Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. a、b 和 c 分別為 ΔABC 的 $\angle A$、$\angle B$ 和 $\angle C$ 的相對邊的長度。
若 $\angle C = 60^\circ$ 及 $\frac{a}{b+c}+\frac{b}{a+c} = P$，求 P 的值。
a, b and c are the lengths of the opposite sides $\angle A$, $\angle B$ and $\angle C$ of the ΔABC respectively. If $\angle C = 60^\circ$ and $\frac{a}{b+c}+\frac{b}{a+c} = P$, find the value of P.

$P =$

2. 已知 $f(x) = x^2 + ax + b$ 是 $x^3 + 4x^2 + 5x + 6$ 和 $2x^3 + 7x^2 + 9x + 10$ 的公因式。
若 $f(P) = Q$，求 Q 的值。
Given that $f(x) = x^2 + ax + b$ is the common factor of $x^3 + 4x^2 + 5x + 6$ and $2x^3 + 7x^2 + 9x + 10$. If $f(P) = Q$, find the value of Q.

$Q =$

3. 已知 $\frac{1}{a}+\frac{1}{b}=\frac{Q}{a+b}$ 及 $\frac{a}{b}+\frac{b}{a}=R$，求 R 的值。
Given that $\frac{1}{a}+\frac{1}{b}=\frac{Q}{a+b}$ and $\frac{a}{b}+\frac{b}{a}=R$, find the value of R.

$R =$

4. 已知 $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ 及 $a^3 + b^3 = S$，求 S 的值。
Given that $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ and $a^3 + b^3 = S$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

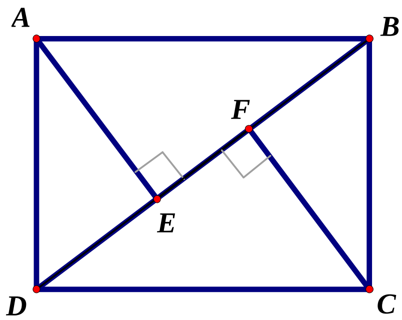
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 P 為整數，及 $5 < P < 20$。
若方程 $x^2 - 2(2P - 3)x + 4P^2 - 14P + 8 = 0$ 的兩個根皆為整數，求 P 的值。
Suppose P is an integer and $5 < P < 20$. If the roots of the equation $x^2 - 2(2P - 3)x + 4P^2 - 14P + 8 = 0$ are integers, find the value of P.

$P =$

2. $ABCD$ 是一長方形。若 $AB = 3P + 4$，$AD = 2P + 6$，AE 和 CF 分別垂直於對角綫 BD，及 $EF = Q$，求 Q 的值。
$ABCD$ is a rectangle. $AB = 3P + 4$, $AD = 2P + 6$.
AE and CF are perpendiculars to the diagonal BD. If $EF = Q$, find the value of Q.

$Q =$



3. 某班學生的人數少於 $4Q$ 人。在一次數學測驗中有 $\frac{1}{3}$ 學生得甲等，$\frac{1}{7}$ 學生得乙等，一半學生得丙等，餘下的學生都不及格。
已知不及格的學生人數是 R，求 R 的值。
There are less than $4Q$ students in a class. In a mathematics test, $\frac{1}{3}$ of the students got grade A, $\frac{1}{7}$ of the students got grade B, half of the students got grade C, and the rest failed. Given that R students failed in the mathematics test, find the value of R.

$R =$

4. $[a]$ 表示不大於 a 的最大整數。例如 $\left[2\frac{1}{3}\right] = 2$。已知方程 $[3x + R] = 2x + \frac{3}{2}$ 的所有根的和為 S，求 S 的值。
$[a]$ represents the largest integer not greater than a. For example, $\left[2\frac{1}{3}\right] = 2$. Given that the sum of the roots of the equation $[3x + R] = 2x + \frac{3}{2}$ is S, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

