Hong Kong Mathematics Olympiad 2011-2012
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求 $2^2 + 3^2 + 4^2 + \cdots + 20122012^2$ 的個位數的值。
   Find the value of the unit digit of $2^2 + 3^2 + 4^2 + \cdots + 20122012^2$.

2. 已知 a、b 及 c 為正偶數，且滿足方程 $a + b + c = 2012$。問該方程共有多少個解？
   Given that a, b and c are positive even integers which satisfy the equation $a + b + c = 2012$. How many solutions does the equation have?

3. 如圖一，$ABCD$ 為一正方形。B 和 D 的坐標分別為 $(5, –1)$ 及 $(–3, 3)$。若 $A(a, b)$位於第一象限內，求 $a + b$ 的值。
   In Figure 1, $ABCD$ is a square. The coordinates of B and D are $(5, –1)$ and $(–3, 3)$ respectively. If $A(a, b)$ lies in the first quadrant, find the value of $a + b$.

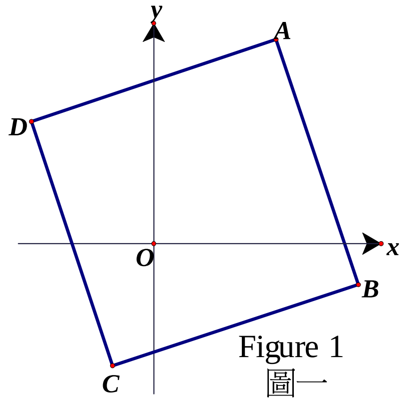


Figure 1
圖一

4. $2^{20}\times25^{12}$ 是一個多少個位的數？
   Find the number of places of the number $2^{20}\times25^{12}$.

5. 已知 $\log_4 N = 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$，求 N 的值。
   Given that $\log_4 N = 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$, find the value of N .

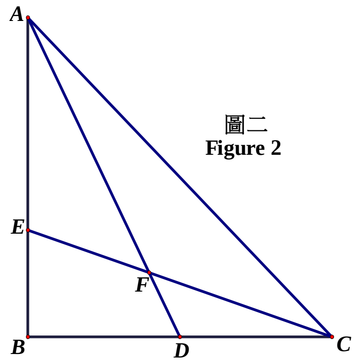
6. 已知 a 及 b 為不同質數，且 $a^2 – 19a + m = 0$ 及 $b^2 – 19b + m = 0$，求 $\frac{a}{b}+\frac{b}{a}$ 的值。
   Given that a and b are distinct prime numbers, $a^2 – 19a + m = 0$ and $b^2 – 19b + m = 0$.
   Find the value of $\frac{a}{b}+\frac{b}{a}$.

7. 已知 a、b、c 是正數，且 $a + b + c = 9$。設 $a + b$、$a + c$、$b + c$ 當中的最大值為 P，求 P 的最小值。
   Given that a, b and c are positive numbers, and $a + b + c = 9$. Suppose the maximum value among $a + b$, $a + c$ and $b + c$ is P, find the minimum value of P .

8. 若方程 $(k^2 – 4)x^2 – (14k + 4)x + 48 = 0$ 有兩個相異的正整數根，求 k 的值。
   If the quadratic equation $(k^2 – 4)x^2 – (14k + 4)x + 48 = 0$ has two distinct positive integral roots, find the value(s) of k .

9. 已知 x、y 為正整數，且 $x > y$，解 $x^3 = 2189 + y^3$。
   Given that x, y are positive integers and $x > y$, solve $x^3 = 2189 + y^3$.

10. 如圖二，$AE = 14$、$EB = 7$、$AC = 29$ 及 $BD = DC = 10$。求 BF^2 的值。
    In figure 2, $AE = 14$, $EB = 7$, $AC = 29$ and $BD = DC = 10$.
    Find the value of BF^2 .



圖二
Figure 2

*** 試卷完 **End of Paper** ***

Hong Kong Mathematics Olympiad 2011-2012
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 已知 x、y 及 z 為三個連續正整數，且 $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ 為整數，求 $x+y+z$ 的值。

   Given that x, y and z are three consecutive positive integers, and $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ is an integer. Find the value of $x+y+z$.

2. 已知 x 是一個實數，且 $\sqrt{x-2012}+\sqrt{(5-x)^2}=x$，求 x 的值。

   Given that x is a real number and $\sqrt{x-2012}+\sqrt{(5-x)^2}=x$. Find the value of x.

