Compounded angles (HKMO Classified Questions by topics)

1994 FG10.3

已知
$$\sin 2\theta = 2 \sin \theta \cos \theta$$
。 求 c ,若 $c = \frac{\sin 20^{\circ} \cos 20^{\circ} \cos 40^{\circ} \cos 60^{\circ} \cos 80^{\circ}}{\sin 160^{\circ}}$ 。

It is given that $\sin 2\theta = 2 \sin \theta \cos \theta$.

Find the value of c, if $c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}$.

1994 FG10.4

已知
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
 , 求 d 的值,若

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ)$$

It is given that $tan(A + B) = \frac{tan A + tan B}{1 - tan A tan B}$. Find the value of d, if

$$d = (1 + \tan 21^\circ)(1 + \tan 22^\circ)(1 + \tan 23^\circ)(1 + \tan 24^\circ).$$

1999 FI1.2

In $\triangle ABC$, AB = 5, AC = 6 and BC = 4. If $\frac{1}{Q} = \cos 2A$, find the value of Q.

 $(\underline{\text{Hint}}: \cos 2A = 2\cos^2 A - 1)$

2001 FG2.4

已知 $\cos 16^{\circ} = \sin 14^{\circ} + \sin d^{\circ}$ 及 0 < d < 90, 求 d 的值。

Given that $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and 0 < d < 90, find the value of d.

2002 HI3

已知
$$\cos 15^{\circ} = \frac{\sqrt{a} + \sqrt{b}}{4}$$
,且 $a \cdot b$ 是自然數。若 $a + b = y$,求 y 的值。

Suppose $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$ and a, b are natural numbers.

If a + b = y, find the value of y.

2003 HI6

若對任意
$$0 < x < \frac{\pi}{2}$$
, $\cot \frac{1}{4}x - \cot x \equiv \frac{\sin kx}{\left(\sin \frac{1}{4}x\right)(\sin x)}$, 其中 k 是一常數,

求 k 的值。

If for any
$$0 < x < \frac{\pi}{2}$$
, $\cot \frac{1}{4}x - \cot x = \frac{\sin kx}{\left(\sin \frac{1}{4}x\right)(\sin x)}$, where k is a constant,

find the value of k.

2003 FG2.4

在ΔABC 中, $\cos A = \frac{4}{5}$ 和 $\cos B = \frac{7}{25}$ 。若 $\cos C = d$,求 d 的值。

In $\triangle ABC$, $\cos A = \frac{4}{5}$ and $\cos B = \frac{7}{25}$. If $\cos C = d$, find the value of d.

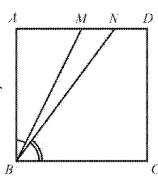
2004 FI2.1

如圖,ABCD 為一正方形,M 是 AD 的中點及 N 是 MD 的中點及 N 是 MD 的中點。

$$\angle CBN: \angle MBA = P:1$$
, 求 P 的值。

In the figure, ABCD is a square, M is the mid-point of AD and N is the mid-point of MD.

If $\angle CBN : \angle MBA = P : 1$, find the value of P.



2004 FG1.4

已知
$$0 \le x_0 \le \frac{\pi}{2}$$
 且 x_0 满足方程 $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin \frac{x}{2}$ 。

若
$$d = \tan x_0$$
, 求 d 的值。

Given that $0 \le x_0 \le \frac{\pi}{2}$ and x_0 satisfies the equation $\sqrt{\sin x + 1} - \sqrt{1 - \sin x} = \sin \frac{x}{2}$.

If $d = \tan x_0$, find the value of d.

2005 FG3.1

設
$$0^{\circ} < \alpha < 45^{\circ}$$
 。若 $\sin \alpha \cos \alpha = \frac{3\sqrt{7}}{16}$ 及 $A = \sin \alpha$,求 A 的值。

Let $0^{\circ} < \alpha < 45^{\circ}$. If $\sin \alpha \cos \alpha = \frac{3\sqrt{7}}{16}$ and $A = \sin \alpha$, find the value of A.

