

## 第四十屆香港數學競賽 (2022/23)

### 決賽規則

1. 競賽共分八項，個人及團體各佔四項。
2. 每隊由已報名參加初賽的同學組成。其中任何四位可參加「個人項目」；又其中任何四位可參加「團體項目」。不足四位同學的隊伍將不獲准出賽。
3. 每隊隊員必須穿著整齊校服，並由負責老師帶領，於上午9時正或以前向會場接待處註冊，同時必須出示身分證/學生證明文件，否則將被撤銷參賽資格。
4. 粵語將會被採用為指示語言。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英並列。
5. 每一「個人項目」包括四部份。每一隊員回答其中一部份，其他隊友不得從旁協助，否則此項目所得分數會被取消。
6. 「個人項目」中，四部份互有關連。解答第二部份之隊員需利用第一部份之答案，如此類推。
7. 每一「團體項目」亦包括四部份。但各部份不一定相關，且可由全隊共同作答。各隊員可進行討論，但必須將聲浪降至最低。
8. 比賽時，參賽者不可使用計算工具，違例者將被取消資格或扣分。
9. 參賽者如有攜帶電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，應把它關掉，並放入手提包內或座位的椅下，否則大會有權取消該隊參賽資格。
10. 除另有聲明外，所有答案須為數字，並應化簡，但無需呈交證明及算草。
11. 每一項目限時五分鐘。
12. 計分辦法如下：

(甲) 準確分:	個人項目	積分	團體項目	積分
	答對第一部分	1	答對任何一部分	2
	答對第二部分	2	答對任何兩部分	4
	答對第三部分	3	答對任何三部分	7
	答對第四部分	4	答對所有四部分	10
	合共	10		

(乙) 快捷分	積分所乘倍數
參賽隊伍完成並交出答案的時間 < 1 分鐘	4
1 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 2 分鐘	3
2 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 3 分鐘	2
參賽隊伍完成並交出答案的時間 ≥ 3 分鐘	1

#### (丙) 獎勵分

任何一隊在某一個人/團體項目競賽中，若全部答對時，可額外獲得 20 分。

#### (丁) 每項目之總分

準確分×倍數 + 獎勵分

13. 如有任何疑問，參賽者須於最後一項個人/團體賽完畢後 10 分鐘內向評判團提出。所有提出之疑問，將由評判團作最後裁決。
14. 得分最高之三隊將獲得獎盃及獎品。
15. 總成績將由評判團作最後裁決。

**The Fortieth Hong Kong Mathematics Olympiad (2022/23)**  
**Regulations (Final Events)**

1. The competition consists of 8 events, which are divided into 4 individual events and 4 group events.
2. Each participating team should consist of students who have enrolled in the Heats. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event. Teams of less than 4 members will not be allowed to participate.
3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception **not later than 9:00 a.m.** Failing to do so, the team **will be disqualified.**
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, instructions written in both Chinese and English will be provided. Question papers are printed in both English and Chinese.
5. Each individual event consists of 4 parts. Each part must be completed by one member of the team. Help from other team members would result in disqualification for that particular event.
6. In an individual event, the four parts are interrelated. When solving Part 2, one has to make use of the answer obtained in Part 1, and so on.
7. In a group event, the four parts are to be done by the whole team and the parts may or may not be interrelated. Discussions are allowed provided that voice level is kept to a minimum.
8. Use of calculating devices will not be allowed; otherwise the team will risk disqualification or deduction of marks.
9. Participants having electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) or any alarm device(s), should turn them off and put them inside their bags or under their chairs. Failing to do so, the team **will risk disqualification.**
10. All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or working is required.
11. The time limit for each event is 5 minutes.
12. The Marking System is as follows:
  - (a) Scores for accuracy:
 

