Division Problem

Created by Mr. Francis Hung Last updated: July 1, 2023 Find all 'x' in the division on the right. x x 7 x xRelabel the 'x' as shown. Let $a = (a_1 a_2 7 a_4 a_5)_x$ $x\;x\;x\;x\;x\;x$ $b = (b_1b_2b_3b_47b_6)_x$ $x \times x \times x \times 7 \times$ $c = (c_1c_27c_4c_5c_6c_7c_8c_9c_{10})_x$ X X X X X X X $d = (d_1d_2d_3d_4d_5d_6)_x$ x 7 x x x x $e = (e_1e_2e_3e_4e_57e_7)_x$ x 7 x x x x $f = (f_1 f_2 f_3 f_4 f_5 f_6 f_7)_x$ X X X X X X X $g = (g_17g_3g_4g_5g_6)_x$ $x \times x \times 7 \times x$ $h = (h_17h_3h_4h_5h_6)_x$ X X X X X X $i = (i_1 i_2 i_3 i_4 i_5 i_6 i_7)_x$ X X X X X X $j = (j_1 j_2 j_3 j_4 7 j_6 j_7)_x$ $a_1 \ a_2 \ 7 \ a_4 \ a_5$ $k = (k_1k_2k_3k_4k_5k_6)_x$ $b_1 b_2 b_3 b_4 7 b_6$) $c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}$ $m = (m_1 m_2 m_3 m_4 m_5 m_6)_x$ $d_1 d_2 d_3 d_4 d_5 d_6$ $e_1 e_2 e_3 e_4 e_5 7 e_7$ $b \times 7 = h$, a 6-digits number. f_1 f_2 f_3 f_4 f_5 f_6 f_7 f and j are 7-digits numbers. $g_1 7 g_3 g_4 g_5 g_6$ so a_2 , $a_4 = 8$ or 9 $h_1 7 h_3 h_4 h_5 h_6$ $b \times 8$ or $b \times 9 = 7$ -digits number *i*₁ *i*₂ *i*₃ *i*₄ *i*₅ *i*₆ *i*₇ $b \times 7 \le 979999$ and $b \times 9 \ge 1000700$ *j*₁ *j*₂ *j*₃ *j*₄ 7 *j*₆ *j*₇ $111189 \le b \le 139999 \Rightarrow b_1 = 1$ k_1 k_2 k_3 k_4 k_5 k_6 $b_5 = 7 \Rightarrow 111270 \le b \le 139979 \dots (1)$ $m_1 m_2 m_3 m_4 m_5 m_6$ $778890 \le 7b \le 979853$ $a_1 \ a_2 \ 7 \ a_4 \ a_5$ $h_1 = 7, 8 \text{ or } 9$ $1 b_2 b_3 b_4 7 b_6$) $c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}$ when $h_1 = 9$, i_1 has no solution, reject $d_1 d_2 d_3 d_4 d_5 d_6$ e1 e2 e3 e4 e5 7 e7 when $h_1 = 8$, $g_1 = 9$, $i_1 = 1$ (2) when $h_1 = 7$, $g_1 = 9$, $i_1 = 1$ or $2 \dots (3)$ f_1 f_2 f_3 f_4 f_5 f_6 f_7 when $h_1 = 7$, $g_1 = 8$, $i_1 = 1$ (4) $g_1 7 g_3 g_4 g_5 g_6$ $b \times a_4 = i$ and $a_4 = 8$ or 9 $h_1 7 h_3 h_4 h_5 h_6$ $890160 \le 8b \le 1119832$, $1001430 \le 9b \le 1259811$ 1 i2 i3 i4 i5 i6 i7 $\Rightarrow 1000700 \le j \le 1259799$ $1 \ j_2 \ j_3 \ j_4 \ 7 \ j_6 \ j_7$

