**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 1 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $7^{2x} = 36$ 及 $7^{-x} = (6)^{\frac{-a}{2}}$，求 $a$ 的值。

Given that $7^{2x} = 36$ and $7^{-x} = (6)^{\frac{-a}{2}}$, find the value of $a$.

$a =$

(ii) 若 $\log_2\{\log_2[\log_2(2b) + a] + a\} = a$，求 $b$ 的值。

Find the value of $b$ if $\log_2\{\log_2[\log_2(2b) + a] + a\} = a$.

$b =$

(iii) 若方程 $(x - b)(x - 2)(x + 1) = 3(x - b)(x + 1)$ 正根的總數為 $c$，求 $c$ 的值。

If $c$ is the total number of positive roots of the equation $(x - b)(x - 2)(x + 1) = 3(x - b)(x + 1)$, find the value of $c$.

$c =$

(iv) 若 $\sqrt{3-2\sqrt{2}} = \sqrt{c} - \sqrt{d}$，求 $d$ 的值。

If $\sqrt{3-2\sqrt{2}} = \sqrt{c} - \sqrt{d}$, find the value of $d$.

$d =$

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**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

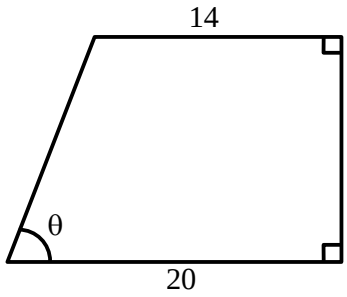
**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 2 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\sin\theta = \frac{4}{5}$，求四邊形面積 $a$。

If $\sin\theta = \frac{4}{5}$, find $a$, the area of the quadrilateral.




$a =$

(ii) 若 $b = 126^2 - a^2$，求 $b$ 的值。

If $b = 126^2 - a^2$, find the value of $b$.

$b =$

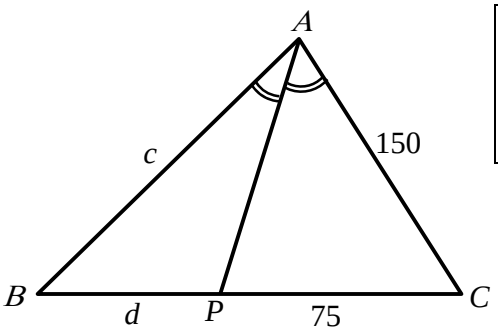
(iii) 將$(3000 + $b$)按 5 : 6 : 8 分成 3 份，最小的一份為$c$。求 $c$ 的值。

Dividing $(3000 + $b$) in a ratio 5 : 6 : 8, the smallest part is $ $c$.
Find the value of $c$.

$c =$

(iv) 圖中 $AP$ 等分$\angle BAC$。已知 $AB = c$，$BP = d$，$PC = 75$ 及 $AC = 150$，求 $d$ 的值。

In the figure, $AP$ bisects $\angle BAC$.
Given that $AB = c$, $BP = d$, $PC = 75$ and $AC = 150$, find the value of $d$.




$d =$

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**FOR OFFICIAL USE**

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Time [ ] [ ]

Total score [ ]

Min. Sec.

**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 3 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a$ 為以 13 除 2614303940317 的餘數，求 $a$ 的值。
If $a$ is the remainder when 2614303940317 is divided by 13, find the value of $a$.

$a =$

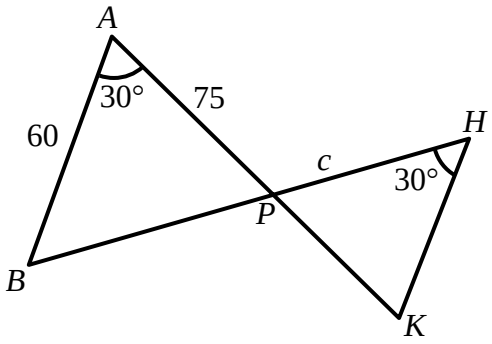
(ii) 設 $P(x, b)$ 為直綫 $x + y = 30$ 上的點且滿足 $OP$ 斜率為 $a$ ($O$ 乃原點)。求 $b$ 的值。
Let $P(x, b)$ be a point on the straight line $x + y = 30$ such that slope of $OP = a$ ($O$ is the origin). Determine the value of $b$.