<u>Individual Events</u>	<u>Scores</u>	<u>Group Events</u>	<u>Scores</u>
Part 1 correct ...	1	Any 1 part correct	...2
Part 2 correct ...	2	Any 2 parts correct	...4
Part 3 correct ...	3	Any 3 parts correct	...7
Part 4 correct ...	4	All 4 parts correct	...10
Total .....	10		
  - (b) Multiplying factors for speed:
 

<i>Time taken for the teams to hand in their answer &lt; 1 min.</i>	<i>4</i>
<i>1 min. ≤ Time taken for the teams to hand in their answer &lt; 2 min.</i>	<i>3</i>
<i>2 min. ≤ Time taken for the teams to hand in their answer &lt; 3 min.</i>	<i>2</i>
<i>Time taken for the teams to hand in their answer ≥ 3 min.</i>	<i>1</i>
  - (c) Bonus Score:  
Teams, which hand in their answers of anyone individual/group event have all the answers in that event correct, will be awarded a bonus score of 20 marks.
  - (d) Total score for each event:  
(Score for accuracy) × (Multiplying factor) + (Bonus score)
13. Any queries should reach the Judging Panel within 10 minutes after the end of the last individual group event. The decision of the Judging Panel on the queries is final.
14. Trophies and prizes will be given to the three schools achieving the highest scores.
15. The decision of the Judging Panel on the overall results is final.

比賽資料 決賽隊伍數目：50 決賽日期：2023 年 5 月 13 日星期六  
決賽名單：

地點：香港教育大學

<u>School ID</u>	<u>Name of School</u>
FE-01	Assembly of God Hebron Secondary School
FE-02	Baptist Lui Ming Choi Secondary School
FE-03	Buddhist Sin Tak College
FE-04	C. & M.A. Sun Kei Secondary School
FE-05	Cheung Sha Wan Catholic Secondary School
FE-06	Carmel Holy Word Secondary School
FE-07	CCC Heep Woh College
FE-08	Cheung Chuk Shan College
FE-09	Cheung Sha Wan Catholic Secondary School
FE-10	Christian Alliance Cheng Wing Gee College
FE-11	Christian Alliance S.C. Chan Memorial College
FE-12	CNEC Christian College
FE-13	Diocesan Boys' School
FE-14	Diocesan Girls' School
FE-15	Fukien Secondary School
FE-16	Good Hope School
FE-17	Heung To Middle School Kiangsu-Chekiang College (Shatin)
FE-18	HKBUS Wong Kam Fai Secondary and Primary School
FE-19	HKTA Tang Hin Memorial Secondary School
FE-20	Ho Yu College and Primary School (Sponsored by Sik Sik Yuen)
FE-21	Hoi Ping Chamber of Commerce Secondary School
FE-22	Hong Kong Chinese Women's Club College
FE-23	King's College
FE-24	Maryknoll Convent School (Secondary Section)
FE-25	PHC Wing Kwong College
FE-26	Po Leung Kuk Centenary Li Shiu Chung Memorial College
FE-27	Po Leung Kuk Laws Foundation College
FE-28	Po Leung Kuk No. 1 WH Cheung College
FE-29	Po Leung Kuk Tang Yuk Tien College
FE-30	Po On Commerce Association Wong Siu Ching Secondary School
FE-31	Pui Ching Middle School
FE-32	Queen Elizabeth School
FE-33	Queen's College
FE-34	Sha Tin Methodist College
FE-35	Shatin Tsung Tsin Secondary School
FE-36	Sing Yin Secondary School
FE-37	Singapore International School (Hong Kong)
FE-38	SKH Bishop Mok Sau Tseng Secondary School
FE-39	St. Francis Xavier's College
FE-40	St. Mark's School
FE-41	St Paul's Co-Educational College
FE-42	STFA Leung Kau Kui College
FE-43	STFA Yung Yau College
FE-44	The Mission Covenant Church Holm Glad College
FE-45	Tseung Kwan O Government Secondary School
FE-46	Tsuen Wan Public Ho Chuen Yiu Memorial College
FE-47	Tung Wah Group of Hospitals Lo Kon Ting Memorial College
FE-48	Wah Yan College, Kowloon
FE-49	Wong Shiu Chi Secondary School
FE-50	Ying Wa College

**Hong Kong Mathematics Olympiad (2022 – 2023)**  
**Final Event 1 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.  
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 若  $A$  是  $2023^{2024}$  的個位數，求  $A$  的值。

If  $A$  is the units digit of  $2023^{2024}$ , find the value of  $A$ .

