Examples on Mathematical Induction: divisibility 49

Created by Mr. Francis Hung Last updated: September 1, 2021

1. **1973 香港中文中學會考高級數學試卷一 Q2(b)**

   Prove, by mathematical induction, that $8^n - 7n - 1$ is a divisible by 49 for all positive integers. Hence find the remainder when 8^{100} is divided by 49.

1. Let $P(n)$ be the statement "$8^n - 7n - 1$ is a divisible by 49 for all positive integers n ."

   $n = 1$, $8^1 - 7(1) - 1 = 0$, which is divisible by 49.

   $\therefore P(1)$ is true.

   Assume $P(k)$ is true for some positive integer $k \geq 1$.

   i.e. $8^k - 7k - 1 = 49M$, where M is an integer.

   Then $8^{k+1} - 7(k + 1) - 1 = 8(8^k) - 7k - 8$

   $= 8(49M + 7k + 1) - 7k - 8$ (induction assumption)

   $= 8\times49M + 56k + 8 - 7k - 8$

   $= 8\times49M + 49k$

   $= 49(8M + k)$

   Since $8M + k$ is an integer, so $8^{k+1} - 7(k + 1) - 1$ is divisible by 49 .

   Assume P(k) is true, then $P(k + 1)$ is also true.

   By mathematical induction, $8^n - 7n - 1$ is a divisible by 49 for all positive integers.

   $8^{100} - 7(100) - 1 = 49M$, where M is an integer.

   $8^{100} = 49M + 701$

   $= 49M + 14(49) + 15$

   The remainder is 15 .