

**Individual Events**

|           |          |    |           |          |    |           |          |                |           |          |                       |           |          |     |           |          |               |
|-----------|----------|----|-----------|----------|----|-----------|----------|----------------|-----------|----------|-----------------------|-----------|----------|-----|-----------|----------|---------------|
| <b>SI</b> | <i>a</i> | 2  | <b>I1</b> | <i>a</i> | 6  | <b>I2</b> | <i>A</i> | $\frac{5}{16}$ | <b>I3</b> | <i>a</i> | 6                     | <b>I4</b> | <i>a</i> | 8   | <b>I5</b> | <i>A</i> | 1             |
|           | <i>b</i> | 54 |           | <i>b</i> | 12 |           | <i>B</i> | 60             |           | <i>b</i> | *9<br>see the remarks |           | <i>b</i> | 10  |           | <i>B</i> | 36            |
|           | <i>c</i> | 2  |           | <i>c</i> | 10 |           | <i>C</i> | 15             |           | <i>c</i> | 16                    |           | <i>c</i> | 3   |           | <i>C</i> | $\frac{1}{2}$ |
|           | <i>d</i> | 1  |           | <i>d</i> | 20 |           | <i>D</i> | 68             |           | <i>d</i> | 48                    |           | <i>d</i> | 203 |           | <i>D</i> | 50            |

**Group Events**

|           |          |                |           |          |                  |           |          |      |           |          |        |           |          |   |            |          |                |
|-----------|----------|----------------|-----------|----------|------------------|-----------|----------|------|-----------|----------|--------|-----------|----------|---|------------|----------|----------------|
| <b>SG</b> | <i>a</i> | 19             | <b>G6</b> | <i>a</i> | $4, \frac{1}{2}$ | <b>G7</b> | <i>a</i> | 24   | <b>G8</b> | <i>a</i> | 56     | <b>G9</b> | <i>A</i> | 2 | <b>G10</b> | <i>a</i> | 10             |
|           | <i>b</i> | 8              |           | <i>b</i> | 16               |           | <i>b</i> | 1024 |           | <i>b</i> | 83     |           | <i>B</i> | 1 |            | <i>b</i> | $\sqrt{37}$    |
|           | <i>c</i> | $\frac{1}{50}$ |           | <i>c</i> | $\frac{3}{7}$    |           | <i>c</i> | 2    |           | <i>c</i> | 256    |           | <i>C</i> | 4 |            | <i>c</i> | $\frac{1}{16}$ |
|           | <i>d</i> | 200            |           | <i>d</i> | 186              |           | <i>d</i> | -1   |           | <i>d</i> | 711040 |           | <i>D</i> | 9 |            | <i>d</i> | 4              |

**Sample Individual Event (1985 Final Sample Individual Event)**

**SI.1** The sum of two numbers is 40, their product is 20 .

If the sum of their reciprocals is  $a$ , find the value of  $a$  .

Let the two numbers be  $x, y$  .

$$x + y = 40 ; xy = 20$$

$$a = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x + y}{xy} = \frac{40}{20}$$

$$= 2$$

**SI.2** If  $b \text{ cm}^2$  is the total surface area of a cube of side  $(a+1) \text{ cm}$ , find the value of  $b$  .

$$b = 6(2+1)^2 = 54$$

**SI.3** One ball is taken at random from a bag containing  $(b-4)$  white balls and  $(b+46)$  red balls.

If  $\frac{c}{6}$  is the probability that the ball is white, find the value of  $c$  .

There are 50 white balls and 100 red balls.

$$P(\text{white ball}) = \frac{50}{150} = \frac{1}{3} = \frac{2}{6} = \frac{c}{6}$$

$$\Rightarrow c = 2$$

**SI.4** The length of a side of an equilateral triangle is  $c \text{ cm}$ .

If its area is  $d\sqrt{3} \text{ cm}^2$ , find the value of  $d$  .

$$\frac{1}{2}(2)^2 \sin 60^\circ = d\sqrt{3}$$

$$\sqrt{3} = d\sqrt{3}$$

$$\Rightarrow d = 1$$

**Individual Event 1**

**I1.1** The equation  $x^2 - ax + (a + 3) = 0$  has equal roots. Find  $a$ , if  $a$  is a positive integer.

$$\Delta = (-a)^2 - 4(a + 3) = 0$$

$$a^2 - 4a - 12 = 0$$

$$(a - 6)(a + 2) = 0$$

$$a = 6 \text{ or } a = -2 \text{ (rejected)}$$

**I1.2** In a test, there are 20 questions.  $a$  marks will be given to a correct answer and 3 marks will be deducted for each wrong answer. A student has done all the 20 questions and scored 48 marks. Find  $b$ , the number of questions that he has answered correctly.