$A =$

2. 若  $B$  是  $336^A$  和  $528^A$  的正公因數的數量，求  $B$  的值。

If  $B$  is the number of positive common factors of  $336^A$  and  $528^A$ , find the value of  $B$ .

$B =$

3. 下圖是一個未完成的九宮格，每一格須填入一個正整數使得每一行、每一列和每一對角線上的三個數字總和相等。求  $C$  的值。

A  $3 \times 3$  grid is partially completed as shown below. Fill each square of the grid with a positive integer such that the sum of the three numbers in each row, column and each diagonal are equal. Find the value of  $C$ .

$C =$

$C$	16	$2B$
4		

4. 有  $\frac{C}{2}$  對夫婦參加了一個派對，即在派對上共有  $C$  人。在這個派對上，沒有人會和同一位客人重複地握手。此外，每位丈夫都會和他妻子以外的所有客人握手，而妻子們不會與其他妻子握手，但會和其他客人握手。 $D$  是在這派對上  $C$  人之間握手的總數，求  $D$  的值。

$D =$

$\frac{C}{2}$  couples are attending a party, which means that there are  $C$  people present. At this party, no one will shake hands repeatedly with the same guest. The party also has the condition that each husband will shake hands with every guest except his own wife, and wives will shake hands with every guest except other wives.  $D$  represents the total number of handshakes between the  $C$  people at the party. Find value of  $D$ .

**FOR OFFICIAL USE**

Score for accuracy

×

Mult. factor for speed

=

Team No.

+

Bonus score

Time



Total score

Min.

Sec.

**Hong Kong Mathematics Olympiad (2022 – 2023)**  
**Final Event 2 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 找出一個能被 11 整除，且各數位之和是 38 的最小正整數  $\alpha$ 。

Find the smallest positive integer  $\alpha$  that is divisible by 11 and the sum of its digits is equal to 38.

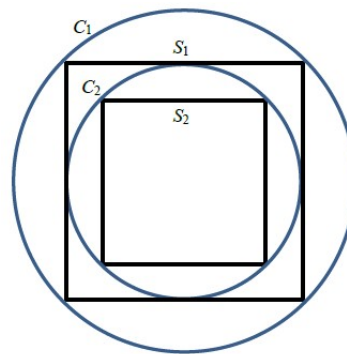
$\alpha =$

2. 若  $\alpha$  的最後一位數字是  $\alpha'$ 。  $C_1$  是正方形  $S_1$  的外接圓，它的半徑為  $\alpha'$ ，  $C_2$  是正方形  $S_1$  的內切圓；同時也是正方形  $S_2$  的外接圓，如此類推。求正方形  $S_6$  的面積  $\beta$ 。

Let  $\alpha'$  be the last digit of  $\alpha$ . A circle  $C_1$  of radius  $\alpha'$  circumscribes a square  $S_1$  which inscribes a circle  $C_2$ .

$C_2$  circumscribes square  $S_2$  and so forth indefinitely.

Find the area  $\beta$  of the square  $S_6$ .



$\beta =$

3. 設  $\beta$  的整數部分是  $[\beta]$ 。若  $m, n$  為整數，方程  $x^3 + nx^2 + mx + [\beta] = 0$  有三個整數根。假設這三個根不全是正整數，若  $\gamma = n - m$ ，求  $\gamma$  的值。

Let  $[\beta]$  be the integral part of  $\beta$ .

The equation  $x^3 + nx^2 + mx + [\beta] = 0$ , where  $m, n$  are integers, has three **integral** roots.

