Hong Kong Mathematics Olympiad (1991 – 92)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 若 $(\log_{10} x)^4 - 3(\log_{10} x)^2 - 4 = 0$，且 $x > 1$，求 x 的值。
If $(\log_{10} x)^4 - 3(\log_{10} x)^2 - 4 = 0$ and $x > 1$, find the value of x.

2. 若 $\begin{cases} 28x + 15y = 19xy \\ 18x - 21y = 2xy \end{cases}$，且 $xy \neq 0$，求 x 的值。
If $\begin{cases} 28x + 15y = 19xy \\ 18x - 21y = 2xy \end{cases}$ and $xy \neq 0$, find the value of x.

3. 由 0 至 9 之中隨機取一整數 a，已知方程 $x^2 - ax + 3 = 0$ 無實根的概率為 $\frac{p}{10}$，求 p 的值。
An integer a lying between 0 and 9 inclusive is randomly selected. It is known that the probability that the equation $x^2 - ax + 3 = 0$ has no real root is $\frac{p}{10}$, find the value of p.

4. $x°$ 為一滿足 $\frac{1}{2}\cos x° \geq \frac{1}{2}(5 - \cos x°) - 2$ 的銳角，求 x 的最大值。
$x°$ is an acute angle satisfying $\frac{1}{2}\cos x° \geq \frac{1}{2}(5 - \cos x°) - 2$.
Determine the largest possible value of x.

5. 設 $f(x)$ 為 $x^4 + 64$ 和 $x^3 + 6x^2 + 16x + 16$ 的最大公因式，求 $f(2)$ 的值。
Let $f(x)$ be the highest common factor of $x^4 + 64$ and $x^3 + 6x^2 + 16x + 16$, find the value of $f(2)$.

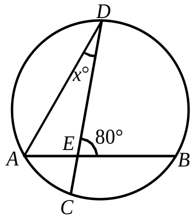
6. 果商把一堆橙分成 A、B、C、D 四個等級。A 級和 B 級橙的數目合起來是 C 級的兩倍；B 級和 D 級橙的數目合起來是 A 級的兩倍。若將 B 級橙中的 7 個升格為 A 級，則 A 級的橙數便是 B 級的兩倍。已知 A、B、C、D 四級橙中，其中某級有橙 54 個，問這是哪一級？
A fruit merchant divides a large lot of oranges into four classes: A, B, C, D. The number of oranges in class A and class B doubles that in class C while the number of oranges in class B and class D doubles that in class A. If 7 oranges from class B are upgraded to class A, class A will then contain twice as many oranges as class B. It is known that one of the four classes contains 54 oranges. Determine which one class it belongs to.

7. 已知 n 為一正整數，求 $x^{2^n} - 10^{2^n} = 0$ 的所有實根。
Given that n is a positive integer, find **ALL** the real roots of $x^{2^n} - 10^{2^n} = 0$.

8. 若 n 是從 1 至 100 中隨意選取的整數，且 5678^n 的個位數大於 3 的概率是 $\frac{3}{x}$，求 x 的值。
If n is an integer randomly selected from 1 to 100, and the probability that the unit digit of 5678^n is greater than 3 is $\frac{3}{x}$, find the value of x.

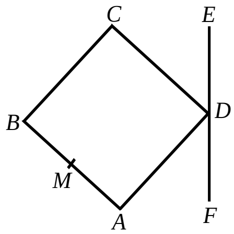
9. 在 ΔABC 中，$AB = 8$ cm、$BC = 6$ cm、$\angle ABC = 90°$，
若 $\angle ACB$ 的角平分綫與 AB 交於 R，且 $CR = 3\sqrt{a}$ cm，求 a 的值。
In ΔABC, $AB = 8$ cm, $BC = 6$ cm and $\angle ABC = 90°$. If the bisector of $\angle ACB$ cuts AB at R and $CR = 3\sqrt{a}$ cm, find the value of a.

10. 在圖一中，弧 BD 的長度是弧 AC 的 4 倍，$\angle DEB = 80°$及$\angle ADC = x°$，求 x 的值。
In figure 1, arc BD is 4 times the arc AC, $\angle DEB = 80°$ and $\angle ADC = x°$, find the value of x .



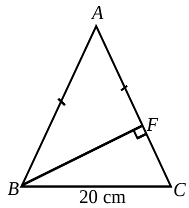
(Figure 1) (圖一)

11. 在圖二中，$ABCD$ 是一正方形，EDF 是一直綫，M 是 AB 的中點。若 A、M 和 C 到直綫 EF 的距離依次為 5 cm、11 cm 和 x cm，求 x 的值。
In figure 2, $ABCD$ is a square. EDF is a straight line. M is the mid-point of AB. If the distances of A, M and C from the line EF are 5 cm, 11 cm and x cm respectively, find the value of x .



