Individual Events (Remark: The Individual Events are interchanged with Group Events)

I1	*a*	40	I2	*a*	7	I3	*a*	1	I4	*a*	1
	b	70		*b*	5		****b*** see the remark	0.0625 or $\frac{1}{16}$		*b*	9
	C	4		*C*	35		*c*	0.5 or $\frac{1}{2}$		*c*	20
	d	*20 see the remark		*d*	6		*d*	6		*d*	6

Group Events (Remark: The Group Events are interchanged with Individual Events)

G1	*a*	9	G2	*a*	15	G3	*A*	$\frac{\sqrt{7}}{4}$	G4	*a*	3
	b	125		*b*	999985		*b*	$\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$		*b*	2
	c	5		*c*	4		*c*	7		*c*	6
	d	$2\frac{1}{2}$ or 2.5		*d*	$\frac{509}{256}$		*d*	$\frac{100}{11}$		*d*	171

Individual Event 1

I1.1 There are a camels in a zoo. The number of one-hump camels exceeds that of two-hump camels by 10. If there have 55 humps altogether, find the value of a.

Suppose there are x one-hump camels, y two-hump camels.

$x - y = 10$ .... (1)

$x + 2y = 55$ .... (2)

(2) – (1) $3y = 45 \Rightarrow y = 15$

sub. $y = 15$ into (1): $x - 15 = 10 \Rightarrow x = 25$

$a = x + y = 25 + 15 = 40$

I1.2 If LCM(a, b) = 280 and HCF(a, b) = 10, find the value of b.

Reference: 2016 FI2.4

HCF $\times$ LCM = ab

$2800 = 40b$

$b = 70$

I1.3 Let C be a positive integer less than $\sqrt{b}$. If b is divided by C, the remainder is 2; when divided by $C + 2$, the remainder is C, find the value of C.

$C < \sqrt{70} \;\Rightarrow C \le 8$ ....(1)

$70 = mC + 2$ ....(2)

$70 = n(C + 2) + C$ ....(3)

From (2), $mC = 68$ $\because 2 < C \le 8$, $\therefore C = 4$ ($C \ne 1, 2$, otherwise remainder > divisor !!!)

I1.4 A regular $2C$-sided polygon has d diagonals, find the value of d.

Reference: 1984 FG10.3, 1985 FG8.3, 1988 FG6.2, 1989 FG6.1, 1991 FI2.3, 2001 FI4.2

The number of diagonals of a convex n-sided polygon is $\frac{n(n-3)}{2}$.

$d = \frac{8\times 5}{2} = 20$

Remark: The following note was put at the end of the original question:

(註：對角線是連接兩個不在同一邊上的頂點的直線。)

(NB: a diagonal is a straight line joining two vertices not on the same side.)

The note is very confusing. As the definition of diagonal is well known, there is no need to add this note.

Individual Event 2

I2.1 Mr. Chan has 8 sons and a daughters. Each of his sons has 8 sons and a daughters. Each of his daughters has a sons and 8 daughters. It is known that the number of his grand sons is one more than the number of his grand daughters and a is a prime number, find the value of a.

Grandsons = $8\times8 + a\times a = a^2 + 64$

Grand daughters = $8\times a + a\times 8 = 16a$

$a^2 + 64 = 16a + 1$

$a^2 - 16a + 63 = 0$

$(a - 7)(a - 9) = 0$

$a = 7$ or $a = 9$

a is a prime number, $a = 7$

I2.2 Let $\frac{a}{7} = \sqrt[3]{2+\sqrt{b}} + \sqrt[3]{2-\sqrt{b}}$. Find the value of b.

Reference: 1999 FI3.2, 2016 FG3.3, 2019 HI10

$1 = \sqrt[3]{2+\sqrt{b}} + \sqrt[3]{2-\sqrt{b}}$

$1 = \left(\sqrt[3]{2+\sqrt{b}} + \sqrt[3]{2-\sqrt{b}}\right)^3$

$1 = 2+\sqrt{b}+3\left(2+\sqrt{b}\right)^{\frac{2}{3}}\left(2-\sqrt{b}\right)^{\frac{1}{3}}+3\left(2+\sqrt{b}\right)^{\frac{1}{3}}\left(2-\sqrt{b}\right)^{\frac{2}{3}}+2-\sqrt{b}$

$1 = 4 + 3\left(4-b\right)^{\frac{1}{3}}\left(2+\sqrt{b}\right)^{\frac{1}{3}}+3\left(4-b\right)^{\frac{1}{3}}\left(2-\sqrt{b}\right)^{\frac{1}{3}}$

