第三十九屆香港數學競賽(2021/22)
比賽規則

1. 比賽包括兩個個人環節，分為兩張卷，卷一限時**六十分鐘**，卷二則限時**二十分鐘**。
2. 每間學校可提名**最多六位中五或以下**同學參賽。其中任何四位可作答卷一；又其中任何四位可作答卷二。不足四位同學的隊伍將被撤銷參賽資格。
3. 所有參賽學生**必須穿著整齊校服**，**並由負責教師帶領**。比賽將會準時於上午9時開始。
4. 比賽題目以中、英文並列。
5. 每一隊員於卷一中須解答15條問題（當中**甲部佔10題**、**乙部佔5題**）；而每一隊員則於卷二中須解答8條問題（當中**甲部佔4題**、**乙部佔4題**）。
6. 比賽時，**不准使用**計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，違例學生將被撤銷參賽資格或扣分。
7. **除非另有聲明**，**否則所有問題的答案均為數字**，**並應化至最簡**，**但無須呈交證明及算草**。
8. 參賽者須關掉所有電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，否則大會有權**取消該學生參賽資格**。
9. 卷一中，甲部和乙部的每一正確答案分別可得 1分及 2分。每校可得之最高積分為80分。
10. 卷二中，甲部和乙部的每一正確答案分別可得 2分及 3分。每校可得之最高積分為80分。
11. 比賽中，並不給予快捷分。
12. 參賽者必須自備工具，例如：**原子筆**及**鉛筆**。
13. 獎項：
    (a) 於每卷中，根據參賽者所得分數由高至低排列後
        (i) 取得滿分者將獲頒予最佳表現及積分獎狀；
        (ii) 除上述(i)中取得最佳表現的參賽者外，
            (1) 成績最佳的首 2% 參賽者將獲頒予一等榮譽獎狀；
            (2) 隨後的 5% 參賽者將獲頒予二等榮譽獎狀；
            (3) 隨後的 10% 參賽者將獲頒予三等榮譽獎狀。
    (b) 總成績（卷一及卷二的積分總和）於各分區（即港島、九龍東、九龍西、新界東及新界西）最高之首10%的參賽隊伍將獲頒予獎狀。
14. 如有任何疑問，參賽者須於比賽完畢後，立即透過負責教師致電 2153 7436 向籌委會的教育局代表鄭仕文先生提出。所提出之疑問，將由籌委會作最後裁決。

The Thirty-ninth Hong Kong Mathematics Olympiad (2021/22)
Regulations

1. The competition consists of two individual rounds, each with one paper: **60 minutes** for Paper 1 and **20 minutes** for Paper 2.
2. Each school may nominate **at most 6 participants** who are students of **Secondary 5 or below**.
3. All student participants, **accompanied by the teacher-in-charge, should wear proper school uniform**. The competition will commence at 9:00 a.m. sharp.
4. Question papers are printed in both Chinese and English.
5. Each participant has to solve 15 questions in Paper 1 (***10 questions in Part A*** and ***5 questions in Part B***), and 8 questions in Paper 2 (***4 questions in Part A*** and ***4 questions in Part B***).
6. During the competition, devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will **not** be allowed to use throughout the competition, otherwise the participant will be disqualified or risk deduction of marks.
7. **All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**
8. All electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) and any alarm device(s), should be turned off during the competition. Failing to do so, the participant **will risk disqualification**.
9. For Paper 1, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 80.
10. For Paper 2, 2 marks and 3 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 80.
11. No mark for speed will be awarded in the competition.
12. Participants should bring along their own instruments, e.g. **ball pens** and **pencils**.
13. Awards:
    - (a) For each of Paper1 and Paper 2, after arranging all participant's scores in the order from the highest to the lowest
        - (i) participants obtaining full score will be awarded Best Performance honour certificates;
        - (ii) after deducting those participants with full score achievement as in (i),
            - (1) the first 2% from the remaining particpants in the said order will be awarded First-class honour certificates ;
            - (2) the next 5% from the remaining participants will be awarded Second-class honour certificates ; and
            - (3) the next 10% from the remaining participants will be awarded Third-class honour certificates ;
    - (b) About 10% of participating schools with the highest aggregate scores (sum of the scores in Paper 1 and Paper 2) in each of the following regions, namely Hong Kong Island, Kowloon East, Kowloon West, the New Territories East, and the New Territories West, will be awarded certificates of merit.
14. Should there be any queries, participants should reach Mr CHENG Sze-man, the representative of EDB in the Organising Committee, via the teacher-in-charge at 2153 7436 immediately after the competition. The decision of the Organising Committee on the queries is final.

