<u>第三十八屆香港數學競賽**(2020/21)**</u>
<u>比賽規則</u>

1. 比賽包括兩個個人環節，分為兩張卷，卷一限時**<u>六十分鐘</u>**，卷二則限時**<u>五十分鐘</u>**。
2. 每校可提名**<u>最多四位中五或以下</u>**同學參賽。
3. 所有參賽學生**<u>必須穿著整齊校服</u>**，**<u>並由負責教師帶領</u>**，於上午8時45分或以前向會場接待處註冊，同時必須出示**<u>身分證/學生證明文件</u>**，否則該學生參賽資格**<u>將被取消</u>**。
4. 指示語言將採用粵語。若參賽者不諳粵語，則可獲發一份中英對照的指示列印本。比賽題目則中、英文並列。
5. 每一參賽學生於卷一中須解答15條問題（當中甲**<u>部佔 10 題</u>**、**<u>乙部佔 5 題</u>**），並於卷二中須解答 7 條問題（當中**<u>甲部佔 4 題</u>**、**<u>乙部佔 3 題</u>**）。
6. 比賽時，**<u>不准使用</u>**計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，違例學生將被撤銷參賽資格或扣分。
7. **<u>除非另有聲明</u>**，**<u>否則所有問題的答案均為數字</u>**，**<u>並應化至最簡</u>**，**<u>但無須呈交證明及算草</u>**。
8. 參賽者如有攜帶電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，應把它關掉，並放入手提包內或座位的椅下，否則大會有權**<u>取消該學生參賽資格</u>**。
9. 卷一中，甲部和乙部的每一正確答案分別可得 1分及 2分。每校可得之最高積分為80分。
10. 卷二中，甲部和乙部的每一正確答案分別可得 2分及 4分。每校可得之最高積分為80分。
11. 比賽中，並不給予快捷分。
12. 參賽者必須自備工具，例如：**<u>原子筆</u>**及**<u>鉛筆</u>**。
13. 獎項：
    (a) 於每卷中，
        (i) 取得滿分的參賽者將獲頒予最佳表現榮譽獎狀；
        (ii) 對於未能取得滿分的參賽者，
            (1) 成績最佳的首 2% 參賽者將獲頒予一等榮譽獎狀；
            (2) 隨後的 5% 參賽者將獲頒予二等榮譽獎狀；
            (3) 緊接著的 10% 參賽者將獲頒予三等榮譽獎狀。
    (b) 總分最高的首 3 所學校將獲頒予總冠軍、亞軍及季軍獎盃及獎狀，隨後的 47 所學校將獲頒予表現優異獎狀。
    (c) 於各比賽場地中，總分最高的首 3 所學校將獲頒予獎狀。
14. 第三十八屆香港數學競賽籌備委員會保留有關比賽結果和獎項的最終裁決權。
15. 如有任何疑問，參賽者須於比賽完畢後，立即向會場主任提出。所提出之疑問，將由籌委會作最後裁決。
16. 比賽結果將於 2021 年 5 月 31 日(星期一)或以前透過教育局數學教育組網站公佈，並於 2021 年 6 月以書面形式通知各得獎者及得獎學校。

