

# Hong Kong Mathematics Olympiad (1993 – 94)

## Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 某兩數之和為 40，其積為 20。若該兩數倒數之和為  $a$ ，求  $a$  的值。

The sum of two numbers is 40, their product is 20.

If the sum of their reciprocals is  $a$ , find the value of  $a$ .

$a =$

- (ii) 若一邊長  $(a+1)$  厘米之正方體之總表面積為  $b$  平方厘米，求  $b$  的值。

If  $b \text{ cm}^2$  is the total surface area of a cube of side  $(a+1) \text{ cm}$ , find the value of  $b$ .

$b =$

- (iii) 一袋內有  $(b-4)$  個白球， $(b+46)$  個紅球。若隨意於袋內取一球，而該球為白色之概率為  $\frac{c}{6}$ ，求  $c$  的值。

One ball is taken at random from a bag containing  $(b-4)$  white balls and  $(b+46)$  red balls. If  $\frac{c}{6}$  is the probability that the ball is white, find the value of  $c$ .

$c =$

- (iv) 若一邊長  $c$  厘米之正三角形之面積  $d\sqrt{3}$  平方厘米，求  $d$  的值。

The length of a side of an equilateral triangle is  $c \text{ cm}$ . If its area is  $d\sqrt{3} \text{ cm}^2$ , find the value of  $d$ .

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

$\times$

Mult. factor for  
speed

$=$

Team No.

$+$

Bonus  
score

Time



Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 方程式  $x^2 - ax + (a + 3) = 0$  有等根。若  $a$  為一正整數，求  $a$  的值。

The equation  $x^2 - ax + (a + 3) = 0$  has equal roots. Find the value of  $a$ , if  $a$  is a positive integer.

- (ii) 在一次測驗中，共 20 題。做對一題給  $a$  分，做錯一題要倒扣 3 分。一學生做了全部的 20 題，而得到 48 分。他答對了的題目數目是  $b$ 。求  $b$  的值。

In a test, there are 20 questions.  $a$  marks will be given to a correct answer and 3 marks will be deducted for each wrong answer. A student has done all the 20 questions and scored 48 marks. Find  $b$ , the number of questions that he has answered correctly.

- (iii) 若

$$x : y = 2 : 3$$

$$x : z = 4 : 5$$

$$y : z = b : c,$$

求  $c$ 。

If

$$x : y = 2 : 3$$

$$x : z = 4 : 5$$

$$y : z = b : c,$$

find  $c$ .

- (iv) 設  $P(x, d)$  為直線  $x + y = 22$  上的點，且  $OP$  的斜率為  $c$  ( $O$  為原點)。求  $d$  的值。

Let  $P(x, d)$  be a point on the straight line  $x + y = 22$  such that the slope of  $OP$  equals to  $c$  ( $O$  is the origin). Determine the value of  $d$ .

### FOR OFFICIAL USE

Score for  
accuracy

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Mult. factor for  
speed

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Team No.

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Bonus  
score

Time

Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

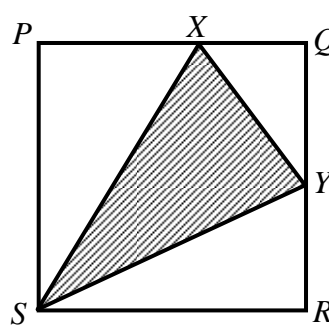
## Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在正方形  $PQRS$  中， $Y$  為  $QR$  之中點，且  $PX = \frac{3}{4}PQ$ 。若  $A$  為陰影部分三角形面積與正方形面積的比，求  $A$  的值。

In square  $PQRS$ ,  $Y$  is the mid-point of the side  $QR$  and  $PX = \frac{3}{4}PQ$ . If  $A$  is the ratio of the area of the shaded triangle to the area of the square, find the value of  $A$ .



