Hong Kong Mathematics Olympiad (1996 – 97)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 設 n 為一正整數。若 $n^2 = 29p + 1$，其中 p 為質數，試求 n 之值。
Let n be a positive integer. If $n^2 = 29p + 1$, where p is a prime number, find the value of n .

2. 若一長方形之闊度增加 $\frac{1}{3}$ 米，其面積增加 $\frac{5}{3}$ 平方米。若其長度減少 $\frac{1}{2}$ 米，則面積減少 $\frac{9}{5}$ 平方米。設該長方形之面積為 x 平方米，求 x 之值。
If the width of a rectangle is increased by $\frac{1}{3}$ m, its area will be increased by $\frac{5}{3}$ m^2. If its length is decreased by $\frac{1}{2}$ m, its area will be decreased by $\frac{9}{5}$ m^2. Let the area of the rectangle be x m^2, find the value of x .

3. 若 s 為 1234 之所有正因數的總和，求 s 之值。
If s is the sum of all positive factors of 1234, find the value of s .

4. 設 $x=\frac{1}{x}$，求 $\frac{x^2+2x-3}{x-1}\div\frac{x+5}{x^2+3x-6}$ 的值。
Let $x=\frac{1}{x}$, find the value of $\frac{x^2+2x-3}{x-1}\div\frac{x+5}{x^2+3x-6}$.

5. Find the value of $1^2 – 2^2 + 3^2 – 4^2 + … + 99^2 – 100^2$.
求 $1^2 – 2^2 + 3^2 – 4^2 + … + 99^2 – 100^2$ 的值。

6. 若 $yz : zx : xy = 1 : 2 : 3$，求 $\frac{x}{yz}:\frac{y}{zx}$ 的值。
If $yz : zx : xy = 1 : 2 : 3$, find the value of $\frac{x}{yz}:\frac{y}{zx}$.

7. 求下列方程的實根： $x(x + 1)(x^2 + x + 1) = x$。
Find the real roots of the equation: $x(x + 1)(x^2 + x + 1) = x$.

8. 某班有 6 位學生。每位學生送給班中其餘各位同學一張聖誕咭，求該班學生寄出聖誕咭的總數。
There are 6 students in a class. Everyone sends one Christmas card to each of the rest of the class. Find the total number of cards sent out by the class.

9. 若 $2x^2-8x+k\equiv 2(x-2)^2+9$，求 k 之值。
If $2x^2-8x+k\equiv 2(x-2)^2+9$, find the value of k .

10. 若十位數 $1357p1357p$ 可被 9 整除，求 p 之值。
If the ten-digit number $1357p1357p$ is divisible by 9, find the value of p .

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad (1996 – 97)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 已知 $a_1, a_2, a_3, \cdots$ 和 $b_1, b_2, b_3, \cdots$ 為等差數列，其中 $a_1 = 25$，$b_1 = 75$ 及 $a_{100} + b_{100} = 100$。

   求數列 $a_1 + b_1, a_2 + b_2, \cdots$ 的前 100 項的和。

   If $a_1, a_2, a_3, \cdots$ and $b_1, b_2, b_3, \cdots$ are arithmetic sequences, where $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$. Find the sum of the first 100 terms of the sequence $a_1 + b_1, a_2 + b_2, \cdots$.

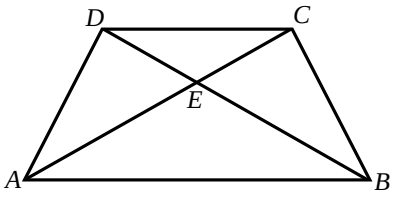
2. 已知 $f(x) = \dfrac{2x}{x+2}$，及 $x_1 = 1$，$x_n = f(x_{n-1})$，求 x_{99} 的值。

   If $f(x) = \dfrac{2x}{x+2}$ and $x_1 = 1$, $x_n = f(x_{n-1})$, find the value of x_{99}.

3. $ABCD$ 為一梯形，其中 AB // DC 及 ΔDCE 的面積: ΔDCB 的面積= 1 : 3。

   求 ΔDEC 的面積 : ΔABD 的面積。

   $ABCD$ is a trapezium, where AB // DC and area of ΔDCE : area of ΔDCB = 1 : 3, find area of ΔDEC : area of ΔABD .



4. 設 x 為一正整數。若 $\dfrac{2}{3}\left(\dfrac{2}{3}\left(\dfrac{2}{3}(x-1)-1\right)-1\right)$ 能被 3 整除，試求 x 之最小可能數值。

   Let x be a positive integer.

   If $\dfrac{2}{3}\left(\dfrac{2}{3}\left(\dfrac{2}{3}(x-1)-1\right)-1\right)$ is divisible by 3 , find the least possible value of x .

5. 水管 A 能於 20 小時內獨自盛滿某一水池，而水管 B 則於 5 小時內完成此工作。若兩水管同時使用時盛滿這水池所需的時間則為 x 小時，求 x 的值。

   Pipe A alone takes 20 hours to fill a tank and pipe B takes 5 hours to fill the same tank alone. If pipes A and B together take x hours to fill the tank, find the value of x .

6. 一正多邊形的每一內角被外角大 150°。求此多邊形的邊的數目。

Each interior angle of a regular polygon exceeds the exterior angle by 150°.

Find the number of sides of the polygon.

7. 若 $x+\frac{1}{x}=3$，求 $x^2+\frac{1}{x^2}$ 的值。

If $x+\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$.

8. 已知 5 個算術級數中之最大值為最小值之 7 倍，及該 5 個數之平均值為 32。

求 5 個數中之最小值。

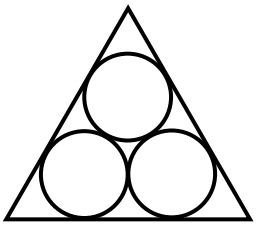
Five numbers are in arithmetic progression. If the largest number is 7 times the smallest one and the average of the five numbers is 32, find the smallest number.

9. 圖中三個半徑為 r cm 之全等圓被一三角形緊緊圍著。

若三角形之周界為 $\left(180+180\sqrt{3}\right)$cm，求 r 的值。

In the figure, three identical circles with radius r cm are tightly enclosed in a triangle.

If the perimeter of the triangle is $\left(180+180\sqrt{3}\right)$cm, find the value of r.



10. 投擲兩粒公平的骰子。求其總和為小於 5 及至少一粒骰子為‘2’的機會率。

Two fair dice are thrown.

Find the probability that the sum is less than 5 and at least one die is a ‘2’.

***** 試卷完 End of Paper *****