Hong Kong Mathematics Olympiad (1989 – 90)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求下式的值：$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$ 。

   Find the value of $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$.

2. 若 $b<0$ 及 $2^{2b+4}-20\times 2^b+4=0$，求 b 的值。

   If $b<0$ and $2^{2b+4}-20\times 2^b+4=0$, find the value of b.

3. 若 $f(a)=a-2$，且 $F(a,b)=a+b^2$，求 $F(3, f(4))$。

   If $f(a)=a-2$ and $F(a,b)=a+b^2$, find $F(3, f(4))$.

4. 對正整數 a 及 b，定義 $a \# b=a^b+b^a$，若 $2 \# w=100$，求 w 的值。

   For positive integers a and b, define $a \# b=a^b+b^a$. If $2 \# w=100$, find the value of w.

5. a 及 b 為常數。直綫 $2ax+3by=4a+12b$ 恆過一定點 P (其座標與 a 和 b 無關)。求 P 點的座標。

   a and b are constants. The straight line $2ax+3by=4a+12b$ passes through a fixed point P whose coordinates do not depend on a and b. Find the coordinates of P.

6. 某三角形各內角正弦的比為 3 : 4 : 5。若 A 為該三角形的最小內角，且 $\cos A=\frac{x}{5}$，求 x 的值。

   The sines of the angles of a triangle are in the ratio 3 : 4 : 5. If A is the smallest interior angle of the triangle and $\cos A=\frac{x}{5}$, find the value of x.

7. 若 $x+y=9$、$y+z=11$ 及 $z+x=10$，求 xyz 的值。

   If $x+y=9$, $y+z=11$ and $z+x=10$, find the value of xyz.

8. 若 α、β 是方程 $2x^2+4x-3=0$ 的根，且 α^2、β^2 是方程 $x^2+px+q=0$ 的根，求 p 的值。

   If α, β are the roots of the equation $2x^2+4x-3=0$ and α^2, β^2 are the roots of the equation $x^2+px+q=0$, find the value of p.

9. 若 $x^{\log_{10} x}=\frac{x^3}{100}$，且 $x>10$，求 x 的值。

   If $x^{\log_{10} x}=\frac{x^3}{100}$ and $x>10$, find the value of x.

10. 已知 $a_0=1$，$a_1=3$ 及 $a_n{}^2-a_{n-1}a_{n+1}=(-1)^n$，其中 n 為正整數。求 a_4 的值。

    Given that $a_0=1$, $a_1=3$ and $a_n{}^2-a_{n-1}a_{n+1}=(-1)^n$ for positive integers n.
    Find the value of a_4.

11. 求 2137^{754} 的個位數。

    Find the units digit of 2137^{754}.

12. 若 $\left(r+\frac{1}{r}\right)^2=3$，求 $r^3+\frac{1}{r^3}$ 的值。

    If $\left(r+\frac{1}{r}\right)^2=3$, find the value of $r^3+\frac{1}{r^3}$.

13. 正整數 N 被 10、9、8、7、6、5、4、3 及 2 除所得的餘數依次是 9、8、7、6、5、4、3、2 及 1，求 N 的最小值。

A positive integer N, when divided by 10, 9, 8, 7, 6, 5, 4, 3 and 2, leaves remainders 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively. Find the least value of N.

14. 若 $\frac{1}{A}=\frac{\cos 45°\sin 70°\cos 60°\tan 40°}{\cos 340°\sin 135°\tan 220°}$，求 A 的值。

If $\frac{1}{A}=\frac{\cos 45°\sin 70°\cos 60°\tan 40°}{\cos 340°\sin 135°\tan 220°}$, find the value of A.

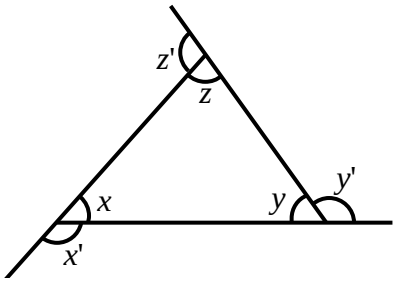
15. 若 10 人需要 5 天製成 20 張檯，請問 15 人需要多少天製成 60 張檯?

If 10 men can make 20 tables in 5 days,

how many days are required to make 60 tables by 15 men ?