 $\Rightarrow j_1 = 1, j_2 = 0, 1 \text{ or } 2$

 $i_2 = 1, 2 \text{ or } 3 \dots (5)$

 $\Rightarrow i_1 = 1, i_2 = j_2 \text{ or } j_2 + 1$

k1 k2 k3 k4 k5 k6

*m*₁ *m*₂ *m*₃ *m*₄ *m*₅ *m*₆

| If there is a borrow digit for $g_3 - h_3$, then $i_2 = 9$ | $a_1 \ a_2 \ 7 \ a_4 \ a_5$ |
|---|---|
| contradict to (5), so there is no borrow digit. | $1 \ b_2 \ b_3 \ b_4 \ 7 \ b_6 \) \ c_1 \ c_2 \ 7 \ c_4 \ c_5 \ c_6 \ c_7 \ c_8 \ c_9 \ c_{10}$ |
| $g_1 - h_1 = 1$ and $g_2 - h_2 = 7 - 7 = 0 = i_2$ | $d_1 d_2 d_3 d_4 d_5 d_6$ |
| $j_2 = 0$ | e ₁ e ₂ e ₃ e ₄ e ₅ 7 e ₇ |
| If $a_4 = 9$, $1000700 \le 9b \le 1099799$ | f1 f2 f3 f4 f5 f6 f7 |
| $111189 \le b \le 122200$ | g ₁ 7 g ₃ g ₄ g ₅ g ₆ |
| $b_5 = 7 \Rightarrow 111270 \le b \le 122179$ | $h_1 7 h_3 h_4 h_5 h_6$ |
| $778890 \le 7b \le 855253$ | 1 0 <i>i</i> ₃ <i>i</i> ₄ <i>i</i> ₅ <i>i</i> ₆ <i>i</i> ₇ |
| $h_2 = 7 \Longrightarrow 778890 \le 7b \le 779999$ | 1 0 <i>j</i> 3 <i>j</i> 4 7 <i>j</i> 6 <i>j</i> 7 |
| $111270 \le b \le 111428$ | $k_1 \ k_2 \ k_3 \ k_4 \ k_5 \ k_6$ |
| $b_5 = 7 \Rightarrow 111270 \le b \le 111379$ | $m_1 m_2 m_3 m_4 m_5 m_6$ |
| $1001430 \le 9b \le 1002411$ | $a_1 \ a_2 \ 7 \ 8 \ a_5$ |
| $j_5 = 7 \Rightarrow 1001700 \le 9b \le 1001799$ | $\frac{a_1 \ a_2 \ 7 \ 8 \ a_3}{1 \ b_2 \ b_3 \ b_4 \ 7 \ b_6) \ c_1 \ c_2 \ 7 \ c_4 \ c_5 \ c_6 \ c_7 \ c_8 \ c_9 \ c_{10}}$ |
| $111300 \le b \le 111311$ | $d_1 d_2 d_3 d_4 + d_6 + d_1 d_2 d_3 d_4 d_5 d_6$ |
| $b_5 = 7 \Rightarrow \text{no solution}$ | |
| $\therefore a_4 \neq 9 \Rightarrow a_4 = 8$ | $e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ 7 \ e_7$ |
| By (1): $111270 \le b \le 139979$ | $ \frac{f_1 \ f_2 \ f_3 \ f_4 \ f_5 \ f_6 \ f_7}{2 \ r_6 \ r_6} $ |
| $890160 \le 8b \le 1119832$ | $g_1 7 g_3 g_4 g_5 g_6$ |
| j = 8b, a 7-digits number | $\frac{h_1 7 \ h_3 \ h_4 \ h_5 \ h_6}{1 \ 0 \ i \ i \ i \ i}$ |
| $\Rightarrow 1000700 \le 8b \le 1099799$ | 1 0 <i>i</i> ₃ <i>i</i> ₄ <i>i</i> ₅ <i>i</i> ₆ <i>i</i> ₇ |
| $3000700 \le 8b \le 1099799$ $125088 \le b \le 137474$ | 1 0 j ₃ j ₄ 7 j ₆ j ₇ |
| | $k_1 \ k_2 \ k_3 \ k_4 \ k_5 \ k_6$ |
| $b_5 = 7 \Rightarrow 125170 \le b \le 137474$ | $m_1 m_2 m_3 m_4 m_5 m_6$ |
| $1001360 \le 8b$ | $a_1 \ a_2 \ 7 \ 8 \ a_5$ |
| $j_5 = 7 \Longrightarrow 1001700 \le 8b$ | $1 b_2 b_3 b_4 7 b_6) c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}$ |
| $125213 \le b$ | $d_1 d_2 d_3 d_4 d_5 d_6$ |
| $b_5 = 7 \Rightarrow 125270 \le b$ | $e_1 e_2 e_3 e_4 e_5 7 e_7$ |
| $1002160 \le 8b$ | f1 f2 f3 f4 f5 f6 f7 |
| $j_5 = 7 \Rightarrow 1002700 \le 8b$ | 9 7 g ₃ g ₄ g ₅ g ₆ |
| $125338 \le b$ | 8 7 h ₃ h ₄ h ₅ h ₆ |
| $b_5 = 7 \Longrightarrow 125370 \le b$ | $\frac{1}{1} 0 i_3 i_4 i_5 i_6 i_7$ |
| $1002960 \le 8b$ | 1 0 j ₃ j ₄ 7 j ₆ j ₇ |
| $j_5 = 7 \Rightarrow 1003700 \le 8b$ | $k_1 \ k_2 \ k_3 \ k_4 \ k_5 \ k_6$ |
| $125463 \le b$ | m_1 m_2 m_3 m_4 m_5 m_6 |
| $b_5 = 7 \Rightarrow 125470 \le b \le 137474 \dots (6)$ | |
| $1003760 \le 8b \le 1099792$ | |
| $b \times 7 = h$ and $g_1 - h_1 = 1 \Rightarrow h_1 = 7$ or 8 | |

