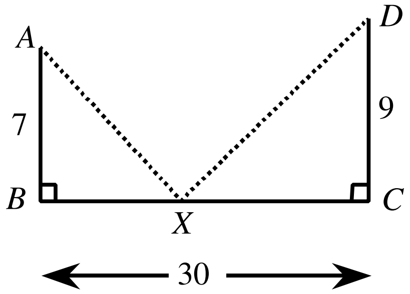
Hong Kong Mathematics Olympiad (1992 – 93)
Heat Event (Individual)

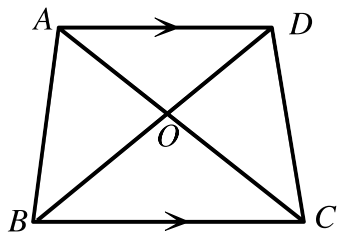
除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. X is a point on the line segment BC as shown in figure 1.
If $AB = 7$, $CD = 9$ and $BC = 30$,
find the minimum value of $AX + XD$.
在圖一中，X 為 BC 上一點。已知 $AB = 7$，$CD = 9$ 及 $BC = 30$，求 $AX + XD$ 的最小值。



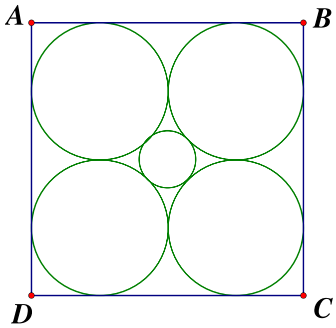
(Figure 1) (圖一)

2. 在圖二中，$ABCD$ 為一四邊形，其中 $AD // BC$，而 AC、BD 交於 O。已知ΔBOC 的面積 $= 36$，ΔAOD 的面積 $= 25$，求四邊形 $ABCD$ 的面積。
In quadrilateral $ABCD$, $AD // BC$, and AC, BD intersect at O (as shown in figure 2). Given that area of $\Delta BOC = 36$, area of $\Delta AOD = 25$, determine the area of the quadrilateral $ABCD$.



(Figure 2) (圖二)

3. 在圖三中，$ABCD$ 是一邊長為 $8(\sqrt{2}+1)$ 的正方形。求正方形中央小圓的半徑。
In figure 3, $ABCD$ is a square of side $8(\sqrt{2}+1)$. Find the radius of the small circle at the centre of the square.



(Figure 3) (圖三)

4. 從分別寫上 1 到 30 的三十張紙牌中隨意抽取一張。求點數是 2 或 5 的倍數的概率。
Thirty cards are marked from 1 to 30 and one is drawn at random. Find the probability of getting a multiple of 2 or a multiple of 5.

5. 一長方形盒子的三塊不同面的面積分別為 120、72 和 60。求該盒子的體積。
The areas of three different faces of a rectangular box are 120, 72 and 60 respectively.
Find its volume.

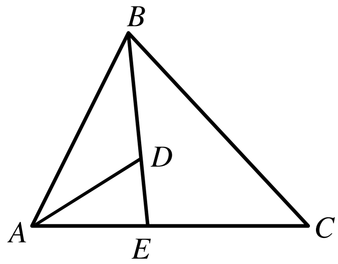
6. 已知對任何正整數 n，$1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$。
求 $12^2 + 14^2 + 16^2 + \cdots + 40^2$ 的值。
For any positive integer n, it is known that $1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.
Find the value of $12^2 + 14^2 + 16^2 + \cdots + 40^2$.

7. 若 x 和 y 為質數，且滿足 $x^2 - y^2 = 117$，求 x 的值。
If x and y are prime numbers such that $x^2 - y^2 = 117$, find the value of x .

8. 若 54000 的正因數有 m 個，求 m 的值。
If m is the total number of positive divisors of 54000, find the value of m .

9. 若 a 為一實數，且 $a^2 - a - 1 = 0$，求 $a^4 - 2a^3 + 3a^2 - 2a + 10$ 的值。
If a is a real number such that $a^2 - a - 1 = 0$, find the value of $a^4 - 2a^3 + 3a^2 - 2a + 10$.

