1990 HI10

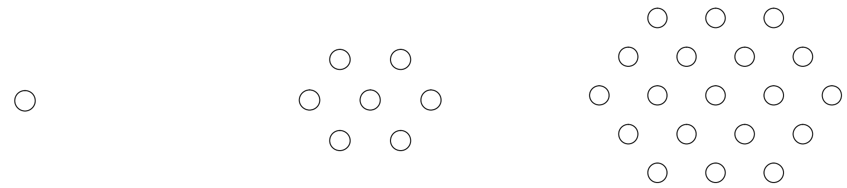
已知 $a_0 = 1$，$a_1 = 3$ 及 $a_n^2 - a_{n-1}a_{n+1} = (-1)^n$，其中 n 為正整數。求 a_4 的值。

Given that $a_0 = 1$, $a_1 = 3$ and $a_n^2 - a_{n-1}a_{n+1} = (-1)^n$ for positive integers n.

Find the value of a_4 .

1991 FG8.1-2

細看以下之六邊形數：Consider the following hexagonal numbers :



$H_1 = 1$ $H_2 = 7$ $H_3 = 19$

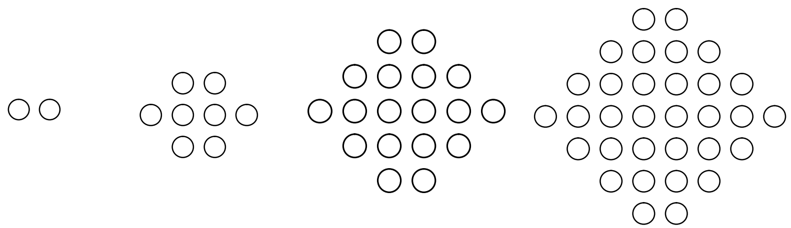
G8.1 求 H_5 的值。Find the value of H_5 .

G8.2 若 $H_n = an^2 + bn + c$，其中 n 為正整數，求 a 的值。

If $H_n = an^2 + bn + c$, where n is any positive integer, find the value of a .

1992 FG9.3-4

細看以下之數形：Consider the following number pattern:



$T_1 = 2$ $T_2 = 8$ $T_3 = 18$ $T_4 = 32$

G9.3 求 T_{10} 的值。Find the value of T_{10} .

G9.4 若 $T_n = 722$，求 n 的值。If $T_n = 722$, find the value of n .

1995 FI4.3

已知 $F_1 = F_2 = 1$ 且 $F_n = F_{n-1} + F_{n-2}$，其中 $n \ge 3$。若 $F_t = 5$，求 t 的值。

It is given that $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$, where $n \ge 3$.

If $F_t = 5$, find the value of t .

1997 HG2

已知 $f(x) = \dfrac{2x}{x+2}$，及 $x_1 = 1$，$x_n = f(x_{n-1})$，求 x_{99} 的值。

If $f(x) = \dfrac{2x}{x+2}$ and $x_1 = 1$, $x_n = f(x_{n-1})$, find the value of x_{99} .

1999 FI2.4

設 $f_0(x) = \dfrac{1}{c-x}$，且 $f_n(x) = f_0(f_{n-1}(x))$， $n = 1, 2, 3, \cdots$

若 $f_{2000}(2000) = d$，求 d 之值。

Let $f_0(x) = \dfrac{1}{c-x}$ and $f_n(x) = f_0(f_{n-1}(x))$, $n = 1, 2, 3, \cdots$

If $f_{2000}(2000) = d$, find the value of d .

1999 FI3.4

數列$\{a_n\}$的定義如下：$a_1 = 2$，$a_{n+1} = a_n + 2n$ $(n \ge 1)$。若 $a_{100} = d$，求 d 之值。

The sequence $\{a_n\}$ is defined as $a_1 = 2$, $a_{n+1} = a_n + 2n$ $(n \ge 1)$.

If $a_{100} = d$, find the value of d .

1999 FI5.4

設 $f(1) = 1$ 及 $f(n) = (n-1)\, f(n-1)$，其中 $n > 1$。若 $d = f(4)$，求 d 之值。

Let $f(1) = 1$ and $f(n) = (n-1)\, f(n-1)$, where $n > 1$. If $d = f(4)$, find d .

1999 FG3.2

數列$\{a_k\}$定義如下：$a_1 = 1$, $a_2 = 1$ 及 $a_k = a_{k-1} + a_{k-2}$ $(k > 2)$。

若 $a_1 + a_2 + \ldots + a_{10} = 11\, a_b$，求 b 之值。

The sequence $\{a_k\}$ is defined as: $a_1 = 1$, $a_2 = 1$ and $a_k = a_{k-1} + a_{k-2}$ $(k > 2)$.

If $a_1 + a_2 + \ldots + a_{10} = 11\, a_b$, find the value of b .

