# Compounded angles (HKMO Classified Questions by topics)

#### 1994 FG10.3

已知 
$$\sin 2\theta = 2 \sin \theta \cos \theta \circ$$
求  $c$  ,若  $c = \frac{\sin 20^{\circ} \cos 20^{\circ} \cos 40^{\circ} \cos 60^{\circ} \cos 80^{\circ}}{\sin 160^{\circ}}$  。  $\Delta ABC$  中, $\cos A = \frac{4}{5}$ 和  $\cos B = \frac{7}{25}$  。若  $\cos C = d$  ,求  $d$  的值。

It is given that  $\sin 2\theta = 2 \sin \theta \cos \theta$ .

Find the value of c, if  $c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}$ .

#### 1994 FG10.4

已知 
$$tan(A + B) = \frac{tan A + tan B}{1 - tan A tan B}$$
 , 求  $d$  的值,若

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ)$$

It is given that  $tan(A + B) = \frac{tan A + tan B}{1 - tan A tan B}$ . Find the value of d, if

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ).$$

#### 1999 FI1.2

In  $\triangle ABC$ , AB = 5, AC = 6 and BC = 4. If  $\frac{1}{Q} = \cos 2A$ , find the value of Q.

(Hint:  $\cos 2A = 2 \cos^2 A - 1$ )

#### 2001 FG2.4

已知 
$$\cos 16^{\circ} = \sin 14^{\circ} + \sin d^{\circ}$$
 及  $0 < d < 90$ ,求  $d$  的值。

Given that  $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$  and 0 < d < 90, find the value of d.

## 2002 HI3

已知 
$$\cos 15^{\circ} = \frac{\sqrt{a} + \sqrt{b}}{4}$$
 ,且  $a \cdot b$  是自然數。若  $a + b = y$  ,求  $y$  的值。

Suppose  $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$  and a, b are natural numbers.

If a + b = y, find the value of y.

## 2003 HI6

若對任意 
$$0 < x < \frac{\pi}{2}$$
,  $\cot \frac{1}{4}x - \cot x = \frac{\sin kx}{\left(\sin \frac{1}{4}x\right)(\sin x)}$ , 其中  $k$  是一常數,

If for any 
$$0 < x < \frac{\pi}{2}$$
,  $\cot \frac{1}{4}x - \cot x = \frac{\sin kx}{\left(\sin \frac{1}{4}x\right)(\sin x)}$ , where  $k$  is a constant,

find the value of k.

## 2003 FG2.4

在ΔABC 中,
$$\cos A = \frac{4}{5}$$
和  $\cos B = \frac{7}{25}$ 。若  $\cos C = d$ ,求  $d$  的值。

In  $\triangle ABC$ ,  $\cos A = \frac{4}{5}$  and  $\cos B = \frac{7}{25}$ . If  $\cos C = d$ , find the value of d.

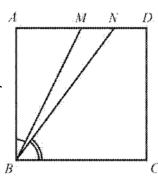
## 2004 FI2.1

如圖,ABCD 為一正方形,M 是AD 的中點及N是MD 的中點及N 是MD 的中點。

$$\angle CBN: \angle MBA = P:1$$
, 求  $P$  的值。

In the figure, ABCD is a square, M is the mid-point of AD and N is the mid-point of MD.

If  $\angle CBN : \angle MBA = P : 1$ , find the value of P.



#### 2004 FG1.4

已知 
$$0 \le x_0 \le \frac{\pi}{2}$$
 且  $x_0$  满足方程  $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin \frac{x}{2}$  。

若 
$$d = \tan x_0$$
, 求  $d$  的值。

Given that  $0 \le x_0 \le \frac{\pi}{2}$  and  $x_0$  satisfies the equation  $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin \frac{x}{2}$ .

If  $d = \tan x_0$ , find the value of d.

# 2005 FG3.1

設 
$$0^{\circ} < \alpha < 45^{\circ}$$
 。若  $\sin \alpha \cos \alpha = \frac{3\sqrt{7}}{16}$  及  $A = \sin \alpha$  ,求  $A$  的值。

Let  $0^{\circ} < \alpha < 45^{\circ}$ . If  $\sin \alpha \cos \alpha = \frac{3\sqrt{7}}{16}$  and  $A = \sin \alpha$ , find the value of A.

# 2006 HG6

設  $a \cdot b \cdot c$  和 d 是實數且滿足  $a^2 + b^2 = c^2 + d^2 = 1$  及 ac + bd = 0。若  $R = c^2 + d^2 = 1$ ab + cd, 求 R 的值。

Let a, b, c and d be real numbers such that  $a^2 + b^2 = c^2 + d^2 = 1$  and ac + bd = 0. If R = ab + cd, find the value of R.

