第三十五屆香港數學競賽(2017/18)
初賽規則

1. 初賽分個人項目、團體項目和幾何作圖項目三部分，個人項目限時**六十分鐘**，團體項目限時二十分鐘，而幾何作圖項目則限時二十分鐘。

2. 每隊由四至六位**中五**或以下同學組成。其中任何四位可參加個人項目；又其中任何四位可參加團體項目及幾何作圖項目。不足四位同學的隊伍將被撤銷參賽資格。

3. 每隊隊員**必須穿著整齊校服**，**並由負責教師帶領**，於上午9時或以前向會場接待處註冊，同時必須出示**身分證/學生證明文件**，否則**將被撤銷參賽資格**。

4. 指示語言將採用粵語。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英文並列。

5. 每一隊員於個人項目中須解答15條問題（當中**甲部佔10題、乙部佔5題**）；而每一隊伍則須於團體項目中解答10條問題；並在幾何作圖項目中解答**所有**問題。

6. 團體項目及幾何作圖項目中，各參賽隊員可進行討論，但必須將聲浪降至最低。

7. 各參賽隊伍須注意：
   (a) 個人項目及團體項目比賽時，**不准使用**計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，
   (b) 幾何作圖項目比賽時，**只准使用**書寫工具（例如：原子筆、鉛筆等）、圓規及大會提供的直尺，

   違例隊伍將被撤銷參賽資格或扣分。

8. **除非另有聲明，否則所有個人項目及團體項目中問題的答案均為數字，並應化至最簡，但無須呈交證明及算草**。

9. 參賽者如有攜帶電子通訊器材，應把它關掉(包括響鬧功能)並放入手提包內或座位的椅下。

10. 個人項目中，甲部和乙部的每一正確答案分別可得1分及2分。每隊可得之最高積分為80分。

11. 團體項目中，每一正確答案均可得2分。每隊可得之最高積分為20分。

12. 至於幾何作圖項目，每隊可得之最高積分為20分（必須詳細列出所有步驟，包括作圖步驟）。

13. 初賽中，並不給予快捷分。

14. 參賽者必須自備工具，例如：**原子筆、鉛筆及圓規**。

15. 籌委會將根據各參賽隊伍的總成績（個人項目、團體項目及幾何作圖項目的積分總和）選出最高積分的五十隊進入決賽。

16. 初賽獎項：
    (a) 於個人項目比賽中，
        (i) 取得滿分者將獲頒予最佳表現及積分獎狀；
        (ii) 除上述 (i) 中取得最佳表現的參賽者外，
            (1) 成績最佳的首 2% 參賽者將獲頒予一等榮譽獎狀；
            (2) 隨後的 5% 參賽者將獲頒予二等榮譽獎狀；
            (3) 緊接著的 10% 參賽者將獲頒予三等榮譽獎狀。
    (b) 於團體項目中取得滿分的隊伍將獲頒予最佳表現及積分獎狀。
    (c) 於幾何作圖項目中表現優秀的隊伍將獲頒予獎狀。
    (d) 於各分區的比賽中，總成績（個人項目、團體項目及幾何作圖項目的積分總和）最高之首10%的參賽隊伍將獲頒予獎狀。

17. 如有任何疑問，參賽者須於比賽完畢後，立即向會場主任提出。所提出之疑問，將由籌委會作最後裁決。

The Thirty-fifth Hong Kong Mathematics Olympiad (2017/18)
Regulations (Heat Events)

1. The Heat Events consists of three parts: **60 minutes** for the individual event, 20 minutes for the group event and 20 minutes for the geometric construction event.

2. Each team should consist of 4 to 6 members who are students of **Secondary 5** level or below. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event and the geometric construction event. Teams of less than 4 members will be disqualified.

3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception not later than 9:00 a.m. Failing to do so, the team **will be disqualified**.

4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.

5. Each member of a team has to solve 15 questions in the individual event (**_10 questions in Part A_** and **_5 questions in Part B_**) and each team has to solve 10 questions in the group event and **ALL** questions in the geometric construction event.

6. In the group event and geometric construction event, discussions among participating team members are allowed provided that the voice level is kept to a minimum.

