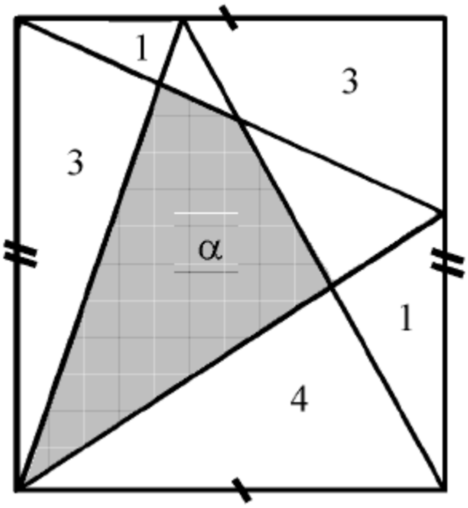
Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 求下圖中陰影部分的面積 α。
Determine the area of the shaded region, α , in the figure below.

α =



圖一 Figure 1

2. 如果 10 個不同的正整數的平均值是 2α，
求這 10 個數中，最大的一個數 β 最大可能值。
If the average of 10 distinct positive integers is 2α,
what is the largest possible value of the largest integer, β, of the ten integers?

β =

3. 考慮兩組由正整數組成的有限數列：$1, 3, 5, 7, \cdots, \beta$ 和 $1, 6, 11, 16, \cdots, \beta+1$。
求它們之間相同數字的數目 γ。
Given that $1, 3, 5, 7, \cdots, \beta$ and $1, 6, 11, 16, \cdots, \beta + 1$ are two finite sequences of positive integers.
Determine γ, the numbers of positive integers common to both sequences.

γ =

4. 若 $\log_2 a + \log_2 b \geq \gamma$，求 $a + b$ 的最小值 δ。
If $\log_2 a + \log_2 b \geq \gamma$, determine the smallest positive value δ for $a + b$.

δ =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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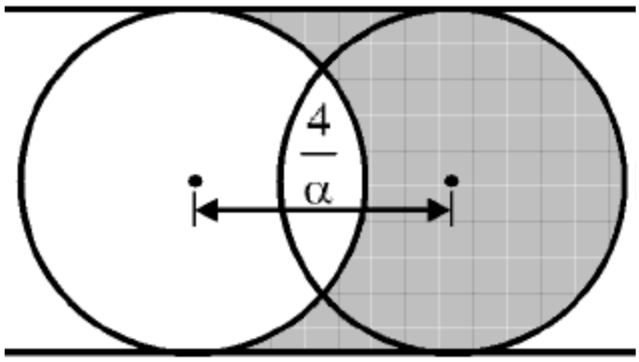
1. 求方程 $\sqrt{\left(x+\sqrt{x}\right)}-\sqrt{\left(x-\sqrt{x}\right)}=\sqrt{x}$ 的正實根 α。
Determine the positive real root, α, of $\sqrt{\left(x+\sqrt{x}\right)}-\sqrt{\left(x-\sqrt{x}\right)}=\sqrt{x}$.

$\alpha =$

2. 下圖為兩個半徑為 4 的圓，其圓心相隔 $\frac{4}{\alpha}$。求陰影部分的面積 β。
In the figure below, two circles of radii 4 with their centres placed apart by $\frac{4}{\alpha}$.
Determine the area β, of the shaded region.

$\beta =$



3. 求正整數 γ 的最小值，以使得方程 $\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ 對 x 有正整數解。
Determine the smallest positive integer γ such that the equation
$\sqrt{x}-\sqrt{\beta\gamma}=4\sqrt{2}$ has an integer solution in x.

$\gamma =$

4. 求 $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$ 的個位數 δ。
Determine the units digit, δ, of $\left(\left(\gamma^{\gamma}\right)^{\gamma}\right)^{\gamma}$.

$\delta =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若數列 $10^{\frac{1}{11}}$、$10^{\frac{2}{11}}$、$10^{\frac{3}{11}}$、…、$10^{\frac{\alpha}{11}}$ 中所有數字的乘積為 1 000 000，求正整數 α 的值。

   If the product of numbers in the sequence $10^{\frac{1}{11}}$, $10^{\frac{2}{11}}$, $10^{\frac{3}{11}}$, … , $10^{\frac{\alpha}{11}}$ is 1 000 000, determine the value of the positive integer α .

   $\alpha =$

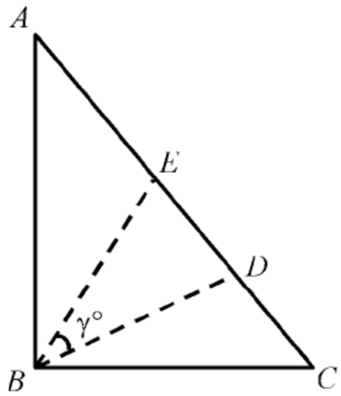
2. 若 $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$，求 β 的值。

   Determine the value of β if $\frac{\beta}{1\times2\times3}+\frac{\beta}{2\times3\times4}+\cdots+\frac{\beta}{8\times9\times10}=\alpha$.

