Individual Events

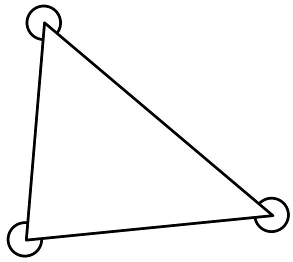
SI	***a***	900	**I1**	***a***	10	**I2**	***a***	$\frac{1}{2}$ (=0.5)	**I3**	***a***	–7	**I4**	***a***	15	**I5**	***a***	80
	b	7		***b***	1		***b***	5		***b***	6		***b***	8		***b***	4
	c	2		***c***	4		***c***	10		***x***	$\frac{1}{2}$ (=0.5)		***c***	4		***N***	10
	d	9		***d***	–5		***d***	15		***y***	–1		***d***	12		***x***	144

Group Events

SG	***a***	2	**G6**	***p***	10	**G7**	***p***	75	**G8**	***M***	1	**G9**	***x***	$\frac{1}{100}$	**G10**	***A***	50
	b	*136 see the remark		***q***	15		***q***	$\frac{1}{2}$ (=0.5)		***N***	6		***A***	52		***S***	2
	c	–6		***r***	24		***a***	2		***R***	8		***m***	501		***n***	7
	d	7		***s***	27		***m***	14		***Y***	2		***P***	36		***d***	5

Sample Individual Event (1988 Sample Individual Event)

SI.1 In the given diagram, the sum of the three marked angles is $a°$. Find a.



Reference: 1987 FSG.3, 1989 FSI.1

Sum of interior angles of a triangle = 180°

angle sum of three vertices = 3×360° = 1080°

a = 1080 – 180 = 900

SI.2 The sum of the interior angles of a regular b-sided polygon is $a°$. Find b.

Reference 1989 FSI.2

$a = 900 = 180\times(b-2)$

$b = 7$

SI.3 If $8^b = c^{21}$, find c.

$8^7 = c^{21}$

$2^{21} = c^{21}$

$c = 2$

SI.4 If $c = \log_d 81$, find d.

$2 = c = \log_d 81$ and $d > 0$

$d^2 = 81$

$d = 9$

Individual Event 1

I1.1 If $100a = 35^2 - 15^2$, find a.

Reference: 1987 FSG.1, 1988 FI2.2

$100a = (35 + 15)(35 - 15) = 50\times20 = 1000$

$a = 10$

I1.2 If $(a-1)^2 = 3^{4b}$, find b.

$9^2 = 3^{4b}$

$4b = 4$

$\Rightarrow b = 1$

I1.3 If b is a root of $x^2 + cx - 5 = 0$, find c.

Put $x = 1$ into the equation: $1 + c - 5 = 0$

$c = 4$

I1.4 If $x + c$ is a factor of $2x^2 + 3x + 4d$, find d.

$x + 4$ is a factor

Put $x = -4$ into the polynomial: $2(-4)^2 + 3(-4) + 4d = 0$

$d = -5$

Individual Event 2

I2.1 If α, β are roots of $x^2 - 10x + 20 = 0$, find a, where $a = \frac{1}{\alpha} + \frac{1}{\beta}$.

$\alpha + \beta = 10$, $\alpha\beta = 20$

$a = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{10}{20} = \frac{1}{2}$

I2.2 If $\sin\theta = a$ ($0° < \theta < 90°$), and $10\cos 2\theta = b$, find b.

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

$b = 10\cos 60° = 5$

I2.3 The point $A(b, c)$ lies on the line $2y = x + 15$. Find c.

Reference: 1984 FI2.3

Put $x = b = 5$, $y = c$ into the line: $2c = 5 + 15$

$c = 10$

I2.4 If $x^2 - cx + 40 \equiv (x + k)^2 + d$, find d.

Reference: 1985 FG10.2, 1986 FG7.3, 1987 FSI.1, 1988 FG9.3

$x^2 - 10x + 40 \equiv (x - 5)^2 + 15$

$k = -5$, $d = 15$

Individual Event 3

I3.1 If a is the remainder when $2x^3 - 3x^2 + x - 1$ is divided by $x + 1$, find a.

