# Hong Kong Mathematics Olympiad (1997-98)
## Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ 及 $\frac{2}{a}-\frac{3}{u}=6$ 為 $a$ 與 $u$ 的聯立方程。求 $a$ 的解。

Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in $a$ and $u$. Solve for $a$.

$a$ =

(ii) 方程 $px + qy + bz = 1$ 的根分別為(0, 3$a$, 1)、(9$a$, –1, 2)和(0, 3$a$, 0)。求係數 $b$ 的值。

Three solutions of the equation $px + qy + bz = 1$ are (0, 3$a$, 1), (9$a$, –1, 2) and (0, 3$a$, 0). Find the value of the coefficient $b$.

$b$ =

(iii) 若 $y = mx + c$ 的圖像經過 ($b$ + 4, 5) 及 (–2, 2) 兩點。求 $c$ 的值。

Find the value of $c$ so that the graph of $y = mx + c$ passes through the two points ($b$ + 4, 5) and (–2, 2).

$c$ =

(iv) 不等式 $x^2 + 5x - 2c \le 0$ 的解為 $d \le x \le 1$。求 $d$ 的值。

The solution of the inequality $x^2 + 5x - 2c \le 0$ is $d \le x \le 1$. Find the value of $d$.

$d$ =

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 1 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a$ 是 $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$ 的最大值，求 $a$ 的值。

If $a$ is the maximum value of $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$, find the value of $a$.

$a$ =

(ii) 若 $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$，求 $b$ 的值。

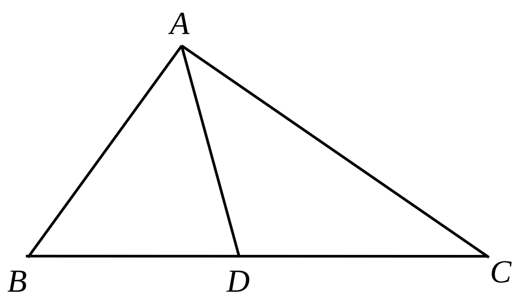
If $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$, find the value of $b$.

$b$ =

(iii) 在圖中，$BD = b$ cm，$DC = c$ cm，且$\Delta ABD$ 的面積$=\frac{1}{3}\times\Delta ABC$ 的面積，求 $c$ 的值。

In the figure, $BD = b$ cm, $DC = c$ cm and area of $\Delta ABD = \frac{1}{3}\times$area of $\Delta ABC$, find the value of $c$.

$c$ =




(iv) 設 $d$ 為 $500 + c$ 的正因數的數目，求 $d$ 的值。
Suppose $d$ is the number of positive factors of $500 + c$, find the value of $d$.

$d$ =

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 2 (Individual)**

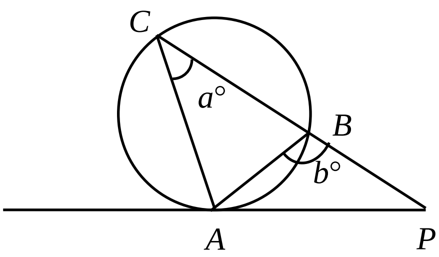
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $A(1, 3)$、$B(5, 8)$及 $C(29, a)$共綫，求 $a$ 的值。
If $A(1, 3)$, $B(5, 8)$ and $C(29, a)$ are collinear, find the value of $a$.

$a$ =

(ii) 在圖中，$PA$ 切圓 $ABC$ 於 $A$。$PBC$ 為一直綫、$AB = BP$、$\angle ACB = a°$。若 $\angle ABP = b°$，求 $b$ 的值。
In the figure, $PA$ touches the circle $ABC$ at $A$, $PBC$ is a straight line, $AB = PB$, $\angle ACB = a°$. If $\angle ABP = b°$, find the value of $b$ .

$b$ =




(iii) 若 $c$ 為二次函數 $y = x^2 + 4x + b$ 之最小值，求 $c$ 的值。
If $c$ is the minimum value of the quadratic function $y = x^2 + 4x + b$, find the value of $c$.

$c$ =

(iv) 若 $d = 1 - 2 + 3 - 4 + \ldots - c$，求 $d$ 的值。
If $d = 1 - 2 + 3 - 4 + \ldots - c$, find the value of $d$ .

$d$ =

**FOR OFFICIAL USE**

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.

