Individual Events

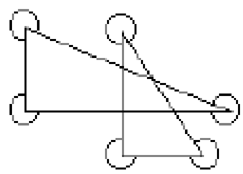
I1	***a***	1800	**I2**	missing	**I3**	***a***	10	**I4**	**no. of routes**	6	**I5**	***a***	$x^2 + 2x + 1$
	b	12				***b***	10		***b***	–2		***b***	–2
	c	*8 See the remark				***c***	30		***c***	3		***c***	2
	d	$\frac{1600}{3}$				***d***	90		**angle**	57°		***d***	1000

Group Events

G6	***a***	R	**G7**	**sum**	360	**G8**	AC	15 m	**G9**	***a***	$\frac{5}{4}$	**G10**	***A***	3578
	b	80		$S_{\Delta ABC}$	5 cm^2		***x***	60		**step**	2		***N***	10
	c	$\frac{1}{2}$		$a^3 + \frac{1}{a^3}$	18			$2x - 1$		***c***	–6		$\angle OAB$	56°
	d	6			$\frac{8}{9}$		***d***	220		**Probability**	$\frac{144}{343}$		***X***	46

Individual Event 1

I.1.1 In the following figure, the sum of the marked angles is a°, find a.



Angle sum of a triangle = 180°

Angles sum of 2 triangles = 360°

Angle at a point = 360°

Angles sum at 6 vertices = 6×360° = 2160°

$\therefore$ a = 2160 – 360 = 1800

I.1.2 The sum of the interior angles of a regular b-sided polygon is a°. Find b.

$180\times(b - 2) = 1800$

$b = 12$

I1.3 Find c, if $2^b = c^4$ and $c > 0$

$2^{12} = (2^3)^4 = 8^4$

$c = 8$

Remark Original question: Find c, if $2^b = c^4$.

$c = \pm 8$

I1.4 Find d, if $\frac{b}{c} = k$ and $c : d = k : 100$.

$$k = \frac{12}{8} = \frac{3}{2}$$

$$8 : d = \frac{3}{2} : 100$$

$\Rightarrow 8 : d = 3 : 200$

$$d = \frac{200}{3} \times 8 = \frac{1600}{3}$$

Individual Event 3

I3.1 If $a = 1.8 \times 5.0865 + 1 - 0.0865 \times 1.8$, find a.

$a = 1.8 \times(5 + 0.0865) + 1 - 0.0865 \times 1.8$

$= 9 + 1$

$= 10$

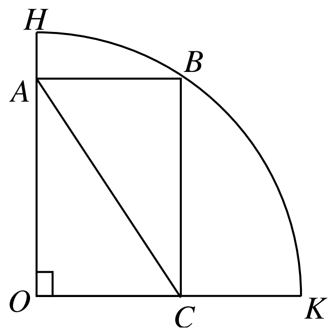
I3.2 In the diagram shown, $OH = OK = a$ units and $OABC$ is a rectangle. $AC = b$ units. What is b?

$b = OB$

$= OH$

$= a$

$= 10$



I3.3 In the expression shown, what is c when it is expanded to the term with $x^{(b-2)}$ as the numerator?

$$\cfrac{x^0}{2+\cfrac{x^1}{6+\cfrac{x^2}{10+\cfrac{x^3}{14+\cfrac{\cdots\cdots\cdots}{\cdots\, c+\cfrac{x^{(b-2)}}{\ddots}}}}}}$$

$b - 2 = 10 - 2 = 8$

$T(1) = 2$

$T(2) = 6$

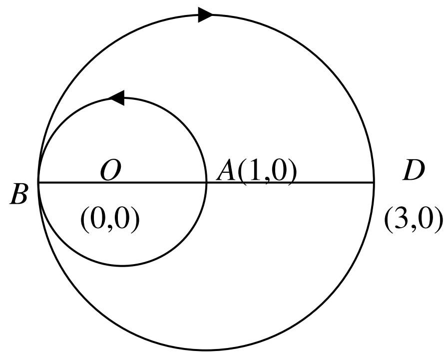
$T(3) = 10$

This is an arithmetic sequence with first term = 2, common difference = 4 .

$T(8) = 2 + (8 - 1)\times 4$

$= 30$

I3.4 As shown a rabbit spends c minutes in travelling from A to B along half circle. With the same speed, it spends d minutes in travelling from $A \to B \to D$ along half circles. What is d?



