 b 的最大值。 If the equation  $x^2 + ax + 3b - a + 2 = 0$  has real root(s) for any real number a, find the maximum value of b.

## 2009 FI4.4

設 
$$x$$
 及  $y$  為實數並滿足方程  $(x-3)^2 + (y-3)^2 = 1$ 。

Let x and y be real numbers satisfying the equation  $(x-3)^2 + (y-3)^2 = 1$ .

If  $k = \frac{y}{x-3}$  and q is the least possible values of  $k^2$ , find the value of q.

# 2010 HG9

若 
$$\log_4(x+2y) + \log_4(x-2y) = 1$$
, 求  $|x|-|y|$  的最小值。

If 
$$\log_4(x+2y) + \log_4(x-2y) = 1$$
, find the minimum value of  $|x| - |y|$ .

#### 2010 FI2.2

若 b 及 h 為正整數,且滿足 b < h 及  $b^2 + h^2 = b(2+h) + 2h$ ,求 b 的值。 If b and h are positive integers with b < h and  $b^2 + h^2 = b(2 + h) + 2h$ , find the value of b.

已知 
$$c$$
 為  $f(x) = \frac{x^2 - 2x - 3}{2x^2 + 2x + 1}$  的最小值。求  $c$  的值。

Given that c is the minimum value of  $f(x) = \frac{x^2 - 2x - 3}{2x^2 + 2x + 1}$ . Find the value of c.

#### 2011 FI3.1

Assume that the curve  $x^2 + 3y^2 = 12$  and the straight line mx + y = 16 intersect at 若 P 為一質數,而且方程  $x^2 + 2(P+1)x + P^2 - P - 14 = 0$  的根為整數,求 P 的最小值。 If P is a prime number and the roots of the equation  $x^2 + 2(P+1)x + P^2 - P - 14 = 0$  are integers, find the least value of P.

#### 2013 HG1

已知一個百角三角形三邊的長度皆為整數,且其中兩邊的長度為方程  $x^{2} - (m+2)x + 4m = 0$  的根。求第三邊長度的最大值。

Given that the length of the sides of a right-angled triangle are integers, and two of them are the roots of the equation  $x^2 - (m+2)x + 4m = 0$ .

Find the maximum length of the third side of the triangle.

# 2013 FI3.4

若 
$$x$$
 為實數及  $d$  為函數  $y = \frac{3x^2 + 3x + 4}{x^2 + x + 1}$  的最大值,求  $d$  的值。

If x is a real number and d is the maximum value of the function  $y = \frac{3x^2 + 3x + 4}{x^2 + x + 1}$ ,

find the value of d.

## 2013 FG2.3

設 
$$f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$$
 ,  $x$  為實數且  $f(x)$  的最大值和最小值分別是  $\frac{1}{2}$  和  $-1$  。

若 
$$t = f(0)$$
, 求  $t$  的值。

Let 
$$f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$$
, where x is a real number and the maximum value of  $f(x)$  is  $\frac{1}{2}$ 

and the minimum value of f(x) is -1. If t = f(0), find the value of t.

#### 2016 FG3.4

若 
$$a \cdot b$$
 及  $y$  為實數 , 並滿足 
$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases}$$
 , 求  $y$  的最大值。 If  $a,b$  and  $y$  are real numbers and satisfy 
$$\begin{cases} a+b+y=5 \\ ab+by+ay=3 \end{cases}$$

determine the greatest possible value of y.

#### 2017 FI4.1

若 a 為正整數, 求 a 的最大值使得  $ax^2 - (a-3)x + (a-2) = 0$  有實根。 If a is a positive integer, determine the greatest value of a such that  $ax^{2} - (a-3)x + (a-2) = 0$  has real root(s).

**2023 HI3** 設 *m* 為一個整數常數,其中 4 < *m* < 40。若方程  $x^2 - 2(2m - 3)x + 4m^2 - 14m + 8 = 0$  有兩個整數根,求 x 的最大可能值。 Let m be an integral constant, where  $4 \le m \le 40$ . If the equation  $x^{2}-2(2m-3)x+4m^{2}-14m+8=0$  has two integral roots, find the largest possible value of x.

#### 2024 HG8

設a為實數。

若方程  $x^2 + ax + 6a = 0$  有兩個整數解,求 a 的最大和最小值之差。 Let a be a real number. If the equation  $x^2 + ax + 6a = 0$  has two integral roots, find the difference between the largest and the smallest values of a.

### Answers

| Allsweis   |             |             |            |                     |
|------------|-------------|-------------|------------|---------------------|
| 1986 FI2.1 | 1987 FI4.1  | 1989 FI4.4  | 1989 FSG.1 | 1990 FI3.4          |
| 4          | 8           | 12          | 2          | 1                   |
| 1991 FG7.2 | 1992 FI1.2  | 1992 FG10.4 | 1994 FI1.1 | 1994 FG7.3          |
| 2          | 25          | 5           | 6          | 2                   |
| 1996 FG9.2 | 1997 FI2.3  | 1999 FI4.3  | 1999 FI5.3 | 1999 FG5.2 2013 HG6 |
| 2          | 12          | 2           | 0          | 6                   |
| 2000 FI5.2 | 2000 FG2.4  | 2001 FI2.1  | 2002 FI2.3 | 2002 FG1.1          |
| 1          | -40         | 12          | 3          | 21                  |
| 2002 FG3.2 | 2003 HG5    | 2009 FI4.4  | 2010 HG9   | 2010 FI2.2          |
| 2          | -1          | 8           | $\sqrt{3}$ | 2                   |
| 2010 FIG 2 | 2011 FI2 1  | 2012 HC1    | 2013 FI3.4 | 2013 FG2.3          |
| 2010 FIS.3 | 2011 FI3.1  | 2013 HG1    | 13         | 1                   |
| _4         | /           | 13          | 3          | $-\frac{1}{2}$      |
| 2016 FG3.4 | 2017 EI 4 1 | 2022 1112   | 2024 HC9   |                     |
| 13         | 2017 FI4.1  | 2023 HI3    | 2024 HG8   |                     |
| 3          | 2           | 52          | 74         |                     |
|            | ı           |             | ı          | 1                   |