Locus Parabola

Reference: HKU AL Pure Mathematics 1964 Paper 2 Q2 (a)

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Find the locus of the centres of circles which passes through $A(0, 4)$ and touches x-axis.

Let the centre of circle be $C(h, k)$. Suppose the circle touches x-axis at $B(h, 0)$.

Then the radius is k (where $k > 0$). The equation of circle is $(x - h)^2 + (y - k)^2 = k^2$.

It passes through $A(0, 4)$: $(0 - h)^2 + (4 - k)^2 = k^2$

$h^2 + 16 - 8k = 0$

$\Rightarrow$ The locus is $x^2 = 8(y - 2)$

which is a vertical parabola with vertex translated to (0, 2).