

Hong Kong Mathematics Olympiad (1990 – 1991)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 $a = -1 + 2 - 3 + 4 - 5 + 6 - \dots + 100$ ，求 a 的值。

If $a = -1 + 2 - 3 + 4 - 5 + 6 - \dots + 100$, find the value of a .

$a =$

- (ii) 首 b 個正奇數之和是 $2a$ 。求 b 的值。

The sum of the first b positive odd numbers is $2a$. Find the value of b .

$b =$

- (iii) 袋中有白球 b 個，黑球 3 個。現任意取出二球。

若得到兩個不同顏色的球的概率為 $\frac{c}{13}$ ，求 c 的值。

A bag contains b white balls and 3 black balls. Two balls are drawn from the bag at random. If the probability of getting 2 balls of different colours is $\frac{c}{13}$, find the value of c .

$c =$

- (iv) 若直線 $cx + 10y = 4$ 及 $dx - y = 5$ 互相垂直，求 d 的值。

If the lines $cx + 10y = 4$ and $dx - y = 5$ are perpendicular to each other, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 1 (Individual)

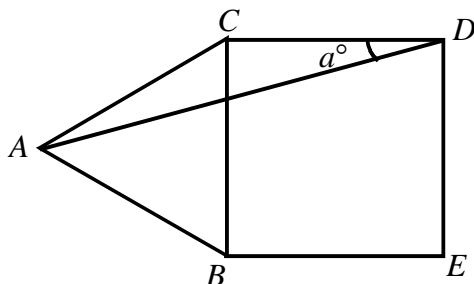
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 如圖所示， ABC 是等邊三角形， $BCDE$ 是正方形。若 $\angle ADC = a^\circ$ ，求 a 的值。

In the figure, ABC is an equilateral triangle and $BCDE$ is a square.

If $\angle ADC = a^\circ$, find the value of a .



$a =$

- (ii) 若 $rb = 15$ ，且 $br^4 = 125a$ ，其中 r 是整數，求 b 的值。

If $rb = 15$ and $br^4 = 125a$, where r is an integer, find the value of b .

$b =$

- (iii) 若方程 $bx^2 - 252x - 13431 = 0$ 之正根是 c ，求 c 的值。

If the positive root of the equation $bx^2 - 252x - 13431 = 0$ is c , find the value of c .

$c =$

- (iv) 已知 $x \# y = \frac{y-1}{x} - x + y$ 。若 $d = 10 \# c$ ，求 d 的值。

Given $x \# y = \frac{y-1}{x} - x + y$. If $d = 10 \# c$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
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×

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Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 $a^2 - 1 = 123 \times 125$ ，且 $a > 0$ ，求 a 的值。

If $a^2 - 1 = 123 \times 125$ and $a > 0$, find the value of a .

$a =$

- (ii) 若 $x^3 - 16x^2 - 9x + a$ 除以 $x - 2$ 之餘數為 b ，求 b 的值。

If the remainder of $x^3 - 16x^2 - 9x + a$ when divided by $x - 2$ is b , find the value of b .

$b =$

- (iii) 若一凸 n 邊形有 $(b + 4)$ 條對角線，求 n 的值。

If an n -sided polygon has $(b + 4)$ diagonals, find the value of n .

$n =$

- (iv) 若點 $(3, n)$ 、 $(5, 1)$ 、 $(7, d)$ 共線，求 d 的值。

If the points $(3, n)$, $(5, 1)$ and $(7, d)$ are collinear, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

\times

Mult. factor for
speed

$=$

Team No.

$+$
Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 6 位數 $168a26$ 可被 3 整除，求 a 之最大可能值。
 If the 6-digit number $168a26$ is divisible by 3,
 find the greatest possible value of a .

- (ii) 一個邊長 a cm 之正方體在全部面上都塗上紅色後，再被分割為邊長 1 cm 之正方體。若所有面都未有被塗上顏色之正方體數目為 b ，求 b 的值。

A cube with edge a cm long is painted red on all faces.

It is then cut into cubes with edge 1 cm long.

