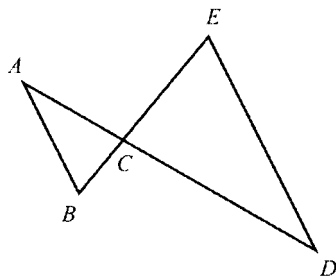


Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event Sample (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a, b, c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 的根。若 $P = a^2 + b^2 + c^2 + d^2$ ，求 P 的值。
 Let a, b, c and d be the roots of the equation $x^4 - 15x^2 + 56 = 0$.
 If $P = a^2 + b^2 + c^2 + d^2$, find the value of P .

2. 如圖一， $AB = AC$ 及 $AB \parallel ED$ 。若 $\angle ABC = P^\circ$ 及 $\angle ADE = Q^\circ$ ，求 Q 的值。
 In Figure 1, $AB = AC$ and $AB \parallel ED$. If $\angle ABC = P^\circ$ and $\angle ADE = Q^\circ$,
 find the value of Q .



圖一
Figure 1

3. 設 $F = 1 + 2 + 2^2 + 2^3 + \dots + 2^Q$ 及 $R = \sqrt{\frac{\log(1+F)}{\log 2}}$ ，求 R 的值。
 Let $F = 1 + 2 + 2^2 + 2^3 + \dots + 2^Q$ and $R = \sqrt{\frac{\log(1+F)}{\log 2}}$, find the value of R .

4. 設 $f(x)$ 是一個函數使得對所有正整數 n ， $f(n) = (n-1)f(n-1)$ 及 $f(1) \neq 0$ 。
 若 $S = \frac{f(R)}{(R-1)f(R-3)}$ ，求 S 的值。

Let $f(x)$ be a function such that $f(n) = (n-1)f(n-1)$ and $f(1) \neq 0$ for all positive integers n . If $S = \frac{f(R)}{(R-1)f(R-3)}$, find the value of S .

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	
			+	Bonus score		
			Total score			

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Hong Kong Mathematics Olympiad (2012 – 2013)

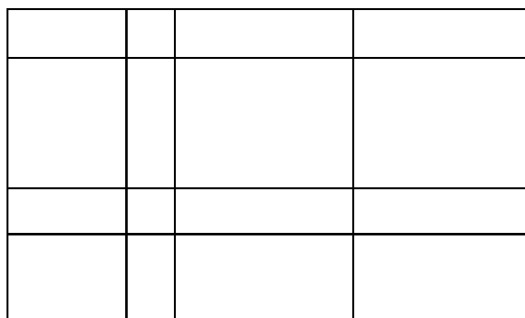
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 圖一共有 a 個長方形，求 a 的值。

Figure 1 has a rectangles, find the value of a .



圖一 Figure 1

$a =$

2. 已知 111111 能被 7 整除。若 b 為 $\underbrace{111111 \dots 111111}_{a \text{ 個}}$ 除以 7 的餘數，求 b 的值。

Given that 7 divides 111111.

If b is the remainder when $\underbrace{111111 \dots 111111}_{a \text{ times}}$ is divided by 7, find the value of b .

$b =$

3. 若 c 為 $\left[(b-2)^{4b^2} + (b-1)^{2b^2} + b^{b^2} \right]$ 除以 3 的餘數，求 c 的數值。

If c is the remainder of $\left[(b-2)^{4b^2} + (b-1)^{2b^2} + b^{b^2} \right]$ divided by 3, find the value of c .

$c =$

4. 若 $|x+1| + |y-1| + |z| = c$ ，求 $d = x^2 + y^2 + z^2$ 的值。

If $|x+1| + |y-1| + |z| = c$, find the value of $d = x^2 + y^2 + z^2$.

