IV Powers of a root.

Theory If α is a root of a polynomial equation $f(x) = 0$, then α^m is a root of a polynomial equation $f(y^{\frac{1}{m}}) = 0$.

That is to say, change $x \to y^{\frac{1}{m}}$.

Class Work IV.1

Let α, β be the roots of $x^2 - 2x + 5 = 0$.

Find a new equation whose roots are α^2, β^2

Transform $x \to \sqrt{y}$.

The new equation is $y^2 + 6y +$ _____ $= 0$

Class Work IV.2

If α, β and $\gamma \neq 0$ are the roots of $x^3 + 3px + q = 0$, find the new equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$.

$\frac{\beta\gamma}{\alpha} = \frac{\alpha\beta\gamma}{\alpha^2} = -\frac{q}{\alpha^2}$

$\therefore$ The roots of the new equation are $-\frac{q}{\alpha^2}$, _____, _____.

Transformation is: _____.

( )$^3 + 3p($ $) + q = 0$

$qy^3 + 9p^2y^2 - 6pqy + q^2 = 0$