7. Please note that
   (a) for the individual and group events, devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will not be allowed to be used; and
   (b) for the geometric construction event, only writing instruments (pens, pencils, etc), **straightedge provided and compasses** will be allowed to be used;
   otherwise the team will be disqualified or risk deduction of marks.

8. **All answers in the individual event and the group event should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**

9. Participants having electronic communication devices should have them turned off (including the alarm function) and put them inside their bags or under their chairs.

10. For the individual event, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The maximum score for a team should be 80.

11. For the group event, 2 marks will be given to each correct answer. The maximum score for a team should be 20.

12. For the geometric construction event, the maximum score for a team should be 20 (all working, including construction work, must be clearly shown).

13. No mark for speed will be awarded in the Heat Event.

14. Participants should bring along their own instruments, e.g. **ball pens**, **pencils and compasses**.

15. The 50 teams with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) will be qualified for the Final Event.

16. Awards of the Heat Event:
    (a) For the individual event,
        (i) candidates obtaining full score will be awarded Best Performance and Score certificates ;
        (ii) apart from the best performer(s) in (i),
            (1) the first 2% of top scoring candidates will be awarded First-class honour certificates ;
            (2) the next 5% of top scoring candidates will be awarded Second-class honour certificates ; and
            (3) the next 10% of top scoring candidates will be awarded Third-class honour certificates;
    (b) For the group event, teams obtaining full marks will be awarded Best Performance and Score certificates.
    (c) For the geometric construction event, teams having outstanding performance will be awarded certificates of merit.
    (d) About 10% of participating schools with the highest aggregate scores (sum of the scores in the individual event, the group event and the geometric construction event) in each region will be awarded certificates of merit.

17. Should there be any queries, participants should reach the Centre Supervisor immediately after the competition. The decision of the Organising Committee on the queries is final.

除非特別聲明，答案須用數字表達，並化至最簡。　　　時限：1 小時 Time allowed: 1 hour
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
Q1- Q10 每題 1 分，Q11-Q15 每題 2 分。Q1- Q10 1 mark each, Q11-Q15 2 marks each.
全卷滿分 20 分。The maximum mark for this paper is 20.

1. 若 a 及 b 均為實數，求 $a^2 + b^2 + 12a - 8b + 2018$ 的最小值。

   If a and b are real numbers, find the minimum value of $a^2 + b^2 + 12a - 8b + 2018$.

2. 設 a 及 k 均為常數。若 $(6x^3 + ax^2 + 7x - 3) \div (2x^2 + kx - 1)$ 的商和餘式分別為 $3x + 5$ 及 $-5x + 2$，求 a 的值。

   Let a and k be constants. If the quotient and the remainder of $(6x^3 + ax^2 + 7x - 3) \div (2x^2 + kx - 1)$ are $3x + 5$ and $-5x + 2$ respectively, find the value of a .

3. 在編制某雜誌中每頁的頁碼時，總共用去了 2,046 個數字，問該雜誌總共有多少頁？
   (假設該雜誌第一頁的頁碼是 1。)

   In numbering the pages of a magazine, 2046 digits were used. How many pages are there in the magazine? (Assume the page number of the magazine starts from 1 .)

4. 解 $\log\left(1 + \dfrac{1}{1}\right) + \log\left(1 + \dfrac{1}{2}\right) + \log\left(1 + \dfrac{1}{3}\right) + \cdots + \log\left(1 + \dfrac{1}{n}\right) = 5$。

   Solve $\log\left(1 + \dfrac{1}{1}\right) + \log\left(1 + \dfrac{1}{2}\right) + \log\left(1 + \dfrac{1}{3}\right) + \cdots + \log\left(1 + \dfrac{1}{n}\right) = 5$.

