Examples on Mathematical Induction: divisibility 13

1. **1970 香港中文中學會考高級數學試卷一 Q6(b)**

    利用數學歸納法證明當 n 為正整數時，$3^{3n} - 1$ 恆為 13 之倍數。

    $n = 1$, $3^3 - 1 = 26 = 2\times13$, which is a multiple of 13

    Suppose $3^{3k} - 1 = 13m$ for some positive integer k, and m is an integer

    $$
    \begin{aligned}
    3^{3(k+1)} - 1 &= 3^{3k}\times3^3 - 1 \\
    &= 27(3^{3k} - 1 + 1) - 1 \\
    &= 27(13m + 1) - 1 \\
    &= 27\times13m + 26 \\
    &= 13(27m + 2)
    \end{aligned}
    $$

    $\because$ $27m + 2$ is an integer

    $\therefore$ $3^{3(k+1)} - 1$ is a multiple of 13.

    If it is true for $n = k$, then it is also true for $n = k + 1$

    By the principle of mathematical induction, it is true for all positive integer n.