Hong Kong Mathematics Olympiad (1994-95)

Event 1 (Individual)

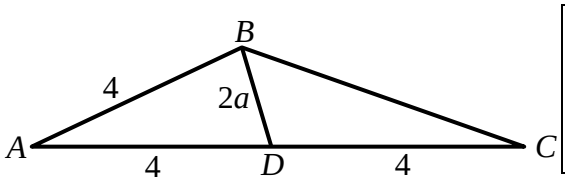
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = \log_{\frac{1}{4}} \frac{1}{2}$，求 a 的值。

Find the value of a, if $a = \log_{\frac{1}{4}} \frac{1}{2}$.

$a =$

(ii) 如圖示，$AB = AD = DC = 4$, $BD = 2a$。
若 BC 之長為 b，求 b 的值。
In the figure, $AB = AD = DC = 4$,
$BD = 2a$. Find b, the length of BC.



$b =$

(iii) 已知 $f(x) = px^3 + qx + 5$ 且 $f(-7) = \sqrt{2}\, b + 1$。若 $c = f(7)$，求 c 的值。
It is given that $f(x) = px^3 + qx + 5$ and $f(-7) = \sqrt{2}\, b + 1$.
Find the value of c, if $c = f(7)$.

$c =$

(iv) 若 $d^c + 1000$ 可被 $10 + c$ 所整除，求 d 的最小正整數值。
Find the least positive integer d, such that $d^c + 1000$ is divisible by $10 + c$.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1994-95)

Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\frac{x}{(x-1)(x-4)} = \frac{x}{(x-2)(x-3)}$，求 x 的值。

If $\frac{x}{(x-1)(x-4)} = \frac{x}{(x-2)(x-3)}$, find the value of x.

$x =$

(ii) 若 $f(t) = 3\times 52^t$ 且 $y = f(x)$。求 y 的值。

If $f(t) = 3\times 52^t$ and $y = f(x)$, find the value of y.

$y =$

(iii) 甲可在 y 日完成某一項工程，乙可在$(y + 3)$日完成同一工程。
假如甲乙二人合作，可在 z 日完成，求 z 的值。
A can finish a job in y days, B can finish a job in $(y + 3)$ days.
If they worked together, they can finish the job in z days, find the value of z.

$z =$

(iv) 用 z 粒骰子擲得 7 點的概率是 w，求 w 的值。
The probability of throwing z dice to score 7 is w, find the value of w.

$w =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1994-95)

Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = \sin 30° + \sin 300° + \sin 3000°$，求 a 的值。
If $a = \sin 30° + \sin 300° + \sin 3000°$, find the value of a.

$a =$

(ii) 已知 $\frac{x+y}{2} = \frac{z+x}{3} = \frac{y+z}{4}$ 且 $x + y + z = 36a$。求 b 之值，若 $b = x + y$。
It is given that $\frac{x+y}{2} = \frac{z+x}{3} = \frac{y+z}{4}$ and $x + y + z = 36a$.
Find the value of b, if $b = x + y$.

$b =$

(iii) 已知方程 $x + 6 + 8k = k(x + b)$有正整數解。求 k 的最小值 c。
It is given that the equation $x + 6 + 8k = k(x + b)$ has positive integral solution. Find c, the least value of k.

$c =$

(iv) 一輛汽車以平均時速 $40c$ km/h 完成了旅程的 40 %。為著使全程的平均速 度為 100 km/h，車速被調至 d km/h 行畢全程。求 d 的值。
A car has already travelled 40% of its journey at an average speed of $40c$ km/h. In order to make the average speed of the whole journey become 100 km/h, the speed of the car is adjusted to d km/h to complete the rest of the journey.
Find the value of d.

$d =$

FOR OFFICIAL USE

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Time [ ] [ ]

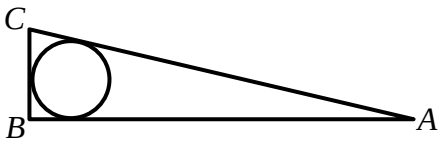
Min. Sec.