5. 已知 $\dfrac{1 - 2^{-\frac{1}{x}}}{2^{-\frac{1}{x}} - 2^{-\frac{2}{x}}} = 4$。求 x 的值。

   Given that $\dfrac{1 - 2^{-\frac{1}{x}}}{2^{-\frac{1}{x}} - 2^{-\frac{2}{x}}} = 4$. Find the value of x .
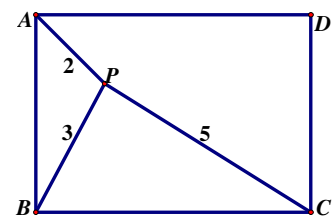
6. 若 x 為有理數，求 x 的值滿足聯立方程 $\begin{cases} y = 2x^2 - 11x + 15 \\ y = 2x^3 - 17x^2 + 16x + 35 \end{cases}$。

   If x is a rational number, find the value of x satisfying the simultaneous equations
   $\begin{cases} y = 2x^2 - 11x + 15 \\ y = 2x^3 - 17x^2 + 16x + 35 \end{cases}$.

7. 如圖一所示，P 為長方形 $ABCD$ 內的一點，使得 $PA = 2$，$PB = 3$ 及 $PC = 5$。求 PD 的長度。
   As shown in Figure 1, P is a point inside a rectangle $ABCD$ such that $PA = 2$, $PB = 3$ and $PC = 5$. Find the length of PD .

圖一　Figure 1

8. 如圖二所示，兩個邊長為 x cm 的正方形於一角重疊。若兩個正方形的非重疊部分與重疊部分面積的比是 $a:1$，求 a 的值。
As shown in Figure 2, two squares with side x cm coincides at one corner. If the ratio of the non-overlapping area to the overlapping area of the two squares is $a:1$, find the value of a.
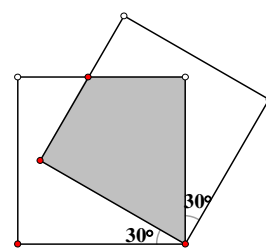
圖二 Figure 2

9. 如圖三所示，ABC 是一個等腰三角形，其中 $AB = AC = 8$ 及 $BC = 4$。D 及 E 分別為 BC 及 AC 上的點使得 $BD = 1$ 及 $\angle ABC = \angle ADE$。求 AE 的值。
As shown in Figure 3, ABC is an isosceles triangle with $AB = AC = 8$ and $BC = 4$. D and E are points lying on BC and AC respectively such that $BD = 1$ and $\angle ABC = \angle ADE$. Find the length of AE.
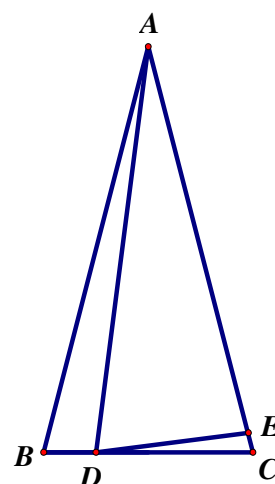
圖三 Figure 3

10. PQR 是一個三角形，其中 $PQ = 13$、$QR = 14$ 及 $PR = 15$。以 PR 為直徑繪畫出圓 C，C 相交 QR 於點 T。求 ΔPTR 的面積。
PQR is a triangle with $PQ = 13$, $QR = 14$ and $PR = 15$. The circle C is drawn with diameter PR. C intersects QR at a point T. Find the area of ΔPTR.

11. 求 $3^x + 5 + \dfrac{36}{3^x + 4}$ 的最小值。

Find the minimum value of $3^x + 5 + \dfrac{36}{3^x + 4}$.

12. 如圖四所示，$ABCD$ 及 $PQRC$ 為 兩個連接的正方形。以 C 為圓心及 CB 為半徑繪畫出弧 BD。已知 $BC = 8$ 及 $RC = 6$。求弧 BD 及綫段 DQ 與 BQ 所圍成的區域的面積。
As shown in Figure 4, two squares $ABCD$ and $PQRC$ are joined together. An arc BD is drawn with centre C and radius CB. Given that $BC = 8$ and $RC = 6$. Find the area of the region bounded by the arc BD, line segments DQ and BQ.
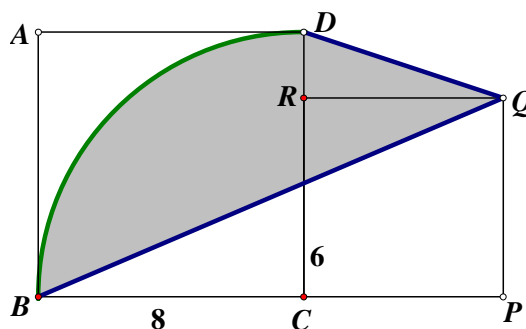
圖四 Figure 4

