Arithmetic series (HKMO Classified Questions by topics)

1982 FI1.1

求 a 的值,若 $a = 5 + 8 + 11 + \dots + 38$ 。

Find the value of *a* if $a = 5 + 8 + 11 + \dots + 38$.

1983 FG7.1

1984 FG9.3

若m為1、2、3、...、1001之平均數,求m的值。

The average of the integers 1, 2, 3, \cdots , 1001 is m. Find the value of m.

1985 FI5.1

 $若 1 + 2 + 3 + 4 + \dots + t = 36$, 求 t 的值。

If $1 + 2 + 3 + 4 + \dots + t = 36$, find the value of t.

1986 FI4.1

已知
$$\begin{cases} 1 = 1^2 \\ 1 + 3 = 2^2 \\ 1 + 3 + 5 = 3^2 \\ 1 + 3 + 5 + 7 = 4^2 \end{cases}$$
。若 $1 + 3 + 5 + \dots + n = 20^2$,求 n 的值。

 $\begin{cases}
1 + 3 + 5 + 7 - 1 \\
1 + 3 + 5 - 2
\end{cases}$ It is known that $\begin{cases}
1 = 1^2 \\
1 + 3 = 2^2 \\
1 + 3 + 5 = 3^2 \\
1 + 3 + 5 + 7 = 4^2
\end{cases}$

If $1 + 3 + 5 + \dots + n = 20^2$, find the value of *n*.

1987 FI1.1

若 $A = 11 + 12 + 13 + \dots + 29$, 求 A 的值。

If $A = 11 + 12 + 13 + \dots + 29$, find the value of A.

1991 FSI.2

首 b 個正奇數之和是 100。求 b 的值。

The sum of the first b positive odd numbers is 100. Find the value of b.

1992 FSG

細看下列各組數字:Consider the following groups of numbers:

(2)

(4, 6)

(8, 10, 12)

(14, 16, 18, 20)

(22, 24, 26, 28, 30)

.....

SG.1 求第 50 組的最後一個數字。Find the last number of the 50th group.

SG.2 求第 50 組的第一個數字。Find the first number of the 50^{th} group.

SG.3 若第 50 組的數字之和為 50P,求 P 的值。

Find the value of P if the sum of the numbers in the 50^{th} group is 50P.

SG.4 若第 100 組的數字之和為 100Q,求 Q 的值。

Find the value of Q if the sum of the numbers in the 100^{th} group is 100Q.

1993 FG8.3-4

設 n 為由 1 至 2000 內被 3 或 7 除時,餘數都為 1 的整數的總數。

Let n be the total number of integers between 1 and 2000 such that each of them gives a remainder of 1 when it is divided by 3 or 7.

G8.3 Find the value of n. 求 n 的值。

G8.4 If s is the sum of all these n integers, find the value of s.

若S為上述n個整數的總和,求S的值。

1994 FI2.4

30 個連續偶數之和為 1170。若 D 為其中最大之偶數, 求 D 的值。

The sum of 30 consecutive even numbers is 1170.

If D is the largest of them, find the value of D.

1995 HG3

已知
$$\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n-1}{n} = \frac{n-1}{2}$$
,

求
$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{10} + \dots + \frac{9}{10}\right)$$
的值。

Given that
$$\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n-1}{n} = \frac{n-1}{2}$$
,

find the value of $\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{10} + \dots + \frac{9}{10}\right)$.

1995 FG7.3

長度同為1的火柴被排成下列圖案。 若以c表示用去火柴枝的總長, 求c的值。



Identical matches of length l are used to arrange the following pattern, if c denotes the total length of matches used, find the value of c.

Rows . .

1996 FG9.4

$$\stackrel{\star}{\cancel{=}} d = \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{60}\right) + \left(\frac{2}{3} + \frac{2}{4} + \dots + \frac{2}{60}\right) + \left(\frac{3}{4} + \frac{3}{5} + \dots + \frac{3}{60}\right) + \dots + \left(\frac{58}{59} + \frac{58}{60}\right) + \frac{59}{60} ,$$

求 d 的值。

Find the sum *d* where

$$d = \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{60}\right) + \left(\frac{2}{3} + \frac{2}{4} + \dots + \frac{2}{60}\right) + \left(\frac{3}{4} + \frac{3}{5} + \dots + \frac{3}{60}\right) + \dots + \left(\frac{58}{59} + \frac{58}{60}\right) + \frac{59}{60} .$$

1997 HI5

求
$$1^2 - 2^2 + 3^2 - 4^2 + \dots + 99^2 - 100^2$$
 的值。

Find the value of $1^2 - 2^2 + 3^2 - 4^2 + \dots + 99^2 - 100^2$.

1997 HG1

已知 a_1, a_2, a_3, \dots 和 b_1, b_2, b_3, \dots 為等差數列,其中 $a_1 = 25$, $b_1 = 75$ 及 $a_{100} + b_{100} = 100$ 。求數列 $a_1 + b_1 \cdot a_2 + b_2 \cdot \dots$ 的前 100 項的和。

If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are arithmetic sequences, where $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$.

