

1991 HI2

某科學家發現某樣本中細菌的數量每小時增加一倍。

於下午四時，他發現細菌的數量為 3.2×10^8 ，

若於同日正午該樣本中細菌的數量為 $N \times 10^7$ ，求 N 的值。

A scientist found that the population of a bacteria culture doubled every hour. At 4:00 pm, he found that the number of bacteria was 3.2×10^8 . If the number of bacteria in that culture at noon on the same day was $N \times 10^7$, find the value of N .

1994 HI1

設 $\log_3 p = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ 至無窮項，求 p 的值。

Suppose $\log_3 p = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ to an infinite number of terms.

Find the value of p .

1997 FG1.3

若 $1 + 3 + 3^2 + \dots + 3^8 = \frac{3^c - 1}{2}$ ，求 c 的值。

If $1 + 3 + 3^2 + \dots + 3^8 = \frac{3^c - 1}{2}$, find the value of c .

1998 HI2

已知 $8, a, b$ 形成一等差級數，且 $a, b, 36$ 形成一等比級數。

若 a 和 b 皆為正數，求 a, b 的和。

Given that $8, a, b$ form an A.P. and $a, b, 36$ form a G.P.

If a and b are both positive numbers, find the sum of a and b .

1998 FG4.3

圖形 S_0, S_1, S_2, \dots 用以下方法構成：把綫段 $[0, 1]$ 的中間三分之一取去，得到 S_0 ，把 S_0 的兩條組成綫段，每段的中間三分之一取去，得到 S_1 ，把 S_1 的四條組成綫段，每段的中間三分之一取去，得到 $S_2, S_3, S_4 \dots$ 等用類似方法獲得。求在構成 S_5 的過程中取去的綫段的總長度 c (答案以分數表示)。

A sequence of figures S_0, S_1, S_2, \dots are constructed as follows. S_0 is obtained by removing the middle third of $[0, 1]$ interval; S_1 by removing the middle third of each of the two intervals in S_0 ; S_2 by removing the middle third of each of the four intervals in S_1 ; S_3, S_4, \dots are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 (Give your answer in fraction).

—————()—————| S_0

0 $\frac{1}{3}$ $\frac{2}{3}$ 1
 |——()——()——| S_1
 0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{7}{9}$ $\frac{8}{9}$ 1
 |——()——()——()——()——| S_2

2001 FI3.3

若 $\sin 30^\circ + \sin^2 30^\circ + \dots + \sin^7 30^\circ = 1 - \cos^R 45^\circ$ ，求 R 的值。

If $\sin 30^\circ + \sin^2 30^\circ + \dots + \sin^7 30^\circ = 1 - \cos^R 45^\circ$, find the value of R .

2002 FI2.2

已知 $99Q = \frac{99}{100} \times (1 + \frac{99}{100} + \frac{99^2}{100^2} + \frac{99^3}{100^3} + \dots)$ ，求 Q 的值。

Given that $99Q = \frac{99}{100} \times (1 + \frac{99}{100} + \frac{99^2}{100^2} + \frac{99^3}{100^3} + \dots)$, find the value of Q .

2005 FG2.4

設 $d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots + \frac{10}{2^{10}}$ ，求 d 的值。

Let $d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots + \frac{10}{2^{10}}$, find the value of d .

2006 HG3

已知 $0^\circ < \theta < 90^\circ$ 及 $1 + \sin \theta + \sin^2 \theta + \dots = \frac{3}{2}$ 。若 $y = \tan \theta$ ，求 y 的值。

Given that $0^\circ < \theta < 90^\circ$ and $1 + \sin \theta + \sin^2 \theta + \dots = \frac{3}{2}$.

If $y = \tan \theta$, find the value of y .

2007 FG2.1

若 $R = 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + \dots + 10 \times 2^{10}$ ，求 R 的值。

If $R = 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + \dots + 10 \times 2^{10}$, find the value of R .

2009 FI1.3

設 $F = 1 + 2 + 2^2 + 2^3 + \dots + 2^{120}$ 及 $T = \sqrt{\frac{\log(1+F)}{\log 2}}$ ，求 T 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \dots + 2^{120}$ and $T = \sqrt{\frac{\log(1+F)}{\log 2}}$, find the value of T .

2010 FG2.1

若 $p = 2 - 2^2 - 2^3 - 2^4 - \dots - 2^9 - 2^{10} + 2^{11}$ ，求 p 的值。

If $p = 2 - 2^2 - 2^3 - 2^4 - \dots - 2^9 - 2^{10} + 2^{11}$, find the value of p .

2012 HI5

已知 $\log_4 N = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ ，求 N 的值。

Given that $\log_4 N = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$, find the value of N .

2015 FI1.4

若 n 為正整數及 $f(n) = 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^2 + 2^1 + 1$ ，求 $\delta = f(10)$ 的值。

If n is a positive integer and $f(n) = 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^2 + 2^1 + 1$,

determine the value of $\delta = f(10)$.

2017 FI3.4

若 $f(x) = 2^0 + 2^1 + 2^2 + \dots + 2^{x-2} + 2^{x-1}$ ，求 $d = f(10)$ 的值。

If $f(x) = 2^0 + 2^1 + 2^2 + \dots + 2^{x-2} + 2^{x-1}$, determine the value of $d = f(10)$.

2019 HI4

設 n 為正整數。若 $a_n = 1 + 2 + \dots + 2^n$ 及 $b = a_{10} - a_5 + a_1$ ，求 b 的值。

Let n be a positive integer. If $a_n = 1 + 2 + \dots + 2^n$ and $b = a_{10} - a_5 + a_1$, find the value of b .

2023 FI2.2

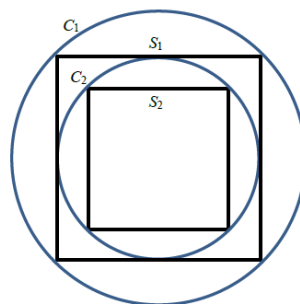
C_1 是正方形 S_1 的外接圓，它的半徑為 9， C_2 是正方形 S_1 的內切圓；

同時也是正方形 S_2 的外接圓，如此類推。求正方形 S_6 的面積 β 。

A circle C_1 of radius 9 circumscribes a square S_1 which inscribes a circle C_2 .

C_2 circumscribes square S_2 and so forth indefinitely.

Find the area β of the square S_6 .

**2024 HI13**

求 $S = \frac{1}{2024} - \frac{3}{2024^2} + \frac{5}{2024^3} - \frac{7}{2024^4} + \frac{9}{2024^5} - \dots$ 的值。

Find the value of $S = \frac{1}{2024} - \frac{3}{2024^2} + \frac{5}{2024^3} - \frac{7}{2024^4} + \frac{9}{2024^5} - \dots$.

Answers

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|------------------|---|-------------------------------------|----------------------------------|---------------------------------|
| 1991 HI2 2 | 1994 HI1 9 | 1997 FG1.4 9 | 1998 HI2 40 | 1998 FG4.3 $\frac{665}{729}$ |
| 2001 FI3.3 14 | 2002 FI2.2 1 | 2005 FG2.4 $\frac{509}{256}$ | 2006 HG3 $\frac{\sqrt{2}}{4}$ | 2007 FG2.1 18434 |
| 2009 FI1.3 11 | 2010 FG2.1 6 | 2012 HI5 8 | 2015 FI1.4 2047 | 2017 FI3.4 1023 |
| 2019 HI4 1987 | 2023 FI2.2 $\frac{81}{16} = 5\frac{1}{16}$ | 2024 HI13 $\frac{2023}{4100625}$ | | |