 $878290 \le 7b \Rightarrow h_1 = 8, g_1 = 9$

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h = (87h_3h_4h_5h_6)_x = 7b
                                                                                                                              a_1 \ a_2 \ 7 \ 8 \ a_5
                                                                                   125b_47b_6) c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}
870000 \le 7b \le 879999
124286 \le b \le 125714
                                                                                                        d_1 d_2 d_3 d_4 d_5 d_6
combine with (6): 125470 \le b \le 125714
                                                                                                         e1 e2 e3 e4 e5 7 e7
b_5 = 7 \Rightarrow 125470 \le b \le 125679
                                                                                                        f<sub>1</sub> f<sub>2</sub> f<sub>3</sub> f<sub>4</sub> f<sub>5</sub> f<sub>6</sub> f<sub>7</sub>
                                                                                                                 9 7 g<sub>3</sub> g<sub>4</sub> g<sub>5</sub> g<sub>6</sub>
1003760 \le 8b \le 1005432
j_5 = 7 \Rightarrow 8b \le 1004799
                                                                                                                 8 7 h<sub>3</sub> h<sub>4</sub> h<sub>5</sub> h<sub>6</sub>
125470 \le b \le 125599 \dots (7)
                                                                                                                 1 0 i3 i4 i5 i6 i7
878290 \le 7b \le 879193 \dots (8)
                                                                                                                 1 0 j<sub>3</sub> j<sub>4</sub> 7 j<sub>6</sub> j<sub>7</sub>
1003760 \le 8b \le 1004792 \dots (9)
                                                                                                                         k_1 k_2 k_3 k_4 k_5 k_6
b_2 = 2, b_3 = 5, b_4 = 4 or 5
                                                                                                                         m<sub>1</sub> m<sub>2</sub> m<sub>3</sub> m<sub>4</sub> m<sub>5</sub> m<sub>6</sub>
h_3 = 8 \text{ or } 9
                                                                                                                          a_1 \ a_2 \ 7 \ 8 \ 1
j_3 = 0, j_4 = 3 \text{ or } 4
                                                                                   12547b_6) c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}
when h_3 = 9, g_3 = 9, i_3 = 0, i_3 - 0 = k_1, no solution
                                                                                                       d_1 d_2 d_3 d_4 d_5 d_6
when h_3 = 8, g_3 = 8 or 9, i_3 = 0 or 1
                                                                                                       e<sub>1</sub> e<sub>2</sub> e<sub>3</sub> e<sub>4</sub> e<sub>5</sub> 7 e<sub>7</sub>
i_3 - j_3 = k_1 \implies k_1 = 1, i_3 = 1, j_3 = 0 \implies a_5 = 1
                                                                                                       f<sub>1</sub> f<sub>2</sub> f<sub>3</sub> f<sub>4</sub> f<sub>5</sub> f<sub>6</sub> f<sub>7</sub>