13. 一個四位數可以透過把它的所有數字加起來，變成另一個數。例如：1234 可以變成 10，因為 $1 + 2 + 3 + 4 = 10$。究竟從 1998 至 4998 (包括此兩個數)有多少個四位數，經上述變換後不可以被 3 整除？

A 4-digit number can be transformed into another number by adding its digits. For example, 1234 is transformed into 10 as $1 + 2 + 3 + 4 = 10$. How many transformed numbers from 1998 to 4998 inclusive are **NOT** divisible by 3?

14. 對任意實數 x $(x \neq 1)$，定義函數 $f(x) = \dfrac{x}{1-x}$ 及 $f \circ f(x) = f(f(x))$。

求 $2017 \underbrace{f \circ f \circ f \circ \cdots \circ f}_{2018 個 f}(2018)$ 的值。

For any real number x $(x \neq 1)$, define a function $f(x) = \dfrac{x}{1-x}$ and $f \circ f(x) = f(f(x))$.

Find the value of $2017 \underbrace{f \circ f \circ f \circ \cdots \circ f}_{2018 \text{ copies of } f}(2018)$.

15. 設 $N^2 = \overline{abcdefabc}$ 為一個 9 位整數，其中 N 是 4 個相異質數的積及 a、b、c、d、e、f 均為非零數字且滿足 $\overline{def} = 2 \times \overline{abc}$。求 N^2 的最小值。

Let $N^2 = \overline{abcdefabc}$ be a nine-digit positive integer, where N is the product of four distinct primes and a, b, c, d, e, f are non-zero digits that satisfy $\overline{def} = 2 \times \overline{abc}$.

Find the least value of N^2.

*** 試卷完 End of Paper ***

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘 Time allowed: 20 minutes
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得兩分。Each correct answer will be awarded 2 marks.
全卷滿分 20 分。The maximum mark for this paper is 20 .

1. 設 f(x) 為二次多項式，其中 f(1) $= \dfrac{1}{2}$ ，f(2) $= \dfrac{1}{6}$ ，f(3) $= \dfrac{1}{12}$ 。求 f(6) 的值。

   Let f(x) be a polynomial of degree 2, where f(1) $= \dfrac{1}{2}$, f(2) $= \dfrac{1}{6}$, f(3) $= \dfrac{1}{12}$. Find the value of f(6) .

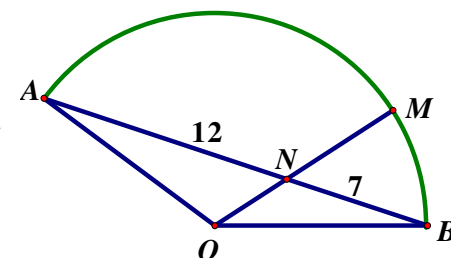2. 求 $\sqrt{2018 \times 2012 \times 1988 \times 1982 + 8100}$ 。

   Evaluate $\sqrt{2018 \times 2012 \times 1988 \times 1982 + 8100}$.

3. 如圖一所示，OAB 是一個以 O 為圓心的扇形。N 為
   半徑 OM 與 AB 的交點。已知 $AN = 12$ ，$BN = 7$ 及
   $3ON = 2MN$ 。求 OM 的長度。
   As shown in Figure 1, OAB is a sector with centre O . N
   is the intersecting point of radius OM and AB .
   Given that $AN = 12$, $BN = 7$ and $3ON = 2MN$.
   Find the length of OM .

   圖一 Figure 1

4. 對任意非零實數 x，函數 f(x) 有以下特性： $2f(x) + f\left(\dfrac{1}{x}\right) = 11x + 4$ 。

   設 S 為所有滿足於 f(x) $= 2018$ 的根之和。求 S 之值。
   For any non-zero real number x, the function f(x) has the following property:
   $2f(x) + f\left(\dfrac{1}{x}\right) = 11x + 4$. Let S be the sum of all roots satisfying the equation f(x) = 2018 .
   Find the value of S .

