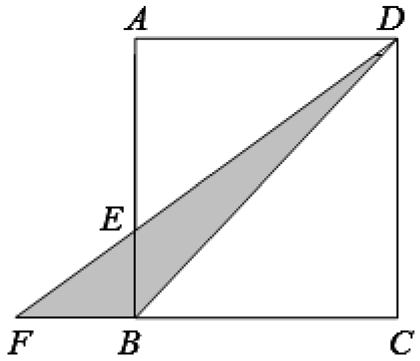
Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在右圖中，$ABCD$ 是一邊長為 10 cm 的正方形，AEB、FED 及 FBC 為直綫，ΔAED 的面積比ΔFEB 的面積大 10 cm^2。若ΔDFB 的面積為 P cm^2，求 P 的值。
In the following figure, $ABCD$ is a square of length 10 cm. AEB, FED and FBC are straight lines. The area of ΔAED is larger than that of ΔFEB by 10 cm^2. If the area of ΔDFB is P cm^2, find the value of P .

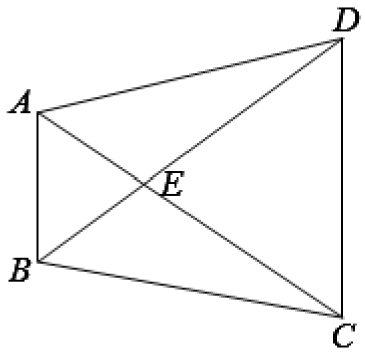


P =

2. 一件工程，甲單獨需時 90 天完成，而乙則需時 Q 天。若甲、乙二人合做只需 P 天完成，求 Q 的值。
Workman A needs 90 days to finish a task independently while workman B needs Q days for the same task. If they only need P days to finish the task when working together, find the value of Q .

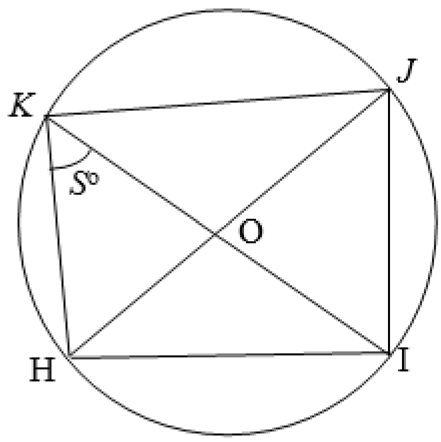
Q =

3. 在右圖中，$AB // CD$，梯形 $ABCD$ 的面積為 R cm^2。已知 ΔABE 和 ΔCDE 的面積分別為 Q cm^2 和 $4Q$ cm^2，求 R 的值。
In the following figure, $AB // CD$, the area of trapezium $ABCD$ is R cm^2. Given that the areas of ΔABE and ΔCDE are Q cm^2 and $4Q$ cm^2 respectively, find the value of R .



R =

4. 在右圖中，O 為圓心，HJ 和 IK 為圓的直徑以及 $\angle HKI = S°$。
已知$\angle HKI + \angle HOI + \angle HJI = \frac{1}{4}R°$，求 S 的值。
In the following figure, O is the centre of the circle, HJ and IK are diameters and $\angle HKI = S°$.
Given that $\angle HKI + \angle HOI + \angle HJI = \frac{1}{4}R°$,
find the value of S .



S =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

\+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $P=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}$，求 P 的值。
Given that $P=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}$, find the value of P.

$P =$

2. 已知 $99Q = P\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\ldots)$，求 Q 的值。
Given that $99Q = P\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\ldots)$, find the value of Q.

$Q =$

3. 已知 x 及 R 為實數。若對所有 x，$\frac{2x^2+2Rx+R}{4x^2+6x+3}\le Q$，求 R 的最大值。
Given that x and R are real numbers and $\frac{2x^2+2Rx+R}{4x^2+6x+3}\le Q$ for all x, find the maximum value of R.

