<u>第四十一屆香港數學競賽 (2023/24)</u>
<u>決賽規則</u>

1. 競賽共分八項，個人及團體各佔*四*項。
2. 每隊由已報名參加初賽的同學組成。其中任何四位可參加「個人項目」；又其中任何四位可參加「團體項目」。不足四位同學的隊伍將不獲准出賽。
3. 每隊隊員必須**<u>穿著整齊校服，並由負責老師帶領</u>**，於**<u>上午 9 時正或以前</u>**向會場接待處註冊，同時**<u>必須出示身分證/學生證明文件</u>**，否則將被**<u>撤銷參賽資格</u>**。
4. 粵語將會被採用為指示語言。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英並列。
5. 每一「個人項目」包括四部份。每一隊員回答其中一部份，其他隊友不得從旁協助，否則此項目所得分數會被取消。
6. 「個人項目」中，四部份互有關連。解答第二部份之隊員需利用第一部份之答案，如此類推。
7. 每一「團體項目」亦包括四部份。但各部份不一定相關，且可由全隊共同作答。各隊員可進行討論，但必須將聲浪降至最低。
8. 比賽時，參賽者不可使用計算工具，違例者將被取消資格或扣分。
9. 參賽者如有攜帶電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，應把它關掉，並放入手提包內或座位的椅下，否則大會有權**<u>取消該隊參賽資格</u>**。
10. 除另有聲明外，所有答案須為數字，並應化簡，但無需呈交證明及算草。
11. 每一項目限時五分鐘。
12. 計分辦法如下：

(甲) 準確分:

個人項目	積分	團體項目	積分
答對第一部分	1	答對任何一部分	2
答對第二部分	2	答對任何兩部分	4
答對第三部分	3	答對任何三部分	7
答對第四部分	4	答對所有四部分	10
合共	10		

(乙) 快捷分

	積分所乘倍數
參賽隊伍完成並交出答案的時間 <1 分鐘	**4**
1 分鐘 ≤ 參賽隊伍完成並交出答案的時間 <2 分鐘	**3**
2 分鐘 ≤ 參賽隊伍完成並交出答案的時間 <3 分鐘	**2**
參賽隊伍完成並交出答案的時間 ≥3 分鐘	**1**

(丙) 獎勵分

任何一隊在某一個人/團體項目競賽中，若全部答對時，可額外獲得 20 分。

(丁) 每項目之總分

準確分×倍數 ＋ 獎勵分

13. 如有任何疑問，參賽者須於最後一項個人/團體賽完畢後 10 分鐘內向評判團提出。所有提出之疑問，將由評判團作最後裁決。
14. 得分最高之三隊將獲得獎盃及獎牌。
15. 總成績將由評判團作最後裁決。

<u>The Forty-First Hong Kong Mathematics Olympiad (2023/24)</u>
<u>Regulations (Final Events)</u>

1. The competition consists of ***8*** events, which are divided into ***4*** individual events and ***4*** group events.
2. Each participating team should consist of students who have enrolled in the Heats. An`y 4 of them may take part in the individual event and any 4 of them may take part in the group event. Teams of less than 4 members will not be allowed to participate.
3. Members of each team, <u>**accompanied by the teacher-in-charge, should wear proper school uniform**</u> and present <u>**ID Card or student identification document**</u> when registering at the venue reception <u>**not later than 9:00 a.m.**</u> Failing to do so, the team <u>**will be disqualified.**</u>
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, instructions written in both Chinese and English will be provided. Question papers are printed in both English and Chinese.
5. Each individual event consists of 4 parts. Each part must be completed by one member of the team. Help from other team members would result in disqualification for that particular event.
6. In an individual event, the four parts are interrelated. When solving Part 2, one has to make use of the answer obtained in Part 1, and so on.
7. In a group event, the four parts are to be done by the whole team and the parts may or may not be interrelated. Discussions are allowed provided that voice level is kept to a minimum.
8. Use of calculating devices will not be allowed; otherwise the team will risk disqualification or deduction of marks.
9. Participants having electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) or any alarm device(s), should turned them off and put them inside their bags or under their chairs. Failing to do so, the team <u>**will risk disqualification.**</u>
10. All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or working is required.
11. The time limit for each event is 5 minutes.
12. The Marking System is as follows:
    (a) Scores for accuracy:

<u>Individual Events</u>	<u>Scores</u>	<u>Group Events</u>	<u>Scores</u>
Part 1 correct ⋯	1	Any 1 part correct	⋯2
Part 2 correct ⋯	2	Any 2 parts correct	⋯4
Part 3 correct ⋯	3	Any 3 parts correct	⋯7
Part 4 correct ⋯	4	All 4 parts correct	⋯10
Total ⋯⋯⋯⋯	10		

    (b) Multiplying factors for speed:

Time taken for the teams to hand in their answer < 1 min.	***4***
1 min. ≤ Time taken for the teams to hand in their answer < 2 min.	***3***
2 min. ≤ Time taken for the teams to hand in their answer < 3 min.	***2***
Time taken for the teams to hand in their answer ≥ 3 min.	***1***

    (c) Bonus Score:
    Teams, which hand in their answers of anyone individual/group event have all the answers in that event correct, will be awarded a bonus score of 20 marks.
    (d) Total score for each event:
    (Score for accuracy) × (Multiplying factor) + (Bonus score)
13. Any queries should reach the Judging Panel within 10 minutes after the end of the last individual group event. The decision of the Judging Panel on the queries is final.
14. Trophies and medals will be given to the three schools achieving the highest scores.
15. The decision of the Judging Panel on the overall results is final.

比賽資料 決賽隊伍數目：50　　決賽日期：2024 年 4 月 6 日星期六　　地點：香港教育大學
決賽名單：

School ID	Name of School
FE-01	Baptist Lui Ming Choi Secondary School
FE-02	Bishop Hall Jubilee School
FE-03	Buddhist Sin Tak College
FE-04	Carmel Pak U Secondary School
FE-05	CNEC Christian College
FE-06	CNEC Lau Wing Sang Secondary School
FE-07	Diocesan Boys' School
FE-08	Diocesan Girls' School
FE-09	Fukien Secondary School
FE-10	G.T. (Ellen Yeung) College
FE-11	Good Hope School
FE-12	Heung To Middle School
FE-13	HKTA Tang Hin Memorial Secondary School
FE-14	Ho Fung College (Sponsored by Sik Sik Yuen)
FE-15	Hoi Ping Chamber of Commerce Secondary School
FE-16	HKBU Affiliated School Wong Kam Fai Secondary and Primary School
FE-17	Hong Kong Chinese Women's Club College
FE-18	Inno Secondary School
FE-19	King's College
FE-20	La Salle College
FE-21	Munsang College (Hong Kong Island)
FE-22	NTHYK Yuen Long District Secondary School
FE-23	Po Leung Kuk No. 1 W.H. Cheung College
FE-24	Po Leung Kuk Centenary Li Shiu Chung Memorial College
FE-25	Po Leung Kuk Laws Foundation College
FE-26	Po Leung Kuk Ma Kam Ming College
FE-27	Po Leung Kuk Tang Yuk Tien College
FE-28	Pui Ching Middle School
FE-29	Pui Kiu College
FE-30	Pui Kiu Middle School
FE-31	Queen Elizabeth School
FE-32	Queen's College
FE-33	SKH Tsang Shiu Tim Secondary School
FE-34	Shun Tak Fraternal Association Lee Shau Kee College
FE-35	Shun Tak Fraternal Association Yung Yau College
FE-36	Sing Yin Secondary School
FE-37	Singapore International School (Hong Kong)
FE-38	SKH Lam Woo Memorial Secondary School
FE-39	St Paul's Co-Educational College
FE-40	St Paul's College
FE-41	The Mission Covenant Church Holm Glad College
FE-42	The Y.W.C.A. Hioe Tjo Yoeng College
FE-43	True Light Girls' College
FE-44	Tseung Kwan O Government Secondary School
FE-45	Tsuen Wan Government Secondary School
FE-46	Tung Wah Group of Hospital Lo Kon Ting Memorial College
FE-47	Tung Wah Group of Hospital Wong Fut Nam College
FE-48	Wa Ying College
FE-49	Wong Shiu Chi Secondary School
FE-50	Ying Wa College

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 m 和 n 均為正整數。如果 $m+n+mn=54$ 及 $A=m+n$，求 A 的值。
Given that m and n are positive integers.
If $m+n+mn=54$ and $A=m+n$, find the value of A.

