Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event Sample (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $A = 15\times\tan 44°\times\tan 45°\times\tan 46°$，求 A 的值。
   Let $A = 15\times\tan 44°\times\tan 45°\times\tan 46°$, find the value of A.

   $A =$

2. 設 n 為正整數及 $\overbrace{20082008\cdots2008}^{n\text{ 個}2008}15$ 能被 A 整除。
   若 n 的最小可能值是 B，求 B 的值。

   Let n be a positive integer and $\overbrace{20082008\cdots2008}^{n\ 2008\text{'s}}15$ is divisible by A.
   If the least possible value of n is B, find the value of B.

   $B =$

3. 已知有 C 個整數滿足方程 $|x-2| + |x+1| = B$，求 C 的值。
   Given that there are C integers that satisfy the equation $|x-2| + |x+1| = B$, find the value of C.

   $C =$

4. 在座標平面上，點 $(-C, 0)$ 與直綫 $y = x$ 的距離是 $\sqrt{D}$，求 D 的值。
   In the coordinate plane, the distance from the point $(-C, 0)$ to the straight line $y = x$ is $\sqrt{D}$, find the value of D.

   $D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 1 (Individual)

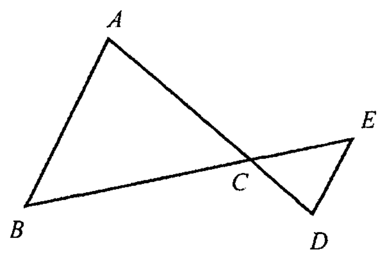
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a、b、c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 相異的根。
若 $R = a^2 + b^2 + c^2 + d^2$，求 R 的值。
Let a, b, c and d be the distinct roots of the equation $x^4 - 15x^2 + 56 = 0$.
If $R = a^2 + b^2 + c^2 + d^2$, find the value of R.

$R =$

2. 如圖一，AD 及 BE 為直綫且 $AB = AC$ 及 $AB \,//\, ED$。
若 $\angle ABC = R°$ 及$\angle ADE = S°$，求 S 的值。
In Figure 1, AD and BE are straight lines with $AB = AC$ and $AB \,//\, ED$.
If $\angle ABC = R°$ and $\angle ADE = S°$, find the value of S.

$S =$



圖一
Figure 1

3. 設 $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^S$ 及 $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$，求 T 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^S$ and $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$, find the value of T.

$T =$

4. 設 f(x)是一個函數使得對所有整數 $n \geq 6$ 時，$f(n) = (n-1)\, f(n-1)$及 $f(n) \neq 0$。
若 $U = \dfrac{f(T)}{(T-1)\, f(T-3)}$，求 U 的值。
Let f(x) be a function such that $f(n) = (n-1)\, f(n-1)$
and $f(n) \neq 0$ hold for all integers $n \geq 6$. If $U = \dfrac{f(T)}{(T-1)\, f(T-3)}$, find the value of U.

$U =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $[x]$ 是不超過 x 的最大整數。若 $a=\left[\left(\sqrt{3}-\sqrt{2}\right)^{2009}\right]+16$，求 a 的值。

   Let $[x]$ be the largest integer not greater than x.

   If $a=\left[\left(\sqrt{3}-\sqrt{2}\right)^{2009}\right]+16$, find the value of a.

   $a=$

2. 在坐標平面上，若 x-軸、y-軸與直綫 $3x+ay=12$ 所圍成三角形的面積是 b 平方單位，求 b 的值。

   In the coordinate plane, if the area of the triangle formed by the x-axis, y-axis and the line $3x+ay=12$ is b square units, find the value of b.