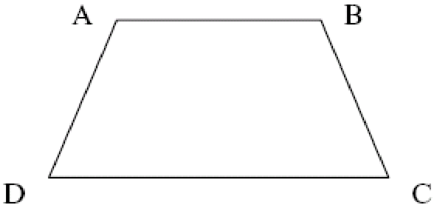
Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. $ABCD$ 是一個梯形，其中 $\angle ADC = \angle BCD = 60°$ 及 $AB = BC = AD = \frac{1}{2}CD$。
若把這梯形分割為 P 等份 $(P > 1)$，使其分割所得的每份與梯形 $ABCD$ 相似。求 P 的最小值。

$ABCD$ is a trapezium such that $\angle ADC = \angle BCD = 60°$ and $AB = BC = AD = \frac{1}{2}CD$.

If this trapezium is divided into P equal portions $(P > 1)$ and each portion is similar to trapezium $ABCD$ itself, find the minimum value of P.



$P =$

2. $(P + 1)^{2001}$ 的個位數字與十位數字的和是 Q，求 Q 的值。
The sum of tens and units digits of $(P + 1)^{2001}$ is Q. Find the value of Q.

$Q =$

3. 若 $\sin 30° + \sin^2 30° + \ldots + \sin^Q 30° = 1 - \cos^R 45°$，求 R 的值。
If $\sin 30° + \sin^2 30° + \ldots + \sin^Q 30° = 1 - \cos^R 45°$, find the value of R.

$R =$

4. 設方程 $x^2 - 8x + (R + 1) = 0$ 的根為 α 和 β。
若 $\frac{1}{\alpha^2}$ 和 $\frac{1}{\beta^2}$ 是方程 $225x^2 - Sx + 1 = 0$ 的根，求 S 的值。
Let α and β be the roots of the equation $x^2 - 8x + (R + 1) = 0$.
If $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ are the roots of the equation $225x^2 - Sx + 1 = 0$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$，$x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$，$y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$。
若 $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$，求 P 的值。
Let $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$, $x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$ and $y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$.
If $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$, find the value of P.

$P =$

2. 若一正 Q 邊形有 P 條對角綫，求 Q 的值。
If a regular Q-sided polygon has P diagonals, find the value of Q.

$Q =$

3. 已知 $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$，$y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$。若 $R=\frac{x^6+y^6}{40}$，求 R 的值。
Let $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$ and $y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$. If $R=\frac{x^6+y^6}{40}$, find the value of R.

$R =$

4. 已知 $[a]$ 表示不大於 a 的最大整數。例如 $[2.5]=2$。
若 $S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$，求 S 的值。
$[a]$ represents the largest integer not greater than a. For example, $[2.5]=2$.
If $S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

\+ Bonus score

Total score

Team No.

Time

Min. Sec.

Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $(a+b+c)^2 = 3(a^2+b^2+c^2)$ 及 $a+b+c=12$。求 a 的值。
Given that $(a+b+c)^2 = 3(a^2+b^2+c^2)$ and $a+b+c=12$, find the value of a.

$a =$

2. 已知 $b\left[\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{1999\times 2001}\right]=2\times\left[\frac{1^2}{1\times 3}+\frac{2^2}{3\times 5}+\cdots+\frac{1000^2}{1999\times 2001}\right]$，求 b 的值。

Given that $b\left[\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{1999\times 2001}\right]=2\times\left[\frac{1^2}{1\times 3}+\frac{2^2}{3\times 5}+\cdots+\frac{1000^2}{1999\times 2001}\right]$,

find the value of b.

$b =$

3. 一六位數 $1234xy$ 能同時被 8 和 9 整除。已知 $x+y=c$，求 c 的值。
A six-digit number $1234xy$ is divisible by both 8 and 9. Given that $x+y=c$, find the value of c.

$c =$

4. 已知 $\log_x t = 6$，$\log_y t = 10$，$\log_z t = 15$。若 $\log_{xyz} t = d$，求 d 的值。
Suppose $\log_x t = 6$, $\log_y t = 10$ and $\log_z t = 15$. If $\log_{xyz} t = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $x=\sqrt{7-4\sqrt{3}}$ 及 $\dfrac{x^2-4x+5}{x^2-4x+3}=a$，求 a 的值。

   Given that $x=\sqrt{7-4\sqrt{3}}$ and $\dfrac{x^2-4x+5}{x^2-4x+3}=a$, find the value of a.

