

# Hong Kong Mathematics Olympiad 2004-2005

## Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 若  $p$  和  $q$  是正整數且  $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$ ，求  $q$  最小可能的值。

Suppose  $p, q$  are positive integers and  $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$ , find the smallest possible value of  $q$ .

2. 已知  $x = 2005$  及  $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$ ，求  $y$  的值。

Given that  $x = 2005$  and  $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$ , find the value of  $y$ .

3. 若  $x$  是實數且滿足  $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$ ，求  $x$  的最小可能的值。

If  $x$  is a real number satisfying the equation  $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$ ,

find the smallest possible value of  $x$ .

4. 設  $t$  為實數且滿足  $(1 + \sin t)(1 + \cos t) = \frac{5}{4}$ 。若  $N = \sin t + \cos t$ ，求  $N$  的值。

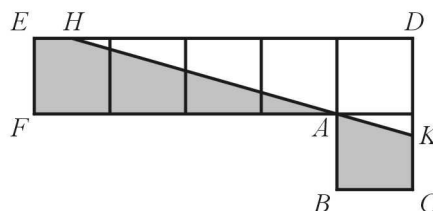
Let  $t$  be a real number satisfying  $(1 + \sin t)(1 + \cos t) = \frac{5}{4}$ .

If  $N = \sin t + \cos t$ , find the value of  $N$ .

5. 如圖一， $ABCDEF$  是由六個正方形所組成的“L 形”圖案。 $HAK$  是一直線，陰影部分的面積是  $ABCDEF$  的面積的  $\frac{1}{2}$ 。若各小正方形的邊長是 1 cm， $HK$  的長度是  $m$  cm，求  $m$  的值。

In Figure 1,  $ABCDEF$  is a “L shape” figure formed by six squares.  $HAK$  is a straight line and the area of the shaded region is equal to  $\frac{1}{2}$  of the area of  $ABCDEF$ .

If the length of each small square is 1 cm and the length of  $HK$  is  $m$  cm, find the value of  $m$ .



圖一

Figure 1

6. 設  $n$  是自然數，直線  $nx + (n+1)y = \sqrt{2}$  與兩坐標軸所圍成的三角形的面積是  $S_n$ 。

若  $K = S_1 + S_2 + \dots + S_{2005}$ ，求  $K$  的值。

Let  $n$  be a natural number, the area of the triangle bounded by the line  $nx + (n+1)y = \sqrt{2}$  and the two coordinate axes is  $S_n$ . If  $K = S_1 + S_2 + \dots + S_{2005}$ , find the value of  $K$ .

7. 設  $[x]$  表示不大於  $x$  的最大整數，例如  $[2.5] = 2$ 。若  $M = \sum_{n=1}^{1024} [\log_2 n]$ ，求  $M$  的值。

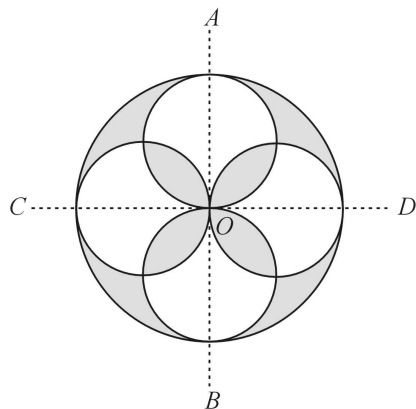
Let  $[x]$  be the largest integer not greater than  $x$ , for example,  $[2.5] = 2$ .

If  $M = \sum_{n=1}^{1024} [\log_2 n]$ , find the value of  $M$ .

8. 如圖二， $AB$  垂直於  $CD$ ，其交點  $O$  是大圓的圓心，而四個小圓的圓心分別在  $AB$  和  $CD$  上。已知大圓的半徑是 1 cm，四個小圓的半徑是  $\frac{1}{2}$  cm。若陰影部分的面積是  $R \text{ cm}^2$ ，求  $R$  的值。(取  $\pi = 3$ )

In Figure 2,  $AB$  is perpendicular to  $CD$ , their intersection point  $O$  is the center of the large circle and the centers of the four circles lies on either  $AB$  or  $CD$ . Given also that the radius of the large circle is 1 cm and the radius of each of the four small circles is  $\frac{1}{2}$  cm.

If the area of the shaded region is  $R \text{ cm}^2$ , find the value of  $R$ . (take  $\pi = 3$ )



圖二

Figure 2

9. 已知  $60^a = 3$  及  $60^b = 5$ 。若  $R = 12^{\frac{1-a-b}{2(1-b)}}$ ，求  $R$  的值。

Given that  $60^a = 3$  and  $60^b = 5$ . If  $R = 12^{\frac{1-a-b}{2(1-b)}}$ , find the value of  $R$ .

10. 已知 2005 年 1 月 29 日是星期六，那麼 2008 年 1 月 29 日是星期幾？  
Given that 29<sup>th</sup> January 2005 is Saturday, on what day is 29<sup>th</sup> January 2008?

**Hong Kong Mathematics Olympiad 2004-2005**  
**Heat Event (Group)**

除非特別聲明，答案須用數字表達，並化至最簡。

時限：20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 若  $x = \frac{19}{97} + \frac{19}{97} \times 2 + \frac{19}{97} \times 3 + \cdots + \frac{19}{97} \times 10$  及  $a$  是最接近  $x$  的整數，求  $a$  的值。

Let  $x = \frac{19}{97} + \frac{19}{97} \times 2 + \frac{19}{97} \times 3 + \cdots + \frac{19}{97} \times 10$  and  $a$  is the integer that is the closest to  $x$ ,  
find the value of  $a$ .