$0 = 3 + 3\left(4-b\right)^{\frac{1}{3}}\left[\left(2+\sqrt{b}\right)^{\frac{1}{3}}+\left(2-\sqrt{b}\right)^{\frac{1}{3}}\right]$

$0 = 1 + \left(4-b\right)^{\frac{1}{3}}$

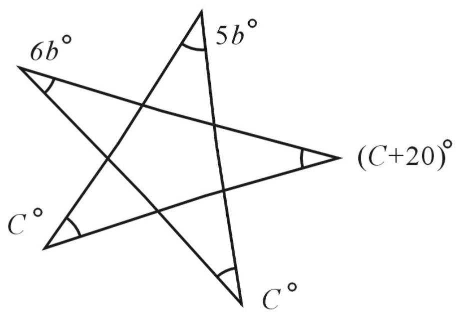
$\left(4-b\right)^{\frac{1}{3}} = -1$

$4 - b = -1$

$b = 5$

I2.3 In Figure 1, find the value of C.

Reference: 1989 FI5.1, 1997 FG1.1



$6b° + 5b° + C° + C° + (C + 20)° = 180°$

$11\times5 + 3C + 20 = 180$

$C = 35$

I2.4 Given that $P_1, P_2, \ldots, P_d$ are d consecutive prime numbers.

If $P_1 + P_2 + \ldots + P_{d-2} = P_{d-1} + P_d = C + 1$, find the value of d.

By trial and error $5 + 7 + 11 + 13 = 17 + 19 = 36$, $d = 6$.

Individual Event 3

I3.1 Given that a is a positive real root of the equation $2^{x+1}=8^{\frac{1}{x}-\frac{1}{3}}$. Find the value of a.

$2^{x+1}=8^{\frac{1}{x}-\frac{1}{3}} \Rightarrow 2^{x+1}=2^{\frac{3}{x}-1} \Rightarrow x+1=\frac{3}{x}-1$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3$ or 1, $a=1$ is a real positive root.

I3.2 The largest area of the rectangle with perimeter a meter is b square meter, find the value of b.

The perimeter = 1 m.

Let the length of the rectangle be x m, the width is $\frac{1}{2}(1-2x)$m.

Its area is $\frac{1}{2}(1-2x)\cdot x\ \text{m}^2=-\left(x^2-\frac{1}{2}x\right)\text{m}^2=\left[-\left(x-\frac{1}{4}\right)^2+\frac{1}{16}\right]\text{m}^2$.

$b=\frac{1}{16}=0.0625$

Remark: The original version is: The area of the largest rectangle …

It is ambiguous to define the largest rectangle. It should be changed to "The largest area of the rectangle ……"

I3.3 If $c=(1234^3-1232\times(1234^2+2472))\times b$, find the value of c.

$c=(1234^3-1232\times(1234^2+2472))\times\frac{1}{16}$, let $x=1234$

$=\frac{1}{16}\{x^3-(x-2)\times[x^2+2(x+2)]\}=\frac{1}{16}\{x^3-(x-2)\times(x^2+2x+4)\}$

$=\frac{1}{16}\{x^3-(x^3-8)\}=\frac{8}{16}=\frac{1}{2}$

I3.4 If $\frac{1}{(c+1)(c+2)}+\frac{1}{(c+2)(c+3)}+\cdots+\frac{1}{(c+d)(c+d+1)}=\frac{8}{15}$, find the value of d.

$\frac{1}{(c+1)(c+2)}+\frac{1}{(c+2)(c+3)}+\cdots+\frac{1}{(c+d)(c+d+1)}=\frac{8}{15}$

$\left(\frac{1}{c+1}-\frac{1}{c+2}\right)+\left(\frac{1}{c+2}-\frac{1}{c+3}\right)+\cdots+\left(\frac{1}{c+d}-\frac{1}{c+d+1}\right)=\frac{8}{15}$

$\frac{1}{c+1}-\frac{1}{c+d+1}=\frac{8}{15} \Rightarrow \frac{1}{\frac{1}{2}+1}-\frac{1}{\frac{1}{2}+d+1}=\frac{8}{15}$

$\frac{2}{3}-\frac{8}{15}=\frac{2}{3+2d}$

$\frac{2}{15}=\frac{2}{3+2d}$

$3+2d=15$

$d=6$

Individual Event 4

I4.1 If $A^2 + B^2 + C^2 = AB + BC + CA = 3$ and $a = A^2$, find the value of a.