**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 3 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\{p, q\} = q \times a + p$ 且 $\{2, 5\} = 52$，求 $a$ 的值。
If $\{p, q\} = q \times a + p$ and $\{2, 5\} = 52$, find the value of $a$.

$a =$

(ii) 若數列 $a$，$\frac{37}{2}$，$b$ 為一等差數列，求 $b$ 的值。
If $a, \frac{37}{2}, b$ is an arithmetic progression, find the value of $b$.

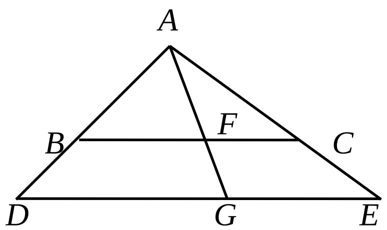
$b =$

(iii) 若 $b^2 - c^2 = 200$ 及 $c > 0$，求 $c$ 的值。
If $b^2 - c^2 = 200$ and $c > 0$, find the value of $c$.

$c =$

(iv) 在圖中，已知 $BC \parallel DE$、$BC : DE = 10 : c$ 及 $AF : FG = 20 : d$，求 $d$ 的值。
Given that in the figure, $BC \parallel DE$, $BC : DE = 10 : c$ and $AF : FG = 20 : d$, find the value of $d$.

$d =$




**FOR OFFICIAL USE**

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.

# Hong Kong Mathematics Olympiad (1997-98)
## Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\dfrac{10x-3y}{x+2y}=2$ 且 $p=\dfrac{y+x}{y-x}$，求 $p$ 的值。
Given that $\dfrac{10x-3y}{x+2y}=2$ and $p=\dfrac{y+x}{y-x}$, find the value of $p$.

$p =$

(ii) 已知 $a \neq b$ 且 $ax = bx$。若 $p + q = 19(a - b)^x$，求 $q$ 的值。
Given that $a \neq b$ and $ax = bx$. If $p + q = 19(a - b)^x$, find the value of $q$.

$q =$

(iii) 已知 $q$ 個連續數之和為 222，其中最大的是 $r$，求 $r$ 的數值。
Given that the sum of $q$ consecutive numbers is 222, and the largest of these consecutive numbers is $r$, find the value of $r$.

$r =$

(iv) 若 $\tan^2(r + s)^\circ = 3$ 且 $0 \leq r + s \leq 90$，求 $s$ 的值。
If $\tan^2(r + s)^\circ = 3$ and $0 \leq r + s \leq 90$, find the value of $s$.

$s =$

**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

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Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 5 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若方程 $5x^2 + ax - 2 = 0$ 的根的和為它的根的積的兩倍，求 $a$ 的值。
If the sum of roots of $5x^2 + ax - 2 = 0$ is twice the product of its roots, find the value of $a$.

$a$ =

(ii) 已知 $y = ax^2 - bx - 13$ 穿過(3, 8)，求 $b$ 的值。
Given that $y = ax^2 - bx - 13$ passes through (3, 8), find the value of $b$.

$b$ =

(iii) 若有 $c$ 種排法把 $b$ 位女孩排成一圓，求 $c$ 的值。
If there are $c$ ways of arranging $b$ girls in a circle, find the value of $c$.

$c$ =

(iv) 若 $\frac{c}{4}$ 條直綫和 3 個圓畫於一白紙上，且它們的最多交點數量為 $d$，求 $d$ 的值。
If $\frac{c}{4}$ straight lines and 3 circles are drawn on a paper, and $d$ is the largest numbers of points of intersection, find the value of $d$.

$d$ =

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1997-98)**
**Sample Event (Group)**

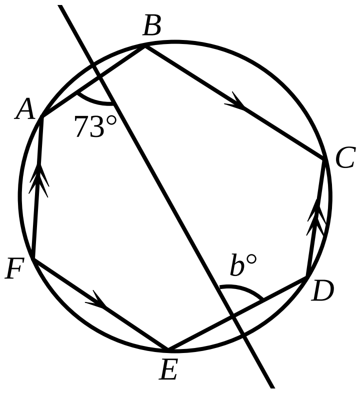
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a$ 是最小的正整數被 3 除時餘 1 而能被 5 整除，求 $a$ 的值。
If $a$ is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find the value of $a$.