   $b=$

3. 已知 $x-\dfrac{1}{x}=2b$ 及 $x^3-\dfrac{1}{x^3}=c$，求 c 的值。

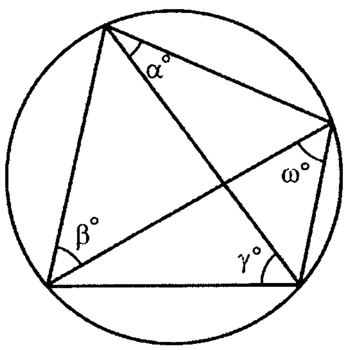
   Given that $x-\dfrac{1}{x}=2b$ and $x^3-\dfrac{1}{x^3}=c$, find the value of c.

   $c=$

4. 如圖一，$\alpha=c$, $\beta=43$, $\gamma=59$ 及 $\omega=d$，求 d 的值。

   In Figure 1, $\alpha=c$, $\beta=43$, $\gamma=59$ and $\omega=d$, find the value of d.

   $d=$



圖一
Figure 1

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+	Bonus score			Time
				Total score			Min. / Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

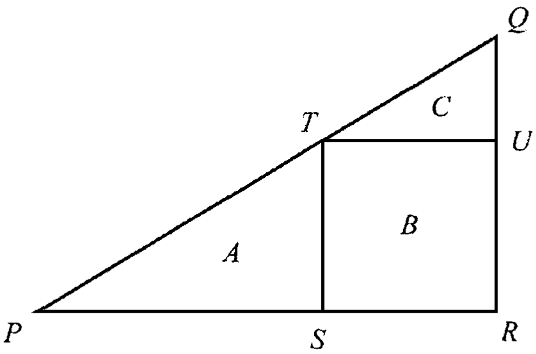
1. 已知 $\dfrac{4}{\sqrt{6}+\sqrt{2}}-\dfrac{1}{\sqrt{3}+\sqrt{2}}=\sqrt{a}-\sqrt{b}$。若 $m=a-b$，求 m 的值。

   Given that $\dfrac{4}{\sqrt{6}+\sqrt{2}}-\dfrac{1}{\sqrt{3}+\sqrt{2}}=\sqrt{a}-\sqrt{b}$. If $m = a - b$, find the value of m.

   $m =$

2. 如圖一，PQR 為直角三角形及 $RSTU$ 為矩形。設 A，B 及 C 是相對圖形的面積。若 $A:B=m:2$ 及 $A:C=n:1$，求 n 的值。
   In figure 1, PQR is a right-angled triangle and $RSTU$ is a rectangle.
   Let A, B and C be the areas of the corresponding regions.
   If $A : B = m : 2$ and $A : C = n : 1$, find the value of n.

   $n =$



圖一
Figure 1

3. 設 x_1、x_2、x_3、x_4 為實數及 $x_1 \neq x_2$。若 $(x_1+x_3)(x_1+x_4)=(x_2+x_3)(x_2+x_4)=n-10$ 及 $p=(x_1+x_3)(x_2+x_3)+(x_1+x_4)(x_2+x_4)$，求 p 的值。
   Let x_1, x_2, x_3, x_4 be real numbers and $x_1 \neq x_2$.
   If $(x_1+x_3)(x_1+x_4)=(x_2+x_3)(x_2+x_4)=n-10$ and
   $p=(x_1+x_3)(x_2+x_3)+(x_1+x_4)(x_2+x_4)$, find the value of p.

   $p =$

4. 已知某校學生人數是 7 的倍數且不少於 1000。若學生人數被 $p+1$、$p+2$ 及 $p+3$ 除後的餘數均是 1。設學生人數的最小可能值為 q，求 q 的值。
   The total number of students in a school is a multiple of 7 and not less than 1000. Given that the same remainder 1 will be obtained when the number of students is divided by $p + 1$, $p + 2$ and $p + 3$. Let q be the least of the possible numbers of students in the school, find the value of q.

   $q =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $x_0^2+x_0-1=0$。若 $m=x_0^3+2x_0^2+2$，求 m 的值。

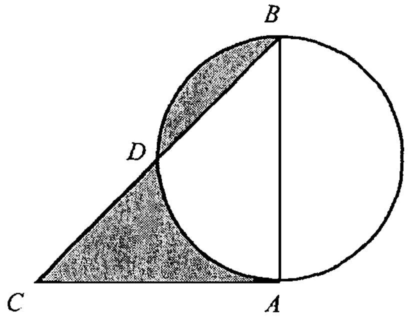
Given that $x_0^2+x_0-1=0$. If $m=x_0^3+2x_0^2+2$, find the value of m.

