Hong Kong Mathematics Olympiad (1984 – 1985)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某兩數之和為 40，其積為 20。若該兩數倒數之和為 a，求 a 的值。
The sum of two numbers is 40, and their product is 20.
If the sum of their reciprocals is a, find the value of a.

$a =$

(ii) 若一邊長 $(a + 1)$ cm 之正方體之總表面積為 b cm^2，求 b 的值。
If b cm^2 is the total surface area of a cube of side $(a + 1)$ cm, find the value of b.

$b =$

(iii) 一袋內有 $b - 4$ 個白球，$b + 46$ 個紅球。若隨意於袋內取一球，而該球為白色之概率為 $\frac{c}{6}$，求 c 的值。
One ball is taken at random from a bag containing $b - 4$ white balls and $b + 46$ red balls. If $\frac{c}{6}$ is the probability that the ball is white, find the value of c.

$c =$

(iv) 若一邊長 c cm 之正三角形之面積為 $d\sqrt{3}$ cm^2，求 d 的值。
The length of a side of an equilateral triangle is c cm. If its area is $d\sqrt{3}$ cm^2, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = \log_5 \frac{(125)(625)}{25}$，求 a 的值。
Find the value of a if $a = \log_5 \frac{(125)(625)}{25}$.

$a =$

(ii) 若 $\left(r+\frac{1}{r}\right)^2 = a-2$ 且 $r^3+\frac{1}{r^3} = b$，求 b 的值。
If $\left(r+\frac{1}{r}\right)^2 = a-2$ and $r^3+\frac{1}{r^3} = b$, find the value of b.

$b =$

(iii) 若 2 為方程 $x^3 + cx + 10 = b$ 之一根，求 c 的值。
If one root of the equation $x^3 + cx + 10 = b$ is 2, find the value of c.

$c =$

(iv) 若 $9^{d+2} = (6489 + c) + 9^d$，求 d 的值。
Find the value of d if $9^{d+2} = (6489 + c) + 9^d$.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在以下數列中，求 a 的值：

1, 8, 27, 64, a, 216, ……

Find a in the following sequence:

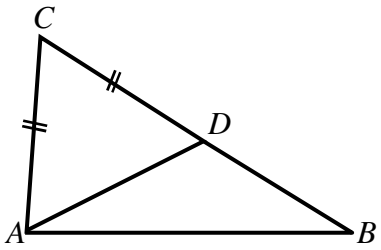
1, 8, 27, 64, a, 216, ……

$a =$

(ii) 在圖一中，$AC = CD$，$\angle CAB - \angle ABC = (a - 95)°$。若$\angle BAD = b°$，求 b 的值。

In Figure 1, $AC = CD$ and $\angle CAB - \angle ABC = (a - 95)°$.
If $\angle BAD = b°$, find the value of b.

$b =$



圖一 Figure 1

(iii) 一直綫過$(-1, 1)$及$(3, b - 6)$。若其 y 截距為 c，求 c 的值。

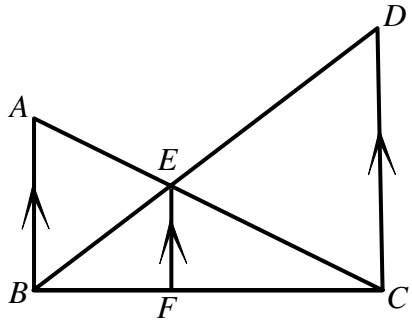
A line passes through the points $(-1, 1)$ and $(3, b - 6)$. If the y-intercept of the line is c, find the value of c.

$c =$

(iv) 在圖二中，$AB = c + 17$，$BC = 100$，$CD = 80$。若 $EF = d$，求 d 的值。

In Figure 2, $AB = c + 17$, $BC = 100$, $CD = 80$. If $EF = d$, find the value of d.