$a$ =

(ii) 下圖中，$FA//DC$ 及 $FE//BC$。求 $b$ 的值。
In the following diagram, $FA//DC$ and $FE//BC$. Find the value of $b$.

$b$ =




(iii) 若 $c$ 是一兩位正整數，其兩位之和是 10 而兩位之積是 25，求 $c$ 的值。
If $c$ is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of $c$.

$c$ =

(iv) 若 $S_1, S_2, \ldots, S_{10}$ 是一由正整數組成的 A.P.的頭十項使得
$S_1 + S_2 + \ldots + S_{10} = 55$ 及$(S_{10} - S_8) + (S_9 - S_7) + \ldots + (S_3 - S_1) = d$。求 $d$ 的值。
If $S_1, S_2, \ldots, S_{10}$ are the first ten terms of an A.P. consisting of positive integers such that $S_1 + S_2 + \ldots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \ldots + (S_3 - S_1) = d$, find the value of $d$.

$d$ =

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 1 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若扇形面積 $s = 4$ cm$^2$、扇形半徑 $r = 2$ cm 及扇形的弧長 $A = p$ cm，求 $p$ 的值。
If the area of a given sector $s = 4$ cm$^2$, the radius of this sector $r = 2$ cm and the arc length of this sector $A = p$ cm, find the value of $p$.

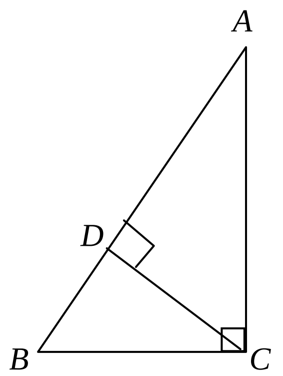
$p =$

(ii) 已知 $\frac{a}{2b+c}=\frac{b}{2c+a}=\frac{c}{2a+b}$ 且 $a+b+c \neq 0$。若 $q=\frac{2b+c}{a}$，求 $q$ 的值。
Given that $\frac{a}{2b+c}=\frac{b}{2c+a}=\frac{c}{2a+b}$ and $a+b+c \neq 0$.
If $q=\frac{2b+c}{a}$, find the value of $q$.

$q =$

(iii) 設直角三角形 $ABC$ 中，$CD$ 是斜邊 $AB$ 上的高，$AC = 3$, $DB=\frac{5}{2}$, $AD = r$，求 $r$ 的值。
Let $ABC$ be a right-angled triangle, $CD$ is the altitude on $AB$, $AC = 3$, $DB=\frac{5}{2}$, $AD = r$, find the value of $r$.

$r =$




(iv) 若 $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$，求 $a$ 的值。
If $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$, find the value of $a$.

$a =$

**FOR OFFICIAL USE**

Score for accuracy $\times$ Mult. factor for speed $=$ | Team No.

$+$ Bonus score | Time

Total score | Min. | Sec.

**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 2 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\frac{137}{a}=0.1\dot{2}3\dot{4}$ ，求 $a$ 的值。

If $\frac{137}{a}=0.1\dot{2}3\dot{4}$, find the value of $a$.

$a =$

(ii) 若 $b = 1999\times19981998 - 1998\times19991999 + 1$，求 $b$ 的數值。
If $b = 1999\times19981998 - 1998\times19991999 + 1$, find the value of $b$.

$b =$

(iii) 若參數方程 $\begin{cases} x=\sqrt{3-t^2} \\ y=t-3 \end{cases}$ 可轉換為 $x^2 + y^2 + cx + dy + 6 = 0$，求 $c$ 及 $d$ 的值。

If the parametric equation $\begin{cases} x=\sqrt{3-t^2} \\ y=t-3 \end{cases}$ can be transformed into $x^2 + y^2 + cx + dy + 6 = 0$, find the values of $c$ and $d$.

$c =$

$d =$

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 3 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在$\Delta ABC$ 中，$\angle ABC = 2\angle ACB$，$BC = 2AB$。若$\angle BAC = a^{\circ}$，求 $a$ 的值。
In $\Delta ABC$, $\angle ABC = 2\angle ACB$, $BC = 2AB$. If $\angle BAC = a^{\circ}$, find the value of $a$.

$a$ =

(ii) 已知 $x+\frac{1}{x}=\sqrt{2}$，$\frac{x^2}{x^4+x^2+1}=b$，求 $b$ 的值。
Given that $x+\frac{1}{x}=\sqrt{2}$, $\frac{x^2}{x^4+x^2+1}=b$, find the value of $b$.