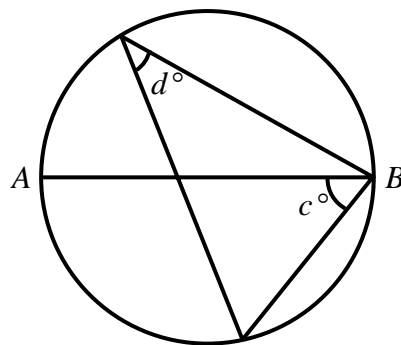
If the number of cubes with all the faces not painted is b , find the value of b .

- (iii) 若 $(x - 85)(x - c) \equiv x^2 - bx + 85c$ ，求 c 的值。

If $(x - 85)(x - c) \equiv x^2 - bx + 85c$, find the value of c .

- (iv) 在圖中， AB 是該圓形的直徑。求 d 的值。

In the figure, AB is a diameter of the circle. Find the value of d .



FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	
				+ Bonus score		
				Total score		

Team No.	
Time	
Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) Given $x - \frac{1}{x} = 3$. If $a = x^2 + \frac{1}{x^2}$, find the value of a .

已知 $x - \frac{1}{x} = 3$ 。若 $a = x^2 + \frac{1}{x^2}$ ，求 a 的值。

$a =$

- (ii) 若 $f(x) = \log_2 x$ ，且 $f(a + 21) = b$ ，求 b 。

If $f(x) = \log_2 x$ and $f(a + 21) = b$, find b .

$b =$

- (iii) 若 $\cos \theta = \frac{8b}{41}$ ，其中 θ 為銳角，且 $c = \frac{1}{\sin \theta} + \frac{1}{\tan \theta}$ ，求 c 的值。

If $\cos \theta = \frac{8b}{41}$, where θ is an acute angle, and $c = \frac{1}{\sin \theta} + \frac{1}{\tan \theta}$, find the value of c .

$c =$

- (iv) 兩骰同擲，得和為 7 或 c 之概率為 $\frac{d}{18}$ ，求 d 的值。

Two dice are tossed. If the probability of getting a sum of 7 or c is $\frac{d}{18}$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

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Mult. factor for
speed

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Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在圖一中，若多邊形之內角和是 a° ，求 a 的值。

In Figure 1, if the sum of the interior angles is a° , find the value of a .

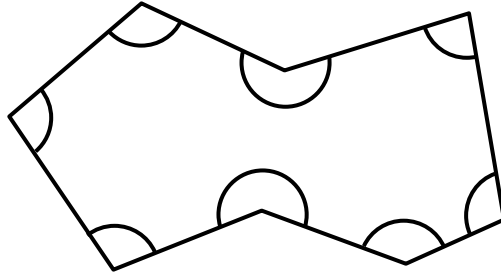


Figure 1 (圖一)

$a =$

- (ii) 若算術級數 80, 130, 180, 230, 280, ... 之第 n 項是 a ，求 n 的值。

If the n^{th} term of the arithmetic progression 80, 130, 180, 230, 280, ... is a , find the value of n .

$n =$

- (iii) 在圖二中， $AP : PB = 2 : 1$ 。若 $AC = 33$ cm, $BD = n$ cm, $PQ = x$ cm，求 x 的值。

In Figure 2, $AP : PB = 2 : 1$. If $AC = 33$ cm, $BD = n$ cm, $PQ = x$ cm, find the value of x .

$x =$

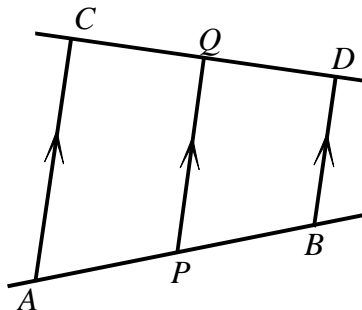


Figure 2 (圖二)

- (iv) 若 $K = \frac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos x^\circ}$ ，求 K 的值。

$K =$

If $K = \frac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos x^\circ}$, find the value of K .

