

1985 FG7.2

正整數 N 分別被 6、5、4、3 及 2 除時，其餘數依次為 5、4、3、2 及 1。求 N 之最小值。

When the positive integer N is divided by 6, 5, 4, 3 and 2, the remainders are 5, 4, 3, 2 and 1 respectively. Find the least value of N .

1985 FG8.1-2

M 、 N 均為小於 10 之正整數，且 $258024M8 \times 9 = 2111110N \times 11$ 。

求 M 及 N 的值。

M, N are positive integers less than 10 and $258024M8 \times 9 = 2111110N \times 11$. Find the value of M and N .

1988 FG7.1-2

M 、 N 是小於 10 的正整數，且 $8M420852 \times 9 = N9889788 \times 11$ 。

求 M 及 N 的值。

M, N are positive integers less than 10 and $8M420852 \times 9 = N9889788 \times 11$. Find the value of M and N .

1990 HI13 2013FG4.3

正整數 N 被 10、9、8、7、6、5、4、3 及 2 除所得的餘數依次是 9、8、7、6、5、4、3、2 及 1，求 N 的最小值。

A positive integer N , when divided by 10, 9, 8, 7, 6, 5, 4, 3 and 2, leaves remainders 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively. Find the least value of N .

1990 FG7.3-4

A 、 B 均為小於 10 的正整數，且 $21A104 \times 11 = 2B8016 \times 9$ 。求 A 及 B 的值。

A, B are positive integers less than 10 such that $21A104 \times 11 = 2B8016 \times 9$.

Find the value of A and B .

1995 FG6.1-2

$2^a \cdot 9^b$ 為一四位數，其千位數是 2，百位數是 a ，十位數是 9，個位數是 b ，求 a 及 b 的值。

$2^a \cdot 9^b$ is a four digit number and its thousands digit is 2, its hundreds digit is a , its tens digit is 9 and its units digit is b , find the value of a and b .

1996 FG6.4

若 r, s, t, u 是正整數及 $r^5 = s^4, t^3 = u^2, t - r = 19$ 及 $d = u - s$ ，求 d 的值。
If r, s, t, u are positive integers and $r^5 = s^4, t^3 = u^2, t - r = 19$ and $d = u - s$, find the value of d .

1998 HG4

給出整數 a, b, c ，使得 $a^2 = b^3 = c$ 。若 $c > 1$ ，求 c 的最小值。

Let a, b, c be integers such that $a^2 = b^3 = c$. If $c > 1$, find the smallest value of c .

1999 FI4.1

李先生今年 a 歲， $a < 100$ 。若把李先生的出生月份與 a 相乘，其結果是 253。求 a 的值。

Mr. Lee is a years old, $a < 100$.

If the product of a and his month of birth is 253, find the value of a .

1999 FIS.3

W 和 F 為兩大於 20 的整數。

若 W 與 F 之積為 770， W 與 F 之和為 c ，求 c 之值。

W and F are two integers which are greater than 20. If the product of W and F is 770 and the sum of W and F is c , find the value of c .

1999 FG3.1

設小於 100 的正整數，同時又是完全平方及完全立方的數目共有 a 個，求 a 之值。

Let a be the number of positive integers less than 100 such that they are both square and cubic numbers, find the value of a .

2000 FG4.1

如果 a 是一個三位數，駁在 504 之後，新組成的六位數可被 7、9、11 整除，求 a 的值。

Let a be a 3-digit number. If the 6-digit number formed by putting a at the end of the number 504 is divisible by 7, 9, and 11, find the value of a .

2000 FG5.1

如果 a 是可被 810 整除的最小立方數，求 a 的值。

If a is the smallest cubic number divisible by 810, find the value of a .

2001 FI2.3

某班學生的人數少於 56 人。在一次數學測驗中有 $\frac{1}{3}$ 學生得甲等， $\frac{1}{7}$ 學生得乙等，一半學生得丙等，餘下的學生都不及格。已知不及格的學生人數是 R ，求 R 的值。

There are less than 56 students in a class. In a mathematics test, $\frac{1}{3}$ of the students got grade A, $\frac{1}{7}$ of the students got grade B, half of the students got grade C, and the rest failed. Given that R students failed in the mathematics test, find the value of R .

2002 HI2

已知 $a^3 = 150b$ ，且 a 和 b 都是正整數。求 b 的最小值。

Given $a^3 = 150b$ and a, b are positive integers, find the least value of b .

2002 HG2

已知 a, b, c 為三個連續奇數且 $b^3 = 3375$ ，求 ac 的數值。

Given that a, b, c are three consecutive odd numbers and $b^3 = 3375$, find the value of ac .

2003 HI9

已知六位數 $N = \overline{x1527y}$ 是 4 的倍數，且 N 被 11 除餘 5。求 $x + y$ 的值。

It is given that the 6-digit number $N = \overline{x1527y}$ is a multiple of 4, and the remainder is 5 when N is divided by 11. Find the value of $x + y$.