Hong Kong Mathematics Olympiad (2021/22)
Individual Paper 1
香港數學競賽 (2021/22)
個人項目卷一

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。Q1- Q10 每題 1 分，Q11-Q15 每題 2 分。全卷滿分 20 分。 時限：1 小時

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q10 1 mark each, Q11-Q15 2 marks each. The maximum mark for this paper is 20. Time allowed: 1 hour

1. α 及 β 是方程 $x^2 - 100x + k = 0$ 的實根。若 $\alpha - 7 = 30\beta$，求 k 的值。

   α and β are the real roots of the equation $x^2 - 100x + k = 0$. If $\alpha - 7 = 30\beta$, find the value of k.

2. 在圖一中，ACD 是一個三角形。B 是 CD 上的一點使 $AB = AC = 2$ 及 $AD = 4$。若 $BC : BD = 1 : 3$，求 CD 的長。

   In Figure 1, ACD is a triangle. B is a point on CD such that $AB = AC = 2$ and $AD = 4$. If $BC : BD = 1 : 3$, find the length of CD.

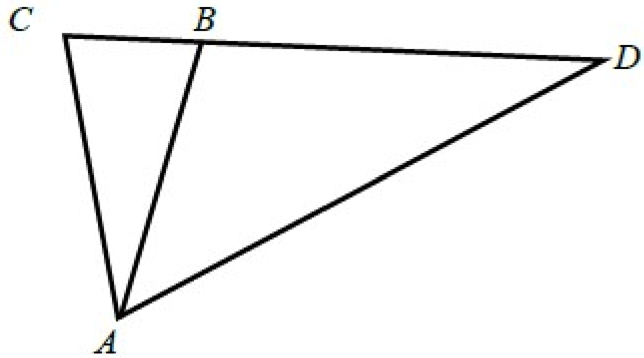

Figure 1 圖一

3. 在圖二中，$ABCD$ 是一個矩形。E 是 AC 上的一點使 $AE = 25$ 及 $CE = 144$。若 $p = AD + DE + CD$，求 p 的值。

   In Figure 2, $ABCD$ is a rectangle. E is a point on AC such that $AE = 25$ and $CE = 144$. If $p = AD + DE + CD$, find the value of p.

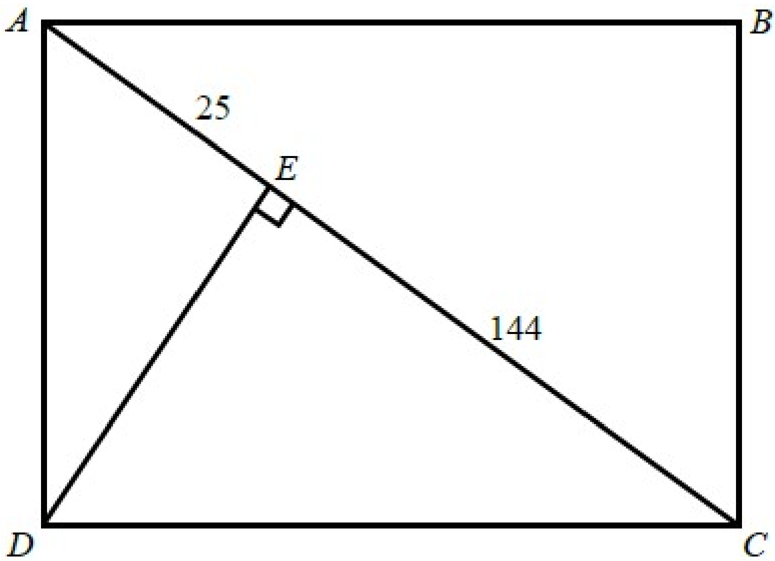

Figure 2 圖二

4. 設 x、y 及 z 是非零數。若 $2^x = 3^y = 18^z$，求 $\dfrac{xz}{5y(x-z)}$ 的值。

Let x, y and z are non-zero numbers. If $2^x = 3^y = 18^z$, find the value of $\dfrac{xz}{5y(x-z)}$.