3. 求 $\sqrt{2^2+2^{1008}+2^{2012}}$ 的值。(答案可以指數表示。)

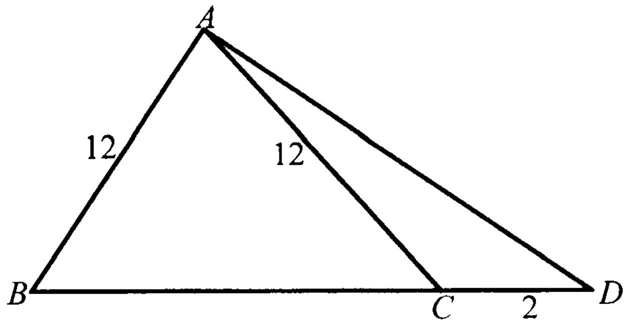
   Evaluate $\sqrt{2^2+2^{1008}+2^{2012}}$. (Answer can be expressed in index form.)

4. 求 $\frac{1}{\sqrt{2012}+\sqrt{2011}}+\frac{1}{\sqrt{2011}+\sqrt{2010}}+\cdots+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{1}}$ 的值。(答案可以根式表示。)

   Evaluate $\frac{1}{\sqrt{2012}+\sqrt{2011}}+\frac{1}{\sqrt{2011}+\sqrt{2010}}+\cdots+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{1}}$.
   (Answer can be expressed in surd form.)

5. 求 $x^2+y^2-10x-6y+2046$ 的最小值。
   Find the minimum value of $x^2+y^2-10x-6y+2046$.

6. 如圖三，ΔABC 為一等腰三角形，設 $AB = AC = 12$。若 D 是 BC 延伸線上的一點，使 $\angle DAB = 90°$ 及 $CD = 2$，求 BC 的長。
   In Figure 3, ΔABC is an isosceles triangle. Suppose $AB = AC = 12$. If D is a point on BC produced such that $\angle DAB = 90°$ and $CD = 2$, find the length of BC.



7. 已知 $a^x = b^y = c^z = 30^w$ 及 $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{w}$，當中 a、b、c 為正整數($a\le b\le c$)及 x、y、z、w 為實數。求 $a+b+c$ 的值。

   Given that $a^x = b^y = c^z = 30^w$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{w}$, where a, b, c are positive integers ($a\le b\le c$) and x, y, z, w are real numbers, find the value of $a+b+c$.

8. 已知方程 $x^2+px+q=0$ 的兩個根為正整數，且 $q>0$。若 $p+q=60$，求 q 的值。
   Given that the roots of the equation $x^2+px+q=0$ are positive integers and $q>0$.
   If $p+q=60$, find the value of q.

9. 求 $\sin^2 1°+\sin^2 2°+\sin^2 3°+\ldots+\sin^2 359°+\sin^2 360°$ 的值。
   Evaluate $\sin^2 1°+\sin^2 2°+\sin^2 3°+\ldots+\sin^2 359°+\sin^2 360°$.

10. 在一集會中，原先安排每位賓客與其他賓客各握手一次，但小明只和他認識的人握手。如果集會中實際握手的總數為 60 次，那麼小明在集會中認識多少人？
    (註：當兩人相互握手，握手的總次數是一次(而不是兩次)。)
    In a gathering, originally each guest will shake hands with every other guest, but Steven only shakes hands with people whom he knows. If the total number of handshakes in the gathering is 60, how many people in the gathering does Steven know? (Note: when two persons shake hands with each other, the total number of handshakes will be one (not two).)

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2011 – 2012
Heat Event (Geometric Construction)
香港數學競賽 2011 – 2012
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: ____

School Name: ____

第一題 Question No. 1

在下面的空位上，試構作一等邊三角形 *ABC*，當中每邊的長等於下圖中 *MN* 的長度。

In the space provided, construct an equilateral triangle *ABC* with sides equal to the length of *MN* below.

