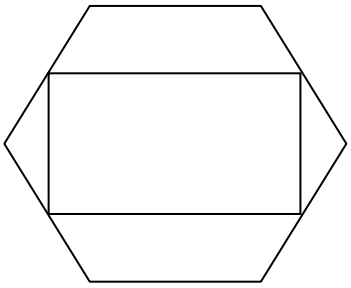
Hong Kong Mathematics Olympiad 2008-2009
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 設 $x = 0.\dot{2}\dot{3} + 0.00\dot{2}\dot{3} + 0.0000\dot{2}\dot{3} + 0.00000\dot{2}\dot{3} + \cdots$，求 x 的值。
Let $x = 0.\dot{2}\dot{3} + 0.00\dot{2}\dot{3} + 0.0000\dot{2}\dot{3} + 0.00000\dot{2}\dot{3} + \cdots$, find the value of x.

2. 如圖一，給定一正六邊形及一矩形，矩形的頂點是六邊形的四條邊的中點。若矩形與六邊形的面積之比為 $1 : q$，求 q 的值。
In Figure 1, a regular hexagon and a rectangle are given. The vertices of the rectangle are the midpoints of four sides of the hexagon. If the ratio of the area of the rectangle to the area of the hexagon is $1 : q$, find the value of q .



圖一 Figure 1

3. 設 $16 \sin^4 \theta° = 5 + 16 \cos^2 \theta°$ 且 $0 \le \theta \le 90$，求 θ 的值。
Let $16 \sin^4 \theta° = 5 + 16 \cos^2 \theta°$ and $0 \le \theta \le 90$, find the value of θ .

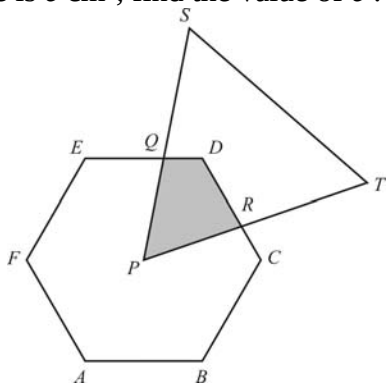
4. 設 m 為 gcd(2008, 4518)的正因數個數，其中 gcd(2008, 4518)是 2008 與 4518 的最大公因數。求 m 的值。
Let m be the number of positive factors of gcd(2008, 4518), where gcd(2008, 4518) is the greatest common divisor of 2008 and 4518. Find the value of m .

5. 已知 $x^2 + (y - 3)^2 = 7$，其中 x 及 y 為實數。若 $5y + x^2$ 的極大值為 k，求 k 的值。
Given that $x^2 + (y - 3)^2 = 7$, where x and y are real numbers. If the maximum value of $5y + x^2$ is k, find the value of k .

6. 設 $f_1(x) = \dfrac{1}{1-x}$ 及 $f_n(x) = f_1(f_{n-1}(x))$，其中 $n = 2、3、4、\cdots$。求 $f_{2009}(2008)$ 的值。
Let $f_1(x) = \dfrac{1}{1-x}$ and $f_n(x) = f_1(f_{n-1}(x))$, where $n = 2, 3, 4, \cdots$. Find the value of $f_{2009}(2008)$.

7. 在圖二中，*ABCDEF* 是一正六邊形，其中心點是 *P*。Δ*PST* 是一等邊三角形。已知 *AB* = 6 cm，*QD* = 2 cm 及 *PT* = 12 cm。若六邊形與三角形的公共部分面積為 *c* cm^2，求 *c* 的值。

In Figure 2, *ABCDEF* is a regular hexagon centered at the point *P*. Δ*PST* is an equilateral triangle. It is given that *AB* = 6 cm, *QD* = 2 cm and *PT* = 12 cm. If the area of the common part of the hexagon and triangle is *c* cm^2, find the value of *c* .



圖二 Figure 2

8. 求 7^{2009} 的個位值。

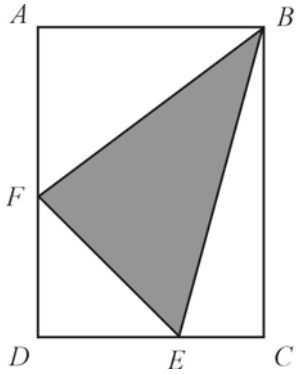
Find the unit digit of 7^{2009} .

9. 已知 *a* 和 *b* 是整數。設 $a - 7b = 2$ 及 $\log_{2b} a = 2$，求 $a \times b$ 的值。

Given that *a* and *b* are integers. Let $a - 7b = 2$ and $\log_{2b} a = 2$, find the value of $a \times b$.