**Reference: 1998 HG10**

$$6b - 3(20 - b) = 48$$

$$9b = 108$$

$$\Rightarrow b = 12$$

**I1.3** If  $x : y = 2 : 3$ ,  $x : z = 4 : 5$ ,  $y : z = b : c$ , find the value of  $c$ .

$$x : y : z = 4 : 6 : 5$$

$$y : z = 6 : 5 = 12 : 10$$

$$\Rightarrow c = 10$$

**I1.4** Let  $P(x, d)$  be a point on the straight line  $x + y = 22$  such that the slope of  $OP$  equals to  $c$  ( $O$  is the origin). Determine the value of  $d$ .

**Reference: 1993 FI3.2**

$$x + d = 22$$

$$\Rightarrow x = 22 - d$$

$$m_{OP} = \frac{d}{x} = c$$

$$\Rightarrow \frac{d}{22 - d} = 10$$

$$d = 220 - 10d$$

$$\Rightarrow d = 20$$

## Individual Event 2

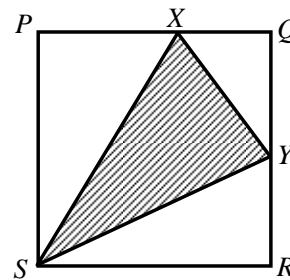
- I2.1** In square  $PQRS$ ,  $Y$  is the mid-point of the side  $QR$  and  $PX = \frac{3}{4}PQ$ .

If  $A$  is the ratio of the area of the shaded triangle to the area of the square, find  $A$ .

Let  $PQ = 4x$ ,  $PX = 3x$ ,  $QX = x$ ,  $QY = YR = 2x$

$$A = \frac{(4x)^2 - \frac{1}{2} \cdot 4x(3x) - \frac{1}{2} \cdot x(2x) - \frac{1}{2} \cdot 4x(2x)}{(4x)^2}$$

$$= \frac{5}{16}$$



- I2.2** A man bought a number of ping-pong balls where a 16% sales tax is added. If he did not have to pay tax he could have bought 3 more balls for the same amount of money. If  $B$  is the total number of balls that he bought, find  $B$ .

Let the price of 1 ping-pong ball be  $x$ . Sales tax = 5%

$$Bx(1 + 5\%) = (B + 3)x$$

$$\frac{21}{20}B = B + 3$$

$$\Rightarrow 21B = 20B + 60$$

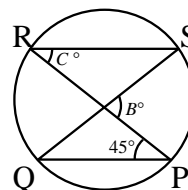
$$\Rightarrow B = 60$$

- I2.3** Refer to the diagram, find  $C$ .

$\angle PQS = C^\circ$  ( $\angle$ s in the same segment)

$$C + 45 = B \text{ (ext. } \angle \text{ of } \Delta)$$

$$C = 60 - 45 = 15$$



- I2.4** The sum of  $2C$  consecutive even numbers is 1170. If  $D$  is the largest of them, find  $D$ .

$$\frac{30}{2}[2D + (30 - 1) \cdot (-2)] = 1170$$

$$\Rightarrow D = 68$$

**Individual Event 3****I3.1** If  $183a8$  is a multiple of 287, find the value of  $a$ .

$$287 = 7 \times 41 \text{ and } 2614 \times 7 = 18298$$

$$183a8 - 18298 = 10(a + 1), \text{ a multiple of } 7$$

$$a = 6$$

**Method 2** The quotient  $\frac{183a8}{287}$  should be a two digit number.The first digit of  $\frac{183}{3}$  (approximate value) is 6. The last digit must be 4 ( $\because 7 \times 4 = 28$ )

$$\therefore 287 \times 64 = 18368$$

$$\Rightarrow a = 6$$

**I3.2** The number of positive factors of  $a^2$  is  $b$ , find the value of  $b$ .**Reference** 1993 HI8, 1997 HI3, 1998 HI10, 1998 FI1.4, 2002 FG4.1, 2005 FI4.4**Remark:** The original question is: The number of factors of  $a^2$  ....., which may include negative factors.

$$6^2 = 2^2 \times 3^2$$

Factors of 36 are in the form  $2^x \times 3^y$ , where  $0 \leq x \leq 2, 0 \leq y \leq 2$ .