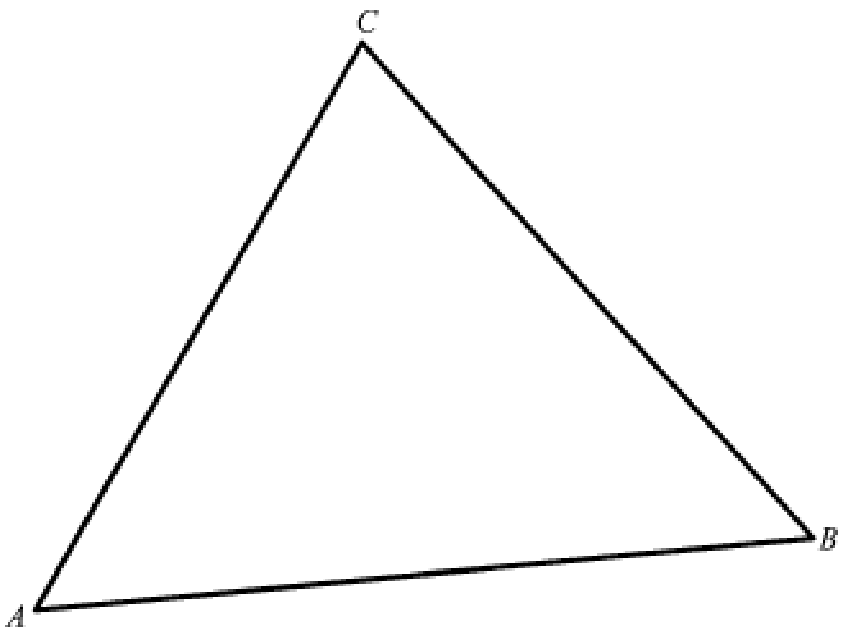
School Code: ____

School Name: ____

第二題 Question No. 2

如圖一，在ΔABC 內構作一圓使得 *AB*、*BC* 及 *CA* 均為該圓的切綫。

As shown in Figure 1, construct a circle inside the triangle *ABC*, so that *AB*, *BC* and *CA* are tangents to the circle.



圖一 Figure 1

Hong Kong Mathematics Olympiad 2011 – 2012
Heat Event (Geometric Construction)
香港數學競賽 2011 – 2012
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: ____
School Name: ____

第三題 Question No. 3

圖二所示為一三角形 PQR。試構作一綫段 MN 平行於 QR 使得

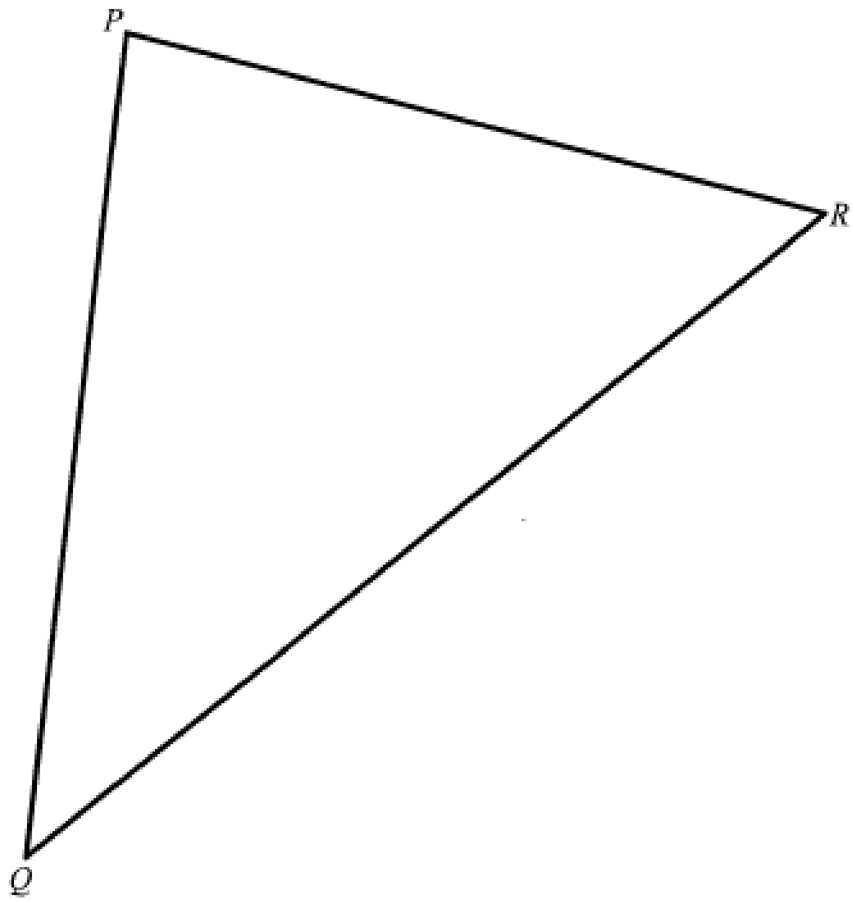
(i) M 及 N 分別位於 PQ 及 PR 上;且

(ii) ΔPMN 的面積$=\frac{1}{2}\times\Delta PQR$ 的面積。

Figure 2 shows a triangle PQR. Construct a line MN parallel to QR so that

(i) M and N lie on PQ and PR respectively; and

(ii) the area of $\Delta PMN=\frac{1}{2}\times$ the area of ΔPQR.



圖二 Figure 2

*** 試卷完 **End of Paper** ***