$d =$



圖二 Figure 2

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

\+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在二時十五分，時鐘兩針所構成之銳角為 $\left(18\frac{1}{2}+a\right)^\circ$，求 a 的值。

The acute angle formed by the hands of a clock at 2:15 is $\left(18\frac{1}{2}+a\right)^\circ$.

Find the value of a.

$a =$

(ii) 若 $(x+y)^a$ 的展開式之係數總和是 b，求 b 的值。

If the sum of the coefficients in the expansion of $(x+y)^a$ is b, find the value of b.

$b =$

(iii) 若 $f(x) = x - 2$，$F(x, y) = y^2 + x$，且 $c = F(3, f(b))$，求 c 的值。

If $f(x) = x - 2$, $F(x, y) = y^2 + x$ and $c = F(3, f(b))$, find the value of c.

$c =$

(iv) x，y 為實數。若 $x + y = c - 195$ 及 d 為 xy 之最大值，求 d 的值。

x, y are real numbers. If $x + y = c - 195$ and d is the maximum value of xy, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若兩綫 $x + 2y + 3 = 0$ 及 $4x - ay + 5 = 0$ 互相垂直，求 a 的值。
If the lines $x + 2y + 3 = 0$ and $4x - ay + 5 = 0$ are perpendicular to each other, find the value of a.

$a =$

(ii) 在圖一中，$ABCD$ 為一梯形，AB 與 DC 平行且 $\angle ABC = \angle DCB = 90°$。
若 $AB = a$，$BC = CD = 8$ 及 $AD = b$，求 b 的值。
In Figure 1, $ABCD$ is a trapezium with AB parallel to DC and $\angle ABC = \angle DCB = 90°$. If $AB = a$, $BC = CD = 8$ and $AD = b$, find the value of b.

$b =$

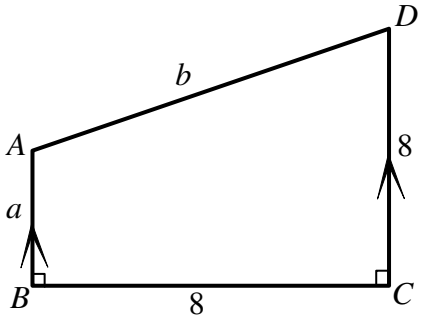


Figure 1
圖一

(iii) 在圖二中，$BD = \frac{b}{2}$，$DE = 4$，$EC = 3$。若 ΔAEC 之面積為 24 及 ΔABC 之面積為 c，求 c 的值。
In Figure 2, $BD = \frac{b}{2}$, $DE = 4$, $EC = 3$.
If the area of ΔAEC is 24 and the area of ΔABC is c, find the value of c.

$c =$

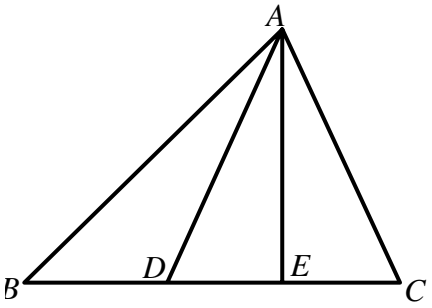


Figure 2
圖二

(iv) 若 $3x^3 - 2x^2 + dx - c$ 可被 $x - 1$ 整除，求 d 的值。
If $3x^3 - 2x^2 + dx - c$ is divisible by $x - 1$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $1 + 2 + 3 + 4 + \ldots + t = 36$，求 t 的值。
If $1 + 2 + 3 + 4 + \ldots + t = 36$, find the value of t.

$t =$

(ii) 若 $\sin u° = \frac{2}{\sqrt{t}}$ 且 $90 < u < 180$，求 u 的值。
If $\sin u° = \frac{2}{\sqrt{t}}$ and $90 < u < 180$, find the value of u.

