Hong Kong Mathematics Olympiad 2000-2001
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

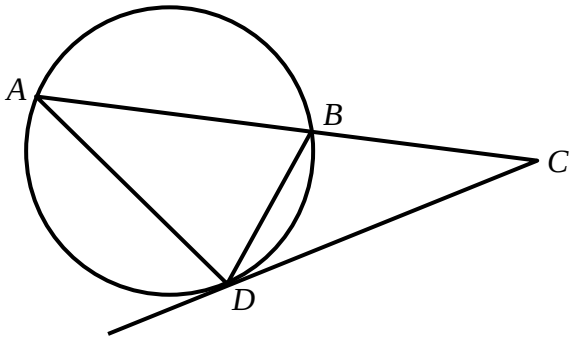
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 如果 $4^a = 25^b = 10$，求 $\frac{1}{a}+\frac{1}{b}$ 的值。

   If $4^a = 25^b = 10$, find the value of $\frac{1}{a}+\frac{1}{b}$.

2. 如圖一，ABC 為一直綫，$AB = AD$，$\angle BDC = 38°$，CD 切圓 ABD 於 D。設 $\angle BCD = x°$，求 x 的值。

   In figure 1, ABC is a straight line, $AB = AD$, $\angle BDC = 38°$, CD is a tangent to the circle ABD. Let $\angle BCD = x°$, find the value of x.



圖一 Figure 1

3. 如果 $p = 10x - 4xy - 5x^2 - y^2 - 8$，其中 x 和 y 為實數，求 p 的最大值。

   If $p = 10x - 4xy - 5x^2 - y^2 - 8$, where x and y are real numbers, find the largest value of p.

4. 如果下列三條直綫相交於一點，求 c 的值。

   L_1: $6x + 6y - 19 = 0$
   L_2: $18x + 12y + c = 0$
   L_3: $2x + 3y - 8 = 0$

   If the following three straight lines intersect at one point, find the value of c.

   L_1: $6x + 6y - 19 = 0$
   L_2: $18x + 12y + c = 0$
   L_3: $2x + 3y - 8 = 0$

5. 已知 $2 - 6\cos^2\theta = 7\sin\theta\cos\theta$，求 $\tan\theta$ 的最大值。

   It is known that $2 - 6\cos^2\theta = 7\sin\theta\cos\theta$, find the largest value of $\tan\theta$.

6. 88 張成人車票總值為 $\$\square 293\square$，由於列印機壞了，五位數字的首尾兩個數字印不出來。已知每張車票的價值為 $\$P$，其中 P 為一整數，求 P 的值。

   The total cost for 88 tickets was $\$\square 293\square$. Because the printing machine was not functioning well, the first and the last digits of the 5-digit number were missing. If the cost for each ticket is $\$P$, where P is an integer, find the value of P.

7. 如果 p 為方程式 $2x^3 + 7x^2 - 29x - 70 = 0$ 的正實數根，求 p 的值。

   If p is the positive real root of $2x^3 + 7x^2 - 29x - 70 = 0$, find the value of p.

8. 甲、乙二人合作做一件工程，30 天便可完工。如果兩人只合作了 6 天，甲便退出，乙須獨自繼續做 40 天才能完工。如果甲每天完成工程的 $\frac{1}{q}$，求 q 的數值。

Two persons *A*, *B* can complete a task in 30 days when they work together. If they work together for 6 days and then *A* quits, *B* needs 40 days more in order to complete the task. If the proportion of the task *A* can finish each day is $\frac{1}{q}$, find the value of *q*.

9. 設 a、b、c 為三個相異常數。已知

$$\frac{a^2}{(a-b)(a-c)(a+x)}+\frac{b^2}{(b-c)(b-a)(b+x)}+\frac{c^2}{(c-a)(c-b)(c+x)}\equiv\frac{p+qx+rx^2}{(a+x)(b+x)(c+x)},$$

其中 p、q、r 為常數，且 $s = 7p + 8q + 9r$，求 s 的值。

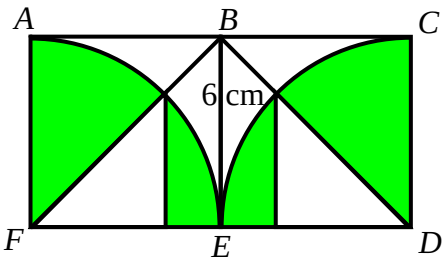
Let *a*, *b*, *c* be three distinct constants. It is given that

$$\frac{a^2}{(a-b)(a-c)(a+x)}+\frac{b^2}{(b-c)(b-a)(b+x)}+\frac{c^2}{(c-a)(c-b)(c+x)}\equiv\frac{p+qx+rx^2}{(a+x)(b+x)(c+x)}$$

where *p*, *q r* are constants, and $s = 7p + 8q + 9r$, find the value of *s* .

