Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $\left|x+\sqrt{5}\right|+\left|y-\sqrt{5}\right|+|z|=0$，求 $\alpha=x^2+y^2+z^2$。

   If $\left|x+\sqrt{5}\right|+\left|y-\sqrt{5}\right|+|z|=0$, determine $\alpha=x^2+y^2+z^2$.

   $\alpha=$

2. 若 β 為乘積 $\underbrace{11111\cdots11}_{\alpha\text{個}1}\times\underbrace{99999\cdots99}_{\alpha\text{個}9}$ 所有數位的數字之和，求 β 的值。

   If β is the sum of all digits of the product $\underbrace{11111\cdots11}_{\alpha\ 1\text{'s}}\times\underbrace{99999\cdots99}_{\alpha\ 9\text{'s}}$, determine the value of β.

   $\beta=$

3. 設實函數 $\mathrm{f}(x)$ 對於所有實數 x 及 y 滿足 $\mathrm{f}(xy)=\mathrm{f}(x)\,\mathrm{f}(y)$，且 $\mathrm{f}(1)<1$。求 $\gamma=\mathrm{f}(\beta)+100-\beta$ 的值。

   Suppose that the real function $\mathrm{f}(x)$ satisfies $\mathrm{f}(xy)=\mathrm{f}(x)\,\mathrm{f}(y)$ for all real numbers x and y, and $\mathrm{f}(1)<1$. Determine the value of $\gamma=\mathrm{f}(\beta)+100-\beta$.

   $\gamma=$

4. 若 n 為正整數及 $\mathrm{f}(n)=2^n+2^{n-1}+2^{n-2}+\cdots+2^2+2^1+1$，求 $\delta=\mathrm{f}(\gamma)$ 的值。

   If n is a positive integer and $\mathrm{f}(n)=2^n+2^{n-1}+2^{n-2}+\cdots+2^2+2^1+1$, determine the value of $\delta=\mathrm{f}(\gamma)$.

   $\delta=$

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Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 x_0, y_0, z_0 為以下方程組的解，求 $\alpha = x_0 + y_0 + z_0$ 的值。
If x_0, y_0, z_0 is a solution to the simultaneous equations below, determine the value of $\alpha = x_0 + y_0 + z_0$.
$$\begin{cases} x - y - z = -1 \\ y - x - z = -2 \\ z - x - y = -4 \end{cases}$$

$\alpha =$

2. 若 β 為 $\underbrace{111\cdots111}_{100個1} \div \alpha$ 的餘數。求 β 的值。
If β is the reminder of $\underbrace{111\cdots111}_{100\ 1's} \div \alpha$, determine the value of β.

$\beta =$

3. 若 γ 為 $[(\beta - 2)^{100} + \beta^{50} + (\beta + 2)^{25}] \div 3$ 的餘數，求 γ 的值。
If γ is the remainder of $[(\beta - 2)^{100} + \beta^{50} + (\beta + 2)^{25}] \div 3$, determine the value of γ.

$\gamma =$

4. 若方程 $x^4 + ax^2 + bx + \delta = 0$ 有四實根，且已知其中三個為 1、γ 及 γ^2，求 δ 的值。
If the equation $x^4 + ax^2 + bx + \delta = 0$ has four real roots with three of them being 1, γ and γ^2, determine the value of δ.

$\delta =$

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Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 由 1 至 1000 的正整數中包括 1 及 1000，有 α 個不能被 5 或 7 整除。求 α 的值。
Of the positive integers from 1 to 1000, including 1 and 1000, there are α of them that are not divisible by 5 or 7 . Determine the value of α.

α =

2. 求 $\beta = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{\alpha}(\alpha + 1)^2$ 的值。
Determine the value of $\beta = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{\alpha}(\alpha + 1)^2$.

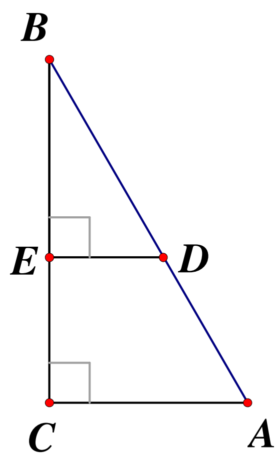
β =

3. 若 γ 為當 β 除以以下數列中的第 1993 項時的餘數：
1，2，2，3，3，3，4，4，4，4，5，5，5，5，5，…
求 γ 的值。
If γ is the remainder of β divided by the 1993rd term of the following sequence:
1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ….
Determine the value of γ.

γ =

4. 在下圖中，$BE = AC$，$BD = \frac{1}{2}$ 及 $DE + BC = 1$。若 δ 是 ED 的長度的 γ 倍，求 δ 的值。
In the figure below, $BE = AC$, $BD = \frac{1}{2}$ and $DE + BC = 1$.
If δ is γ times the length of ED, determine the value of δ.

