Examples on Mathematical Induction: divisibility 32

Created by Mr. Francis Hung Last updated: September 1, 2021

1. Prove by mathematical induction $n^6 + 3n^4 + 7n^2 - 1$ is divisible by 32 for all odd positive integer n.

Let $P(n) \equiv$ "$n^6 + 3n^4 + 7n^2 - 11$ is divisible by 32 for all odd positive integer n ."

$n = 1$, $1 + 3 + 7 - 11 = 0$, which is divisible by 32 .

Suppose $k^6 + 3k^4 + 7k^2 - 11 = 32m$, where k is an odd positive integer, m is an integer.

$(k + 2)^6 + 3(k + 2)^4 + 7(k + 2)^2 - 11 - (k^6 + 3k^4 + 7k^2 - 11)$

$= [(k + 2)^6 - k^6] + 3[(k + 2)^4 - k^4] + 7[(k + 2)^2 - k^2]$

$= [(k + 2)^2 - k^2][(k + 2)^4 + (k + 2)^2k^2 + k^4] + 3[(k + 2)^2 + k^2][(k + 2)^2 - k^2] + 7[(k + 2)^2 - k^2]$

$= (4k+4)(3k^4 + 12k^3 + 28k^2 + 32k + 16 + 6k^2 + 12k + 12 + 7)$

$= 4(k + 1)(3k^4 + 12k^3 + 34k^2 + 44k + 35)$

$= 4(k + 1)(3k^4 + 6k^2 + 3 + 12k^3 + 28k^2 + 44k + 32)$

$= 4(k + 1)[3(k^2 + 1)^2 + 4(3k^3 + 7k^2 + 11k + 8)]$

$\because$ k is odd, $\therefore k + 1$ and $k^2 + 1$ are even.

$3(k^2 + 1)^2 + 4(3k^3 + 7k^2 + 11k + 8)$ is divisible by 4

$4(k + 1)[3(k^2 + 1)^2 + 4(3k^3 + 7k^2 + 11k + 8)]$ is divisible by 32 .

$(k + 2)^6 + 3(k + 2)^4 + 7(k + 2)^2 - 11$ is divisible by 32 .

If $P(k)$ is true then $P(k+2)$ is also true.

By the principle of Mathematical Induction, $P(n)$ is true for all positive integer n .