2000 FI3.4

設 $f(0) = 0$；$f(n) = f(n-1) + 3$ 當 $n = 1$、2、3、4、$\cdots$。

如果 $2f(S) = 3996$，求 S 的值。

Let $f(0) = 0$; $f(n) = f(n-1) + 3$ when $n = 1, 2, 3, 4, \cdots$.

If $2f(S) = 3996$, find the value of S .

2001 FG4.1

$x_1 = 2001$。當 $n > 1$，$x_n = \dfrac{n}{x_{n-1}}$。已知 $x_1x_2x_3\ldots x_{10} = a$，求 a 的值。

$x_1 = 2001$. When $n > 1$, $x_n = \dfrac{n}{x_{n-1}}$. Given that $x_1x_2x_3\ldots x_{10} = a$, find the value of a .

2004 FGS.1

對任意整數 n，F_n 的定義如下：$F_n = F_{n-1} + F_{n-2}$，$F_0 = 0$ 及 $F_1 = 1$。

若 $a = F_{-5} + F_{-4} + \ldots + F_4 + F_5$，求 a 的值。

For all integers n, F_n is defined by $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$ and $F_1 = 1$.

If $a = F_{-5} + F_{-4} + \ldots + F_4 + F_5$, find the value of a .

2005 FG4.4

已知 $f_1 = 0$，$f_2 = 1$及對正整數$n \geq 3$，$f_n = f_{n-1} + 2f_{n-2}$。

若$d = f_{10}$，求d的值。

Given that $f_1 = 0$, $f_2 = 1$, and for any positive integer $n \geq 3$, $f_n = f_{n-1} + 2f_{n-2}$.
If $d = f_{10}$, find the value of d.

2009 HI6

設 $\mathrm{f}_1(x) = \frac{1}{1-x}$ 及 $\mathrm{f}_n(x) = \mathrm{f}_1(\mathrm{f}_{n-1}(x))$，其中 $n = 2$、3、4、…。求 $\mathrm{f}_{2009}(2008)$的值。

Let $\mathrm{f}_1(x) = \frac{1}{1-x}$ and $\mathrm{f}_n(x) = \mathrm{f}_1(\mathrm{f}_{n-1}(x))$, where $n = 2, 3, 4, \cdots$.

Find the value of $\mathrm{f}_{2009}(2008)$.

2009 FI1.4

設 f(x)是一個函數使得對所有整數 $n \geq 6$ 時，$\mathrm{f}(n) = (n-1)\mathrm{f}(n-1)$及 $\mathrm{f}(n) \neq 0$。

若 $U = \frac{\mathrm{f}(11)}{10\mathrm{f}(8)}$，求 U 的值。

Let f(x) be a function such that $\mathrm{f}(n) = (n-1)\mathrm{f}(n-1)$ and $\mathrm{f}(n) \neq 0$ hold for all integers $n \geq 6$. If $U = \frac{\mathrm{f}(11)}{10\mathrm{f}(8)}$, find the value of U.

2012 FG3.3

設 k 為正整數及函數 f(k)的定義是若 $\frac{k-1}{k} = 0.k_1k_2k_3\cdots\cdots$，則 $\mathrm{f}(k) = \overline{k_1k_2k_3}$，

例如 $\mathrm{f}(3) = 666$ 因為 $\frac{3-1}{3} = 0.666\cdots\cdots$，求 $D = \mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(112)))))$的值。

Let k be positive integer and f(k) a function that if $\frac{k-1}{k} = 0.k_1k_2k_3\cdots\cdots$,

then $\mathrm{f}(k) = \overline{k_1k_2k_3}$, for example, $\mathrm{f}(3) = 666$ because $\frac{3-1}{3} = 0.666\cdots\cdots$,

find the value of $D = \mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(112)))))$.

2012 FG3.4

若 F_n 為一整數值函數，其定義為 $F_n(k) = F_1(F_{n-1}(k))$，$n \geq 2$ 且 $F_1(k)$是 k 的所有位數的平方之和，求 $F_{2012}(7)$的值。

If F_n is an integral valued function defined recursively by $F_n(k) = F_1(F_{n-1}(k))$ for $n \geq 2$ where $F_1(k)$ is the sum of squares of the digits of k,
find the value of $F_{2012}(7)$.

2013 HG10

對所有正整數 n，定義函數 f 為

(i) $\mathrm{f}(1) = 2012$,

(ii) $\mathrm{f}(1) + \mathrm{f}(2) + \cdots + \mathrm{f}(n-1) + \mathrm{f}(n) = n^2\,\mathrm{f}(n)$，$n > 1$

求 f(2012) 的值。

For all positive integers n, define a function f as

(i) $\mathrm{f}(1) = 2012$,

(ii) $\mathrm{f}(1) + \mathrm{f}(2) + \cdots + \mathrm{f}(n-1) + \mathrm{f}(n) = n^2\,\mathrm{f}(n)$，$n > 1$.