δ =



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Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 α 為 2^{1000} 除以 13 的餘數，求 α 的值。
Let α be the remainder of 2^{1000} divided by 13, determine the value of α.

$\alpha =$

2. 求 $\beta = \dfrac{\left(7+4\sqrt{\alpha}\right)^{\frac{1}{2}} - \left(7-4\sqrt{\alpha}\right)^{\frac{1}{2}}}{\sqrt{\alpha}}$ 的值。
Determine the value of $\beta = \dfrac{\left(7+4\sqrt{\alpha}\right)^{\frac{1}{2}} - \left(7-4\sqrt{\alpha}\right)^{\frac{1}{2}}}{\sqrt{\alpha}}$.

$\beta =$

3. 若 $f(a) = a - \beta$ 且 $F(a, b) = b^2 + a$，求 $\gamma = F(3, f(4))$ 的值。
If $f(a) = a - \beta$ and $F(a, b) = b^2 + a$, determine the value of $\gamma = F(3, f(4))$.

$\gamma =$

4. 若 δ 是方程 $x^{\log_\gamma x} = 10$ 所有實根的積，求 δ 的值。
If δ is the product of all real roots of $x^{\log_\gamma x} = 10$, determine the value of δ.

$\delta =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (2014– 2015)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 化簡$\left(\dfrac{1\times 3\times 9+2\times 6\times 18+\cdots+n\times 3n\times 9n}{1\times 5\times 25+2\times 10\times 50+\cdots+n\times 5n\times 25n}\right)^{\frac{1}{3}}$。

   Simplify $\left(\dfrac{1\times 3\times 9+2\times 6\times 18+\cdots+n\times 3n\times 9n}{1\times 5\times 25+2\times 10\times 50+\cdots+n\times 5n\times 25n}\right)^{\frac{1}{3}}$.

2. 在 50 隊香港數學競賽的參賽隊伍中，沒有一隊能答對一團體項目中的全部共四個題目。若該項目中的第一題有 45 隊答中，第二題有 40 隊答中，第三題有 35 隊答中，及第四題有 30 隊答中。請計算有多少隊伍同時答中第三及第四題。
   Among 50 school teams joining the HKMO, no team answered all four questions correctly in the paper of a group event. If the first question was solved by 45 teams, the second by 40 teams, the third by 35 teams and the fourth by 30 teams.
   How many teams solved both the third and the fourth questions?

3. 設 n 為 3659893456789325678 和 342973489379256 的乘積。
   求 n 中數字的位數。
   Let n be the product 3659893456789325678 and 342973489379256.
   Determine the number of digits of n.

4. 三個半徑分別為 2、3 及 10 單位的圓同時放於另一大圓內，
   使得四個圓剛好彼此接觸。求大圓的半徑的值。
   Three circles of radii 2, 3 and 10 units are placed inside another big circle in such a way that all circles are touching one another.
   Determine the value of the radius of the big circle.

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				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

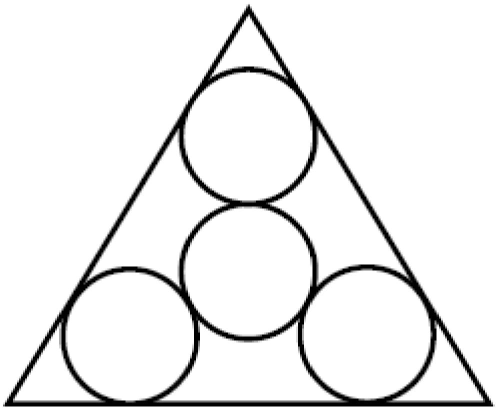
1. 在一個 3×3 的方格內的九個正方形上，分別填上紅色或藍色。
若α為不同着色方法的數量而使得所有 2×2 方格中所包含的正方形都不是全為紅色，求 α 的值。
On a 3×3 grid of 9 squares, each squares is to be painted with either Red or Blue.
If α is the total number of possible colouring in which no 2×2 grid consists of only Red squares, determine the value of α.

α =

2. 若 25 個連續正整數之和剛好等於三個質數的積，這三個質數之和最小是多少？
If the sum of 25 consecutive positive integers is the product of 3 prime numbers, what is the minimum sum of these 3 prime numbers?

3. 求以下方程的所有實根之和
$$|x + 3| - |x - 1| = x + 1\text{。}$$
Determine the sum of all real roots of the following equation
$$|x + 3| - |x - 1| = x + 1.$$

4. 在下圖中，四個大小相同的圓形剛好放入一等邊三角形內。
若圓的半徑為 1 單位，求三角形的面積的值。
In the figure below, there are 4 identical circles placed inside an equilateral triangle. If the radii of the circles are 1 unit, what is the value of the area of the triangle?