Radius of the smaller circle = 1

Radius of the larger circle = 2

Circumference of the smaller semi-circle $A \to B = \pi$

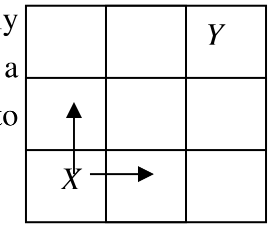
Circumference of the larger semi-circle $B \to D = 2\pi$

Speed $= \dfrac{\pi}{c} = \dfrac{\pi + 2\pi}{d}$

$\Rightarrow d = 3c = 90$

Individual Event 4

I4.1 The figure shows a board consisting of nine squares. A counter originally on square X can be moved either upwards or to the right one square at a time. By how many different routes may the counter be moved from X to Y?



Reference: 1998 HG6, 2000 HI4, 2007 HG5

By adding numbers on the right as shown (Pascal triangle), the number of different routes = 6

1	3	6
1	2	3
1	1	1

I4.2 Given $\sqrt{2a} = -b\tan\frac{\pi}{3}$. Find b.

$\sqrt{12} = -b \cdot \sqrt{3}$

$b = -2$

I4.3 Given that $p * q = \frac{p-q}{p}$, find c if $c = (a + b) * (b - a)$.

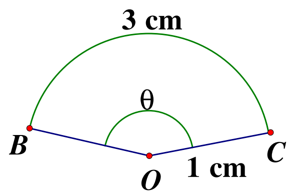
$c = (6 - 2)*(-2 - 6)$

$= 4*(-8)$

$= \frac{4+8}{4}$

$= 3$

I4.4 A wire of c cm is bent to form a sector of radius 1 cm. What is the angle of the sector in degrees (correct to the nearest degree)?



Let the angle at centre be θ radians.

$2 + 1\times\theta = 3$

$\theta = 1$ radian

$= \frac{180^\circ}{\pi}$

$\approx 57.3^\circ$

$= 57^\circ$ (correct to the nearest degree)

Individual Event 5

I5.1 If $a(x+1) = x^3 + 3x^2 + 3x + 1$, find a in terms of x.

$a(x+1) = (x+1)^3$

$a = (x+1)^2 = x^2 + 2x + 1$

I5.2 If $a - 1 = 0$, then the value of x is 0 or b, what is b?

$a = 1 \Rightarrow 1 = (x+1)^2$

$x^2 + 2x = 0 \Rightarrow x = 0$ or -2

$\Rightarrow b = -2$

I5.3 If $pc^4 = 32$, $pc = b^2$ and c is positive, what is the value of c?

$pc^4 = 32$ …… (1)

$pc = (-2)^2 = 4$ …… (2)

(1)÷(2): $c^3 = 8$

$c = 2$

I5.4 P is an operation such that $P(A \cdot B) = P(A) + P(B)$.

$P(A) = y$ means $A = 10^y$. If $d = A \cdot B$, $P(A) = 1$ and $P(B) = c$, find d.

$P(A) = 1 \Rightarrow A = 10^1 = 10$

$P(B) = c \Rightarrow B = 10^2 = 100$

$d = A \cdot B = 10 \cdot 100 = 1000$

Group Event 6

G6.1 The table shows the results of the operation * on P, Q, R, S taken two at a time.

*	P	Q	R	S
P	Q	R	S	P
Q	R	S	P	Q
R	S	P	Q	R
S	P	Q	R	S

Let a be the inverse of P. Find a.

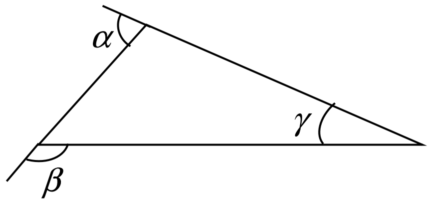
$P*S = P = S*P$, $Q*S = Q = S*Q$, $R*S = R = S*R$, $S*S = S$

The identity element is S.

$P*R = S = R*P$

The inverse of P is R .

G6.2 The average of α and β is 105°, the average of α, β and γ is b°. Find b.



Reference: 1991 FG6.3

$(\alpha + \beta)\div 2 = 105°$

$\Rightarrow \alpha + \beta = 210° \cdots\cdots (1)$

$180° - \beta + \gamma = \alpha$ (adj. $\angle$s on st. line, ext. $\angle$ of Δ)

$\gamma = \alpha + \beta - 180° \cdots\cdots (2)$

Sub. (1) into (2): $\gamma = 210° - 180° = 30°$

$b = (210 + 30)\div 3 = 80$

G6.3 The sum of two numbers is 10, their product is 20. The sum of their reciprocal is c. What is c?

Reference 1984 FSG.1, 1985 FSGI.1, 1986 FSG.1

Let the two numbers be x, y.

