Individual Events

SI	***a***	900	**I1**	***a***	35	**I2**	***a***	7	**I3**	α	8	**I4**	***t***	13	**I5**	***a***	30
	b	7		***b***	7		***b***	3		***b***	16		***s***	4		***b***	150
	c	3		***c***	10		***c***	9		***A***	128		***a***	3		***n***	12
	d	5		***d***	2		***d***	5		***d***	7		***c***	12		***k***	24

Group Events

SG	***a***	2	**G6**	***n***	8	**G7**	***G***	1	**G8**	***y***	7	**G9**	***x***	40	**G10**	***a***	6
	b	9		***k***	5		***D***	8		***k***	–96		***y***	3		***x***	3
	p	23		***u***	35		***L***	2		***a***	1		***k***	8		***k***	2
	k	3		***a***	1		***E***	5		***m***	2		***r***	5		***y***	4

Sample Individual Event

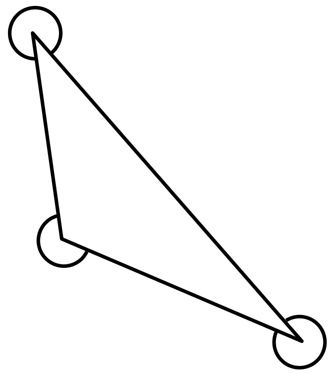
SI.1 In the given diagram, the sum of the three marked angles is $a°$. Find the value of a.

Reference: 1984 FSI.1 1987 FSG.3

Sum of interior angles of a triangle = 180°

angle sum of three vertices = 3×360° = 1080°

$a = 1080 - 180 = 900$



SI.2 The sum of the interior angles of a convex b-sided polygon is $a°$. Find the value of b.

Reference 1984 FSI.2

$a = 900 = 180\times(b-2)$

$b = 7$

SI.3 If $27^{b-1} = c^{18}$, find the value of c.

$3^{3(7-1)} = c^{18}$

$c = 3$

SI.4 If $c = \log_d 125$, find the value of d.

$3 = c = \log_d 125$

$d^3 = 125$

$d = 5$

Individual Event 1

I1.1 The obtuse angle formed by the hands of a clock at 10:30 is $(100 + a)°$. Find the value of a.

Reference 1984 FG7.1, 1985 FI3.1, 1987 FG7.1, 1990 FG6.3, 2007 HI1

At 10:00, the angle between the arms of the clock = 60°

From 10:00 to 10:30, the hour-hand had moved $360°\times\frac{1}{12}\times\frac{1}{2}=15°$.

The minute hand had moved 180°.

$100 + a = 180 - 60 + 15 = 135 \Rightarrow a = 35$

I1.2 The lines $ax + by = 0$ and $x - 5y + 1 = 0$ are perpendicular to each other. Find the value of b.

$-\frac{35}{b}\times\frac{1}{5}=-1$

$\Rightarrow b = 7$

I1.3 If $(b + 1)^4 = 2^{c+2}$, find the value of c.

$8^4 = 2^{c+2}$

$2^{3(4)} = 2^{c+2}$

$\Rightarrow c = 10$

I1.4 If $c - 9 = \log_c (6d - 2)$, find the value of d.

$10 - 9 = 1 = \log_{10} (6d - 2)$

$\Rightarrow 6d - 2 = 10$

$\Rightarrow d = 2$

Individual Event 2

I2.1 If $1000a = 85^2 - 15^2$, find the value of a.

$1000a = (85 + 15)(85 - 15) = 100\times70$

$\Rightarrow a = 7$

I2.2 The point (a, b) lies on the line $5x + 2y = 41$. Find the value of b.

$5(7) + 2b = 41$

$\Rightarrow b = 3$

I2.3 $x + b$ is a factor of $x^2 + 6x + c$. Find the value of c.

Put $x = -3$ into $x^2 + 6x + c = 0$

$(-3)^2 + 6(-3) + c = 0$

$\Rightarrow c = 9$

I2.4 If d is the distance between the points $(c, 1)$ and $(5, 4)$, find the value of d.

$d^2 = (9 - 5)^2 + (1 - 4)^2 = 25$

$\Rightarrow d = 5$

Individual Event 3

I3.1 If $\alpha + \beta = 11$, $\alpha\beta = 24$ and $\alpha > \beta$, find the value of α.

α and β are the roots of the equation $x^2 - 11x + 24 = 0$

$(x - 3)(x - 8) = 0$

$\because \alpha > \beta$

$\therefore \alpha = 8$

I3.2 If $\tan\theta = \frac{-\alpha}{15}$, $90° < \theta < 180°$ and $\sin\theta = \frac{b}{34}$, find the value of b.

