Isosceles trapezium

Given a trapezium *ABCD* with *AB* = *DC*, *AD* // *BC* and *AB* is not parallel to *DC*. Prove that

(a)   ∠*ABC* = ∠*DCB*

(b)   ∠*BAD* = ∠*CDA*

(c)   *AC* = *BD*
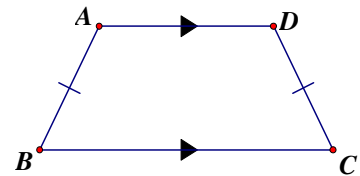
(a)   Draw *AE* // *DC*, cutting *BC* at *E*.

ADCE is a //-gram.	(2 pairs of // lines)
AE = *DC*	(opp. sides of //-gram)
AB = *DC*	(given)
∴ *AB* = *AE*	
Δ*ABE* is isosceles	(2 sides equal)
∠*ABC* = ∠*AEB*	(base ∠s isos. Δ)
∠*AEB* = ∠*DCE*	(corr. ∠s *AE* // *DC*)
∴ ∠*ABC* = ∠*DCB*	

(b)

∠*BAD* = 180° − ∠*ABC*	(int. ∠s *AD* // *BC*)
= 180° − ∠*DCB*	(by (a))
= ∠*CDA*	(int. ∠s *AD* // *BC*)

(c)   Join *AC* and *BD*.

AB = *DC*	(given)
∠*ABC* = ∠*DCB*	(by (a))
BC = *CB*	(common side)
∴ Δ*ABC* ≅ Δ*DCB*	(S.A.S.)
AC = *BD*	(corr. sides ≅ Δs)