   $\beta =$

3. 在下圖的三角形 ABC 中，$\angle ABC = 2\beta°$，$AB = AD$ 及 $CB = CE$。設 $\gamma° = \angle DBE$，求 γ 的值。
   In the figure below, triangle ABC has $\angle ABC = 2\beta°$, $AB = AD$ and $CB = CE$.
   If $\gamma° = \angle DBE$, determine the value of γ .

   $\gamma =$



4. 考慮數列 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, …，求首 γ 項的和 δ。
   For the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, … ,
   determine the sum δ of the first γ terms.

   $\delta =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
				Total score				Min.	Sec.

Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 3\sqrt{\alpha} - 6$，求 α 的值。
   If $\frac{6\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 3\sqrt{\alpha} - 6$, determine the value of α.

   $\alpha =$

2. 考慮形如 $\frac{n}{n+1}$ 的分數，當中 n 是一個正整數。若同時把該分數的分子和分母減去 1，得出的分數是小於 $\frac{\alpha}{7}$，且大於 0，求這樣的分數的數目 β。
   Consider fractions of the form $\frac{n}{n+1}$, where n is a positive integer. If 1 is subtracted from both the numerator and the denominator, and the resultant fraction remains positive and is strictly less than $\frac{\alpha}{7}$, determine, β, the number of these fractions.

   $\beta =$

3. 一個等邊三角形和一個正六邊形的周長相同。若該等邊三角形的面積為 β 平方單位，求正六邊形的面積 γ(平方單位)。
   The perimeters of an equilateral triangle and a regular hexagon are equal. If the area of the triangle is β square units, determine the area, γ, of the hexagon in square units.

   $\gamma =$

4. 求 $\delta = \frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}-\gamma$ 的值。
   Determine the value of $\delta = \frac{3}{2}+\frac{5}{4}+\frac{9}{8}+\frac{17}{16}+\frac{33}{32}+\frac{65}{64}-\gamma$.

   $\delta =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2013– 2014)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若一個等腰三角形對應底邊(不是兩條等腰邊)的高是 8，且周長是 32，求該三角形的面積。
If an isosceles triangle has height 8 from the base, not the legs, and perimeters 32, determine the area of the triangle.

area =

2. 若 $\mathrm{f}(x)=\dfrac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)}$ 當中 x 是一個正實數，求 $\mathrm{f}(x)$ 的最小值。
If $\mathrm{f}(x)=\dfrac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)}$ where x is a positive real number, determine the minimum value of $\mathrm{f}(x)$.

minimum =

3. 求 81 位數 $\overline{111\cdots1}$ 除以 81 的餘數。
Determine the remainder of the 81-digit integer $\overline{111\cdots1}$ divided by 81 .

remainder =

4. 給定一實數數列 $a_1, a_2, a_3, \cdots$ ，它滿足
1) $a_1=\dfrac{1}{2}$，及
2) 對 $k\geq 2$，有 $a_1+a_2+\cdots+a_k=k^2a_k$。
求 a_{100} 的值。
Given a sequence of real numbers $a_1, a_2, a_3, \cdots$ that satisfy
1) $a_1=\dfrac{1}{2}$, and
2) $a_1+a_2+\cdots+a_k=k^2a_k$, for $k\geq 2$.
Determine the value of a_{100} .

$a_{100}=$

FOR OFFICIAL USE

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			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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1. 若在 $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$ 中刪去若干項後剩 1，求刪去各項的乘積。

By removing certain terms from the sum, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$, we can get 1 .

What is the product of the removed term(s) ?

Product =

2. 若 $S_n = 1 - 2 + 3 - 4 + \ldots + (-1)^{n-1} n$，當中 n 是正整數，求 $S_{17} + S_{33} + S_{50}$ 的值。
If $S_n = 1 - 2 + 3 - 4 + \ldots + (-1)^{n-1} n$, where n is a positive integer, determine the value of $S_{17} + S_{33} + S_{50}$.

$S_{17}+S_{33}+S_{50} =$

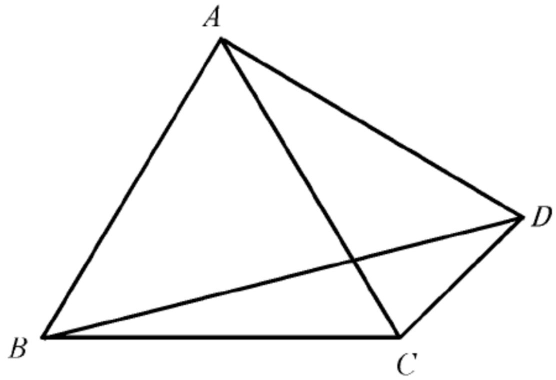
3. A, B, C, D, E 和 F 六人根據英文字母的順序輪班工作。A 在第一個星期日當值，然後 B 在星期一當值，如此類推。A 於第 50 個星期的哪一天當值？(答案以數字 0 代表星期日，數字 1 代表星期一，……，數字 6 代表星期六）。

Six persons A, B, C, D, E and F are to rotate for night shifts in alphabetical order with A serving on the first Sunday, B on the first Monday and so on. In the fiftieth week, which day does A serve on? (Represent Sunday by 0, Monday by 1, … , Saturday by 6 in your answer.)