16. 圖一的三角形的三個外角的比是 $x':y':z'=4:5:6$，而三個內角的比是 $x:y:z=a:b:3$，求 b 的值。

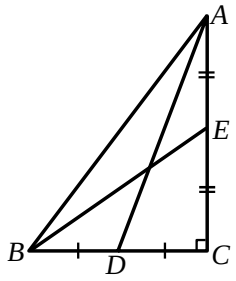
In figure 1, the exterior angles of the triangle are in the ratio $x':y':z'=4:5:6$ and the interior angles are in the ratio $x:y:z=a:b:3$. Find the value of b.



(Figure 1)(圖一)

17. 在 ΔABC 中，$\angle C=90°$ 及 D、E 分別為 BC 及 CA 的中點。若 $AD=7$ 及 $BE=4$，求 AB 的長度。(參考圖二)

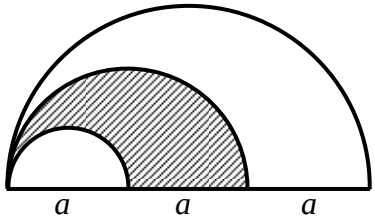
In ΔABC, $\angle C = 90°$ and D, E are the mid-points of BC and CA respectively. If $AD = 7$ and $BE = 4$, find the length of AB. (See figure 2.)



(Figure 2)(圖二)

18. 如圖三，三個半圓的直徑分別為 a、$2a$ 及 $3a$。求陰影部分的面積與沒有陰影部分的面積的比值。

Figure 3 shows 3 semi-circles of diameters a, $2a$ and $3a$ respectively. Find the ratio of the area of the shaded part to that of the unshaded part.



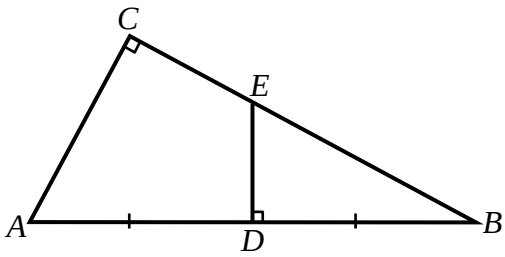
(Figure 3)(圖三)

19. 求 $\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\cdots+\frac{1}{19\times 20}$ 的值

Find the value of $\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\cdots+\frac{1}{19\times 20}$.

20. 在圖四中，$\angle C = 90°$、$AD = DB$ 及 DE 垂直於 AB。若 $AB=20$ 及 $AC=12$，求四邊形 $ADEC$ 的面積。

In figure 4, $\angle C = 90°$, $AD = DB$ and DE is perpendicular to AB. If $AB = 20$ and $AC = 12$, find the area of the quadrilateral $ADEC$.



(Figure 4)(圖四)

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Hong Kong Mathematics Olympiad (1989 – 90)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
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1. 若 $\frac{1}{a}+\frac{1}{b}=5$ 及 $\frac{1}{a^2}+\frac{1}{b^2}=13$，求 $\frac{1}{a^5}+\frac{1}{b^5}$ 的值。

   If $\frac{1}{a}+\frac{1}{b}=5$ and $\frac{1}{a^2}+\frac{1}{b^2}=13$, find the value of $\frac{1}{a^5}+\frac{1}{b^5}$.

2. 某班有學生 N 人。
   若將學生分為每 4 人一組，有 1 人餘下，
   若將學生分為每 5 人一組，有 3 人餘下，
   若將學生分為每 7 人一組，有 3 人餘下。
   求 N 的最小值。
   There are N pupils in a class.
   When they are divided into groups of 4, 1 pupil is left behind.
   When they are divided into groups of 5, 3 pupils are left behind.
   When they are divided into groups of 7, 3 pupils are left behind.
   Find the least value of N.

3. A、B、C 及 D 的座標依次是(10, 1)、(1, 7)、(–2, 1)及(1, 3)。AB 與 CD 相交於 P。
   求 $\frac{AP}{PB}$ 的值。
   The coordinates of A, B, C and D are (10, 1), (1, 7), (–2, 1) and (1, 3) respectively. AB and CD meet at P. Find the value of $\frac{AP}{PB}$.

