Individual Events

SI	***a***	10	**I1**	***A***	380	**I2**	***r***	3	**I3**	***x***	30	**I4**	***a***	8	**I5**	***a***	1968
	b	280		***B***	70		***x***	1		***a***	6		***b***	17		***b***	2
	c	400		***n***	60		***y***	15		***c***	8		***d***	287		***c***	25
	d	120		***m***	5		***p***	40		***f*(4)**	63		***K***	280		***d***	95

Group Events

SG	***A***	10	**G6**	***p***	60	**G7**	***A***	75	**G8**	***A***	2	**G9**	***A***	1	**G10**	***A***	20
	B	4		***k***	100		***B***	3		***B***	52		***B***	0		***B***	30
	C	900		***N***	9		***C***	2		***N***	5		***C***	8		***C***	2
	D	18		***M***	999999		***D***	5		***K***	16		***D***	9		***D***	18

Sample Individual Event

SI.1 If $x^2 - 8x + 26 \equiv (x + k)^2 + a$, find a.

Reference: 1984 FI2.4, 1985 FG10.2, 1986 FG7.3, 1988 FG9.3

$x^2 - 8x + 26 \equiv (x - 4)^2 + 26 - 16$

$a = 10$

SI.2 If $\sin a° = \cos b°$, where $270 < b < 360$, find b.

$\sin 10° = \cos b°$

$\cos b° = \cos 80°$

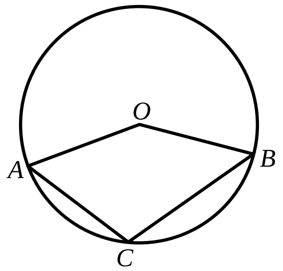
$b = 360 - 80 = 280$

SI.3 X sold an article to Y for \$$b$ at a loss of 30%. If the cost price of the article for X is \$$c$, find c.

$c\cdot(1 - 30\%) = 280$

$c = 400$

SI.4 In the figure, O is the centre of the circle. If $\angle ACB = \frac{3c°}{10}$ and $\angle AOB = d°$, find d.



$\angle ACB = 120°$

reflex $\angle AOB = 240°$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle AOB = 120°$ ($\angle$s at a point)

$d = 120$

Individual Event 1

I1.1 If $A = 11 + 12 + 13 + \dots + 29$, find A.

$A = \frac{1}{2}(11 + 29) \cdot 19 = 380$

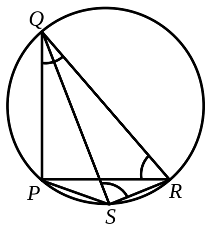
I1.2 If $\sin A° = \cos B°$, where $0 < B < 90$, find B.

$\sin 380° = \cos B°$

$\sin 20° = \cos B°$

$B = 70$

I1.3 In the given figure, $\angle PQR = B°$, $\angle PRQ = 50°$. If $\angle QSR = n°$, find n.



$\angle PQR = 70°$

$\angle QPR = 60°$ ($\angle$s sum of Δ)

$n = 60$ ($\angle$s in the same segment)

I1.4 n cards are marked from 1 to n and one is drawn at random. If the chance of it being a multiple of 5 is $\frac{1}{m}$, find m.

Favourable outcome = $\{5, 10, \dots, 60\}$

$\frac{1}{m} = \frac{12}{60} = \frac{1}{5}$

$\Rightarrow m = 5$

Individual Event 2

I2.1 The volume of a sphere with radius r is 36π, find r.

$\frac{4\pi}{3}r^3 = 36\pi$

$r = 3$

I2.2 If $r^x + r^{1-x} = 4$ and $x > 0$, find x.

$3^x + \frac{3}{3^x} = 4$ (It is straight forward by guessing $x = 1$)

$(3^x)^2 - 4\cdot 3^x + 3 = 0$

$(3^x - 1)(3^x - 3) = 0$

$3^x = 1$ or 3

$x = 0$ (rejected, as $x > 0$) or 1

I2.3 In $a : b = 5 : 4$, $b : c = 3 : x$ and $a : c = y : 4$, find y.

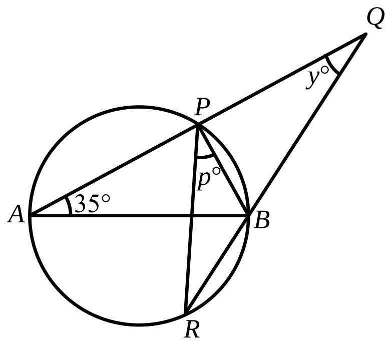
$a : b : c = 15 : 12 : 4$

$a : c = 15 : 4$

$\Rightarrow y = 15$

I2.4 In the figure, AB is a diameter of the circle. APQ and RBQ are straight lines.

