Independent Events

7.1 Introduction

Consider the following 3 examples

e.g. 1 RE = throw a die twice
A = first thrown is "3"
B = second thrown is "5"
To find P(B | A)

Solution: Since the occurrence of B **<u>does not depend</u>** on A

$\therefore$ P(B | A) = P(B) = $\frac{1}{6}$

e.g. 2 The probability of bearing a boy baby is $\frac{1}{2}$. Given that the second baby is a girl, what is the probability that the first baby is a boy?

Solution: A = first baby is a boy.
B = second baby is a girl.
The occurrence of A <u>does not depend</u> on B

$\therefore$ P(A|B) = P(A) = $\frac{1}{2}$.

e.g. 3 Two cards are drawn successively <u>without</u> replacement from ten cards numbers from 1 to 10. Given that the first card drawn is even, what is the probability that the second card drawn is also even?

Solution: A = first card drawn is even
B = second card drawn is even.
In this case, the probability of B **<u>depends</u>** on the occurrence of A.
After the first drawn (the card drawn is even), there are 9 cards remaining, of which 4 are even.

$\therefore$ P(B|A) = $\frac{4}{9}$

Discarding the result of the first card drawn, P(B) = $\frac{1}{2}$

$\therefore$ P(B|A) $\neq$ P(B)

Conclusion: If B is **<u>independent</u>** of A, then
(i) P(B | A) = P(B)
(ii) P(A $\cap$ B) = P(A) P(B) - this is known as multiplication law.

7.2 Application of multiplication Law

Example 4 A coin is loaded so that the probability of a head is $\frac{1}{3}$. The coin is tossed 5 times. Find the probability that exactly 3 heads appear.

Solution We consider there are “5” random experiments.
RE = tossed the loaded coin.
The “5” random experiments are independent.

H= getting a head, P(H) = $\frac{1}{3}$

T = getting a tail, P(T) = $1-\frac{1}{3}=\frac{2}{3}$

To obtain heads exactly 3 times, there must be tails 2 times.
They can be arranged in these forms:
HHHTT,HHTHT,HTHHT,THHHT, HHTTH, HTHTH, THHTH, HTTHH, THTHH, TTHHH.
A total of 10 permutations.

P(3 heads and 2 tails) = $10\times\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2=\frac{40}{243}$

Example 5 Two cards are drawn from tens cards numbered from 1 to 10, find the probability that the sum is even.

Solution: E = event that the sum is even

F = the first is odd and the second is odd

G = the first is even and the second is even

$E = F \cup G$

$F \cap G = \phi$

P(E) = P(F) + P(G) (by axiom 3, chapter 3)

$$= \frac{5}{10}\times\frac{4}{9}+\frac{5}{10}\times\frac{4}{9}$$

$$= 2\times\frac{1}{2}\times\frac{4}{9}=\frac{4}{9}$$

(note: the case that the first card is odd and the second card is odd is dependent.)

7.3 Sometimes, a **<u>tree diagram</u>** is helpful:

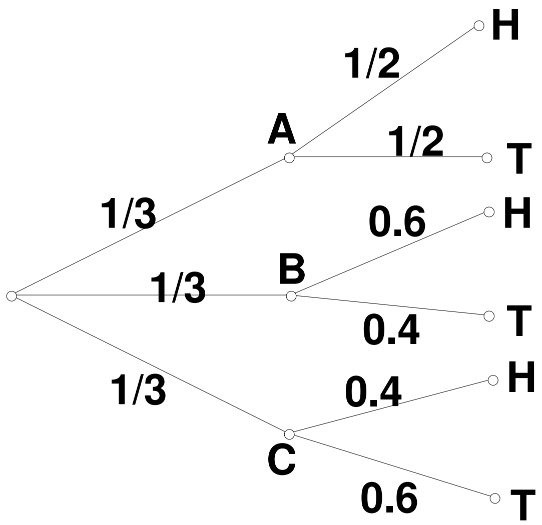
Example 6 An urn contains 3 coins. One coin is fair, one falls head with probability 0.6, and the other falls head with probability 0.4.

(*a*) Find the chance that a coin chosen at random and the flipped will come up heads.

(*b*) Find the chance that a coin chosen was the fair one, given that it came up tails.

Solution: (*a*) Let the first coin be A, the second coin be B, the third coin be C.