2004 FI1.1

已知有 a 個少於 200 的正整數，它們每個都只有三個正因數，求 a 的值。

Given that there are a positive integers less than 200 and each of them has exactly three positive factors, find the value of a .

2005 FG1.1

若在 1 至 200 內能同時被 3 和 7 整除的數有 a 個，求 a 的值。

Suppose there are a numbers between 1 and 200 that can be divisible by 3 and 7, find the value of a .

2006 HI7

已知在數列 $1001, 1001001, 1001001001, \dots, 1001001\dots1001, \dots$ 中有 R 個質數，

求 R 的值。

Given that the number of prime numbers in the sequence $1001, 1001001, 1001001001, \dots, 1001001\dots1001, \dots$ is R , find the value of R .

2007 HG1

若由 1 至 50 內與 50 互質的整數有 N 個，求 N 的值。

If there are N integers from 1 to 50 that are relatively prime to 50, find the value of N .

2007 FI3.3

若有 c 個正整數 n 使得 $\frac{n+17}{n-7}$ 也是正整數，求 c 的值。

If there is (are) c positive integer(s) n such that $\frac{n+17}{n-7}$ is also a positive integer, find the value of c .

2008 HG2

有一批糖共 x 粒， x 為正整數，這批糖能分別為 851 人及 943 人所均分。求 x 的最小可能值。

A total number of x candies, x is a positive integer, can be evenly distributed to 851 people as well as 943 people. Find the least possible value of x .

2008 HG4

已知 x 為正整數及 $x < 60$ 。若 x 恰有 10 個正因子，求 x 的值。

Given that x is a positive integer and $x < 60$. If x has exactly 10 positive factors, find the value of x .

2008 FIS.1

若 28 的所有正因子是 d_1, d_2, \dots, d_n 及 $a = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}$ ，求 a 的值。

If all the positive factors of 28 are d_1, d_2, \dots, d_n and $a = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}$,

find the value of a .

2010 HG1

已知六位數 $503xyz$ 可以被 7, 9, 11 整除。求三位數 xyz 的最小值。

Given that the six-digit number $503xyz$ is divisible by 7, 9, 11.

Find the minimum value of the three-digit number xyz .

2010 FI3.1

若 a 為 15147 的相異質因數的數目。求 a 的值。

If a is the number of distinct prime factors of 15147, find the value of a .

2011 HG1

若 $(1000 - a)(1000 - b)(1000 - c)(1000 - d)(1000 - e) = 24^2$ ，其中 a, b, c, d 及 e 為偶數，且 $a > b > c > d > e$ ，求 a, b, c, d 及 e 的值。

If $(1000 - a)(1000 - b)(1000 - c)(1000 - d)(1000 - e) = 24^2$, where a, b, c, d and e are even numbers and $a > b > c > d > e$, find the values of a, b, c, d and e .

2011 FG3.1

若 a 為一正整數及 $a^2 + 100a$ 為一質數，求 a 的最大值。

If a is a positive integer and $a^2 + 100a$ is a prime number, find the maximum value of a .

2011 FG3.4

把數字 1, 2, ..., 10 分成兩組並設 P_1 及 P_2 分別為該兩組的乘積。

若 P_1 為 P_2 的倍數，求 $\frac{P_1}{P_2}$ 的最小值。

Spilt the numbers 1, 2, ..., 10 into two groups and let P_1 be the product of the first group and P_2 the product of the second group.

If P_1 is a multiple of P_2 , find the minimum value of $\frac{P_1}{P_2}$.

2013 HI2

一個平行四邊形可被分成 178 個邊長為 1 單位的等邊三角形，若該平行四邊形的周界為 P 單位，求 P 的最大值。

A parallelogram is cut into 178 pieces of equilateral triangles with sides 1 unit. If the perimeter of the parallelogram is P units, find the maximum value of P .

2013 FG2.1

若 $4^3 + 4^r + 4^4$ 是一平方數，其中 r 是正整數，求 r 的最小值。

If $4^3 + 4^r + 4^4$ is a perfect square and r is a positive integer, find the minimum value of r .

2014 HI6

設 n 為正整數，且 $n < 1000$ 。若 $(n-1)^2$ 整除 $(n^{2014}-1)$ ，求 n 的最大值。

Let n be a positive integer and $n < 1000$.

If $(n^{2014} - 1)$ is divisible by $(n - 1)^2$, find the maximum value of n .

2014 HI8

設 $\overline{xy} = 10x + y$ 。若 $\overline{xy} + \overline{yx}$ 為一個平方數，這樣的數有多少個？

Let $\overline{xy} = 10x + y$.

If $\overline{xy} + \overline{yx}$ is a square number, how many numbers of this kind exist?

2015 HI3

設正 n 邊形的內角為 x° ，其中 x 為整數。問 n 有多少個可能值？

Let x° be the measure of an interior angle of an n -sided regular polygon, where x is an integer, how many possible values of n are there?

