第四十屆香港數學競賽(2022/23)
初賽規則

1. 初賽分個人項目和團體項目兩部分。個人項目限時**六十分鐘**，團體項目限時**二十分鐘**。
2. 每間學校可提名4至6位**中五**或以下同學參賽。其中任何4位可參加個人項目；又其中任何4位可參加團體項目。不足4位同學的隊伍將被撤銷參賽資格。
3. 所有參賽學生**必須穿著整齊校服**，**並由負責教師帶領**，初賽將會準時於上午9時開始。
4. 初賽題目以中、英文並列。指示語言將採用粵語。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英文並列。
5. 每一隊員於個人項目中須解答15條問題（當中甲部佔**10**題、乙部佔**5**題）；而每一隊員則於團體項目中須解答10條問題（當中甲部佔**5**題、乙部佔**5**題）。
6. 團體項目中，各參賽隊員可進行討論，但必須將聲浪降至最低。
7. 初賽時，不准使用計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，違例學生將被撤銷參賽資格或扣分。
8. 除非另有聲明，否則所有問題的答案均為數字，並應化至最簡，但無須呈交證明及算草。
9. 參賽者須關掉所有電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，否則大會有權取消該學生參賽資格。
10. 個人項目中，甲部和乙部的每一正確答案分別可得1分及2分。每隊可得之最高總積分為80分。
11. 團體項目中，甲部和乙部的每一正確答案分別可得2分及3分。每隊可得之最高總積分為25分。
12. 初賽中，並不給予快捷分。
13. 參賽者必須自備書寫工具，例如: 原子筆及鉛筆。
14. 籌委會將根據各參賽隊伍的總成績（個人項目及團體項目的積分總和）選出最高積分的五十隊進入決賽。
15. 獎項：

    (a) 於個人項目和團體項目中，根據參賽者所得分數由高至低排列後

    (i) 取得滿分者將獲頒予最佳表現及積分獎狀；

    (ii) 除上述(i) 中取得最佳表現的參賽者外，

    (1) 成績最佳的首2%參賽者將獲頒予一等榮譽獎狀；

    (2) 隨後的5%參賽者將獲頒予二等榮譽獎狀；

    (3) 隨後的10%參賽者將獲頒予三等榮譽獎狀。

    (4) 隨後的13%參賽者將獲頒予優秀表現獎狀。

    (b) 總成績（個人項目及團體項目的積分總和）於各分區（即港島、九龍東、九龍西、新界東及新界西）最高之首10% 的參賽隊伍將獲頒予獎狀。

16. 如有任何疑問，參賽者須於初賽完畢後，立即透過負責教師致電2153 7436向籌委會的教育局代表鄭仕文先生提出。所提出之疑問，將由籌委會作最後裁決。

The Fortieth Hong Kong Mathematics Olympiad (2022/23)
Regulations (Heat Events)

1. The Heats consists of two parts, namely, individual and Group Events. Individual Event will last for **60 minutes** and Group Event will last for **20 minutes**.
2. Each school may nominate **4 to 6 student participants** of **Secondary 5 level or below**. Any 4 of them may take part in the Individual Event and any 4 of them may take part in the Group Event. Teams of less than 4 members will be disqualified.
3. All student participants, **accompanied by the teacher-in-charge, should wear proper school uniform**. The competition will commence at 9:00 a.m. sharp.
4. Question papers are printed in both Chinese and English. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.
5. Each participant has to solve 15 questions in the Individual Event (***10 questions in Part A*** and ***5 questions in Part B***), and 10 questions in the Group Event (***5 questions in Part A*** and ***5 questions in Part B***).
6. In the Group Event, discussions among participating team members are allowed provided that the voice level is kept to a minimum.
7. Devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will **NOT** be allowed to use throughout the Heats, otherwise the participant will be disqualified or risk deduction of marks.
8. **All answers in the Individual and the Group Events should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**
9. All electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) and alarm device(s) should be turned off during the Heats. Failing to do so, the participant **will risk disqualification**.
10. For the Individual Event, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 80.
11. For the Group Event, 2 marks and 3 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 25.
12. No mark for speed will be awarded in the Heats.
13. Participants should bring along their own writing instruments, e.g. **ball pens** and **pencils**.
14. Based on the highest aggregate scores (sum of the scores in the Individual and the Group Events), the Organising Committee will select the 50 highest scored teams entering the Finals.
15. Awards:
    (a) For each of Individual and Group Events, after arranging all participant's scores in the order from the highest to the lowest
        (i) participants obtaining full score will be awarded Best Performance honour certificates ;
        (ii) after deducting those participants with full score achievement as in (i),
            (1) the first 2% from the remaining participants in the said order will be awarded First-class Honour certificates ;
            (2) the next 5% from the remaining participants will be awarded Second-class Honour certificates ; and
            (3) the next 10% from the remaining participants will be awarded Third-class Honour certificates ;
            (4) the next 13% from the remaining participants will be awarded Honourable Mention certificates ;
    (b) About 10% of participating schools with the highest aggregate scores (sum of the scores in the Individual and Group Events) in each of the regions (Hong Kong Island, Kowloon East, Kowloon West, the New Territories East, and the New Territories West)will be awarded certificates of merit.
16. Should there be any queries, participants should reach Mr CHENG Sze-man, the representative of EDB in the Organising Committee, **via the teacher-in-charge** at 2153 7436 immediately after the Heats. The decision of the Organising Committee on the queries is final.