$x + y = 10 \ldots\ldots (1)$

$x\,y = 20 \ldots\ldots (2)$

$$c = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x+y}{xy}$$

$$= \frac{10}{20} = \frac{1}{2}$$

G6.4 It is given that $\sqrt{90} = 9.49$, to 2 decimal places.

If $d < 7\sqrt{0.9} < d + 1$, where d is an integer, what is d?

$7\sqrt{0.9} = 0.7\sqrt{90} = 0.7\times 9.49$ (correct to 2 decimal places)

$= 6.643$

$d = 6$

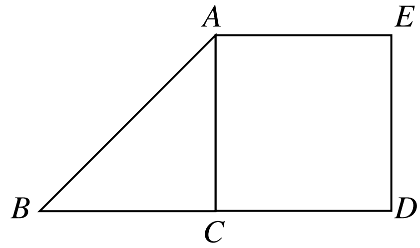
Group Event 7

G7.1 Find 3 + 6 + 9 + … + 45.

The above is an arithmetic series with first term = 3, common difference =3, no. of terms =15.

$S_{15} = \frac{15}{2}\cdot(3+45) = 360$

G7.2 In the figure shown, $ACDE$ is a square and $AC = BC$, $\angle ACB = 90°$. Find the area of ΔABC if the area of $ACDE$ is 10 cm^2.



$\Delta ABC \cong \Delta CED \cong \Delta ECA$ (S.A.S.)

The area of $\Delta ABC = \frac{1}{2}\times$area of $ACDE$

$= 5$ cm^2

G7.3 Given that $a + \frac{1}{a} = 3$. Evaluate $a^3 + \frac{1}{a^3}$.

Reference: 1996 FI1.2, 1998 FG5.2, 2010 FI3.2

$\left(a+\frac{1}{a}\right)^2 = 9$

$\Rightarrow\ a^2 + \frac{1}{a^2} = 7$

$a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$

$= 3\times(7-1)$

$= 18$

G7.4 Given that $\sum_{y=1}^{n}\frac{1}{y} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$.

Find $\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1}$. (Express your answer in fraction.)

Reference: 1991 FSG.1

$\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{8} - \left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{9}\right)$

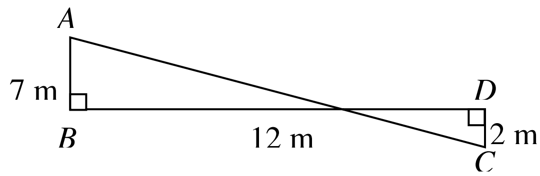
$= 1 - \frac{1}{9}$

$= \frac{8}{9}$

Group Event 8

G8.1 Peter is standing at A and John is at C. The distance between B and D is 12 m. What is the shortest distance between John and Peter?

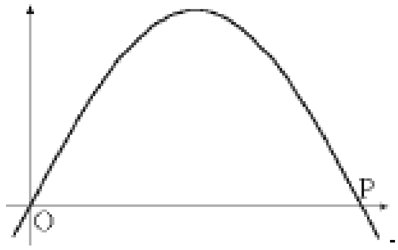
Reference: 1991 HG9, 1993 HI1, 1996 HG9



$AC = \sqrt{(7+2)^2 + 12^2}$ m

$= 15$ m

G8.2 The following figure shows a part of the graph $y = \sin 3x°$. What is the x-coordinate of P?



$\sin 3x° = 0$

$3x° = 180°$

$x = 60$

G8.3 If $f(x) = x^2$, then express $f(x) - f(x-1)$ in terms of x.

$f(x) - f(x-1) = x^2 - (x-1)^2 = 2x - 1$

G8.4 If mnp, nmp, mmp and nnp are numbers in base 10 composed of the digits m, n and p, such that: $mnp - nmp = 180$ and $mmp - nnp = d$. Find d.

$100m + 10n + p - (100n + 10m + p) = 180$

$100(m - n) - 10(m - n) = 180$

$m - n = 2$

$d = mmp - nnp$

$= 100m + 10m + p - (100n + 10n + p)$

$= 110(m - n)$

$= 220$

Group Event 9

G9.1 If $\sin\theta = \dfrac{3}{5}$, $a = \sqrt{\tan^2\theta + 1}$, find a.

$\cos^2\theta = 1 - \sin^2\theta = 1 - \dfrac{9}{25} = \dfrac{16}{25}$

$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\frac{9}{25}}{\frac{16}{25}} = \dfrac{9}{16}$

$a = \sqrt{\tan^2\theta + 1} = \sqrt{\dfrac{9}{16} + 1} = \dfrac{5}{4}$

G9.2 Examine the following proof carefully: To prove $\dfrac{1}{8} > \dfrac{1}{4}$.

