Examples on Mathematical Induction: divisibility 30

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1. Prove by mathematical induction that $16\cdot 3^{2n-1} + 21\cdot 2^{2n-1}$ is divisible by 30 for all positive integers n.

   Let $P(n)$ be the statement: $16\cdot 3^{2n-1} + 21\cdot 2^{2n-1}$ is divisible by 30 for all positive integers n.

   $n = 1$, $16\cdot 3^1 + 21\cdot 2^1 = 90$ which is divisible by 30.

   Suppose $16\cdot 3^{2k-1} + 21\cdot 2^{2k-1} = 30m$ where m is an integer and k is a positive integer.

   $16\cdot 3^{2k+1} + 21\cdot 2^{2k+1} = 9\cdot 16\cdot 3^{2k-1} + 4\cdot 21\cdot 2^{2k-1} = 9\cdot(30m - 21\cdot 2^{2k-1}) + 4\cdot 21\cdot 2^{2k-1}$

   $= 270m - 5\cdot 21\cdot 2^{2k-1} = 30\cdot(9m - 7\cdot 2^{2k-2})$ which is divisible by 30.

   If $P(k)$ is true then $P(k+1)$ is also true.

   By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

2. Prove by mathematical induction $5^{2n-1} - 3^{2n-1} - 2^{2n-1}$ is divisible by 30 for all positive integers n.

   Let $P(n) \equiv$ "$5^{2n-1} - 3^{2n-1} - 2^{2n-1}$ is divisible by 30 for all positive integers n."

   $n = 1$, $5 - 3 - 2 = 0$, which is divisible by 30.

   Suppose $5^{2k-1} - 3^{2k-1} - 2^{2k-1} = 30m$, where m is an integer.

   $5^{2k+1} - 3^{2k+1} - 2^{2k+1} = 25\cdot 5^{2k-1} - 9\cdot 3^{2k-1} - 4\cdot 2^{2k-1} = 25\cdot(30m + 3^{2k-1} + 2^{2k-1}) - 9\cdot 3^{2k-1} - 4\cdot 2^{2k-1}$

   $= 750m + 16\cdot 3^{2k-1} + 21\cdot 2^{2k-1}$

   $= 750\,m + 30q$, by Q1, where m and q are integers.

   If $P(k)$ is true then $P(k+1)$ is also true.

   By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

3. Prove that $\dfrac{n^5}{5} + \dfrac{n^4}{2} + \dfrac{n^3}{3} - \dfrac{n}{30}$ is an integer.