If $\angle PAB = 35°$, $\angle PQB = y°$ and $\angle RPB = p°$, find p.



$\angle ABR = 35° + y° = 50°$ (ext. $\angle$ of Δ)

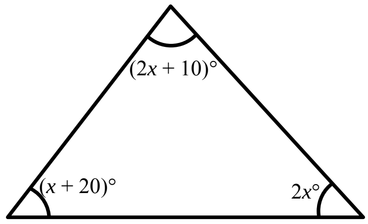
$\angle APR = \angle ABR = 50°$ ($\angle$s in the same segment)

$p + 50 = 90$ ($\angle$ in semi-circle)

$p = 40$

Individual Event 3

I3.1 In the figure, find x.



$x + 20 + 2x + 10 + 2x = 180$ ($\angle$s sum of Δ)

$x = 30$

I3.2 The coordinates of the points P and Q are $(a, 2)$ and $(x, -6)$ respectively.

If the coordinates of the mid-point of PQ is $(18, b)$, find a.

$\frac{1}{2}(a + 30) = 18$

$a = 6$

I3.3 A man travels from X to Y at a uniform speed of a km/h and returns at a uniform speed of $2a$ km/h. If his average speed is c km/h, find c.

Let the distance between X and Y be s km.

$c = \dfrac{2s}{\frac{s}{a} + \frac{s}{2a}} = \dfrac{2}{\frac{1}{6} + \frac{1}{12}} = 8$

I3.4 If $f(y) = 2y^2 + cy - 1$, find $f(4)$.

$f(4) = 2(4)^2 + 8(4) - 1 = 63$

Individual Event 4

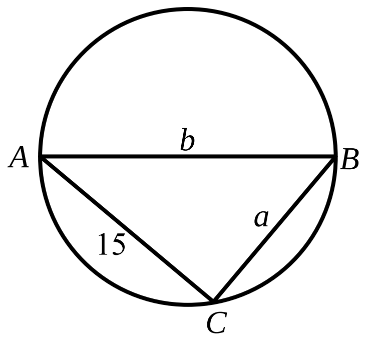
I4.1 If the curve $y = 2x^2 - 8x + a$ touches the x-axis, find a.

$\Delta = (-8)^2 - 4(2)a = 0$

$a = 8$

I4.2 In the figure, AB is a diameter of the circle.

If $AC = 15$, $BC = a$ and $AB = b$, find b.



$\angle ACB = 90°$ ($\angle$ in semi-circle)

$b^2 = 15^2 + 8^2$ (Pythagoras' theorem)

$b = 17$

I4.3 The line $5x + by + 2 = d$ passes through $(40, 5)$. Find d.

Reference: 1984 FI2.3

$d = 5(40) + 17(5) + 2 = 287$

I4.4 X sold an article to Y for \$$d$ at a profit of 2.5%.

If the cost price of the article for X is \$$K$, find K.

$K = 287 \div (1 + 2.5\%) = 280$

Individual Event 5

I5.1 Let $x = 19.\dot{8}\dot{7}$. If $19.\dot{8}\dot{7} = \frac{a}{99}$, find a. (Hint: $99x = 100x - x$)

$99x = 100x - x = 1987 + 0.\dot{8}\dot{7} - 19.\dot{8}\dot{7} = 1968$

$x = \frac{1968}{99}$

$\Rightarrow a = 1968$

I5.2 If $f(y) = 4 \sin y°$ and $f(a - 18) = b$, find b.

$b = f(a - 18) = f(1950)$

$= 4 \sin 1950°$

$= 4 \sin(360°\times 5 + 150°)$

$= 4 \sin 150° = 2$

I5.3 If $\frac{\sqrt{3}}{b\sqrt{7} - \sqrt{3}} = \frac{2\sqrt{21} + 3}{c}$, find c.

$$\frac{\sqrt{3}}{2\sqrt{7} - \sqrt{3}} \cdot \frac{2\sqrt{7} + \sqrt{3}}{2\sqrt{7} + \sqrt{3}} = \frac{2\sqrt{21} + 3}{c}$$

$c = 4(7) - 3 = 25$

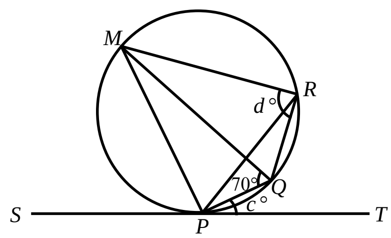
I5.4 In the figure, ST is a tangent to the circle at P.