Find the sum of the first 100 terms of the sequence $a_1 + b_1$, $a_2 + b_2$, ...

1997 FG5.4

設 $S_1 \times S_2 \times \cdots \times S_{10}$ 是一個由正整數組成的 A.P.之首 10 項。

若
$$S_1 + S_2 + \dots + S_{10} = 55$$
 及 $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d \circ 求 d$ 的值。

Let S_1, S_2, \dots, S_{10} be the first 10 terms of an A.P., which are positive integers.

If
$$S_1 + S_2 + \cdots + S_{10} = 55$$
 and $(S_{10} - S_8) + (S_9 - S_7) + \cdots + (S_3 - S_1) = d$,

find the value of d.

1998 FI4.3

已知 4 個連續數之和為 222, 其中最大的是 r, 求 r 的值。

Given that the sum of 4 consecutive numbers is 222, and the largest of these consecutive numbers is r, find the value of r.

1999 HI9

25

已知下列序列的第 1001 項的分母為 46, 求該項的分子。

Given that the denominator of the 1001^{th} term of the following sequence is 46, find the numerator of this term. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, ...

1999 FI1.1

若一個P-邊的多邊形的內角形成一算術級數,且最小和最大的角分別為 20° 及 160° ,求 P 之值。

If the interior angles of a P-sided polygon form an Arithmetic Progression and the smallest and the largest angles are 20° and 160° respectively.

Find the value of P.

2000 FI4.4

設 $a_1 \cdot a_2 \cdot \cdots \cdot a_{12}$ 為正整數,其中 $a_1 < a_2 < a_3 < \cdots < a_{11} < a_{12}$ 。已知這 12 個正整數的和為 90 及 a_1 的最大值為 S,求 S 的值。

Let a_1, a_2, \ldots, a_{12} be positive integers such that $a_1 < a_2 < a_3 < \cdots < a_{11} < a_{12}$. Given that the sum of these 12 integers is 90 and the maximum value of a_1 is S, find the value of S.

2002 FG2.3

已知 $2002^2 - 2001^2 + 2000^2 - 1999^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2 = c$, 求 c 的值。

Given that $2002^2 - 2001^2 + 2000^2 - 1999^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2 = c$,

find the value of c.

2003 FI1.4

設 $x_1 \cdot x_2 \cdot \cdots \cdot x_K(K>1)$ 為 K 個不相同的正整數且 $x_1 + x_2 + \cdots + x_K = 2003$ 。 若 S 是 K 的最大可能的值,求 S 的值。

Let x_1, x_2, \dots, x_K (K > 1) be K distinct positive integers and $x_1 + x_2 + \dots + x_K = 2003$. If S is the maximum possible value of K, find the value of S. Arithmetic series (HKMO Classified Questions by topics)

2003 FG1.1

已知 $n \cdot k$ 皆為自然數,且1 < k < n。

Given that n and k are natural numbers and 1 < k < n.

If
$$\frac{(1+2+3+\cdots+n)-k}{n-1} = 10$$
 and $n+k=a$, find the value of a .

2004 HI1

設
$$A = 1^2 - 2^2 + 3^2 - 4^2 + \dots + 2003^2 - 2004^2$$
,求 A 的值。
Let $A = 1^2 - 2^2 + 3^2 - 4^2 + \dots + 2003^2 - 2004^2$, find the value of A .

2004 HI4

把自然數按下列方式排列,其中 9 的位置是第 3 列第 2 行。 若 2003 的位置是第 x 列第 y 行,求 xy 的值。

Arrange the natural numbers in the following order. In this arrangement, 9 is in the row 3 and the column 2. If the number 2003 is in the row x and the column y, find the value of xy.

2004 HG1

若
$$x = \frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \dots + \left(\frac{1}{100} + \frac{2}{100} + \dots + \frac{99}{100}\right)$$
,
求 x 的值。

If
$$x = \frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \dots + \left(\frac{1}{100} + \frac{2}{100} + \dots + \frac{99}{100}\right)$$
, 如果 $P = a_2 + a_4 + a_6 + \dots + a_{100}$,求 P 的值。

find the value of x.

2004 HG5

若 R 個連續正整數之和是 1000 (其中 R > 1), 求 R 的最小值。 If the sum of R consecutive positive integers is 1000 (where R > 1), find the least value of R.

2004 FI2.3

If
$$1 + (1 + 2) + (1 + 2 + 3) + \dots + (1 + 2 + 3 + \dots + 6) = R$$
, find the value of R.

2004 FG1.3

若在 200 至 500 之間有 c 個數是 7 的倍數, 求 c 的值。

If there are c multiples of 7 between 200 and 500, find the value of c.

2005 HG1

If
$$x = \frac{19}{97} + \frac{19}{97} \times 2 + \frac{19}{97} \times 3 + \dots + \frac{19}{97} \times 10$$
 and a is the integer that is the closest to x, find the value of a.

