Challenge Problem 2

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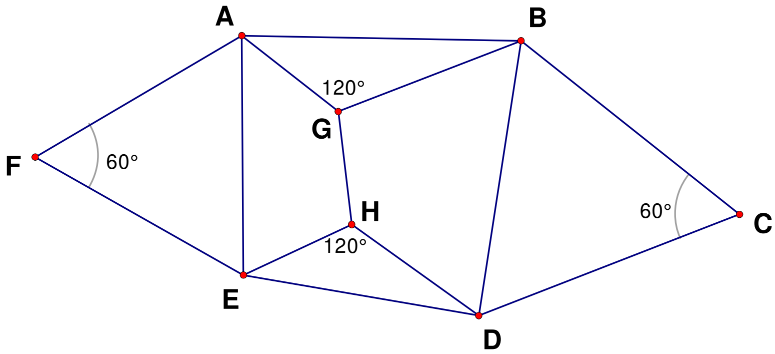
ABCDEF is a convex hexagon. $AB = BC = CD$, $DE = EF = FA$, $\angle BCD = \angle EFA = 60°$.

G and *H* are two points inside the hexagon such that $\angle AGB = \angle DHF = 120°$.

Prove that $AG + GB + GH + DH + HE \geq CF$.



Construct two equilateral Δs *PAB* and *QDE* outside the hexagon.

By the theorem of circle 7,

$PG = AG + BG$ ········· (1)

$QH = EH + DH$ ········· (2)

It is easy to see that $\Delta APB \cong \Delta BCD$ and $\Delta AFE \cong \Delta DQQ$ (S.A.S.)

$\Delta ABE \cong \Delta DBE$ (S.S.S.)

The figure is symmetric about *BE*.

$\therefore CF = PQ$

$AG + GB + GH + DH + HE$

$= PG + GH + HQ$ by (1) and (2)

$\geq PQ$ (shortest distance between *P* and *Q*)

$= CF$.

The problem is solved.