Individual Events

SI	***h***	4	**I1**	***a***	5	**I2**	***p***	3	**I3**	***a***	1000	**I4**	***a***	5	**I5**	***a***	17
	k	32		***b***	4		***q***	36		***b***	8		***b***	12		***b***	5
	p	3		***c***	10		***k***	12		***c***	16		***c***	4		***c***	23
	q	16		***d***	34		***m***	150		***d***	1		***d***	12		***d***	9

Group Events

SG	***a***	2	**G6**	***a***	150	**G7**	***C***	47	**G8**	***A***	2	**G9**	***S***	1000	**G10**	***A***	1584
	b	–3		***b***	10		***K***	2		***B***	3		***K***	98		***k***	14
	p	60		***k***	37.5		***A***	1		***C***	7		***t***	20		***x***	160
	q	136		***d***	6		***B***	5		***k***	9		***d***	5		***n***	15

Sample Individual Event (1986 Final Individual Event 2)

SI.1 Given that $3x^2 - 4x + \frac{h}{3} = 0$ has equal roots, find h.

$\Delta = (-4)^2 - 4(3)\cdot\frac{h}{3} = 0$

$h = 4$

SI.2 If the height of a cylinder is doubled and the new radius is h times the original, then the new volume is k times the original. Find k.

Let the old height be x, old radius be r, then the old volume is $\pi r^2 x$.

The new height is $2x$, the new radius is $4r$,

then the new volume is $\pi(4r)^2(2x) = 32\pi r^2 x$

$k = 32$

SI.3 If $\log_{10} 210 + \log_{10}k - \log_{10}56 + \log_{10}40 - \log_{10}120 + \log_{10}25 = p$, find p.

$p = \log_{10}\left(\frac{210\times 32\times 40\times 25}{56\times 120}\right)$

$= \log_{10} 1000 = 3$

SI.4 If $\sin A = \frac{p}{5}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find q.

$\sin A = \frac{3}{5}$

$\frac{\cos A}{\tan A} = \frac{q}{15}$

$\frac{\cos^2 A}{\sin A} = \frac{q}{15}$

$\frac{1-\sin^2 A}{\sin A} = \frac{1-\left(\frac{3}{5}\right)^2}{\frac{3}{5}} = \frac{16}{15} = \frac{q}{15}$

$q = 16$

Individual Event 1

I1.1 Find a if $2t + 1$ is a factor of $4t^2 + 12t + a$.

Let $f(t) = 4t^2 + 12t + a$

$$f\left(-\frac{1}{2}\right)=4\left(-\frac{1}{2}\right)^2+12\left(-\frac{1}{2}\right)+a=0$$

$a = 5$

I1.2 $\sqrt{K}$ denotes the nonnegative square root of K, where $K \geq 0$. If b is the root of the equation $\sqrt{a-x} = x - 3$, find b.

$\left(\sqrt{5-x}\right)^2 = (x-3)^2$

$\Rightarrow 5 - x = x^2 - 6x + 9$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow x = 1$ or 4

When $x = 1$, LHS $= 2 \neq -1 =$ RHS

When $x = 4$, LHS $= 1 =$ RHS.

$\therefore x = b = 4$

I1.3 If c is the greatest value of $\dfrac{20}{b+2\cos\theta}$, find c.

$$\frac{20}{b+2\cos\theta}=\frac{20}{4+2\cos\theta}=\frac{10}{2+\cos\theta}$$

c = the greatest value $=\dfrac{10}{2-1} = 10$

I1.4 A man drives a car at $3c$ km/h for 3 hours and then $4c$ km/h for 2 hours. If his average speed for the whole journey is d km/h, find d.

Total distance travelled = $(30\times3 + 40\times2)$ km = 170 km

$d=\dfrac{170}{3+2} = 34$

Individual Event 2

I2.1 If $0° \le \theta < 360°$, the equation in θ: $3\cos\theta + \dfrac{1}{\cos\theta} = 4$ has p roots. Find p.

$3\cos^2\theta + 1 = 4\cos\theta$

$\Rightarrow 3\cos^2\theta - 4\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = \dfrac{1}{3}$ or 1

$p = 3$

I2.2 If $x - \dfrac{1}{x} = p$ and $x^3 - \dfrac{1}{x^3} = q$, find q.

