

1991 FG6.1-2

某兩位數 x 之個位數字是 M ，十位數字是 N 。另一兩位數 y 之個位數字是 N ，十位數字是 M 。若 $x > y$ ，且他們的和是他們的差的十一倍，求 M 及 N 的值。

A 2-digit number x has M as the units digit and N as the tens digit. Another 2-digit number y has N as the units digit and M as the tens digit. If $x > y$ and their sum is equal to eleven times their differences, find the values of M and N .

1993 FG8.1-2

已知方程 $x^2 + (m+1)x - 2 = 0$ 有兩整數根 $(\alpha+1)$ 及 $(\beta+1)$ ，且 $\alpha < \beta$ 及 $m \neq 0$ 。設 $d = \beta - \alpha$ 。求 m 及 d 的值。

Given that the equation $x^2 + (m+1)x - 2 = 0$ has 2 integral roots $(\alpha+1)$ and $(\beta+1)$ with $\alpha < \beta$ and $m \neq 0$. Let $d = \beta - \alpha$. Find the values of m and d .

1995 FI3.3

已知方程 $x + 6 + 8k = k(x+8)$ 有正整數解。求 k 的最小值 c 。

It is given that the equation $x + 6 + 8k = k(x+8)$ has positive integral solution. Find c , the least value of k .

1996 HG8

若質數 a, b 為二次方程 $x^2 - 21x + t = 0$ 的根，求 $\left(\frac{b}{a} + \frac{a}{b}\right)$ 的值。

If prime numbers a, b are the roots of the quadratic equation $x^2 - 21x + t = 0$, find the value of $\left(\frac{b}{a} + \frac{a}{b}\right)$.

1996 FG7.1

若方程 $ax^2 - mx + 1996 = 0$ 的兩個不等根是質數，求 a 的值。

If the two distinct roots of the equation $ax^2 - mx + 1996 = 0$ are primes, find the value of a .

1999 FG2.4

設 d 為奇質數，若 $89 - (d+3)^2$ 是一整數之平方，求 d 之值。

Let d be an odd prime number.

If $89 - (d+3)^2$ is the square of an integer, find the value of d .

2001 HI6

88 張成人車票總值為 \$□293□，由於列印機壞了，五位數字的首尾兩個數字印不出來。已知每張車票的價值為 \$ P ，其中 P 為一整數，求 P 的值。

The total cost for 88 tickets was \$□293□. Because the printing machine was not functioning well, the first and the last digits of the 5-digit number were missing. If the cost for each ticket is \$ P , where P is an integer, find the value of P .

2001 FG3.2

已知方程 $x^2y - x^2 - 3y - 14 = 0$ 只得一組正整數解 (x_0, y_0) 。

若 $x_0 + y_0 = b$ ，求 b 的值。

Suppose the equation $x^2y - x^2 - 3y - 14 = 0$ has only one positive integral solution (x_0, y_0) . If $x_0 + y_0 = b$, find the value of b .

2001 FG4.4

方程 $x^2 - 45x + m = 0$ 的兩個根皆為質數。

已知兩根的平方和為 d ，求 d 的值。

The roots of the equation $x^2 - 45x + m = 0$ are prime numbers.

Given that the sum of the squares of the roots is d , find the value of d .

2004 FG3.2

已知質數 p 和 q 滿足方程 $18p + 30q = 186$ 。

若 $\log_8 \frac{p}{3q+1} = b \geq 0$ ，求 b 的值。

Given that p and q are prime numbers satisfying the equation $18p + 30q = 186$.

If $\log_8 \frac{p}{3q+1} = b \geq 0$, find the value of b .

2005 FG1.2

設質數 p 和 q 是方程 $x^2 - 13x + R = 0$ 的兩個不同的根，其中 R 是實數。

若 $b = p^2 + q^2$ ，求 b 的值。

Let p and q be prime numbers that are the two distinct roots of the equation $x^2 - 13x + R = 0$, where R is a real number. If $b = p^2 + q^2$, find the value of b .

