Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 P 是 $3^{2003}\times 5^{2002}\times 7^{2001}$ 的個位數。求 P 的值。
Let P be the units digit of $3^{2003}\times 5^{2002}\times 7^{2001}$. Find the value of P .

P =

2. 若方程 $(x^2-x-1)^{x+P-1}=1$ 有 Q 個整數解，求 Q 的值。
If the equation $(x^2-x-1)^{x+P-1}=1$ has Q integral solutions, find the value of Q .

Q =

3. 設 x、y 為實數且 $xy=1$。若 $\frac{1}{x^4}+\frac{1}{Qy^4}$ 的最小值是 R，求 R 的值。
Let x, y be real numbers and $xy=1$. If the minimum value of $\frac{1}{x^4}+\frac{1}{Qy^4}$ is R, find the value of R .

R =

4. 設 x_R、x_{R+1}、…、x_K $(K>R)$ 為 $K-R+1$ 個不相同的正整數且 $x_R+x_{R+1}+\ldots+x_K=2003$。若 S 是 K 的最大可能的值，求 S 的值。
Let $x_R, x_{R+1}, \ldots, x_K$ $(K>R)$ be $K-R+1$ distinct positive integers and $x_R+x_{R+1}+\ldots+x_K=2003$.
If S is the maximum possible value of K, find the value of S .

S =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若一個兩位數 P 的 50 次方是一個 69 位數，求 P 的值。
(已知 $\log 2 = 0.3010$，$\log 3 = 0.4771$，$\log 11 = 1.0414$)
If the 50^{th} power of a two-digit number P is a 69-digit number, find the value of P.
(Given that $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 11 = 1.0414$.)

$P =$

2. 方程式 $x^2 + ax - P + 7 = 0$ 的根是 α 和 β；而方程式 $x^2 + bx - r = 0$ 的根是 $-\alpha$ 和 $-\beta$。若方程式 $(x^2 + ax - P + 7) + (x^2 + bx - r) = 0$ 的正根是 Q，求 Q 的值。
The roots of the equation $x^2 + ax - P + 7 = 0$ are α and β, whereas the roots of the equation $x^2 + bx - r = 0$ are $-\alpha$ and $-\beta$. If the positive root of the equation $(x^2 + ax - P + 7) + (x^2 + bx - r) = 0$ is Q, find the value of Q.

$Q =$

3. 已知ΔABC 為一等腰三角形，$AB = AC = \sqrt{2}$ 及 BC 上有 Q 個點 D_1、D_2、 …、D_Q。設 $m_i = AD_i^2 + BD_i \times D_iC$。若 $m_1 + m_2 + m_3 + \ldots + m_Q = R$，求 R 的值。
Given that ΔABC is an isosceles triangle, $AB = AC = \sqrt{2}$, and $D_1, D_2, \ldots, D_Q$ are Q points on BC. Let $m_i = AD_i^2 + BD_i \times D_iC$.
If $m_1 + m_2 + m_3 + \ldots + m_Q = R$, find the value of R.

$R =$

4. 有 2003 個袋從左至右排列。已知最左面的袋裝有 R 個球，而且每 7 個相鄰的袋共裝有 19 個球。若最右面的袋有 S 個球，求 S 的值。
There are 2003 bags arranged from left to right. It is given that the leftmost bag contains R balls, and every 7 consecutive bags contains 19 balls altogether.
If the rightmost bag contains S balls, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]   Team No. [ ]

+ Bonus score [ ]   Time [ ] [ ]

Total score [ ]   Min.   Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $\begin{cases} wxyz = 4 \\ w - xyz = 3 \end{cases}$ 且 $w > 0$。若 w 的解是 P，求 P 的值。

Given that $\begin{cases} wxyz = 4 \\ w - xyz = 3 \end{cases}$ and $w > 0$. If the solution of w is P, find the value of P.

$P =$

2. 設 $[y]$ 表示小數 y 的整數部分，如 $[3.14] = 3$。若 $\left[\left(\sqrt{2}+1\right)^P\right] = Q$，求 Q 的值。

Let $[y]$ represents the integral part of the decimal number y.