<u>Steps</u>

1	$3 > 2$
2	Multiply both sides by $\log\left(\frac{1}{2}\right)$, then $3\log\left(\frac{1}{2}\right) > 2\log\left(\frac{1}{2}\right)$
3	$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$
4	$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$

$\therefore \dfrac{1}{8} > \dfrac{1}{4}$

Which step is incorrect?

Step 2 is incorrect because $\log\left(\dfrac{1}{2}\right) < 0$.

Multiply both sides by $\log\left(\dfrac{1}{2}\right)$, then $3\log\left(\dfrac{1}{2}\right) < 2\log\left(\dfrac{1}{2}\right)$.

G9.3 If the lines $2y + x + 3 = 0$ and $3y + cx + 2 = 0$ are perpendicular, find the value of c.

Reference: 1984 FSG.3, 1985 FI4.1, 1986 FSG.2, 1987 FG10.2, 1988 FG8.2

Product of slopes = –1

$-\dfrac{1}{2} \times \left(-\dfrac{c}{3}\right) = -1$

$c = -6$

G9.4 There are 4 red balls and 3 black balls in a box. If 3 balls are chosen one by one with replacement, what is the probability of choosing 2 red balls and 1 black ball?

P(2 red, 1 black) $= 3 \times \left(\dfrac{4}{7}\right)^2 \times \dfrac{3}{7} = \dfrac{144}{343}$

Group Event 10

G10.1 $1^2 - 1 = 0 \times 2$

$2^2 - 1 = 1 \times 3$

$3^2 - 1 = 2 \times 4$

$4^2 - 1 = 3 \times 5$

………………

$A^2 - 1 = 3577 \times 3579$

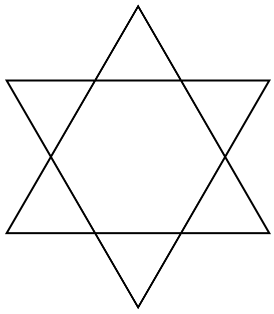
If $A > 0$, find A.

Reference: 1984 FSG..2, 1991 FI2.1

$A^2 - 1 = (3578 - 1) \times (3578 + 1)$

$A = 3578$

G10.2 The sides of an N-sided regular polygon are produced to form a "star". If the angle at each point of that "star" is 108°, find N. (For example, the "star" of a six-sided polygon is given as shown in the diagram.)



6-sided regular polygon.

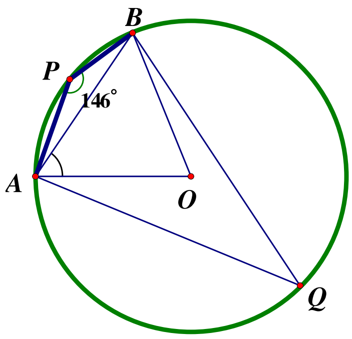
Consider an isosceles triangle formed by each point. The vertical angle is 108°.

Each of the base angle $= \dfrac{180^\circ - 108^\circ}{2} = 36^\circ$

$36N = 360$ (sum of ext. $\angle$s of polygon) $\Rightarrow N = 10$

G10.3 A, P, B are three points on a circle with centre O.

If $\angle APB = 146^\circ$, find $\angle OAB$.



Add a point Q as shown in the diagram.

$\angle AQB = 180^\circ - 146^\circ = 34^\circ$ (opp. $\angle$s cyclic quad.)

$\angle AOB = 2\times 34^\circ = 68^\circ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$OA = OB =$ radii

$\angle OAB = \angle OBA$ (base $\angle$s isos. Δ)

$= \dfrac{180^\circ - 68^\circ}{2} = 56^\circ$ ($\angle$s sum of Δ)

G10.4 A number X consists of 2 digits whose product is 24. By reversing the digits, the new number formed is 18 greater than the original one. What is X? (**Reference: 1991 FG6.1-2**)

Let the tens digit of X be a and the units digit be b.

$X = 10a + b$, reversed number $= 10b + a$

$ab = 24 \Rightarrow b = \dfrac{24}{a}$ …… (1)

$10b + a - (10a + b) = 18 \Rightarrow b - a = 2$ …… (2)

Sub. (1) into (2): $\dfrac{24}{a} - a = 2$

$24 - a^2 = 2a$

$a^2 + 2a - 24 = 0$

$(a - 4)(a + 6) = 0$

$a = 4$ or -6 (rejected)

$b = 6$

$X = 46$