## 2006 FI2.3

已知 
$$T = \sin 50^{\circ} \times (1 + \sqrt{3} \times \tan 10^{\circ})$$
,求  $T$  的值。

Given that  $T = \sin 50^{\circ} \times (1 + \sqrt{3} \times \tan 10^{\circ})$ , find the value of T.

#### 2006 FG3.3

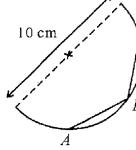
已知  $\tan x + \tan y + 1 = \cot x + \cot y = 6$ 。若  $z = \tan(x + y)$ ,求 z 的值。 Given that  $\tan x + \tan y + 1 = \cot x + \cot y = 6$ .

If  $z = \tan(x + y)$ , find the value of z.

#### 2007 HG8

如圖三,已知半圓的直徑為  $10 \text{ cm} \circ A \circ B \Rightarrow C$ 是半圓上任意的三點使 B 在 $\widehat{AC}$  上。設x 為綫段 AB 及 BC 的長度之和,求x可取的最大值。

In figure 3, given that the diameter of the semicircle is 10 cm. A, B and C are three arbitrary points on the semicircle where B is on  $\widehat{AC}$ . If x is the sum of the length of the line segments AB and BC,



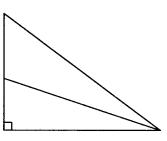
find the greatest possible value of x.

### 2010 HI6

若 
$$x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$$
,其中  $0 \le x, y \le 1$ ,求  $x^2+y^2$  的值。  
If  $x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$ , where  $0 \le x, y \le 1$ , find the value of  $x^2+y^2$ .

## 2010 FI4.4

在圖二中,EFG 為一直角三角形。已知 H 為 F FG 上的一點,使得 GH: HF=4:5 及 $\angle GEH=\angle FEH$ 。若 EG=1 及 FG=d,求 d 的值。 In Figure 2, EFG is a right-angled triangle. Given H that H is a point on FG, such that GH: HF=4:5 and  $\angle GEH=\angle FEH$ . If EG=1 and FG=d, find the value of d.

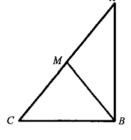


## 2011 HG6

如下圖,M 為 AC上的一點,且 AM = MC = BM = 3。 求 AB + BC 的最大值。

In the figure below, M is a point on AC, AM = MC = BM = 3.

Find the maximum value of AB + BC.



## 2011 FI4.1

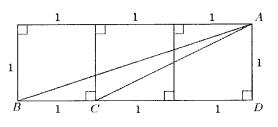
考慮函數  $y = \sin x + \sqrt{3} \cos x$ 。設 a 為 y 的最大值。求 a 的值。 Consider the function  $y = \sin x + \sqrt{3} \cos x$ .

Let a be the maximum value of y. Find the value of a.

#### 2012 FI3.1

在圖中,長方形由三個邊長為1之正方形組成。

若 $\alpha$ ° =  $\angle ABD$  +  $\angle ACD$  ,求α的值。 1 In the figure, a rectangle is subdivided into 3 identical squares of side length 1.



If  $\alpha^{\circ} = \angle ABD + \angle ACD$ , find the value of  $\alpha$ .

## 2012 FI3.2

設 ABC 為一銳角三角形。若  $\sin A = \frac{36}{45}$ ,  $\sin B = \frac{12}{13}$  及  $\sin C = \frac{\beta}{y}$ ,求  $\beta$  的值,

其中 β 及 y 是最簡化之代表形式。

Let ABC be an acute-angled triangle. If  $\sin A = \frac{36}{45}$ ,  $\sin B = \frac{12}{13}$  and  $\sin C = \frac{\beta}{y}$ ,

find the value of  $\beta$ , where  $\beta$  and y are in the lowest terms.

# 2012 FG2.4

在圖中,圓有直徑 BC,圓心在 O,P 、 B 及 C 皆為圓周上的點。若 AB=BC = CD 及 AD 為一綫段, $\alpha = \angle APB$  及  $\beta = \angle CPD$ ,求 $(\tan \alpha)(\tan \beta)$ 的值。

In the figure, P, B and C are points on a circle with centre O and diameter BC. If A, B, C, D are collinear such that AB = BC = CD,  $\alpha = \angle APB$  and



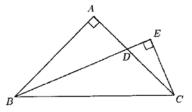
find the value of  $(\tan \alpha)(\tan \beta)$ .