$A =$

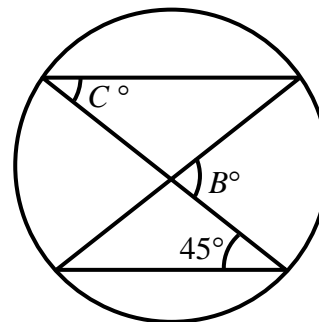
- (ii) 某甲買了一些乒乓球，需多付出銷售稅  $16A\%$ 。若他毋須付稅，則可用同等金錢多買 3 個乒乓球。假設  $B$  是他所買乒乓球的個數，求  $B$  的值。

A man bought a number of ping-pong balls where a  $16A\%$  sales tax is added. If he did not have to pay tax he could have bought 3 more balls for the same amount of money. If  $B$  is the total number of balls that he bought, find the value of  $B$ .

$B =$

- (iii) 如圖，求  $C$  的值。

Refer to the diagram, find the value of  $C$ .



$C =$

- (iv)  $2C$  個連續偶數之和為 1170。若  $D$  為其中最大之偶數，求  $D$  的值。

The sum of  $2C$  consecutive even numbers is 1170. If  $D$  is the largest of them, find the value of  $D$ .

$D =$

### FOR OFFICIAL USE

Score for accuracy

$\times$

Mult. factor for speed

$=$

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Bonus score

Time



Total score

Min.

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# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $183a8$  為  $287$  的倍數，求  $a$  的值。

If  $183a8$  is a multiple of  $287$ , find the value of  $a$ .

$a =$

- (ii)  $a^2$  這個數共有  $b$  個正因數，求  $b$  的值。

The number of positive factors of  $a^2$  is  $b$ , find the value of  $b$ .

$b =$

- (iii) 瓶中有球  $c$  個，其中  $b$  個是黑色或紅色的， $(b + 2)$  個是紅色或白色的，而黑色或白色的有  $12$  個。求  $c$  的值。

In an urn, there are  $c$  balls,  $b$  of them are either black or red,  $(b + 2)$  of them are either red or white and  $12$  of them are either black or white. Find the value of  $c$ .

$c =$

- (iv) 已知對所有  $x$ ， $f(3 + x) = f(3 - x)$ ，且方程式  $f(x) = 0$  有  $c$  個不等根，求所有根的總和  $d$ 。

Given  $f(3 + x) = f(3 - x)$  for all values of  $x$ , and the equation  $f(x) = 0$  has exactly  $c$  distinct roots. Find  $d$ , the sum of these roots.

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

×

Mult. factor for  
speed

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Team No.

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Bonus  
score

Time

Total score

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Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 4 (Individual)

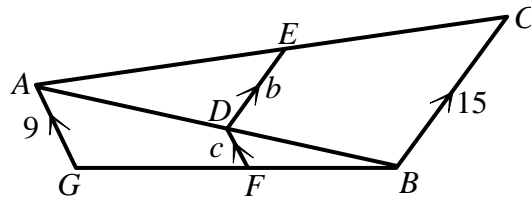
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i)  $x^6 - 8x^3 + 6$  除以  $(x-1)(x-2)$ ，其餘數為  $7x - a$ ，求  $a$  的值。  
The remainder when  $x^6 - 8x^3 + 6$  is divided by  $(x-1)(x-2)$  is  $7x - a$ , find the value of  $a$ .

- (ii) 若  $x^2 - x + 1 = 0$  及  $b = x^3 - 3x^2 + 3x + a$ ，求  $b$  的值。  
If  $x^2 - x + 1 = 0$  and  $b = x^3 - 3x^2 + 3x + a$ , find the value of  $b$ .

- (iii) 如圖，求  $c$  的值。  
Refer to the diagram, find the value of  $c$ .




- (iv) 有  $c$  個兒童，他們均生於一九九零年六月，若果他們生於不同日子的概率是  $\frac{d}{225}$ ，求  $d$  的值。  
If  $c$  boys were all born in June 1990 and the probability that their birthdays are all different is  $\frac{d}{225}$ , find the value of  $d$ .

### FOR OFFICIAL USE

Score for accuracy

×

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Bonus score

Time



Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知  $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ 。若  $A = \frac{2}{x}$ ，求  $A$  的值。

Given  $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ . If  $A = \frac{2}{x}$ , find the value of  $A$ .