P(H|B) = 0.6, P(H|C) = 0.4



$$P(H) = \frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times0.6+\frac{1}{3}\times0.4=\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}$$

(*b*) $P(A \mid T) = \dfrac{P(A\cap T)}{P(T)}$

$$= \frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times0.4+\frac{1}{3}\times0.6}$$

$$= \frac{\frac{1}{6}}{\frac{1}{2}}$$

$$= \frac{1}{3}$$

Exercise 1 Mr. Fan and Mr. Chan plan to hold a meeting on some day in September, 1996. Mr. Fan is available on the days whose numbers are multiples of 6 while Mr. Chan is available only on Wednesdays. They select randomly and separately a day in September according to their own schedules. The calendar of September 1996 is shown below.

September , 1996						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

What is the probability that

(a) Mr. Fan selects a Wednesday?

(b) Mr. Chan selects a day whose number is a multiple of 6?

(c) Mr. Fan and Mr. Chan select the same day in (a) and (b)?

Answers (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{20}$

Exercise 2 Find the probability of obtaining each of the following poker hands. (A poker hand is a set of five cards chosen at random from a deck of 52 playing cards.)

(a) Royal flush (ten, jack, queen, king, ace in a single suit.)

(b) Straight flush (five in a sequence in a single suit, but not a royal flush.)

(c) Four of a kind (four cards of the same face value.)

(d) Full house (one pair and one triple of the same face value.)

(e) Flush (five cards in a single suit but not a straight or royal flush.)

Answers (a) 0.0000015 (b) 0.000014 (c) 0.00024 (d) 0.0014 (e) 0.0020

Exercise 3 A bag contains 5 red balls, 4 blue balls and 3 yellow balls. A ball is drawn at random. If it is not red, another ball is drawn. If the ball is not yellow, a third ball is drawn. All the balls drawn are not put back into the bag. What is the probability of getting at least one red ball?

Answer: $\frac{17}{22}$

Exercise 2

(a) $\frac{4}{C_5^{52}}$ or $4\times\frac{5}{52}\times\frac{4}{51}\times\frac{3}{50}\times\frac{2}{49}\times\frac{1}{48}=0.000001539077169$

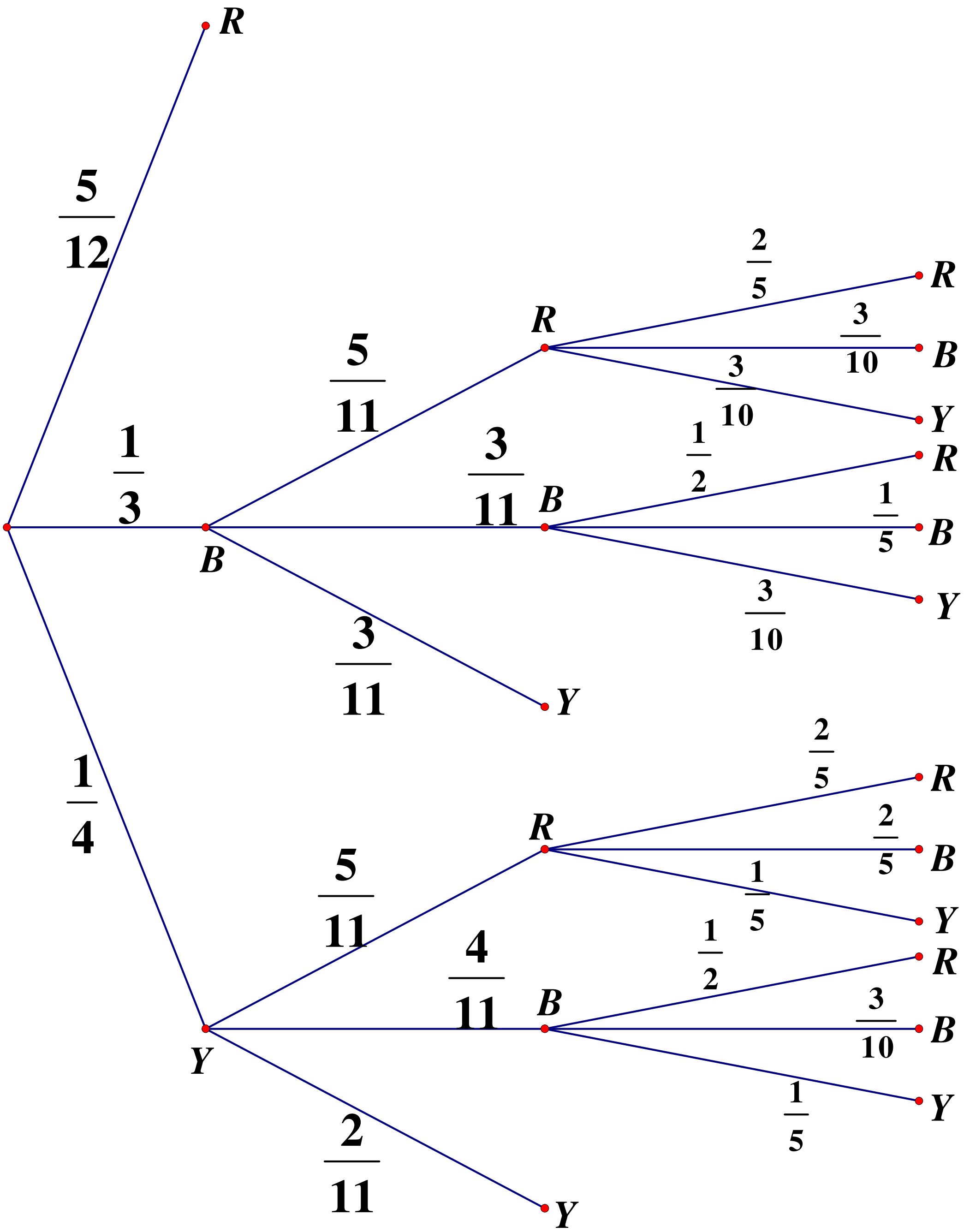
(b) $\frac{4\times 9}{C_5^{52}}=0.00001385169452$ or $4\times 9\times\frac{5}{52}\times\frac{4}{51}\times\frac{3}{50}\times\frac{2}{49}\times\frac{1}{48}$