Reference: 2009 FI2.3

$x - \dfrac{1}{x} = 3$; $\left(x - \dfrac{1}{x}\right)^2 = 9$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 11$

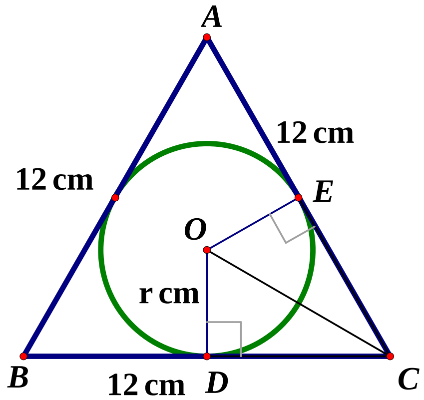
$q = x^3 - \dfrac{1}{x^3}$

$= \left(x - \dfrac{1}{x}\right)\left(x^2 + 1 + \dfrac{1}{x^2}\right)$

$= 3(11 + 1) = 36$

I2.3 A circle is inscribed in an equilateral triangle of perimeter q cm. If the area of the circle is $k\pi$ cm^2, find k.

Reference: 1984 FG9.4



Let the equilateral triangle be ABC, the centre of the inscribed circle is O, which touches the triangle at D and E, with radius r cm

Perimeter = 36 cm

$\Rightarrow$ Each side = 12 cm

$\angle ACB = 60°$ ($\angle$s of an equilateral Δ)

$\angle ODC = 90°$ (tangent $\perp$ radius)

$\angle OCD = 30°$ (tangent from ext. pt.)

$CD = 6$ cm (tangent from ext. pt.)

$r = 6\tan 30° = 2\sqrt{3}$

Area of circle $= \pi\left(2\sqrt{3}\right)^2$ cm^2 = 12π cm^2

$k = 12$

I2.4 Each interior angle of a regular polygon of k sides is $m°$. Find m.

Angle sum of 12-sides polygon = $180°(12 - 2) = 1800°$

Each interior angle = $m° = 1800° \div 12 = 150°$

$m = 150$

Individual Event 3

I3.1 If $998a + 1 = 999^2$, find a.

$998a = 999^2 - 1$

$= (999 - 1)(999 + 1)$

$= 998\times1000$

$a = 1000$

I3.2 If $\log_{10}a = \log_2 b$, find b.

$\log_{10} 1000 = \log_2 b$

$\log_2 b = 3$

$\Rightarrow b = 2^3 = 8$

I3.3 The area of the triangle formed by the x-axis, the y-axis and the line $2x + y = b$ is c sq. units. Find c.

Reference: 1994 FI5.3

$2x + y = 8$; x-intercept = 4, y-intercept = 8

$c = \text{area} = \frac{1}{2}\cdot 4\times 8 = 16$

I3.4 If $64t^2 + ct + d$ is a perfect square, find d.

$64t^2 + 16t + d$ has a double root

$\Delta = 16^2 - 4\times64d = 0$

$d = 1$

Individual Event 4

I4.1 Solve for a in the equation $2^{a+1} + 2^a + 2^{a-1} = 112$.

$2^a \cdot (2 + 1 + \frac{1}{2}) = 112$

$2^a = 32$

$a = 5$

Method 2

$112 = 64 + 32 + 16 = 2^6 + 2^5 + 2^4$

$a = 5$

I4.2 If a is one root of the equation $x^2 - bx + 35 = 0$, find b.

One root of $x^2 - bx + 35 = 0$ is 5

$\Rightarrow 5^2 - 5b + 35 = 0$

$\Rightarrow b = 12$

I4.3 If $\sin\theta = \frac{-b}{15}$, where $180° < \theta < 270°$, and $\tan\theta = \frac{c}{3}$, find c.

$\sin\theta = -\frac{12}{15} = -\frac{4}{5}$

$\Rightarrow \tan\theta = \frac{4}{3}$

$\Rightarrow c = 4$

I4.4 The probability of getting a sum of c in throwing two dice is $\frac{1}{d}$. Find d.

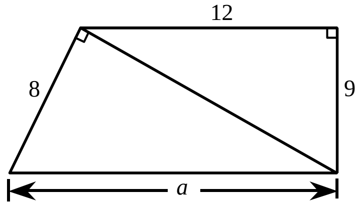
P(sum = 4) = P((1,3), (2, 2), (3, 1))

$= \frac{3}{36} = \frac{1}{12} = \frac{1}{d}$

$\Rightarrow d = 12$

Individual Event 5

I5.1 In the figure, find *a*.



$a^2 - 8^2 = 12^2 + 9^2$ (Pythagoras' Theorem)

$a = 17$

I5.2 If the lines $ax + by = 1$ and $10x - 34y = 3$ are perpendicular to each other, find *b*.

$17x + by = 1$ is perpendicular to $10x - 34y = 3$

$\Rightarrow$ product of slopes = –1

$$-\frac{17}{b}\times\frac{10}{34}=-1$$

$\Rightarrow b = 5$

I5.3 If the b^{th} day of May in a year is Friday and the c^{th} day of May in the same year is Tuesday, where $16 < c < 24$, find *c*.

5^{th} May is a Friday

$\Rightarrow 9^{\text{th}}$ May is Tuesday

$\Rightarrow 16^{\text{th}}$ May is Tuesday

$\Rightarrow 23^{\text{rd}}$ May is Tuesday

$c = 23$

I5.4 *c* is the d^{th} prime number. Find *d*.