Hong Kong Mathematics Olympiad (2022-2023)
Heats – Individual Event
香港數學競賽 (2022 – 2023)
初賽個人項目

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。第一至十題，每題 1 分，第十一至十五題，每題 2 分。全卷滿分 20 分。

時限：1 小時

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q10 1 mark each, Q11-Q15 2 marks each. The maximum mark for this paper is 20. Time allowed: 1 hour

1. 已知 a 和 b 均為實數。若 $a^2 + b^2 - 8a + 34b + 305 = 0$，求 $a + b$ 的值。
Given that a and b are real numbers. If $a^2 + b^2 - 8a + 34b + 305 = 0$, find the value of $a + b$.

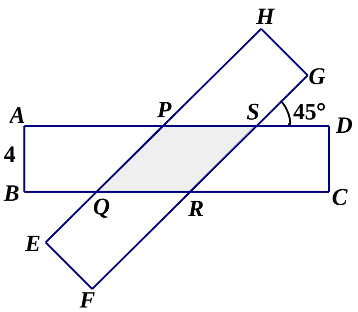
2. 若 x 及 y 均為正整數且滿足 $x + 8xy + y = 28$，求 $x + 2y$ 的最大可能值。
If x and y are positive integers satisfying $x + 8xy + y = 28$,
find the largest possible value of $x + 2y$.

3. 設 m 為一個整數常數，其中 $4 < m < 40$。若方程 $x^2 - 2(2m - 3)x + 4m^2 - 14m + 8 = 0$ 有兩個整數根，求 x 的最大可能值。
Let m be an integral constant, where $4 < m < 40$. If the equation
$x^2 - 2(2m - 3)x + 4m^2 - 14m + 8 = 0$ has two integral roots, find the largest possible value of x .

4. 設 a 為一正實數。若 $a^2 + \dfrac{1}{a^2} = 14$，求 $a^3 + \dfrac{1}{a^3}$ 的值。
Let a be a positive real number. If $a^2 + \dfrac{1}{a^2} = 14$, find the value of $a^3 + \dfrac{1}{a^3}$.

5. 若干正整數之和是 60。最大正整數為 15 及其中有一個正整數是 12。除卻這正整數 12，其餘正整數恰好組成一個等差數列。求最小的正整數。
The sum of certain number of positive integers is 60. The largest positive integer is 15 and one of the positive integers is 12. Apart from this positive integer 12, the remaining positive integers form an arithmetic sequence. Find the smallest positive integer.

6. 在圖一中，把長方形 $ABCD$ 繞它的中心逆時針轉 45° 得長方形 $EFGH$。若 $AB = 4$，求陰影部分 $PQRS$ 的面積。
In Figure 1, the rectangle $ABCD$ is rotated about its centre 45° anticlockwise to obtain the rectangle $EFGH$. If $AB = 4$, find the area of the shaded region $PQRS$.



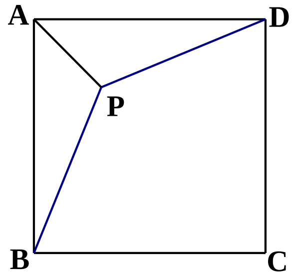
圖一 Figure 1

7. 求 $\left(\dfrac{1\times4\times16\times64+2\times8\times32\times128+3\times12\times48\times192+\cdots+2023\times8092\times32368\times129472}{1\times5\times25\times125+2\times10\times50\times250+3\times15\times75\times375+\cdots+2023\times10115\times50575\times252875}\right)^{\frac{1}{6}}$ 的值。

Evaluate $\left(\dfrac{1\times4\times16\times64+2\times8\times32\times128+3\times12\times48\times192+\cdots+2023\times8092\times32368\times129472}{1\times5\times25\times125+2\times10\times50\times250+3\times15\times75\times375+\cdots+2023\times10115\times50575\times252875}\right)^{\frac{1}{6}}$.