$d =$

FOR OFFICIAL USE

Score for accuracy

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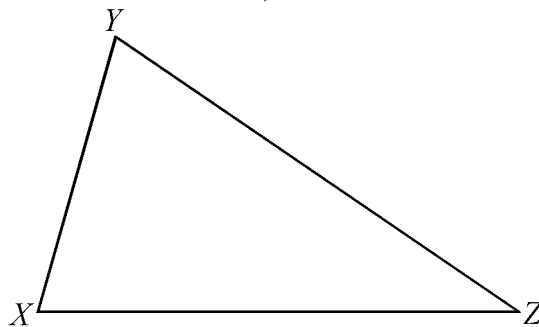
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Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知函數 $f(x) = x^2 + rx + s$ 和 $g(x) = x^2 - 9x + 6$ 有以下特性：
 $f(x)$ 的根之和是 $g(x)$ 的根之積，且 $f(x)$ 的根之積是 $g(x)$ 的根之和。
 若 $f(x)$ 的最小值取值於 $x = a$ ，求 a 的值。
 Given that functions $f(x) = x^2 + rx + s$ and $g(x) = x^2 - 9x + 6$ have the properties that the sum of roots of $f(x)$ is the product of the roots of $g(x)$, and the product of roots of $f(x)$ is the sum of roots of $g(x)$. If $f(x)$ attains its minimum at $x = a$, find the value of a .
2. 一正方體的表面積是 $b \text{ cm}^2$ 。
 若它每一條邊的長度增加 3 cm ，它的體積隨之增加 $(2b - a) \text{ cm}^3$ ，求 b 的值。
 The surface area of a cube is $b \text{ cm}^2$. If the length of each side is increased by 3 cm , its volume is increased by $(2b - a) \text{ cm}^3$, find the value of b .
3. 設 $f(1) = 3$, $f(2) = 5$ 且對所有正整數 n , $f(n + 2) = f(n + 1) + f(n)$ 。
 當 $f(b)$ 除以 3 的餘數是 c ，求 c 的值。
 Let $f(1) = 3$, $f(2) = 5$ and $f(n + 2) = f(n + 1) + f(n)$ for positive integers n .
 If c is the remainder of $f(b)$ divided by 3, find the value of c .
4. 如圖二，三角形 XYZ 的角度滿足 $\angle Z \leq \angle Y \leq \angle X$ 且 $c \cdot \angle X = 6 \cdot \angle Z$ 。
 若 $\angle Z$ 的最大可能值是 d° ，求 d 的值。
 In Figure 2, the angles of triangle XYZ satisfy $\angle Z \leq \angle Y \leq \angle X$ and $c \cdot \angle X = 6 \cdot \angle Z$.
 If the maximum possible value of $\angle Z$ is d° , find the value of d .



圖二 Figure 2

FOR OFFICIAL USE

Score for accuracy

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Team No.

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Total score

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Hong Kong Mathematics Olympiad (2012 – 2013)

Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $a = \frac{(7+4\sqrt{3})^{\frac{1}{2}} - (7-4\sqrt{3})^{\frac{1}{2}}}{\sqrt{3}}$ ，求 a 的值。

$\alpha =$

If $a = \frac{(7+4\sqrt{3})^{\frac{1}{2}} - (7-4\sqrt{3})^{\frac{1}{2}}}{\sqrt{3}}$, find the value of a .

2. 設 $f(x) = x - a$ 及 $F(x, y) = y^2 + x$ 。如果 $b = F(3, f(4))$ ，求 b 的值。

Suppose $f(x) = x - a$ and $F(x, y) = y^2 + x$. If $b = F(3, f(4))$, find the value of b .

$b =$

3. 已知 392 除以一個兩位正整數的餘數是 b ，
符合這個條件的兩位正整數共有 c 個，求 c 的值。

The remainder when 392 is divided by a 2-digit positive integer is b .

If c is the number of such 2-digit positive integers, find the value of c .

