

## 1990 FI5.3

某年五月第 5 日為星期五，而同年五月第  $c$  日為星期二，且  $16 < c < 24$ ，求  $c$  的值。

If the 5<sup>th</sup> day of May in a year is Friday and the  $c^{\text{th}}$  day of May in the same year is Tuesday, where  $16 < c < 24$ , find the value of  $c$ .

## 1990 FI5.4

23 是第  $d$  個質數。求  $d$  的值。 23 is the  $d^{\text{th}}$  prime number. Find the value of  $d$ .

1993 HG4

What is the maximum number of regions produced by drawing 20 straight lines on a plane?

# 1994 HG6

圖一包含  $n$  個長方形，求  $n$  的值。

There are  $n$  rectangles in figure 1. Find the value of  $n$ .


## 1994 FI5.4

十五塊邊長為 5 單位的正方形磚如圖排列。一蟻沿磚之邊緣爬行，而其左邊必為一黑磚。求  $D$  的值，此蟻由  $P$  爬至  $Q$  之最短距離。

Fifteen square tiles with side 5 units long are arranged as shown. An ant walks along the edges of the tiles, always keeping a black tile on its left. Find the shortest distance  $D$  that the ant would walk in going from  $P$  to  $Q$ .

## 1994 FG8.3-4

用 1、2、3、4 這四個數字，而每個數字均可重複使用，則可組成一些 4 位數。求

**G8.3** 共可組成的 4 位數的個數  $c$  的值。

**G8.4** 所組成的 4 位數的總和  $d$  的值。

From the digits 1, 2, 3, 4, when each digit can be used repeatedly, 4-digit numbers are formed. Find the values of

**G8.3**  $c$ , the number of 4-digit numbers that can be formed.

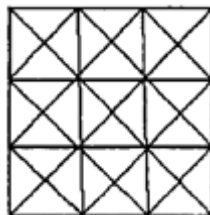
**G8.4**  $d$ , the sum of all these 4-digit numbers.

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**1995 HG9**

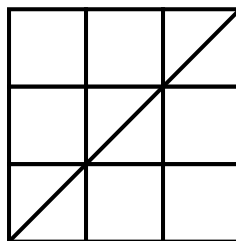
圖中是一個  $3 \times 3$  的正方形，求圖中三角形的總數。

In the figure, find the total number of triangles in the  $3 \times 3$  square.

**1995 FG8**

在方格紙上繪畫尺寸為  $\ell \times b$  的長方形，其中  $\ell$ 、 $b$  為正整數並添上對角線一條。以  $V$  代表相交的端點總數(不包括首尾兩點在內)。(如圖示)

Rectangles of length  $\ell$  and breadth  $b$  where  $\ell$ ,  $b$  are positive integers, are drawn on square grid paper. For each of these rectangles, a diagonal is drawn and the number of vertices  $V$  intersected (excluding the two end points) is counted (see figure).



$$\ell = b = 3$$

$$V = 2$$

**G8.1** 當  $\ell = 6$ ,  $b = 4$  時，求  $V$  的值。

Find the value of  $V$ , when  $\ell = 6$ ,  $b = 4$ .

**G.8.2** 當  $\ell = 5$ ,  $b = 3$  時，求  $V$  的值。

Find the value of  $V$ , when  $\ell = 5$ ,  $b = 3$ .

**G8.3** 當  $\ell = 12$  且  $1 < b < 12$  時，求使  $V = 0$  時， $b$  的不同個數  $r$ 。

When  $\ell = 12$  and  $1 < b < 12$ , find  $r$ , the number of different values of  $b$  that makes  $V = 0$ ?

**G8.4** 當  $\ell = 108$ ,  $b = 72$  時，求  $V$  的值。

Find the value of  $V$ , when  $\ell = 108$ ,  $b = 72$ .

**1996 HI7**

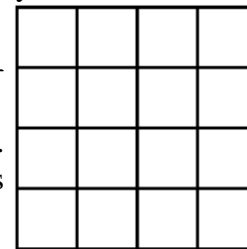
若各數字可重複選用，從數字 4, 5, 6, 7, 8, 9 中，可組成多少個 3 位數？

How many 3-digit numbers can be made from the figures 4, 5, 6, 7, 8, 9 when repetitions are allowed?