Find the value of f(2012).

2013 FI2.3

設 $\mathrm{f}(1) = 3$，$\mathrm{f}(2) = 5$ 且對所有正整數n，$\mathrm{f}(n+2) = \mathrm{f}(n+1) + \mathrm{f}(n)$。

當 f(600) 除以 3 的餘數是 c，求 c 的值。

Let $\mathrm{f}(1) = 3$, $\mathrm{f}(2) = 5$ and $\mathrm{f}(n+2) = \mathrm{f}(n+1) + \mathrm{f}(n)$ for positive integers n.
If c is the remainder of f(600) divided by 3, find the value of c.

2013 FI4.2

設函數 $F(n)$滿足 $F(1) = F(2) = F(3) = 1$ 及 $F(n+1) = \frac{F(n)\cdot F(n-1)+1}{F(n-2)}$，

其中 $n \geq 3$ 為正整數。求 $b = F(6)$的值。

Let $F(n)$ be a function with $F(1) = F(2) = F(3) = 1$ and
$F(n+1) = \frac{F(n)\cdot F(n-1)+1}{F(n-2)}$ for positive integer $n \geq 3$,
find the value of $b = F(6)$.

2014 HG8

設 $a_1 = 215$，$a_2 = 2014$ 及 $a_{n+2} = 3a_{n+1} - 2a_n$，其中 n 為一正整數。

求 $a_{2014} - 2a_{2013}$ 的值。

Let $a_1 = 215$, $a_2 = 2014$ and $a_{n+2} = 3a_{n+1} - 2a_n$, where n is a positive integer.
Find the value of $a_{2014} - 2a_{2013}$.

2014 FG1.4

給定一實數數列 $a_1, a_2, a_3, \cdots$，它滿足

(1) $a_1 = \frac{1}{2}$，及 (2) 對 $k \geq 2$，有 $a_1 + a_2 + \cdots + a_k = k^2 a_k$。求 a_{100} 的值。

Given a sequence of real numbers $a_1, a_2, a_3, \cdots$ that satisfy

(1) $a_1 = \frac{1}{2}$, and (2) $a_1 + a_2 + \cdots + a_k = k^2 a_k$, for $k \geq 2$.

Determine the value of a_{100}.

2015 HI5

已知 $a_1, a_2, \cdots, a_n, \cdots$ 為一正實數序列，其中 $a_1 = 1$ 及 $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$。求 a_{2015} 的值。

It is given that $a_1, a_2, \cdots, a_n, \cdots$ is a sequence of positive real numbers such that $a_1 = 1$ and $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$. Find the value of a_{2015}.

2016 HI15

已知數列$\{a_n\}$，其中 $a_{n+2} = a_{n+1} - a_n$。若 $a_2 = -1$ 及 $a_3 = 1$，求 a_{2016} 的值。

Given a sequence $\{a_n\}$, where $a_{n+2} = a_{n+1} - a_n$.

If $a_2 = -1$ and $a_3 = 1$, find the value of a_{2016}.

2016 HG3

考慮數列 $a_1, a_2, a_3, \cdots$。定義 $S_n = a_1 + a_2 + \cdots + a_n$ 其中 n 為任何正整數。

若 $S_n = 2 - a_n - \frac{1}{2^{n-1}}$，求 a_{2016} 的值。

Consider a sequence of numbers $a_1, a_2, a_3, \cdots$.

Define $S_n = a_1 + a_2 + \cdots + a_n$ for any positive integer n.

Find the value of a_{2016} if $S_n = 2 - a_n - \frac{1}{2^{n-1}}$.

2016 FG1.3

設 $f_{n+1} = \begin{cases} f_n + 3 & \text{若 } n \text{ 是雙數} \\ f_n - 2 & \text{若 } n \text{ 是單數} \end{cases}$。

若 $f_1 = 60$，求 n 的最少可能值，令當 $m \ge n$ 時，滿足 $f_m \ge 63$。

Let $f_{n+1} = \begin{cases} f_n + 3 & \text{if } n \text{ is even} \\ f_n - 2 & \text{if } n \text{ is odd} \end{cases}$.

If $f_1 = 0$, determine the smallest possible value of n satisfying $f_m \ge 63$ for all $m \ge n$.

2016 FG2.2

設 $f_1 = 9$ 及 $f_n = \begin{cases} f_{n-1} + 3 & \text{若 } n \text{ 是 3 的倍數} \\ f_{n-1} - 1 & \text{若 } n \text{ 不是 3 的倍數} \end{cases}$。

若 B 為 k 的值的可能數量，使得 $f_k < 11$，求 B 的值。

Let $f_1 = 9$ and $f_n = \begin{cases} f_{n-1} + 3 & \text{if } n \text{ is a multiple of 3} \\ f_{n-1} - 1 & \text{if } n \text{ is not a multiple of 3} \end{cases}$.