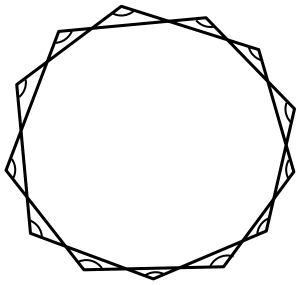
(Figure 2) (圖二)

12. 在圖三中，$AB = AC = 2BC$ 及 $BC = 20$ cm。
若 BF 垂直於 AC，且 $AF = x$ cm，求 x 的值。
In figure 3, $AB = AC = 2BC$ and $BC = 20$ cm.
If BF is perpendicular to AC and $AF = x$ cm, find the value of x .

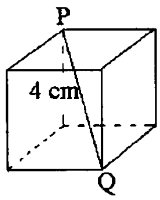


(Figure 3) (圖三)

13. 圖四是延長一個 13 邊形的邊所構成的圖形。若圖中標示的角的和是 $n°$，求 n 的值。
Figure 4 shows a figure obtained by producing the sides of a 13-sided polygon.
If the sum of the marked angles is $n°$, find the value of n .


(Figure 4) (圖四)

14. 在圖五中，PQ 為一正方體的對角綫。
若 $PQ = 4$ cm，且這正方體的總表面面積為 x cm^2，求 x 的值。
In figure 5, PQ is a diagonal of the cube.
If $PQ = 4$ cm and the total surface area of the cube is x cm^2, find the value of x .



(Figure 5) (圖五)

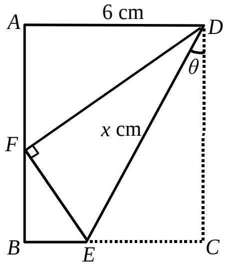
15. 若 $(3x-1)^7 = a_1x^7 + a_2x^6 + a_3x^5 + \cdots + a_8$，求 $a_1 + a_2 + a_3 + \cdots + a_8$ 的值。
If $(3x-1)^7 = a_1x^7 + a_2x^6 + a_3x^5 + \cdots + a_8$, find the value of $a_1 + a_2 + a_3 + \cdots + a_8$.

16. $A(1, 1)$、$B(a, 0)$、$C(1, a)$ 是三角形 ABC 的頂點，若ΔABC 的面積是 2 平方單位，且 $a > 0$，求 a 的值。
$A(1, 1)$, $B(a, 0)$ and $C(1, a)$ are the vertices of the triangle ABC .
Find the value of a if the area of ΔABC is 2 square units and $a > 0$.

17. 若 $N = 2^{12} \times 5^8$，N 是一個多少位的數字？
If $N = 2^{12} \times 5^8$, find the number of digits of N .

18. 若 $a : b = 3 : 4$ 及 $a : c = 2 : 5$，求 $\dfrac{ac}{a^2 + b^2}$ 的值。
If $a : b = 3 : 4$ and $a : c = 2 : 5$, find the value of $\dfrac{ac}{a^2 + b^2}$.

19. 一張闊 6 cm 的長方形紙按圖六所示對摺，使得一角與對邊接觸。若 θ 為 30°，且 $DE = x$ cm，求 x 的值。
A rectangular piece of paper of width 6 cm is folded such that one corner touches the opposite side as shown in figure 6. If $\theta = 30°$ and $DE = x$ cm, find the value of x.



(Figure 6) (圖六)

20. 若 $\sin x + \cos x = \frac{1}{5}$，且 $0 \le x \le \pi$，求 $\tan x$ 的值。
If $\sin x + \cos x = \frac{1}{5}$ and $0 \le x \le \pi$, find the value of $\tan x$.

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad (1991 – 92)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 有甲、乙、丙三人，甲的年齡較乙和丙的年齡之和大了 16 歲，甲年齡的平方較乙和丙的年齡之和的平方大 1632，求甲、乙、丙的年齡之和。
   A, B, C are three men in a team. The age of A is greater than the sum of the ages of B and C by 16. The square of the age of A is greater than the square of the sum of the ages of B and C by 1632. Find the sum of the ages of A, B and C.

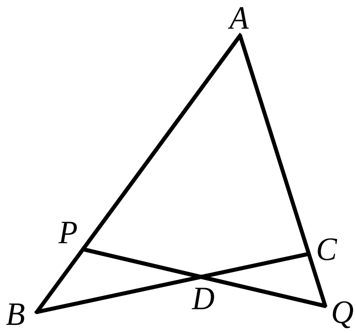
2. a、b、c 為非零實數，且 $\dfrac{a+b-c}{c}=\dfrac{a-b+c}{b}=\dfrac{-a+b+c}{a}$。
   若 $x=\dfrac{(a+b)(b+c)(c+a)}{abc}$ 及 $x<0$，求 x 的值。
   a, b, c are non-zero real numbers such that $\dfrac{a+b-c}{c}=\dfrac{a-b+c}{b}=\dfrac{-a+b+c}{a}$.
   If $x=\dfrac{(a+b)(b+c)(c+a)}{abc}$ and $x<0$, find the value of x.