$A =$

2. 若 $\mathrm{f}(a)=a-2$，$\mathrm{F}(a,b)=b^2+a+A$， $B=\mathrm{F}(4,\mathrm{f}(5))$，求 B 的值。
If $\mathrm{f}(a)=a-2$, $\mathrm{F}(a,b)=b^2+a+A$ and $B=\mathrm{F}(4,\mathrm{f}(5))$, find the value of B .

$B =$

3. 在 x-y 座標平面上，由 $(B+2,0)$、$(-B-2,0)$、$(0,2)$ 及 $(0,-2)$所形成之菱形的面積為 C 平方單位，求 C 的值。
The area of the rhombus on the x-y coordinate plane with vertices $(B+2,0)$, $(-B-2,0)$, $(0,2)$ and $(0,-2)$ is C square units. Find the value of C .

$C =$

4. 如果 D 是正整數且 $\left(\frac{C}{4}+227\right)^{\frac{1}{D}}=D$，求 D 的值。
If D is a positive integer such that $\left(\frac{C}{4}+227\right)^{\frac{1}{D}}=D$, find the value of D .

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 A 是 $2022^{2023^{2024}}$ 的個位數 A。
A is the units digit of $2022^{2023^{2024}}$. Find the value of A.

$A =$

2. 已知 $(x + 20)^2 + (y - 24)^2$ 的最小值是 B，當中 x 和 y 是方程 $19x + 13y = A$ 的整數解。求 B 的值。
B is the minimum value of $(x + 20)^2 + (y - 24)^2$, where x and y are integers that satisfy the equation $19x + 13y = A$. Find the value of B.

$B =$

3. 在袋中有若干顆紅色和藍色的彈珠，它們的總數量是 C。如果加入 B 顆紅色彈珠，紅色和藍色彈珠數量的比例則為 3：2；如果加入 B 顆藍色彈珠，紅色和藍色彈珠數量的比例則為 2：3。求 C 的值。
There is C marbles in a bag, which are either red or blue. If we add B red marbles to the bag, the ratio of red marbles to the blue marbles becomes 3 : 2. If we add B blue marbles to the bag, the ratio of red marbles to the blue marbles becomes 2 : 3.
Find the value of C.

$C =$

4. 若 $5\left(\sqrt{25+2\sqrt{D}}+\sqrt{25-2\sqrt{D}}\right)=C$，求 D 的值。
If $5\left(\sqrt{25+2\sqrt{D}}+\sqrt{25-2\sqrt{D}}\right)=C$, find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 x 和 y 為滿足方程 $\frac{1}{x}+\frac{1}{y}=\frac{2}{5}$ 的不同正整數，求 $A=x+y$ 的值。

If x and y are two different positive integers such that $\frac{1}{x}+\frac{1}{y}=\frac{2}{5}$,

find the value of $A=x+y$.

$A=$

2. 若 B 是所有正整數 N 使得 7 整除 $2^N+(19-A)$ 的數量，求 B 的值。
If B is the number of positive integers N such that $2^N+(19-A)$ is divisible by 7,
find the value of B .

$B=$

3. 已知 a 和 b 為滿足方程組 $a^2-b^2=9$ 及 $ab=3$ 的實數。
若對於正整數 α 和 C，$a+b=\sqrt{\sqrt{\alpha}+C-B}$，求 C 的值。
Given that a and b are real numbers such that $a^2-b^2=9$ and $ab=3$.
If $a+b=\sqrt{\sqrt{\alpha}+C-B}$ for positive integers α and C, find the value of C .

$C=$

4. 若 x 為滿足方程 $(\log_a x)^{\log_a x}=x$ 的實數，其中 a 是常數且 $a>1$。
求 $D=\frac{C\log_a x}{3a}$ 的值。
If x is real root of the equation $(\log_a x)^{\log_a x}=x$, where a is a constant and $a>1$,
find the value of $D=\frac{C\log_a x}{3a}$.

$D=$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如果 $A > 1$ 且 $1 + \frac{1}{A} + \frac{1}{A^2} + \frac{1}{A^3} + \cdots = \frac{A}{3}$，求 A 的值。

   If $A > 1$ and $1 + \frac{1}{A} + \frac{1}{A^2} + \frac{1}{A^3} + \cdots = \frac{A}{3}$, find the value of A.