$$\text{The number of factors} = (1 + 2)(1 + 2) = 9$$

**I3.3** In an urn, there are  $c$  balls,  $b$  of them are either black or red,  $(b + 2)$  of them are either red or white and 12 of them are either black or white. Find the value of  $c$ .Suppose there are  $x$  black balls,  $y$  red balls,  $z$  white balls.

$$x + y = 9 \dots\dots (1)$$

$$y + z = 11 \dots\dots (2)$$

$$z + x = 12 \dots\dots (3)$$

$$(1) + (2) + (3): 2(x + y + z) = 32$$

$$c = x + y + z = 16$$

**I3.4** Given  $f(3 + x) = f(3 - x)$  for all values of  $x$ , and the equation  $f(x) = 0$  has exactly  $c$  distinct roots. Find  $d$ , the sum of these roots.**Reference:** 2010 FG3.4Let one root be  $3 + \alpha$ .

$$f(3 + \alpha) = 0 = f(3 - \alpha)$$

 $\Rightarrow 3 - \alpha$  is also a root.

$$3 + \alpha + 3 - \alpha = 6$$

 $\therefore$  Sum of a pair of roots = 6

There are 16 roots, i.e. 8 pairs of roots

$$\text{Sum of all roots} = 8 \times 6 = 48$$

### Individual Event 4

**I4.1** The remainder when  $x^6 - 8x^3 + 6$  is divided by  $(x - 1)(x - 2)$  is  $7x - a$ , find  $a$ .

$$\text{Let } f(x) = x^6 - 8x^3 + 6$$

$$f(1) = 1 - 8 + 6 = 7 - a$$

$$a = 8$$

**I4.2** If  $x^2 - x + 1 = 0$  and  $b = x^3 - 3x^2 + 3x + a$ , find  $b$ .

$$b = x(x^2 - x + 1) - 2(x^2 - x + 1) + 10$$

$$= 10$$

**I4.3** Refer to the diagram, find  $c$ .

**Reference: 1989 FG10.2**

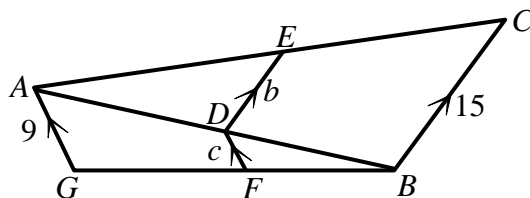
$$\triangle ADE \sim \triangle ABC, AD : AB = 10 : 15 = 2 : 3$$

$$AD : DB = 2 : 1$$

$$BD : AB = 1 : 3$$

$$\triangle BDF \sim \triangle BAG, c : 9 = 1 : 3$$

$$c = 3$$



**I4.4** If  $c$  boys were all born in June 1990 and the probability that their birthdays are all different is

$$\frac{d}{225}, \text{ find the value of } d.$$

$$P(3 \text{ boys were born in different days}) = 1 \times \frac{29}{30} \times \frac{28}{30} = \frac{d}{225}$$

$$d = 203$$

### Individual Event 5

**I5.1** Given  $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ . If  $A = \frac{2}{x}$ , find the value of  $A$ .

**Reference: 1999 FI5.2**

$$\left(1 - \frac{2}{x}\right)^2 = 0$$

$$\Rightarrow A = \frac{2}{x} = 1$$

**I5.2** If  $B$  circular pipes each with an internal diameter of  $A$  cm carry the same amount of water as a pipe with an internal diameter 6 cm, find the value of  $B$ .

$$\pi(1)^2 \cdot B = \pi(6)^2$$

$$\Rightarrow B = 36$$

**I5.3** If  $C$  is the area of the triangle formed by  $x$ -axis,  $y$ -axis and the line  $Bx + 9y = 18$ , find the value of  $C$ .

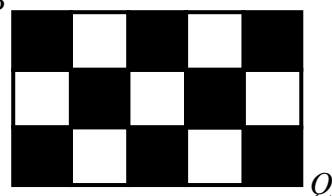
**Reference: 1990 FI3.3**

$$36x + 9y = 18$$

$$x\text{-intercept} = \frac{1}{2}, y\text{-intercept} = 2$$

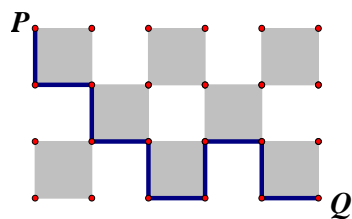
$$C = \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = \frac{1}{2}$$

**I5.4** Fifteen square tiles with side  $10C$  units long are arranged as  $P$  shown. An ant walks along the edges of the tiles, always keeping a black tile on its left. Find the shortest distance  $D$  that the ant would walk in going from  $P$  to  $Q$ .