2008 HI3

已知 x_0 及 y_0 為正整數且滿足方程 $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ 。

若 $35 < y_0 < 50$ 及 $x_0 + y_0 = z_0$ ，求 z_0 的值。

Given that x_0 and y_0 are positive integers satisfying the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$.

If $35 < y_0 < 50$ and $x_0 + y_0 = z_0$, find the value of z_0 .

2009 HG3

已知 p 和 q 為整數。若 $\frac{2}{p} + \frac{1}{q} = 1$ ，求 $p \times q$ 的最大值。

Given that p and q are integers. If $\frac{2}{p} + \frac{1}{q} = 1$, find the maximum value of $p \times q$.

2010 HI4

已知 $x + y + z = 3$ 及 $x^3 + y^3 + z^3 = 3$ ，且 x, y, z 為整數。

若 $x < 0$ ，求 y 的值。

Given that $x + y + z = 3$ and $x^3 + y^3 + z^3 = 3$, where x, y, z are integers.

If $x < 0$, find the value of y .

2010 HI5

已知 a, b, c, d 為正整數，且滿足 $\log_a b = \frac{1}{2}$ 及 $\log_c d = \frac{3}{4}$ 。

若 $a - c = 9$ ，求 $b - d$ 的值。

Given that a, b, c, d are positive integers satisfying $\log_a b = \frac{1}{2}$ and $\log_c d = \frac{3}{4}$.

If $a - c = 9$, find the value of $b - d$.

2010 FG4.1

設 a 為整數及 $a \neq 1$ 。已知方程 $(a - 1)x^2 - mx + a = 0$ 的兩根均為正整數。求 m 的值。

Let a be an integer and $a \neq 1$. Given that the equation $(a - 1)x^2 - mx + a = 0$ has two roots which are positive integers. Find the value of m .

2010 FG4.3

已知 A, B, C 為正整數，且 A, B 和 C 的最大公因數等於 1。

若 A, B, C 滿足 $A \log_{500} 5 + B \log_{500} 2 = C$ ，求 $A + B + C$ 的值。

Given that A, B, C are positive integers with their greatest common divisor equal to 1. If A, B, C satisfy $A \log_{500} 5 + B \log_{500} 2 = C$, find the value of $A + B + C$.

2010 FGS.4

共有多少個正整數 m 使得通過點 $A(-m, 0)$ 及點 $B(0, 2)$ 的直線亦通過 $P(7, k)$ ，其中 k 為一正整數？

How many positive integers m are there for which the straight line passing through points $A(-m, 0)$ and $B(0, 2)$ and also passes through the point $P(7, k)$, where k is a positive integer?

2011 FI2.1

若方程組 $\begin{cases} x + y = P \\ 3x + 5y = 13 \end{cases}$ 的解為正整數，求 P 的值。

If the solution of the system of equations $\begin{cases} x + y = P \\ 3x + 5y = 13 \end{cases}$ are positive integers, find the value of P .

2011 FI2.3

若 a 及 b 為相異質數且 $a^2 - 5a + R = 0$ 及 $b^2 - 5b + R = 0$ ，求 R 的值。

If a and b are distinct prime numbers and $a^2 - 5a + R = 0$ and $b^2 - 5b + R = 0$, find the value of R .

2011 FGS.4

設 F 為方程 $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$ 的整數解的數目。求 F 的值。

Let F be the number of integral solutions of $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$. Find the value of F .

2012 HI6

已知 a 及 b 為不同質數，且 $a^2 - 19a + m = 0$ 及 $b^2 - 19b + m = 0$ ，

求 $\frac{a}{b} + \frac{b}{a}$ 的值。

Given that a and b are distinct prime numbers, $a^2 - 19a + m = 0$

and $b^2 - 19b + m = 0$. Find the value of $\frac{a}{b} + \frac{b}{a}$.

2012 HI8

若方程 $(k^2 - 4)x^2 - (14k + 4)x + 48 = 0$ 有兩個相異的正整數根，求 k 的值。

If the quadratic equation $(k^2 - 4)x^2 - (14k + 4)x + 48 = 0$ has two distinct positive integral roots, find the value(s) of k .