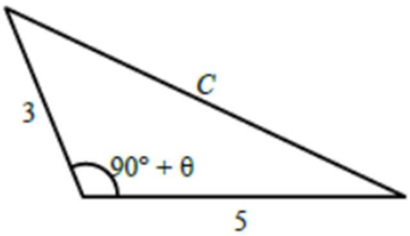
   $A =$

2. 如果 $\frac{1}{A}$ 是二次方程 $x^2 - Bx + \frac{1}{6}B = 0$ 的一個根，求 B 的值。

   If $\frac{1}{A}$ is a root of the quadratic equation $x^2 - Bx + \frac{1}{6}B = 0$, find the value of B.

   $B =$

3. 考慮下圖中的三角形，如果 $\tan\theta = B$，其中 $0° < \theta < 90°$，求 C 的值。
   Consider the triangle in the figure below.
   If $\tan\theta = B$, where $0° < \theta < 90°$, find the value of C.



   $C =$

4. 設 $d = C^2 - 20$，如果 D 滿足方程 $8^D = D^d$，求 D 的值。

   Let $d = C^2 - 20$. If D satisfies the equation $8^D = D^d$, the value of D.

   $D =$

FOR OFFICIAL USE

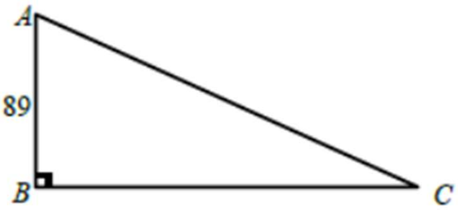
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若直角三角形ABC所有邊長均為正整數，且$AB = 89$，求三角形ABC的周界P。
Find the perimeter P of the right-angled triangle ABC if all the side lengths are positive integers and $AB = 89$.

$P =$



2. 若 A 是 $8888^{20242024}$ 的個位數。求 A 的值。
If A is the units digit of $8888^{20242024}$. Find the value of A .

$A =$

3. 有多少個 5 位數包含最少 1 個「1」和最少 1 個「3」？
How many 5-digit numbers contain at least one “1” and at least one “3” ?

4. 設有 m 對正整數 a 和 b，使 $a^4 + 4b^4$ 為質數，求 m 的值。
Let m be the number of possible pairs of positive integers a and b for which $a^4 + 4b^4$ is a prime number. Find the value of m .

$m =$

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $x>0$。已知 $x-\frac{1}{x}=\sqrt{3}$ 且 $a=x^5+x^3+x+\frac{1}{x}+\frac{1}{x^3}+\frac{1}{x^5}$，求 a 的值。
Let $x>0$. Given that $x-\frac{1}{x}=\sqrt{3}$ and $a=x^5+x^3+x+\frac{1}{x}+\frac{1}{x^3}+\frac{1}{x^5}$, find the value of a.

$a =$

2. 用首2024個正整數：1、2、3、4、5、6、…、2024，造出一個新整數：123456789101112…2024。若b是這個整數裡「0」的數量，求 b 的值。
Using the first 2024 positive integers: 1, 2, 3, 4, 5, 6, ⋯ , 2024, a new integer is formed as 123456789101112⋯2024.
If b is the number of "0" in this integer, find the value of b.

$b =$

3. c 是 2024^2-2023^2 的正因數的數量。求 c 的值。
c is the number of positive factors of 2024^2-2023^2. Find the value of c.

$c =$

4. 假設「0」、「1」、「2」、…及「6」分別為星期日、星期一、星期二、…和星期六，今日是星期一，若 $20^{24^{2024}}$ 天後的那一天是星期幾之代號為「d」，求 d 的值。
Let "0", "1", "2", ⋯ and "6" represent Sunday, Monday, Tuesday, ⋯ and Saturday respectively. Today is Monday. If "d" represents the day of week that comes after $20^{24^{2024}}$ days. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 試找出最小的正整數 n 使得 $2^{10}+2^{13}+2^{n}$ 成為一個完全平方數。
Find the smallest positive integer n such that $2^{10}+2^{13}+2^{n}$ is a perfect square number.

smallest n =

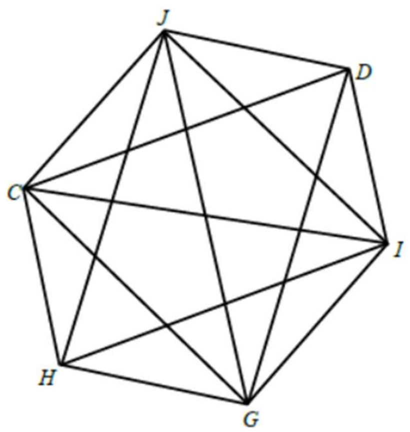
2. 設 $a^2+b^2+6a-14b+58=0$。求 $b-a$ 的值。
Suppose $a^2+b^2+6a-14b+58=0$. Find the value of $b-a$.