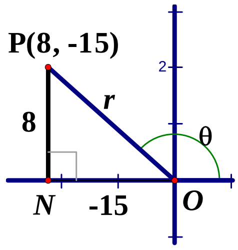
In the figure, $P = (8, -15)$

$r^2 = 8^2 + (-15)^2$ (Pythagoras' theorem)

$r = 17$

$\sin\theta = \frac{8}{17} = \frac{16}{34}$

$b = 16$



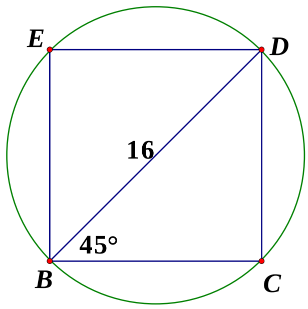
I3.3 If A is the area of a square inscribed in a circle of diameter b, find the value of A.

Reference: 1984 FG10.1, 1985 FSG.4

Let the square be *BCDE*.

$BC = 16\cos 45° = 8\sqrt{2}$

$A = \left(8\sqrt{2}\right)^2 = 128$



I3.4 If $x^2 + 22x + A \equiv (x + k)^2 + d$, where k, d are constants, find the value of d.

$x^2 + 22x + 128 \equiv (x + 11)^2 + 7$

$d = 7$

Individual Event 4

I4.1 The average of p, q, r is 12. The average of $p, q, r, t, 2t$ is 15. Find the value of t.

$p + q + r = 36$

$p + q + r + t + 2t = 75$

$3t = 75 - 36 = 39$

$t = 13$

I4.2 k is a real number such that $k^4 + \frac{1}{k^4} = t + 1$, and $s = k^2 + \frac{1}{k^2}$. Find the value of s.

$k^4 + \frac{1}{k^4} = 14$

$k^4 + 2 + \frac{1}{k^4} = 16$

$(k^2 + \frac{1}{k^2})^2 = 16$

$\Rightarrow s = k^2 + \frac{1}{k^2} = 4$

I4.3 M and N are the points (1, 2) and (11, 7) respectively. $P(a, b)$ is a point on MN such that $MP : PN = 1 : s$. Find the value of a.

$MP : PN = 1 : 4$

$a = \frac{4+11}{1+4} = 3$

I4.4 If the curve $y = ax^2 + 12x + c$ touches the x-axis, find the value of c.

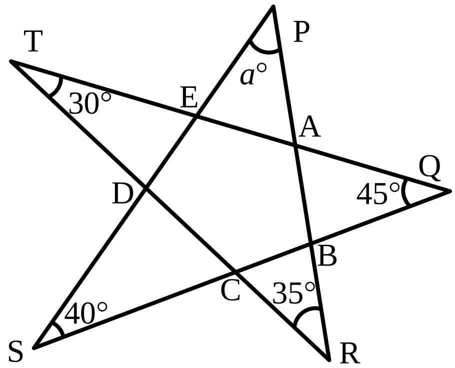
$y = 3x^2 + 12x + c$

$\Delta = 12^2 - 4(3)c = 0$

$\Rightarrow c = 12$

Individual Event 5

I5.1 In the figure, find the value of a.



Reference: 1997 FG1.1, 2005 FI2.3

Label the vertices *A*, *B*, *C*, *D*, *E*, *P*, *Q*, *R*, *S*, *T* as shown.

$\angle AEP = 40° + 45° = 85°$ (ext. $\angle$ of ΔSQE)

$\angle EAP = 30° + 35° = 65°$ (ext. $\angle$ of ΔTRA)

In ΔAEP, $85° + 65° + a° = 180°$ ($\angle$s sum of Δ)

$a = 30$

I5.2 If $\sin(a° + 210°) = \cos b°$, and $90° < b < 180°$, find the value of b.

$\sin 240° = -\frac{\sqrt{3}}{2} = \cos b°$

$b = 150$

I5.3 Each interior angle of an n-sided regular polygon is $b°$. Find the value of n.

Each exterior angle = 30° (adj. $\angle$s on st. line)

$\frac{360}{n} = 30$ (sum of exterior angles of polygon)

$\Rightarrow n = 12$

I5.4 If the n^{th} day of March in a year is Friday. The k^{th} day of March in the same year is Wednesday, where $20 < k < 25$, find the value of k.

12^{th} March is Friday

17^{th} March is Wednesday

24^{th} March is Wednesday

$\Rightarrow k = 24$

Sample Group Event

SG.1 If $2at^2 + 12t + 9 = 0$ has equal roots, find the value of a.

