Hong Kong Mathematics Olympiad (1998-99)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若一個 P-邊的多邊形的內角形成一算術級數，且最小和最大的角分別為 $20°$ 及 $160°$，求 P 之值。

If the interior angles of a P-sided polygon form an Arithmetic Progression and the smallest and the largest angles are $20°$ and $160°$ respectively. Find the value of P .

$P =$

(ii) 在 ΔABC 中，$AB = 5$, $AC = 6$ 及 $BC = P$，若 $\frac{1}{Q} = \cos 2A$，求 Q 之值。

(<u>提示</u>: $\cos 2A = 2\cos^2 A - 1$)

In ΔABC, $AB = 5$, $AC = 6$ and $BC = P$. If $\frac{1}{Q} = \cos 2A$, find the value of Q.

(<u>Hint</u>: $\cos 2A = 2\cos^2 A - 1$)

$Q =$

(iii) 若 $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$，求 R 之值。

If $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$, find the value of R .

$R =$

(iv) 若兩數 R 和 $\frac{11}{S}$ 的積等於它們的和，求 S 之值。

If the product of the numbers R and $\frac{11}{S}$ is the same as their sum, find the value of S.

$S =$

<u>FOR OFFICIAL USE</u>

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 x、y 及 z 為正實數使得 $\frac{x+y-z}{z}=\frac{x-y+z}{y}=\frac{-x+y+z}{x}$，且 $a=\frac{(x+y)\cdot(y+z)\cdot(z+x)}{xyz}$，求 a 之值。

If x, y and z are positive real numbers such that $\frac{x+y-z}{z}=\frac{x-y+z}{y}=\frac{-x+y+z}{x}$ and $a=\frac{(x+y)\cdot(y+z)\cdot(z+x)}{xyz}$, find the value of a.

$a =$

(ii) 設 u 和 t 為正整數使得 $u+t+ut=4a+2$，若 $b=u+t$，求 b 之值。
Let u and t be positive integers such that $u+t+ut=4a+2$.
If $b=u+t$, find the value of b.

$b =$

(iii) 在圖一，OAB 為四分之一圓，且以 OA、OB 為直徑繪出兩個半圓，若 p、q 代表陰影部分之面積，其中 $p=(b-9)$ cm^2 及 $q=c$ cm^2，求 c 之值。
In Figure 1, OAB is a quadrant of a circle and semi-circles are drawn on OA and OB. If p, q denotes the areas of the shaded regions, where $p=(b-9)$ cm^2 and $q=c$ cm^2, find the value of c.

$c =$

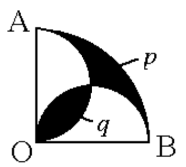


Figure 1 圖一

(iv) 設 $f_0(x)=\frac{1}{c-x}$，且 $f_n(x)=f_0(f_{n-1}(x))$，$n=1, 2, 3, \cdots\cdots$
若 $f_{2000}(2000)=d$，求 d 之值。
Let $f_0(x)=\frac{1}{c-x}$ and $f_n(x)=f_0(f_{n-1}(x))$, $n=1, 2, 3, \cdots\cdots$
If $f_{2000}(2000)=d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+ Bonus score				Time
			Total score				Min. / Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 對任意整數 m 及 n，$m \otimes n$ 之定義如下：$m \otimes n = m^n + n^m$。
若 $2 \otimes a = 100$，求 a 之值。
For all integers m and n, $m \otimes n$ is defined as: $m \otimes n = m^n + n^m$.
If $2 \otimes a = 100$, find the value of a.

$a =$

(ii) 若 $\sqrt[3]{13b+6a+1} - \sqrt[3]{13b-6a-1} = \sqrt[3]{2}$，其中 $b > 0$，求 b 之值。
If $\sqrt[3]{13b+6a+1} - \sqrt[3]{13b-6a-1} = \sqrt[3]{2}$, where $b > 0$, find the value of b.

$b =$

(iii) 在圖二，$AB = AC$ 和 $KL = LM$。若 $LC = b - 6$ cm 及 $KB = c$ cm，求 c 之值。
In figure 2, $AB = AC$ and $KL = LM$. If $LC = b - 6$ cm and $KB = c$ cm, find the value of c.