5. 求可滿足下列方程組的 x 的值： $\begin{cases} x^2 + 9x - 10y - 220 = 0 \\ y^2 - 5x + 6y - 166 = 0 \\ \qquad\qquad xy = 195 \end{cases}$.

   Find the value of x that satisfy the following system of equations:
   $\begin{cases} x^2 + 9x - 10y - 220 = 0 \\ y^2 - 5x + 6y - 166 = 0 \\ \qquad\qquad xy = 195 \end{cases}$.

6. 已知 $n^4 + 104 = 3^m$，其中 n、m 為正整數。求 n 的最小值。
   Given that $n^4 + 104 = 3^m$, where n, m are positive integers. Find the least value of n .

7. 如圖二所示，A、B、C、D 及 E 為圓上的點。T 是該圓外的一點。TA 是該圓在點 A 的切綫， TBE 及 TCD 為直綫。已知 TBE 是 $\angle ATD$ 的角平分綫、$TA = 12$、$TB = 6$ 及 $TC = 8$。
求 ΔABE 與四邊形 $BCDE$ 的面積比。
As shown in Figure 2, A, B, C, D and E are points on the circle. T is a point outside the circle such that TA is a tangent to the circle at A and TBE and TCD are straight lines. It is given that TBE is the angle bisector of $\angle ATD$, $TA = 12$, $TB = 6$ and $TC = 8$. Find the ratio of the area of ΔABE to the area of quadrilateral $BCDE$.
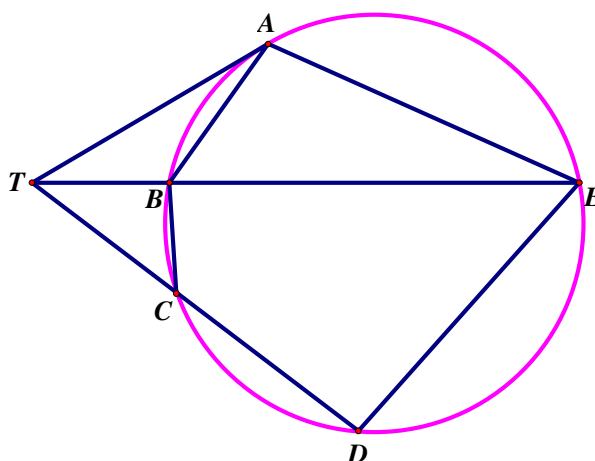
圖二 Figure 2

8. 已知 a、b、c、d、e、f、g 及 h 為正整數，使得 $a > b > c > d > e > f > g > h$ 及 $a + h = b + g = c + f = d + e = 35$，問有多少組可行答案 $\{a, b, c, d, e, f, g, h\}$ 存在？
Given that a, b, c, d, e, f, g and h are positive integers such that $a > b > c > d > e > f > g > h$ and $a + h = b + g = c + f = d + e = 35$. How many possible solution sets of $\{a, b, c, d, e, f, g, h\}$ exist?

9. 求 $\left(\dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{100}\right) + \left(\dfrac{2}{3} + \dfrac{2}{4} + \cdots + \dfrac{2}{100}\right) + \left(\dfrac{3}{4} + \dfrac{3}{5} + \cdots + \dfrac{3}{100}\right) + \cdots + \left(\dfrac{98}{99} + \dfrac{98}{100}\right) + \dfrac{99}{100}$ 的值。
Find the value of

$$\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{100}\right) + \left(\frac{2}{3} + \frac{2}{4} + \cdots + \frac{2}{100}\right) + \left(\frac{3}{4} + \frac{3}{5} + \cdots + \frac{3}{100}\right) + \cdots + \left(\frac{98}{99} + \frac{98}{100}\right) + \frac{99}{100}.$$