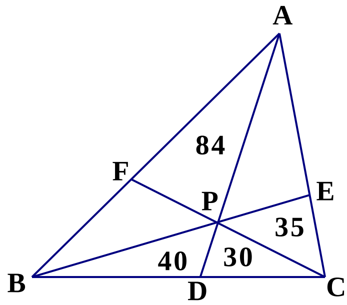
8. 若一個等邊三角形的面積與其在周界在數值上相等，求該正三角形的外接圓的半徑。
If the area of an equilateral triangle is numerically equal to its perimeter, find the radius of the circumcircle of this equilateral triangle.

9. 在圖二中，P 為正方形 $ABCD$ 內的一點使得 $\Delta ABP \cong \Delta ADP$，$AP = 5\sqrt{2}$ 及 $BP = 13$。求正方形 $ABCD$ 的面積。
In Figure 2, P is a point inside the square $ABCD$ such that $\Delta ABP \cong \Delta ADP$. $AP = 5\sqrt{2}$ and $BP = 13$. Find the area of square $ABCD$.



圖二 Figure 2

10. 在圖三中，D、E 及 F 分別為 BC、AC 及 AB 上的點。AD、BE 及 CF 相交於 P 使得 ΔAPF 的面積$= 84$、ΔBPD 的面積$= 40$、ΔCPD 的面積$= 30$ 及 ΔCPE 的面積$= 35$。求ΔABC 的面積。
In Figure 3, D, E and F are points lying on BC, AB and AB respectively. AD, BE and CF intersect at P such that such that area of $\Delta APF = 84$, area of $\Delta BPD = 40$, area of $\Delta CPD = 30$ and area of $\Delta CPE = 35$. Find the area of ΔABC.



圖三 Figure 3

11. 已知 n 是一個少於 2023 正整數。若 n 只有三個不同的因數，求 n 的可能性的總數。
Given that n is a positive integer less than 2023.
If n has only 3 distinct factors, find the number of possible values of n.

12. 已知 p 及 q 為正實數。若 $\log_9 p = \log_{15} q = \log_{25}(3p+2q)$，求 $\frac{p}{q}$ 的值。
Given that p and q are positive numbers. If $\log_9 p = \log_{15} q = \log_{25}(3p+2q)$, find the value of $\frac{p}{q}$.

13. 數列 $\{a_n\}$ 定義為 $a_1 = 1$、$a_2 = \frac{3}{7}$ 及對所有 $n \geq 3$，$a_n = \frac{a_{n-2}a_{n-1}}{2a_{n-2} - a_{n-1}}$。求 $\frac{1}{a_{2023}}$ 的值。
A sequence of numbers $\{a_n\}$ is defined by $a_1 = 1$, $a_2 = \frac{3}{7}$ and $a_n = \frac{a_{n-2}a_{n-1}}{2a_{n-2} - a_{n-1}}$ for all $n \geq 3$.
Find the value of $\frac{1}{a_{2023}}$.

14. ABC 是一個等腰三角形，其中 $AB = AC = 18$ 及 $BC = 12$。P 為 ΔABC 內的任意一點使得 $\angle ABP + \angle ACP = 90°$ 及 $AP = 15$。求 $BP^2 + CP^2$ 的值。
ABC is an isosceles triangle with $AB = AC = 18$ and $BC = 12$. P is any interior point of ΔABC such that $\angle ABP + \angle ACP = 90°$ and $AP = 15$. Find the value of $BP^2 + CP^2$.

15. 求方程 $\sqrt[3]{x} + \sqrt[3]{x-4} = \sqrt[3]{x-2}$ 的根之積。
Find the product of roots of the equation $\sqrt[3]{x} + \sqrt[3]{x-4} = \sqrt[3]{x-2}$.

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Hong Kong Mathematics Olympiad (2022-2023)
Heats – Group Event
香港數學競賽 (2022 – 2023)
初賽團體項目

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。第一至五題，每題 2 分，第六至十題，每題 3 分。全卷滿分 25 分。
本卷由四位參賽同學共同作答。 時限：20 分鐘
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted. The diagrams are not necessarily drawn to scale.
Q1- Q5 2 marks each, Q6-Q10 3 marks each. The maximum mark for this paper is 25.
This paper is answered by 4 participants together. Time allowed: 20 minutes

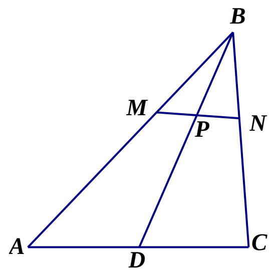
1. 求 3^{2023} 的最尾兩位數字。
Find the last two digits of 3^{2023}.