$A =$

- (ii) 若  $B$  條內直徑為  $A$  厘米的圓形水管的輸水量與一內直徑為 6 厘米的圓形水管相等，求  $B$  的值。

If  $B$  circular pipes each with an internal diameter of  $A$  cm carry the same amount of water as a pipe with an internal diameter 6 cm, find the value of  $B$ .

$B =$

- (iii) 若一個由  $x$  軸、 $y$  軸及直線  $Bx + 9y = 18$  所圍成之三角形之面積為  $C$ ，求  $C$  的值。

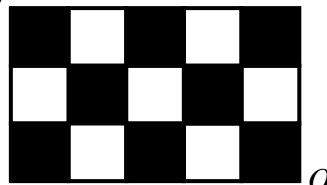
If  $C$  is the area of the triangle formed by  $x$ -axis,  $y$ -axis and the line  $Bx + 9y = 18$ , find the value of  $C$ .

$C =$

- (iv) 十五塊邊長為  $10C$  單位的正方形磚如圖排列。一  $P$  蟻沿磚之邊緣爬行，而其左邊必為一黑磚。求  $D$ ，此蟻由  $P$  爬至  $Q$  之最短距離。

Fifteen square tiles with side  $10C$  units long are arranged as shown. An ant walks along the edges of the tiles, always keeping a black tile on its left.

Find the shortest distance  $D$  that the ant would walk in going from  $P$  to  $Q$ .



$D =$

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# Hong Kong Mathematics Olympiad (1993 – 94)

## Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $x*y = xy + 1$ ，且  $a = (2*4)*2$ ，求  $a$  的值。

If  $x*y = xy + 1$  and  $a = (2*4)*2$ , find the value of  $a$ .

$a =$

- (ii) 若第  $b$  個質數為  $a$ ，求  $b$  的值。

If the  $b^{\text{th}}$  prime number is  $a$ , find the value of  $b$ .

$b =$

- (iii) 若  $c = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{50}\right)$ ，試以最簡單之分數表  $c$ 。

If  $c = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{50}\right)$ , find  $c$  in the simplest fractional form.

$c =$

- (iv) 一正方形內接於一個半徑為 10 之圓。若正方形之面積為  $d$ ，求  $d$  的值。

If  $d$  is the area of a square inscribed in a circle of radius 10, find the value of  $d$ .

$d =$

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score

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Total score

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Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若  $\log_2 a - 2 \log_a 2 = 1$ ，求  $a$  的值。

If  $\log_2 a - 2 \log_a 2 = 1$ , find the value of  $a$ .

$a =$

- (ii) 若  $b = \log_3[2(3+1)(3^2+1)(3^4+1)(3^8+1)+1]$ ，求  $b$  的值。

If  $b = \log_3[2(3+1)(3^2+1)(3^4+1)(3^8+1)+1]$ , find the value of  $b$ .

$b =$

- (iii) 若任意選擇一個有三十一日的月份，求該月有五個星期天的機率  $c$ 。

If a 31-day month is taken at random,  
find  $c$ , the probability that there are 5 Sundays in the month.

$c =$

- (iv) 從六名男士及四名女士中選出五人，組成一組。若其間共有  $d$  種選法，使男士必多於女士，求  $d$  的值。

A group of 5 people is to be selected from 6 men and 4 women.

Find  $d$ , the number of ways that there are always more men than women.

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

×

Mult. factor for  
speed

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Team No.

+

Bonus  
score

Time

Total score

Min.

Sec.



# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在  $1 \times 2 \times 3 \times \dots \times 100$  的積數中，最末的  $a$  個位都是 0。求  $a$  的值。

There are  $a$  zeros at the end of the product  $1 \times 2 \times 3 \times \dots \times 100$ . Find the value of  $a$ .

$a =$

- (ii)  $1998^{10}$  除以  $10^4$ ，所得餘數為  $b$ ，求  $b$  的值。

Find the value of  $b$ , if  $b$  is the remainder when  $1998^{10}$  is divided by  $10^4$ .

