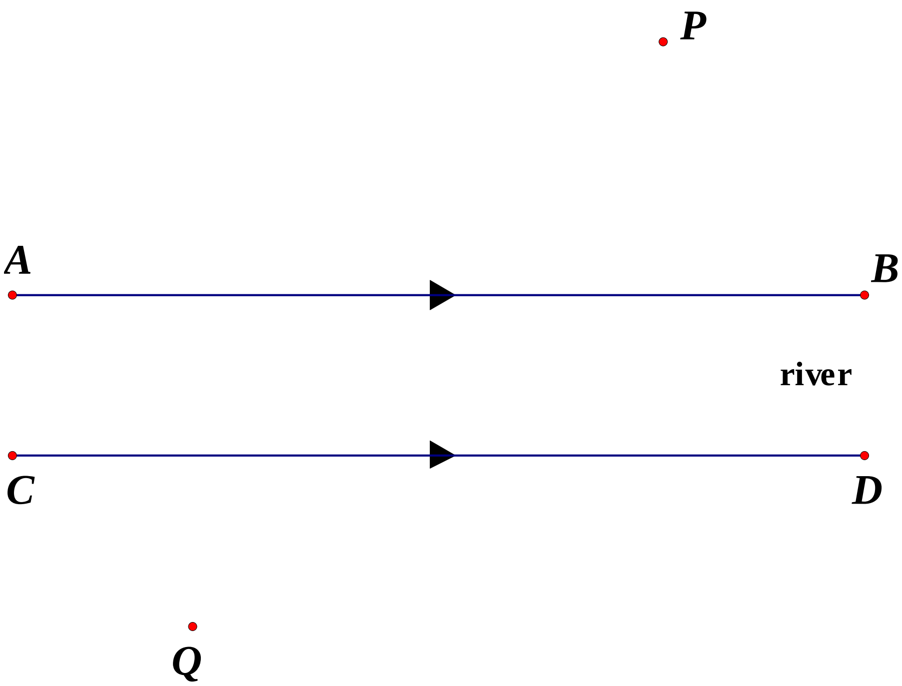
Bridge

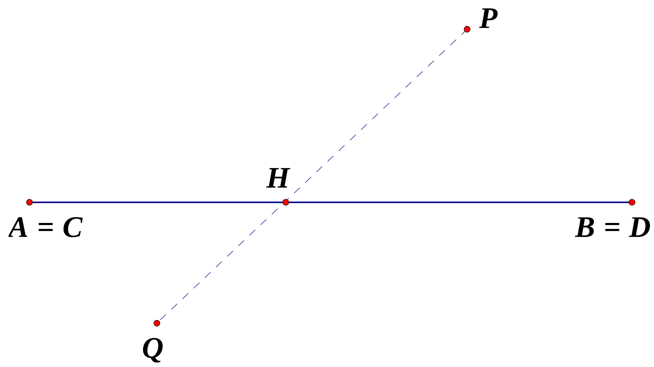
Created by Mr. Francis Hung on 20111008. Last updated: 2011-10-09

In the following diagram, towns P and Q are situated in the opposite shores of two parallel river banks AB and CD. You are asked to build a bridge across the river, which must be perpendicular to the river. In order to minimise the walking distance from P to Q via the bridge, what is the best position to build the bridge? Show your works clearly.



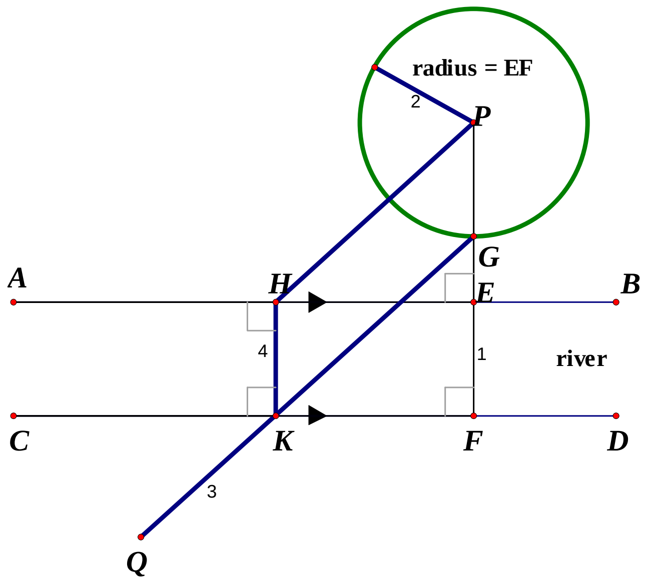
Consider the special case:

When the width of the river is zero, then the diagram is as follows:



In order to minimise the walking distance from P to Q via the bridge, we join P and Q to meet AB (hence CD) at H. Then P, H, Q are collinear. $PH + HQ$ will be the minimum. H is the location of the bridge.

The solution to the general case is as follows:



(1) From P draw a line $PEF \perp AB$. E and F are the feet of perpendiculars on AB and CD respectively.

(2) Use P as centre, EF as radius to draw a circle, cutting PEF at G.

(3) Join QG, which cuts CD at K.

(4) Draw $HK \perp AB$, where H lies on AB.

Then HK is the position of the bridge.

Proof: Join PH.

$HK = EF = PG$ by construction

$HK \parallel PG$ by construction

$\therefore$ $HKGP$ is a parallelogram

(opp. sides are eq. and parallel)

$PH = GK$ (opp. sides of parallelogram)

G, K, Q are collinear.

$PH + KQ = GK + KQ$ which is the minimum distance between G and Q.

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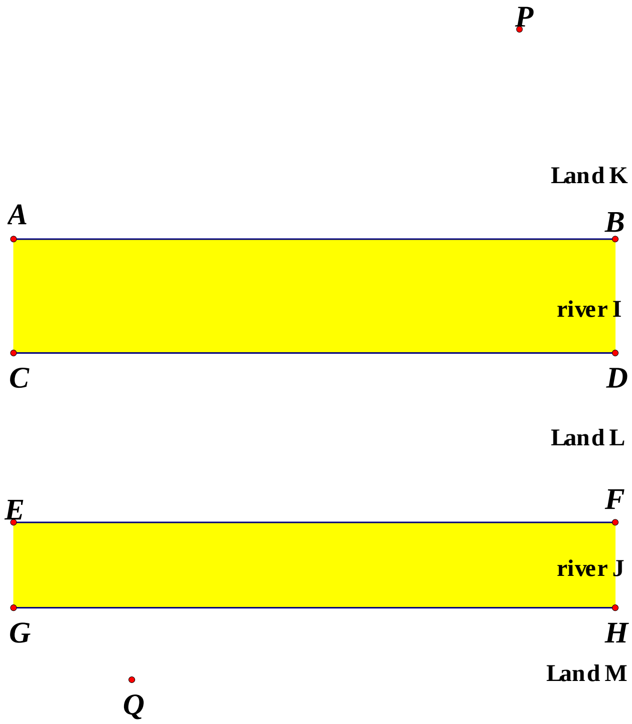
Follow-up question:



As shown in the diagram, towns P, Q are separated by two parallel rivers I and J. The widths of river are i, j respectively. You are asked to build two bridges across the rivers. Each one must be perpendicular to the rivers. In order to minimise the walking distance from P to Q via the bridges, what is the best position to build the two bridges? Show your works clearly.

Solution

(1) From P draw a line $PW \perp GH$. T, U, V and W are the feet of perpendiculars on AB, CD, EF and GH respectively.

(2) Use P as centre, $TU + VW$ as radius to draw a circle, cutting PW at M.

(3) Join QG, which cuts GH at N.

(4) Draw $NO \perp EF$, where O lies on EF.

(5) OR // QM, cutting CD at R.

(6) Draw $RS \perp AB$, where S lies on AB.

(7) Join PS.

SR and ON are the positions of the bridges.

Proof: omitted.