For example, $[3.14] = 3$. If $\left[\left(\sqrt{2}+1\right)^P\right] = Q$, find the value of Q.

$Q =$

3. 已知 $x_0y_0 \neq 0$ 及 $Qx_0^2 - 22\sqrt{3}\,x_0y_0 + 11y_0^2 = 0$。若 $\dfrac{6x_0^2 + y_0^2}{6x_0^2 - y_0^2} = R$,，求 R 的值。

Given that $x_0y_0 \neq 0$ and $Qx_0^2 - 22\sqrt{3}\,x_0y_0 + 11y_0^2 = 0$.
If $\dfrac{6x_0^2 + y_0^2}{6x_0^2 - y_0^2} = R$, find the value of R.

$R =$

4. 四邊形 $ABCD$ 兩對角綫 AC 和 BD 互相垂直。$AB = 5$，$BC = 4$，$CD = R$。若 $DA = S$，求 S 的值。
The diagonals AC and BD of a quadrilateral $ABCD$ are perpendicular to each other. Given that $AB = 5$, $BC = 4$, $CD = R$. If $DA = S$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如果 9 位數 $\overline{32x35717y}$ 是 72 的倍數，$P = xy$，求 P 的值。
Suppose the 9-digit number $\overline{32x35717y}$ is a multiple of 72, and $P = xy$, find the value of P.

$P =$

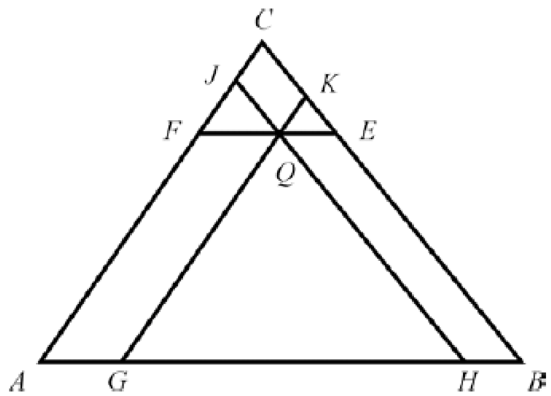
2. 已知三條直綫 $4x + y = \frac{P}{3}$，$mx + y = 0$ 和 $2x - 3my = 4$ 不能構成一個三角形。若 $m > 0$ 及 Q 是 m 的最小可能的值，求 Q 的值。
Given that the lines $4x + y = \frac{P}{3}$, $mx + y = 0$ and $2x - 3my = 4$ cannot form a triangle. Suppose that $m > 0$ and Q is the minimum possible value of m, find Q.

$Q =$

3. 已知 R，x，y 及 z 是整數且 $R > x > y > z$。若 R，x，y 及 z 滿足方程 $2^R + 2^x + 2^y + 2^z = \frac{495Q}{16}$，求 R 的值。
Given that R, x, y, z are integers and $R > x > y > z$.
If R, x, y, z satisfy the equation $2^R + 2^x + 2^y + 2^z = \frac{495Q}{16}$, find the value of R.

$R =$

4.



圖一 Figure 1

如圖一，ΔABC 內任選一點 Q，通過 Q 作三條分別平行於各邊的直綫，其中 $FE // AB$，$GK // AC$ 及 $HJ // BC$。ΔKQE，ΔJFQ 及 ΔQGH 的面積分別是 R，9 及 49。若 ΔABC 的面積是 S，求 S 的值。
In Figure 1, Q is the interior point of ΔABC. Three straight lines passing through Q are parallel to the sides of the triangle such that $FE // AB$, $GK // AC$ and $HJ // BC$.
Given that the areas of ΔKQE, ΔJFQ and ΔQGH are R, 9 and 49 respectively.
If the area of ΔABC is S, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

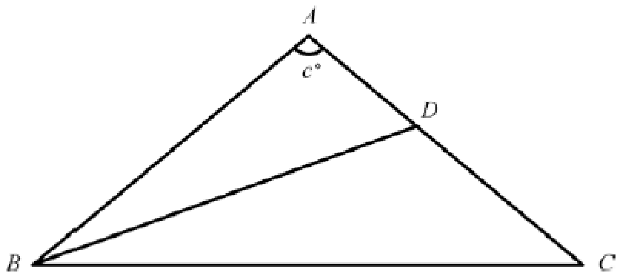
1. 已知 n、k 皆為自然數，且 $1 < k < n$。
若 $\frac{(1+2+3+\cdots+n)-k}{n-1}=10$ 及 $n+k=a$，求 a 的值。
Given that n and k are natural numbers and $1 < k < n$.
If $\frac{(1+2+3+\cdots+n)-k}{n-1}=10$ and $n + k = a$, find the value of a.