$R =$

4. 已知 $S=\log_{144}\sqrt[R]{2}+\log_{144}\sqrt[2R]{R}$，求 S 的值。
Given that $S=\log_{144}\sqrt[R]{2}+\log_{144}\sqrt[2R]{R}$, find the value of S.

$S =$

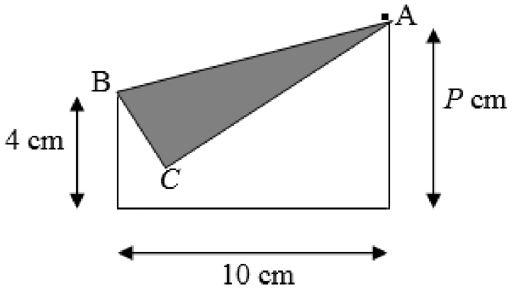
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 將一長方形紙摺出以下的圖形。若△ABC 的面積是原長方形紙面積的 $\frac{1}{3}$，求 P 的值。
A rectangular piece of paper is folded into the following figure. If the area of △ABC is $\frac{1}{3}$ of the area of the original rectangular piece of paper, find the value of P .



P =

2. 已知 $\frac{P}{2}(4^x + 4^{-x}) - 35(2^x + 2^{-x}) + 62 = 0$。若 Q 是此方程的正整數解，求 Q 的值。
If Q is the positive integral solution of the equation $\frac{P}{2}(4^x + 4^{-x}) - 35(2^x + 2^{-x}) + 62 = 0$, find the value of Q .

Q =

3. 設 $[a]$ 表示不大於 a 的最大整數，例如 $[2.5] = 2$。
若 $R = [\sqrt{1}\,] + [\sqrt{2}\,] + \ldots + [\sqrt{99Q}\,]$，求 R 的值。
Let $[a]$ be the largest integer not greater than a. For example, $[2.5] = 2$.
If $R = [\sqrt{1}\,] + [\sqrt{2}\,] + \ldots + [\sqrt{99Q}\,]$, find the value of R .

R =

4. 一個凸多邊形，除了內角 A 以外，其他內角的和是 $4R°$。若 $\angle A = S°$，求 S 的值。
In a convex polygon, other than the interior angle A, the sum of all the remaining interior angles is equal to $4R°$. If $\angle A = S°$, find the value of S .

S =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

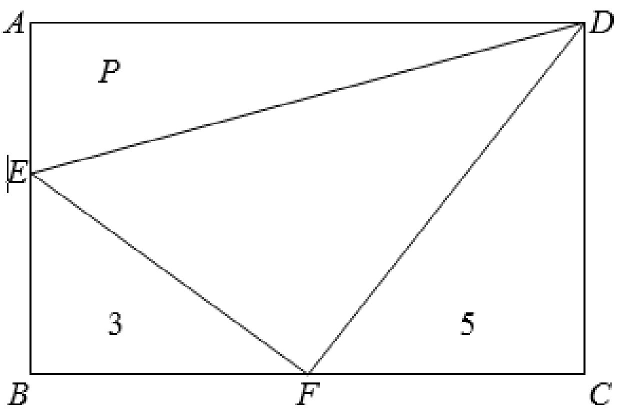
1. 已知 $f(x) = (x^2 + x - 2)^{2002} + 3$ 及 $f\left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) = P$，求 P 的值。

Given that $f(x) = (x^2 + x - 2)^{2002} + 3$ and $f\left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) = P$, find the value of P.

P =

2. 在下圖中，$ABCD$ 為一長方形。E 和 F 分別是 AB 和 BC 上的點。三角形 AED、EBF 和 FCD 的面積分別為 P、3 和 5。若 ΔEFD 的面積為 Q，求 Q 的值。

In the following figure, $ABCD$ is a rectangle. E and F are points on AB and BC respectively. The areas of triangles AED, EBF and FCD are P, 3 and 5 respectively. If the area of ΔEFD is Q, find the value of Q.