2006 HG5

已知連續 k 個正整數之和是 2006,求 k 最大可能的值。

Given that the sum of k consecutive positive integers is 2006, find the maximum possible value of k.

2006 FG4.4

若
$$W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2$$
, 求 W 的值。

If
$$W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2$$
, find the value of W .

2010 FI1.4

在一個有31行的演奏廳中,每一行都比前一行多兩個座位。

若中間的行有64個座位,這演奏廳共有多少個座位?

There are 31 rows in a concert hall and each succeeding row has two more seats than the previous row.

If the middle row has 64 seats, how many seats does the concert have?

2012 FG1.2

設 $a_1 \cdot a_2 \cdot a_3 \cdot ...$ 為一等差數列,公差是 1 及 $a_1 + a_2 + a_3 + ... + a_{100} = 2012$ 。

如果
$$P = a_2 + a_4 + a_6 + \dots + a_{100}$$
, 求 P 的值

Let a_1, a_2, a_3, \cdots be an arithmetic sequence with common difference 1 and

$$a_1 + a_2 + a_3 + \dots + a_{100} = 2012$$
. If $P = a_2 + a_4 + a_6 + \dots + a_{100}$, find the value of P.

2014 FI3.1

若數列 $10^{\frac{1}{11}}$ 、 $10^{\frac{2}{11}}$ 、 $10^{\frac{3}{11}}$ 、...、 $10^{\frac{\alpha}{11}}$ 中所有數字的乘積為 $1\,000\,000$,求正整數 α 的值。

If the product of numbers in the sequence $10^{\frac{1}{11}}$, $10^{\frac{2}{11}}$, $10^{\frac{3}{11}}$, \cdots , $10^{\frac{\alpha}{11}}$ is 1 000 000, determine the value of the positive integer α .

2015 FI3.2

求
$$\beta = 1^2 - 2^2 + 3^2 - 4^2 + \dots + 685^2 - 686^2 + 687^2$$
 的值。

Determine the value of $\beta = 1^2 - 2^2 + 3^2 - 4^2 + \dots + 685^2 - 686^2 + 687^2$.

2015 FG2.2

若25個連續正整數之和剛好等於三個質數的積,這三個質數之和最小是多少? If the sum of 25 consecutive positive integers is the product of 3 prime numbers, what is the minimum sum of these 3 prime numbers?

2015 FG4.1

設
$$b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots - 2012^2 + 2013^2$$
 , 求 b 除以 2015 的餘數。

Let
$$b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots - 2012^2 + 2013^2$$
.

Determine the remainder of b divided by 2015.

2018 HG9

求
$$\left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{100}\right) + \left(\frac{2}{3} + \frac{2}{4} + \dots + \frac{2}{100}\right) + \left(\frac{3}{4} + \frac{3}{5} + \dots + \frac{3}{100}\right) + \dots + \left(\frac{98}{99} + \frac{98}{100}\right) + \frac{99}{100}$$
 的 值。

Find the value of

$$\left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{100}\right) + \left(\frac{2}{3} + \frac{2}{4} + \dots + \frac{2}{100}\right) + \left(\frac{3}{4} + \frac{3}{5} + \dots + \frac{3}{100}\right) + \dots + \left(\frac{98}{99} + \frac{98}{100}\right) + \frac{99}{100}.$$

2023 HI5

若干正整數之和是 60。最大正整數為 15 及其中有一個正整數是 12。除卻 這正整數 12,其餘正整數恰好組成一個等差數列。求最小的正整數。

The sum of certain number of positive integers is 60. The largest positive integer is 15 and one of the positive integers is 12. Apart from this positive integer 12, the remaining positive integers form an arithmetic sequence. Find the smallest positive integer.

Answers

Answers				
1982 FI1.1	1983 FG7.1	1984 FG9.3	1985 FI5.1	1986 FI4.1
258	360	501	8	39
1987 FI1.1	1991 FSI.2	1992 FSG.1	1992 FSG.2	1992 FSG.3
380	10	2550	2452	2501
1992 FSG.4 10001	1993 FG8.3 96	1993 FG8.4 95856	1994 FI2.4 68	1995 HG3 45 2
1995 FG7.3	1996 FG9.4	1997 HI5	1997 HG1	1997 FG5.4
700	885	-5050	10000	16
1998 FI4.3	1999 HI9	1999 FI1.1	2000 FI4.4	2002 FG2.3
57	11	4	2	2005003
2003 FI1.4	2003 FG1.1	2004 HI1	2004 HI4	2004 HG1
62	29	-2009010	700	2475
2004 HG5	2004 FI2.3	2004 FG1.3	2005 HG1	2006 HG5
5	56	43	11	59
2006 FG4.4	2010 FI1.4	2012 FG1.2	2014 FI3.1	2015 FI3.2
2013021	1984	1031	11	236328
2015 FG2.2	2015 FG4.1	2018 HG9	2023 HI5	
23	1	2475	9	