Challenge Problem 1

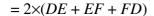D, E and F are points of BC, CA and AB of an equilateral ΔABC.

Prove that the perimeter of $\Delta DEF \geq \dfrac{1}{2}$ perimeter of ΔABC.

Reflect ΔABC, ΔDEF 5 times as shown.

Then ΔABC, ΔAB_1C, ΔA_1B_1C, $\Delta A_1B_1C_1$, $\Delta A_1B_2C_1$, $\Delta A_2B_2C_1$ are identical;

ΔDEF, ΔD_1EF_1, $\Delta D_1E_1F_2$, $\Delta D_2E_2F_2$, $\Delta D_3E_2F_3$, $\Delta D_3E_3F_4$ are identical.

    2×perimeter of ΔDEF

$= 2\times(DE + EF + FD)$

$= FE + ED_1 + D_1F_2 + F_2E_2 + E_2D_3 + D_3F_4$

$\geq FF_4 = BB_2 =$ perimeter of ΔABC

Hence result follows.