$c =$

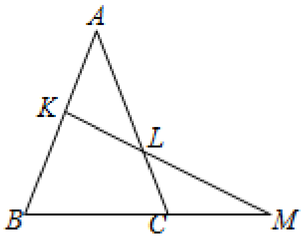


Figure 2 圖二

(iv) 數列$\{a_n\}$的定義如下：$a_1 = c$，$a_{n+1} = a_n + 2n$ ($n \ge 1$)。若 $a_{100} = d$，求 d 之值。
The sequence $\{a_n\}$ is defined as $a_1 = c$, $a_{n+1} = a_n + 2n$ ($n \ge 1$).
If $a_{100} = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 李先生今年 a 歲，$a < 100$。若把李先生的出生月份與 a 相乘，其結果是 253。求 a 的值。

Mr. Lee is a years old, $a < 100$.

If the product of a and his month of birth is 253, find the value of a.

$a =$

(ii) 李先生有糖 $a + b$ 粒，若平均分給 10 人，則餘下 5 粒。若平均分給 7 人，則欠 3 粒。求 b 之最小值。

Mr. Lee has $a + b$ sweets. If he divides them equally among 10 persons, 5 sweets will be remained. If he divides them equally among 7 persons, 3 more sweets are needed. Find the minimum value of b.

$b =$

(iii) 設 c 為一正實數，若 $x^2 + 2\sqrt{c}\,x + b = 0$ 僅有一實數解，求 c 之值。

Let c be a positive real number.

If $x^2 + 2\sqrt{c}\,x + b = 0$ has one real root only, find the value of c.

$c =$

(iv) 在圖三，正方形 $ABCD$ 之面積為 d。若 E，F，G，H 分別是 AB，BC，CD，DA 之中心點，及 $EF = c$，求 d 之值。

In figure 3, the area of the square $ABCD$ is equal to d. If E, F, G, H are the mid-points of AB, BC, CD and DA respectively and $EF = c$, find the value of d.

$d =$

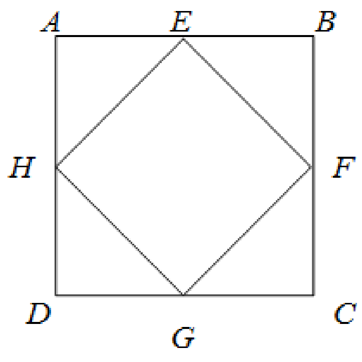


Figure 3 圖三

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=		Team No.	
		+ Bonus score				Time	
		Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $144^p = 10$，$1728^q = 5$ 及 $a = 12^{2p-3q}$，求 a 之值。

If $144^p = 10$, $1728^q = 5$ and $a = 12^{2p-3q}$, find the value of a.

a =

(ii) 若 $1-\frac{4}{x}+\frac{4}{x^2}=0$，及 $b=\frac{a}{x}$，求 b 之值。

If $1-\frac{4}{x}+\frac{4}{x^2}=0$, $b=\frac{a}{x}$, find the value of b.

b =

(iii) 若方程 $x^2 - bx + 1 = 0$ 有 c 個實數解，求 c 之值。

If the number of real roots of the equation $x^2 - bx + 1 = 0$ is c, find the value of c.

c =

(iv) 設 $f(1) = c + 1$ 及 $f(n) = (n-1)\, f(n-1)$，其中 $n > 1$。若 $d = f(4)$，求 d 之值。

Let $f(1) = c + 1$ and $f(n) = (n-1)\, f(n-1)$, where $n > 1$.

If $d = f(4)$, find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

\+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (1998-99)
Spare Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 為能整除 $3^{11}+5^{13}$ 的最小質數，求 a 之值。
If a is the smallest prime number which can divide the sum $3^{11}+5^{13}$, find the value of a.

$a =$

(ii) 對任意實數 x 及 y, $x\oplus y$ 之定義如下：$x\oplus y=\frac{1}{xy}$。
若 $b=4\oplus(a\oplus 1540)$，求 b 之值。
For all real number x and y, $x\oplus y$ is defined as: $x\oplus y=\frac{1}{xy}$.
If $b=4\oplus(a\oplus 1540)$, find the value of b.

$b =$

(iii) W 和 F 為兩大於 20 的整數。
若 W 與 F 之積為 b，W 與 F 之和為 c，求 c 之值。
W and F are two integers which are greater than 20.
If the product of W and F is b and the sum of W and F is c, find the value of c.

$c =$

(iv) 若 $\frac{d}{114}=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{c^2}\right)$，求 d 之值。
If $\frac{d}{114}=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{c^2}\right)$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 $x * y = x + y - xy$，其中 x, y 為實數，若 $a = 1 * (0 * 1)$，求 a 之值。
Let $x * y = x + y - xy$, where x, y are real numbers.
If $a = 1 * (0 * 1)$, find the value of a.