10. 如圖三，*ABCD* 是一矩形。*E* 及 *F* 分別在 *CD* 及 *AD* 上使得 *AF* = 8 cm 及 *EC* = 5 cm。已知陰影部分的面積是 80 cm^2。設矩形 *ABCD* 的面積為 *g* cm^2，求 *g* 的值。

In Figure 3, *ABCD* is a rectangle. Points *E* and *F* lie on *CD* and *AD* respectively, such that *AF* = 8 cm and *EC* = 5 cm. Given that the area of the shaded region is 80 cm^2.
Let the area of the rectangle *ABCD* be *g* cm^2, find the value of *g* .



圖三 Figure 3

Spare Question

1. 已知 *a* 為負實數。若 $\dfrac{1}{a+\frac{1}{a+2}} = 2$，求 *a* 的值。

Given that *a* is a negative real number. If $\dfrac{1}{a+\frac{1}{a+2}} = 2$, find the value of *a* .

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2008-2009
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。　　時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark.　　Time allowed: 20 minutes

1. 若 a 是正整數及 $\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}+\cdots+\frac{1}{2008\times 2009}=\frac{272}{30135}$，求 a 的值。

   If a is a positive integer and $\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}+\cdots+\frac{1}{2008\times 2009}=\frac{272}{30135}$,
   find the value of a.

2. 設 $x=1+\sqrt{2}$，求 $x^5-2x^4+3x^3-4x^2-10x-6$ 的值。

   Let $x=1+\sqrt{2}$, find the value of $x^5-2x^4+3x^3-4x^2-10x-6$.

3. 已知 p 和 q 為整數。若 $\frac{2}{p}+\frac{1}{q}=1$，求 $p\times q$ 的最大值。

   Given that p and q are integers. If $\frac{2}{p}+\frac{1}{q}=1$, find the maximum value of $p\times q$.

4. 已知 $0\le x\le 180$。若方程 $\cos 7x^\circ=\cos 5x^\circ$ 有 r 個不同的根，求 r 的值。

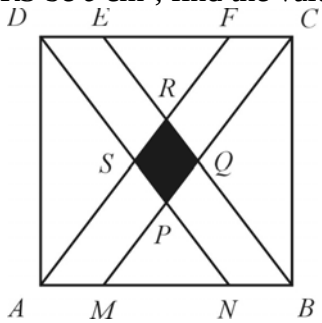
   Given that $0\le x\le 180$. If the equation $\cos 7x^\circ=\cos 5x^\circ$ has r distinct roots,
   find the value of r.

5. 設 x、y 及 z 為正整數且滿足 $\sqrt{z-\sqrt{28}}=\sqrt{x}-\sqrt{y}$。求 $x+y+z$ 的值。

   Let x, y and z be positive integers and satisfy $\sqrt{z-\sqrt{28}}=\sqrt{x}-\sqrt{y}$.
   Find the value of $x+y+z$.

6. 如圖一，$ABCD$ 是一正方形且 $AM = NB = DE = FC = 1$ cm 及 $MN = 2$ cm。設四邊形 $PQRS$ 的面積為 c cm^2，求 c 的值。

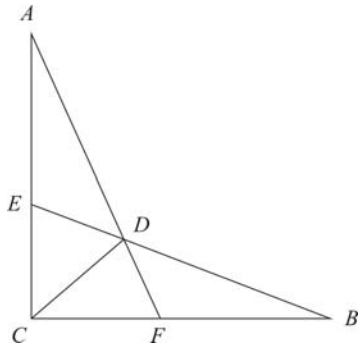
   In Figure 1, $ABCD$ is a square and $AM = NB = DE = FC = 1$ cm and $MN = 2$ cm.
   Let the area of quadrilateral $PQRS$ be c cm^2, find the value of c.



圖一 Figure 1

7. 已知 x 為實數且滿足 $2^{2x+8}+1=32\times 2^x$，求 x 的值。
Given that x is a real number and satisfies $2^{2x+8}+1=32\times 2^x$. Find the value of x.

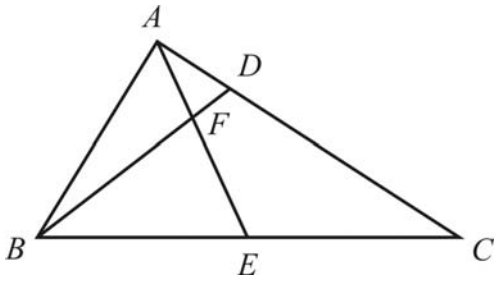
8. 在圖二中，$\angle ACB$ 為直角，$AC = BC = 14$ cm 及 $CE = CF = 6$ cm。
若 $CD = d$ cm，求 d 的值。
In Figure 2, $\angle ACB$ is a right angle, $AC = BC = 14\,\text{cm}$ and $CE = CF = 6\,\text{cm}$.
If $CD = d$ cm, find the value of d.