$m =$

2. 如圖一，ΔBAC 是一直角三角形，$AB = AC = m$ cm。已知直徑為 AB 的圓與 BC 相交於 D 且陰影部分的面積是 n cm^2，求 n 的值。

In Figure 1, ΔBAC is a right-angled triangle, $AB = AC = m$ cm. Suppose that the circle with diameter AB intersects the line BC at D, and the total area of the shaded region is n cm^2. Find the value of n.

$n =$



圖一
Figure 1

3. 已知 $p=4n\left(\frac{1}{2^{2009}}\right)^{\log(1)}$，求 p 的值。

Given that $p=4n\left(\frac{1}{2^{2009}}\right)^{\log(1)}$, find the value of p.

$p =$

4. 設 x 及 y 為實數並滿足方程 $\left(x-\sqrt{p}\right)^2+\left(y-\sqrt{p}\right)^2=1$。
若 $k=\frac{y}{x-3}$ 及 q 是 k^2 的最小可能值，求 q 的值。

Let x and y be real numbers satisfying the equation $\left(x-\sqrt{p}\right)^2+\left(y-\sqrt{p}\right)^2=1$.
If $k=\frac{y}{x-3}$ and q is the least possible values of k^2, find the value of q.

$q =$

FOR OFFICIAL USE

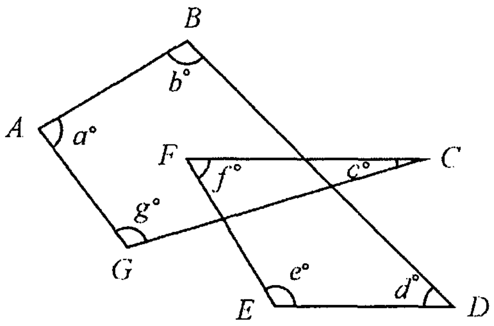
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event Sample (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，BD、FC、GC 及 FE 為直綫。
若 $z = a + b + c + d + e + f + g$，求 z 的值。
In Figure 1, BD, FC, GC and FE are straight lines.
If $z = a + b + c + d + e + f + g$, find the value of z.

$z =$



圖一

Figure 1

2. 若 $1^6 + 2^6 + 3^6 + 4^6 + 5^6 + 6^6$ 被 7 除後的餘數是 R，求 R 的值。
If R is the remainder of $1^6 + 2^6 + 3^6 + 4^6 + 5^6 + 6^6$ divided by 7, find the value of R.

$R =$

3. 若 14!能被 6^k 整除，其中 k 為整數，求 k 的最大可能值。
If 14! is divisible by 6^k, where k is an integer, find the largest possible value of k.

$k =$

4. 設實數 x、y 及 z 滿足 $x+\frac{1}{y}=4$，$y+\frac{1}{z}=1$ 及 $z+\frac{1}{x}=\frac{7}{3}$。求 xyz 的值。
Let x, y and z be real numbers that satisfy $x+\frac{1}{y}=4$, $y+\frac{1}{z}=1$ and $z+\frac{1}{x}=\frac{7}{3}$.
Find the value of xyz.

$xyz =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知三角形三邊的長度分別是 a cm、2 cm 及 b cm，其中 a 和 b 是整數且 $a \le 2 \le b$。若有 q 種不全等的三角形滿足上述條件，求 q 的值。
Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \le 2 \le b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q.

$q =$

2. 已知方程 $|x|-\frac{4}{x}=\frac{3|x|}{x}$ 有 k 個相異實根，求 k 的值。
Given that the equation $|x|-\frac{4}{x}=\frac{3|x|}{x}$ has k distinct real root(s), find the value of k.