Reference: 2018 FG4.4

$2[A^2 + B^2 + C^2 - (AB + BC + CA)] = 6 - 6 = 0$

$A^2 - 2AB + B^2 + B^2 - 2BC + C^2 + C^2 - 2AC + A^2 = 0$

$(A - B)^2 + (B - C)^2 + (C - A)^2 = 0$ (sum of three non-negative numbers = 0)

$A - B = B - C = C - A = 0$

$A = B = C = 1$

$a = A^2 = B^2 = C^2 = 1$

I4.2 Given that n and b are integers satisfying the equation $29n + 42b = a$. If $5 < b < 10$, find the value b.

$42 = 29 + 13 \Rightarrow 13 = 42 - 29$ ....(1)

$29 = 13\times 2 + 3 \Rightarrow 3 = 29 - 13\times 2$ ....(2)

$13 = 3\times 4 + 1 \Rightarrow 1 = 13 - 3\times 4$ ....(3)

Sub. (1) into (2): $3 = 29 - (42 - 29)\times 2 = 29\times 3 - 42\times 2$ ......(4)

Sub. (1), (4) into (3) $1 = 42 - 29 - (29\times 3 - 42\times 2)\times 4$

$1 = 29\times(-13) + 42\times 9$

$\therefore n = -13$, $b = 9$

Method 2

$b = 6, 7, 8, 9$.

By trial and error,

when $b = 9$,

$29n + 42\times 9 = 1$

$n = -13$

$\therefore$ $b = 9$ satisfies the equation.

I4.3 If $\dfrac{\sqrt{3}-\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \dfrac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+b}{59}$, find the value of c.

$$\frac{\sqrt{3}-\sqrt{5}+\sqrt{7}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{\sqrt{3}+\sqrt{7}-\sqrt{5}}{\sqrt{3}+\sqrt{7}+\sqrt{5}}\cdot\frac{\sqrt{3}+\sqrt{7}-\sqrt{5}}{\sqrt{3}+\sqrt{7}-\sqrt{5}} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{\left(\sqrt{3}+\sqrt{7}\right)^2-2\sqrt{5}\left(\sqrt{3}+\sqrt{7}\right)+5}{\left(\sqrt{3}+\sqrt{7}\right)^2-5} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{3+7+2\sqrt{21}-2\sqrt{15}-2\sqrt{35}+5}{3+7+2\sqrt{21}-5} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{15+2\sqrt{21}-2\sqrt{15}-2\sqrt{35}}{5+2\sqrt{21}}\cdot\frac{2\sqrt{21}-5}{2\sqrt{21}-5} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{30\sqrt{21}+84-12\sqrt{35}-28\sqrt{15}-75-10\sqrt{21}+10\sqrt{15}+10\sqrt{35}}{4\times 21-25} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$$\frac{20\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59} = \frac{c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9}{59}$$

$c = 20$

Method 2 Cross multiplying $59\left(\sqrt{3}-\sqrt{5}+\sqrt{7}\right) = \left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)\cdot\left(c\sqrt{21}-18\sqrt{15}-2\sqrt{35}+9\right)$

Compare coefficient of $\sqrt{3}$: $9 - 90 + 7c = 59 \Rightarrow c = 20$

I4.4 If c has d positive factors, find the value of d.

Reference 1993 HI8, 1994 FI3.2, 1997 HI3, 1998 HI10, 1998 FI1.4, 2002 FG4.1

The positive factors of 20 are 1, 2, 4, 5, 10 and 20.

$d = 6$

Group Event 1

G1.1 Suppose there are a numbers between 1 and 200 that can be divisible by 3 and 7, find the value of a.

The number which can be divisible be 3 and 7 are multiples of 21. $200 \div 21 = 9.5$, $a = 9$

G1.2 Let p and q be prime numbers that are the two distinct roots of the equation $x^2 - 13x + R = 0$, where R is a real number. If $b = p^2 + q^2$, find the value of b.

