Transformation Example

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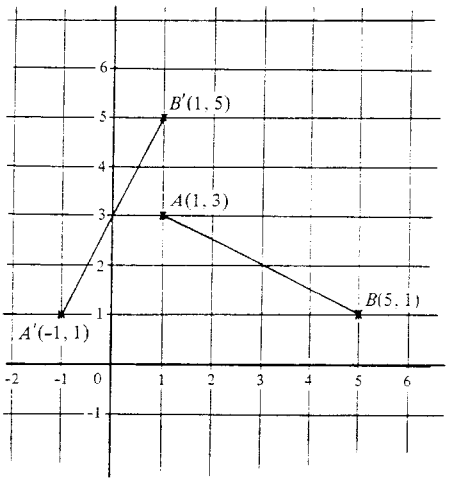
HKCEE 1974 Mathematics (中文版), Syllabus *A* Paper 2 Q7



Given four points $A(1, 3)$, $B(5, 1)$, $A'(-1, 1)$, $B'(1, 5)$.

(*a*) (*i*) Prove that AB and $A'B'$ are equal in length.

(*ii*) Find the equation of the perpendicular bisector of AA'.

(*b*) Let C be the centre of rotation when the line segment AB is rotated to $A'B'$ in anticlockwise direction. (Note: A to A', B to B')

(*i*) Find the coordinates of C.

(*ii*) Prove that the angle of rotation is 90°.

(*a*) (*i*) $AB = \sqrt{(1--1)^2 + (5-1)^2} = \sqrt{20} = 2\sqrt{5}$

$A'B' = \sqrt{(1-3)^2 + (5-1)^2} = \sqrt{20} = 2\sqrt{5}$

$\therefore AB = A'B'$

(*ii*) Mid-point of $AA' = \left(\frac{-1+1}{2}, \frac{1+3}{2}\right) = (0, 2)$

Slope of $AA' = \frac{3-1}{1--1} = 1$

$\therefore$ Equation of the perpendicular bisector: $\frac{y-2}{x} \cdot 1 = -1$

$y - 2 = -x$

$x + y - 2 = 0$ .....(1)

(*b*) (*i*) Next we find the perpendicular bisector of BB'.

mid point of $BB' = \left(\frac{5+1}{2}, \frac{1+5}{2}\right) = (3, 3)$

Slope of $BB' = \frac{5-1}{1-5} = -1$

$\therefore$ Equation of the perpendicular bisector: $\frac{y-3}{x-3} \cdot (-1) = -1$

$y - 3 = x - 3$

$x - y = 0$ .....(2)

The centre of rotation C lies on the intersection of these two perpendicular bisectors.

From (2), $y = x$, sub. into (1): $x + x - 2 = 0$

$x = 1, y = 1$

$\therefore C = (1, 1)$

(*ii*) $A'(-1, 1)$, $C(1,1)$, $A(1, 3)$

The x-coordinate of C is the same as that of A, while the y-coordinate of C is the same as that of A'

$\therefore \angle ACA' = 90°$

The angle of rotation is 90°.