$(12)^2 - 4(2a)(9) = 0$

$\Rightarrow a = 2$

SG.2 If $ax + by = 1$ and $4x + 18y = 3$ are parallel, find the value of b.

Reference: 1986 FI4.2, 1987 FSG.4

$-\frac{2}{b} = -\frac{4}{18}$

$\Rightarrow b = 9$

SG.3 The b^{th} prime number is p. Find the value of p.

Reference: 1985 FSG.2, 1990 FI5.4

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, $\cdots$

$p = 23$

SG.4 If $k = \frac{4\sin\theta + 3\cos\theta}{2\sin\theta - \cos\theta}$ and $\tan\theta = 3$, find the value of k.

Reference: 1986 FG10.3, 1987 FG8.1, 1989 FG10.3, 1990 FG7.2

$$k = \frac{(4\sin\theta + 3\cos\theta) \div \cos\theta}{(2\sin\theta - \cos\theta) \div \cos\theta}$$

$$= \frac{4\tan\theta + 3}{2\tan\theta - 1}$$

$$= \frac{4(3) + 3}{2(3) - 1}$$

$$= 3$$

Group Event 6

G6.1 An n-sided convex polygon has 20 diagonals. Find the value of n.

Reference: 1984 FG10.3, 1985 FG8.3, 1988 FG6.2, 1991 FI2.3, 2001 FI4.2, 2005 FI1.4

Number of diagonals $= C_2^n - n = \frac{n(n-1)}{2} - n = 20$

$n^2 - 3n - 40 = 0$

$(n - 8)(n + 5) = 0$

$\Rightarrow n = 8$

G6.2 Two dice are thrown. The probability of getting a total of n is $\frac{k}{36}$. Find the value of k.

Total = 8

Favourable outcomes = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

P(total = 8) $= \frac{5}{36}$

$k = 5$

G6.3 A man drives at 25 km/h for 3 hours and then at 50 km/h for 2 hours.

His average speed for the whole journey is u km/h. Find the value of u.

$u = \frac{25\times 3 + 50\times 2}{3+2} = 35$

G6.4 If $a\Delta b = ab + 1$ and $(2\Delta a)\Delta 3 = 10$, find the value of a.

$2\Delta a = 2a + 1$

$(2\Delta a)\Delta 3 = (2a + 1)\Delta 3 = 3(2a + 1) + 1 = 10$

$6a + 4 = 10$

$a = 1$

Group Event 7

In the attached calculation, different letters represent different integers ranging from 1 to 9. If the letters O and J represent 4 and 6 respectively, find the values of **G7.1** G. **G7.2** D. **G7.3** L. **G7.4** E. Carry digit in the 100000 digit is 2 $G = 1, D = 8$	$\begin{array}{rcccccc} & G & O & L & D & E & N \\ \times & & & & & & J \\ \hline & D & E & N & G & O & L \end{array}$ $\begin{array}{rcccccc} & 1 & 4 & L & 8 & E & N \\ \times & & & & & & 6 \\ \hline & 8 & E & N & 1 & 4 & L \end{array}$
Carry digit in the hundreds digit is 3 $E = 5$ Carry digit in the tens digit is 4 $N = 7, L = 2$ $\therefore G = 1, D = 8, L = 2, E = 5$	$\begin{array}{rcccccc} & 1 & 4 & 2 & 8 & 5 & 7 \\ \times & & & & & & 6 \\ \hline & 8 & 5 & 7 & 1 & 4 & 2 \end{array}$

Group Event 8

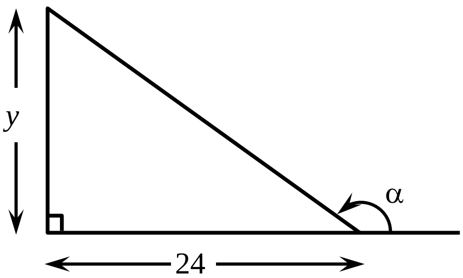
G8.1 If y is the greatest value of $\frac{14}{5+3\sin\theta}$, find the value of y.

$2 \le 5 + 3\sin\theta \le 8$

$\frac{14}{8} \le \frac{14}{5+3\sin\theta} \le \frac{14}{2}$

$\Rightarrow y = 7$

G8.2 In the figure, $100\cos\alpha = k$. Find the value of k.



Hypotenuse = 25

$k = -100\cos(\alpha - 180°)$

$= -100 \cdot \frac{24}{25} = -96$

G8.3 When $3x^2 + 4x + a$ is divided by $x + 2$, the remainder is 5. Find the value of a.