10. 在圖四中，BDE 及 AEC 為直綫、$AB = 2$、$BC = 3$、$\angle ABC = 60°$、$AE : EC = 1 : 2$。若 $BD : DE = 9 : 1$ 及三角形 ΔDBA 的面積 $= \frac{a\sqrt{3}}{20}$，求 a 的值。
In figure 4, BDE and AEC are straight lines, $AB = 2$, $BC = 3$, $\angle ABC = 60°$, $AE : EC = 1 : 2$. If $BD : DE = 9 : 1$ and area of $\Delta DBA = \frac{a\sqrt{3}}{20}$, find the value of a .



(Figure 4) (圖四)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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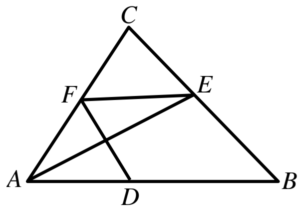
1. 一汽車 P 位於另一汽車 Q 以北 $10\sqrt{2}$ km。兩車同時起步，其中 P 以 4 km/h 速度向東南方走，而 Q 則以 3 km/h 速度向東北方走。求兩車最接近時的距離並以 km 表示。
A car P is $10\sqrt{2}$ km north of another car Q. The two cars start to move at the same time with P moving south-east at 4 km/h and Q moving north-east at 3 km/h. Find their smallest distance of separation in km .

2. 若 α、β 為方程 $x^2 - 3x - 3 = 0$ 的兩根，求 $\alpha^3 + 12\beta$ 的值。
If α, β are the roots of the equation $x^2 - 3x - 3 = 0$, find the value of $\alpha^3 + 12\beta$.

3. 在圖一中，三角形 ABC 的面積為 10。D、E 及 F 分別為 AB、BC 及 CA 上的點且滿足 $AD : DB = 2 : 3$，
且 ΔABE 的面積 = 四邊形 $BEFD$ 的面積。求 ΔABE 的面積。
As shown in figure 1, the area of ΔABC is 10. D, E, F are points on AB, BC and CA respectively such that $AD : DB = 2 : 3$, and area of ΔABE = area of quadrilateral $BEFD$. Find the area of ΔABE .



(Figure 1) (圖一)

4. 在一平面上畫 20 條直綫，最多可將平面分成幾個區域 ?
What is the maximum number of regions produced by drawing 20 straight lines on a plane?

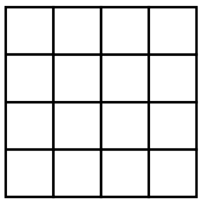
5. 若四個連續正整數的乘積為 3024，求其中最大的一個。
The product of 4 consecutive positive integers is 3024. Find the largest integer among the four.

6. 求方程 $(x + 2)(x + 3)(x + 4)(x + 5) = 3$ 的實根的總和。
Find the sum of all real roots of the equation $(x + 2)(x + 3)(x + 4)(x + 5) = 3$.

7. 若 a 為一整數，且 $a^7 = 8031810176$，求 a 的值。
If a is an integer and $a^7 = 8031810176$, find the value of a .

8. 若 x 及 y 為實數，且 $\begin{cases} x^2 - xy + y^2 - 3x - 3y = 1 \\ xy = 1 \end{cases}$ 及 $x > y > 0$，求 x 的值。
If x and y are real numbers satisfying $\begin{cases} x^2 - xy + y^2 - 3x - 3y = 1 \\ xy = 1 \end{cases}$ and $x > y > 0$, find the value of x .

9. 一正方形的每邊被均分為四份，且以直綫連接如圖二。
求非正方形的長方形數目。
Each side of a square is divided into four equal parts and straight lines are joined as shown in figure 2. Find the number of rectangles which are not squares.


(Figure 2) (圖二)

10. 若 $0° \le \theta \le 90°$，且 $\cos\theta - \sin\theta = \frac{\sqrt{5}}{3}$，求 $\cos\theta + \sin\theta$ 的值。
If $0° \le \theta \le 90°$ and $\cos\theta - \sin\theta = \frac{\sqrt{5}}{3}$, find the value of $\cos\theta + \sin\theta$.

*** 試卷完 **End of Paper** ***