Reference: 1985 FSG.2, 1989 FSG.3

The first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23

23 is the 9^{th} prime number

$d = 9$

Sample Group Event (1986 Sample Group Event)

SG.1 The sum of two numbers is 50, and their product is 25.

If the sum of their reciprocals is a, find a.

Let the 2 numbers be x, y.

$x + y = 50$, $xy = 25$

$$\Rightarrow a = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x+y}{xy}$$

$$= \frac{50}{25} = 2$$

SG.2 If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular, find b.

$2x + 2y + 1 = 0$ is $\perp$ to $3x + by + 5 = 0$

$\Rightarrow$ product of slopes = –1

$$-\frac{2}{2} \times \frac{-3}{b} = -1$$

$\Rightarrow b = -3$

SG.3 The area of an equilateral triangle is $100\sqrt{3}$ cm^2. If its perimeter is p cm, find p.

Let the length of one side be x cm.

$$\frac{1}{2}x^2 \sin 60^\circ = 100\sqrt{3}$$

$\Rightarrow x = 20$

$\Rightarrow p = 60$

SG.4 If $x^3 - 2x^2 + px + q$ is divisible by $x + 2$, find q.

Let $f(x) = x^3 - 2x^2 + 60x + q$

$f(-2) = -8 - 8 - 120 + q = 0$

$q = 136$

Group Event 6

G6.1 If $a = \dfrac{(68^3 - 65^3)\cdot(32^3 + 18^3)}{(32^2 - 32\times 18 + 18^2)\cdot(68^2 + 68\times 65 + 65^2)}$, find a.

$a = \dfrac{(32+18)(32^2 - 32\times 18 + 18^2)\cdot(68-65)(68^2 + 68\times 65 + 65^2)}{(32^2 - 32\times 18 + 18^2)\cdot(68^2 + 68\times 65 + 65^2)}$

$= 50\times 3 = 150$

G6.2 If the 3 points (a, b), (10, –4) and (20, –3) are collinear, find b.

The slopes are equal: $\dfrac{b+4}{150-10} = \dfrac{-3+4}{20-10}$

$\Rightarrow b = 10$

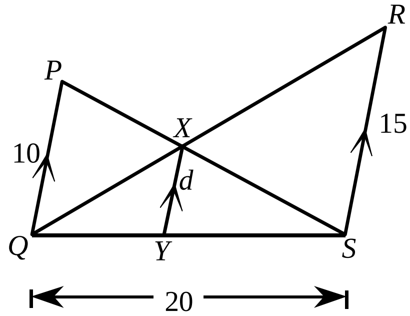
G6.3 If the acute angle formed by the hands of a clock at 4:15 is $k°$, find k.

Reference 1984 FG7.1, 1985 FI3.1, 1987 FG7.1, 1989 FI1.1, 2007 HI1

$k = 30 + 30\times\dfrac{1}{4} = 37.5$

G6.4 In the figure, PQ = 10, RS = 15, QS = 20. If $XY = d$, find d.

Reference: 1985 FI2.4, 1989 HG8



$\dfrac{1}{d} = \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{25}{150} = \dfrac{1}{6}$

$d = 6$

Group Event 7

G7.1 2 apples and 3 oranges cost 6 dollars.

4 apples and 7 oranges cost 13 dollars.

16 apples and 23 oranges cost C dollars. Find C.

Let the cost of one apple be $\$x$ and one orange be $\$y$.

$2x + 3y = 6$ …… (1)

$4x + 7y = 13$…… (2)

(2) – 2(1): $y = 1$, $x = 1.5$

$C = 16x + 23y = 24 + 23 = 47$

G7.2 If $K = \dfrac{6\cos\theta + 5\sin\theta}{2\cos\theta + 3\sin\theta}$ and $\tan\theta = 2$, find K.

Reference: 1986 FG10.3, 1987 FG8.1, 1989 FSG.4, 1989 FG10.3

$$K = \frac{6\frac{\cos\theta}{\cos\theta} + 5\frac{\sin\theta}{\cos\theta}}{2\frac{\cos\theta}{\cos\theta} + 3\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{6 + 5\tan\theta}{2 + 3\tan\theta}$$

$$= \frac{6 + 5\times 2}{2 + 3\times 2} = 2$$

G7.3 and G7.4 A, B are positive integers less than 10 such that $21A104 \times 11 = 2B8016 \times 9$.

Similar Questions 1985 FG8.1-2, 1988 FG8.3-4

G7.3 Find A.

11 and 9 are relatively prime, $21A104$ is divisible by 9.

$2 + 1 + A + 1 + 0 + 4 = 9m$

$\Rightarrow 8 + A = 9m$

$\Rightarrow A = 1$

G7.4 Find B.