$3(-2)^2 + 4(-2) + a = 5$

$a = 1$

G8.4 The solution for $3t^2 - 5t - 2 < 0$ is $-\frac{1}{3} < t < m$. Find the value of m.

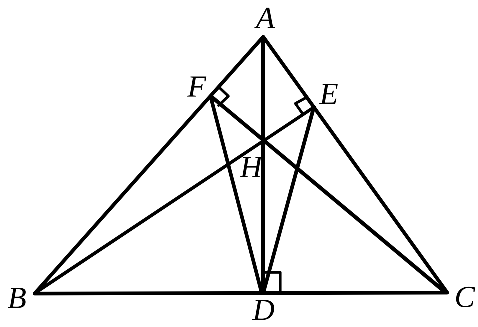
$(3t + 1)(t - 2) < 0$

$\Rightarrow -\frac{1}{3} < t < 2$

$\Rightarrow m = 2$

Group Event 9

G9.1 In the figure, $\angle BAC = 70°$ and $\angle FDE = x°$. Find the value of x.



$\angle AFC = 90° = \angle ADC$ (given)

$ACDF$ is a cyclic quad (converse, $\angle$s in the same seg.)

$\angle BDF = \angle BAC = 70°$ (ext. $\angle$, cyclic quad.)

$\angle AEB = 90° = \angle ADB$ (given)

$ABDE$ is a cyclic quad (converse, $\angle$s in the same seg.)

$\angle CDE = \angle BAC = 70°$ (ext. $\angle$, cyclic quad.)

$\angle FDE = 180° - \angle BDF - \angle CDE$ (adj. $\angle$s on st. line)

$= 180° - 70° - 70° = 40°$

$\Rightarrow x = 40$

G9.2 A cuboid is y cm wide, 6 cm long and 5 cm high. Its surface area is 126 cm^2. Find the value of y.

$2(5y + 6y + 5\times 6) = 126$

$11y = 33$

$y = 3$

G9.3 If $\log_9(\log_2 k) = \frac{1}{2}$, find the value of k.

$\log_2 k = \sqrt{9} = 3$

$k = 2^3 = 8$

G9.4 If $a : b = 3 : 8$, $b : c = 5 : 6$ and $a : c = r : 16$, find the value of r.

$a : b : c = 15 : 40 : 48$

$a : c = 15 : 48 = 5 : 16$

$\Rightarrow r = 5$

Group Event 10

G10.1 If $\frac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=3\sqrt{a}+6$, find the value of a.

Reference: 2014 FI4.1

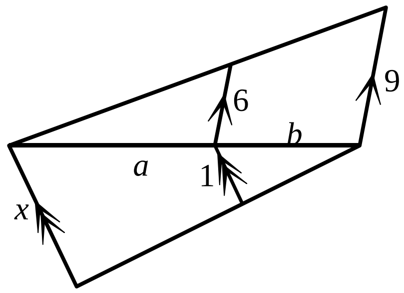
$\frac{6\sqrt{3}\left(3\sqrt{2}+2\sqrt{3}\right)}{18-12}=3\sqrt{a}+6$

$3\sqrt{6}+6=3\sqrt{a}+6$

$a = 6$

G10.2 In the figure, find the value of x.

Reference: 1994 FI4.3



By similar triangles

$6 : 9 = a : (a + b)$

$a = 2k$, $b = k$

$x : 1 = (a + b) : b = 3 : 1$

$x = 3$

G10.3 If $k=\frac{6\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta}{\cos^2\theta+\sin\theta\cos\theta+\sin^2\theta}$ and $\tan\theta = 2$, find the value of k.

Reference: 1986 FG10.3, 1987 FG8.1, 1989 FSG.4, 1990 FG7.2

$k=\frac{\left(6\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta\right)\div\cos^2\theta}{\left(\cos^2\theta+\sin\theta\cos\theta+\sin^2\theta\right)\div\cos^2\theta}$

$=\frac{6+2\tan\theta+\tan^2\theta}{1+\tan\theta+\tan^2\theta}$

$=\frac{6+2(2)+2^2}{1+2+2^2}$

$= 2$

G10.4 If $y=\frac{3\left(2^k\right)-4\left(2^{k-2}\right)}{2^k-2^{k-1}}$, find the value of y.

$y=\frac{3\left(2^k\right)-4\left(2^{k-2}\right)}{2^k-2^{k-1}}$

$=\frac{3-1}{1-\frac{1}{2}}$

$= 4$