Hong Kong Mathematics Olympiad (1994-95)

Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在三角形 ABC 中，$\angle B = 90°$，$BC = 7$ 且 $AB = 24$。
若 r 為內切圓之半徑，求 r 的值。
In triangle ABC, $\angle B = 90°$, $BC = 7$ and $AB = 24$. If r is the radius of the inscribed circle, find the value of r.



$r =$

(ii) 若 $x^2 + x - 1 = 0$ 且 $s = x^3 + 2x^2 + r$，求 s 的值。
If $x^2 + x - 1 = 0$ and $s = x^3 + 2x^2 + r$, find the value of s.

$s =$

(iii) 已知 $F_1 = F_2 = 1$ 且 $F_n = F_{n-1} + F_{n-2}$，其中 $n \geq 3$。若 $F_t = s + 1$，求 t 的值。
It is given that $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$, where $n \geq 3$. If $F_t = s + 1$, find the value of t.

$t =$

(iv) 若 $u = \sqrt{t(t+1)(t+2)(t+3)+1}$，求 u 的值。
If $u = \sqrt{t(t+1)(t+2)(t+3)+1}$, find the value of u.

$u =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (1994-95)

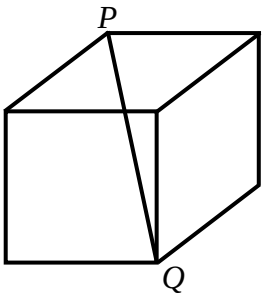
Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) It is given that $\log_7(\log_3(\log_2 x)) = 0$. Find the value of a, if $a = x^{\frac{1}{3}}$.
已知 $\log_7(\log_3(\log_2 x)) = 0$。若 $a = x^{\frac{1}{3}}$，求 a 的值。

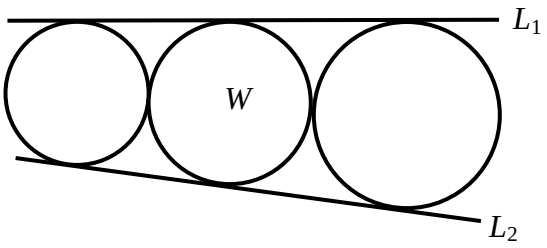
$a =$

(ii) 如圖示，PQ 是正方體的一條對角綫，且 $PQ = \frac{a}{2}$。
若 b 為此正方體的總表面積，求 b 的值。
In the figure, PQ is a diagonal of the cube and $PQ = \frac{a}{2}$.
Find the value of b, if b is the total surface area of the cube.



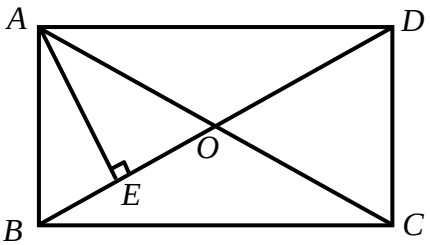
$b =$

(iii) 如圖示，L_1、L_2 為三個圓的切綫。如果最大圓的半徑是 18，最小圓半徑是 $4b$，求 c，若 c 為圓 W 的半徑。
In the figure, L_1 and L_2 are tangents to the three circles. If the radius of the largest circle is 18 and the radius of the smallest circle is $4b$, find c, where c is the radius of the circle W.



$c =$

(iv) 如圖，$ABCD$ 為一長方形。$AE \perp BD$
且 $BE = EO = \frac{c}{6}$。求長方形 $ABCD$ 之面積 d。
Refer to the figure, $ABCD$ is a rectangle. $AE \perp BD$ and $BE = EO = \frac{c}{6}$.
Find d, the area of the rectangle $ABCD$.



$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ] Team No. [ ]

\+ Bonus score [ ] Time [ ] [ ]

Total score [ ] Min. Sec.

Hong Kong Mathematics Olympiad (1994-95)

Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) $2^a \cdot 9^b$ 為一四位數，其千位數是 2，百位數是 a，十位數是 9，個位數是 b，求 a 及 b 的值。
$2^a \cdot 9^b$ is a four digit number and its thousands digit is 2, its hundreds digit is a, its tens digit is 9 and its units digit is b, find the values of a and b.