$b =$

- (iii) 若  $c = 2 - x + 2\sqrt{x-1}$  且  $x > 1$ ，求  $c$  之最大值。

Find the largest value of  $c$ , if  $c = 2 - x + 2\sqrt{x-1}$  and  $x > 1$ .

$c =$

- (iv) 若  $\left| \frac{3-2d}{5} + 2 \right| \leq 3$ ，求  $d$  的最小值。

Find the least value of  $d$ , if  $\left| \frac{3-2d}{5} + 2 \right| \leq 3$ .

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

$\times$

Mult. factor for  
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$=$

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Bonus  
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Time



Total score

Min.

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# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 由 1 至 121，有  $a$  個數是 3 或是 5 的倍數。求  $a$  的值。  
From 1 to 121, there are  $a$  numbers which are multiplies of 3 or 5.  
Find the value of  $a$ .

$a =$

- (ii) 由 1 至 121，有  $b$  個數不能被 5 或 7 整除。求  $b$  的值。  
From 1 to 121, there are  $b$  numbers which are not divisible by 5 nor 7.  
Find the value of  $b$ .

$b =$

用 1、2、3、4 這四個數字，而每個數字均可重複使用，則可組成一些 4 位數。求  
From the digits 1, 2, 3, 4, when each digit can be used repeatedly, 4-digit numbers are formed. Find

- (iii) 共可組成的 4 位數的個數  $c$ 。  
 $c$ , the number of 4-digit numbers that can be formed.

$c =$

- (iv) 所組成的 4 位數的總和  $d$ 。  
 $d$ , the sum of all these 4-digit numbers.

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

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Mult. factor for  
speed

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Team No.

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Bonus  
score

Time

Total score

Min.

Sec.

# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

$A$ 、 $B$ 、 $C$ 、 $D$  為由 0 至 9 間的不同整數，而

$$\begin{array}{r} \phantom{\times} \phantom{A} \phantom{B} \phantom{A} \\ \phantom{\times} \phantom{A} \phantom{B} \phantom{A} \\ \times \phantom{A} \phantom{B} \phantom{A} \\ \hline C \phantom{C} D \phantom{C} C \end{array}$$

求  $A$ 、 $B$ 、 $C$  及  $D$  的值。

$A$ ,  $B$ ,  $C$ ,  $D$  are different integers ranging from 0 to 9 and

$$\begin{array}{r} \phantom{\times} \phantom{A} \phantom{B} \phantom{A} \\ \phantom{\times} \phantom{A} \phantom{B} \phantom{A} \\ \times \phantom{A} \phantom{B} \phantom{A} \\ \hline C \phantom{C} D \phantom{C} C \end{array}$$

Find the values of  $A$ ,  $B$ ,  $C$  and  $D$ .

$A =$

$B =$

$C =$

$D =$

### FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	
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				Total score		

Team No.

Time

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# Hong Kong Mathematics Olympiad (1993 – 94)

## Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

在長方形  $ABCD$  中， $AD = 10$ ， $CD = 15$ ， $P$  為長方形內一點，使  $PB = 9$ ， $PA = 12$ 。求

In rectangle  $ABCD$ ,  $AD = 10$ ,  $CD = 15$ ,  $P$  is a point inside the rectangle such that  $PB = 9$ ,  $PA = 12$ . Find

- (i)  $PD$  之長  $a$ ，及  
 $a$ , the length of  $PD$  and

$a =$

- (ii)  $PC$  之長  $b$ 。  
 $b$ , the length of  $PC$ .

$b =$

- (iii) 已知  $\sin 2\theta = 2 \sin \theta \cos \theta$ 。求  $c$ ，若  $c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}$  的值。

It is given that  $\sin 2\theta = 2 \sin \theta \cos \theta$ . Find the value of  $c$ , if

$$c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}.$$

$c =$

- (iv) 已知  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ ，求  $d$  的值，若  
 $d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ)$ 。

It is given that  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ . Find the value of  $d$ , if

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ).$$

$d =$

### FOR OFFICIAL USE

Score for  
accuracy

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