

Limit Example

Created by Mr. Francis Hung on 20210426. Last updated: 11 February 2022.

If A and B are real numbers, evaluate $\lim_{x \rightarrow 0} \frac{\sin^2 Ax - \sin^2 Bx}{x^2}$ (type $\frac{0}{0}$)

$$\lim_{x \rightarrow 0} \frac{\sin^2 Ax - \sin^2 Bx}{x^2}$$
$$= \lim_{x \rightarrow 0} \frac{1 - \cos 2Ax}{2} - \frac{1 - \cos 2Bx}{2} \quad \text{by using double angle formula} \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$= \lim_{x \rightarrow 0} \frac{\cos 2Bx - \cos 2Ax}{2x^2}$$
$$= \lim_{x \rightarrow 0} \frac{-2 \sin(A+B)x \sin(B-A)x}{2x^2} \quad \text{by using sum to product formula}$$

$$= \lim_{x \rightarrow 0} \frac{[\sin(A+B)x]}{x} \cdot \frac{[\sin(A-B)x]}{x}$$

If $A = B$, then the limit = 0

If $A \neq B$,

$$\lim_{x \rightarrow 0} \frac{[\sin(A+B)x]}{x} \cdot \frac{[\sin(A-B)x]}{x}$$
$$= (A^2 - B^2) \lim_{x \rightarrow 0} \frac{[\sin(A+B)x]}{(A+B)x} \cdot \lim_{x \rightarrow 0} \frac{[\sin(A-B)x]}{(A-B)x}$$
$$= A^2 - B^2$$