$c =$

4. 若 x 為實數及 d 為函數 $y = \frac{3x^2 + 3x + c}{x^2 + x + 1}$ 的最大值，求 d 的值。

If x is a real number and d is the maximum value of the function $y = \frac{3x^2 + 3x + c}{x^2 + x + 1}$,

find the value of d .

$d =$

FOR OFFICIAL USE

Score for
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Hong Kong Mathematics Olympiad (2012 – 2013)

Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 設實函數 $f(x)$ 對於所有實數 x 及 y 滿足 $f(xy) = f(x) \cdot f(y)$ ，且 $f(0) \neq 0$ 。

求 $a = f(1)$ 的值。

Let $f(x)$ be a real value function that satisfies $f(xy) = f(x) \cdot f(y)$ for all real numbers x and y and $f(0) \neq 0$. Find the value of $a = f(1)$.

$a =$

2. 設函數 $F(n)$ 滿足 $F(1) = F(2) = F(3) = a$ 及 $F(n+1) = \frac{F(n) \cdot F(n-1) + 1}{F(n-2)}$ ，

其中 $n \geq 3$ 為正整數。求 $b = F(6)$ 的值。

Let $F(n)$ be a function with $F(1) = F(2) = F(3) = a$ and $F(n+1) = \frac{F(n) \cdot F(n-1) + 1}{F(n-2)}$ for

positive integer $n \geq 3$, find the value of $b = F(6)$.

$b =$

3. 若 $b-6$ 、 $b-5$ 及 $b-4$ 為方程 $x^4 + rx^2 + sx + t = 0$ 的根，求 $c = r + t$ 的值。

If $b-6$, $b-5$, $b-4$ are three roots of the equation $x^4 + rx^2 + sx + t = 0$, find the value of $c = r + t$.

$c =$

4. 設 (x_0, y_0) 是以下方程組的一個解：

$$\begin{cases} xy = 6 \\ x^2y + xy^2 + x + y + c = 2 \end{cases}$$

求 $d = x_0^2 + y_0^2$ 的值。

Suppose that (x_0, y_0) is a solution of the system:

$$\begin{cases} xy = 6 \\ x^2y + xy^2 + x + y + c = 2 \end{cases}$$

Find the value of $d = x_0^2 + y_0^2$.

$d =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (2012 – 2013)

Final Event Sample (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知三角形三邊的長度分別是 a cm、 2 cm 及 b cm，其中 a 和 b 是整數且 $a \leq 2 \leq b$ 。若有 q 種不全等的三角形滿足上述條件，求 q 的值。

$q =$

Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \leq 2 \leq b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q .

2. 已知方程 $|x| - \frac{4}{x} = \frac{3|x|}{x}$ 有 k 個相異實根，求 k 的值。

$k =$

Given that the equation $|x| - \frac{4}{x} = \frac{3|x|}{x}$ has k distinct real root(s), find the value of k .

3. 已知 x 及 y 為非零實數且滿足方程 $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ 及 $x - y = 7$ 。

$w =$

若 $w = x + y$ ，求 w 的值。

Given that x and y are non-zero real numbers satisfying the equations $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$

and $x - y = 7$. If $w = x + y$, find the value of w .

4. 已知 x 及 y 為實數且 $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$ 。設 $p = |x| + |y|$ ，求 p 的值。

$p =$

Given that x and y are real numbers and $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$.

Let $p = |x| + |y|$, find the value of p .

FOR OFFICIAL USE

Score for
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Total score

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Hong Kong Mathematics Olympiad (2012 – 2013)

Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 求 $(2^{13} + 1)(2^{14} + 1)(2^{15} + 1)(2^{16} + 1)$ 的個位數字。
Find the units digit of $(2^{13} + 1)(2^{14} + 1)(2^{15} + 1)(2^{16} + 1)$.

unit digit =

2. 求 $16 \div (0.40 + 0.41 + 0.42 + \dots + 0.59)$ 的值的整數部分。
Find the integral part of $16 \div (0.40 + 0.41 + 0.42 + \dots + 0.59)$.

integral part

3. 從 1、2、4、6、7 中選三個數字組成三位數。
這些三位數有多少個能被 3 整除？
Choose three digits from 1, 2, 4, 6, 7 to construct three-digit numbers.
Of these three-digit numbers, how many of them are divisible by 3?