   $a =$

2. E 是長方形 $ABCD$ 內一點。已知 EA、EB、EC 和 ED 的長度分別為 2、$\sqrt{11}$、4 和 b，求 b 的值。

   E is an interior point of the rectangle $ABCD$. Given that the lengths of EA, EB, EC and ED are 2, $\sqrt{11}$, 4 and b respectively, find the value of b.

   $b =$

3. 已知 $111111222222 = c\times(c+1)$，求 c 的值。

   Given that $111111222222 = c\times(c+1)$, find the value of c.

   $c =$

4. 已知 $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ 及 $0 < d < 90$，求 d 的值。

   Given that $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and $0 < d < 90$, find the value of d.

   $d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知方程 $\sqrt{3x+1}+\sqrt{3x+6}=\sqrt{4x-2}+\sqrt{4x+3}$ 的解為 a，求 a 的值。
Given that the solution of the equation $\sqrt{3x+1}+\sqrt{3x+6}=\sqrt{4x-2}+\sqrt{4x+3}$ is a, find the value of a.

a =

2. 已知方程 $x^2y - x^2 - 3y - 14 = 0$ 只得一組正整數解 (x_0, y_0)。若 $x_0 + y_0 = b$，求 b 的值。
Suppose the equation $x^2y - x^2 - 3y - 14 = 0$ has only one positive integral solution (x_0, y_0). If $x_0 + y_0 = b$, find the value of b.

b =

3. $ABCD$ 是一圓內接四邊形。AC 和 BD 相交於 G。
已知 AC =16 cm，$BC = CD$ = 8 cm，$BG = x$ cm 和 $GD = y$ cm。
若 x 和 y 皆為整數且 $x + y = c$，求 c 的值。
$ABCD$ is a cyclic quadrilateral. AC and BD intersect at G.
Suppose AC =16 cm, $BC = CD$ = 8 cm, $BG = x$ cm and $GD = y$ cm.
If x and y are integers and $x + y = c$, find the value of c.

c =

4. 已知 $5^{\log 30}\times\left(\frac{1}{3}\right)^{\log 0.5}=d$。求 d 的值。
Given that $5^{\log 30}\times\left(\frac{1}{3}\right)^{\log 0.5}=d$, find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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Hong Kong Mathematics Olympiad (2000 – 2001)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. $x_1 = 2001$。當 $n > 1$，$x_n = \dfrac{n}{x_{n-1}}$。已知 $x_1x_2x_3\ldots x_{10} = a$，求 a 的值。

   $x_1 = 2001$. When $n > 1$, $x_n = \dfrac{n}{x_{n-1}}$. Given that $x_1x_2x_3\ldots x_{10} = a$, find the value of a .

   $a =$

2. 已知 $1^3 + 2^3 + 3^3 + \ldots + 2001^3$ 的個位數字為 b，求 b 的值。
   Given that the units digit of $1^3 + 2^3 + 3^3 + \ldots + 2001^3$ is b, find the value of b .

   $b =$

3. 甲乙兩人在一圓形跑道上同時同地相背以均速開跑。他們第一次相遇後，乙再跑 1 分鐘到達原起步點。已知甲和乙分別需要 6 分鐘和 c 分鐘繞跑道一周，求 c 的值。

   A and B ran around a circular path with constant speeds. They started from the same place and at the same time in opposite directions. After their first meeting, B took 1 minute to go back to the starting place. If A and B need 6 minutes and c minutes respectively to complete one round of the path, find the value of c .

   $c =$

4. 方程 $x^2 - 45x + m = 0$ 的兩個根皆為質數。已知兩根的平方和為 d，求 d 的值。
   The roots of the equation $x^2 - 45x + m = 0$ are prime numbers.
   Given that the sum of the squares of the roots is d, find the value of d .

   $d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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			Total score				Min.	Sec.