The Thirty-eighth Hong Kong Mathematics Olympiad (2020/21)
Regulations

1. The competition consists of two individual rounds, each with one paper: **60 minutes** for Paper 1 and **50 minutes** for Paper 2.
2. Each school may nominate **at most 4 participants** who are students of **Secondary 5 or below**.
3. All student participants, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception not later than 8:45 a.m. Failing to do so, **the participant will risk disqualification**.
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, a printed copy of instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.
5. Each participant has to solve 15 questions in Paper 1 (***10 questions in Part A*** and ***5 questions in Part B***), and 7 questions in Paper 2 (***4 questions in Part A*** and ***3 questions in Part B***).
6. Please note that devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers **will not be allowed** to use throughout the competition, otherwise the participant will be disqualified or risk deduction of marks.
7. **All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**
8. Participants having electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) or any alarm devices, should turn them off and put them inside their bags or under their chairs. Failing to do so, the participant **will risk disqualification**.
9. For Paper 1, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The maximum score for a school should be 80.
10. For Paper 2, 2 marks and 4 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school should be 80.
11. No mark for speed will be awarded in the competition.
12. Participants should bring along their own instruments, e.g. **ball pens** and **pencils**.
13. Awards:
    (a) For each of Paper1 and Paper 2,
        (i) participants obtaining full score will be awarded the Best Performance honour certificates;
        (ii) for participants not attaining the full score,
            (1) the top 2% will be awarded the First-class honour certificates ;
            (2) the next 5% will be awarded the Second-class honour certificates ; and
            (3) the next 10% will be awarded the Third-class honour certificates.
    (b) For the aggregated score, schools obtaining the top 3 highest marks will be awarded the Overall Champion, First-Runner Up and Second-Runner Up cups and certificates, the next 47 schools will be awarded Good Performance certificates.
    (c) The 3 participating schools with the highest aggregate scores in each centre will be awarded certificates of merit.
14. The decision of the Organising Committee of the 38th Hong Kong Mathematics Olympiad on the results and awards of the competition is final.
15. Should there be any queries, participants should reach the Centre Supervisor immediately after the competition. The decision of the Organising Committee on the queries is final.
16. The competition results will be announced through the website of the Mathematics Education Section, EDB on or before 31 May 2021 (Monday). A separate letter will be sent to all winners and winning schools in June 2021.

Hong Kong Mathematics Olympiad 2020-2021
Individual Paper 1
香港數學競賽 2020 – 2021
個人項目卷一

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。Q1- Q10 每題 1 分，Q11-Q15 每題 2 分。全卷滿分 20 分。　　時限：1 小時

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q10 1 mark each, Q11-Q15 2 marks each. The maximum mark for this paper is 20.　　Time allowed: 1 hour

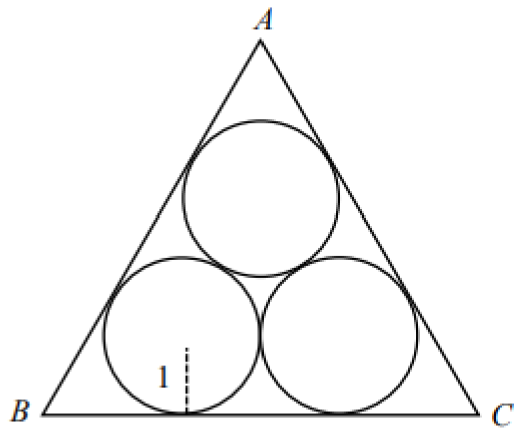
1. 已知 $W = a^b - b^a$，其中 $a \neq b \neq 0$。若 W 為一非負整數，求 W 的最小值。
Given that $W = a^b - b^a$, where $a \neq b \neq 0$. If W is a non-negative integer, find the least value of W.

2. 求 2^{2021} 的最尾兩位數字。
Find the last two digits of 2^{2021}.

3. α 及 β 為方程 $x^2 - 7x + 4 = 0$ 的根。求 $\alpha^3 + \beta^3$ 的值。
α and β are the roots of the equation $x^2 - 7x + 4 = 0$. Find the value of $\alpha^3 + \beta^3$.

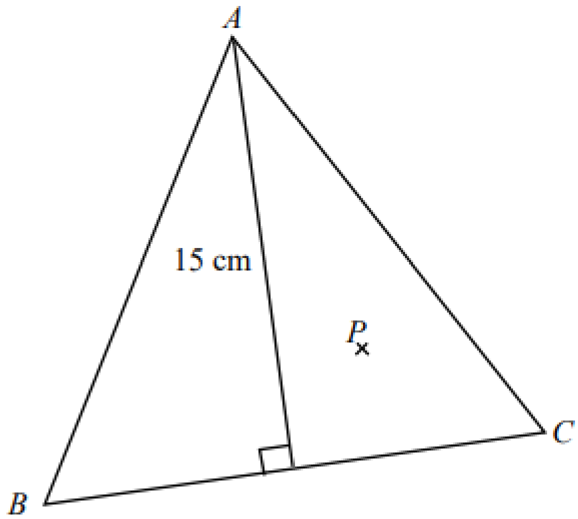
4. 求 $8\cos^2 15° \cos^2 30° - 8\sin^2 15° \cos^2 30°$ 的值。
Find the value of $8\cos^2 15° \cos^2 30° - 8\sin^2 15° \cos^2 30°$.