If $\angle MQP = 70°$, $\angle QPT = c°$ and $\angle MRQ = d°$, find d.

$\angle MRP = 70°$ ($\angle$s in the same segment)

$\angle PRQ = c° = 25°$ ($\angle$ in alt. segment)

$d = 70 + 25 = 95$



Sample Group Event

SG.1 If $100A = 35^2 - 15^2$, find A.

Reference: 1984 FI1.1

$100A = (35 - 15)(35 + 15) = 1000$

$A = 10$

SG.2 If $(A - 1)^6 = 27^B$, find B.

$(10 - 1)^6 = 27^B$

$3^{12} = 3^{3B}$

$\Rightarrow B = 4$

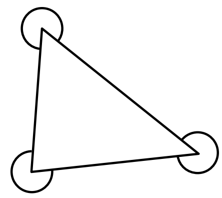
SG.3 In the given diagram, the sum of the three marked angles is $C°$. Find C.

Reference: 1984 FSI.1, 1989 FSI.1

Sum of interior angles of a triangle = 180°

angle sum of three vertices = 3×360° = 1080°

$C = 1080 - 180 = 900$



SG.4 If the lines $x + 2y + 1 = 0$ and $9x + Dy + 1 = 0$ are parallel, find D.

Reference: 1986 FI4.2, 1989 FSG.2

$-\frac{1}{2} = -\frac{9}{D}$

$\Rightarrow D = 18$

Group Event 6

G6.1 If α, β are the roots of $x^2 - 10x + 20 = 0$, and $p = \alpha^2 + \beta^2$, find p.

$\alpha + \beta = 10$, $\alpha\beta = 20$

$p = (\alpha + \beta)^2 - 2\alpha\beta$

$= 10^2 - 2(20) = 60$

G6.2 The perimeter of an equilateral triangle is p. If its area is $k\sqrt{3}$, find k.

Reference: 1984FI4.4, 1985 FSI.4, 1986 FSG.3, 1988 FG9.1

Length of one side = 20

$$\frac{1}{2}\cdot 20^2 \sin 60^\circ = k\sqrt{3}$$

$k = 100$

G6.3 Each interior angle of an N-sided regular polygon is 140°. Find N.

Reference: 1997 FI4.1

Each exterior angle = 40° (adj. ∠s on st. line)

$$\frac{360^\circ}{N} = 40^\circ \quad \text{(sum of ext. ∠s of polygon)}$$

$\Rightarrow N = 9$

G6.4 If $M = (10^2 + 10\times 1 + 1^2)(10^2 - 1^2)(10^2 - 10\times 1 + 1^2)$, find M.

$M = (10^2 + 10\times 1 + 1^2)(10 - 1)(10 + 1)(10^2 - 10\times 1 + 1^2)$

$= (10^3 - 1)(10^3 + 1)$

$= 10^6 - 1 = 999999$

Group Event 7

G7.1 The acute angle formed by the hands of a clock at 3:30 p.m. is $A°$. Find A.

Reference 1984 FG7.1, 1985 FI3.1, 1989 FI1.1, 1990 FG6.3, 2007 HI1

At 3:00 p.m., the angle between the arms of the clock = 90°

From 3:00 p.m. to 3:30 p.m., the hour-hand had moved $360°\times\frac{1}{12}\times\frac{1}{2} = 15°$.

The minute hand had moved 180°.

$A = 180 - 90 - 15 = 75$

G7.2 If $\tan(3A + 15)° = \sqrt{B}$, find B.

$\tan(225 + 15)° = \sqrt{B}$

$\Rightarrow B = 3$

G7.3 If $\log_{10}AB = C \log_{10}15$, find C.

$\log_{10} (75\times3) = C \log_{10} 15$

$\log_{10} 225 = C \log_{10} 15$

$\Rightarrow C = 2$

G7.4 The points (1, 3), (4, 9) and (2, D) are collinear. Find D.

Reference: 1984 FSG.4, 1984 FG7.3, 1986 FG6.2, 1989 HI8

$\frac{D-9}{2-4} = \frac{9-3}{4-1}$

$D - 9 = -4$

$\Rightarrow D = 5$

Group Event 8

G8.1 If $A = \dfrac{5\sin\theta + 4\cos\theta}{3\sin\theta + \cos\theta}$ and $\tan\theta = 2$, find A.