                                                                                                                9 7 9 g<sub>4</sub> g<sub>5</sub> g<sub>6</sub>
g_3 = 9, k_1 = m_1 = 1, k_2 = m_2 = 2, k_3 = m_3 = 5
By (8) and h_3 = 8 \Rightarrow 878290 \le 7b \le 878999
                                                                                                                8 7 8 h_4 h_5 h_6
125470 \le b \le 125571
                                                                                                                1 0 1 i4 i5 i6 i7
1003760 \le 8b \le 1004568
                                                                                                                1 0 0 j<sub>4</sub> 7 j<sub>6</sub> j<sub>7</sub>
j_5 = 7 \Rightarrow 8b \le 1003799
                                                                                                                        1 2 5 4 7 k<sub>6</sub>
125470 \le b \le 125474 \dots (10)
                                                                                                                        1 \quad 2 \quad 5 \quad 4 \quad 7 \quad m_6
\Rightarrow b_4 = k_4 = m_4 = 4, k_5 = m_5 = 7
                                                                                                                            a_1 \ a_2 \ 7 \ 8 \ 1
878290 \le 7b \le 878318 \dots (11)
                                                                                  12547b_6) c_1 c_2 7 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}
\Rightarrow h_4 = 2 \text{ or } 3
                                                                                                       d_1 d_2 d_3 d_4 d_5 d_6
1003760 \le 8b \le 1003792 \dots (12)
                                                                                                       e<sub>1</sub> e<sub>2</sub> e<sub>3</sub> e<sub>4</sub> e<sub>5</sub> 7 e<sub>7</sub>
\Rightarrow 6 \leq j_6 \leq 9
                                                                                                       f_1 f_2 f_3 f_4 f_5 f_6 f_7
                                                                                                                9 7 9 g<sub>4</sub> g<sub>5</sub> g<sub>6</sub>
j_4 = 3
j_6 = 7 > 5 = k_3 \Rightarrow there is a borrow digit for i_4
                                                                                                                8 7 8 h<sub>4</sub> h<sub>5</sub> h<sub>6</sub>
i_4 - 1 - j_4 = 2 \implies i_4 = 6
                                                                                                                1 0 1 6 i<sub>5</sub> i<sub>6</sub> i<sub>7</sub>
i_5 = 2 \text{ or } 3 \dots (13)
                                                                                                                1 \ 0 \ 0 \ 3 \ 7 \ j_6 \ j_7
when h_4 = 2, by (11), 878290 \le 7b \le 878299
                                                                                                                        1 2 5 4 7 k<sub>6</sub>
h_5 = 9
                                                                                                                        1 \quad 2 \quad 5 \quad 4 \quad 7 \quad m_6
g_4 - h_4 = 6 \Rightarrow g_4 = 8
\Rightarrow 10 + e_6 - f_6 = g_4 or 10 + e_6 - 1 - f_6 = g_4
\Rightarrow f_6 = 9 \text{ or } 8 \dots (14)