$a =$

(ii) 在圖一，AB 平行於 DC，$\angle ACB$ 為一直角，$AC = CB$ 及 $AB = BD$，
若 $\angle CBD = b°$，求 b 之值。
In figure 1, AB is parallel to DC, $\angle ACB$ is a right angle, $AC = CB$ and $AB = BD$.
If $\angle CBD = b°$, find the value of b.

$b =$

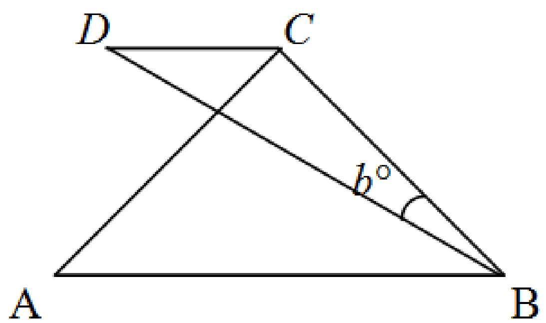


Figure 1 圖一

(iii) 設 x, y 為非零實數，若 x 是 y 的 250%，而 $2y$ 是 x 的 c %，求 c 之值。
Let x, y be non-zero real numbers.
If x is 250% of y and $2y$ is c % of x, find the value of c.

$c =$

(iv) 若 $\log_p x = 2$，$\log_q x = 3$，$\log_r x = 6$ 及 $\log_{pqr} x = d$，求 d 之值。
If $\log_p x = 2$, $\log_q x = 3$, $\log_r x = 6$ and $\log_{pqr} x = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = x^4 + x^{-4}$ 及 $x^2 + x + 1 = 0$ ，求 a 之值。

If $a = x^4 + x^{-4}$ and $x^2 + x + 1 = 0$, find the value of a.

$a =$

(ii) 若 $6^b + 6^{b+1} = 2^b + 2^{b+1} + 2^{b+2}$，求 b 之值。

If $6^b + 6^{b+1} = 2^b + 2^{b+1} + 2^{b+2}$, find the value of b.

$b =$

(iii) 設 c 為質數，若 $11c + 1$ 是一正整數之平方，求 c 之值。

Let c be a prime number.

If $11c + 1$ is the square of a positive integer, find the value of c.

$c =$

(iv) 設 d 為奇質數，若 $89 - (d + 3)^2$ 是一整數之平方，求 d 之值。

Let d be an odd prime number.

If $89 - (d + 3)^2$ is the square of an integer, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設小於 100 的正整數，同時又是完全平方及完全立方的數目共有 a 個，求 a 之值。

Let a be the number of positive integers less than 100 such that they are both square and cubic numbers, find the value of a.

a =

(ii) 數列$\{a_k\}$定義如下：$a_1 = 1$、$a_2 = 1$ 及 $a_k = a_{k-1} + a_{k-2}$ $(k > 2)$。
若 $a_1 + a_2 + \cdots + a_{10} = 11\,a_b$，求 b 之值。

The sequence $\{a_k\}$ is defined as:
$a_1 = 1$, $a_2 = 1$ and $a_k = a_{k-1} + a_{k-2}$ $(k > 2)$.
If $a_1 + a_2 + \cdots + a_{10} = 11\,a_b$, find the value of b.

b =

(iii) 若 c 是 $\log(\sin x)$的最大值，其中 $0 < x < \pi$，求 c 之值。
If c is the maximum value of $\log(\sin x)$, where $0 < x < \pi$, find the value of c.

c =

(iv) 設 $x \ge 0$ and $y \ge 0$。已知 $x + y = 18$。若 $\sqrt{x} + \sqrt{y}$ 之最大值是 d，求 d 之值。
Let $x \ge 0$ and $y \ge 0$. Given that $x + y = 18$.
If the maximum value of $\sqrt{x} + \sqrt{y}$ is d, find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	
			+ Bonus score			
			Total score			

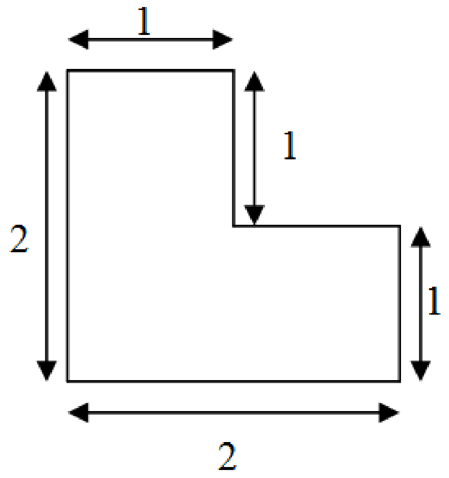
Team No.		
Time		
	Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若以 a 塊 L 形的瓷磚（圖二），不重疊地拼出一幅與之相似，但面積較大的圖形，求 a 的最小可能值。
If a tiles of L-shape are used to form a larger similar figure (figure 2) without overlapping, find the least possible value of a.