Day

4. 在下圖中，D 以直綫連接著等邊三角形 ABC 的頂點，當中 $AB = AD$。
設 $\angle BDC = \alpha°$，求 α 的值。
In the figure below, vertices of equilateral triangle ABC are connected to D in straight line segments with $AB = AD$. If $\angle BDC = \alpha°$, determine the value of α .

$\alpha =$



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Score for accuracy		×	Mult. factor for speed		=		Team No.	
		+	Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 求乘積 $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$ 的值。

   Determine the value of the product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$.

   Product =

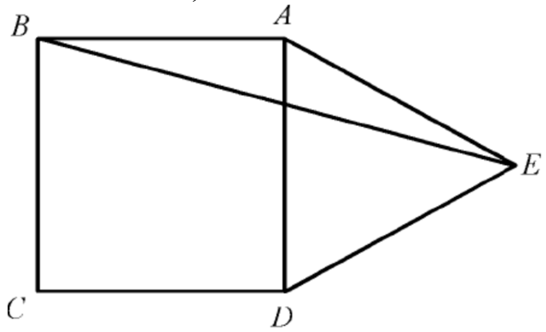
2. 求和 $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$ 的值，
   當中 $100! = 100\times99\times98\times\ldots\times3\times2\times1$。

   Determine the value of the sum $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$ where $100! = 100\times99\times98\times\ldots\times3\times2\times1$.

   Sum =

3. 在下圖中，*ABCD* 是一個正方形，*ADE* 是一個等邊三角形，且 *E* 是正方形 *ABCD* 外的一點。設 $\angle AEB = \alpha°$，求 α 的值。
   In the figure below, *ABCD* is a square, *ADE* is an equilateral triangle and *E* is a point outside of the square *ABCD*. If $\angle AEB = \alpha°$, determine the value of α .

   α =



4. 把不同的非零個位數填進下表白色的正方格內，使所有橫、直的等式均成立。
   求 α 的值。
   Fill the white squares in the figure below with distinct non-zero digits so that the arithmetical expressions, read both horizontally and vertically, are correct.
   What is the value of α ?

   α =

	÷		=	
+		×		
	+		=	α
=		=		

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (2013 – 2014)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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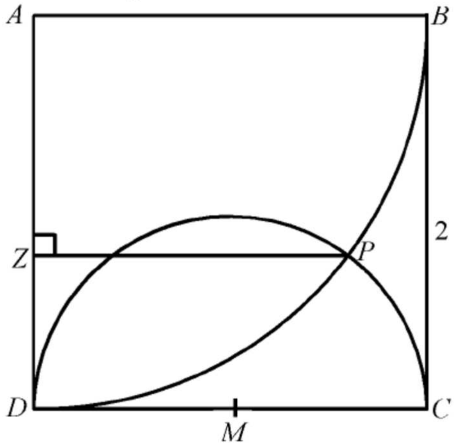
1. 在下圖，$ABCD$ 是一個邊長為 2 的正方形。先以 A 為圓心畫出弧 BD，再以 CD 的中點 M 為圓心從 C 到 D 畫出一個半圓。弧 BD 和弧 DC 相交於 P。求 P 與 AD 的最短距離，即 PZ 的長度。
In the figure below, $ABCD$ is a square of side length 2. A circular arc with centre at A is drawn from B to D. A semicircle with centre at M, the midpoint of CD, is drawn from C to D and sits inside the square. Determine the shortest distance from P, the intersection of the two arcs, to side AD, that is, the length of PZ.

$PZ =$



2. 若 $x=\dfrac{\sqrt{5}+1}{2}$ 及 $y=\dfrac{\sqrt{5}-1}{2}$，求 $x^3y+2x^2y^2+xy^3$ 的值。
If $x=\dfrac{\sqrt{5}+1}{2}$ and $y=\dfrac{\sqrt{5}-1}{2}$，determine the value of $x^3y+2x^2y^2+xy^3$.

$x^3y+2x^2y^2+xy^3=$

3. 若 a, b, c 及 d 是不同的個位數，且

$$\begin{array}{r} a\,a\,b\,c\,d \\ -\,d\,a\,a\,b\,c \\ \hline 2\,0\,1\,4\,d \end{array}$$

求 d 的值。
If a, b, c and d are distinct digits and

$$\begin{array}{r} a\,a\,b\,c\,d \\ -\,d\,a\,a\,b\,c \\ \hline 2\,0\,1\,4\,d \end{array}$$

determine the value of d .

$d =$

4. 求方程 $x^4+(x-4)^4=32$ 所有實根的乘積。
Determine the product of all real roots of the equation $x^4+(x-4)^4=32$.

Product =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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			Total score				Min.	Sec.