Reference: 1986 FG10.3, 1989 FSG.4, 1989 FG10.3, 1990 FG7.2

$$A = \frac{(5\sin\theta + 4\cos\theta) \div \cos\theta}{(3\sin\theta + \cos\theta) \div \cos\theta}$$

$$= \frac{5\tan\theta + 4}{3\tan\theta + 1}$$

$$= \frac{5(2) + 4}{3(2) + 1} = 2$$

G8.2 If $x + \dfrac{1}{x} = 2A$, and $x^3 + \dfrac{1}{x^3} = B$, find B.

Reference: 1983 FG7.3, 1984 FG10.2, 1985 FI1.2, 1989 HI1, 1990 HI12, 2002 FG2.2

$$x + \frac{1}{x} = 4$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 4^2 - 2 = 14$$

$$B = x^3 + \frac{1}{x^3}$$

$$= \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$$

$$= 4(14 - 1) = 52$$

G8.3 There are exactly N values of α satisfying the equation $\cos^3\alpha - \cos\alpha = 0$, where $0° \le \alpha \le 360°$. Find N.

$\cos\alpha (\cos\alpha + 1)(\cos\alpha - 1) = 0$

$\cos\alpha = 0, -1$ or 1

$\alpha = 90, 270, 180, 0, 360$

$\Rightarrow N = 5$

G8.4 If the N^{th} day of May in a year is Thursday and the K^{th} day of May in the same year is Monday, where $10 < K < 20$, find K.

Reference: 1984 FG6.3, 1985 FG9.3, 1988 FG10.2

5^{th} May is Thursday

9^{th} May is Monday

16^{th} May is Monday

$\Rightarrow K = 16$

Group Event 9

In the given multiplication, different letters represent different integers ranging from 0 to 9.

$$\begin{array}{rcccc} & A & B & C & D \\ \times & & & & 9 \\ \hline & D & C & B & A \end{array}$$

G9.1 Find A.
G9.2 Find B.
G9.3 Find C.
G9.4 Find D.

Reference: 1994 HI6

$$\begin{array}{rcccc} & 1 & B & C & 9 \\ \times & & & & 9 \\ \hline & 9 & C & B & 1 \end{array}$$

As there is no carry digit in the thousands digit multiplication, A = 1, D = 9

Consider the tens digit: $9C + 8 \equiv B \pmod{10}$ …… (1)

As there is no carry digit in the thousands digit, let the carry digit in the hundreds digit be x.

$9B + x = C$ and B, C are distinct integers different from 1 and 9

$\Rightarrow B = 0, C = x$

Sub. B = 0 into (1): $9C + 8 \equiv 0 \pmod{10}$

$\Rightarrow 9C \equiv 2 \pmod{10}$

$\Rightarrow C = 8$

$\therefore A = 1, B = 0, C = 8, D = 9$

Group Event 10

G10.1 The average of p, q, r and s is 5. The average of p, q, r, s and A is 8. Find A.

Reference: 1985 FG6.1, 1986 FG6.4, 1988 FG9.2

$p + q + r + s = 20$

$p + q + r + s + A = 40$

$A = 20$

G10.2 If the lines $3x - 2y + 1 = 0$ and $Ax + By + 1 = 0$ are perpendicular, find B.

Reference: 1983 FG9.3, 1984 FSG.3, 1985 FI4.1, 1986 FSG.2, 1988 FG8.2

$\frac{3}{2}\times\left(-\frac{20}{B}\right) = -1 \Rightarrow B = 30$

G10.3 When $Cx^3 - 3x^2 + x - 1$ is divided by $x + 1$, the remainder is -7. Find C.

$C(-1) - 3 - 1 - 1 = -7$

$C = 2$

G10.4 If P, Q are positive integers such that $P + Q + PQ = 90$ and $D = P + Q$, find D.
(Hint: Factorise $1 + P + Q + PQ$)

Reference: 2002 HG9, 2012 FI4.2

WLOG assume $P \le Q$, $1 + P + Q + PQ = 91$

$(1 + P)(1 + Q) = 1\times 91 = 7\times 13$

$1 + P = 1 \Rightarrow P = 0$ (rejected)

or $1 + P = 7 \Rightarrow P = 6$

$1 + Q = 13 \Rightarrow Q = 12$

$D = 6 + 12 = 18$