Suppose that the roots are not all positive, if  $\gamma = n - m$ , find **the value of  $\gamma$** .

$\gamma =$

4. 在  $x$ - $y$  座標平面上，每一步移動都包含  $x$  座標和  $y$  座標分別增加(或減少)1 個單位(即對角綫移動)。若  $\delta$  是由  $(0, 0)$  開始行走 12 步後到達  $(\gamma, \gamma)$  的方法的數目，求  $\delta$  的值。

On the  $x$ - $y$  **coordinate** plane, a move consists of independently increasing (or decreasing)  $x$ -coordinate and  $y$ -coordinate by 1 (i.e. moving diagonally). If  $\delta$  is the number of ways to start from  $(0, 0)$ , make 12 moves and end at  $(\gamma, \gamma)$ , find the value of  $\delta$ .

$\delta =$

**FOR OFFICIAL USE**

Score for accuracy

×

Mult. factor for speed

=

Team No.

+  
Bonus  
score

Time



Total score

Min.

Sec.

**Hong Kong Mathematics Olympiad (2022 – 2023)**  
**Final Event 3 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.  
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知  $m$  和  $n$  均為正整數。如果  $m + n + mn = 76$  及  $A = m + n$ ，求  $A$  的值。

Given that  $m$  and  $n$  are positive integers.

If  $m + n + mn = 76$  and  $A = m + n$ , find the value of  $A$ .

$A =$

2. 如果  $B = \sqrt{(401)^2 - 100A}$ ，求  $B$  的值。

If  $B = \sqrt{(401)^2 - 100A}$ , find the value of  $B$ .

$B =$

3. 在  $x$ - $y$  座標平面上，由  $(B + 1, 0)$ 、 $(-B - 1, 0)$ 、 $(0, 1)$  及  $(0, -1)$  所形成的面積為  $C$  平方單位，求  $C$  的值。

The area of the rhombus on the  $x$ - $y$  coordinate plane with vertices  $(B + 1, 0)$ ,  $(-B - 1, 0)$ ,  $(0, 1)$  and  $(0, -1)$  is  $C$  square units. Find the value of  $C$ .

$C =$

4. 如果  $D$  是正整數且  $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$ ，求  $D$  的值。

If  $D$  is a positive integer such that  $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$ , find the value of  $D$ .

$D =$

**FOR OFFICIAL USE**

Score for  
accuracy

×

Mult. factor for  
speed

=

Team No.

+  
Bonus  
score

Time



Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (2022 – 2023)

## Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

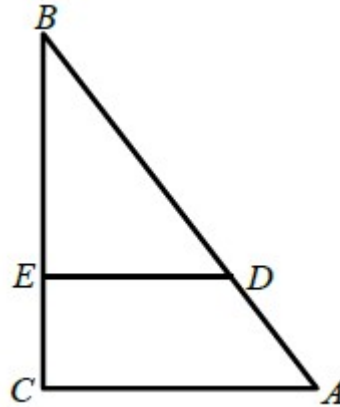
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在三角形  $ABC$  中， $\angle C = 90^\circ$ ， $DE \perp BC$ ， $BE = AC$ ， $BD = \frac{1}{2}$

及  $DE + BC = 1$ 。如果  $\alpha = 4\angle EDC$ ，求  $\alpha$  的值。

In triangle  $ABC$ ,  $\angle C = 90^\circ$ ,  $DE \perp BC$ ,  $BE = AC$ ,  $BD = \frac{1}{2}$

and  $DE + BC = 1$ . If  $\alpha = 4\angle EDC$ , find the value of  $\alpha$ .



$\alpha =$

2. 若  $f(a) = a - 2$ ，且  $F(a, b) = b^2 + a + \alpha$  及  $\beta = F(3, f(4))$ ，求  $\beta$  的值。

If  $f(a) = a - 2$ ,  $F(a, b) = b^2 + a + \alpha$  and  $\beta = F(3, f(4))$ , find the value of  $\beta$ .