2. 已知正方形  $ABCD$  的面積是  $130 \text{ cm}^2$  及圓  $O$  經過點  $A$ 、 $B$ 、 $C$  及  $D$ 。  
若圓  $O$  的面積是  $b \text{ cm}^2$ ，求  $b$  的值。(取  $\pi = 3.14$ )

Given that the area of a square  $ABCD$  is equal to  $130 \text{ cm}^2$  and a circle  $O$  passes through the points  $A$ ,  $B$ ,  $C$  and  $D$ . If the area of the circle  $O$  is  $b \text{ cm}^2$ , find the value of  $b$ . (Take  $\pi = 3.14$ )

3. 已知  $p$ 、 $q$  和  $r$  是方程  $x^3 - x^2 + x - 2 = 0$  的三個不同的根。若  $Q = p^3 + q^3 + r^3$ ，求  $Q$  的值。  
Given that  $p$ ,  $q$  and  $r$  are distinct roots of the equation  $x^3 - x^2 + x - 2 = 0$ .  
If  $Q = p^3 + q^3 + r^3$ , find the value of  $Q$ .

4. 當一個三位數減去它的各個數位的數字的和，其差還是一個三位數  $\overline{46x}$ ，求  $x$  的值。

When a 3-digit number minus the sum of the values of the three digits, the difference is a 3-digit number  $\overline{46x}$ , find the value of  $x$ .

5. 若  $B$  是整數且  $B > (\sqrt{2} + \sqrt{3})^6$ ，求  $B$  最小可能的值。

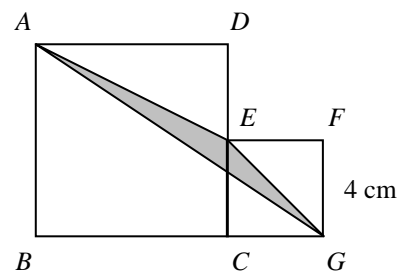
If  $B$  is an integer and  $B > (\sqrt{2} + \sqrt{3})^6$ , find the smallest possible value of  $B$ .

6. 若一正八面體的邊長為  $1 \text{ cm}$ ，其體積為  $f \text{ cm}^3$ ，求  $f$  的值。

Suppose the side of a regular octahedron is equal to  $1 \text{ cm}$  and the volume is equal to  $f \text{ cm}^3$ ,  
find the value of  $f$ .

7. 如圖一， $ABCD$  和  $CEFG$  是兩個正方形， $FG = 4$  cm。若  $\triangle AEG$  的面積是  $g$   $\text{cm}^2$ ，求  $g$  的值。

In Figure 1,  $ABCD$  and  $CEFG$  are two squares and  $FG = 4$  cm. If the area of  $\triangle AEG$  is equal to  $g$   $\text{cm}^2$ , find the value of  $g$ .



圖一

Figure 1

8. 設  $x$  為實數。若  $h$  是  $x$  的最大值使得  $2(\log_{\frac{1}{2}} x)^2 + 9\log_{\frac{1}{2}} x + 9 \leq 0$ ，求  $h$  的值。

Let  $x$  be a real number. If  $h$  is the greatest value of  $x$  such that  $2(\log_{\frac{1}{2}} x)^2 + 9\log_{\frac{1}{2}} x + 9 \leq 0$ ,

find the value of  $h$ .

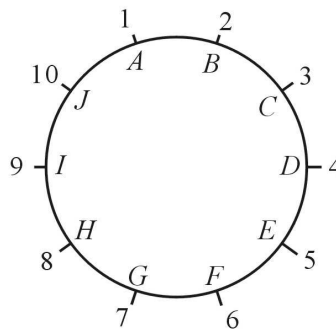
9. 已知在三角形  $ABC$  內的一點  $O$  到三角形三邊的垂線的長度均為 2 cm，而  $\triangle ABC$  的周界為 21 cm。若  $\triangle ABC$  的面積是  $k$   $\text{cm}^2$ ，求  $k$  的值。

Given that the perpendicular distances from the point  $O$  to three sides of a triangle  $ABC$  are all equal to 2 cm and the perimeter of  $\triangle ABC$  is equal to 21 cm.

If the area of  $\triangle ABC$  is equal to  $k$   $\text{cm}^2$ , find the value of  $k$ .

10. 如圖二，十人圍成一圈，並依座號 1, 2, 3, ..., 10 而坐。每人選擇一個整數，分別是  $A, B, C, \dots, J$ ，並將這個數字告訴他左右兩個鄰座的人。每人跟着算出他左右兩個鄰座所選的數的算術平均數。若各人所算出的平均數與其座號相等，求  $F$  的值。

In Figure 2, ten people are sitting in a round table with sitting numbers 1, 2, 3, ..., 10 respectively. Each of them chooses an integer  $A, B, C, \dots, J$  respectively and tells the people on his left and right about his chosen number. Then each of them calculates the average number of the chosen numbers of his two neighborhoods and announces this average number. If all the announced average numbers are the same as the corresponding sitting numbers, find the value of  $F$ .



圖二

Figure 2