$b =$

(iii) 兩人踏單車，起始時相距$(b + 26)$ km，以時速 40 km/h 及 60 km/h 相向而行。一蒼蠅以時速 100 km/h 往返兩人鼻尖，若牠在兩人碰上前共飛 $c$ km，求 $c$ 的值。
Two cyclists, initially $(b + 26)$ km apart travelling towards each other with speeds 40 km/h and 60 km/h respectively. A fly flies back and forth between their noses at 100 km/h. If the fly flied $c$ km before crushed between the cyclists, find the value of $c$.

$c =$

(iv) 圖中 $APK$ 及 $BPH$ 為直綫。若 $d = \Delta HPK$ 的面積，求 $d$ 的值。
In the figure, $APK$ and $BPH$ are straight lines.
If $d$ = area of triangle $HPK$, find the value of $d$.




$d =$

**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

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Time [ ] [ ]

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Min. Sec.

**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 4 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

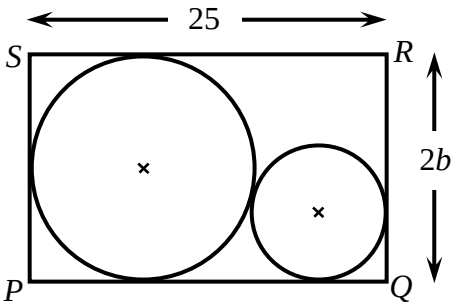
(i) 已知 $x$ 和 $y$、$y$ 和 $z$、$z$ 和 $x$ 的平均值分別為 5、9、10。
若 $x$、$y$、$z$ 的平均值是 $a$，求 $a$ 的值。
Given that the means of $x$ and $y$, $y$ and $z$, $z$ and $x$ are respectively 5, 9, 10.
If $a$ is the mean of $x$, $y$, $z$, find the value of $a$ .

$a =$

(ii) 某兩數的比例為 5 : $a$。當每邊加 12 時，兩數的比例變為 3 : 4。
若 $b$ 為原本兩數之差及 $b > 0$，求 $b$ 的值。
The ratio of two numbers is 5 : $a$. If 12 is added to each of them, the ratio becomes 3 : 4.
If $b$ is the difference of the original numbers and $b > 0$, find the value of $b$ .

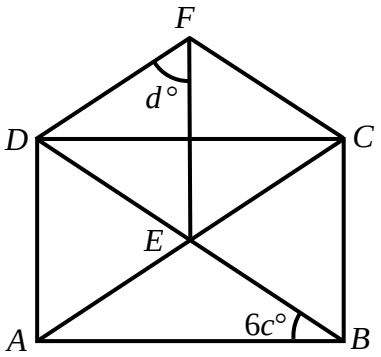
$b =$

(iii) $PQRS$ 為一長方形，
若細圓的半徑為 $c$，求 $c$ 的值。
$PQRS$ is a rectangle. If $c$ is the radius of the smaller circle, find the value of $c$.




$c =$

(iv) $ABCD$ 為一長方形及 $CEF$ 為一等邊三角形，
$\angle ABD = 6c°$，求 $d$ 的值。
$ABCD$ is a rectangle and $CEF$ is an equilateral triangle, $\angle ABD = 6c°$, find the value of $d$ .




$d =$

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**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 5 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 長方形兩對邊同時加長 50%，而其餘兩對邊則縮短 20%。
若長方形的面積增加 $a$%，求 $a$ 的值。
Two opposite sides of a rectangle are increased by 50% while the other two are decreased by 20%. If the area of the rectangle is increased by $a$%, find the value of $a$ .

$a =$

(ii) 設 $f(x) = x^3 - 20x^2 + x - a$ 及 $g(x) = x^4 + 3x^2 + 2$。若 $h(x)$為 $f(x)$和 $g(x)$的最大公因子，求 $b = h(1)$的值。
Let $f(x) = x^3 - 20x^2 + x - a$ and $g(x) = x^4 + 3x^2 + 2$.
If $h(x)$ is the highest common factor of $f(x)$ and $g(x)$, find the value of $b = h(1)$ .