$2B8016$ is divisible by 11.

$2 + 8 + 1 – (B + 0 + 6) = 11n$

$\Rightarrow 11 – (B + 6) = 11n$

$\Rightarrow B = 5$

Group Event 8

In the multiplication shown, the letters A, B, C and K ($A < B$) represent different integers from 1 to 9.

$$\begin{array}{rrr} & A & C \\ \times) & B & C \\ \hline K & K & K \end{array}$$

(Hint: $KKK = K\times 111$.)

G8.1 Find A.

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$

Possible $K = 1, 4, 5, 6, 9$

$100K + 10K + K = 111K = 3\times 37K$, 37 is a prime number

Either $10A + C$ or $10B + C$ is divisible by 37

$10B + C = 37$ or 74

When $B = 3$, $C = 7$, $K = 9$

$999 \div 37 = 27$

$\therefore A = 2$

G8.2 Find B.

$B = 3$

G8.3 Find C.

$C = 7$

G8.4 Find K.

$K = 9$

Group Event 9

G9.1 If $S = ab - 1 + a - b$ and $a = 101$, $b = 9$, find S.

Reference: 1985 FG8.4, 1986 FG9.3, 1988 FG6.3

$S = (a - 1)(b + 1) = 100\times10 = 1000$

G9.2 If $x = 1.9\dot{8}\dot{9}$ and $x - 1 = \frac{K}{99}$, find K.

$x = 1.9 + \frac{89}{990}$

$x - 1 = \frac{K}{99} = \frac{9}{10} + \frac{89}{990}$

$= \frac{9\times99+89}{990} = \frac{980}{990} = \frac{98}{99}$

$K = 98$

G9.3 The average of p, q and r is 18. The average of $p + 1$, $q - 2$, $r + 3$ and t is 19. Find t.

$\frac{p+q+r}{3} = 18$

$\Rightarrow p + q + r = 54$

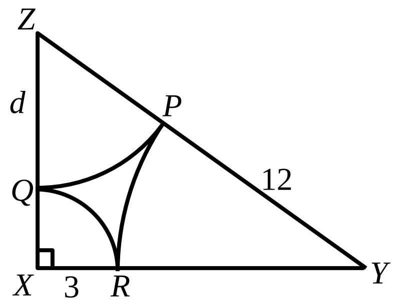
$\frac{p+1+q-2+r+3+t}{4} = 19$

$\Rightarrow p + q + r + 2 + t = 76$

$\Rightarrow 54 + 2 + t = 76$

$t = 20$

G9.4 In the figure, $\overset{\frown}{QR}$, $\overset{\frown}{RP}$, $\overset{\frown}{PQ}$ are 3 arcs, centres at X, Y and Z respectively, touching one another at P, Q and R. If $ZQ = d$, $XR = 3$, $YP = 12$, $\angle X = 90°$, find d.



Reference: 1986 FG7.1

$XZ = 3 + d$, $XY = 3 + 12 = 15$, $YZ = 12 + d$

$XZ^2 + XY^2 = YZ^2$ (Pythagoras' theorem)

$(3 + d)^2 + 15^2 = (12 + d)^2$

$9 + 6d + d^2 + 225 = 144 + 24d + d^2$

$18d = 90$

$\Rightarrow d = 5$

Group Event 10

G10.1 If $A = 1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \dots + 97 + 98 - 99$, find A.

Reference: 1985 FG7.4, 1988 FG6.4, 1991 FSI.1, 1992 FI1.4

$A = (1 + 2 - 3) + (4 + 5 - 6) + (7 + 8 - 9) + \dots + (97 + 98 - 99)$

$A = 0 + 3 + 6 + \dots + 96 = \frac{3+96}{2}\times 32 = 99\times 16 = 1584$

G10.2 If $\log_{10}(k - 1) - \log_{10}(k^2 - 5k + 4) + 1 = 0$, find k.

$10(k - 1) = k^2 - 5k + 4$

$k^2 - 15k + 14 = 0$

$k = 1$ or 14

When $k = 1$, LHS is undefined $\therefore$ rejected

When $k = 14$, LHS $= \log_{10} 13 - \log_{10}(14 - 1)(14 - 4) + 1 =$ RHS

$\therefore k = 14$

G10.3 and **G10.4** One interior angle of a convex n-sided polygon is $x°$. The sum of the remaining interior angles is $2180°$.

Reference: 1989 HG2, 1992 HG3, 2002 FI3.4, 2013 HI6

G10.3 Find x.

$2180 + x = 180(n - 2)$ ($\angle$s sum of polygon)

$2160 + 20 + x = 180\times 12 + 20 + x = 180(n - 2)$

$\because x < 180$

$\therefore 20 + x = 180$

$x = 160$

G10.4 Find n.

$n - 2 = 12 + 1$

$n = 15$