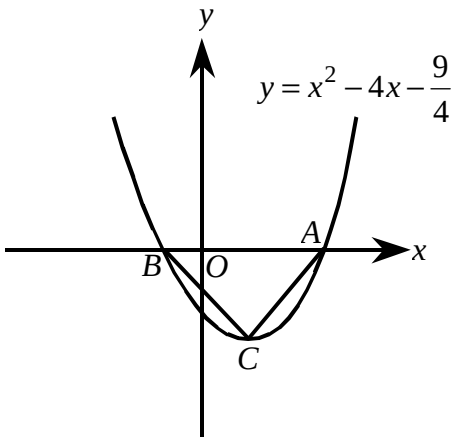
4. 求 $2^{1989}+1$ 被 3 除所得的餘數。
   Find the remainder when $2^{1989}+1$ is divided by 3.

5. 歐拉在 1700 A.D.和 1800 A.D.之間出生和去世。在 n^3 A.D.時，他剛好 $n+9$ 歲，而他在 76 歲時去世。求歐拉去世的年份。
   Euler was born and died between 1700 A.D. and 1800 A.D. He was $n+9$ years old in n^3 A.D. and died at the age of 76. Find the year in which Euler died.

6. 設 $N!$ 為首 N 個自然數的乘積，即 $N! = 1\times2\times3\times\cdots\times N$。
   若 k 是正整數使得 $30! = 2^k\times$一奇數，求 k 的值。
   Let $N!$ denote the product of the first N natural numbers, i.e. $N! = 1\times2\times3\times\cdots\times N$.
   If k is a positive integer such that $30! = 2^k\times$ an odd integer, find the value of k.

7. 拋物綫 $y=x^2-4x-\frac{9}{4}$ 的圖像交 x-軸於 A 及 B(圖一)。若 C 是拋物綫的頂點，求 ΔABC 的面積。

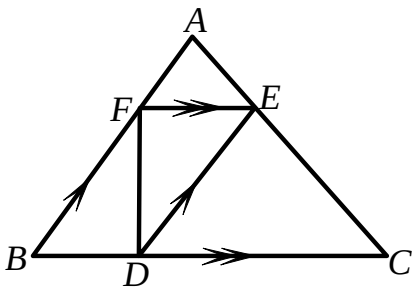
The graph of the parabola $y=x^2-4x-\frac{9}{4}$ cuts the x-axis at A and B (figure 1).
If C is the vertex of the parabola,
find the area of ΔABC.



(Figure 1)(圖一)

8. 在圖二中，FE // BC 及 ED // AB。若 $AF:FB=1:4$，求 ΔEDC 的面積 : ΔDEF 的面積。

In figure 2, FE // BC and ED // AB. If $AF:FB=1:4$, find the ratio of area of ΔEDC : area of ΔDEF.



(Figure 2)(圖二)

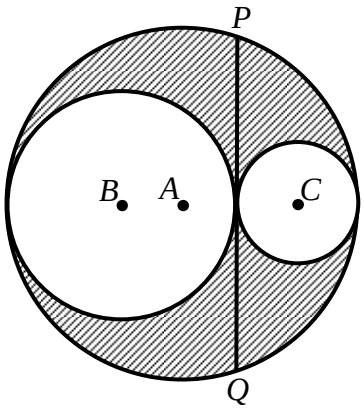
9. 在所附乘法算式中(圖三)，字母 O、L、Y、M、P、I、A 及 D 代表由 1 至 9 的不同整數，求 A 所代表的整數。

In the attached multiplication (figure 3), the letters O, L, Y, M, P, I, A and D represent different integers ranging from 1 to 9. Find the integer represented by A.

$$\begin{array}{r} OLYMPIAD \\ \times \quad\quad\quad D \\ \hline OOOOOOOOO \end{array}$$
(Figure 3)(圖三)

10. 以 A、B 及 C 為圓心的三個圓兩兩相切如圖四。若 A、B 及 C 共綫，且 PQ 是兩個較小圓的公切綫，其中 $PQ=4$，試以 π 表陰影面積。

Three circles, with centres A, B and C respectively, touch one another as shown in figure 4. If A, B and C are collinear and PQ is a common tangent to the two smaller circles, where $PQ = 4$, find the area of the shaded part in terms of π.



(Figure 4)(圖四)

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