$b =$

(iii) It is known that $b^{16} - 1$ has four distinct prime factors, determine the largest one, denoted by $c$ .
已知 $b^{16} - 1$ 共有四質因子，求其中最大的一個，以 $c$ 表它。

$c =$

(iv) When $c$ is represented in binary scale, there are $d$ '0's. Find the value of $d$ .
當以二進制表示 $c$，則其中有 $d$ 個'0'。求 $d$ 的值。

$d =$

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**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

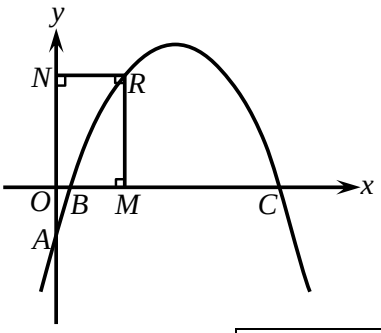
**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 6 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

右圖所示為 $y = px^2 + 5x + p$ 的圖像。$A = (0, -2)$、$B = \left(\frac{1}{2}, 0\right)$、$C = (2, 0)$、$O = (0, 0)$。

The following shows the graph of $y = px^2 + 5x + p$. $A = (0, -2)$, $B = \left(\frac{1}{2}, 0\right)$, $C = (2, 0)$, $O = (0, 0)$.




(i) 求 $p$ 的值。
Find the value of $p$.

$p =$

(ii) 若 $y$ 的最大值為 $\frac{9}{m}$，求 $m$ 的值。
If $\frac{9}{m}$ is the maximum value of $y$, find the value of $m$.

$m =$

(iii) 設 $R$ 為曲綫上一點且 $OMRN$ 為一正方形。若 $R$ 的 $x$ 坐標為 $r$，求 $r$ 的值。
Let $R$ be a point on the curve such that $OMRN$ is a square.
If $r$ is the $x$-coordinate of $R$, find the value of $r$.

$r =$

(iv) 一斜率為 $-2$ 及通過原點的直綫與上述曲綫相交於兩點 $E$ 及 $F$。若 $EF$ 中點的 $y$ 坐標為 $\frac{7}{s}$，求 $s$ 的值。
A straight line with slope = $-2$ passes through the origin cutting the curve at two points $E$ and $F$.
If $\frac{7}{s}$ is the $y$-coordinate of the midpoint of $EF$, find the value of $s$.

$s =$

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**FOR OFFICIAL USE**

| Score for accuracy | | × | Mult. factor for speed | | = | | Team No. | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | + Bonus score | | Time | | |
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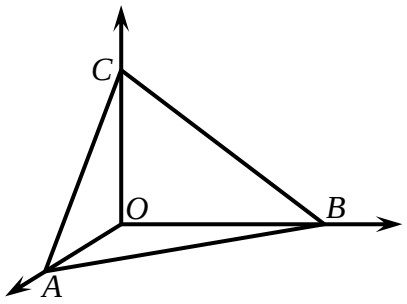
**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 7 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

*OABC* 為一四面體，其中 *OA*、*OB* 及 *OC* 互相垂直。
已知 $OA = OB = OC = 6x$。
*OABC* is a tetrahedron with *OA*, *OB* and *OC* being mutually perpendicular. Given that $OA = OB = OC = 6x$.




(i) 若 *OABC* 的體積為 $ax^3$，求 *a* 的值。
If the volume of *OABC* is $ax^3$, find the value of $a$.

$a =$

(ii) 若 $\Delta ABC$ 的面積為 $b\sqrt{3}x^2$，求 *b* 的值。
If the area of $\Delta ABC$ is $b\sqrt{3}x^2$, find the value of $b$.

$b =$

(iii) 若由 *O* 至 $\Delta ABC$ 的距離為 $c\sqrt{3}x$，求 *c* 的值。
If the distance from *O* to $\Delta ABC$ is $c\sqrt{3}x$, find the value of $c$.

$c =$

(iv) 若由 *C* 至 *AB* 中點的俯角為θ，且 $\sin\theta = \frac{\sqrt{d}}{3}$，求 *d* 的值。
If θ is the angle of depression from *C* to the midpoint of *AB* and $\sin\theta = \frac{\sqrt{d}}{3}$, find the value of $d$.

$d =$

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**FOR OFFICIAL USE**

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Team No. [ ]

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Time [ ] [ ]

Total score [ ]

Min. Sec.

**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 8 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

已知方程 $x^2 + (m + 1)x - 2 = 0$ 有兩整數根$(\alpha + 1)$及$(\beta + 1)$，且$\alpha < \beta$及 $m \neq 0$。設 $d = \beta - \alpha$。
Given that the equation $x^2 + (m + 1)x - 2 = 0$ has 2 integral roots $(\alpha + 1)$ and $(\beta + 1)$ with $\alpha < \beta$ and $m \neq 0$.
Let $d = \beta - \alpha$.