5. 在圖一中，三個單位圓位於一等邊三角形 ABC 內，使得每個圓均與另外兩圓及三角形的兩邊相 切。求 ΔABC 的面積。
In Figure 1, three unit circles are placed inside an equilateral triangle ABC such that any circle is tangential to two sides of the triangle and to the other two circles. Find the area of ΔABC.



圖一 Figure 1

6. 在圖二中，等邊三角形 ABC 的高為 15 cm。P 為 ΔABC 內的一點。從 P 與 AB、BC 和 AC 的垂直距離分別為 h cm、4 cm 和 5 cm。求 h 的值。
In Figure 2, the altitude of an equilateral triangle ABC is 15 cm. P is a point inside ΔABC. The perpendicular distances from P to AB, BC and AC are h cm, 4 cm and 5 cm respectively. Find the value of h.



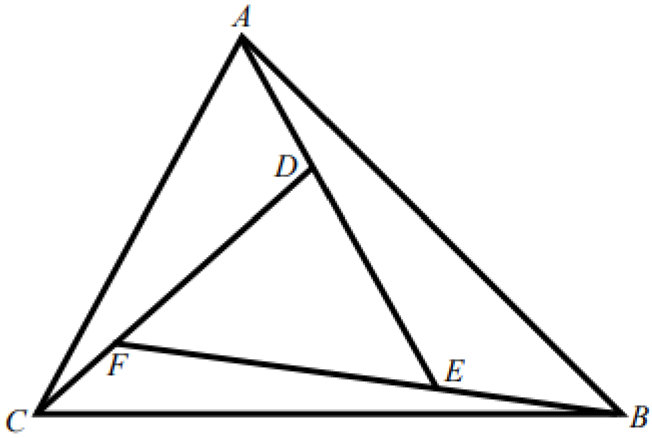
圖二 Figure 2

7. p、q 及 r 為質數。若 $pqr = 7(p + q + r)$，求 $p + q + r$ 的值。
p, q and r are prime numbers. If $pqr = 7(p + q + r)$, find the value of $p + q + r$.

8. 求 $\dfrac{1001\times1002}{\dfrac{1}{1+\dfrac{1}{1002}}+\dfrac{2}{2+\dfrac{2}{1002}}+\dfrac{3}{3+\dfrac{3}{1002}}+\cdots+\dfrac{1001}{1001+\dfrac{1001}{1002}}}$ 的值。
Find the value of $\dfrac{1001\times1002}{\dfrac{1}{1+\dfrac{1}{1002}}+\dfrac{2}{2+\dfrac{2}{1002}}+\dfrac{3}{3+\dfrac{3}{1002}}+\cdots+\dfrac{1001}{1001+\dfrac{1001}{1002}}}$.

9. 在 4000 和 7000 之間 4 個數位各不相同的偶數有多少個？
How many even numbers between 4000 and 7000 have four different digits?

10. 在圖三中，BEF、ADE 及 CFD 是直線，使得 $BE : EF = 1 : 2$，$AD : DE = 1 : 3$ 及 $CF : FD = 1 : 4$。若 ΔDEF 的面積是 24 平方單位，求 ΔABC 的面積。
In Figure 3, BEF, ADE and CFD are straight lines such that $BE : EF = 1 : 2$, $AD : DE = 1 : 3$ and $CF : FD = 1 : 4$. If the area of ΔDEF is 24 square unit, find the area of ΔABC.

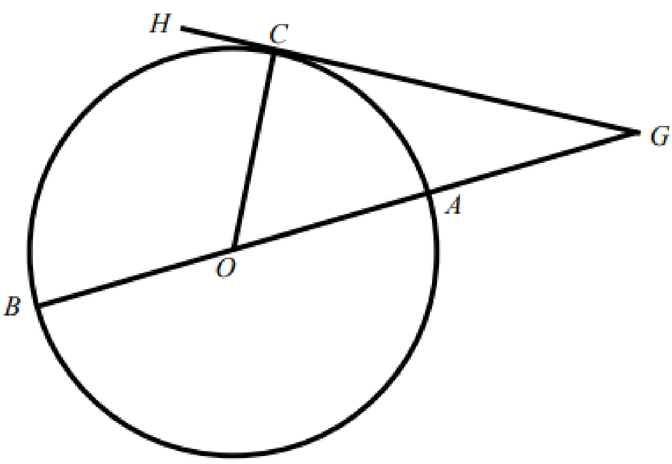


圖三 Figure 3

11. 若 $\log_9 x^{18} = (\log_3 x)^3$，求 x 的最小值。
If $\log_9 x^{18} = (\log_3 x)^3$, find the least value of x.