5. 設 $N = 24x + 216y$，其中x及y均為正整數。若N為完全立方數，求$x + y$的最小值。
Let $N = 24x + 216y$, where both x and y are positive integers.
If N is a cube number, find the minimum value of $x + y$.

6. 小馬參加數學比賽，解其中一條題目
$$\begin{cases} ax + by = -16 \\ cx + 20y = -224 \end{cases}$$
，其中 a，b 及c 是實數。
題目的正確答案為 $x = 8$ 及 $y = -10$。
怎料小馬抄錯 c 的數值，得出答案 $x = 12$ 及 $y = -13$。求原題中 $a^2 + b^2 + c^2$ 的值。
John participated in a mathematics competition, in which one of the questions was to solve
$$\begin{cases} ax + by = -16 \\ cx + 20y = -224 \end{cases},$$
where a, b and c are real numbers.
The correct answer to the question was $x = 8$ and $y = -10$. However, John copied a wrong value for c and then gave an answer of $x = 12$ and $y = -13$. Find the value of $a^2 + b^2 + c^2$ in the original question.

7. 已知 $459 + x^3 = 3^y$，其中 x 及 y 均為正整數。求 y 的 最小值。
Given that $459 + x^3 = 3^y$, where both x and y are positive integers. Find the least value of y.

8. 在圖三中，D 為四邊形$ABCE$ 內的一點使得 $AD \parallel BC$，$AB \perp AD$，$CD \perp DE$，$CD = ED$，$AD = 4$ cm 及 $BC = 6$ cm。若 ΔADE 的面積為 P cm^2，求 P 的值。
In Figure 3, D is a point inside the quadrilateral $ABCE$ such that $AD \parallel BC$, $AB \perp AD$, $CD \perp DE$, $CD = ED$, $AD = 4$ cm and $BC = 6$ cm . If P cm^2 is the area of ΔADE, find the value of P .

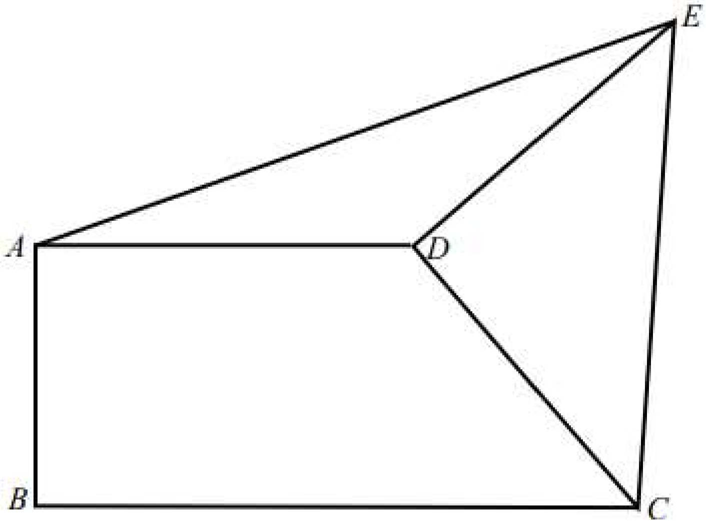

Figure 3 圖三

9. $ABCD$ 是一個圓內接四邊形，其中$AB = 7$, $BC = 15$, $CD = 20$ and $DA = 24$。
求圓$ABCD$ 的半徑。
$ABCD$ is a cyclic quadrilateral with $AB = 7$, $BC = 15$, $CD = 20$ and $DA = 24$.
Find the radius of the circle $ABCD$.