$a =$

2. 已知$(x-1)^2+y^2=4$，其中 x 和 y 是實數。若 $2x+y^2$ 的極大值是 b，求 b 的值。
Given that $(x-1)^2+y^2=4$, where x and y are real numbers.
If the maximum value of $2x+y^2$ is b, find the value of b.

$b =$

3.



圖一
Figure 1

如圖一，ΔABC 是一個等腰三角形，其中 $AB = AC$。
若 $\angle B$ 的角平分綫交 AC 於 D 且 $BC = BD + AD$。設 $\angle A = c°$，求 c 的值。
In Figure 1, ΔABC is an isosceles triangle and $AB = AC$. Suppose the angle bisector of $\angle B$ meets AC at D and $BC = BD + AD$. Let $\angle A = c°$, find the value of c.

$c =$

4. 兩質數之和為 105。若這兩質數之積為 d，求 d 的值。
Given that the sum of two prime numbers is 105.
If the product of these prime numbers is d, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

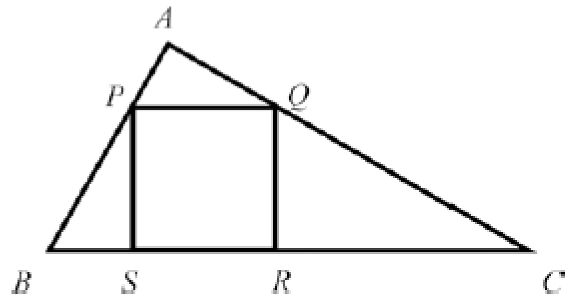
1. 設方程 $ax(x+1) + bx(x+2) + c(x+1)(x+2) = 0$ 有根 1 和 2。若 $a + b + c = 2$，求 a 的值。
Given that the equation $ax(x + 1) + bx(x + 2) + c(x + 1)(x + 2) = 0$ has roots 1 and 2. If $a + b + c = 2$, find the value of a .

$a =$

2. 設 $48^x = 2$，$48^y = 3$。若 $8^{\frac{x+y}{1-x-y}} = b$，求 b 的值。
Given that $48^x = 2$ and $48^y = 3$. If $8^{\frac{x+y}{1-x-y}} = b$, find the value of b .

$b =$

3.



圖一
Figure 1

如圖一，正方形 $PQRS$ 內接於 ΔABC。ΔAPQ、ΔPBS 和ΔQRC 的面積分別為 4、4 和 12。若正方形 $PQRS$ 的面積為 c，求 c 的值。
In Figure 1, the square $PQRS$ is inscribed in ΔABC. The areas of ΔAPQ, ΔPBS and ΔQRC are 4, 4 and 12 respectively. If the area of the square is c, find the value of c .

$c =$

4. 在ΔABC 中，$\cos A = \frac{4}{5}$ 和 $\cos B = \frac{7}{25}$。若 $\cos C = d$，求 d 的值。
In ΔABC, $\cos A = \frac{4}{5}$ and $\cos B = \frac{7}{25}$. If $\cos C = d$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 f 為一函數，f(1) = 1，並對任意整數 m 及 n，$f(m+n) = f(m) + f(n) + mn$。若 $a = \frac{f(2003)}{6}$，求 a 的值。
Let f be a function such that f(1) = 1 and for any integers m and n, $f(m+n) = f(m) + f(n) + mn$. If $a = \frac{f(2003)}{6}$, find the value of a.

