Hong Kong Mathematics Olympiad 2010-2011
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。　　時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark.　　Time allowed: 40 minutes

1. 求 2^{2011} 除以 13 的餘數。
   Find the remainder when 2^{2011} is divided by 13.

2. 已知 $x^2 + y^2 = 1$，求 $2x + 5y^2$ 的極大值。
   Given that $x^2 + y^2 = 1$, find the maximum value of $2x + 5y^2$.

3. 已知 $a+b=\sqrt{\sqrt{2011}+\sqrt{2010}}$ 及 $a-b=\sqrt{\sqrt{2011}-\sqrt{2010}}$，求 ab 的值。(答案以根式表示)
   Given that $a+b=\sqrt{\sqrt{2011}+\sqrt{2010}}$ and $a-b=\sqrt{\sqrt{2011}-\sqrt{2010}}$, find the value of ab.
   (Give your answer in surd form)

4. 在 ΔABC 內，分別垂直於三條邊 AB、BC 及 CA 的高的比是 3 : 4 : 5。若三條邊的長均為整數，求 AB 的最小值。
   In ΔABC, the ratio of the altitudes perpendicular to the three sides AB, BC and CA is 3 : 4 : 5. If the lengths of the three sides are integers, find the minimum value of AB .

5. 整數 x 減去 12 後是一個整數的平方。將 x 加上 19 後則是另一個整數的平方。求 x 的值。
   An integer x minus 12 is the square of an integer. x plus 19 is the square of another integer. Find the value of x.

6. 甲、乙及丙三人互相傳球。甲首先將球傳出。有多少不同方案使得經過 5 次傳球後，球會
   ？
   A, B and C pass a ball among themselves. A is the first one to pass the ball to other one.
   In how many ways will the ball be passed back to A after 5 passes?

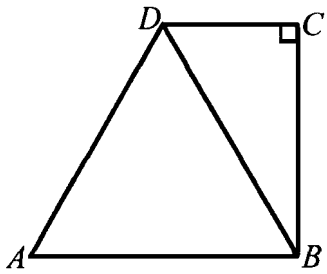
7. 求 $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$ 的值。
   Find the value of $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$.

8. 學校推出每張面值為\$10、\$15、\$25 及 \$40 的四種賣物券。甲班用若干張\$100 紙幣買了 30 張賣物券，包括其中兩種賣物券各 5 張及另外兩種賣物券各 10 張。問甲班共用了多少張 \$100 紙幣購買賣物券？
   A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40.
   Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types.
   How many one-hundred dollars notes Class A use to buy the raffle tickets?

9. 某長方形的長和濶均為整數。若面積比周界大 9，求周界的值。
   The length and the width of a rectangle are integers.
   If its area is larger than its perimeter by 9, find the perimeter.

***** 試卷完 End of Paper *****

10. 如圖，*ABCD* 為個梯形，其中 $\angle C = 90°$。
若等邊三角形 *ABD* 的面積為 $16\sqrt{3}$，求 梯形 *ABCD* 的面積。
In the figure, *ABCD* is a trapezium with $\angle C = 90°$.
If the area of the equilateral triangle *ABD* is $16\sqrt{3}$,
find the area of trapezium *ABCD* .



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Hong Kong Mathematics Olympiad 2010-2011
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。　　時限：20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 若$(1000-a)(1000-b)(1000-c)(1000-d)(1000-e)=24^2$，其中 a、b、c、d 及 e 為偶數，且 $a>b>c>d>e$，求 a、b、c、d 及 e 的值。

   If $(1000-a)(1000-b)(1000-c)(1000-d)(1000-e)=24^2$, where a, b, c, d and e are even numbers and $a>b>c>d>e$, find the values of a, b, c, d and e .

2. 以 $\overline{ab}$ 表示一個兩位數，其十位是 a，個位是 b，且 $R_{\overline{ab}}$ 表示 $\overline{ab}$ 除以 $a+b$ 的餘數。求 $R_{\overline{ab}}$ 的最大值。

   $\overline{ab}$ denotes a two digit number with a as tens digit and b as the unit digit. $R_{\overline{ab}}$ is the remainder when $\overline{ab}$ is divided by $a+b$. Find the maximum value of $R_{\overline{ab}}$.

3. 已知 a、b、c 為整數，且 $a+b=2011$，$c-a=2010$，$a<b$。求 $a+b+c$ 的可能最大值。

   Given that a, b and c are integers, and $a+b=2011$, $c-a=2010$, $a<b$.
   Find the greatest possible value of $a+b+c$.