2016 HI13

設 A 、 B 和 C 為三個數字。利用這三個數字組成的三位數有以下性質：

- (a) ACB 可以被 3 整除；
 - (b) BAC 可以被 4 整除；
 - (c) BCA 可以被 5 整除；及
 - (d) CBA 的因數數目為單數。
- 求三位數 ABC 。

Let A , B and C be three digits. The number formed by these three digits has the following properties:

- (a) ACB is divisible by 3;
- (b) BAC is divisible by 4;
- (c) BCA is divisible by 5;
- (d) CBA has an odd number of factors.

Find the 3-digit number ABC .

2017 HI7

設 N 為完全立方數，已知 $N = 161x + 23y$ ，其中 x 和 y 均為正整數。求 $x + y$ 的最小值。

Let N be a perfect cube number. Given that $N = 161x + 23y$, where x and y are positive integers. Find the minimum value of $x + y$.

2018 HI15

設 $N^2 = \overline{abcdefabc}$ 為一個 9 位整數，其中 N 是 4 個相異質數的積及 a 、 b 、 c 、 d 、 e 、 f 均為非零數字且滿足 $\overline{def} = 2 \times \overline{abc}$ 。求 N^2 的最小值。

Let $N^2 = \overline{abcdefabc}$ be a nine-digit positive integer, where N is the product of four distinct primes and a, b, c, d, e, f are non-zero digits that satisfy $\overline{def} = 2 \times \overline{abc}$. Find the least value of N^2 .

2018 FI2.4

求整數 d ，使得對於任何實數 x ， $x^{13} + x + 90$ 可被 $x^2 - x + d$ 整除。

Determine the integral value of d , so that $x^{13} + x + 90$ is divisible by $x^2 - x + d$ for any real number x .

2018 FI4.2

若 x 、 y 為正整數及 b 為 x 、 y 組合的數量使得它們的乘積 $x \times y = 100$ ，求 b 的值。

If x, y are positive integers numbers and b is the number of groups of x, y such that the product $x \times y = 100$, determine the value of b .

2019 HI13

已知正整數 a 、 b 及 c 滿足下列條件：

- (i) $a > b > c$,
- (ii) $(a - b)(b - c)(a - c) = 84$,
- (iii) $abc < 100$.

設 M 為 a 的最大值。求 M 的值。

Given that a, b and c are positive integers satisfying the following conditions:

- (i) $a > b > c$,
- (ii) $(a - b)(b - c)(a - c) = 84$,
- (iii) $abc < 100$.

Let M be the maximum value of a . Find the value of M .

2021 P2Q4

已知 a 、 b 、 c 、 d 及 e 是連續正整數，其中 $a < b < c < d < e$ 。若 $a + b + c + d + e$ 是一個立方數及 $b + c + d$ 是一個平方數，求 c 的最小可能值。

Given that a, b, c, d and e are consecutive positive integers, where $a < b < c < d < e$. If $a + b + c + d + e$ is a perfect cube and $b + c + d$ is a perfect square, find the smallest possible value of c .

2022 P1Q5

設 $N = 24x + 216y$ ，其中 x 及 y 均為正整數。

若 N 為完全立方數，求 $x + y$ 的最小值。

Let $N = 24x + 216y$, where both x and y are positive integers.

If N is a cube number, find the minimum value of $x + y$.

2023 HI11

已知 n 是一個少於 2023 正整數。

若 n 只有三個不同的因數，求 n 的可能性的總數。

Given that n is a positive integer less than 2023.

If n has only 3 distinct factors, find the number of possible values of n .

Answers

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|--------------------------------|--------------------------------|--|----------------------------|--------------------------------|
| 1985 FG7.2 59 | 1985 FG8.1-2 $M = 5, N = 2$ | 1988 FG7.1-2 $M = 5, N = 6$ | 1990HI13 2013FG4.3 2519 | 1990 FG7.3-4 $A = 1, B = 5$ |
| 1995 FG6.1-2 $a = 5, b = 2$ | 1996 FG6.4 757 | 1998 HG4 64 | 1999 FI4.1 23 | 1999 FIS.3 57 |
| 1999 FG3.1 2 | 2000 FG4.1 504 | 2000 FG5.1 729000 | 2001 FI2.3 1 | 2002 HI2 180 |
| 2002 HG2 221 | 2003 HI9 9 | 2004 FI1.1 6 | 2005 FG1.1 9 | 2006 HI7 0 |
| 2007 HG1 20 | 2007 FI3.3 8 | 2008 HG2 34891 | 2008 HG4 48 | 2008 FIS.1 2 |
| 2010 HG1 118 | 2010 FI3.1 3 | 2011 HG1 $a = 1006, b = 1002,$ $c = 998, d = 996, e = 994$ | 2011 FG3.1 1 | 2011 FG3.4 7 |
| 2013 HI2 180 | 2013 FG2.1 1 | 2014 HI6 107 | 2014 HI8 8 | 2015 HI3 22 |
| 2016 HI13 522 | 2017 HI7 79 | 2018 HI15 289578289 | 2018 FI2.4 2 | 2018 FI4.2 9 |
| 2019 HI13 9 | 2021 P2Q4 75 | 2022 P1Q5 16 | 2023 HI11 14 | |