$\beta =$

3. 如果方程組  $\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$  整數解的數量是  $\gamma$ ，求  $\gamma$  的值。

If the system of equations  $\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$  has  $\gamma$  sets of **integral**

solutions, find **the value of**  $\gamma$ .

$\gamma =$

4. 在三角形  $ABC$  中， $AB = AC$ ， $\angle A = 40^\circ + \gamma^\circ$ 。點  $O$  在三角形  $ABC$  內且  $\angle OBC = \angle OCA$ 。如果  $\angle BOC = \delta^\circ$ ，求  $\delta$  的值。

In a triangle  $ABC$ ,  $AB = AC$ ,  $\angle A = 40^\circ + \gamma^\circ$ . Point  $O$  is inside the triangle  $ABC$  with  $\angle OBC = \angle OCA$ . If  $\angle BOC = \delta^\circ$ , the value of  $\delta$ .

$\delta =$

### FOR OFFICIAL USE

Score for  
accuracy

$\times$

Mult. factor for  
speed

$=$

Team No.

$+$

Bonus  
score

Time



Total score

Min.

Sec.

**Hong Kong Mathematics Olympiad (2022– 2023)**  
**Final Event 1 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.  
除非特別聲明，答案須用數字表達，並化至最簡。

1. 有 100 個燈泡，編號從 1 到 100。班上有 100 名學生。每個學生輪流按下燈泡開關，情序如下：第一個學生按下編號為 1 及其倍數的燈泡開關，第二個學生按下編號為 2 及其倍數的燈泡開關，以此類推。每個學生只出來一次。如果燈泡亮著，按下開關後就會熄滅，反之亦然。一開始所有燈泡都是熄滅的。 $X$  代表在第 100 個學生按下開關後，燈泡亮著的數量。求  $X$  的值。

$X =$

There are 100 light bulbs labeled from 1 to 100, and there are 100 students in the class. Each student takes a turn to press the switch buttons of the light bulbs with a label that is a multiple of their assigned number. For example, the first student presses the switch buttons of the light bulb with label 1 and all of its multiples, the second student presses the switch buttons of the light bulb with label 2 and all of its multiples, and so on. Each student will only come out once, and if a light bulb is on, it becomes off after being pressed, and vice versa. All the light bulbs are off at the beginning.  $X$  is the number of light bulbs that are on after the 100th student presses. Find the value of  $X$ .

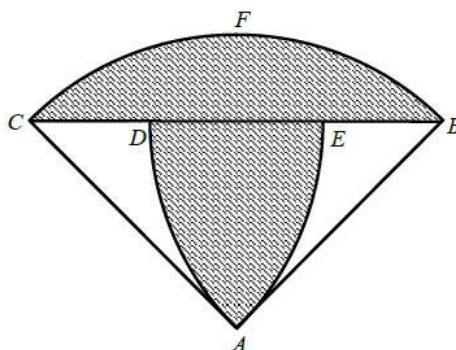
2. 已知  $x + \frac{1}{x} = 2\sqrt{5}$ 。求  $x^5 - \frac{1}{x^5}$  的值。

Given that  $x + \frac{1}{x} = 2\sqrt{5}$ . Find the value of  $x^5 - \frac{1}{x^5}$ .

$x^5 - \frac{1}{x^5} =$

3. 右圖中， $ABC$  是一個等腰三角形，其中  $\angle A = 90^\circ$  及  $AB = 2$ 。圖中有三個弧，它們分別是弧  $BFC$ 、弧  $AD$  和弧  $AE$ 。弧  $BFC$  是以  $A$  為圓心、 $AB$  為半徑畫出的。弧  $AD$  是以  $B$  為圓心、 $AB$  為半徑畫出的。弧  $AE$  是以  $C$  為圓心、 $AC$  為半徑畫出的。

求這個圖形的陰影面積。（取  $\pi = 3$ ）



Shaded area =

In the above figure,  $ABC$  is an isosceles triangle, where  $\angle A = 90^\circ$  and  $AB = 2$ . The figure includes three arcs: arc  $BFC$ , arc  $AD$ , and arc  $AE$ . Arc  $BFC$  has a radius of  $AB$  and is drawn from centre  $A$ . Arc  $AD$  is drawn from centre  $B$  with radius  $AB$ , while arc  $AE$  is drawn from centre  $C$  with radius  $AC$ . Find the area of this shaded region.

