The formula P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

By axiom 3 of chapter 3, if $A \cap B = \phi$, $P(A \cup B) = P(A) + P(B)$

Consider the following **example 1**:
RE = draw a card from 52 playing cards.
S = {♠A, ⋯ , ♥A, ⋯ , ♦A, ⋯ , ♣K}, n(S) = 52

E = the event that the card drawn is a spade ={♠A, ⋯ , ♠K}, n(E) = 13

F = the event that the card drawn is a King = {♠K, ♥K, ♦K, ♣K}, n(F) = 4
E ∩ F = {♠K} ≠ ϕ
E ∪ F = {♠A, ⋯ , ♠K, ♥K, ♦K, ♣K}, n(E ∪ F) = ___________

P(E ∪ F) = ___________

P(E) + P(F) = _________

∴ P(E ∪ F) ≠ P(E) + P(F)
In fact, we have proved in Chapter 3, theorem e that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
In this case, if $A \cap B = \phi$, $n(A \cap B) = 0$, so that $P(A \cap B) = 0$
The formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ reduces to $P(A \cup B) = P(A) + P(B)$, axiom 3 of chapter 3.

Example 2 Two fair (公平) dice are thrown.
S = {(x, y): x, y = 1, 2, 3, 4, 5, 6}, n(S) = 6 × 6 = 36
A = event that the first die is 6 = {(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}
B = event that the second die is 6 = {(1,6), (2,6), (3,6), (4,6), (5,6), (6,6)}
Find P(A ∪ B)

Solution A ∩ B = event that both are 6 ={(6, 6)}

$n(A \cap B) = 1$, $P(A \cap B) = \frac{1}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= _____ + _____ – _______ = ______

Example 3 In a class of 40 students, each of them must take either Chinese or French or both. If 35 take Chinese and 12 take French, find the chance that a student picked at random takes both languages.
Solution: S = {class}, n(S) = 40
A = {students take Chinese}, n(A) = _______

B = {students take French}, n(B) = _______
A ∪ B = S
P(both languages) = P(A ∩ B) = P(A) + P(B) – P(A ∪ B)

= ________________________________