Reference: 1996 HG8, 1996FG7.1, 2001 FG4.4, 2012 HI6

$x^2 - 13x + R = 0$, roots p and q are prime numbers. $p + q = 13$, $pq = R$

The sum of two prime numbers is 13, so one is odd and the other is even, $p = 2$, $q = 11$

$b = p^2 + q^2 = 2^2 + 11^2 = 125$

G1.3 Given that $\tan\alpha = -\frac{1}{2}$. If $c = \frac{2\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha}$, find the value of c.

$$\tan\alpha = -\frac{1}{2}.\quad c = \frac{2\cos\alpha - \sin\alpha}{\sin\alpha + \cos\alpha} = \frac{2 - \tan\alpha}{\tan\alpha + 1} = \frac{2 + \frac{1}{2}}{-\frac{1}{2} + 1} = 5$$

G1.4 Let r and s be the two distinct real roots of the equation $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) = 1$. If $d = r + s$, find the value of d.

$2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) = 1$, real roots r, s. Let $t = x + \frac{1}{x}$, then $x^2 + \frac{1}{x^2} = t^2 - 2$.

$2(t^2 - 2) - 3t = 1$

$2t^2 - 3t - 5 = 0$

$(2t - 5)(t + 1) = 0$

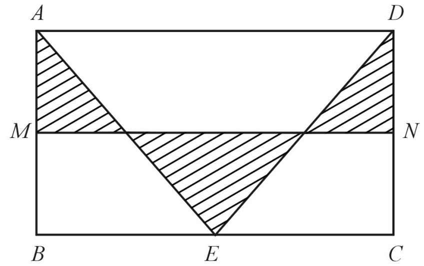
$t = \frac{5}{2}$ or -1

$x + \frac{1}{x} = \frac{5}{2}$ or $x + \frac{1}{x} = -1$

$x = 2$ or $\frac{1}{2}$ $\Rightarrow r = 2$, $s = \frac{1}{2} \Rightarrow d = r + s = \frac{5}{2}$

Group Event 2

G2.1 In Figure 1, *ABCD* is a rectangle, AB = 6 cm and BC = 10 cm. M and N are the midpoints of AB and DC respectively. If the area of the shaded region is a cm^2, find the value of a.



$a = \frac{1}{4}$ area of rectangle $= \frac{1}{4} \times 6 \times 10 = 15$

G2.2 Let $b = 89 + 899 + 8999 + 89999 + 899999$, find the value of b.

$b = 89 + 899 + 8999 + 89999 + 899999$

$= (90 - 1)+(900 - 1)+ (9000 - 1) + (90000 - 1) + (900000 - 1)$

$= 999990 - 5 = 999985$

G2.3 Given that $2x + 5y = 3$. If $c = \sqrt{4^{x+\frac{1}{2}} \times 32^y}$, find the value of c.

$2x + 5y = 3$. $c = \sqrt{4^{x+\frac{1}{2}} \times 32^y} = \sqrt{2^{2x+1} \times 2^{5y}} = \sqrt{2^{2x+5y+1}} = \sqrt{2^{3+1}} = 4$

G2.4 Let $d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots + \frac{10}{2^{10}}$, find the value of d.

Reference: 2005 HI7, 2007 FG2.1

$d = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots + \frac{10}{2^{10}}$, $2d = 1 + 1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \ldots + \frac{10}{2^9}$

$2d - d = 1 + 1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \ldots + \frac{10}{2^9} - \left(\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots + \frac{10}{2^{10}}\right)$

$= 1 + (1 - \frac{1}{2}) + (\frac{3}{4} - \frac{2}{4}) + (\frac{4}{8} - \frac{3}{8}) + (\frac{5}{16} - \frac{4}{16}) + \ldots\ldots + (\frac{10}{2^9} - \frac{9}{2^9}) - \frac{10}{2^{10}}$

$d = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots + \frac{1}{2^9} - \frac{10}{1024} = \frac{1 - \frac{1}{2^{10}}}{1 - \frac{1}{2}} - \frac{5}{512}$

$= \frac{1023}{512} - \frac{5}{512} = \frac{1018}{512} = \frac{509}{256}$

Group Event 3

G3.1 Let $0° < \alpha < 45°$. If $\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16}$ and $A = \sin\alpha$, find the value of A.