$b-a$ =

3. 在正方形土地的某一個角落裡埋著一個裝有$8,000 的箱子。在一次比賽中，你和另一個叫「倒霉先生」的人一起挖箱子。倒霉先生有一個特點：他總是做出錯誤的選擇。你贏了擲骰子先選。你選了一個角落，倒霉先生選了另一個角落。在你準備開始時，你發現倒霉先生沒有找到箱子。遊戲規則允許你換另一個角落，但要罰$200。計算換角落的期望收益。
There was a chest containing $8,000 buried in one of the corners of a square piece of land. In a contest, you and another man called "Mr. Badluck" were digging for the chest. Mr. Badluck had one peculiarity: he always made the wrong choice. You won the toss and chose first. You picked a corner, and Mr. Badluck picked another. Before you started, you observed that Mr. Badluck found no chest. The rules of the game allowed you to make a switch to another corner, but with a penalty of $200. Calculate the expected gain from making the switch in dollars.

expected gain =

4. 一個凸六邊形有以下性質：
(i) 由任意頂點與相鄰兩個頂點組成的三角形的面積都是1,000 cm^2；及
(ii) $CH = DI$。
求六邊形的面積。
A convex hexagon has the following property:
(i) all the triangles formed from any vertex with the two adjacent vertices have an area of 1,000 cm^2 ; and
(ii) $CH = DI$.
Find the area of the hexagon.



Area =

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Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (2023 – 2024)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a、b 為非零整數，且滿足方程 $a - ab + b = 18$。求 $a + b$ 的值。
Let a , b be non-zero integers satisfying the equation $a - ab + b = 18$.
Find the value of $a + b$.

$a + b =$

2. 設 x 為一正整數，且滿足 $x(x + 1)(x + 2)(x + 3) = 3024$。求 x 的值。
Let x be a positive integer satisfying $x(x + 1)(x + 2)(x + 3) = 3024$. Find the value of x .

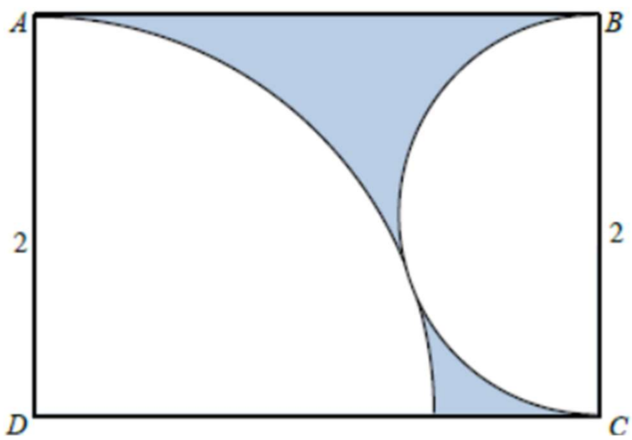
$x =$

3. 設 α、β 為二次方程 $x^2 + 6x + 2 = 0$ 的兩個根，求以 $\dfrac{\alpha^2}{\beta}$ 和 $\dfrac{\beta^2}{\alpha}$ 為根及 x^2 的系數為 1 的二次方程。
Let α, β be the two roots of the quadratic equation $x^2 + 6x + 2 = 0$.
Find the quadratic equation whose roots are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$, and coefficient of x^2 is 1 .

equation:

4. 下圖空白部分由一個四分一圓和一個半圓互相外切組成。*ABCD*是一個長方形。求陰影部分的面積。
The unshaded part in the diagram below is made up of a quarter-circle and a semi-circle which touch each other externally. *ABCD* is a rectangle.
Find the area of the shaded part.

shaded area =



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Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
			Total score				Min.	Sec.

成績 Result

Champion	Diocesan Boys' School 拔萃男書院
1st runner up	Hong Kong Chinese Women's Club College 香港中國婦女會中學
2nd runner up	Pui Ching Middle School 香港培正中學