**1996 HG7**

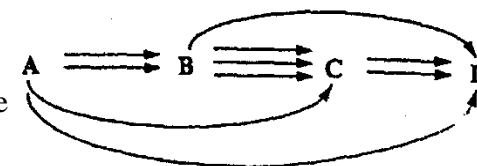
圖中每個小正方形的邊長為 1 單位。求圖中所有可能組成之長方形(包括正方形)的面積之和。

In the figure, the side of each smaller square is 1 unit long. Find the sum of the area of all possible rectangles (squares included) that can be formed in the figure.

**1996 FIS.1**

看下圖，由  $A$  到  $D$  共有  $a$  條路徑，求  $a$  的值。

From the following figure, determine the number of routes  $a$  from  $A$  to  $D$ .

**1997 HI8**

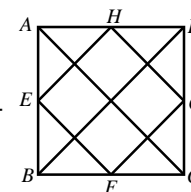
某班有 6 位學生。每位學生送給班中其餘各位同學一張聖誕咭，求該班學生寄出聖誕咭的總數。

There are 6 students in a class. Everyone sends one Christmas card to each of the rest of the class. Find the total number of cards sent out by the class.

**1998 HG9**

在圖五， $ABCD$  為正方形。點  $E$ 、 $F$ 、 $G$ 、 $H$  分別為邊  $AB$ 、 $BC$ 、 $CD$ 、 $DA$  之中點，求圖中直角三角形的數目。

In figure 5,  $ABCD$  is a square and points  $E$ ,  $F$ ,  $G$ ,  $H$  are the mid-points of sides  $AB$ ,  $BC$ ,  $CD$ ,  $DA$  respectively, find the number of right-angled triangles in the figure.

**1998 FI5.4**

若 6 條直線和 3 個圓畫於一白紙上，且它們的最多交點數量為  $d$ ，求  $d$  的值。  
If 6 straight lines and 3 circles are drawn on a paper, and  $d$  is the largest numbers of points of intersection, find the value of  $d$ .

**1999 HI5**

在 0 至  $10^6$  之間，有多少個整數  $n$ ，使得  $n^3$  的個位數字是 1？

How many integers  $n$  are there between 0 and  $10^6$ , such that the unit digit of  $n^3$  is 1?

**2000 HI3**

用數字 0、1、2、5 可以組成多少個能被 5 整除的三位數？  
(若數字不可以重複使用。)

Using digits 0, 1, 2, and 5, how many 3-digit numbers can be formed, which are divisible by 5? (If no digit may be repeated.)

## 2007 FG3.4

某數學比賽共有 4 條題目。以下述方式為每個題目評分：答對得 2 分、答錯扣一分、不作答得零分。若至少有  $S$  名參賽者才可保證比賽中有三人同分，求  $S$  的值。

There are 4 problems in a mathematics competition. The scores of each problem are allocated in the following ways: 2 marks will be given for a correct answer, 1 mark will be deducted from a wrong answer and 0 mark will be given for a blank answer. To ensure that 3 candidates will have the same scores, there should be at least  $S$  candidates in the competition. Find the value of  $S$ .

2008 HI6

在圖三中， $\triangle ABC$ 為等邊三角形。它由多個相同的等邊三角形組成。若圖中共有 $N$ 個等邊三角形，求 $N$ 的值。

In Figure 3,  $\triangle ABC$  is an equilateral triangle. It is formed by several identical equilateral triangles. If there are altogether  $N$  equilateral triangles in the figure, find the value of  $N$ .

A large equilateral triangle is shown, composed of 16 smaller equilateral triangles. The vertices of the large triangle are labeled A (top), B (bottom-left), and C (bottom-right). The triangle is divided into four rows of smaller triangles: the top row has 1 triangle, the second row has 3 triangles, the third row has 5 triangles, and the bottom row has 7 triangles. The lines forming the triangles are blue.

2010 HI9

把  $1, 2, \dots, n (n \geq 3)$  作環形排列，使得每兩個相鄰的數字相差為 1 或 2。求有多少個此類的環形排列。

Arrange the numbers  $1, 2, \dots, n$  ( $n \geq 3$ ) in a circle so that adjacent numbers always differ by 1 or 2. Find the number of possible such circular arrangements.