125470 \le b \le 125471, b_6 = 0 or 1
a_2 = 8 \text{ or } 9 \Rightarrow f = 8b \text{ or } 9b
when b = 125470, a_2 = 8, f = 1003760 contradict (14)
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| when $b=125470$, $a_2=9$, $f=1129230$, contradict (14) | | | | | | | a_1 | 8 | 7 | 8 | 1 |
|---|---------------------------------|-------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------|-----------------------|
| when $b=125471$, $a_2=8$, $f=1003768$, contradict (14) | 1 2 5 4 7 <i>b</i> ₆ | c_1 | c_2 | 7 | <i>C</i> 4 | C5 | <i>C</i> ₆ | <i>C</i> 7 | <i>C</i> ₈ | C 9 | C ₁₀ |
| when $b=125471$, $a_2=9$, $f=1129239$, contradict (14) | | d_1 | d_2 | d_3 | d_4 | d_5 | d_6 | | | | |
| so $h_4 \neq 2$, $\Rightarrow h_4 = 3$ | | e_1 | e_2 | <i>e</i> ₃ | <i>e</i> ₄ | <i>e</i> ₅ | 7 | <i>e</i> 7 | | | |
| $g_4 = 9, f_6 = 8 \text{ or } 7 \dots (15)$ | | f_1 | f_2 | f3 | f_4 | f_5 | f_6 | f7 | | | |
| By (11) and $h_4 = 3$, $878300 \le 7b \le 878318$ | | | | 9 | 7 | 9 | 9 | g 5 | g 6 | | |
| $125472 \le b \le 125474$ | | | | 8 | 7 | 8 | 3 | <i>h</i> ₅ | h_6 | | |
| when $a_2 = 9$, $1129248 \le 9b \le 1129266$ | | | | 1 | 0 | 1 | 6 | i ₅ | i_6 | i7 | |
| $1129248 \le f \le 1129266$ | | | | 1 | 0 | 0 | 3 | 7 | j_6 | j_7 | |
| contradict with (15) | | | | | | 1 | 2 | 5 | 4 | 7 | <i>k</i> ₆ |
| so $a_2 = 8$ | | | | | | 1 | 2 | 5 | 4 | 7 | <i>m</i> ₆ |
| $1003776 \le 8b \le 1003792, f = 8b$ | | | | | | | a_1 | 8 | 7 | 8 | 1 |
| $f_6 = 8 \text{ or } 7 \Rightarrow 1003776 \le 8b \le 1003789$ | 1 2 5 4 7 <i>b</i> ₆ | c_1 | <i>c</i> ₂ | 7 | <i>C</i> 4 | C 5 | <i>C</i> 6 | <i>C</i> 7 | C8 | C 9 | C10 |
| $125472 \le b \le 125473$ | | | | | | | d_6 | | | | |
| $f_1 = 1, f_2 = 0, f_3 = 0, f_4 = 3, f_5 = 7$ | | | | | | | 7 | <i>e</i> ₇ | | | |
| $878304 \le 7b \le 878311$ | | 1 | 0 | 0 | 3 | 7 | f_6 | f7 | | | |
| $1003776 \le 8b \le 1003784$ | | _ | | | | | 9 | | g ₆ | | |
| If $b_6 = 2$, then $k = m = 125472$ | | | | | | | 3 | | | | |
| j = 1003776 = f | | | | _ | | | 6 | | | | |
| i = j + 12547 = 1016323 | | | | | | | 3 | | | | |
| h = 7b = 878304 | | | | | | | 2 | | | | k_6 |
| g = h + 101632 = 979936 | | | | | | 1 | 2 | 5 | 4 | 7 | m_6 |
| $e = f + 97993 = 1101769$ contradict $e_6 = 7$ | | | | | | | | | | | |
| Therefore $b_6 = k_6 = m_6 = 3$ | | | | | | | 5 | 8 | 7 | 8 | 1 |
| j = 8b = 1003784 = f | 125473 | 7 | 3 | 7 | 5 | 4 | 2 | 8 | 4 | 1 | 3 |
| i = j + 12547 = 1016331 | | 6 | 2 | 7 | 3 | 6 | 5 | | | | |
| h = 7b = 878311 | | 1 | 1 | 0 | 1 | 7 | 7 | 8 | | | |
| g = h + 101633 = 979944 | | 1 | 0 | 0 | 3 | 7 | 8 | 4 | | | |
| e = f + 97994 = 1101778 | | | | 9 | 7 | 9 | 9 | 4 | 4 | | |
| $7 - d_3 = 0 \text{ or } 6 - d_3 = 0 \Rightarrow d_3 = 6 \text{ or } 7$ | | | | 8 | 7 | 8 | 3 | 1 | 1 | | |
| By the method of exhaustion | | | | 1 | 0 | 1 | 6 | 3 | 3 | 1 | |
| 125473×3=376419, 125473×5=627365 | | | | 1 | 0 | 0 | 3 | 7 | 8 | 4 | |
| $a_1 = 3 \text{ or } 5$ | | | | | | 1 | 2 | 5 | 4 | 7 | 3 |
| When $a_1 = 3$, $d = 376419$ | | | | | | 1 | 2 | 5 | 4 | 7 | 3 |
| $c = (d+110177) \times 10000 + 8413 = 4865968413$ | | | | | | | | | | | |
| contradict $c_3 = 7$ | | | | | | | | | | | |
| So $a_1 = 5$, $d = 627365$, | | | | | | | | | | | |
| $c = (d + 110177) \times 10000 + 8413 = 7375428413$ | | | | | | | | | | | |