Hong Kong Mathematics Olympiad (1994 – 95) Heat Event (Individual)

除非特別聲明,答案須用數字表達,並化至最簡。

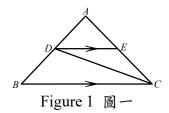
時限:40分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form. 每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

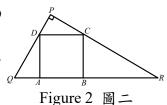
- 1. 求 1234567654321 的平方根。 Find the positive square root of 1234567654321.
- 2. 己知 $f\left(\frac{1}{x}\right) = \frac{x}{1-x^2}$,求 f(2) 的值。

 Given that $f\left(\frac{1}{x}\right) = \frac{x}{1-x^2}$, find the value of f(2).
- 3. 解 $3^{2x} + 9 = 10(3^x)$ 。 Solve $3^{2x} + 9 = 10(3^x)$.
- 4. 隨意抽出一個三位數。求這個數是一完全平方數的機會。
 A three-digit number is selected at random. Find the probability that the number selected is a perfect square.
- 5. 已知 $\sin x + \cos x = \frac{1}{5}$,且 $0 \le x \le \pi$,求 $\tan x$ 的值。

 Given that $\sin x + \cos x = \frac{1}{5}$ and $0 \le x \le \pi$, find the value of $\tan x$.
- 6. 有多少對正整數 $x \cdot y$ 可满足 xy 3x 2y = 10? How many pairs of positive integers x, y are three satisfying xy - 3x - 2y = 10?
- 7. $x \cdot y$ 為正整數,且 3x + 5y = 123。求 |x y| 的最小值。 x, y are positive integers and 3x + 5y = 123. Find the least value of |x y|.
- 8. 求 1997⁹¹³ 被 10 除時所得的餘數。 Find the remainder when 1997⁹¹³ is divided by 10.
- 9. 如圖一,若 BC = 3DE,求 r 的值,其中 $r = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDC}$ 。
 In figure 1, if BC = 3DE, find the value of r,
 where $r = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BCD}$.



10. 如圖二, $A \cdot B \cdot C \cdot D$ 為直角三角形 PQR 各邊上的點。若 ABCD 為一正方形,且 QA = 8 及 BR = 18,求 AB 的值。 A, B, C, D are points on the sides of the right-angled triangle PQR as shown in figure 2. If ABCD is a square, QA = 8 and BR = 18, find the value of AB.



P.1

*** 試卷完 End of Paper ***

Hong Kong Mathematics Olympiad (1994 – 95) Heat Event (Group)

除非特別聲明,答案須用數字表達,並化至最簡。

時限:20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form. 每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

- 1. 求方程 $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$ 的正整數解數目。 Find the number of positive integral solutions of the equation $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$.
- 2. 如圖一,四邊形 ABCD 的對角綫交於 O。 若 $\angle AOB = 30^\circ$ 、AC = 24 及 BD = 22, 求四邊形 ABCD 的面積。

In figure 1, ABCD is a quadrilateral whose diagonals intersect at O. If $\angle AOB = 30^{\circ}$, AC = 24 and BD = 22, find the area of the quadrilateral ABCD.

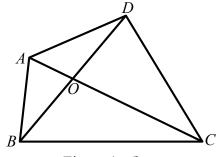


Figure 1 圖一

3. 已知
$$\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n-1}{n} = \frac{n-1}{2}$$
,求 $\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{10} + \dots + \frac{9}{10}\right)$ 的值。

Given that $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n-1}{n} = \frac{n-1}{2}$, find the value of
$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{10} + \dots + \frac{9}{10}\right).$$

4. 若
$$x$$
 及 y 為正整數,且 $x^2 = y^2 + 2000$,求 x 的最小值。

Suppose x and y are positive integers such that $x^2 = y^2 + 2000$, find the least value of x.

5. 已知
$$37^{100}$$
 為一 157 位數,且 37^{15} 為一 n 位數,求 n 的值。

Given that 37^{100} is a 157-digit number, and 37^{15} is an *n*-digit number. Find the value of *n* .

6. 已知
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1)$$
,求 $19 \times 21 + 18 \times 22 + 17 \times 23 + \dots + 1 \times 39$ 的值。

Given that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1)$,

find the value of $19 \times 21 + 18 \times 22 + 17 \times 23 + \dots + 1 \times 39$.

7. 在圖中, ABCD 為一正方形,且 AB=1 及 CPQ 為一等邊三 D 角形。求 ΔCPQ 的面積。

In figure 2, ABCD is a square where AB = 1 and CPQ is an equilateral P triangle. Find the area of ΔCPQ .

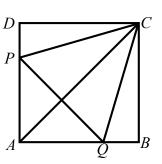


Figure 2 圖二

8. 用一元、二元及五元硬幣,以湊合十七元,且每次均須使用各種硬幣,其方法有 n 種,求 n 的值。

The number of ways to pay a sum of \$17 by using \$1 coins, \$2 coins and \$5 coins is n. Find the value of n. (Assume that all types of coins must be used each time.)

9. 圖三是一個 3×3 的正方形,求圖中三角形的總數。 In figure 3, find the total number of triangles in the 3×3 square.

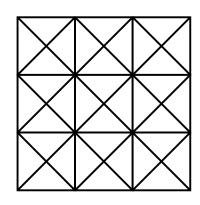


Figure 3 圖三

10. 在圖四中,象限的半徑和大半圓的直徑是 2,求小半圓的半徑。 In figure 4, the radius of the quadrant and the diameter of the large semi-circle is 2. Find the radius of the small semi-circle.

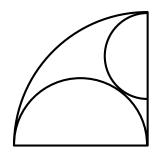


Figure 4 圖四