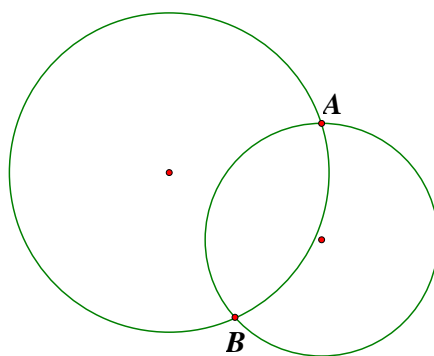


二不等圓交於 A, B 。求過 A 作一直線分別交此二圓於 C 與 D ，使 $CA = AD$ 。

1968 香港中文中學會考普通數學 Paper 2 Q16 (a)

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Let the centres of the two circles be P and Q respectively.

- (1) Join PQ .
- (2) Draw the perpendicular bisector of PQ . M is the mid-point.
- (3) Join MA .
- (4) Through A draw a chord perpendicular to MA , cutting the two circles at C and D respectively.
- (5) Draw $PH \perp CD$, $QK \perp CD$. H and K are the feet of perpendiculars.

Then CAD is the required chord.

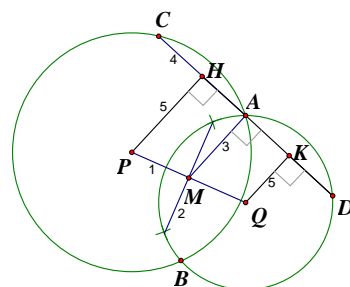
Proof: $PH \parallel MK \parallel QK$ (corr. \angle s eq.)

$PM = MQ$ (by construction)

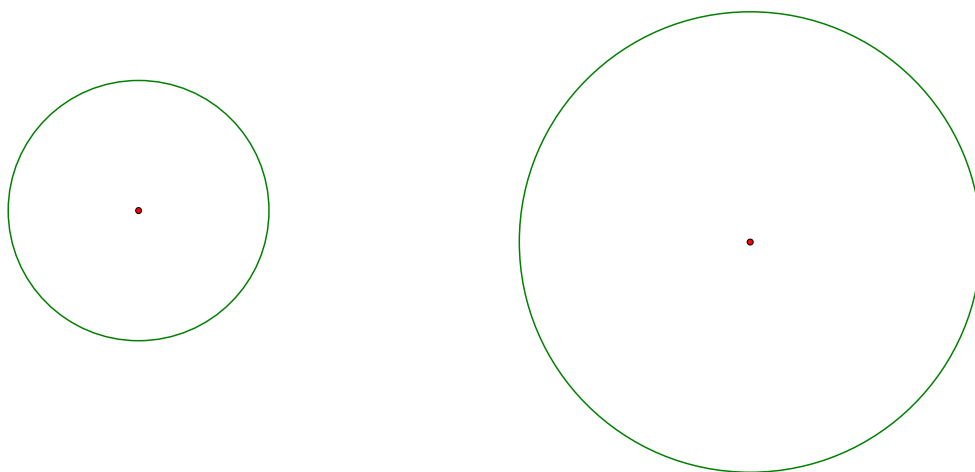
$AH = AK$ (intercept theorem)

$CH = HA$ and $AK = KD$ (\perp from centre bisect chord)

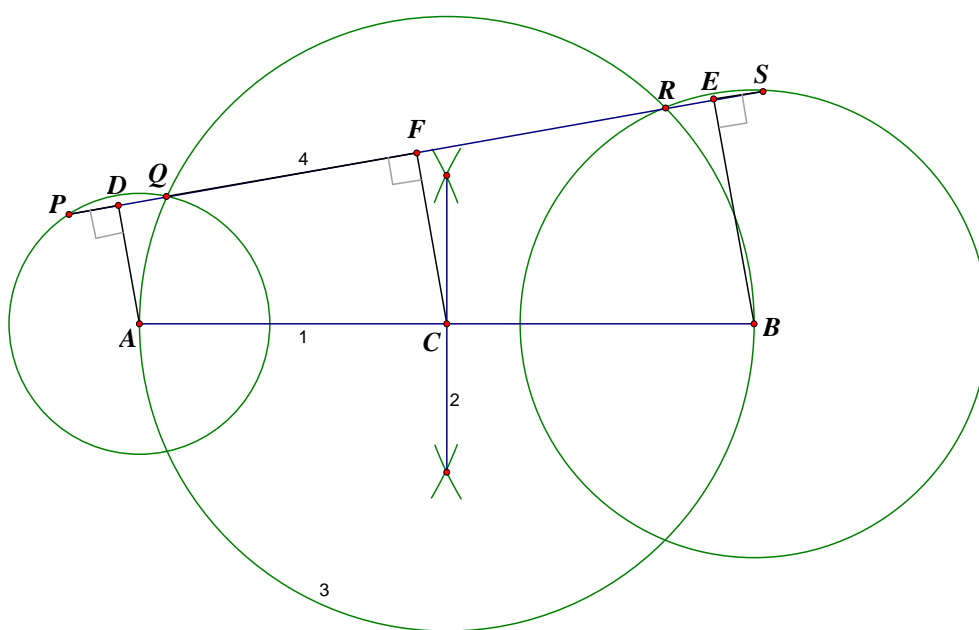
$\therefore AC = 2AH = 2AK = AD$



二不等圓不相交。求作一直線分別交此二圓於 P 、 Q 、 R 及 S ，使 $PQ = RS$ 。



Construction steps:



Let A and B be the centres of the two non-intersecting circles.

- (1) Join A , B .
- (2) Draw the perpendicular bisector of AB , let C be the mid-point of AB .
- (3) Use C as centre, CA as radius to draw a circle, intersecting the original circles at Q , R .
- (4) Join QR and extend the line both ways, cutting the 2 circles again at P and S as shown.

Then $PQ = RS$.

Proof: Draw $AD \perp PQ$, $CF \perp QR$, $BE \perp RS$ as shown.

$$PD = DQ, QF = FR, RE = ES \quad (\perp \text{ from centre bisects chord})$$

$$AD \parallel CF \parallel BE \quad (\text{corr. } \angle\text{s eq.})$$

$$DF = FE \quad (\text{intercept theorem})$$

$$DQ = DF - QF = FE - FR = RE$$

$$2 DQ = 2 RE$$

$$PQ = RS \quad (\perp \text{ from centre bisects chord})$$

Remark: The circle in step 3 does not necessary pass through A and B . The only requirement is cutting the two circles.