Length of a square =  $10C = 5$

As shown in the figure,  $D = 10(10C) = 50$



**Sample Group Event (1985 Sample Group Event)****SG.1** If  $x*y = xy + 1$  and  $a = (2*4)*2$ , find the value of  $a$ .

$$2*4 = 2(4) + 1 = 9$$

$$(2 * 4)*2 = 9*2 = 9(2) + 1 = 19$$

**SG.2** If the  $b^{\text{th}}$  prime number is  $a$ , find the value of  $b$ .

List the prime number in ascending order: 2, 3, 5, 7, 11, 13, 17, 19.

$$b = 8$$

**SG.3** If  $c = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{50}\right)$ , find the value of  $c$  in the simplest fractional form.

$$c = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdots \frac{49}{50} = \frac{1}{50}$$

**SG.4** If  $d$  is the area of a square inscribed in a circle of radius 10, find the value of  $d$ .

Diameter = 20 = diagonal of the square

Let the side of the square be  $x$ .By Pythagoras' Theorem,  $2x^2 = 20^2 = 400$ 

$$d = x^2 = 200$$

**Group Event 6****G6.1** If  $\log_2 a - 2 \log_a 2 = 1$ , find the value of  $a$ .

$$\frac{\log a}{\log 2} - \frac{2 \log 2}{\log a} = 1$$

$$(\log a)^2 - 2 (\log 2)^2 = \log 2 \log a$$

$$(\log a)^2 - \log 2 \log a - 2 (\log 2)^2 = 0$$

$$(\log a - 2 \log 2)(\log a + \log 2) = 0$$

$$\log a = 2 \log 2 \text{ or } -\log 2$$

$$a = 4 \text{ or } \frac{1}{2}$$

**G6.2** If  $b = \log_3[2(3+1)(3^2+1)(3^4+1)(3^8+1)+1]$ , find the value of  $b$ .**Reference: 2016 FG1.4**

$$b = \log_3[(3-1)(3+1)(3^2+1)(3^4+1)(3^8+1)+1]$$

$$= \log_3[(3^2-1)(3^2+1)(3^4+1)(3^8+1)+1]$$

$$= \log_3[(3^4-1)(3^4+1)(3^8+1)+1]$$

$$= \log_3[(3^8-1)(3^8+1)+1]$$

$$= \log_3(3^{16}-1+1) = 16$$

**G6.3** If a 31-day month is taken at random, find  $c$ , the probability that there are 5 Sundays in the month.

$$1^{\text{st}} \text{ day} = \text{Sunday} \Rightarrow 29^{\text{th}} \text{ day} = 5^{\text{th}} \text{ Sunday}$$

$$1^{\text{st}} \text{ day} = \text{Saturday} \Rightarrow 30^{\text{th}} \text{ day} = 5^{\text{th}} \text{ Sunday}$$

$$1^{\text{st}} \text{ day} = \text{Friday} \Rightarrow 31^{\text{st}} \text{ day} = 5^{\text{th}} \text{ Sunday}$$

$$\text{Probability} = \frac{3}{7}$$

**G6.4** A group of 5 people is to be selected from 6 men and 4 women. Find  $d$ , the number of ways that there are always more men than women.

$$3 \text{ men and } 2 \text{ women, number of combinations} = C_3^6 \cdot C_2^4 = 120$$

$$4 \text{ men and } 1 \text{ woman, number of combinations} = C_4^6 \cdot C_1^4 = 60$$

$$5 \text{ men, number of combinations} = C_5^6 = 6$$

$$\text{Total number of ways} = 120 + 60 + 6 = 186$$



**Group Event 7****G7.1** There are  $a$  zeros at the end of the product  $1 \times 2 \times 3 \times \dots \times 100$ . Find the value of  $a$ .**Reference: 1990 HG6, 1996 HI3, 2004 FG1.1, 2011 HG7, 2012 FI1.4, 2012 FG1.3**

When each factor of 5 is multiplied by 2, a trailing zero will appear in the product.

The number of factors of 2 is clearly more than the number of factors of 5 in 100!

It is sufficient to find the number of factors of 5.

5, 10, 15, ..., 100; altogether 20 numbers, have at least one factor of 5.

