xy + x + y (HKMO Classified Questions by topics)

#### 1985 FG8.4

If y = ab + a + b + 1 and a = 99, b = 49, find the value of y.

## 1986 FG9.3

If S = ab + a - b - 1 and a = 101, b = 49, find the value of S.

## 1987 FG10.4

If P, Q are positive integers such that P + Q + PQ = 90 and D = P + Q, find the value of D. (Hint: Factorise 1 + P + Q + PQ)

## 1988 FG6.3

若 y = ab - a + b - 1,且 a = 49, b = 21,求 y 的值。

If y = ab - a + b - 1 and a = 49, b = 21, find the value of y.

## 1990 FG9.1

If S = ab - 1 + a - b and a = 101, b = 9, find the value of S.

#### 1995 HI6

有多少對正整數  $x \cdot y$  可滿足 xy - 3x - 2y = 10?

How many pairs of positive integers x, y are there satisfying xy - 3x - 2y = 10?

# 1998 FG3.3

若方程x+y+2xy=141有c個正整數解,求c的值。

If the number of positive integral root(s) of the equation x + y + 2xy = 141 is c, find the value of c.

# 1999 FI2.2

設  $u \rightarrow t$  為正整數使得 u + t + ut = 34,若 b = u + t,求 b 之值。

Let u and t be positive integers such that u + t + ut = 34.

If b = u + t, find the value of b.

# 2002 HG9

已知正整數  $a \cdot b$  满足方程 ab-a-b=12, 求 ab 的值。

Given that a and b are positive integers satisfying the equation ab - a - b = 12, find the value of ab.

# 2004 HG6

 $\ddot{a}$  a b b c 是正整數且 abc+ab+bc+ac+a+b+c=2003,求 abc 的最小值。

If a, b and c are positive integers such that

abc + ab + bc + ac + a + b + c = 2003, find the least value of abc.

2006 FI3.3

已知整數 x 和 y 满足 15xy = 21x + 20y - 13。若 T = xy,求 T 的值。 Given that x and y are integers satisfying the equation 15xy = 21x + 20y - 13. If T = xy, find the value of T.

## 2006 FG2.4

已知 x 和 y 是正整數及 x+y+xy=54。若 t=x+y,求 t 的值。 Given that x and y are positive integers and x+y+xy=54.

If t = x + y, find the value of t.

## 2011 HI9

某長方形的長和濶均為整數。若面積比周界大9,求周界的值。

The length and the width of a rectangle are integers.

If its area is larger than its perimeter by 9, find the perimeter.

#### 2011 HG8

已知  $a \cdot b \cdot c$  及 d 為非負整數,且  $ac+bd+ad+bc=2011 \circ 求 <math>a+b+c+d$  的值。 Given that a, b, c and d are non-negative integers and ac+bd+ad+bc=2011. Find the value of a+b+c+d.

#### 2012 HG8

已知方程 $x^2+px+q=0$ 的兩個根為正整數,且q>0。若p+q=60,求q的值。 Given that the roots of the equation  $x^2+px+q=0$  are integers and q>0. If p+q=60, find the value of q.

# 2012 FI4.2

If x and y be positive integers such that x > y > 1 and xy = x + y + 22.

Let  $B = \frac{x}{v}$ , find the value of B.

# 2016 HG4

設 x 及 y 為正整數且滿足  $\log x + \log y = \log(2x - y) + 1$ ,求 (x, y) 的數量。 If x and y are positive integers that satisfy  $\log x + \log y = \log(2x - y) + 1$ , find the number of possible pairs of (x, y).

# 2018 FI4.3

若對於正整數 x>y>z,  $xyz+xy+xz+yz+x+y+z+1=30\times9+87$ 。 求 c=x+y+z 的值。

If  $xyz + xy + xz + yz + x + y + z + 1 = 30 \times 9 + 87$  for positive integers x > y > z, determine the value of c = x + y + z.

#### 2019 HG9

有多少對正整數  $x \cdot y$  可满足  $xy = 6\left(x + y + \sqrt{x^2 + y^2}\right)$ ?

How many pairs of positive integers x, y are there satisfying  $xy = 6(x + y + \sqrt{x^2 + y^2})$ ?

#### 2019 FI1.2

若 x 和 y 為正整數,並且滿足以下等式  $\log_{10} x + \log_{10} y = \log_{10} (2x - 80y) + 1$ , 而 B 是 (x, y) 所有可能組合的數量,求 B 的值。

If x and y are positive integers that satisfy  $\log_{10} x + \log_{10} y = \log_{10} (2x - 80y) + 1$ , and B is the number of possible pairs of (x, y), determine the value of B.

# 2021 P1Q7

 $p \cdot q$  及 r 為質數。若 pqr = 7(p+q+r),求 p+q+r 的值。 p,q and r are prime numbers. If pqr = 7(p+q+r), find the value of p+q+r. 2022 P2O2

# 2022 P2Q2

 $\overline{AB}$  和  $\overline{CB}$  均為兩位正整數,其中  $A \cdot B$  和 C 是不同的數字。 設  $d = \overline{AB} + \overline{CB}$  。若  $\overline{AB} \times \overline{CB} = \overline{BCBB}$  是四位數,求 d 的值。

Both  $\overline{AB}$  and  $\overline{CB}$  are two-digit positive integers, where A, B and C are different digits. Let  $d = \overline{AB} + \overline{CB}$ . If  $\overline{AB} \times \overline{CB} = \overline{BCBB}$  is a four-digit number, find the value of d.

# 2022 P2Q3

假設方程  $x^2y - 2x^2 - 3y - 13 = 0$  只有一對正整數解  $(x_0, y_0)$ 。 若  $a = v_0 - x_0$ ,求 a 的值。

Suppose the equation  $x^2y - 2x^2 - 3y - 13 = 0$  has only one pair of positive integral solution  $(x_0, y_0)$ . If  $a = y_0 - x_0$ , find the value of a.

# 2023 HI2

若 x 及 y 均為正整數且滿足 x + 8xy + y = 28,求 x + 2y 的最大可能值。 If x and y are positive integers satisfying x + 8xy + y = 28, find the largest possible value of x + 2y.

# 2023 FI3.1

已知 m 和 n 均為正整數。如果 m+n+mn=76 及 A=m+n,求 A 的值。 Given that m and n are positive integers. If m+n+mn=76 and A=m+n, find the value of A.

# Answers

1985 FG8.4	1986 FG9.3	1987 FG10.4	1988 FG6.3	1990 FG9.1
5000	5000	18	1000	1000
1995 HI6	1998 FG3.3	1999 FI2.2	2002 HG9	2004 HG6
5	0	10	28	500
2006 FI3.3	2006 FG2.4	2011 HI9	2011 HG8	2012 HG8
6	14	36	2012	124
2012 FI4.2	2016 HG4	2018 FI4.3	2019 HG9	2019 FI1.2
12	6	24	12	7
2021 P1Q7	2022 P2Q2	2022 P2Q3	2023 HI2	2023 FI3.1
15	112	19	7	16