$a =$

2. 若 $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 3$，$b = \frac{x^{\frac{3}{2}} + x^{-\frac{3}{2}} - 3}{x^2 + x^{-2} - 2}$，求 b 的值。
Suppose $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 3$, $b = \frac{x^{\frac{3}{2}} + x^{-\frac{3}{2}} - 3}{x^2 + x^{-2} - 2}$, find the value of b.

$b =$

3. 已知 $f(n) = \sin\frac{n\pi}{4}$，其中 n 是整數。若 $c = f(1) + f(2) + \cdots + f(2003)$，求 c 的值。
Given that $f(n) = \sin\frac{n\pi}{4}$, where n is an integer.
If $c = f(1) + f(2) + \cdots + f(2003)$, find the value of c.

$c =$

4. 已知函數 $f(x) = \begin{cases} -2x+1, \text{ when } x<1 \\ x^2-2x, \text{ when } x \ge 1 \end{cases}$。若 d 是 $f(x) = 3$ 的最大整數解，求 d 的值。
Given that $f(x) = \begin{cases} -2x+1, \text{ when } x<1 \\ x^2-2x, \text{ when } x \ge 1 \end{cases}$.
If d is the maximum integral solution of $f(x) = 3$, find the value of d.

$d =$

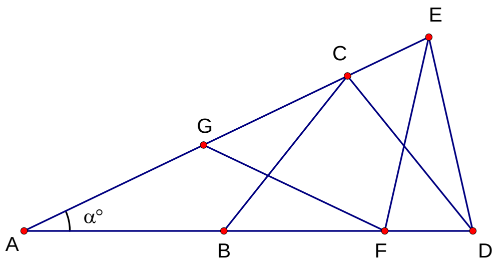
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，AE 、AD 是直綫且 $AB = BC = CD = DE = EF = FG = GA$。若 $\angle DAE = \alpha°$，求 α 的值。
In Figure 1, AE and AD are two straight lines and $AB = BC = CD = DE = EF = FG = GA$. If $\angle DAE = \alpha°$, find the value of α .



圖一
Figure 1

α =

2. 設 $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_8x^8$ 為八次多項式，其中 a_0、a_1、…、a_8 為實數。若 $P(k) = \frac{1}{k}$ 當 $k = 1, 2, \ldots, 9$, 及 $b = P(10)$，求 b 的值。

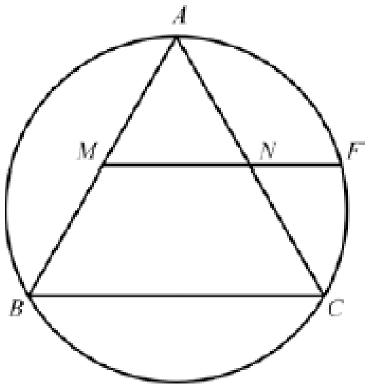
Suppose $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_8x^8$ is a polynomial of degree 8 with real coefficients $a_0, a_1, \ldots, a_8$. If $P(k) = \frac{1}{k}$ when $k = 1, 2, \ldots, 9$, and $b = P(10)$, find the value of b .

b =

3. 已知 x，y 為兩正整數使 $xy - (x + y) = \text{HCF}(x, y) + \text{LCM}(x, y)$，其中 $\text{HCF}(x, y)$ 和 $\text{LCM}(x, y)$ 分別是 x 和 y 的最大公因數和最小公倍數。若 c 是 $x + y$ 的最大可能的值，求 c。
Given two positive integers x and y, $xy - (x + y) = \text{HCF}(x, y) + \text{LCM}(x, y)$, where $\text{HCF}(x, y)$ and $\text{LCM}(x, y)$ are respectively the greatest common divisor and the least common multiple of x and y. If c is the maximum possible value of $x + y$, find c.

c =

4. 如圖二，ΔABC 是等邊三角形，M 及 N 分別是 AB 及 AC 的中點，F 是直綫 MN 與圓 ABC 的交點。若 $d = \frac{MF}{MN}$，求 d 的值。
In Figure 2, ΔABC is an equilateral triangle, points M and N are the midpoints of sides AB and AC respectively, and F is the intersection of the line MN with the circle ABC.
If $d = \frac{MF}{MN}$, find the value of d .



圖二 Figure 2

d =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.