圖二
Figure 2

9. 若滿足 $\left|\left|x^2-6x-16\right|-10\right| = f$ 的相異實數 x 恰有 6 個，求 f 的值。
If there are 6 different values of the real number x that satisfies $\left|\left|x^2-6x-16\right|-10\right| = f$, find the value of f.

10. 如圖三，ABC 是一三角形，E 是 BC 的中點，F 在 AE 上使得 $AE = 3AF$。BF 的延綫與 AC 相交於 D。已知 ΔABC 的面積為 $48\,\text{cm}^2$。設 ΔAFD 的面積為 g cm^2，求 g 的值。
In Figure 3, ABC is a triangle, E is the midpoint of BC, F is a point on AE where $AE = 3AF$.
The extension segment of BF meets AC at D. Given that the area of ΔABC is $48\,\text{cm}^2$.
Let the area of ΔAFD be g cm^2, find the value of g.



圖三
Figure 3

Spare Question

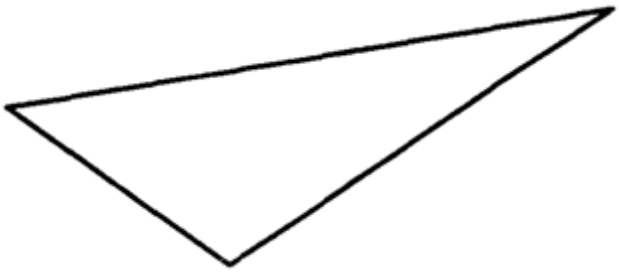
1. 設 R 為 588^{2009} 除以 97 所得的餘數。求 R 的值。
Let R be the remainder of 588^{2009} divided by 97. Find the value of R.

***** 試卷完 End of Paper *****

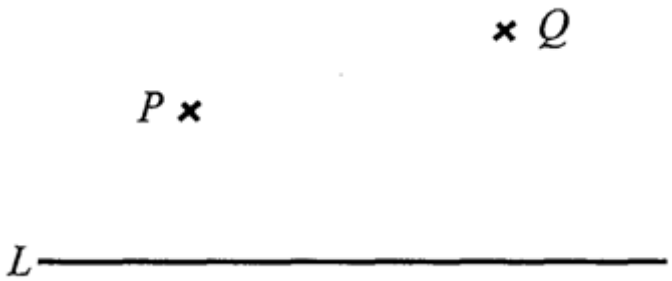
Hong Kong Mathematics Olympiad 2008-2009
Heat Event (Geometric Construction)
Sample Paper
香港數學競賽 – 幾何作圖
樣本題目

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：15 分鐘
All working (including geometric drawing) must be clearly shown.
此部份不計分。This part does not carries any marks. Time allowed: 15 minutes

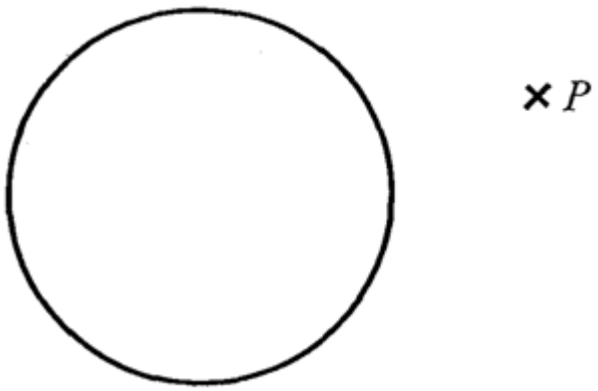
1. 在下列三角形中，試作出一點使它與該三角形各邊的距離相等。



2. 已知一直線 L，及兩點 P、Q 位於 L 的同一方。試在 L 上作一點 T 使得 PT 及 QT 的長度之和最小。



3. 試繪畫一固定圓切線，且該切線通過一固定點 P。



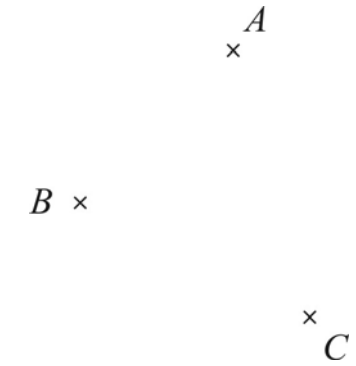
***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2008-2009 香港數學競賽 2008-2009
Heat Event (Geometric Construction) 初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：15 分鐘
All working (including geometric drawing) must be clearly shown.
此部份不計分。This part does not carries any marks. Time allowed: 15 minutes

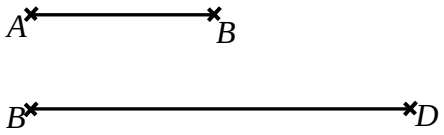
(以下圖形僅作參考，實際圖形以答案紙上者為準)
(The below figures are for reference only, the actual figures to be used are on the answer sheets.)