$u =$

(iii) 在圖一中，$\angle ABC = 30°$，且 $AC = (u - 90)$ cm。若ΔABC 之外接圓半徑為 v cm，求 v 的值。
In Figure 1, $\angle ABC = 30°$ and $AC = (u - 90)$ cm.
If the radius of the circumcircle of ΔABC is v cm, find the value of v.

$v =$

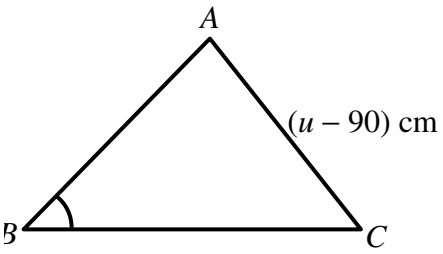


Figure 1
圖一

(iv) 在圖二中，ΔPAB 由切於圓之三切綫形成，且 O 為圓心，
若 $\angle APB = (v - 5)°$，且 $\angle AOB = w°$,，求 w 的值。
In Figure 2, ΔPAB is formed by the 3 tangents of the circle with centre O.
If $\angle APB = (v - 5)°$ and $\angle AOB = w°$, find the value of w.

$w =$

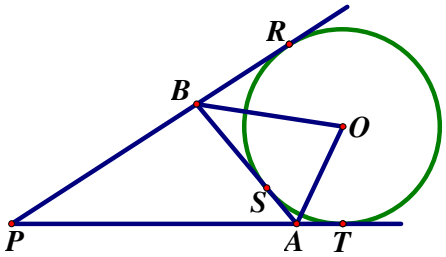


Figure 2
圖二

FOR OFFICIAL USE

Score for accuracy ____ × Mult. factor for speed ____ = ____ Team No. ____

\+ Bonus score ____ Time ____ ____

Total score ____ Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a*b = ab + 1$，且 $s = (2*4)*2$，求 s 的值。
If $a*b = ab + 1$ and $s = (2*4)*2$, find the value of s .

$s =$

(ii) 若第 n 個質數為 s，求 n 的值。
If the n^{th} prime number is s, find the value of n .

$n =$

(iii) 若 $K = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{50}\right)$，試以最簡單之分數表 K。
If $K = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{50}\right)$,
find the value of K in the simplest fractional form.

$K =$

(iv) 一正方形內接於一個半徑為 10 之圓。若正方形之面積為 A，求 A 的值。
If A is the area of a square inscribed in a circle of radius 10, find the value of A .

$A =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) p、q、r 之平均數為 4。 p、q、r、x 之平均數為 5。求 x 的值。
The average of p, q, r is 4 . The average of p, q, r, x is 5. Find the value of x .

$x =$

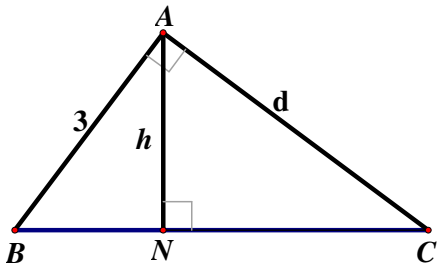
(ii) 一行車速率為 60 km/h 的貨車之一輪每秒轉動 4 周，
若其直徑為 $\frac{y}{6\pi}$ m，求 y 的值。
A wheel of a truck travelling at 60 km/h makes 4 revolutions per second.
If its diameter is $\frac{y}{6\pi}$ m, find the value of y .

$y =$

(iii) If $\sin(55 - y)° = \frac{d}{x}$, find the value of d .
若 $\sin(55 - y)° = \frac{d}{x}$ ，求 d 的值。

$d =$

(iv) 如附圖所示，$BA \perp AC$ 及 $AN \perp BC$。若 $AB = 3$，$AC = d$，$AN = h$，求 h 的值。
In the figure, $BA \perp AC$ and $AN \perp BC$. If $AB = 3$, $AC = d$, $AN = h$,
find the value of h .