Q =

3. 已知 x 和 y 為兩正整數。若不等式 $x^2 + y^2 \le Q$ 的解(x, y)的數目為 R，求 R 的值。

It is given that x and y are positive integers.

If the number of solutions (x, y) of the inequality $x^2 + y^2 \le Q$ is R, find the value of R.

R =

4. 已知 α 和 β 是方程 $x^2 - ax + a - R = 0$ 的兩個根，其中 a 為實數。若 $(\alpha+1)^2 + (\beta+1)^2$ 的最小值為 S，求 S 的值。

It is given that α and β are roots of the equation $x^2 - ax + a - R = 0$, where a is real.
If the minimum value of $(\alpha+1)^2 + (\beta+1)^2$ is S, find the value of S.

S =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

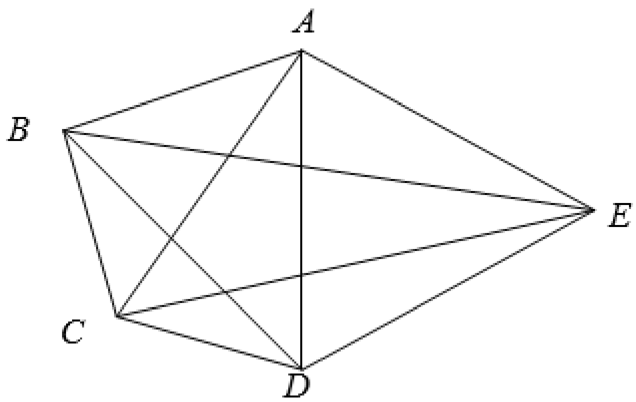
1. 假設曲綫 $x^2 + 3y^2 = 12$ 及直綫 $mx + y = 16$ 只相交於一點。若 $a = m^2$，求 a 的值。
Assume that the curve $x^2 + 3y^2 = 12$ and the straight line $mx + y = 16$ intersect at only one point. If $a = m^2$, find the value of a .

$a =$

2. 已知 $x + y = 1$ 及 $x^2 + y^2 = 2$。若 $x^3 + y^3 = b$，求 b 的值。
It is given that $x + y = 1$ and $x^2 + y^2 = 2$. If $x^3 + y^3 = b$, find the value of b .

$b =$

3. 在右圖中，$AC = AD = AE = ED = DB$ 及 $\angle BEC = c°$。已知 $\angle BDC = 26°$及 $\angle ADB = 46°$，求 c 的值。
In the following figure, $AC = AD = AE = ED = DB$ and $\angle BEC = c°$. Given that $\angle BDC = 26°$ and $\angle ADB = 46°$, find the value of c .



$c =$

4. 已知 $4 \cos^4 \theta + 5 \sin^2 \theta - 4 = 0$，其中 $0° < \theta < 360°$。若 θ 的最大值為 d，求 d 的值。
It is given that $4 \cos^4 \theta + 5 \sin^2 \theta - 4 = 0$, where $0° < \theta < 360°$.
If the maximum value of θ is d, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 2 (Group)

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1. 已知三角形三邊的長分別為 6、8 和 10。若這三角形的面積為 a，求 a 的值。
It is given that the lengths of the sides of a triangle are 6, 8, and 10.
If the area of the triangle is a, find the value of a.

a =

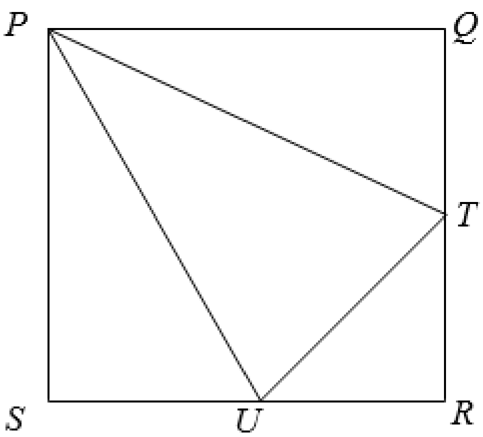
2. 已知 $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}$ 。若 $f(4)=b$，求 b 的值。
Given that $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}$ and $f(4)=b$, find the value of b.