2010 HG8

如果正整數  $a$  的各數位之和等於 7，則  $a$  稱為「幸運數」。例如 7, 61, 12310 都是「幸運數」。將所有「幸運數」從小到大排成一列  $a_1, a_2, a_3, \dots$ 。若  $a_n = 1600$ ，求  $a_{2n}$  的值。

Let  $a$  be a positive integer. If the sum of all digits of  $a$  is equal to 7, then  $a$  is called a “lucky number”. For example, 7, 61, 12310 are lucky numbers.

List all lucky numbers in ascending order  $a_1, a_2, a_3, \dots$ .

If  $a_n = 1600$ , find the value of  $a_{2n}$ .

2011 HI6

甲、乙及丙三人互相傳球。甲首先將球傳出。有多少不同方案使得經過 5 次傳球後，球會回傳給甲？

$A$ ,  $B$  and  $C$  pass a ball among themselves.  $A$  is the first one to pass the ball to other one. In how many ways will the ball be passed back to  $A$  after 5 passes?

2011 FG2.2

從 1 到 100 選取兩整數(容許重覆)其和大於 100。問可選得多少對？

From 1 to 100, take a pair of integers (repetitions allowed) so that their sum is greater than 100. How many ways are there to pick such pairs?

2004 FG2.3

Given that there are  $c$  prime numbers less than 100 such that their unit digits are not square numbers, find the values of  $c$ .

**2005 HI10**

已知 2005 年 1 月 29 日是星期六，那麼 2008 年 1 月 29 日是星期幾？

Given that 29<sup>th</sup> January 2005 is Saturday, on what day is 29<sup>th</sup> January 2008?

**2013 HG9**

從 1、2、3、...、2012、2013 中最多可取出多少個數，使得在取出的數中任意兩數之和都不是這兩個數之差的倍數？

At most how many numbers can be taken from the set of integers: 1, 2, 3, ..., 2012, 2013 such that the sum of any two numbers taken out from the set is not a multiple of the difference between the two numbers?

**2014 FG2.3**

A, B, C, D, E 和 F 六人根據英文字母的順序輪班工作。A 在第一個星期日當值，然後 B 在星期一當值，如此類推。A 於第 50 個星期的哪一天當值？(答案以數字 0 代表星期日、數字 1 代表星期一、...、數字 6 代表星期六)。

Six persons A, B, C, D, E and F are to rotate for night shifts in alphabetical order with A serving on the first Sunday, B on the first Monday and so on. In the fiftieth week, which day does A serve on? (Represent Sunday by 0, Monday by 1, ..., Saturday by 6 in your answer.)

**2015 FG1.2**

在 50 隊香港數學競賽的參賽隊伍中，沒有一隊能答對一團體項目中的全部共四個題目。若該項目中的第一題有 45 隊答中，第二題有 40 隊答中，第三題有 35 隊答中，及第四題有 30 隊答中。請計算有多少隊伍同時答中第三及第四題。

Among 50 school teams joining the HKMO, no team answered all four questions correctly in the paper of a group event. If the first question was solved by 45 teams, the second by 40 teams, the third by 35 teams and the fourth by 30 teams. How many teams solved both the third and the fourth questions?

**2015 FG2.1**

在一個 3×3 的方格內的九個正方形上，分別填上紅色或藍色。若  $\alpha$  為不同着色方法的數量而使得所有 2×2 方格中所包含的正方形都不是全為紅色，求  $\alpha$  的值。

On a 3×3 grid of 9 squares, each squares is to be painted with either Red or Blue. If  $\alpha$  is the total number of possible colouring in which no 2×2 grid consists of only Red squares, determine the value of  $\alpha$ .

**2016 HI7**

在整數 1 至 500 之間出現了多少個數字「2」？

How many '2's are there in the numbers between 1 to 500?