25, 50, 75, 100; altogether 4 numbers, have two factors of 5.

 $\therefore$  Total number of factors of 5 is  $20 + 4 = 24$ 

There are 24 trailing zeros of 100!

 $\Rightarrow a = 24$ **G7.2** Find  $b$ , if  $b$  is the remainder when  $1998^{10}$  is divided by  $10^4$ .

$$1998^{10} = (2000 - 2)^{10}$$

$$= \sum_{k=0}^{10} C_k^{10} \cdot 2000^{10-k} \cdot 2^k$$

$$= \sum_{k=0}^9 C_k^{10} \cdot 2000^{10-k} \cdot 2^k + 2^{10}$$

$$\equiv 2^{10} \pmod{10^4} \quad (\because C_9^{10} \cdot 2000^{10-9} \cdot 2^1 = 10000m, \text{ where } m \text{ is an integer})$$

$$\equiv 1024 \pmod{10^4}$$

 $\Rightarrow b = 1024$ **G7.3** Find the largest value of  $c$ , if  $c = 2 - x + 2\sqrt{x-1}$  and  $x > 1$ .

$$(c + x - 2)^2 = 4(x - 1)$$

$$c^2 + x^2 + 4 + 2cx - 4x - 4c = 4x - 4$$

$$x^2 + 2(c - 4)x + (c^2 - 4c + 8) = 0$$

For real values of  $x$ ,  $\Delta \geq 0$ 

$$4(c - 4)^2 - 4(c^2 - 4c + 8) \geq 0$$

$$c^2 - 8c + 16 - c^2 + 4c - 8 \geq 0$$

$$8 \geq 4c$$

$$\Rightarrow c \leq 2$$

 $\Rightarrow$  The largest value  $c = 2$ **Method 2**Let  $y = \sqrt{x-1}$ , then  $y^2 = x - 1$ 

$$\Rightarrow x = y^2 + 1$$

$$c = 2 - (y^2 + 1) + 2y = 2 - (1 - y)^2 \leq 2$$

The largest value of  $c = 2$ .**G7.4** Find the least value of  $d$ , if  $\left| \frac{3-2d}{5} + 2 \right| \leq 3$ .

$$-3 \leq \frac{3-2d}{5} + 2 \leq 3$$

$$\Leftrightarrow -5 \leq \frac{3-2d}{5} \leq 1$$

$$\Leftrightarrow -25 \leq 3 - 2d \leq 5$$

$$\Leftrightarrow -28 \leq -2d \leq 2$$

$$\Leftrightarrow 14 \geq d \geq -1$$

The least value of  $d = -1$

**Group Event 8**

**G8.1** From 1 to 121, there are  $a$  numbers which are multiples of 3 or 5. Find the value of  $a$ .

**Reference: 1993 FG8.3-4, 1998 HI6, 2015 FI3.1**

Number of multiples of 3 = 40 ( $120 = 3 \times 40$ )

Number of multiples of 5 = 24 ( $120 = 5 \times 24$ )

Number of multiples of 15 = 8 ( $120 = 15 \times 8$ )

Number of multiples of 3 or 5 =  $a = 40 + 24 - 8 = 56$

**G8.2** From 1 to 121, there are  $b$  numbers which are not divisible by 5 nor 7. Find the value of  $b$ .

Number of multiples of 5 = 24 ( $120 = 5 \times 24$ )

Number of multiples of 7 = 17 ( $119 = 7 \times 17$ )

Number of multiples of 35 = 3 ( $105 = 35 \times 3$ )

Number of multiples of 5 or 7 =  $24 + 17 - 3 = 38$

Number which are not divisible by 5 nor 7 =  $121 - 38 = 83$

From the digits 1, 2, 3, 4, when each digit can be used repeatedly, 4-digit numbers are formed. Find

**G8.3**  $c$ , the number of 4-digit numbers that can be formed.

$$c = 4^4 = 256$$

**G8.4**  $d$ , the sum of all these 4-digit numbers.

**Reference: 2002 HI4**

$\therefore$  There are 256 different numbers

$\therefore$  1, 2, 3, 4 each appears 64 times in the thousands, hundreds, tens and units digit.

$$\begin{aligned} d &= [1000(1 + 2 + 3 + 4) + 100(1 + 2 + 3 + 4) + 10(1 + 2 + 3 + 4) + 1 + 2 + 3 + 4] \times 64 \\ &= 1111(10) \times 64 = 711040 \end{aligned}$$