10. 如圖二，*ABEF*、*BCDE* 為正方形，*BE* = 6 cm，$\overset{\frown}{AE}$ 及 $\overset{\frown}{CE}$ 是分別以 *F*、*D* 為圓心畫出來的弧。如果圖中陰影部分的總面積為 *S* cm^2，求 *S* 的數值。(取 $\pi = 3$)

In figure 2, *ABEF*, *BCDE* are two squares, *BE* = 6 cm, and $\overset{\frown}{AE}$ and $\overset{\frown}{CE}$ are the arcs drawn with centres *F* and *D* respectively. If the total area of the shaded parts is *S* cm^2, find the value of *S*. (Assume $\pi = 3$.)



圖二 Figure 2

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Hong Kong Mathematics Olympiad 2000-2001
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。　　時限：20 分鐘

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每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

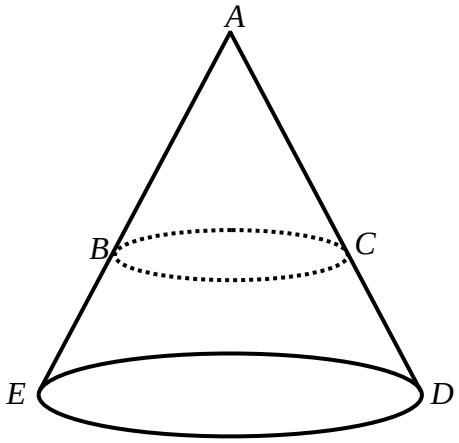
1. 現在鐘面上的時間是一時正。p 分鐘後，分針與時針剛好重疊，求 p 的最小值。
   The time on the clock face is now one o'clock. After p minutes, the minute hand overlaps with the hour hand, find the minimum value of p.

2. 把 10 個完全相同的球放入 3 個不同的盒子裏，使得沒有一個盒子是空的，共有多少種放法？
   In how many ways can 10 identical balls be distributed into 3 different boxes such that no box is to be empty?

3. 設 $x=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$ 及 $y=x^2$，求 y 的值。
   Let $x=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$ and $y=x^2$, find the value of y.

4. 如果 $\dfrac{4a}{1-x^{16}}\equiv\dfrac{2}{1-x}+\dfrac{2}{1+x}+\dfrac{4}{1+x^2}+\dfrac{8}{1+x^4}+\dfrac{16}{1+x^8}$，求 a 的值。
   If $\dfrac{4a}{1-x^{16}}\equiv\dfrac{2}{1-x}+\dfrac{2}{1+x}+\dfrac{4}{1+x^2}+\dfrac{8}{1+x^4}+\dfrac{16}{1+x^8}$, find the value of a.

5. 如圖一，ADE 是一個直立圓錐體。如果從底部向上並在 $\dfrac{1}{4}$ 的高度平行底部橫切，上面細錐體 ABC 斜面與餘下底部 $BCDE$ 斜面的面積的比為 $1:k$，求 k 的值。
   In figure 1, ADE is a right circular cone. Suppose the cone is divided into two parts by a cut running parallel to the base and made $\dfrac{1}{4}$ of the way up, the ratio of the slant surface of the small cone ABC to that of the truncated base $BCDE$ is $1:k$, find the value of k.

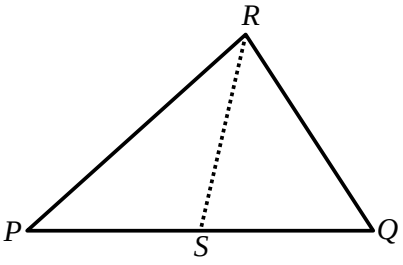


圖一 Figure 1

6. 如果十位數 2468m2468m 可被 3 整除，求 m 的最大值。
   If a ten-digit number 2468m2468m is divisible by 3, find the maximum value of m.

7. 求由 x-軸 及直綫 $x-3y=0$、$x+y-4=0$ 圍出的面積。
   Find the area enclosed by the x-axis and the straight lines $x-3y=0$, $x+y-4=0$.

8. 如圖二，PQR 是一個三角形，S 是 PQ 上的中點，$RQ=PS=SQ$，且 $\angle RQS=2\angle RPS$。設$\angle PSR=x°$，求 x 的值。
   In figure 2, PQR is a triangle, S is the mid-point of PQ, $RQ=PS=SQ$, and $\angle RQS=2\angle RPS$.
   Let $\angle PSR=x°$, find the value of x.



圖二 Figure 2

9. 如果 x 滿足方程 $|x-3|+|x-5|=2$，求 x 的最小值。
   If x satisfies the equation $|x-3|+|x-5|=2$, find the minimum value of x.

10. 從 6 對不同型號的鞋子中任取 3 只，求 3 只鞋子中恰有 2 只是同一型號的概率。
    3 shoes are chosen randomly from 6 pairs of shoes with different models, find the probability that exactly two out of the three shoes are of the same model.

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