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Hong Kong Mathematics Olympiad (1990 – 1991)
Sample Event (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 一等邊三角形的高是 $8\sqrt{3}$ cm，面積是 $a\sqrt{3}$ cm²。求 a 的值。

The height of an equilateral triangle is $8\sqrt{3}$ cm and the area of the triangle is $a\sqrt{3}$ cm². Find the value of a .

$a =$

- (ii) 已知 $\sum_{x=1}^n \frac{1}{x} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ ，及 $\sum_{x=4}^{10} \frac{1}{x-2} - \sum_{x=4}^{10} \frac{1}{x-1} = \frac{b}{18}$ 。求 b 的值。

$b =$

Given that $\sum_{x=1}^n \frac{1}{x} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, and $\sum_{x=4}^{10} \frac{1}{x-2} - \sum_{x=4}^{10} \frac{1}{x-1} = \frac{b}{18}$. Find the value of b .

某童把一平行四邊形兩鄰邊相乘當作該圖形之面積，其結果為正確答案之兩倍。
若該圖形之銳角及鈍角依次為 h° 及 k° 。

A boy tries to find the area of a parallelogram by multiplying together the lengths of two adjacent sides. His answer is double the correct answer.

If the acute angle and the obtuse angle of the figure are h° and k° respectively,

- (iii) 求 h 的值。

find the value of h .

$h =$

- (iv) 求 k 的值。

find the value of k .

$k =$

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Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

某兩位數 x 之個位數字是 M ，十位數字是 N 。另一兩位數 y 之個位數字是 N ，十位數字是 M 。若 $x > y$ ，且他們的和是他們的差的十一倍，

A 2-digit number x has M as the units digit and N as the tens digit. Another 2-digit number y has N as the units digit and M as the tens digit.

If $x > y$ and their sum is equal to eleven times their differences,

- (i) 求 M 的值。
find the value of M .

$M =$

- (ii) 求 N 的值。
find the value of N .

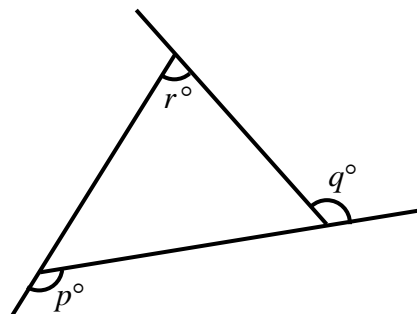
$N =$

- (iii) 兩數之和是 20，積是 5。若該兩數倒數之和是 z ，求 z 的值。
The sum of two numbers is 20 and their product is 5.
If the sum of their reciprocals is z , find the value of z .

$z =$

- (iv) 圖中， p 與 q 的平均值是 $121 + z$ 。求 r 的值。
In the figure, the average of p and q is $121 + z$. Find the value of r .

$r =$



FOR OFFICIAL USE

Score for
accuracy

×

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speed

=

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score

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Total score

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Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 5 部印刷機可在 5 天內印 5 本書。

若要在 100 天內印 100 本書，則需要 n 部印刷機，求 n 的值。

5 printing machines can print 5 books in 5 days. If n printing machines are required in order to have 100 books printed in 100 days, find the value of n .

$n =$

- (ii) 某方程 $x^2 + 2x + c = 0$ 無實根，且 c 為小於 3 之整數，求 c 的值。

If the equation $x^2 + 2x + c = 0$ has no real root and c is an integer less than 3, find the value of c .

$c =$

雞蛋每只 \$0.50，鴨蛋每只 \$0.60，鵝蛋每只 \$0.90。某人賣出 x 只雞蛋， y 只鴨蛋， z 只鵝蛋，共得 \$60。若 x 、 y 、 z 皆為正數，且 $x + y + z = 100$ ，及在 x 、 y 、 z 中有兩數相同，

Chicken eggs cost \$0.50 each, duck eggs cost \$0.60 each and goose eggs cost \$0.90 each.

A man sold x chicken eggs, y duck eggs, z goose eggs and received \$60.