### Group Event 9

$A, B, C, D$  are different integers ranging from 0 to 9 and

$$(ABA)^2 = (CCDCC) < 100000.$$

Find the values of  $A, B, C$  and  $D$ .

$$(ABA) < \sqrt{100000} < 316$$

$A = 1, 2$  or  $3$

When  $A = 1$ , then  $A^2 = 1 = C$  contradict that  $A$  and  $C$  must be different  $\therefore$  rejected

When  $A = 2$ ,  $C = 4$

$$(202 + 10B)^2 = 44044 + 100D$$

$$40804 + 4040B + 100B^2 = 44044 + 100D$$

$$4040B + 100B^2 = 3240 + 100D$$

$$404B + 10B^2 = 324 + 10D$$

$$\therefore B = 1, 414 = 324 + 10D$$

$$\Rightarrow D = 9$$

When  $A = 3$ ,  $C = 9$

$$(303 + 10B)^2 = 99099 + 100D$$

$$91809 + 6060B + 100B^2 = 99099 + 100D$$

$$606B + 10B^2 = 729 + 10D$$

$$\therefore B = 1, 616 = 729 + 10D$$

$$\Rightarrow \text{no solution for } D$$

$$\therefore A = 2, B = 1, C = 4, D = 9$$

$$\begin{array}{r} \phantom{\times} \phantom{00} A \phantom{00} B \phantom{00} A \\ \times \phantom{00} A \phantom{00} B \phantom{00} A \\ \hline C \phantom{00} C \phantom{00} D \phantom{00} C \phantom{00} C \end{array}$$

### Group Event 10

In rectangle  $ABCD$ ,  $AD = 10$ ,  $CD = 15$ ,  $P$  is a point inside the rectangle such that  $PB = 9$ ,  $PA = 12$ . Find (Reference: 2001 FG2.2, 2003 FI3.4, 2018 HI7)

**G10.1**  $a$ , the length of  $PD$  and

$$AP^2 + BP^2 = 12^2 + 9^2 = 144 + 81 = 225 = 15^2 = AB^2$$

$\therefore \angle APB = 90^\circ$  (Converse, Pythagoras' theorem)

$$\text{Let } \angle ABP = \theta, \text{ then } \cos \theta = \frac{9}{15} = \frac{3}{5}, \sin \theta = \frac{4}{5}$$

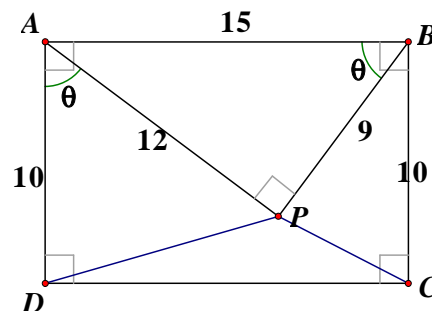
$$\angle BAP = 90^\circ - \theta \text{ (}\angle\text{s sum of } \Delta\text{)}$$

$$\angle DAP = \theta$$

$$\angle PBC = 90^\circ - \theta$$

$$a = PD = \sqrt{10^2 + 12^2 - 2 \cdot 10 \cdot 12 \cdot \frac{3}{5}} \text{ (Cosine rule on } \Delta ADP\text{)}$$

$$a = 10$$



**G10.2**  $b$ , the length of  $PC$ .

$$b = CP = \sqrt{10^2 + 9^2 - 2 \cdot 10 \cdot 9 \cdot \frac{4}{5}} = \sqrt{37}$$

**G10.3** It is given that  $\sin 2\theta = 2 \sin \theta \cos \theta$ . Find  $c$ , if  $c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}$ .

$$\begin{aligned} C &= \frac{2 \sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{2 \sin 160^\circ} = \frac{2 \sin 40^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{4 \sin 160^\circ} \\ &= \frac{2 \sin 80^\circ \cos 60^\circ \cos 80^\circ}{8 \sin 160^\circ} = \frac{\sin 160^\circ \times \frac{1}{2}}{8 \sin 160^\circ} = \frac{1}{16} \end{aligned}$$

**G10.4** It is given that  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ . Find  $d$ , if

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ).$$

$$\text{If } A + B = 45^\circ, 1 = \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$1 - \tan A \tan B = \tan A + \tan B$$

$$2 = 1 + \tan A + \tan B + \tan A \tan B$$

$$(1 + \tan A)(1 + \tan B) = 2$$

$$d = (1 + \tan 21^\circ)(1 + \tan 24^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)$$

$$= 2 \times 2 = 4$$