Method 1 $2\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{8}$

$\sin 2\alpha = \frac{3\sqrt{7}}{8}$

$\cos 2\alpha = \sqrt{1-\sin^2 2\alpha} = \sqrt{1-\left(\frac{3\sqrt{7}}{8}\right)^2}$

$= \frac{1}{8}\sqrt{64-63} = \frac{1}{8}$

$1 - 2\sin^2\alpha = \frac{1}{8} \Rightarrow \sin\alpha = \frac{\sqrt{7}}{4}$

Method 2 $2\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{8}$,

$\sin 2\alpha = \frac{3\sqrt{7}}{8} \Rightarrow \tan 2\alpha = 3\sqrt{7}$

$t = \tan\alpha$, $\tan 2\alpha = \frac{2t}{1-t^2} = 3\sqrt{7}$

$2t = 3\sqrt{7} - 3\sqrt{7}\,t^2$

$3\sqrt{7}\,t^2 + 2t - 3\sqrt{7} = 0$

$(3t - \sqrt{7})(\sqrt{7}\,t + 3) = 0$

$t = \frac{\sqrt{7}}{3}$ or $-\frac{3}{\sqrt{7}}$ (rejected)

$\tan\alpha = \frac{\sqrt{7}}{3}$

$A = \sin\alpha = \frac{\sqrt{7}}{4}$

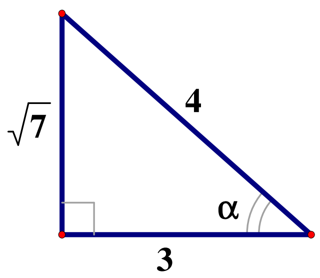
Method 3

$\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16} = \frac{3}{4}\times\frac{\sqrt{7}}{4}$

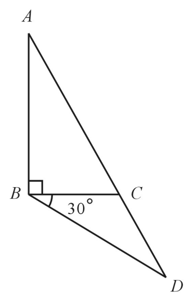
$\sin\alpha = \frac{3}{4}$, $\cos\alpha = \frac{\sqrt{7}}{4}$

or $\sin\alpha = \frac{\sqrt{7}}{4}$, $\cos\alpha = \frac{3}{4}$

$\because 0° < \alpha < 45°$, $\therefore \sin\alpha < \cos\alpha$, $\sin\alpha = \frac{\sqrt{7}}{4}$



G3.2 In figure 1, C lies on AD, $AB = BD = 1$ cm, $\angle ABC = 90°$ and $\angle CBD = 30°$. If $CD = b$ cm, find the value of b.



$AB = BD = 1$ cm, ΔABD is isosceles.

$\angle BAD = \angle BDA = (180° - 90° - 30°) \div 2 = 30°$ ($\angle$s sum of isosceles Δ)

ΔBCD is also isosceles.

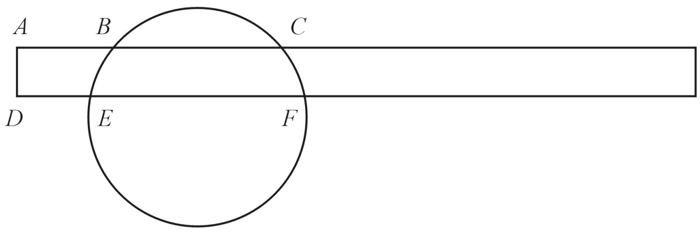
$CD = b$ cm $= BC$

$= AB \tan\angle BAD$

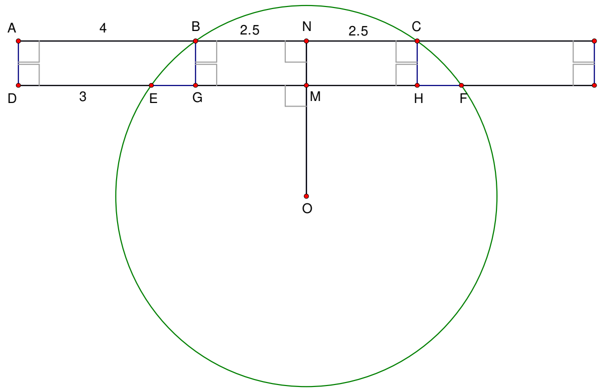
$= 1 \tan 30°$ cm

$= \frac{1}{\sqrt{3}}$ cm

G3.3 In Figure 2, a rectangle intersects a circle at points B, C, E and F. Given that $AB = 4$ cm, $BC = 5$ cm and $DE = 3$ cm. If $EF = c$ cm, find the value of c.



Draw $BG \perp DF$, $CH \perp DF$



$DG = AB = 4$ cm, $GH = BC = 5$ cm

$EG = DG - DE = 4$ cm $- 3$ cm $= 1$cm

Let O be the centre.

Let M be the foot of perpendicular of O on EF and produce OM to N on BC.