$a =$

(ii)

$b =$

(iii) 若 $c = \left(1+\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{3}\right)$，求 c 的值。
Find the value of c, if $c = \left(1+\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{3}\right)$.

$c =$

(iv) 求 d 的值，若
$d = \left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)$。
Find the value of d, if
$d = \left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)$.

$d =$

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Time [ ] [ ]

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Hong Kong Mathematics Olympiad (1994-95)

Event 7 (Group)

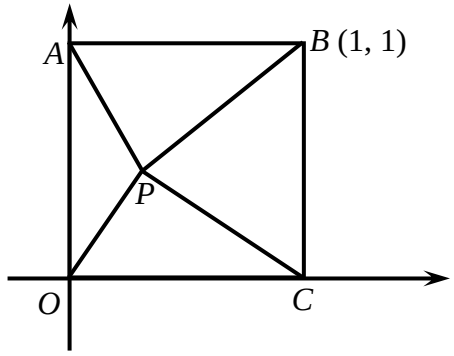
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 p、q、r 為三角形 PQR 的三邊。若 $p^4 + q^4 + r^4 = 2r^2(p^2 + q^2)$，且 $a = \cos^2 R$，其中 R 的對邊為 r，求 a 的值。
Let p, q, r be the three sides of triangle PQR. If $p^4 + q^4 + r^4 = 2r^2(p^2 + q^2)$, find the value of a, where $a = \cos^2 R$ and R denotes the angle opposite r.

$a =$

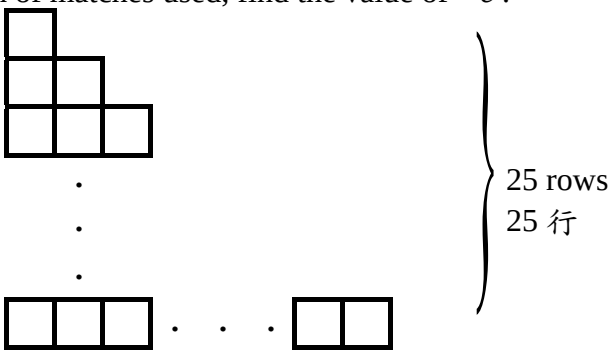
(ii) 如圖，P 為正方形 $OABC$ 內的任意點，且 b 為 $PO + PA + PB + PC$ 之最小值，求 b 的值。
Refer to the diagram, P is any point inside the square $OABC$ and b is the minimum value of $PO + PA + PB + PC$, find the value of b.

$b =$



(iii) 長度同為 1 的火柴被排成下列圖案。若以 c 表示用去火柴枝的總長，求 c 的值。
Identical matches of length l are used to arrange the following pattern, if c denotes the total length of matches used, find the value of c.

$c =$



(iv) 求 d 的值，若 $d = \sqrt{111111 - 222}$。
Find the value of d, where $d = \sqrt{111111 - 222}$.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

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Time [ ] [ ]

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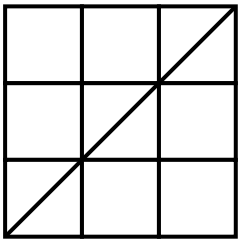
Hong Kong Mathematics Olympiad (1994-95)

Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在方格紙上繪畫尺寸為 $\ell \times b$ 的長方形，其中 ℓ、b 為正整數並添上對角綫一條。以 V 代表相交的端點總數(不包括首尾兩點在內)。(如右圖示)

Rectangles of length ℓ and breadth b where ℓ, b are positive integers, are drawn on square grid paper. For each of these rectangles, a diagonal is drawn and the number of vertices V intersected (excluding the two end points) is counted (see the figure).



$\ell = b = 3$

$V = 2$

(i) 當 $\ell = 6$，$b = 4$ 時，求 V 的值。

Find the value of V, when $\ell = 6$, $b = 4$.

$V =$

(ii) 當 $\ell = 5$，$b = 3$ 時，求 V 的值。

Find the value of V, when $\ell = 5$, $b = 3$.