(i) 求 $m$ 的值。
Find the value of $m$.

$m =$

(ii) 求 $d$ 的值。
Find the value of $d$.

$d =$

設 $n$ 為由 1 至 2000 內被 3 或 7 除時，餘數都為 1 的整數的總數。
Let $n$ be the total number of integers between 1 and 2000 such that each of them gives a remainder of 1 when it is divided by 3 or 7.

(iii) 求 $n$ 的值。
Find the value of $n$.

$n =$

(iv) 若 $s$ 為上述 $n$ 個整數的總和，求 $s$ 的值。
If $s$ is the sum of all these $n$ integers, find the value of $s$.

$s =$

**FOR OFFICIAL USE**

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Total score [ ]

Team No. [ ]

Time [ ] [ ]

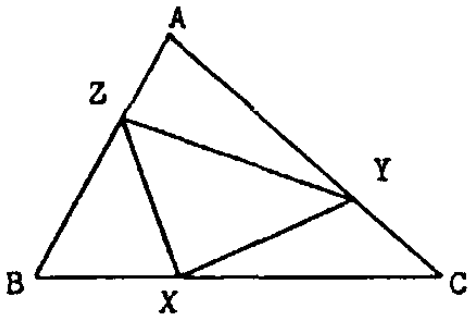
Min. Sec.

**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 9 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

點 $X$、$Y$、$Z$ 依次將 $BC$、$CA$、$AB$ 分成 1 : 2。
設 $\Delta AZY$ 的面積: $\Delta ABC$ 的面積 $= 2 : a$ 及
$\Delta AZY$ 的面積: $\Delta XYZ$ 的面積 $= 2 : b$。
*BC, CA, AB* are divided respectively by the points *X, Y, Z* in the ratio 1 : 2.
Let area of $\Delta AZY$ : area of $\Delta ABC = 2 : a$ and
area of $\Delta AZY$ : area of $\Delta XYZ = 2 : b$ .




(i) 求 $a$ 的值。
Find the value of $a$ .

$a =$

(ii) 求 $b$ 的值。
Find the value of $b$ .

$b =$

擲一枚骰子兩次。設 $\frac{x}{36}$ 為擲得點數總和為 7 或 8 的概率，$\frac{y}{36}$ 為擲得兩數之差為 1 的概率。

A die is thrown 2 times. Let $\frac{x}{36}$ be the probability that the sum of numbers obtained is 7 or 8 and $\frac{y}{36}$ be the probability that the difference of numbers obtained is 1.

(iii) 求 $x$ 的值。
Find the value of $x$ .

$x =$

(iv) 求 $y$ 的值。
Find the value of $y$ .

$y =$

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**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

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Time [ ] [ ]

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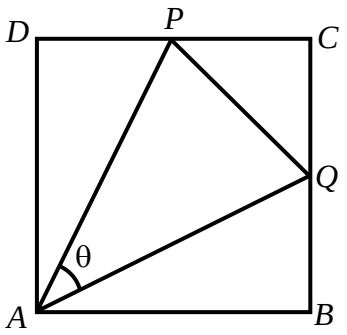
**Hong Kong Mathematics Olympiad (1992 – 93)**

**Event 10 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

*ABCD* 乃一邊長為 $20\sqrt{5}x$ 的正方形。*P*、*Q* 分別為 *DC* 及 *BC* 的中點。
*ABCD* is a square of side length $20\sqrt{5}x$. *P*, *Q* are midpoints of *DC* and *BC* respectively.




(i) 若 $AP = ax$，求 *a* 的值。
If $AP = ax$, find the value of $a$.

$a =$

(ii) 若 $PQ = b\sqrt{10}x$，求 *b* 的值。
If $PQ = b\sqrt{10}x$, find the value of $b$.

$b =$

(iii) 若由 *A* 至 *PQ* 的距離為 $c\sqrt{10}x$，求 *c* 的值。
If the distance from *A* to *PQ* is $c\sqrt{10}x$, find the value of $c$.

$c =$

(iv) 若 $\sin\theta = \frac{d}{100}$，求 *d* 的值。
If $\sin\theta = \frac{d}{100}$, find the value of $d$.

$d =$

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**FOR OFFICIAL USE**

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.