$b$ =

(iii) 若方程 $x + y + 2xy = 141$ 有 $c$ 個正整數解，求 $c$ 的值。
If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is $c$, find the value of $c$.

$c$ =

(iv) 已知 $x + y + z = 0$、$x^2 + y^2 + z^2 = 1$ 及 $d = 2(x^4 + y^4 + z^4)$，求 $d$ 的值。
Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2(x^4 + y^4 + z^4)$, find the value of $d$.

$d$ =

**FOR OFFICIAL USE**

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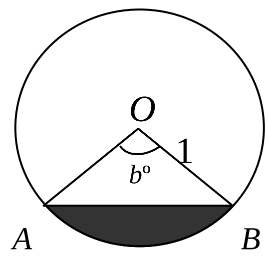
**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 4 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\ldots+0.00000000\dot{9}=a$，求 $a$ 的值。(答案以小數表示。)
If $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\ldots+0.00000000\dot{9}=a$, find the value of $a$.
(Give your answer in decimal)

$a$ =

(ii) 圖中的圓之圓心為 $O$，半徑為 1，$A$ 和 $B$ 是圓形上的點。
已知 $\dfrac{\text{陰影部分}}{\text{沒有陰影部分}}=\dfrac{\pi-2}{3\pi+2}$ 且 $\angle AOB=b^\circ$，求 $b$ 的值。
The circle in the figure has centre $O$ and radius 1, $A$ and $B$ are points on the circle.
Given that $\dfrac{\text{Area of shaded part}}{\text{Area of unshaded part}}=\dfrac{\pi-2}{3\pi+2}$ and $\angle AOB=b^\circ$, find the value of $b$.




$b$ =

(iii) 圖形 $S_0$，$S_1$，$S_2$，…用以下方法構成：把綫段[0, 1]的中間三分之一取去，得到 $S_0$，把 $S_0$ 的兩條組成綫段，每段的中間三分之一取去，得到 $S_1$，把 $S_1$ 的四條組成綫段，每段的中間三分之一取去，得到 $S_2$，$S_3$、$S_4$ …等用類似方法獲得。求在構成 $S_5$ 的過程中取去的綫段的總長度 $c$ (答案以分數表示)。
A sequence of figures $S_0$, $S_1$, $S_2$, … are constructed as follows. $S_0$ is obtained by removing the middle third of [0,1] interval; $S_1$ by removing the middle third of each of the two intervals in $S_0$; $S_2$ by removing the middle third of each of the four intervals in $S_1$; $S_3$, $S_4$, … are obtained similarly. Find the total length $c$ of the intervals removed in the construction of $S_5$ (Give your answer in fraction).

|————————(　　　　　　　　)————————| $S_0$
$0 \quad \frac{1}{3} \quad \frac{2}{3} \quad 1$

|——(　)——(　　　　　　　　)——(　)——| $S_1$
$0 \quad \frac{1}{9} \quad \frac{2}{9} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{7}{9} \quad \frac{8}{9} \quad 1$

|–( )–(　)–( ) –(　　　　　　　　)–( )–(　)–( )–| $S_2$

$c$ =

(iv) 把所有整數用下表的方法編碼。若編碼 101 至 200 的所有整數之和為 $d$，求 $d$ 的值。
All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is $d$, find the value of $d$.

| 整數 Integer | … | … | –3 | –2 | –1 | 0 | 1 | 2 | 3 | … | … |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 編碼 Code | … | … | 7 | 5 | 3 | 1 | 2 | 4 | 6 | … | … |

$d$ =

**FOR OFFICIAL USE**

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐　　Team No. ☐

+ Bonus score ☐　　Time ☐ ☐

Total score ☐　　Min.　Sec.

**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event 5 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $1\times2\times3+2\times3\times4+3\times4\times5+\ldots+10\times11\times12=a$，求 $a$ 的值。
If $1\times2\times3+2\times3\times4+3\times4\times5+\ldots+10\times11\times12=a$, find the value of $a$.

$a=$

(ii) 已知 $5^x+5^{-x}=3$。若 $5^{3x}+5^{-3x}=b$，求 $b$ 的值。
Given that $5^x+5^{-x}=3$. If $5^{3x}+5^{-3x}=b$, find the value of $b$.