10. 已知 $a^2+\frac{1}{a^2}=7$，其中 $a>0$。若 $b=a^5+\frac{1}{a^5}$，求 b 的值。
Given that $a^2+\frac{1}{a^2}=7$, where $a>0$. If $b=a^5+\frac{1}{a^5}$, find the value of b .

11. x_1 及 x_2 是方程 $(\log 2x)(\log 3x) = a$ 的實根，其中 a 為實數。求 x_1x_2 的值。
x_1 and x_2 are the real roots of the equation $(\log 2x)(\log 3x) = a^2$, where a is a real number.
Find the value of x_1x_2 .

12. 由數字 0，1，2，3，4，5，6 組成一個沒有重複數字的 7 位數。若這個數可以被 55 整除，求這個數的最大值。
A 7-digit number is formed by putting the numerals 0, 1, 2, 3, 4, 5, 6 together without repetition. If this number is divisible by 55, find its largest possible value.

13. 已知 $a^{2x}-b^{2y}=1672$，其中 a、b、x 及 y 為正整數。求 $ax+by$ 的最小值。
Given that $a^{2x}-b^{2y}=1672$, where a , b , x and y are positive integers.
Find the minimum value of $ax+by$.

14. 設 a、 b 及 c 為非零數字。有多少個三位數 $\overline{abc}$ 使得 $\overline{ab}<\overline{bc}<\overline{ca}$？
Let a, b and c are non-zero digits. How many three digit numbers $\overline{abc}$ are there such that $\overline{ab}<\overline{bc}<\overline{ca}$?

15. PQR 是一個等腰三角形，其中 $PQ=PR=17$ and $QR=16$。將 I 及 H 分別記為 PQR 的內心及垂心。求 IH 長度的值。
PQR is an isosceles triangle with $PQ=PR=17$ and $QR=16$. Denote the in-centre and the orthocentre of PQR by I and H respectively. Find the length of HI .

*** 試卷完 **End of Paper** ***

Hong Kong Mathematics Olympiad 2020-2021
Individual Paper 2
香港數學競賽 2020 – 2021
個人項目卷二

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。Q1- Q4 每題 2 分，Q5-Q8 每題 3 分。全卷滿分 20 分。 時限：20 分鐘
Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q4 2 marks each, Q5-Q8 3 marks each. The maximum mark for this paper is 20. Time allowed: 20 minutes

1. 設 $\frac{A}{2022}=\frac{1}{1+1\times 2\times 3\times\cdots\times 2022}+\frac{1}{1+\frac{1}{1\times 2\times 3\times\cdots\times 2022}}$。求 A 的值。

Let $\frac{A}{2022}=\frac{1}{1+1\times 2\times 3\times\cdots\times 2022}+\frac{1}{1+\frac{1}{1\times 2\times 3\times\cdots\times 2022}}$. Find the value of A .

2. $\overline{AB}$ 和 $\overline{CB}$ 均為兩位正整數，其中 A、B 和 C 是不同的數字。設 $d=\overline{AB}+\overline{CB}$。

若 $\overline{AB}\times\overline{CB}=\overline{BCBB}$ 是四位數，求 d 的值。

Both $\overline{AB}$ and $\overline{CB}$ are two-digit positive integers, where A , B and C are different digits. Let $d=\overline{AB}+\overline{CB}$. If $\overline{AB}\times\overline{CB}=\overline{BCBB}$ is a four-digit number, find the value of d .

3. 假設方程 $x^2y-2x^2-3y-13=0$ 只有一對正整數解 (x_0, y_0)。若 $a=y_0-x_0$，求 a 的值。
Suppose the equation $x^2y - 2x^2 - 3y - 13 = 0$ has only one pair of positive integral solution (x_0 , y_0) . If $a=y_0-x_0$, find the value of a .

4. 圖一所示為一正方形。每一條邊的中點都連接對邊的兩端點，由此形成一個四角星(著色部分)。求 $\frac{\text{四角星的面積}}{\text{正方形的面積}}$ 的值。

Figure 1 shows a square. The midpoint of each side is joined to the two end points of the opposite side and a four-pointed star is thus formed (the shaded part).