a =



圖二 Figure 2

(ii) 設α、β 是 $x^2 + bx - 2 = 0$ 的根。若 $\alpha > 1$ 及 $\beta < -1$，且 b 為一整數，求 b 之值。
Let α, β be the roots of $x^2 + bx - 2 = 0$.
If $\alpha > 1$ and $\beta < -1$, and b is an integer, find the value of b.

b =

(iii) 已知 m，c 是小於 10 的正整數。若 $m = 2c$，且 $0.\dot{m}\dot{c} = \dfrac{c+4}{m+5}$，求 c 之值。
Given that m, c are positive integers less than 10. If $m = 2c$ and $0.\dot{m}\dot{c} = \dfrac{c+4}{m+5}$,
find the value of c.

c =

(iv) 一個袋子裏有 d 個球，其中 x 個是黑球，$x + 1$ 個是紅球，$x + 2$ 個是白球。若從袋裏隨機抽出一個黑球之概率小於 $\dfrac{1}{6}$，求 d 之值。
A bag contains d balls of which x are black, $x + 1$ are red and $x + 2$ are white.
If the probability of drawing a black ball randomly from the bag is less than $\dfrac{1}{6}$,
find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1998-99)
Final Event 5 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $x^2 - 2x - P = 0$ 的根相差 12，求 P 之值。

If the roots of $x^2 - 2x - P = 0$ differ by 12, find the value of P.

P =

(ii) 已知方程式 $x^2 + ax + 2b = 0$ 及 $x^2 + 2bx + a = 0$ 的根為實數，且 $a, b > 0$。若 $a + b$ 的最小值為 Q，求 Q 之值。

Given that the roots of $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ are both real and $a, b > 0$. If the minimum value of $a + b$ is Q, find the value of Q.

Q =

(iii) If $R^{2000} < 5^{3000}$, where R is a positive integer, find the largest value of R.

若 $R^{2000} < 5^{3000}$，其中 R 為正整數，求 R 之最大值。

R =

(iv) 在圖三，直角三角形 ABC 中，$BH \perp AC$。
若 $AB = 15$，$HC = 16$ 及ΔABC 的面積是 S，求 S 之值。
In figure 3, ΔABC is a right-angled triangle and $BH \perp AC$.
If $AB = 15$, $HC = 16$ and the area of ΔABC is S, find the value of S.

S =

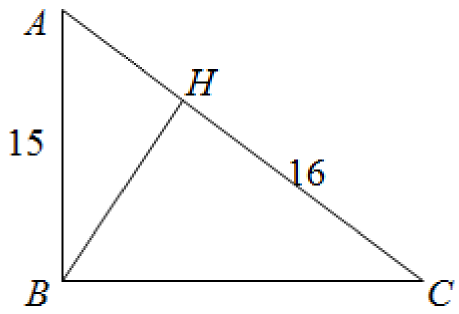


Figure 3 圖三

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1998-99)

Spare Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若從正整數集中任意抽取一數 N，N^4 的個位數字為 1 的概率是 $\frac{P}{10}$，求 P 之值。
If a number N is chosen randomly from the set of positive integers, the probability of the unit digit of N^4 being unity is $\frac{P}{10}$, find the value of P.

$P =$

(ii) 設 $x \geq 0$ and $y \geq 0$。已知 $x + y = 18$。若 $\sqrt{x} + \sqrt{y}$ 的最大值為 Q，求 Q 之值。
Let $x \geq 0$ and $y \geq 0$. Given that $x + y = 18$.
If the maximum value of $\sqrt{x} + \sqrt{y}$ is Q, find the value of Q.

$Q =$

(iii) 若 $x^2 - 2x - R = 0$ 的兩根之差為 12，求 R 之值。
If the roots of $x^2 - 2x - R = 0$ differs by 12, find the value of R.

$R =$

(iv) 若一四位數 ab$S$d 與 9 的積恰為四位數 dSba，求 S 之值。
If the product of a 4-digit number ab$S$d and 9 is equal to another 4-digit number dSba , find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.