b =

3. 已知 $2002^2-2001^2+2000^2-1999^2+\ldots+4^2-3^2+2^2-1^2=c$，求 c 的值。
Given that $2002^2-2001^2+2000^2-1999^2+\ldots+4^2-3^2+2^2-1^2=c$,
find the value of c.

c =

4. $PQRS$ 為一正方形，PTU 為一等腰三角形及 $\angle TPU=30°$。T 及 U 分別為 QR 及 RS 上的點。ΔPTU 之面積為 1。
若正方形 $PQRS$ 之面積為 d，求 d 的值。
$PQRS$ is a square, PTU is an isosceles triangle, and $\angle TPU=30°$. Points T and U lie on QR and RS respectively. The area of ΔPTU is 1.
If the area of $PQRS$ is d, find the value of d.



d =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

\+ Bonus score

Total score

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Time

Min. Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 3 (Group)

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1. 若 $\dfrac{2002^3+4\times 2002^2+6006}{2002^2+2002}=a$，求 a 的值。
If $\dfrac{2002^3+4\times 2002^2+6006}{2002^2+2002}=a$, find the value of a.

$a =$

2. 已知 x 和 y 為兩實數且滿足關係 $y=\dfrac{x}{2x-1}$。若 $\dfrac{1}{x^2}+\dfrac{1}{y^2}$ 的最小值為 b，求 b 的值。
It is given that the real numbers x and y satisfy the relation $y=\dfrac{x}{2x-1}$.
If the minimum value of $\dfrac{1}{x^2}+\dfrac{1}{y^2}$ is b, find the value of b.

$b =$

3. 從 50 個正整數 1，2，3，…，50 中任意抽兩個不同的數。
已知兩數之和不少於 50。若抽取這兩數共有 c 種取法，求 c 的值。
Suppose two different numbers are chosen randomly from the 50 positive integers 1, 2, 3, … , 50, and the sum of these two numbers is not less than 50.
If the number of ways of choosing these two numbers is c, find the value of c.

$c =$

4. 已知 $x-y=1+\sqrt{5}$，$y-z=1-\sqrt{5}$。若 $x^2+y^2+z^2-xy-yz-zx=d$，求 d 的值。
Given that $x-y=1+\sqrt{5}$, $y-z=1-\sqrt{5}$.
If $x^2+y^2+z^2-xy-yz-zx=d$, find the value of d.

$d =$

FOR OFFICIAL USE

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			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2001 - 2002)
Final Event 4 (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a 是 2002 的所有正因數之和，求 a 的值。
If a is the sum of all the positive factors of 2002, find the value of a.

$a =$

2. 設 $x > 0$，$y > 0$ 且 $\sqrt{x}(\sqrt{x}+\sqrt{y}) = 3\sqrt{y}(\sqrt{x}+5\sqrt{y})$。
若 $b = \dfrac{2x+\sqrt{xy}+3y}{x+\sqrt{xy}-y}$，求 b 的值。
It is given that $x > 0$, $y > 0$ and $\sqrt{x}(\sqrt{x}+\sqrt{y}) = 3\sqrt{y}(\sqrt{x}+5\sqrt{y})$.
If $b = \dfrac{2x+\sqrt{xy}+3y}{x+\sqrt{xy}-y}$, find the value of b.

$b =$

3. 若方程 $\left||x-2|-1\right| = c$ 只有 3 個整數解，求 c 的值。
Given that the equation $\left||x-2|-1\right| = c$ $= c$ has only 3 integral solutions, find the value of c.

$c =$

4. 若 d 是方程 $\frac{1}{2}\left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right]+2\right\}=2$ 的正實數解，求 d 的值。
If d is the positive real root of the equation $\frac{1}{2}\left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right]+2\right\}=2$,
find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=	Team No.	
		+ Bonus score			Time	
		Total score			Min.	Sec.