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Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

\+ Bonus score [ ]

Time [ ] Min. [ ] Sec.

Total score [ ]

Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 化簡 $\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}$ 。
Simplify $\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}$.

2. 設 p 為質數及 m 為整數。若 $p(p+m)+2p=(m+2)^3$，找出 m 的最大可能值。
Let p be a prime and m be an integer.
If $p(p+m)+2p=(m+2)^3$, find the greatest possible value of m.

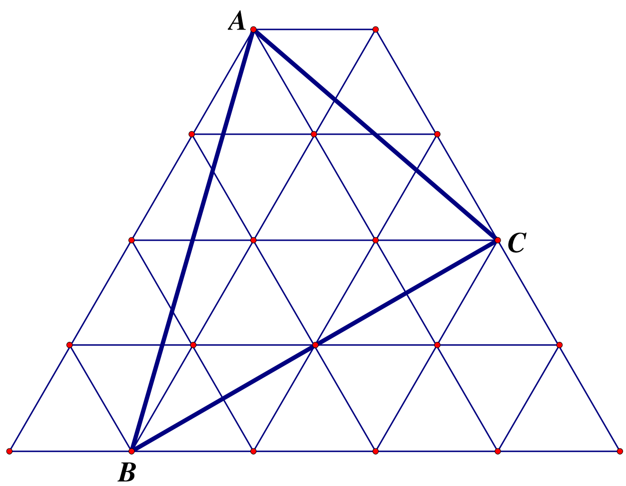
$m =$

3. 求以下方程的根
$$x=\left(x-\frac{1}{x}\right)^{\frac{1}{2}}+\left(1-\frac{1}{x}\right)^{\frac{1}{2}}$$ 。
Determine a root to $x=\left(x-\frac{1}{x}\right)^{\frac{1}{2}}+\left(1-\frac{1}{x}\right)^{\frac{1}{2}}$.

$x =$

4. 下圖中，每個小三角形的面積皆為 1，求三角形 ABC 的面積的值。
In the figure below, the area of each small triangle is 1. Determine the value of the area of the triangle ABC.

area =



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Hong Kong Mathematics Olympiad (2014 – 2015)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

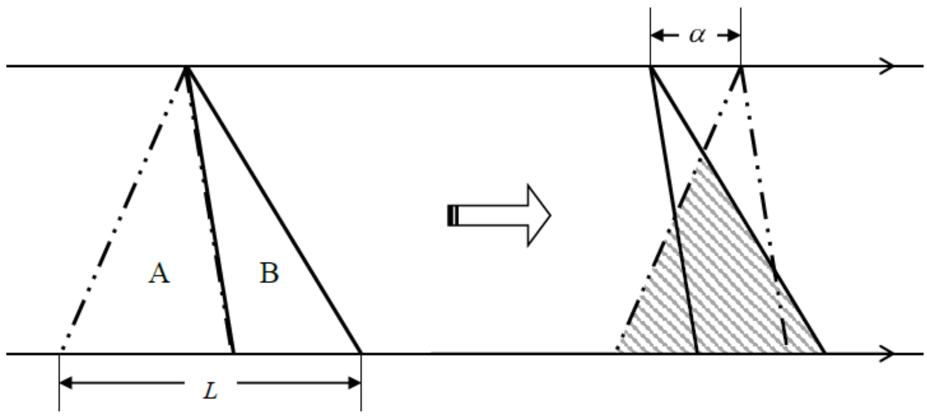
1. 設 $b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \ldots - 2012^2 + 2013^2$，求 b 除以 2015 的餘數。
Let $b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \ldots - 2012^2 + 2013^2$.
Determine the remainder of b divided by 2015 .

2. 考慮所有最大數位為 6 的數，及當把這個最大數位移除後，餘下數值剛為原來數值的 $\frac{1}{25}$ 的正整數。找出在這些正整數中，數值最小的一個。
There are positive integers with leading digits being 6 and upon removing this leading digit, the resulting integer is $\frac{1}{25}$ of the original value.
Determine the least of such positive integers.

3. 若 $x+\frac{1}{x}=1$，求 $x^5+\frac{1}{x^5}$ 的值。
If $x+\frac{1}{x}=1$, determine the value of $x^5+\frac{1}{x^5}$.

4. 在下圖中，若三角形 A 向右移動 α 單位後，所形成的陰影部分的面積為三角形 A 及 B 面積總和的 $\frac{\alpha}{L}$ 倍，求 $\frac{\alpha}{L}$ 的值。
In the figure below, when triangle A shifts α units to the right, the area of shaded region is $\frac{\alpha}{L}$ times of the total area of the triangles A and B. Determine the value of $\frac{\alpha}{L}$.

$\frac{\alpha}{L}=$



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Score for accuracy × Mult. factor for speed =

+ Bonus score

Total score

Team No.

Time

Min. Sec.