

Hong Kong Mathematics Olympiad (1994 – 95)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求 1234567654321 的平方根。

Find the positive square root of 1234567654321.

2. 已知 $f\left(\frac{1}{x}\right) = \frac{x}{1-x^2}$ ，求 $f(2)$ 的值。

Given that $f\left(\frac{1}{x}\right) = \frac{x}{1-x^2}$, find the value of $f(2)$.

3. 解 $3^{2x} + 9 = 10(3^x)$ 。

Solve $3^{2x} + 9 = 10(3^x)$.

4. 隨意抽出一個三位數。求這個數是一完全平方數的機會。

A three-digit number is selected at random. Find the probability that the number selected is a perfect square.

5. 已知 $\sin x + \cos x = \frac{1}{5}$ ，且 $0 \leq x \leq \pi$ ，求 $\tan x$ 的值。

Given that $\sin x + \cos x = \frac{1}{5}$ and $0 \leq x \leq \pi$, find the value of $\tan x$.

6. 有多少對正整數 x, y 可滿足 $xy - 3x - 2y = 10$ ？

How many pairs of positive integers x, y are there satisfying $xy - 3x - 2y = 10$?

7. x, y 為正整數，且 $3x + 5y = 123$ 。求 $|x - y|$ 的最小值。

x, y are positive integers and $3x + 5y = 123$. Find the least value of $|x - y|$.

8. 求 1997^{913} 被 10 除時所得的餘數。

Find the remainder when 1997^{913} is divided by 10.

9. 如圖一，若 $BC = 3DE$ ，求 r 的值，其中 $r = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDC}$ 。

In figure 1, if $BC = 3DE$, find the value of r ,

where $r = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDC}$.

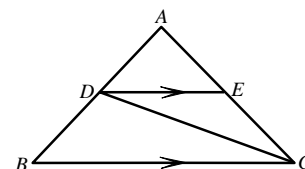


Figure 1 圖一

10. 如圖二， A, B, C, D 為直角三角形 PQR 各邊上的點。若 $ABCD$ 為一正方形，且 $QA = 8$ 及 $BR = 18$ ，求 AB 的值。

A, B, C, D are points on the sides of the right-angled triangle PQR as shown in figure 2.

If $ABCD$ is a square, $QA = 8$ and $BR = 18$, find the value of AB .

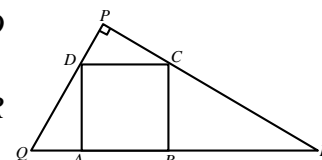


Figure 2 圖二

*** 試卷完 End of Paper ***

Hong Kong Mathematics Olympiad (1994 – 95)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 求方程 $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$ 的正整數解數目。

Find the number of positive integral solutions of the equation $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$.

2. 如圖一，四邊形 $ABCD$ 的對角線交於 O 。

若 $\angle AOB = 30^\circ$ 、 $AC = 24$ 及 $BD = 22$ ，

求四邊形 $ABCD$ 的面積。

In figure 1, $ABCD$ is a quadrilateral whose diagonals intersect at O . If $\angle AOB = 30^\circ$, $AC = 24$ and $BD = 22$, find the area of the quadrilateral $ABCD$.

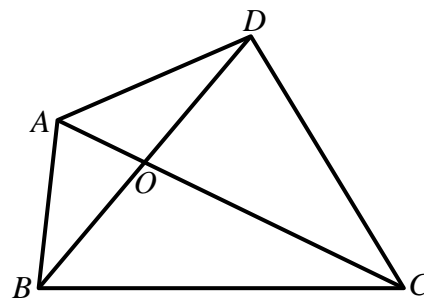


Figure 1 圖一

3. 已知 $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \cdots + \frac{n-1}{n} = \frac{n-1}{2}$ ，求 $\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \cdots + \left(\frac{1}{10} + \cdots + \frac{9}{10}\right)$ 的值。

Given that $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \cdots + \frac{n-1}{n} = \frac{n-1}{2}$, find the value of

$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \cdots + \left(\frac{1}{10} + \cdots + \frac{9}{10}\right).$$

4. 若 x 及 y 為正整數，且 $x^2 = y^2 + 2000$ ，求 x 的最小值。

Suppose x and y are positive integers such that $x^2 = y^2 + 2000$, find the least value of x .

5. 已知 37^{100} 為一 157 位數，且 37^{15} 為一 n 位數，求 n 的值。

Given that 37^{100} is a 157-digit number, and 37^{15} is an n -digit number. Find the value of n .

6. 已知 $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$ ，求 $19 \times 21 + 18 \times 22 + 17 \times 23 + \cdots + 1 \times 39$ 的值。

Given that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$,

find the value of $19 \times 21 + 18 \times 22 + 17 \times 23 + \cdots + 1 \times 39$.

7. 在圖中， $ABCD$ 為一正方形，且 $AB=1$ 及 CPQ 為一等邊三角形。求 $\triangle CPQ$ 的面積。

In figure 2, $ABCD$ is a square where $AB = 1$ and CPQ is an equilateral triangle. Find the area of $\triangle CPQ$.

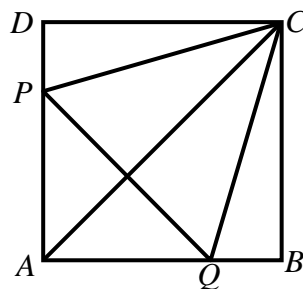


Figure 2 圖二

8. 用一元、二元及五元硬幣，以湊合十七元，且每次均須使用各種硬幣，其方法有 n 種，求 n 的值。

The number of ways to pay a sum of \$17 by using \$1 coins, \$2 coins and \$5 coins is n .

Find the value of n . (Assume that all types of coins must be used each time.)

9. 圖三是一個 3×3 的正方形，求圖中三角形的總數。

In figure 3, find the total number of triangles in the 3×3 square.

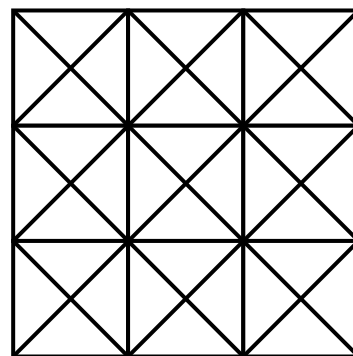


Figure 3 圖三

10. 在圖四中，象限的半徑和大半圓的直徑是 2，求小半圓的半徑。

In figure 4, the radius of the quadrant and the diameter of the large semi-circle is 2. Find the radius of the small semi-circle.

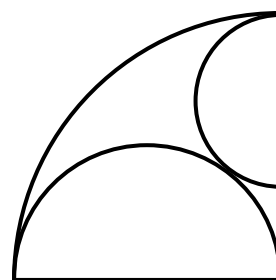


Figure 4 圖四