10. 如圖三所示，ABC 是一個三角形，其中 $AB = 40$、$BC = 30$ 及 $\angle ABC = 150°$。 M 及 N 分別為 AB 及 BC 的中點。$\angle ABC$ 的角平分綫分別相交 MN 及 AC 於 D 及 E。求 $AMDE$ 的面積。
As shown in Figure 3, ABC is a triangle with $AB = 40$, $BC = 30$ and $\angle ABC = 150°$. M and N are the mid-points of AB and BC respectively. The angle bisector of $\angle ABC$ intersects MN and AC at D and E respectively.
Find the area of quadrilateral $AMDE$.
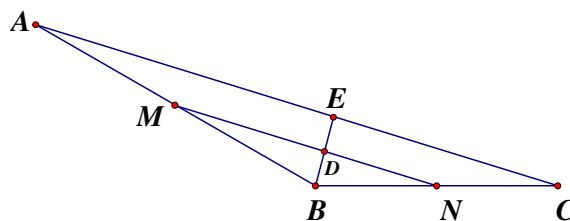
圖三 Figure 3

*** 試卷完 End of Paper ***

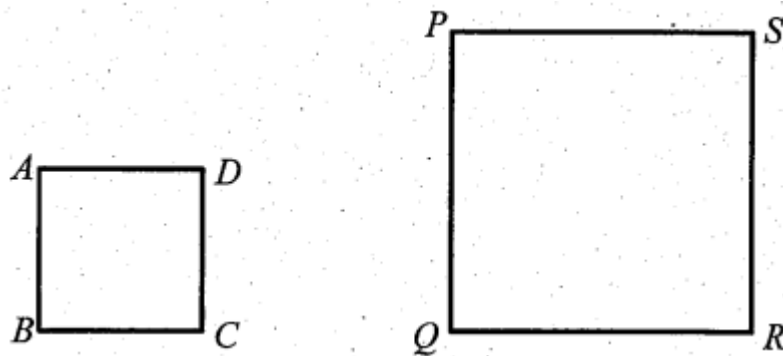|	
每隊必須列出詳細所有步驟(包括作圖步驟)。	時限：20 分鐘
All working (including geometric drawing) must be clearly shown.	
此部份滿分為二十分。The full marks of this part is 20 marks.	Time allowed: 20 minutes

School ID: _____

School Name: _____

第一題 Question No. 1

求作一個正方形使得其面積等於下圖的兩個正方形 *ABCD* 及 *PQRS* 的面積之和。

Construct a square whose area is equal to the sum of the areas of the squares *ABCD* and *PQRS* as shown below.

每隊必須列出詳細所有步驟(包括作圖步驟)。	時限：20 分鐘
All working (including geometric drawing) must be clearly shown.	
此部份滿分為二十分。The full marks of this part is 20 marks.	Time allowed: 20 minutes

School ID: _____

School Name: _____

第二題 Question No. 2

已知 *AC* 是一條通過一個以 *O* 作圓心的綫段，如下圖所示。

求作一個鳶形 *ABCD* 使得∠*BAD* = 2×∠*BCD* 及 *B*、*D* 分別位於圓 *APC* 上。

Given that *AC* is a line segment passing through the centre *O* of a circle, as shown in the figure below. Construct a kite *ABCD* such that ∠*BAD* = 2×∠*BCD* and *B*, *D* lies on the circle *APC*.