4. 已知 n 為一正整數，且 n^4-18n^2+49 為一質數。求 n 的值。

   Given that n is a positive integer and n^4-18n^2+49 is a prime number, find the value of n .

5. 已知 $\mathrm{f}(x)=\dfrac{4^x}{4^x+2}$，其中 x 是實數。

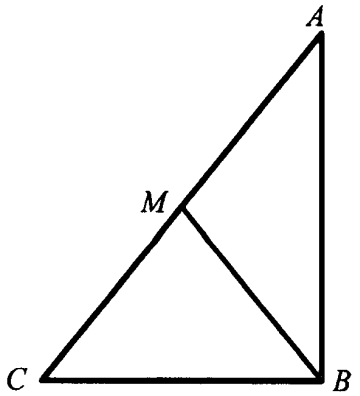
   求 $\mathrm{f}\left(\dfrac{1}{2011}\right)+\mathrm{f}\left(\dfrac{2}{2011}\right)+\mathrm{f}\left(\dfrac{3}{2011}\right)+\cdots+\mathrm{f}\left(\dfrac{2009}{2011}\right)+\mathrm{f}\left(\dfrac{2010}{2011}\right)$ 的值。

   Given that $\mathrm{f}(x)=\dfrac{4^x}{4^x+2}$, where x is a real number, find the value of

   $\mathrm{f}\left(\dfrac{1}{2011}\right)+\mathrm{f}\left(\dfrac{2}{2011}\right)+\mathrm{f}\left(\dfrac{3}{2011}\right)+\cdots+\mathrm{f}\left(\dfrac{2009}{2011}\right)+\mathrm{f}\left(\dfrac{2010}{2011}\right)$.

6. 如下圖，M 為 AC 上的一點，且 $AM=MC=BM=3$。求 $AB+BC$ 的最大值。

   In the figure below, M is a point on AC, $AM=MC=BM=3$. Find the maximum value of $AB+BC$.

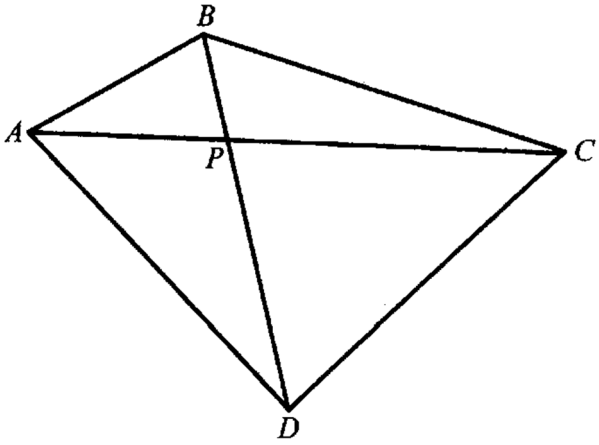


7. 已知 $n!=n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$ 且 $\dfrac{2011!}{10^k}$ 是整數，其中 k 是正整數。若 S 是 k 的所有可能值之和，求 S 的值。

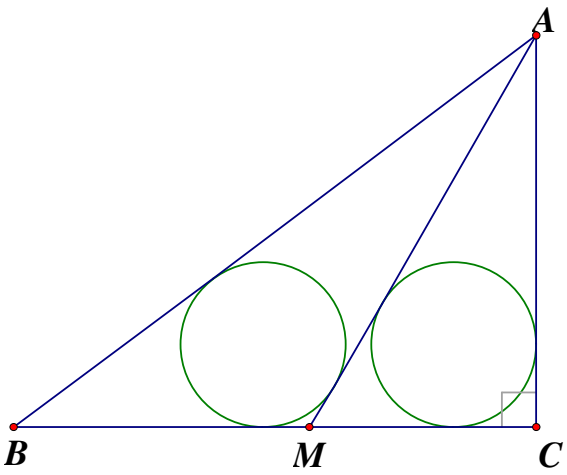
   Given that $n!=n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$ and $\dfrac{2011!}{10^k}$ is an integer, where k is a positive integer. If S is the sum of all possible values of k, find the value of S.

8. 已知 a、b、c 及 d 為非負整數，且 $ac + bd + ad + bc = 2011$。求 $a + b + c + d$ 的值。
Given that a, b, c and d are non-negative integers and $ac + bd + ad + bc = 2011$.
Find the value of $a + b + c + d$.

9. 如圖，$ABCD$ 為一凸四邊形，$\angle BAC = 27°$，$\angle BCA = 18°$，$\angle BDC = 54°$，$\angle BDA = 36°$，且四邊形的對角綫 AC、BD 相交於 P。求 $\angle CPB$。
As shown in the figure, $ABCD$ is a convex quadrilateral, $\angle BAC = 27°$, $\angle BCA = 18°$, $\angle BDC = 54°$, $\angle BDA = 36°$. The diagonals AC and BD intersect at P. Find $\angle CPB$.