**2016 FG1.1**

一項工程包括三個項目：A、B 和 C。若項目 A 開始三天後，項目 B 才可開始進行。項目 C 亦必須在項目 B 開始四天後才可開始進行。若完成項目 A、B 和 C 分別需要四天、六天和五天，求最少天數 (P) 完成全項工程。A project comprises of three tasks, A, B and C. Suppose task B must begin 3 days later than task A begins, and task C must begin 4 days later than task B begins. If the numbers of days to complete tasks A, B and C are 4, 6 and 5, respectively, determine the least number of days (P) to complete the project.

**2016 FG2.1**

一個盒子有五個球，球面上分別印上號碼 3、4、6、9 或 10。由盒中同時隨機取出 2 個球，並得出其號碼的總和。若 A 為不同總和的數量，求 A 的值。A box contains five distinctly marked balls with number markings being 3, 4, 6, 9 or 10. Two balls are randomly drawn without replacement from the box.

If A is the number of possible distinct sums of the selected numbers, determine the value of A.

**2016 FG2.3**

設  $a_1$ 、 $a_2$ 、 $a_3$ 、 $a_4$ 、 $a_5$ 、 $a_6$  為非負整數，並滿足

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}.$$

若  $c$  為方程系統的解的數量，求  $c$  的值。

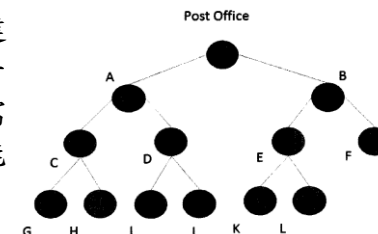
Let  $a_1, a_2, a_3, a_4, a_5, a_6$  be non-negative integers and satisfy

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 = 26 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 5 \end{cases}.$$

If  $c$  is the number of solutions to the system of equations, determine the value of  $c$ .

**2016 FG3.1**

下圖是郵差的送信路線圖：從郵局開始，到達十二個地點送信，最後返回郵局。若郵差從一地點步行到另一地點需要十分鐘及  $K$  為郵差需要的時數來完成整天路線，求  $K$  的最小可能值。

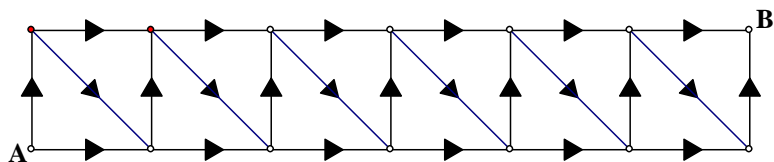


The figure below represents routes of a postman. Starting at the post office, the postman walks through all the 12 points and finally returns to the post office. If he takes 10 minutes from a point to another adjacent point by walk and  $K$  is the number of hours required for the postman to finish the routes, find the smallest possible value of  $K$ .

# 2019 HG3

若從一個正九邊形的 9 個頂點中選 3 點，共可組成多少個等腰三角形？

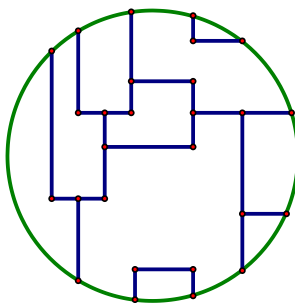
If three vertices are chosen from the nine vertices of a regular nonagon, how many possible isosceles triangles are there ?



## 2019 FI3.3

若今天是某一週的第 2 日，以及已知  $6^{2019}$  日後的當天為該週的第  $u$  日，求  $u$  的值。

If today is the 2<sup>nd</sup> day of a week, and **it is** known that  $6^{2019}$  days later is the  $u^{\text{th}}$  day of the week, determine the value of  $u$ .



## 2021 P1Q9

在 4000 和 7000 之間 4 個數位各不相同的偶數有多少個？

How many even numbers between 4000 and 7000 have four different digits?

## 2022 P1Q14

設  $a$ 、 $b$  及  $c$  為非零數字。有多少個三位數  $\overline{abc}$  使得  $\overline{ab} < \overline{bc} < \overline{ca}$ ？

Let  $a$ ,  $b$  and  $c$  are non-zero digits. How many three digit numbers  $\overline{abc}$  are there such that  $\overline{ab} < \overline{bc} < \overline{ca}$ ?