每隊必須列出詳細所有步驟(包括作圖步驟)。　　　　　　　　　　　時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為二十分。The full marks of this part is 20 marks.　　　Time allowed: 20 minutes
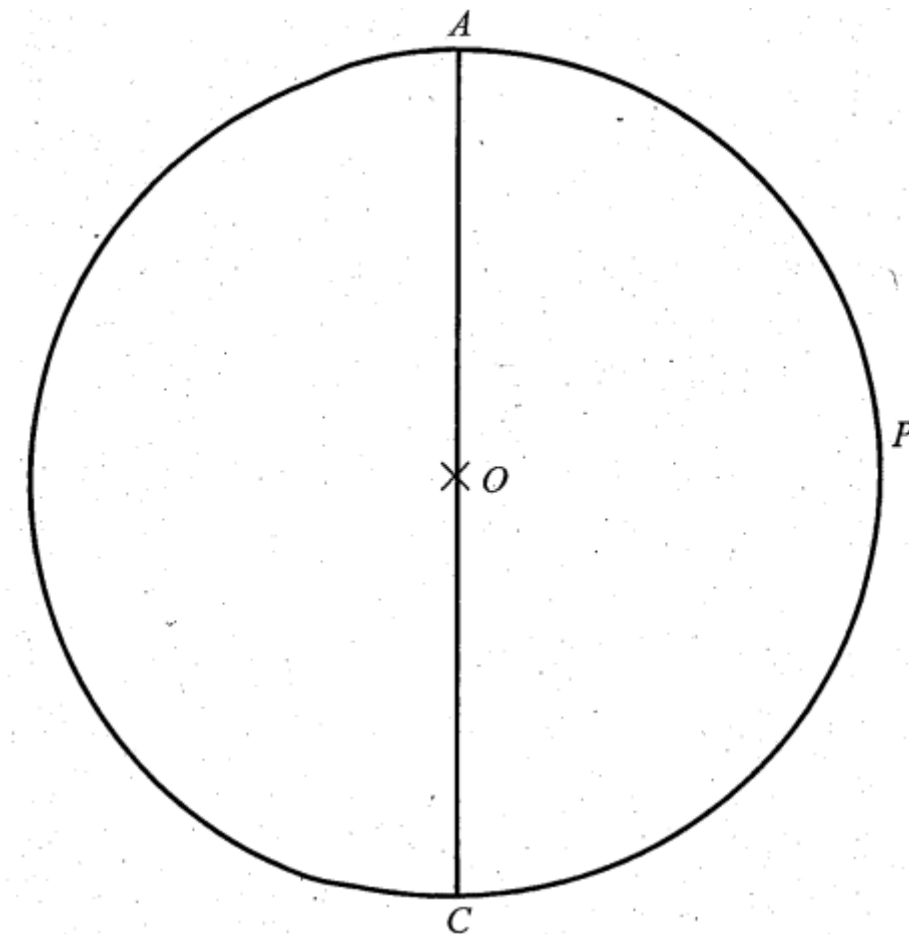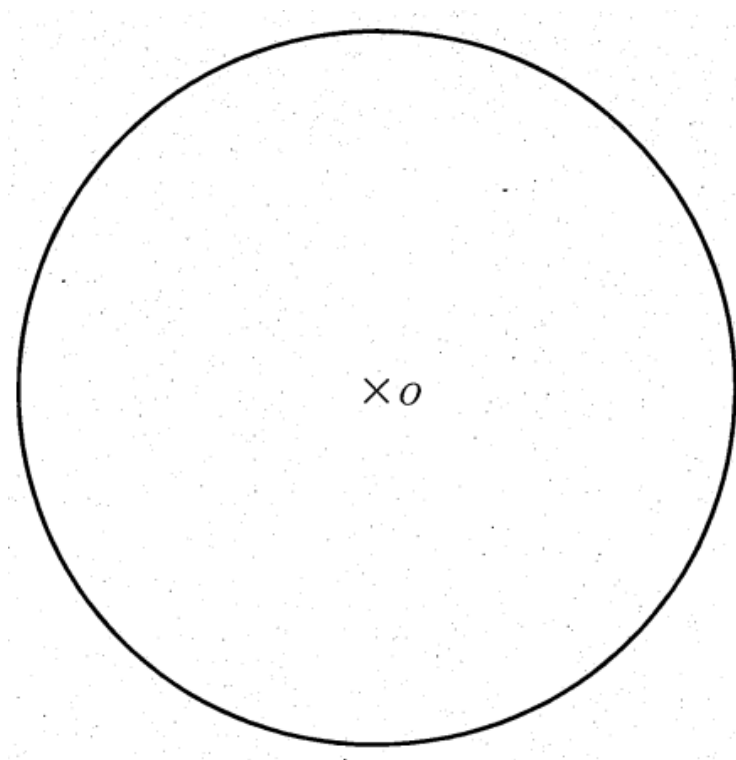
School ID: _____

School Name: _____

第三題 Question No. 3
作一個以 *O* 為圓心的圓上的正八邊形 *ABCDEFGH*。
Construct a regular octagon *ABCDEFGH* on a circle with centre *O* .