(Take  $\pi = 3$ )

4. 使用正整數序列 1、2、3、4、5、6 等等，通過將它們連接起來形成一個新的整數：123456789101112131415161718... 這個整數的最左邊的數位被定義為第一個數位。問在第 2023 數位是 0 至 9 的哪一個數？

Using the sequence of positive integers 1, 2, 3, 4, 5, 6, and so on, a new integer is formed by concatenating them: 123456789101112131415161718... The leftmost digit in this integer is defined as first position. What is the digit at position 2023?



# Hong Kong Mathematics Olympiad (2022 – 2023)

## Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 假如  $x$  和  $y$  都是正整數且它們的和是 15，找出  $x^3 + y^3$  的最小值。  
Find the minimum value of  $x^3 + y^3$  if  $x$  and  $y$  are two positive integers whose sum is 15.

2. 有一顆骰子，它的六個面上分別寫上數字 6 至 11。現投擲這顆骰子兩次，第一次得知四個側面的數字和是 36，第二次的數字和是 33。  
請問數字 10 的對面是甚麼數字？  
A cubic dice has faces marked with numbers from 6 to 11. The dice was rolled twice. At the first time, the sum of the numbers on the four lateral faces was 36. At the second time, the sum was 33.  
What number is on the face opposite to the one with the number 10?

3. 找出  $10^{12} + 809$  和  $10^{10} + 8$  的最大公因數。  
Find the greatest common divisor of  $10^{12} + 809$  and  $10^{10} + 8$ .

4. 在直角坐標平面上，香港的坐標是  $(0, 0)$ ，颱風是  $(4, -2)$ 。假設颱風向西（左）移動時，概率為 0.1，和向北（上）移動時，概率為 0.9，而且只能在移動一個單位距離後才可改變方向，請問這個颱風遇到香港的概率是多少？  
(答案需準確至四位有效數字。)

Hong Kong is located at  $(0, 0)$  of a grid map and a typhoon is at  $(4, -2)$ . Suppose the typhoon will only move to the west (left) with a probability of 0.1 or to the north (up) with a probability of 0.9, and may only change course after moving one unit distance. What is the probability that it will hit Hong Kong?  
(Give your answer in 4 significant figures.)

### FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
				Total score		<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

# Hong Kong Mathematics Olympiad (2022 – 2023)

## Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 設  $a_n$  為序列且  $a_n = \frac{1}{(n+1)\sqrt{n+n\sqrt{n+1}}}$ 。如果  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ ，求  $s$  的值。

$s =$

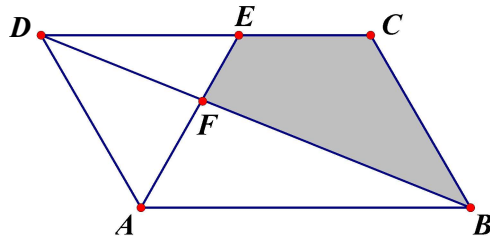
Let  $a_n$  be a sequence such that  $\frac{1}{(n+1)\sqrt{n+n\sqrt{n+1}}}$ .

Find the value of  $s$  where  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ .

2. 設  $ABCD$  為平行四邊形且  $AB = 40$ ， $AD = 24$  及  $DB = 56$ 。  $\angle DAB$  的角平分線與  $DC$  相交於  $E$  點，且對角線  $DB$  與  $AE$  相交於  $F$  點。求四邊形  $ECBF$  的面積。

Let  $ABCD$  be a parallelogram with  $AB = 40$ ,  $AD = 24$  and  $DB = 56$ . The angle bisector of  $\angle DAB$  meets side  $DC$  at the point  $E$ , and the diagonal  $DB$  meets  $AE$  at the point  $F$ .