10. 如圖，$AC = 3$，$BC = 4$ 及 $\angle C = 90°$。M 是 BC 上的一點使得 ΔABM 及 ΔACM 的內切圓的半徑相等。求 AM 的長。
As shown in the figure, $AC = 3$, $BC = 4$ and $\angle C = 90°$. M is a point on BC such that the radii of the incircles in ΔABM and ΔACM are equal. Find the length of AM.



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Hong Kong Mathematics Olympiad 2010 – 2011
Heat Event (Geometric Construction)
香港數學競賽 2010 – 2011
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: _____
School Name: _____

第一題 Question No. 1

已知一直綫 L，及兩點 P、Q 位於 L 的同一方。試在 L 上作一點 T 使得 PT 及 QT 的長度之和最小。(提示：可考慮 P 點於直綫 L 上作反射的影像)

Given a straight line L, and two points P and Q lying on the same side of L. Mark a point T on L so that the sum of the lengths of PT and QT is minimal. (Hint: Consider the reflection image of P about the line L.)

Hong Kong Mathematics Olympiad 2010 – 2011
Heat Event (Geometric Construction)
香港數學競賽 2010 – 2011
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
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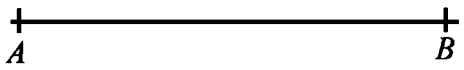
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: _____
School Name: _____

第二題 Question No. 2

如圖所示為一綫段 *AB*。*D* 為一固定點，且 *A*、*B*、*D* 不共綫。試作Δ*ABC*，使得 *C*、*B* 及 *D* 共綫，及其餘兩條邊的長度差(即 *AC* – *BC*)等如 *BD*。

The figure shows a line segment *AB*. *D* is a fixed point such that *A*, *B*, *D* are not collinear. Construct a triangle *ABC* so that *C*, *B* and *D* are collinear and the difference between the other two sides of Δ*ABC* (i.e. *AC* – *BC*) is equal to *BD*.



圖二 Figure 2

Hong Kong Mathematics Olympiad 2010 – 2011
Heat Event (Geometric Construction)
香港數學競賽 2010 – 2011
初賽(幾何作圖)

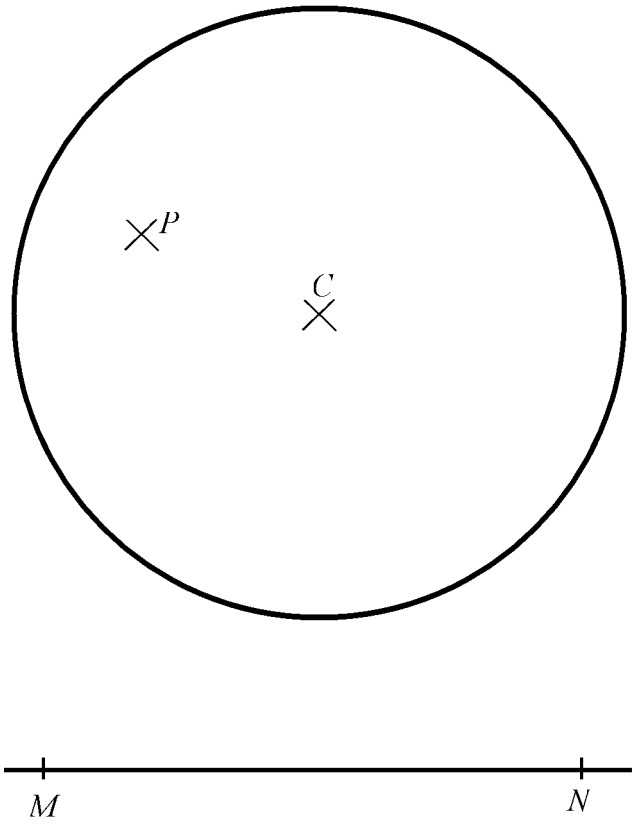
每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: _____
School Name: _____

第三題 Question No. 3

圖三所示為一以 C 為圓心的圓及一綫段 MN。P 為該圓內的一點。試構作一通過 P 的弦 QR，其中 Q 及 R 為圓周上的點，且 QR 的長度與 MN 的長度相等。

Figure 3 shows a circle of centre C and a line segment MN. P is a point lies inside the circle. Construct a chord QR with points Q and R on the circumference of the circle so that it passes through P and its length is equal to that of MN .



圖三 Figure 3

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