Examples on Mathematical Induction: divisibility 8

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1. Prove by mathematical induction $1 + 7^{2n-1}$ is divisible by 8 for all positive integers n.
2. Prove by mathematical induction $3^{2n} + 8n + 7$ is divisible by 8 for all non-negative integers n.
3. Prove by mathematical induction $3^{2n+1} + 5^{2n+1}$ is divisible by 8 for all non-negative integers n.
4. **1969 Paper 1 Q15(a) 2004 Q7**

   Prove by mathematical induction $9^n - 1$ is divisible by 8 for all non-negative integers n.

   By induction on n. $n = 1$, $9 - 1 = 8$ which is divisible by 8. It is true for $n = 1$.

   Suppose $9^k - 1 = 8m$, where k is a positive integer and m is an integer.

   $9^{k+1} - 1 = 9(9^k) - 1 = 9(8m + 1) - 1$

   $= 72m + 8 = 8(9m + 1)$, which is a multiple of 8.

   Therefore, $9^{k+1} - 1$ is also divisible by 8 if $9^k - 1$ is divisible by 8 and k is a positive integer.

   By the principle of mathematical induction, $9^n - 1$ is divisible by 8 for all positive integers n.
5. (a) Prove by mathematical induction $9^n + 7$ is divisible by 8 for all non-negative integers n.

   (b) Find a factor of $3^n + 5$.

   (c) Hence prove that $3^{3n} + 5\cdot 3^{2n} + 7\cdot 3^n + 35$ is divisible by 16. (You are not required to use M.I. in part c)

   (a) Let $P(n) \equiv$"$9^n + 7$ is divisible by 8 for all positive integers n."

   $n = 1$, $9 + 7 = 16$ is divisible by 8

   Suppose $9^k + 7 = 8m$, where m is an integer.

   $9^{k+1} + 7 = 9\times 9^k + 7$

   $= 9\times(8m - 7) + 7$

   $= 9\times 8m - 63 + 7$

   $= 9\times 8m - 56$

   $= 8\times(9m - 7)$, which is also divisible by 8

   If $P(k)$ is true, then $P(k + 1)$ is also true.

   By the principle of mathematical induction, $9^n + 7$ is divisible by 8 for all positive integers n.

   (b) Since both 3^n and 5 are odd integers, $3^n + 5$ is an even integer.

   One factor of $3^n + 5$ is 2.

   (c) $3^{3n} + 5\times 3^{2n} + 7\times 3^n + 35 = 3^n\times(3^{2n} + 7) + 5\times(3^{2n} + 7)$

   $= 3^n\times(9^n + 7) + 5\times(9^n + 7)$

   $= (3^n + 5) \times (9^n + 7)$

   $= 2p\times 8q$, where p and q are integers.

   Therefore $3^{3n} + 5\times 3^{2n} + 7\times 3^n + 35$ is divisible by 16 .