在圖一中,ABC 及 EBC 是兩個直角三角形, $\angle BAC = \angle BEC = 90^\circ$ ,AB = AC 及 EDB 為 $\angle ABC$  的角平分綫。求  $\frac{BD}{CE}$  的值。

In figure 1, ABC and EBC are two right-angled triangles,  $\angle BAC = \angle BEC = 90^{\circ}$ , AB = AC and EDB is the angle bisector of  $\angle ABC$ .

Find the value of  $\frac{BD}{CE}$ .



## 2017 FI4.4

若 cos 2θ = 
$$\frac{3}{44}$$
 , 求  $d = \sin^4 \theta + \cos^4 \theta$  的值。

If  $\cos 2\theta = \frac{3}{44}$ , determine the value of  $d = \sin^4 \theta + \cos^4 \theta$ .

## 2018 FG4.3

求 
$$C = \cos\frac{\pi}{15} \times \cos\frac{2\pi}{15} \times \cos\frac{3\pi}{15} \times \cos\frac{4\pi}{15} \times \cos\frac{5\pi}{15} \times \cos\frac{6\pi}{15} \times \cos\frac{7\pi}{15}$$
 的值。

Determine the value of

$$C = \cos\frac{\pi}{15} \times \cos\frac{2\pi}{15} \times \cos\frac{3\pi}{15} \times \cos\frac{4\pi}{15} \times \cos\frac{5\pi}{15} \times \cos\frac{6\pi}{15} \times \cos\frac{7\pi}{15}.$$

#### 2019 HI14

已知  $3 \sin x + 2 \sin y = 4$ 。設 N 為  $3 \cos x + 2 \cos y$  的最大值。 求 N 的值。

Given that  $3 \sin x + 2 \sin y = 4$ . Let N be the maximum value of  $3 \cos x + 2 \cos y$ .

Find the value of N.

# 2021 P1Q4

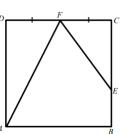
求 8 cos<sup>2</sup> 15° cos<sup>2</sup> 30° - 8 sin<sup>2</sup> 15° cos<sup>2</sup> 30° 的值。

Find the value of  $8\cos^2 15^{\circ}\cos^2 30^{\circ} - 8\sin^2 15^{\circ}\cos^2 30^{\circ}$ .

# 2021 P2Q1

在圖一中,ABCD 是一個邊長為 6 的正方形。F 是  $CD_D$  的中點。若  $\angle FAB = \angle AFE$ ,求 BE 的長度。

In Figure 1, ABCD is a square of sides 6 units. F is the midpoint of CD. If  $\angle FAB = \angle AFE$ , find the length of BE.

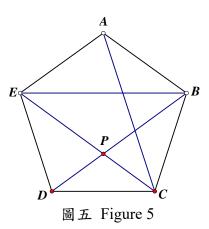


## 2023 HG3

已知  $\tan \alpha$  和  $\tan \beta$  是二次方程  $x^2 - 4x - 2 = 0$  的根。 求  $\sin^2(\alpha + \beta) + 2\sin(\alpha + \beta)\cos(\alpha + \beta) + 3\cos^2(\alpha + \beta)$  的值。

#### 2024 HG10

在圖五中,ABCDE 為一正五邊形,BD 及 CE 相交於 P。 若 $\Delta ABE$  的面積為 1,求 $\Delta BPC$  的面積。 In Figure 5, ABCDE is a regular pentagon, BD and CE intersect at P. If the area of  $\Delta ABE$  is 1, find the area of  $\Delta BPC$ .



# Answers

Allswers				
1994 FG10.3 $\frac{1}{16}$	1994 FG10.4 4	1999 FI1.2 8	2001 FG2.4 46	2002 HI3 8
2003 HI6 $\frac{3}{4}$	2003 FG2.4 44 125	2004 FI2.1 2	2004 FG1.4 0	2005 FG3.1 $\frac{\sqrt{7}}{4}$
2006 HG6 0	2006 FI2.3 1	2006 FG3.3 30	$2007 \text{ HG8}$ $10\sqrt{2}$	2010 HI6 1
$\frac{2010 \text{ FI4.4}}{\frac{3}{4}}$	2011 HG6 6√2	2011 FI4.1 2	2012 FI3.1 45	2012 FI3.2 56
2012 FG2.4 $\frac{1}{4}$	2012 FG4.1 2	2017 FI4.4 1945 3872	2018 FG4.3 1 128	2019 HI14 3
2021 P1Q4 $3\sqrt{3}$	2021 P2Q1 2	2023 HG3 $\frac{67}{25}$	$\frac{2024 \text{ HG10}}{\sqrt{5} - 1}$	