2006 HG6

設 $a \cdot b \cdot c$ 和 d 是實數且滿足 $a^2 + b^2 = c^2 + d^2 = 1$ 及 ac + bd = 0。若 R = ab + cd,求 R 的值。

Let a, b, c and d be real numbers such that $a^2 + b^2 = c^2 + d^2 = 1$ and ac + bd = 0. If R = ab + cd, find the value of R.

2006 FI2.3

已知
$$T = \sin 50^{\circ} \times (1 + \sqrt{3} \times \tan 10^{\circ})$$
,求 T 的值。

Given that $T = \sin 50^{\circ} \times (1 + \sqrt{3} \times \tan 10^{\circ})$, find the value of T.

2006 FG3.3

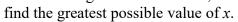
已知 $\tan x + \tan y + 1 = \cot x + \cot y = 6$ 。若 $z = \tan(x + y)$,求 z 的值。 Given that $\tan x + \tan y + 1 = \cot x + \cot y = 6$.

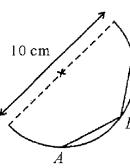
If $z = \tan(x + y)$, find the value of z.

2007 HG8

如圖三,已知半圓的直徑為 $10 \,\mathrm{cm} \circ A \cdot B$ 和 C 是 半圓上任意的三點使 B 在 \widehat{AC} 上。設x 為綫段 AB及 BC 的長度之和,求 x 可取的最大值。

In figure 3, given that the diameter of the semicircle is 10 cm. A, B and C are three arbitrary points on the semicircle where B is on $\stackrel{\frown}{AC}$. If x is the sum of the length of the line segments AB and BC,



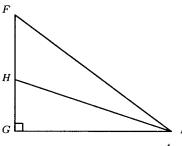


2010 HI6

若
$$x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$$
,其中 $0 \le x, y \le 1$,求 x^2+y^2 的值。
If $x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$, where $0 \le x, y \le 1$, find the value of x^2+y^2 .

2010 FI4.4

在圖二中,EFG 為一直角三角形。已知 H 為 FFG 上的一點,使得 GH:HF=4:5 及 $\angle GEH$ $= \angle FEH \circ$ 若 EG = 1 及 FG = d , 求 d 的 值 。 In Figure 2, EFG is a right-angled triangle. Given Hthat H is a point on FG, such that GH: HF = 4:5and $\angle GEH = \angle FEH$. If EG = 1 and FG = d, find the value of d.

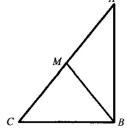


2011 HG6

如下圖,M為AC上的一點,且AM = MC = BM = 3。 求 AB + BC 的最大值。

In the figure below, M is a point on AC, AM = MC = BM = 3.

Find the maximum value of AB + BC.



2011 FI4.1

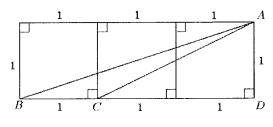
考慮函數 $v = \sin x + \sqrt{3} \cos x$ 。設 $a \triangleq v$ 的最大值。求 a 的值。 Consider the function $y = \sin x + \sqrt{3} \cos x$.

Let a be the maximum value of y. Find the value of a.

2012 FI3.1

在圖中,長方形由三個邊長為1之 正方形組成。

In the figure, a rectangle is subdivided into 3 identical squares of side length 1.



If $\alpha^{\circ} = \angle ABD + \angle ACD$, find the value of α .

2012 FI3.2

設
$$ABC$$
 為一銳角三角形。若 $\sin A = \frac{36}{45}$, $\sin B = \frac{12}{13}$ 及 $\sin C = \frac{\beta}{y}$,求 β 的值,

其中 β 及 ν 是最簡化之代表形式。

Let ABC be an acute-angled triangle. If $\sin A = \frac{36}{45}$, $\sin B = \frac{12}{13}$ and $\sin C = \frac{\beta}{v}$,

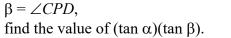
find the value of β , where β and γ are in the lowest terms.