$ON \perp BC$ (corr. $\angle$s AC // DF)

$BN = NC = 2.5$ cm ($\perp$ from centre bisect chord)

$MF = EM$ ($\perp$ from centre bisect chords)

$= EG + GM = 1$ cm $+ BN$

$= 1$ cm $+ 2.5$cm $= 3.5$ cm

$EF = 2EM = 7$ cm

G3.4 Let x and y be two positive numbers that are inversely proportional to each other. If x is increased by 10 %, y will be decreased by d %, find the value of d.

$x y = k$, $x_1 = 1.1 x$

$x_1y_1 = xy$

$\Rightarrow 1.1xy_1 = xy$

$y_1 = \dfrac{10y}{11}$

Percentage decrease $= \dfrac{y - \frac{10y}{11}}{y} \times 100\%$

$= \dfrac{100}{11}\%$

$d = \dfrac{100}{11}$

Group Event 4

G4.1 If $a = \log_{\frac{1}{2}} 0.125$, find the value of a.

$$a = \log_{\frac{1}{2}} 0.125$$
$$= \frac{\log 0.125}{\log \frac{1}{2}}$$
$$= \frac{\log \frac{1}{8}}{\log \frac{1}{2}}$$
$$= \frac{\log 2^{-3}}{\log 2^{-1}}$$
$$= \frac{-3\log 2}{-\log 2} = 3$$

G4.2 Suppose there are b distinct solutions of the equation $\left|x - |2x+1|\right| = 3$, find the value of b.

Reference: 2002 FG.4.3, 2009 HG9, 2012 FG4.2, 2017 FG1.2

$\left|x - |2x+1|\right| = 3$

$x - |2x + 1| = 3$ or $x - |2x + 1| = -3$

$x - 3 = |2x + 1|$ or $x + 3 = |2x + 1|$

$x - 3 = 2x + 1$ or $3 - x = 2x + 1$ or $x + 3 = 2x + 1$ or $2x + 1 = -x - 3$

$x = -4$ or $\frac{2}{3}$ or 2 or $-\frac{4}{3}$

Check: when $x = -4$ or $\frac{2}{3}$, $0 > x - 3 = |2x + 1| \ge 0$, no solution

When $x = 2$ or $-\frac{4}{3}$, $x + 3 = |2x + 1| \ge 0$, accepted

There are 2 distinct solutions. $b = 2$

G4.3 If $c = 2\sqrt{3} \times \sqrt[3]{1.5} \times \sqrt[6]{12}$, find the value of c.

$$c = 2\sqrt{3} \times \sqrt[3]{1.5} \times \sqrt[6]{12} = 2\times 3^{\frac{1}{2}} \times \left(\frac{3}{2}\right)^{\frac{1}{3}} \times \left(2^2\times 3\right)^{\frac{1}{6}}$$
$$= 2^{1-\frac{1}{3}+\frac{2}{6}} \times 3^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = 2\times 3 = 6$$

G4.4 Given that $f_1 = 0$, $f_2 = 1$, and for any positive integer $n \ge 3$, $f_n = f_{n-1} + 2f_{n-2}$. If $d = f_{10}$, find the value of d.

The characteristic equation: $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x + 1)(x - 2) = 0 \Rightarrow x = -1$ or 2

$f_n = A(-1)^n + B\times 2^n$, $n = 1, 2, 3, \cdots\cdots\cdots$

$f_1 = -A + 2B = 0 \cdots\cdots\cdots (1)$

$f_2 = A + 4B = 1 \quad \cdots\cdots\cdots(2)$

$(1) + (2)$ $6B = 1$, $B = \frac{1}{6}$

Sub. into (1): $-A + \frac{1}{3} = 0$, $A = \frac{1}{3}$

$f_n = \frac{1}{3}(-1)^n + \frac{1}{6}\times 2^n$, $d = f_{10} = \frac{1}{3} + \frac{1}{6}\times 1024 = \frac{513}{3} = 171$

Method 2: $f_1 = 0, f_2 = 1$
$f_3 = f_2 + 2f_1 = 1 + 0 = 1; f_4 = f_3 + 2f_2 = 1 + 2 = 3$
$f_5 = f_4 + 2f_3 = 3 + 2\times1 = 5; f_6 = f_5 + 2f_4 = 5 + 2\times3 = 11$
$f_7 = f_6 + 2f_5 = 11 + 2\times5 = 21; f_8 = f_7 + 2f_6 = 21 + 2\times11 = 43$
$f_9 = f_8 + 2f_7 = 43 + 2\times21 = 85; f_{10} = f_9 + 2f_8 = 85 + 2\times43 = 171$
$d = 171$