12. 設 $f(x) = \sqrt{(x-3)^2 + x^2} + \sqrt{(x-6)^2 + (x+5)^2}$，其中 x 為一實數。 求 $f(x)$ 的最小值。
Let $f(x) = \sqrt{(x-3)^2 + x^2} + \sqrt{(x-6)^2 + (x+5)^2}$, where x is a real number. Find the minimum value of $f(x)$.

13. 在圖四中，O 是圓的圓心。直徑 BA 延長至點 G 使得 GH 切圓於 C 點。若 $OA = 5$ 及 $GC = 12$，求 BC 的長度。
In Figure 4, O is the centre of the circle. The diameter BA is produced to a point G such that GH is a tangent to the circle at C. If $OA = 5$ and $GC = 12$, find the length of BC.



圖四 Figure 4

14. 對任意實數 x，函數 $f(x)$有以下性質 $f(x) + f(x - 1) = x^2$。若 $f(19) = 94$，求 $f(94)$的值。
For each real number x, the function $f(x)$ has the following property $f(x) + f(x - 1) = x^2$.
If $f(19) = 94$, find the value of $f(94)$.

15. 已知 $(x + 2y)^2 = 2xy - 3x + 6y - 9$。若 x 及 y 為實數，求 $x + y$ 的值。
Given that $(x + 2y)^2 = 2xy - 3x + 6y - 9$. If x and y are real number, find the value of $x + y$.

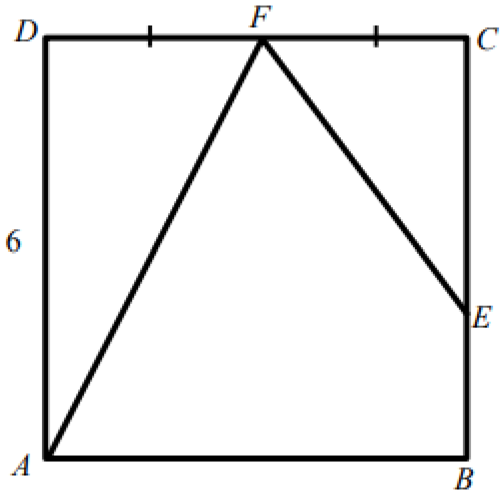
***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2020-2021
Individual Paper 2
香港數學競賽 2020 – 2021
個人項目卷二

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。Q1- Q4 每題 2 分，Q5-Q7 每題 4 分。全卷滿分 20 分。 時限：50 分鐘

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q4 2 marks each, Q5-Q7 4 marks each. The maximum mark for this paper is 20. Time allowed: 50 minutes

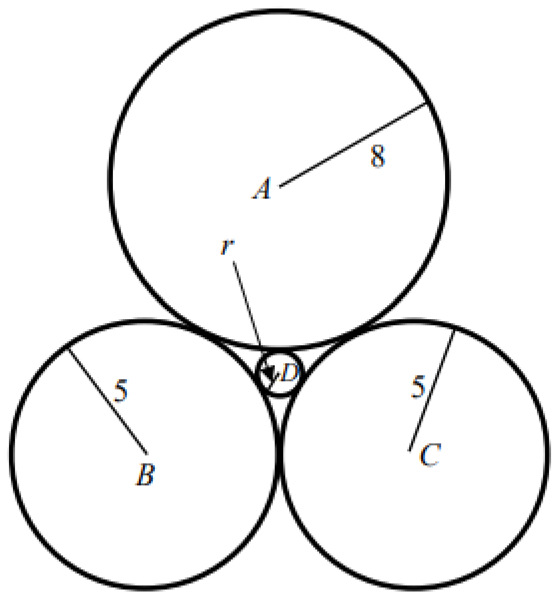
1. 在圖一中，$ABCD$ 是一個邊長為 6 的正方形。F 是 CD 的中點。若 $\angle FAB = \angle AFE$，求 BE 的長度。
In Figure 1, $ABCD$ is a square of sides 6 units. F is the mid-point of CD . If $\angle FAB = \angle AFE$, find the length of BE .