4. 用 1、2、3、4、5、6 組成一個位數： $ABCDEF$ ，使得 A 能被 1 整除， AB 能被 2 整除， ABC 能被 3 整除， $ABCD$ 能被 4 整除， $ABCDE$ 能被 5 整除，及 $ABCDEF$ 能被 6 整除。求 A 的最大值。

Greatest A

Using numbers: 1, 2, 3, 4, 5, 6 to form a six-digit number: $ABCDEF$ such that A is divisible by 1, AB is divisible by 2, ABC is divisible by 3, $ABCD$ is divisible by 4, $ABCDE$ is divisible by 5, $ABCDEF$ is divisible by 6. Find the greatest value of A .

FOR OFFICIAL USE

Score for accuracy

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Mult. factor for speed

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Team No.

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Bonus score

Time

Total score

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Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $4^3 + 4^r + 4^4$ 是一平方數，其中 r 是正整數，求 r 的最小值。

If $4^3 + 4^r + 4^4$ is a perfect square and r is a positive integer,
find the minimum value of r .

minimum r

2. 三男 B_1, B_2, B_3 和三女 G_1, G_2, G_3 就坐一排座位，並滿足以下兩個條件：

- 1) 一男不會坐在另一男旁邊及一女不會坐在另一女旁邊
- 2) B_1 必須坐在 G_1 旁邊

若 s 是這樣就坐的排列數量，求 s 的值。

Three boys B_1, B_2, B_3 and three girls G_1, G_2, G_3 are to be seated in a row according to the following rules:

- 1) A boy will not sit next to another boy and a girl will not sit next to another girl,
- 2) Boy B_1 must sit next to girl G_1

If s is the number of different such seating arrangements, find the value of s .

$s =$

3. 設 $f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$, x 為實數且 $f(x)$ 的最大值和最小值分別是 $\frac{1}{2}$ 和 -1 。

若 $t = f(0)$, 求 t 的值。

Let $f(x) = \frac{x+a}{x^2 + \frac{1}{2}}$, where x is a real number and the maximum value of $f(x)$ is $\frac{1}{2}$ and

the minimum value of $f(x)$ is -1 . If $t = f(0)$, find the value of t .

$t =$

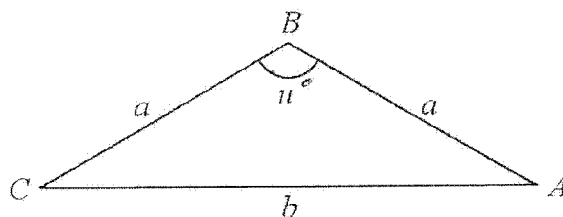
4. 在圖三， ABC 是一等腰三角形，其中 $\angle ABC = u^\circ$, $AB = BC = a$ 和 $AC = b$ 。

若二次方程 $ax^2 - \sqrt{2} \cdot bx + a = 0$ 有兩個實根，它們的絕對差為 $\sqrt{2}$ ，求 u 的值。

In Figure 3, ABC is an isosceles triangle with $\angle ABC = u^\circ$, $AB = BC = a$ and $AC = b$.

If the quadratic equation $ax^2 - \sqrt{2} \cdot bx + a = 0$ has two real roots, whose absolute difference is $\sqrt{2}$, find the value of u .

$u =$



圖三 Figure 3

FOR OFFICIAL USE

Score for
accuracy

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Mult. factor for
speed

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Team No.

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Bonus
score

Time

Total score

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Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 m 和 n 是正整數且 $m^2 - n^2 = 43$ ，求 $m^3 - n^3$ 的值。

If m and n are positive integers with $m^2 - n^2 = 43$, find the value of $m^3 - n^3$.