(c) $13\times 48\times 5\times\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\times\frac{1}{49}\times\frac{1}{48}=0.0002400960384$ or $\frac{C_1^{13}\times C_1^{48}}{C_5^{52}}$

(d) $13\times 12\times C_2^5\times\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\times\frac{4}{49}\times\frac{3}{48}=0.00144057623$

(e) $4\times\frac{13}{52}\times\frac{12}{51}\times\frac{11}{50}\times\frac{10}{49}\times\frac{9}{48}-\frac{4}{C_5^{52}}-\frac{4\times 9}{C_5^{52}}=0.001965401545$

Exercise 3



Probability $=\frac{5}{12}+\frac{1}{3}\times\frac{5}{11}+\frac{1}{3}\times\frac{3}{11}\times\frac{1}{2}+\frac{1}{4}\times\frac{5}{11}+\frac{1}{4}\times\frac{4}{11}\times\frac{1}{2}=\frac{17}{22}$

Class Activity To find the probability of non-duplicated birthday

Evan Maletsky, March 1984, NCTM student Math Notes

Gardner, 1961, Mathematical Puzzles and Diversions, pp 42

The chances of duplication have been of interest. Find the least number of people to ensure the probability of a duplication of birthday is greater than $\frac{1}{2}$. You may be surprised that the number is 23! The calculation is as follows. Assume a year has 365 days and assume that all 365 days are equally likely birthdays.

Instead of finding the probability of a duplication of birthday, we find the probability of complementary event first. The probability of 2 persons to have different birthdays is $1\times\frac{364}{365}=\frac{364}{365}$

The probability of 3 persons to have different birthdays is $1\times\frac{364}{365}\times\frac{363}{365}=0.9918$

Continue in this way, we get a table:

n	chance	n	chance	n	chance
2	0.9973	10	0.8831	18	0.6531
3	0.9918	11	0.8589	19	0.6209
4	0.9836	12	0.8330	20	0.5886
5	0.9729	13	0.8056	21	0.5563
6	0.9595	14	0.7769	22	0.5243
7	0.9438	15	0.7471	23	0.4927
8	0.9257	16	0.7164		
9	0.9054	17	0.6850		

Therefore, for 23 persons, the probability of at least one duplication of birthday is $1 - 0.4927 = 0.5073$, just greater than $\frac{1}{2}$.

A neat illustration of the paradox is provided by the birth and death dates of the presidents of the United States. With a total of 33 presidents, the probability of coincidence in each case is close to 75 percent. Polk and Harding were born on November 2, and three presidents - Jefferson, Adams and Monroe - all died on July 4 .