圖一 Figure 1

2. 設 $S = 2011^n + 2012^n + 2013^n + 2014^n + 2015^n + 2016^n + 2017^n + 2018^n + 2019^n$，其中 n 為一正整數。若 S 未能被 5 整除，求 S 的個位數。
Let $S = 2011^n + 2012^n + 2013^n + 2014^n + 2015^n + 2016^n + 2017^n + 2018^n + 2019^n$, where n is an integer. If S is not divisible by 5 , find the unit digit of S .

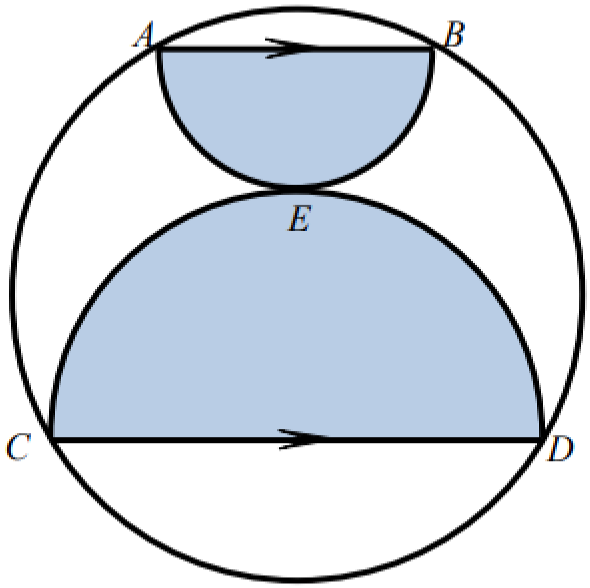
3. 在圖二中，四個半徑分別為 8、5、5 及 r 的圓互相外切。求 r 的值。
In Figure 2, four circles of radii 8, 5, 5 and r are touching each other externally. Find the value of r .



圖二 Figure 2

4. 已知 a、b、c、d 及 e 是連續正整數，其中 $a < b < c < d < e$。
若 $a + b + c + d + e$ 是一個立方數及 $b + c + d$ 是一個平方數，求 c 的最小可能值。
Given that a , b , c , d and e are consecutive positive integers, where $a < b < c < d < e$.
If $a + b + c + d + e$ is a perfect cube and $b + c + d$ is a perfect square, find the smallest possible value of c .

5. $ABCD$ 是圓形而 ABE 及 CED 為半圓形互切於 E 在圓內。已知圓面積為 1 cm^2 及 AB // CD，求半圓形 ABE 及 CED 的面積之和。
$ABCD$ is a circle while ABE and CED are semi-circles touching each other at E inside the circle. Given the area of circle is 1 cm^2 and AB // CD , find the sum of the area of the semi-circles ABE and CED .



6. 如果 $d = \log_2\left(\sqrt{2^2 + 2^{1013} + 2^{2022}} - 2\right)$，求 d 的值。
If $d = \log_2\left(\sqrt{2^2 + 2^{1013} + 2^{2022}} - 2\right)$, find the value of d .

7. 求 $\sqrt{10000 \times 10002 \times 10004 \times 10006 + 16}$ 的值。
Find the value of $\sqrt{10000 \times 10002 \times 10004 \times 10006 + 16}$.

***** 試卷完 End of Paper *****

參加學校數目：101
比賽日期：2021 年 4 月 17 日星期六
在各自學校以 Google Classroom 進行
比賽結果：
全場總冠軍：聖保羅男女中學 (St Paul's Co-Educational College)
亞軍：香港中國婦女會中學 (Hong Kong Chinese Women's Club College)
季軍：拔萃男書院 (Diocesan Boys' School)
區域得獎學校名單(以學校英文名稱排序)
港島區
HKI-03 香港中國婦女會中學 (Hong Kong Chinese Women's Club College)
HKI-10 筲箕灣官立中學 (Shau Kei Wan Government Secondary School)
HKI-15 聖保羅男女中學 (St Paul's Co-Educational College)
九龍東區
KLNW-02 中華基督教會銘賢書院 (CCC Ming Yin College)
KLNW-18 香港培正中學 (Pui Ching Middle School)
KLNW-26 英華書院 (Ying Wa College)
新界區
NT-08 香港道教聯合會鄧顯紀念中學 (HKTA Tang Hin Memorial Secondary School)
NT-14 新界鄉議局元朗區中學 (NTHYK Yuen Long District Secondary School)
NT-21 聖公會曾肇添中學 (SKH Tsang Shiu Tim Secondary School)