If B is the number of possible values of k such that $f_k < 11$,
determine the value of B.

2017 HI6

已知 $a_0 = 2$，$a_1 = -1$ 及 $a_{n+1} = 2a_n - a_{n-1}$，其中 $n \ge 1$，求 a_{2017} 的值。

Given that $a_0 = 2$, $a_1 = -1$ and $a_{n+1} = 2a_n - a_{n-1}$, where $n \ge 1$,
determine the value of a_{2017}.

2018 HI14

對任意實數 x $(x \ne 1)$，定義函數 $\mathrm{f}(x) = \frac{x}{1-x}$ 及 $\mathrm{f}\circ\mathrm{f}(x) = \mathrm{f}(\mathrm{f}(x))$。

求 $2017\underbrace{\mathrm{f}\circ\mathrm{f}\circ\mathrm{f}\circ\cdots\circ\mathrm{f}}_{2018\text{個f}}(2018)$ 的值。

For any real number x $(x \ne 1)$, define a function $\mathrm{f}(x) = \frac{x}{1-x}$ and $\mathrm{f}\circ\mathrm{f}(x) = \mathrm{f}(\mathrm{f}(x))$.

Find the value of $2017\underbrace{\mathrm{f}\circ\mathrm{f}\circ\mathrm{f}\circ\cdots\circ\mathrm{f}}_{2018\text{ copies of f}}(2018)$.

2018 FI3.3

若 n 是正整數、$a_1 = B$ 及 $a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{若 } a_n \text{ 是偶數；} \\ 3a_n + 1 & \text{若 } a_n \text{ 是奇數。} \end{cases}$ 求 $C = a_{2018}$ 的最值。

If n is a positive integer $a_1 = B$ and $a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{if } a_n \text{ is even;} \\ 3a_n + 1 & \text{if } a_n \text{ is odd.} \end{cases}$

determine the value of $C = a_{2018}$.

2019 HG8

設 $\{a_n\}$ 為一個正實數序列使當 $n > 1$ 時，$a_n = a_{n-1}a_{n+1} - 1$。已知 2018 在序列中及 $a_2 = 2019$。若 a_1 的所有可取的數目為 s，求 s 的值。

Let $\{a_n\}$ be a sequence of positive real numbers such that $a_n = a_{n-1}a_{n+1} - 1$ for $n > 1$. It is given that 2018 is in the sequence and $a_2 = 2019$. If the number of all possible values of a_1 is s, find the value of s.

2022 P2Q8

對所有正整數 $n > 1$，函數 f 定義如下：

$f(1) = 2021$ 及 $f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n)$。

求 f(2021) 的值。

For all positive integers $n > 1$, a function f is defined as

$f(1) = 2021$ and $f(1) + f(2) + f(3) + \cdots + f(n) = n^2 f(n)$.

Find the value of f(2021) .

2023 HI13

數列 $\{a_n\}$ 定義為 $a_1 = 1$、$a_2 = \frac{3}{7}$ 及對所有 $n \ge 3$，$a_n = \frac{a_{n-2}a_{n-1}}{2a_{n-2} - a_{n-1}}$。求 $\frac{1}{a_{2023}}$ 的值。

A sequence of numbers $\{a_n\}$ is defined by $a_1 = 1$, $a_2 = \frac{3}{7}$ and

$a_n = \frac{a_{n-2}a_{n-1}}{2a_{n-2} - a_{n-1}}$ for all $n \ge 3$. Find the value of $\frac{1}{a_{2023}}$.

Answers

1990 HI10 109	1991 FG8.1-2 $H_5 = 61$, $a = 3$	1992 FG9.3-4 $T_{10} = 200$, $n =$ 19	1995 FI4.3 5	1997 HG2 $\frac{1}{50}$
1999 FI2.4 2000	1999 FI3.4 9902	1999 FI5.4 6	1999 FG3.2 7	2000 FI3.4 666
2001 FG4.1 3840	2004 FGS.1 16	2005 FG4.4 171	2009 HI6 $\frac{2007}{2008}$	2009 FI1.4 72
2012 FG3.3 998	2012 FG3.4 1	2013 HG10 $\frac{2}{2013}$	2013 FI2.3 2	2013 FI4.2 7
2014 HG8 1584	2014 FG1.4 $\frac{1}{10100}$	2015 HI5 1016064	2016 HI15 -1	2016 HG3 $\frac{63}{2^{2011}}$
2016 FG1.3 127	2016 FG2.2 5	2017 HI6 -6049	2018 HI14 $-\frac{2018}{2019}$	2018 FI3.3 2
2019 HG8 4	2022 P2Q8 $\frac{1}{1011}$	2023 HI13 2697		