$a = 2(-1)^3 - 3(-1)^2 - 1 - 1 = -7$

I3.2 If b cm^2 is the total surface area of a cube of side $(8 + a)$ cm, find b.

Similar Questions: 1984 FG9.2, 1985 FSI.2

$8 + a = 1$

$b = 6$

I3.3 One ball is taken at random from a bag containing $b + 4$ red balls and $2b - 2$ white balls. If x is the probability that the ball is white, find x.

There are $b + 4 = 10$ red balls and $2b - 2 = 10$ white balls

$x = \frac{1}{2}$

I3.4 If $\sin\theta = x$ ($90° < \theta < 180°$) and $\tan(\theta - 15°) = y$, find y.

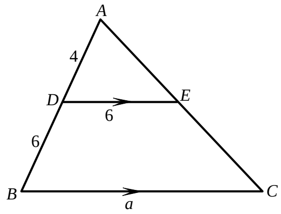
$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 150°$

$y = \tan(\theta - 15°) = \tan 135° = -1$

Individual Event 4

I4.1 In figure 1, DE // BC. If $AD = 4$, $DB = 6$, $DE = 6$ and $BC = a$, find a.

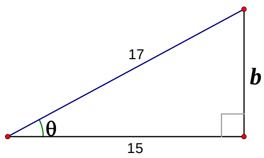


$\Delta ADE \sim \Delta ABC$ (equiangular)

$\frac{a}{6} = \frac{10}{4}$ (ratio of sides, ~Δ's)

$a = 15$

I4.2 θ is an acute angle such that $\cos\theta = \frac{a}{17}$. If $\tan\theta = \frac{b}{15}$, find b.



$b^2 = 17^2 - 15^2$

$b = 8$

I4.3 If $c^3 = b^2$, find c.

$c^3 = 8^2 = 64 = 4^3$

$\Rightarrow c = 4$

I4.4 The area of an equilateral triangle is $c\sqrt{3}$ cm^2. If its perimeter is d cm, find d.

Reference: 1985 FSI.4, 1986 FSG.3, 1987 FG6.2, 1988 FG9.1

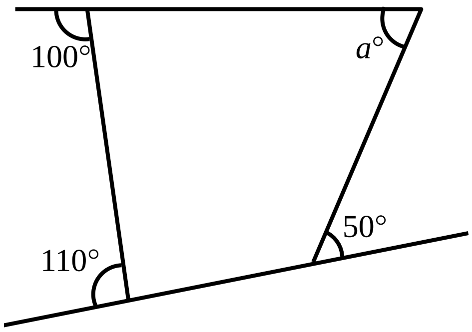
Each side $= \frac{d}{3}$ cm

$\frac{1}{2}\cdot\left(\frac{d}{3}\right)^2 \sin 60^\circ = c\sqrt{3} = 4\sqrt{3}$

$d = 12$

Individual Event 5

I5.1 In Figure 2, find a.



$100 + (180 - a) + 50 + 110 = 360$ (sum of ext. ∠ of Δ)

$a = 80$

I5.2 If $b = \log_2\left(\frac{a}{5}\right)$, find b.

$2^b = 16$

$b = 4$

I5.3 A piece of string, 20 m long, is divided into 3 parts in the ratio of $b - 2 : b : b + 2$. If N m is the length of the longest portion, find N.

$b - 2 : b : b + 2 = 2 : 4 : 6 = 1 : 2 : 3$

$N = 20\times\frac{3}{1+2+3} = 10$

I5.4 Each interior angle of an N-sided regular polygon is $x°$. Find x.

$x = \frac{180\times(10-2)}{10} = 144$

Sample Group Event

SG.1 The sum of 2 numbers is 20, their product is 10. If the sum of their reciprocals is a, find a.

Reference: 1983 FG6.3, 1985 FSI.1, 1986 FSG.1

Let the 2 numbers be x and y.

$x + y = 20$ and $xy = 10$

$a = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = 2$

SG.2 $1^2 - 1 = 0\times2$, $2^2 - 1 = 1\times3$, $3^2 - 1 = 2\times4$, … , $b^2 - 1 = 135\times137$. If $b > 0$, find b.