Find the area of the quadrilateral  $ECBF$ .



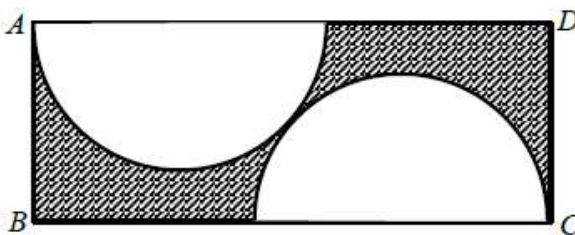
3. 設  $f(x)$  為函數並滿足  $f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right)$ ,  $x \neq 0, 1$ 。求  $f(-1)$  的值。

$f(-1) =$

Let  $f(x)$  be a function such that  $f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right)$ ,  $x \neq 0, 1$ .

Find the value of  $f(-1)$ .

4. 右圖中， $ABCD$  是一個長方形。兩個半圓形完全相等且它們彼此相切。如果  $AB = 2$  及  $BC = 6$ ，求圖中陰影面積（答案以  $\pi$  表示）。



In the following figure,  $ABCD$  is a rectangle.

The two semi-circles are identical and they are tangent to each other.

If  $AB = 2$  and  $BC = 6$ , find the area of the shaded part in terms of  $\pi$ .

### FOR OFFICIAL USE

Score for accuracy

$\times$

Mult. factor for speed

$=$

Team No.

$+$

Bonus score

Time



Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (2022 – 2023)

## Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 求方程  $x^{\log_{10} x} = 10$  所有實根的積。

Find the product of all the real roots of the equation  $x^{\log_{10} x} = 10$ .

2. 設  $p$  為質數及  $m$  為整數。如果  $p(p + m) + 2p = (m + 2)^3$ ，求  $m$  的最大值。

Let  $p$  be a prime and  $m$  be an integer.

If  $p(p + m) + 2p = (m + 2)^3$ , find the greatest possible value of  $m$ .

$m =$

3. 如果正四面體的邊長是 1，求該正四面體的體積。

If the length of one side of a regular tetrahedron is 1, find the volume of such tetrahedron.

4. 設  $P$  為 3659893456789325678 和 342973489379256 的乘積。求  $P$  的位數。

Let  $P$  be the product of 3659893456789325678 and 342973489379256.

Find the number of digits of  $P$ .

### FOR OFFICIAL USE

Score for  
accuracy

×

Mult. factor for  
speed

=

Team No.

+

Bonus  
score

Time



Total score

Min.

Sec.

Final Events (Group)