2. 對於 $0 < x < 2$，求 $\left(\frac{\sqrt{2+x}}{\sqrt{2+x}-\sqrt{2-x}}+\frac{2-x}{\sqrt{4-x^2}+x-2}\right)\left(\sqrt{\frac{4}{x^2}-1}-\frac{2}{x}\right)$ 的值。
For $0 < x < 2$, find the value of $\left(\frac{\sqrt{2+x}}{\sqrt{2+x}-\sqrt{2-x}}+\frac{2-x}{\sqrt{4-x^2}+x-2}\right)\left(\sqrt{\frac{4}{x^2}-1}-\frac{2}{x}\right)$.

3. 已知 $\tan\alpha$ 和 $\tan\beta$ 是二次方程 $x^2-4x-2=0$ 的根。
求 $\sin^2(\alpha+\beta)+2\sin(\alpha+\beta)\cos(\alpha+\beta)+3\cos^2(\alpha+\beta)$ 的值。
Given that $\tan\alpha$ and $\tan\beta$ are the roots of the quadratic equation $x^2-4x-2=0$.
Find the value of $\sin^2(\alpha+\beta)+2\sin(\alpha+\beta)\cos(\alpha+\beta)+3\cos^2(\alpha+\beta)$.

4. 排列 5 個不同的單數及 5 個不同的雙數在同一行使得任意兩個相鄰數的積必為雙數。求所有排列的可能性數目。
Five distinct odd numbers and five distinct even numbers are arranged in a row such that the product of any two consecutive numbers is always even.
Find the number of all possible arrangements.

5. 在圖一中，M 和 N 分別是 ΔABC 的邊 AB 和 BC 上的點。MN 與 ΔABC 的中線相交於 P。若 $\frac{AM}{BM}=\frac{5}{3}$ 及 $\frac{CN}{BN}=\frac{3}{2}$。求 $\frac{DP}{BP}$ 的值。
In Figure 1, M and N are points on AB and BC of ΔABC respectively. MN and the median of ΔABC intersect at P.
If $\frac{AM}{BM}=\frac{5}{3}$ and $\frac{CN}{BN}=\frac{3}{2}$, find the value of $\frac{DP}{BP}$.



圖一 Figure 1

6. 設 x、y 及 z 為實數，且滿足方程 $\begin{cases} x+yz=6 \\ y+zx=6 \\ z+xy=6 \end{cases}$，求 xyz 的最大值。
If x, y and z are real numbers that satisfy the system of equations $\begin{cases} x+yz=6 \\ y+zx=6 \\ z+xy=6 \end{cases}$,
find the largest possible value of xyz.

7. 整數數列$\{a_n\}$定義為$a_n = 100 + n^2$，其中 n 為正整數。設 d_n 為a_n和a_{n+1}的最大公因數。求 d_n的最大值。

A sequence of integers $\{a_n\}$ is defined by $a_n = 100 + n^2$, where n is a positive integer. Let d_n be the greatest common divisor of a_n and a_{n+1}. Find the greatest possible value of d_n.

8. 已知 x 及 y 為正實數且滿足 $x^2 - y^2 = 4$ 及 $xy = 2$。若 $x + y$ 可寫成$a\sqrt{b+\sqrt{c}}$，其中a、b及c均為正整數，求 $100a + 10b + c$的最小值。

Given that x and y are positive real numbers satisfying $x^2 - y^2 = 4$ and $xy = 2$. If the value of $x + y$ can be expressed in the form of $a\sqrt{b+\sqrt{c}}$, where a, b and c are positive integers, find the least value of $100a + 10b + c$.

9. 定義 $\mathrm{f}(z) = z^2 + 4z$，其中 z 是一個複數，設 $z = x + 2i$，當中x 為非零實數。若 $\dfrac{\mathrm{f}(\mathrm{f}(z)) - \mathrm{f}(z)}{z - \mathrm{f}(z)}$ 是一個純虛數，求 x 的值。

Define $\mathrm{f}(z) = z^2 + 4z$, where z is a complex number. let $z = x + 2i$, where x is a non-zero real number. If $\dfrac{\mathrm{f}(\mathrm{f}(z)) - \mathrm{f}(z)}{z - \mathrm{f}(z)}$ is a purely imaginary number, find the value of x.

10. 下列方程有一個實數解：$\begin{cases} 3\log_a\left(\sqrt{x}\log_a x\right) = 26 \\ \log_{\log_a x} x = 24 \end{cases}$，求 a 的值。

The following system of equations has one real number solution $\begin{cases} 3\log_a\left(\sqrt{x}\log_a x\right) = 26 \\ \log_{\log_a x} x = 24 \end{cases}$, find the value of a .

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