Reference: 1983 FI10.1, 1991 FI2.1

$135\times137 = (136 - 1)\times(136 + 1) = 136^2 - 1$

$b = 136$

Remark The original question is:

$1^2 - 1 = 0\times2$, $2^2 - 1 = 1\times3$, $3^2 - 1 = 2\times4$, … , $b^2 - 1 = 135\times137$, find b.

$b = 136$ or -136, there are 2 different answers!

SG.3 If the lines $x + 2y + 1 = 0$ and $cx + 3y + 1 = 0$ are perpendicular, find c.

Reference: 1983 FG9.3, 1985 FI4.1, 1986 FSG.2, 1987 FG10.2, 1988 FG8.2

$-\frac{1}{2}\times\left(-\frac{c}{3}\right) = -1 \Rightarrow c = -6$

SG.4 The points $(2, -1)$, $(0, 1)$, (c, d) are collinear. Find d.

Reference: 1984 FG7.3, 1986FG6.2, 1987 FG7.4, 1989 HG8

$\frac{d-1}{-6} = \frac{1-(-1)}{0-2}$

$d = 7$

Group Event 6

G6.1 If $p = \frac{21^3 - 11^3}{21^2 + 21\times11 + 11^2}$, find p. **(Similar questions: 1985 FG7.1)**

$p = \frac{21^3 - 11^3}{21^2 + 21\times11 + 11^2} = \frac{(21-11)(21^2 + 21\times11 + 11^2)}{21^2 + 21\times11 + 11^2} = 10$

G6.2 If p men can do a job in 6 days and 4 men can do the same job in q days, find q.

10 men can do a job in 6 days.

1 man can do a job in 60 days

4 men can do a job in 15 days $\Rightarrow q = 15$

G6.3 If the q^{th} day of March in a year is Wednesday and the r^{th} day of March in the same year is Friday, where $18 < r < 26$, find r. **(Reference: 1985 FG9.3, 1987 FG6.4, 1988 FG10.2)**

15^{th} March is Wednesday

17^{th} March is Friday

24^{th} March is Friday $\Rightarrow r = 24$

G6.4 If $a*b = ab + 1$, and $s = (3*4)*2$, find s. **(Reference: 1985 FSG.1)**

$3*4 = 3\times4 + 1 = 13$

$s = (3*4)*2 = 13*2 = 13\times2 + 1 = 27$

Group Event 7 (1988 Sample Group Event)

G7.1 The acute angle between the 2 hands of a clock at 3:30 a.m. is $p°$. Find p.

Reference: 1985 FI3.1 1987 FG7.1, 1989 FI1.1, 1990 FG6.3, 2007 HI1

At 3:00 a.m., the angle between the arms of the clock = 90°

From 3:00 a.m. to 3:30 a.m., the hour-hand had moved $360° \times \frac{1}{12} \times \frac{1}{2} = 15°$.

The minute hand had moved 180°.

$p = 180 - 90 - 15 = 75$

G7.2 In ΔABC, $\angle B = \angle C = p°$. If $q = \sin A$, find q.

$\angle B = \angle C = 75°$, $\angle A = 180° - 75° - 75° = 30°$

$q = \sin 30° = \frac{1}{2}$

G7.3 The 3 points (1, 3), (a, 5), (4, 9) are collinear. Find a.

Reference: 1984 FSG.4, 1986FG6.2, 1987 FG7.4, 1989 HG8

$\frac{9-5}{4-a} = \frac{9-3}{4-1} = 2$

$\Rightarrow a = 2$

G7.4 The average of 7, 9, x, y, 17 is 10. If m is the average of $x + 3$, $x + 5$, $y + 2$, 8, y +18, find m.

$\frac{7+9+x+y+17}{5} = 10$

$\Rightarrow x + y = 17$

$m = \frac{x+3+x+5+y+2+8+y+18}{5}$

$= \frac{2(x+y)+36}{5} = 14$

Group Event 8

In the addition shown, each letter represents a different digit ranging from zero to nine. It is already known that

$S = 9$, O = zero, $E = 5$.

$$\begin{array}{cccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array}$$

Find the numbers represented by

(i) M, (ii) N, (iii) R, (iv) Y

Consider the thousands digit and the ten thousands digits.