$h =$

FOR OFFICIAL USE

Score for accuracy ____ × Mult. factor for speed ____ = ____ Team No. ____

\+ Bonus score ____ Time ____ ____

Total score ____ Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 $M=\dfrac{78^3+22^3}{78^2-78\times22+22^2}$ 。求 M 的值。

Let $M=\dfrac{78^3+22^3}{78^2-78\times22+22^2}$. Find the value of M.

$M=$

(ii) 正整數 N 分別被 6、5、4、3 及 2 除時，其餘數依次為 5、4、3、2 及 1。
求 N 之最小值。
When the positive integer N is divided by 6, 5, 4, 3 and 2, the remainders are 5, 4, 3, 2 and 1 respectively. Find the least value of N.

$N=$

(iii) 一人以 4 km/h 之速率步行 10 km，再以 6 km/h 之速率步行另 10 km。
若全程之平均速率為 x km/h，求 x 的值。
A man travels 10 km at a speed of 4 km/h and another 10 km at a speed of 6 km/h.
If the average speed of the whole journey is x km/h, find the value of x.

$x=$

(iv) 若 $S = 1 + 2 – 3 – 4 + 5 + 6 – 7 – 8 + \ldots + 1985$，求 S 的值。
If $S = 1 + 2 – 3 – 4 + 5 + 6 – 7 – 8 + \ldots + 1985$, find the value of S.

$S=$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

M、N 均為小於 10 之正整數，且 $258024\text{M}8 \times 9 = 2111110\text{N} \times 11$。

M, N are positive integers less than 10 and $258024\text{M}8 \times 9 = 2111110\text{N} \times 11$.

(i) 求 M 的值。
Find the value of M.

$M =$

(ii) 求 N 的值。
Find the value of N.

$N =$

(iii) 一凸 20 邊形有 x 條對角線。求 x 的值。
A convex 20-sided polygon has x diagonals. Find the value of x.

$x =$

(iv) 若 $y = ab + a + b + 1$ 且 $a = 99$，$b = 49$，求 y 的值。
If $y = ab + a + b + 1$ and $a = 99$, $b = 49$, find the value of y.

$y =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) ΔLMN 之三邊長分別為 8、15 及 17。若 ΔLMN 之面積為 A，求 A 的值。
The lengths of the 3 sides of ΔLMN are 8, 15 and 17 respectively.
If the area of ΔLMN is A, find the value of A.

$A =$

(ii) 若 ΔLMN 之內接圓之半徑為 r，求 r 的值。
If r is the length of the radius of the circle inscribed in ΔLMN, find the value of r.

$r =$

(iii) 若某年五月第 r 日為星期五，且同年五月第 n 日為星期一，
其中 $15 < n < 25$，求 n 的值。
If the r^{th} day of May in a year is Friday and the n^{th} day of May in the same year is Monday, where $15 < n < 25$, find the value of n.

$n =$

(iv) 若一凸 n 邊形之內角和為 $x°$，求 x 的值。
If the sum of the interior angles of an n-sided convex polygon is $x°$,
find the value of x.

$x =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1984 – 1985)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 三連續奇數(最小者為 k)之和為 51。求 k 的值。
The sum of 3 consecutive odd integers (the smallest being k) is 51.
Find the value of k.

$k =$

(ii) 若 $x^2 + 6x + k \equiv (x + a)^2 + C$，且 a、C 為常數，求 C 的值。
If $x^2 + 6x + k \equiv (x + a)^2 + C$, where a, C are constants, find the value of C.

$C =$

(iii) 若 $\frac{p}{q} = \frac{q}{r} = \frac{r}{s} = 2$ 且 $R = \frac{p}{s}$，求 R 的值。
If $\frac{p}{q} = \frac{q}{r} = \frac{r}{s} = 2$ and $R = \frac{p}{s}$, find the value of R.

$R =$

(iv) 若 $A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$，求 A 的值。
If $A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$, find the value of A.

$A =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.