## 2023 FI1.4

有 21 對夫婦參加了一個派對，即在派對上共有 42 人。在這個派對上，沒有人會和同一位客人重複地握手。此外，每位丈夫都會和他妻子以外的所有客人握手，而妻子們不會與其他妻子握手，但會和其他客人握手。 $D$  是在這派對上 42 人之間握手的總數，求  $D$  的值。

21 couples are attending a party, which means that there are 42 people present. At this party, no one will shake hands repeatedly with the same guest. The party also has the condition that each husband will shake hands with every guest except his own wife, and wives will shake hands with every guest except other wives.  $D$  represents the total number of handshakes between the 42 people at the party.

Find value of  $D$  .

**2023 FG1.1**

有 100 個燈泡，編號從 1 到 100。班上有 100 名學生。每個學生輪流按下燈泡開關，情序如下：第一個學生按下編號為 1 及其倍數的燈泡開關，第二個學生按下編號為 2 及其倍數的燈泡開關，以此類推。每個學生只出來一次。如果燈泡亮著，按下開關後就會熄滅，反之亦然。一開始所有燈泡都是熄滅的。 $X$  代表在第 100 個學生按下開關後，燈泡亮著的數量。求  $X$  的值。

There are 100 light bulbs labeled from 1 to 100, and there are 100 students in the class. Each student takes a turn to press the switch buttons of the light bulbs with a label that is a multiple of their assigned number. For example, the first student presses the switch buttons of the light bulb with label 1 and all of its multiples, the second student presses the switch buttons of the light bulb with label 2 and all of its multiples, and so on. Each student will only come out once, and if a light bulb is on, it becomes off after being pressed, and vice versa. All the light bulbs are off at the beginning.  $X$  is the number of light bulbs that are on after the 100th student presses. Find the value of  $X$ .

**2023 FG1.4**

使用正整數序列 1、2、3、4、5、6 等等，通過將它們連接起來形成一個新的整數：123456789101112131415161718 ... 這個整數的最左邊的數位被定義為第一個數位。問在第 2023 數位是 0 至 9 的哪一個數？

Using the sequence of positive integers 1, 2, 3, 4, 5, 6, and so on, a new integer is formed by concatenating them: 123456789101112131415161718...

The leftmost digit in this integer is defined as first **position**.

What is the digit at position 2023?

**2023 FG2.2**

有一顆骰子，它的六個面上分別寫上數字 6 至 11。現投擲這顆骰子兩次，第一次得知四個側面的數字和是 36，第二次的數字和是 33。請問數字 10 的對面是甚麼數字？

A cubic dice has faces marked with numbers from 6 to 11. The dice was rolled twice. At the first time, the sum of the numbers on the four lateral faces was 36. At the second time, the sum was 33. What number is on the face opposite to the one with the number 10?

**Answers**

1982 FI2.3 15	1984 FG6.3 24	1985 FSG.2 8	1985 FG9.3 20	1987 FG8.4 16
1988 FG10.2 22	1989 FI5.4 24	1989 FSG.3 23	1990 FI5.3 23	1990 FI5.4 9
1993 HG4 211	1994 HG6 27	1994 FI5.4 50	1994 FG8.3 256	1994 FG8.4 711040
1995 HG9 124	1995 FG8 1, 0, 3, 35	1996 HI7 216	1996 HG7 400	1996 FIS.1 17
1997 HI8 30	1998 HG9 20	1998 FI5.4 57	1999 HI5 100000	2000 HI3 10
2000 FI1.3 72	2002 HI4 93324	2002 FG3.3 649	2003 FI2.4 8	2004 FG2.3 15
2005 HI10 Tuesday	2007 FG3.4 25	2008 HI6 27	2010 HI9 2	2010 HG8 10105
2011 HI6 10	2011 FG2.2 2550	2013 HG9 671	2014 FG2.3 5	2015FG1.2 15
2015FG2.1 417	2016 HI7 200	2016 FG1.1 12	2016 FG2.1 9	2016 FG2.3 5
2016 FG3.1 4	2017 FG4.3 377	2017 FG4.4 5	2018 HI3 718	2018 FG2.3 9
2019 HG3 30	2019 FI3.3 1	2021 P1Q9 728	2022 P1Q14 120	2023 FI1.4 1260
2023 FG1.1 10	2023 FG1.4 7	2023 FG2.2 8		