# 成績 Results

School code	School Name	Ind				Isum Group				Gsum Total rank			
		Event 1	Event 2	Event 3	Event 4	Event 1	Event 2	Event 3	Event 4				
FE-01	Assembly of God Hebron Secondary School	1	0	4	0	5	0					5	36
FE-02	Baptist Lui Ming Choi Secondary School	1	1	12	1	15	2					17	15
FE-03	Buddist Sin Tak College	1	0	0	7	8	0					8	29
FE-04	C. & M.A. Sun Kei Secondary School	1	0	0	2	3	4					7	31
FE-05	Cheung Sha Wan Catholic Secondary School	3	0	3	0	6	4					10	23
FE-06	Carmel Holy Word Secondary School	1	0	1	0	2	0					2	48
FE-07	CCC Heep Woh College	3	0	0	0	3	4					7	31
FE-08	Cheung Chuk Shan College	1	0	0	1	2	2					4	42
FE-09	Cheung Sha Wan Catholic Secondary School	1	0	1	0	2	0					2	48
FE-10	Christian Alliance Cheng Wing Gee College	3	0	5	0	8	7					15	17
FE-11	Christian Alliance S.C. Chan Memorial College	3	0	5	0	8	4					12	21
FE-12	CNEC Christian College	0	0	1	2	3	7					10	23
FE-13	Diocesan Boys' School	6	0	40	0	12	30					76	2
FE-14	Diocesan Girls' School	3	1	30	3	37	4					41	8
FE-15	Fukien Secondary School	3	1	6	0	10	7					17	15
FE-16	Good Hope School	1	0	0	0	1	4					5	36
FE-17	Heung To Middle School Kiangsu-Chekiang College (Shatin)	3	0	0	1	4	0					4	42
FE-18	HKBUAS Wong Kam Fai Secondary and Primary School	0	0	1	2	3	2					5	36
FE-19	HKTA Tang Hin Memorial Secondary School	3	3	30	0	36	4					40	9
FE-20	Ho Yu College and Primary School (Sponsored by Sik Sik Yuen)	3	0	30	3	36	30					66	3
FE-21	Hoi Ping Chamber of Commerce Secondary School	1	0	12	0	13	2					15	17
FE-22	Hong Kong Chinese Women's Club College	6	3	40	3	52	7					59	4
FE-23	King's College	1	0	0	0	1	4					5	36
FE-24	MaryKnoll Convent School (Secondary Section)	3	0	0	0	3	4					7	31
FE-25	PHC Wing Kwong College	1	0	1	0	2	2					4	42
FE-26	Po Leung Kuk Centenary Li Shiu Chung Memorial College	6	2	40	3	51	4					55	5
FE-27	Po Leung Kuk Laws Foundation College	1	0	4	3	8	2					10	23

# 成績 Results

FE-28	Po Leung Kuk No. 1 WH Cheung College	2	3	0	9	14	0					14	20
FE-29	Po Leung Kuk Tang Yuk Tien College	6	0	9	0	15	4					19	13
FE-30	Po On Commerce Association Wong Siu Ching Secondary School	1	0	1	1	3	2					5	36
FE-31	Pui Ching Middle School	1	0	40	3	44	4					48	7
FE-32	Queen Elizabeth School	1	1	40	3	45	4					49	6
FE-33	Queen's College	1	3	1	3	8	7					15	17
FE-34	Sha Tin Methodist College	2	0	5	0	7	2					9	27
FE-35	Shatin Tsung Tsin Secondary School	3	0	1	0	4	0					4	42
FE-36	Sing Yin Secondary School	1	0	30	3	34	0					34	11
FE-37	Singapore International School (Hong Kong)	1	0	3	0	4	0					4	42
FE-38	SKH Bishop Mok Sau Tseng Secondary School	0	1	0	2	3	0					3	47
FE-39	St. Francis Xavier's College	3	0	0	1	4	2					6	35
FE-40	St. Mark's School	1	3	2	0	6	2					8	29
FE-41	St Paul's Co-Educational College	6	3	50	2	61	30					91	1
FE-42	STFA Leung Kau Kui College	1	1	0	3	5	4					9	27
FE-43	STFA Yung Yau College	1	0	14	0	15	4					19	13
FE-44	The Mission Covenant Church Holm Glad College	0	1	1	3	5	0					5	36
FE-45	Tseung Kwan O Government Secondary School	0	0	2	3	5	2					7	31
FE-46	Tsuen Wan Public Ho Chuen Yiu Memorial College	0	1	30	0	31	7					38	10
FE-47	Tung Wah Group of Hospitals Lo Kon Ting Memorial College	4	0	6	0	10	2					12	21
FE-48	Wah Yan College, Kowloon	0	0	6	0	6	4					10	23
FE-49	Wong Shiu Chi Secondary School	1	0	0	0	1	0					1	50
FE-50	Ying Wa College	3	1	14	0	18	14					32	12

Champion	Diocesan Boys' School
1st runner up	St Paul's Co-Educational College
2nd runner up	Po Leung Kuk Centenary Li Shiu Chung Memorial College