2012 HI9

已知 x, y 為正整數，且 $x > y$ ，解 $x^3 = 2189 + y^3$ 。

Given that x, y are positive integers and $x > y$, solve $x^3 = 2189 + y^3$.

2012 HG1

已知 x, y 及 z 為三個連續正整數，且 $\frac{y}{x} + \frac{z}{x} + \frac{x}{y} + \frac{z}{y} + \frac{x}{z} + \frac{y}{z}$ 為整數，

求 $x + y + z$ 的值。

Given that x, y and z are three consecutive positive integers, and

$\frac{y}{x} + \frac{z}{x} + \frac{x}{y} + \frac{z}{y} + \frac{x}{z} + \frac{y}{z}$ is an integer. Find the value of $x + y + z$.

2013 HI4

已知 $x^2 + 399 = 2^y$ ，其中 x, y 為正整數。求 x 的值。

Given that $x^2 + 399 = 2^y$, where x, y are positive integers. Find the value of x .

2015 FG3.2

設 p 為質數及 m 為整數。若 $p(p + m) + 2p = (m + 2)^3$ ，找出 m 的最大可能值。

Let p be a prime and m be an integer.

If $p(p + m) + 2p = (m + 2)^3$, find the greatest possible value of m .

2016 HI4

若 x 、 y 為整數，有多少對 x 、 y 且滿足 $(x+1)^2 + (y-2)^2 = 50$?

If x, y are integers, how many pairs of x, y are there which satisfy the equation

$$(x+1)^2 + (y-2)^2 = 50 ?$$

2016 HG9

設整數 a 、 b 及 c 為三角形的邊長。已知 $f(x) = x(x-a)(x-b)(x-c)$ ，且 x 為一個大於 a 、 b 及 c 的整數。若 $x = (x-a) + (x-b) + (x-c)$ 及 $f(x) = 900$ ，求該三角形三條垂高的總和。

Let the three sides of a triangle are of lengths a, b and c where all of them are integers. Given that $f(x) = x(x-a)(x-b)(x-c)$ where x is an integer of size greater than a, b and c .

If $x = (x-a) + (x-b) + (x-c)$ and $f(x) = 900$,

find the sum of the lengths of the three altitudes of this triangle .

2017 HG10

已知方程 $a^2x^2 - (4a - 3a^2)x + 2a^2 - a - 21 = 0$ (其中 $a > 0$) 最少 **有** 一個整數根，求所有 a 的可能整數值之和。

It is given that the equation $a^2x^2 - (4a - 3a^2)x + 2a^2 - a - 21 = 0$ (where $a > 0$) has at least one **integral** root. Find the sum of all possible integral values of a .

2018 HG6

已知 $n^4 + 104 = 3^m$ ，其中 n 、 m 為正整數。求 n 的最小值。

Given that $n^4 + 104 = 3^m$, where n, m are positive integers.

Find the least value of n .

2023 FI4.2

如果方程組
$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + 8z^2 = 100 \end{cases}$$
 整數解的數量是 γ ，求 γ 的值。

If the system of equations
$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + 8z^2 = 100 \end{cases}$$
 has γ sets of **integral**

solutions, find **the value of** γ .

Answers

1991 FG6.1-2 $M = 4, N = 5$	1993 FG8.1-2 $m = -2, d = 3$	1995 FI3.3 2	1996 HG8 $\frac{365}{38}$	1996 FG7.1 2
1999 FG2.4 5	2001 HI6 147	2001 FG3.2 20	2001 FG4.4 1853	2004 FG3.2 0
2005 FG1.2 125	2008 HI3 64	2009 HG3 9	2010 HI4 4	2010 HI5 -3
2010 FG4.1 3	2010 FG4.3 6	2010 FGS.4 4	2011 FI2.1 3	2011 FI2.3 6
2011 FGS.4 208	2012 HI6 $\frac{293}{34}$	2012 HI8 4	2012 HI9 $x = 13, y = 2$	2012 HG1 6
2013 HI4 25	2015 FG3.2 0	2016 HI4 12	2016 HG9 $\frac{281}{13}$	2017 HG10 11
2018 HG6 5	2023 FI4.2 0			