3. 一凸 n 邊形的一個內角是 $x°$，其餘各內角之和等於 2468°，求 x 的值。
   An interior angle of an n-sided convex polygon is $x°$. The sum of the other interior angles is 2468°. Find the value of x.

4. 當正整數 N 除以 4、7、9 時，其餘數分別為 3、2、2。求 N 的最小值。
   When a positive integer N is divided by 4, 7, 9, the remainders are 3, 2, 2 respectively. Find the least value of N.

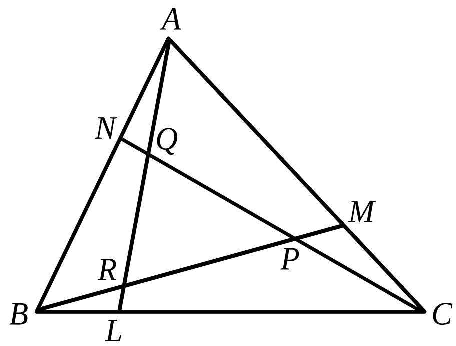
5. 求 10^{1991} 除以 7 的餘數。
   Find the remainder when 10^{1991} is divided by 7.

6. 在圖一中，$BD=DC$、$AP=AQ$。若 $AB=13$ cm、$AC=7$ cm 及 $AP=x$ cm，求 x 的值。
   In figure 1, $BD=DC$, $AP=AQ$. If $AB=13$ cm, $AC=7$ cm and $AP=x$ cm, find the value of x.



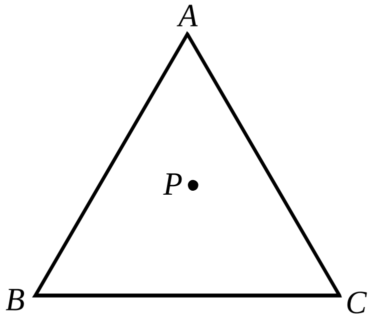
(Figure 1) (圖一)

7. 在圖二中，$BL=\dfrac{1}{3}BC$、$CM=\dfrac{1}{3}CA$ 及 $AN=\dfrac{1}{3}AB$。
   若 ΔPQR 及 ΔABC 的面積分別為 6 cm^2 及 x cm^2，求 x 的值。
   In figure 2, $BL=\dfrac{1}{3}BC$, $CM=\dfrac{1}{3}CA$ and $AN=\dfrac{1}{3}AB$.
   If the areas of ΔPQR and ΔABC are 6 cm^2 and x cm^2 respectively, find the value of x.



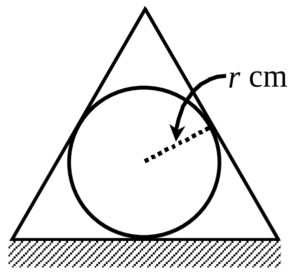
(Figure 2) (圖二)

8. *ABC* 為一邊長 $\sqrt{12}$ cm 的等邊三角形，而 *P* 為此三角形內的任意一點(如圖三所示)。若 *P* 至三邊 *AB*、*BC* 及 *CA* 的垂直距離的總和為 x cm，求 x 的值。
*AB*C is an equilateral triangle of side $\sqrt{12}$ cm, and *P* is any point inside the triangle (as shown in figure 3). If the sum of the perpendicular distances from *P* to the three sides *AB*, *BC* and *CA* is x cm, find the value of x.



(Figure 3) (圖三)

9. 一半徑為 r cm 的球體剛好被一體積為 $\frac{8\pi r^2}{3}$ cm^3 的圓錐形容器覆蓋於桌上 (如圖四所示)。求 r 的最大可能值。
A sphere of radius r cm can just be covered on a table by a conical vessel of volume $\frac{8\pi r^2}{3}$ cm^3 (as shown in figure 4).
Determine the largest possible value of r.



(Figure 4) (圖四)

10. a、b、c、d 為四個數字。已知 (i) a、b、c； (ii) b、c、d；和 (iii) a、b、d 的算術平均數依次為 13、15 和 17。若 a、b、c 和 d 的中位數為 $c+9$，求 c 的最大可能值。
a, b, c, d are four numbers. The arithmetic means of (i) a, b, c; (ii) b, c, d; (iii) a, b, d are respectively 13, 15 and 17.
If the median of a, b, c and d is $c + 9$, find the largest possible value of c.

***** 試卷完 End of Paper *****