$k =$

3. 已知 x 及 y 為非零實數且滿足方程 $\frac{\sqrt{x}}{\sqrt{y}}-\frac{\sqrt{y}}{\sqrt{x}}=\frac{7}{12}$ 及 $x-y=7$。

若 $w=x+y$，求 w 的值。

Given that x and y are non-zero real numbers satisfying the equations $\frac{\sqrt{x}}{\sqrt{y}}-\frac{\sqrt{y}}{\sqrt{x}}=\frac{7}{12}$

and $x-y=7$. If $w=x+y$, find the value of w.

$w =$

4. 已知 x 及 y 為實數且 $\left|x-\frac{1}{2}\right|+\sqrt{y^2-1}=0$。設 $p=|x|+|y|$，求 p 的值。
Given that x and y are real numbers and $\left|x-\frac{1}{2}\right|+\sqrt{y^2-1}=0$.
Let $p=|x|+|y|$, find the value of p.

$p =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	Team No.	
			+ Bonus score			Time	
			Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $\tan\theta = \frac{5}{12}$，其中 $180° \le \theta \le 270°$。若 $A = \cos\theta + \sin\theta$，求 A 的值。

   Given $\tan\theta = \frac{5}{12}$, where $180° \le \theta \le 270°$. If $A = \cos\theta + \sin\theta$, find the value of A.

   A =

2. 設 $[x]$ 是不超過 x 的最大整數。
   若 $B = \left[10+\sqrt{10+\sqrt{10+\sqrt{10+\cdots}}}\right]$，求 B 的值。

   Let $[x]$ be the largest integer not greater than x.
   If $B = \left[10+\sqrt{10+\sqrt{10+\sqrt{10+\cdots}}}\right]$, find the value of B.

   B =

3. 設 $a \oplus b = ab + 10$。若 $C = (1\oplus 2) \oplus 3$，求 C 的值。
   Let $a \oplus b = ab + 10$. If $C = (1\oplus 2) \oplus 3$, find the value of C.

   C =

4. 在座標平面上，用以下直綫所圍成圖形的面積為 D 平方單位，求 D 的值。

$$L_1: y - 2 = 0$$
$$L_2: y + 2 = 0$$
$$L_3: 4x + 7y - 10 = 0$$
$$L_4: 4x + 7y + 20 = 0$$

   In the coordinate plane, the area of the region bounded by the following lines is D square units, find the value of D.

$$L_1: y - 2 = 0$$
$$L_2: y + 2 = 0$$
$$L_3: 4x + 7y - 10 = 0$$
$$L_4: 4x + 7y + 20 = 0$$

   D =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設[x]是不超過 x 的最大整數。
若 $A=\left[\dfrac{2008\times 80+2009\times 130+2010\times 180}{2008\times 15+2009\times 25+2010\times 35}\right]$，求 A 的值。
Let [x] be the largest integer not greater than x.
If $A=\left[\dfrac{2008\times 80+2009\times 130+2010\times 180}{2008\times 15+2009\times 25+2010\times 35}\right]$, find the value of A.

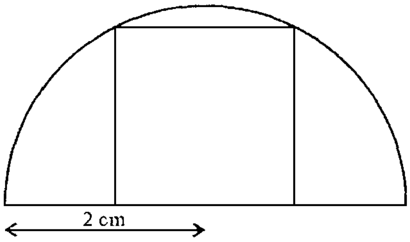
A =

2. 在 $\underbrace{99\ldots9}_{2009\text{個}9}\times\underbrace{99\ldots9}_{2009\text{個}9}+1\underbrace{99\ldots9}_{2009\text{個}9}$ 中，末位的 0 共有 R 個，求 R 的值。
There are R zeros at the end of $\underbrace{99\ldots9}_{2009\text{ of }9's}\times\underbrace{99\ldots9}_{2009\text{ of }9's}+1\underbrace{99\ldots9}_{2009\text{ of }9's}$, find the value of R.