*** 試卷完 End of Paper ***

初賽成績（一）

參加學校數目：209
初賽日期：2018 年 3 月 3 日星期六
試場一(HK1)：新會商會陳白沙紀念中學 (41 隊)
試場二(KLN1)：仁濟醫院王華湘中學 (49 隊)
試場三(KLN2)：長沙灣天主教英文中學 (50 隊)
試場四(NT1)：梁文燕紀念中學（沙田）(35 隊)
試場四(NT2)：伊利沙伯中學舊生會中學 (34 隊)

Regional winners of the Heat Event

Hong Kong Island Region

Hong Kong Chinese Women's Club College

Queen's College

St Paul's Co-Educational College

Wah Yan College, Hong Kong

Kowloon Region 1

Diocesan Boys' School

La Salle College

Po Leung Kuk No. 1 WH Cheung College

Queen Elizabeth School

Sing Ying Secondary School

Kowloon Region 2

Diocesan Girls' School

Heung To Middle School

Pui Ching Middle School

Tsuen Wan Government Secondary School

Ying Wa College

New Territories Region 1

HKTA Tang Hin Memorial Secondary School

Kiangsu-Chekiang College (Shatin)

TWGH Kap Yan Directors' College

Wong Shiu Chi Secondary School

New Territories Region 2

Christian Alliance S.C. Chan Memorial College

NTHYK Yuen Long District Secondary School

PLK Centenary Li Shiu Chung Memorial College

Yuen Long Merchant Association Secondary School

初賽成績（二）決賽名單：

School ID	Name of School school = new school entering final event this year
FE-01	Baptist Lui Ming Choi Secondary School
FE-02	Bishop Hall Jubilee School
FE-03	Buddhist Sin Tak College
FE-04	Carmel Divine Grace Foundation Secondary School
FE-05	Carmel Pak U Secondary School
FE-06	CCC Ming Yin College
FE-07	Chiu Lut Sau Memorial Secondary School
FE-08	Christian Alliance S.C. Chan Memorial College
FE-09	Diocesan Boys' School
FE-10	Diocesan Girls' School
FE-11	G.T. (Ellen Yeung) College
FE-12	Good Hope School
FE-13	Heung To Middle School
FE-14	HKMA K S Lo College
FE-15	HKSYC & IA Wong Tai Shan Memorial College
FE-16	HKTA Tang Hin Memorial Secondary School
FE-17	Hoi Ping Chamber of Commerce Secondary School
FE-18	Hong Kong Chinese Women's Club College
FE-19	Kiangsu-Chekiang College (Shatin)
FE-20	King's College
FE-21	Kwun Tong Government Secondary School
FE-22	La Salle College
FE-23	Maryknoll Convent School (Secondary Section)
FE-24	Munsang College
FE-25	Munsang College (Hong Kong Island)
FE-26	NTHYK Yuen Long District Secondary School
FE-27	Po Leung Kuk Centenary Li Shiu Chung Memorial College
FE-28	Po Leung Kuk Laws Foundation College
FE-29	Po Leung Kuk No. 1 WH Cheung College
FE-30	Po On Commerce Association Wong Siu Ching Secondary School
FE-31	Pui Ching Middle School
FE-32	Queen Elizabeth School
FE-33	Queen's College
FE-34	Shatin Tsung Tsin Secondary School
FE-35	Sing Yin Secondary School
FE-36	SKH Lam Woo Memorial Secondary School
FE-37	St Joseph's College
FE-38	St Paul's Co-Educational College
FE-39	St Paul's College
FE-40	STFA Lee Shau Kee College
FE-41	The ELCHK Yuen Long Lutheran Secondary School
FE-42	Tseung Kwan O Government Secondary School
FE-43	Tsuen Wan Government Secondary School
FE-44	TWGH Kap Yan Directors' College
FE-45	Wa Ying College
FE-46	Wah Yan College, Hong Kong
FE-47	Wah Yan College, Kowloon
FE-48	Wong Shiu Chi Secondary School
FE-49	Ying Wa College
FE-50	Yuen Long Merchants Association Secondary School