Find the value of $\frac{\text{Area of the four point star}}{\text{Area of the square}}$.

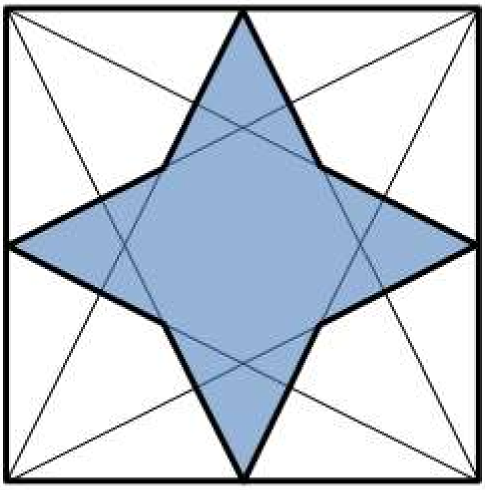

Figure 1 圖一

5. $VABC$ 為一個錐體，其中 $VA = VB = VC$ 及 $AB = BC = CA = a$ m。設它的高為 h m 及它的總表面積及體積相等。若 a 和 h 均為正整數，求 h 的可能值之和。
$VABC$ is a right pyramid with $VA = VB = VC$ and $AB = BC = CA = a$ m . Let its height be h m and its total surface area and volume are the same. If a and h are both positive integers, find the sum of all possible values of h .

6. 圖二中，$ABCD$ 是平行四邊形。E 為 BC 的中點，AE 和 BD 相交於 H，AC 和 DE 相交於 F，AC 和 BD 相交於 G。若四邊形 $EFGH$ 的面積及$ABCD$ 的面積分別為 10 cm^2 及 $k\text{ cm}^2$，求 k 的值。
In Figure 2, $ABCD$ is a parallelogram. E is the midpoint of BC, AE and BD intersect at H , AC and DE intersect at F , AC and BD intersect at G . If the area of the quadrilateral $EFGH$ and $ABCD$ are 10 cm^2 and $k\text{ cm}^2$ respectively, find the value of k .

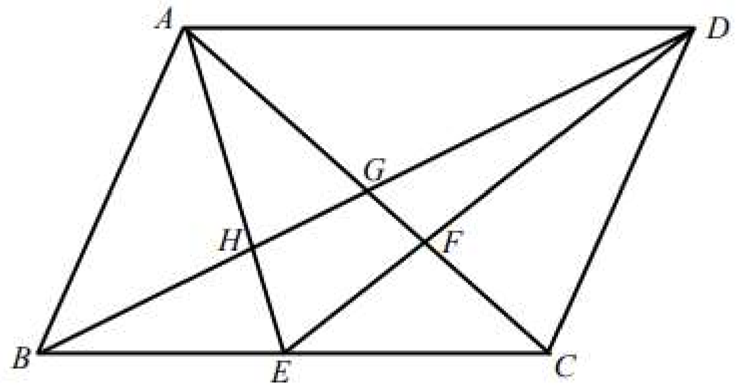

Figure 2 圖二

7. 已知 $x + y + z = 1$，$x^2 + y^2 + z^2 = 2$ 及 $x^3 + y^3 + z^2 = 3$。求 $x^4 + y^4 + z^4$ 的值。
Given that $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$ and $x^3 + y^3 + z^2 = 3$. Find the value of $x^4 + y^4 + z^4$.

8. 對所有正整數 $n > 1$，函數 f 定義如下：

$\mathrm{f}(1) = 2021$ 及 $\mathrm{f}(1) + \mathrm{f}(2) + \mathrm{f}(3) + \cdots + \mathrm{f}(n) = n^2\,\mathrm{f}(n)$。

求 $\mathrm{f}(2021)$ 的值。
For all positive integers $n > 1$, a function f is defined as

$\mathrm{f}(1) = 2021$ and $\mathrm{f}(1) + \mathrm{f}(2) + \mathrm{f}(3) + \cdots + \mathrm{f}(n) = n^2\,\mathrm{f}(n)$.

Find the value of $\mathrm{f}(2021)$.

***** 試卷完 End of Paper *****