R =

3. 如圖一，邊長為 Q cm 的正方形內接於半徑為 2 cm 的半圓中，求 Q 的值。
In Figure 1, a square of side length Q cm is inscribed in a semi-circle of radius 2 cm. Find the value of Q.

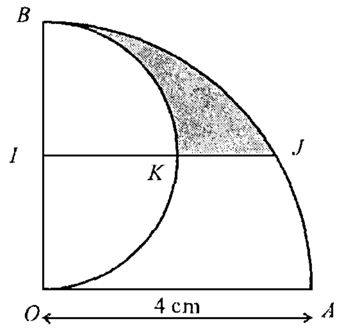
Q =



圖一
Figure 1

4. 如圖二，扇形 OAB 的半徑為 4 cm 及 $\angle AOB$ 為直角。設以 OB 為直徑的半圓，其圓心為 I 且 $IJ\,//\,OA$ 及 IJ 與該半圓相交於 K。若陰影部分的面積為 T cm^2，求 T 的值。(取 $\pi=3$)
In Figure 2, the sector OAB has radius 4 cm and $\angle AOB$ is a right angle.
Let the semi-circle with diameter OB be centred at I with $IJ\,//\,OA$, and IJ intersects the semi-circle at K.
If the area of the shaded region is T cm^2, find the value of T. (Take $\pi = 3$)

T =



圖二
Figure 2

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2008 – 2009)
Final Event 4 (Group)

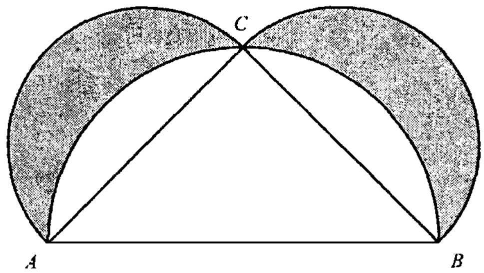
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 P 為實數。若 $\sqrt{3-2P}+\sqrt{1-2P}=2$，求 P 的值。
Let P be a real number. If $\sqrt{3-2P}+\sqrt{1-2P}=2$, find the value of P.

$P =$

2. 如圖一，設 AB、AC 及 BC 為相應半圓的直徑。
若 $AC = BC = 1$ cm 及陰影部分的面積是 R cm^2，求 R 的值。
In Figure 1, let AB, AC and BC be the diameters of the corresponding three semi-circles. If $AC = BC = 1$ cm and the area of the shaded region is R cm^2.
Find the value of R.

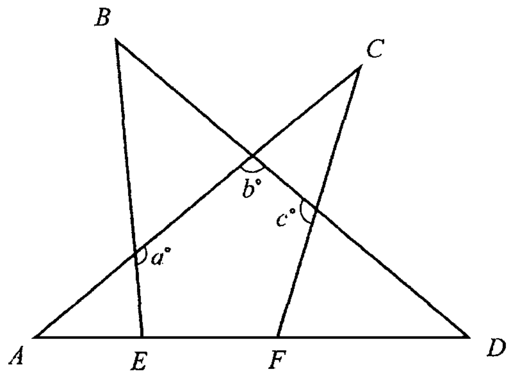
$R =$



圖一
Figure 1

3. 如圖二，AC、AD、BD、BE 及 CF 為直綫。
若 $\angle A+\angle B+\angle C+\angle D=140^\circ$ 及 $a+b+c=S$，求 S 的值。
In Figure 2, AC, AD, BD, BE and CF are straight lines.
If $\angle A+\angle B+\angle C+\angle D=140^\circ$ and $a+b+c=S$, find the value of S.

$S =$



圖二
Figure 2

4. 設 $Q=\log_{2+\sqrt{2^2-1}}\left(2-\sqrt{2^2-1}\right)$，求 Q 的值。
Let $Q=\log_{2+\sqrt{2^2-1}}\left(2-\sqrt{2^2-1}\right)$, find the value of Q.

$Q =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.