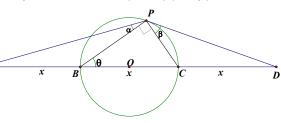
2012 FG2.4

在圖中,圓有直徑 BC,圓心在 O,P、B 及 C 皆為圓周上的點。若 AB=BC= CD 及 AD 為一綫段, $\alpha = \angle APB$ 及 $\beta = \angle CPD$,求(tan α)(tan β)的值。

In the figure, P, B and C are points on a circle with centre O and diameter BC. If A, B, C, Dare collinear such that AB = BC= CD, $\alpha = \angle APB$ and

 $\beta = \angle CPD$,



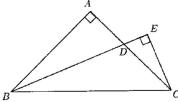


2012 FG4.1

在圖一中,
$$ABC$$
 及 EBC 是兩個直角三角形, $\angle BAC = \angle BEC = 90^\circ$, $AB = AC$ 及 EDB 為 $\angle ABC$ 的角平分綫。求 $\frac{BD}{CE}$ 的值。

In figure 1, ABC and EBC are two right-angled triangles, $\angle BAC = \angle BEC = 90^{\circ}$, AB = AC and EDBis the angle bisector of $\angle ABC$.

Find the value of $\frac{BD}{CE}$.



2017 FI4.4

若
$$\cos 2\theta = \frac{3}{44}$$
 ,求 $d = \sin^4 \theta + \cos^4 \theta$ 的值。

If $\cos 2\theta = \frac{3}{44}$, determine the value of $d = \sin^4 \theta + \cos^4 \theta$.

2018 FG4.3

求
$$C = \cos\frac{\pi}{15} \times \cos\frac{2\pi}{15} \times \cos\frac{3\pi}{15} \times \cos\frac{4\pi}{15} \times \cos\frac{5\pi}{15} \times \cos\frac{6\pi}{15} \times \cos\frac{7\pi}{15}$$
 的值。

Determine the value of

$$C = \cos\frac{\pi}{15} \times \cos\frac{2\pi}{15} \times \cos\frac{3\pi}{15} \times \cos\frac{4\pi}{15} \times \cos\frac{5\pi}{15} \times \cos\frac{6\pi}{15} \times \cos\frac{7\pi}{15}.$$

2019 HI14

已知 $3\sin x + 2\sin y = 4$ 。設 N 為 $3\cos x + 2\cos y$ 的最大值。

求 N 的值。

Given that $3 \sin x + 2 \sin y = 4$. Let N be the maximum value of $3 \cos x + 2 \cos y$.

Find the value of N.

2021 P1Q4

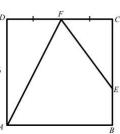
求 8 cos² 15° cos² 30° - 8 sin² 15° cos² 30° 的值。

Find the value of $8\cos^2 15^{\circ} \cos^2 30^{\circ} - 8\sin^2 15^{\circ} \cos^2 30^{\circ}$.

2021 P2Q1

在圖一中,ABCD 是一個邊長為 6 的正方形。F 是 CD^D 的中點。若 $\angle FAB = \angle AFE$,求 BE 的長度。

In Figure 1, ABCD is a square of sides 6 units. F is the midpoint of CD. If $\angle FAB = \angle AFE$, find the length of BE.



2023 HG3

已知 $\tan \alpha$ 和 $\tan \beta$ 是二次方程 $x^2 - 4x - 2 = 0$ 的根。 求 $\sin^2(\alpha + \beta) + 2\sin(\alpha + \beta)\cos(\alpha + \beta) + 3\cos^2(\alpha + \beta)$ 的值。

Answers

Allsweis				
1994 FG10.3 1/16	1994 FG10.4 4	1999 FI1.2 8	2001 FG2.4 46	2002 HI3 8
2003 HI6 $\frac{3}{4}$	2003 FG2.4 44 125	2004 FI2.1 2	2004 FG1.4 0	2005 FG3.1 $\frac{\sqrt{7}}{4}$
2006 HG6 0	2006 FI2.3 1	2006 FG3.3 30	2007 HG8 $10\sqrt{2}$	2010 HI6 1
$ \begin{array}{c} 2010 \text{ FI4.4} \\ \frac{3}{4} \end{array} $	2011 HG6 6√2	2011 FI4.1 2	2012 FI3.1 45	2012 FI3.2 56
2012 FG2.4 $\frac{1}{4}$	2012 FG4.1 2	2017 FI4.4 1945 3872	2018 FG4.3 1 128	2019 HI14 3
2021 P1Q4 $3\sqrt{3}$	2021 P2Q1 2	2023 HG3 $\frac{67}{25}$		