$V =$

(iii) 當 $\ell = 12$ 且 $1 < b < 12$ 時，求使 $V = 0$ 時，b 的不同個數 r。

When $\ell = 12$ and $1 < b < 12$, find r, the number of different values of b that makes $V = 0$?

$r =$

(iv) 當 $\ell = 108$，$b = 72$ 時，求 V 的值。

Find the value of V, when $\ell = 108$, $b = 72$.

$V =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (1994-95)

Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

A、*B*、*C*、D 為自 0 至 9 間的不同整數，且

$$\begin{array}{rcccc} & A & A & B & C \\ - & B & A & C & B \\ \hline & D & A & C & D \end{array}$$

求 *A*、*B*、*C* 及 *D* 之值。

A, *B*, *C*, *D* are different integers ranging from 0 to 9 and

$$\begin{array}{rcccc} & A & A & B & C \\ - & B & A & C & B \\ \hline & D & A & C & D \end{array}$$

Find the values of *A*, *B*, *C* and *D* .

A =	*B* =	*C* =	*D* =

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1994-95)
Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在直角坐標平面上，*x*- 和 *y*- 坐標同為整數的點稱為格點。*P* 是起始時位於(0, 0)的移動點，它每一步必須沿坐標綫的其中一過個方向走 1 個單位的距離。

Lattice points are points on a rectangular coordinate plane having both *x*- and *y*-coordinates being integers. A moving point *P* is initially located at (0, 0). It moves 1 unit along the coordinate lines (in either directions) in a single step.

(i) 若 *P* 走 1 步，它可到達 *a* 個格點，求 *a* 的值。
If *P* moves 1 step then *P* can reach *a* different lattice points, find the value of *a* .

a =

(ii) 若 *P* 可走不超過 2 步，它可到達 *b* 個格點，求 *b* 的值。
If *P* moves not more than 2 steps then *P* can reach *b* different lattice points, find the value of *b* .

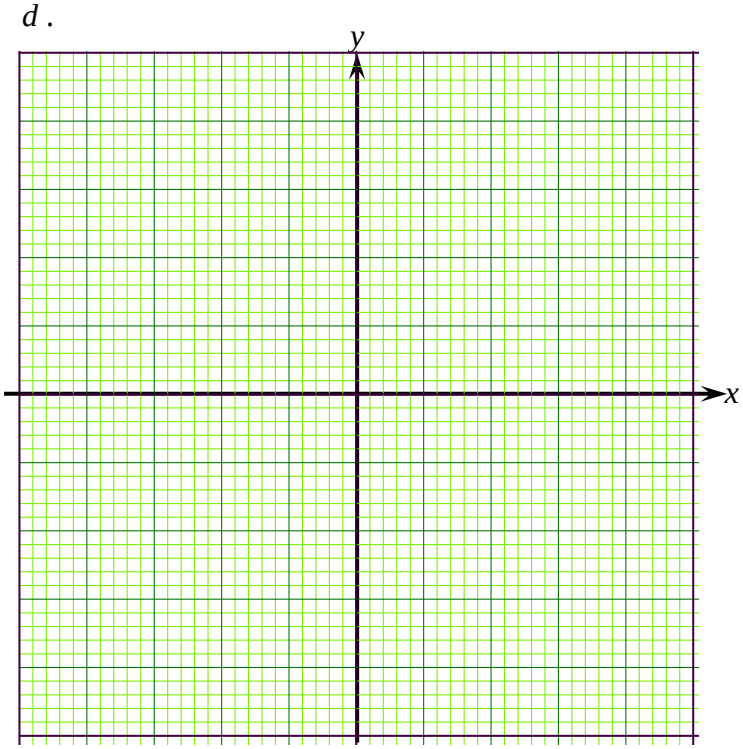
b =

(iii) 若 *P* 走 3 步，它可到達 *c* 個格點，求 *c* 的值。
If *P* moves 3 steps then *P* can reach *c* different lattice points, find the value of *c*.

c =

(iv) 若 *P* 走 9 步，它停在直綫 $x + y = 9$ 上的概率是 *d*，求 *d* 的值。
If *d* is the probability that *P* lies on the straight line $x + y = 9$ when *P* advances 9 steps, find the value of *d* .

d =



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Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.