If x , y , z are all positive numbers with $x + y + z = 100$ and two of the values x , y , z are equal,

- (iii) 求 x 的值。

find the value of x .

$x =$

- (iv) 求 y 的值。

find the value of y .

$y =$

FOR OFFICIAL USE

Score for accuracy

×

Mult. factor for speed

=

Team No.

+

Bonus score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 8 (Group)

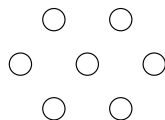
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

細看以下之六邊形數：

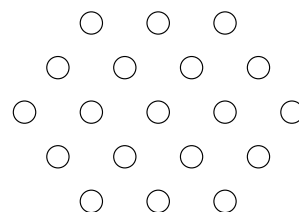
Consider the following hexagonal numbers :



$$H_1 = 1$$



$$H_2 = 7$$



$$H_3 = 19$$

- (i) 求 H_5 的值。

Find the value of H_5 .

$H_5 =$

- (ii) 若 $H_n = an^2 + bn + c$ ，其中 n 為正整數，求 a 的值。

If $H_n = an^2 + bn + c$, where n is any positive integer, find the value of a .

$a =$

- (iii) 若 $p : q = 2 : 3$ ， $q : r = 4 : 5$ ，且 $p : q : r = 8 : t : 15$ ，求 t 的值。

If $p : q = 2 : 3$, $q : r = 4 : 5$ and $p : q : r = 8 : t : 15$, find the value of t .

$t =$

- (iv) 若 $\frac{1}{x} : \frac{1}{y} = 4 : 3$ ，且 $\frac{1}{x+y} : \frac{1}{x} = 3 : m$ ，求 m 的值。

If $\frac{1}{x} : \frac{1}{y} = 4 : 3$ and $\frac{1}{x+y} : \frac{1}{x} = 3 : m$, find the value of m .

$m =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

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+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

圖中， BC 與 DE 平行。

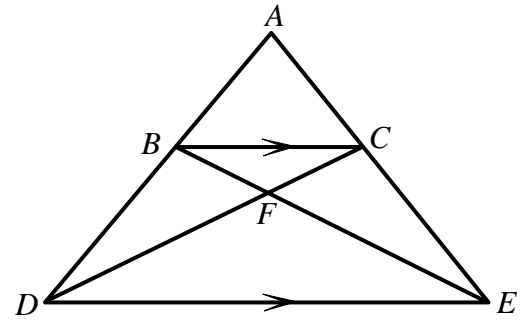
若 $AB : BC : BF : CF : FE = 5 : 4 : 2 : 3 : 5$ ，

且 $\triangle BCF$ 之面積為 12，求

In the figure, BC is parallel to DE .

If $AB : BC : BF : CF : FE = 5 : 4 : 2 : 3 : 5$

and the area of $\triangle BCF$ is 12, find



- (i) $\triangle BDF$ 之面積，
the area of $\triangle BDF$,

Area of $\triangle BDF =$

- (ii) $\triangle FDE$ 之面積，
the area of $\triangle FDE$,

Area of $\triangle FDE =$

- (iii) $\triangle ABC$ 之面積。
the area of $\triangle ABC$.

Area of $\triangle ABC =$

- (iv) 若一球體之體積增加 72.8%，則其表面面積增加 $x\%$ 。求 x 的值。

If the volume of a sphere is increased by 72.8%, then the surface area of the sphere is increased by $x\%$. Find the value of x .

$x =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)

Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

在所附除法算式中

In the attached division

						1	D	E
2	1	5	J	A	7	B	9	C
				F	G	H		
				J	5	K	9	
				L	5	M	5	
						N	4	P
						Q	R	S

- (i) 求 A 的值。

find the value of A .

$$A =$$

- (ii) 求 B 的值。

find the value of B .

$$B =$$

- (iii) 求 C 的值。

find the value of C .

$$C =$$

- (iv) 求 D 的值。

find the value of D .

 $D =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

Team No.

 $+$

Bonus
score

Total score

Time

Min.

Sec.