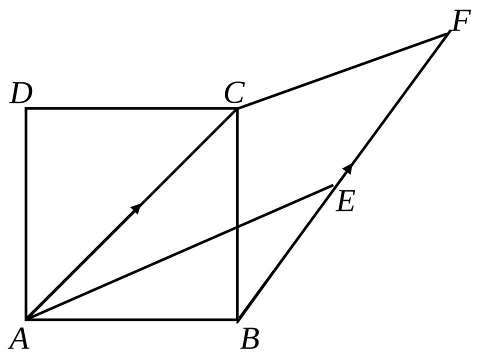
$b=$

(iii) 已知二次方程 $x^2+mx+n=0$ 的根為 98 和 99，且 $y=x^2+mx+n$。
若 $x$ 取 0、1、2、…、100，則有 $c$ 個 $y$ 的數值能被 6 整除。求 $c$ 的值。
Given that the roots of equation $x^2+mx+n=0$ are 98 and 99 and $y=x^2+mx+n$.
If $x$ takes on the values of 0, 1, 2, …, 100, then there are $c$ values of $y$ that can be divisible by 6. Find the value of $c$.

$c=$

(iv) 在圖中，$ABCD$ 為一正方形，$BF \parallel AC$，且 $AEFC$ 為一菱形。
若 $\angle EAC=d^\circ$，求 $d$ 的值。
In the figure, $ABCD$ is a square, $BF \parallel AC$, and $AEFC$ is a rhombus.
If $\angle EAC=d^\circ$, find the value of $d$.

$d=$




**FOR OFFICIAL USE**

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| Score for accuracy | | × | Mult. factor for speed | | = | | Team No. | |
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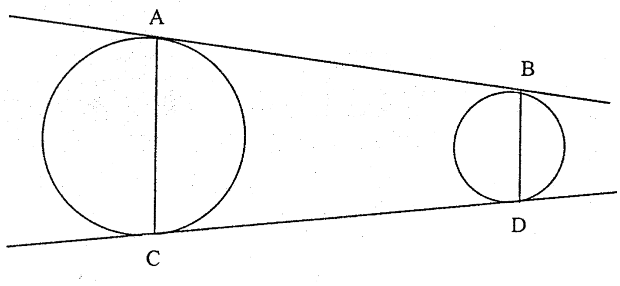
| | | | Total score | | | | Min. | Sec. |

**Hong Kong Mathematics Olympiad (1997-98)**
**Final Event Spare (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

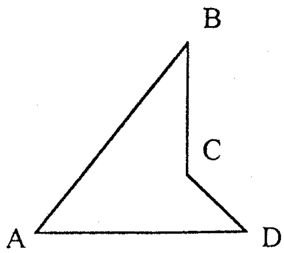
(i) 在圖中，有兩外公切綫，此外公切綫與圓相交於點 $A$、$B$、$C$ 及 $D$。
若 $AC = 9$ cm，$BD = 3$ cm，$\angle BAC = 60°$及 $AB = s$ cm，求 $s$ 的值。
In the figure, there are two common tangents. These common tangents meet the circles at points $A$, $B$, $C$ and $D$. If $AC = 9$ cm, $BD = 3$ cm, $\angle BAC = 60°$ and $AB = s$ cm, find the value of $s$.

$s =$




(ii) 在圖中，$ABCD$ 為一四邊形，其中內角$\angle A$、$\angle B$ 及$\angle D$ 均為 45°。$BC$ 的延綫與 $AD$ 互相垂直。若 $AC = 10$，$BD = b$，求 $b$ 的值。
In the figure, $ABCD$ is a quadrilateral, where the interior angles $\angle A$, $\angle B$ and $\angle D$ are all equal to 45°. When produced, $BC$ is perpendicular to $AD$.
If $AC = 10$ and $BD = b$, find the value of $b$.

$b =$




(iii) 若 $\log_c 27 = 0.75$，求 $c$ 的值。
If $\log_c 27 = 0.75$, find the value of $c$.

$c =$

(iv) 若數據 30，80，50，40，$d$ 的平均數、眾數和中位數都相等，求 $d$ 的值。
If the mean, mode and median of the data 30, 80, 50, 40, $d$ are all equal, find the value of $d$.

$d =$

**FOR OFFICIAL USE**

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.