$0 \le S, M \le 9$, $9 + M = 10M + 0$ or $9 + M + 1 = 10M + 0$

$\Rightarrow M = 1$ and there is no carry digit.

Consider the hundreds digit. $5 + 0 + 1 = N$

$\Rightarrow N = 6$ and there is a carry digit.

For the tens digit. $6 + R = 10 + 5$

$\Rightarrow R = 9$ (same as S, rejected) or $6 + R + 1 = 10 + 5$

$\Rightarrow R = 8$

There is a carry digit in the unit digit

$D + 5 = 10 + Y$, $(D, Y) = (7, 2) \Rightarrow Y = 2$

$\therefore M = 1, N = 6, R = 8, Y = 2$

Group Event 9

G9.1 If $x=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{100}\right)$, find x in the simplest fractional form.

Reference: 1985 FSG.3, 1986 FG10.4

$x=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{99}{100}=\frac{1}{100}$

G9.2 The length, width and height of a rectangular block are 2, 3 and 4 respectively. Its total surface area is A, find A.

Similar Questions: 1984 FI3.2, 1985 FSI.2

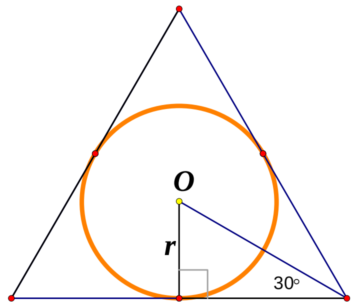
$A = 2\times(2\times3 + 3\times4 + 2\times4) = 52$

G9.3 The average of the integers 1, 2, 3, ... , 1001 is m. Find m.

$m=\frac{1}{1001}(1+2+3+\cdots+1001)$

$=\frac{1}{1001}\cdot\frac{(1+1001)\cdot1001}{2}=501$

G9.4 The area of a circle inscribed in an equilateral triangle is 12π. If P is the perimeter of this triangle, find P.



Reference: 1990 FI2.3

Let the radius be r and the centre be O.

$\pi r^2 = 12\pi$

$\Rightarrow r=2\sqrt{3}$

The length of one side of the equilateral triangle is $\frac{P}{3}$.

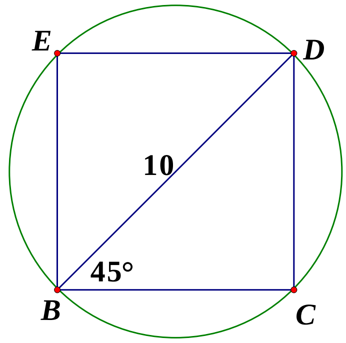
$\frac{P}{3}=2r\cot 30°$

$=2\sqrt{3}r=12$

$P = 36$

Group Event 10

G10.1 If A is the area of a square inscribed in a circle of diameter 10, find A.



Reference: 1985 FSG.4, 1989 FI3.3

Let the square be $BCDE$.

$BC = 10\cos 45° = 5\sqrt{2}$

$A = \left(5\sqrt{2}\right)^2 = 50$

G10.2 If $a + \frac{1}{a} = 2$, and $S = a^3 + \frac{1}{a^3}$, find S.

Reference: 1998 HG1

$a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2 = 4 - 2 = 2$

$S = a^3 + \frac{1}{a^3}$

$= \left(a + \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$

$= 2(2 - 1) = 2$

G10.3 An n-sided convex polygon has 14 diagonals. Find n.

Reference: 1985 FG8.3, 1988 FG6.2, 1989 FG6.1, 1991 FI2.3, 2001 FI4.2, 2005 FI1.4

Number of diagonals $= C_2^n - n = \frac{n(n-1)}{2} - n = 14$

$n^2 - 3n - 28 = 0$

$(n - 7)(n + 4) = 0$

$\Rightarrow n = 7$

G10.4 If d is the distance between the 2 points (2, 3) and (−1, 7), find d.

Reference: 1986 FG9.4

$d = \sqrt{[2-(-1)]^2 + (3-7)^2} = 5$