$m^3 - n^3 =$

2. 設 x_1, x_2, \dots, x_{10} 為非零整數，且滿足 $-1 \leq x_i \leq 2$ ，其中 $i = 1, 2, \dots, 10$ 。

若 $x_1 + x_2 + \dots + x_{10} = 11$ ，求 $x_1^2 + x_2^2 + \dots + x_{10}^2$ 的最大可能值。

Let x_1, x_2, \dots, x_{10} be non-zero integers satisfying $-1 \leq x_i \leq 2$ for $i = 1, 2, \dots, 10$.

If $x_1 + x_2 + \dots + x_{10} = 11$, find the maximum possible value for $x_1^2 + x_2^2 + \dots + x_{10}^2$.

Maximum

3. 若 $f(n) = a^n + b^n$ ，其中 n 是正整數且 $f(3) = [f(1)]^3 + f(1)$ ，求 $a \cdot b$ 的值。

If $f(n) = a^n + b^n$, where n is a positive integer and $f(3) = [f(1)]^3 + f(1)$, find the value of $a \cdot b$.

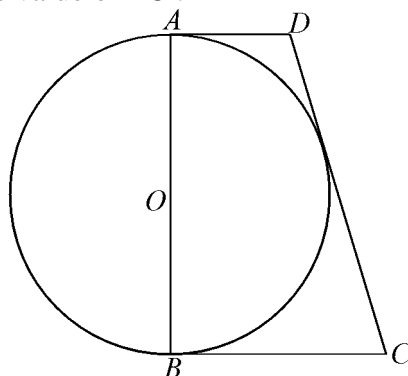
$a \cdot b =$

4. 在圖四， AD 、 BC 和 CD 是以 O 作圓心且直徑 $AB = 12$ 的圓的切線。

若 $AD = 4$ ，求 BC 的值。

In Figure 4, AD , BC and CD are tangents to the circle with centre at O and diameter $AB = 12$. If $AD = 4$, find the value of BC .

$BC =$



圖四 Figure 4

FOR OFFICIAL USE

Score for accuracy

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Mult. factor for speed

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Team No.

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Bonus score

Time

Total score

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Hong Kong Mathematics Olympiad (2012 – 2013)

Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 P 為整數 3,659,893,456,789,325,678 與 342,973,489,379,256 的乘積，求 P 的位數。

In P be the product of 3,659,893,456,789,325,678 and 342,973,489,379,256, find the number of digits of P .

no. of digits

2. 若 $\frac{1}{4} + 4\left(\frac{1}{2013} + \frac{1}{x}\right) = \frac{7}{4}$ ，求 $1872 + 48 \times \left(\frac{2013x}{x+2013}\right)$ 的值。

If $\frac{1}{4} + 4\left(\frac{1}{2013} + \frac{1}{x}\right) = \frac{7}{4}$, find the value of $1872 + 48 \times \left(\frac{2013x}{x+2013}\right)$.

3. 有一個整數被 10 除，餘數為 9；被 9 除，餘數為 8；被 8 除，餘數為 7；等等直至被 2 除，餘數為 1。求此整數的最小值。

The remainders of an integer when divided by 10, 9, 8, ..., 2 are 9, 8, 7, ..., 1 respectively. Find the smallest such an integer.

4. 如圖五， A 、 B 、 C 、 D 、 E 代表不同的個位數字。求 $A + B + C + D + E$ 的值。

In Figure 5, A, B, C, D, E represent different digits.

Find the value of $A + B + C + D + E$.

$$\begin{array}{r} ABCDE \\ \times \quad \quad 9 \\ \hline 1AAA0E \end{array}$$

圖五 Figure 5

FOR OFFICIAL USE

Score for accuracy

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Mult. factor for speed

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Team No.

+ Bonus score

Time

Total score

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