1. 如圖一，*A*、*B* 及 *C* 為三定點。試構作一穿過該三點的圓。
In Figure 1, *A*, *B* and *C* are three fixed points. Construct a circle passing through the given three points.



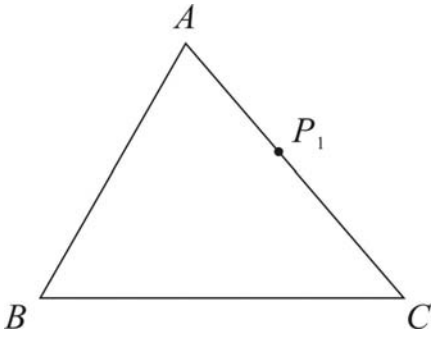
圖一 Figure 1

2. 如圖二，*AB* 為三角形 *ABC* 的底，且 *BD* 的長度是 *BC* 及 *CA* 長度的和。已知∠*ABC* = 60°，試構作三角形 *ABC*。
In Figure 2, *AB* is the base of a triangle *ABC*, and the length of *BD* is the sum of the lengths of *BC* and *CA*. Given that ∠*ABC* = 60°, construct the triangle *ABC*.



圖二 Figure 2

3. 如圖三，Δ*ABC* 為一銳角三角形。 P_1 是 *AC* 上的一點。試構作三角形 P_1XY，使得 *X* 及 *Y* 分別為 *AB* 及 *BC* 上的點，且 Δ P_1XY 的周界為最小。
In Figure 3, Δ*ABC* is an acute angle triangle. P_1 is a point on *AC*. Construct a triangle P_1XY such that *X* is a point on *AB*, *Y* is a point on *BC* and the perimeter of Δ P_1XY is the least.



圖三 Figure 3

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2008-2009 香港數學競賽 2008-2009
Heat Event (Geometric Construction) 初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：15 分鐘
All working (including geometric drawing) must be clearly shown.
此部份不計分。This part does not carries any marks. Time allowed: 15 minutes

School Code : _____

School Name : _____

Students' Name : (1) _____

(2) _____

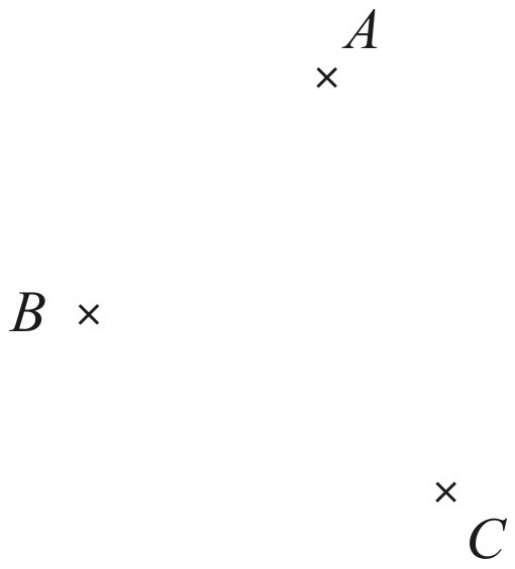
(3) _____

(4) _____

第一題 Question No. 1

如圖一，*A*、*B* 及 *C* 為三定點。試構作一穿過該三點的圖。

In Figure 1, *A*, *B* and *C* are three fixed points. Construct a circle passing through the given three points.



Hong Kong Mathematics Olympiad 2008-2009 香港數學競賽 2008-2009
Heat Event (Geometric Construction) 初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：15 分鐘
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此部份不計分。This part does not carries any marks. Time allowed: 15 minutes

School Code : _____

School Name : _____

Students' Name : (1) _____

(2) _____

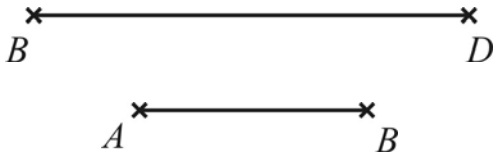
(3) _____

(4) _____

第二題 Question No. 2

如圖二，*AB* 為三角形 *ABC* 的底，且 *BD* 的長度是 *BC* 及 *CA* 長度的和。
已知∠*ABC* = 60°，試構作三角形 *ABC*。

In Figure 2, *AB* is the base of a triangle *ABC*, and the length of *BD* is the sum of the lengths of *BC* and *CA*. Given that ∠*ABC* = 60°, construct the triangle *ABC* .



Hong Kong Mathematics Olympiad 2008-2009 香港數學競賽 2008-2009
Heat Event (Geometric Construction) 初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：15 分鐘
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此部份不計分。This part does not carries any marks. Time allowed: 15 minutes

School Code : _____

School Name : _____

Students' Name : (1) _____

(2) _____

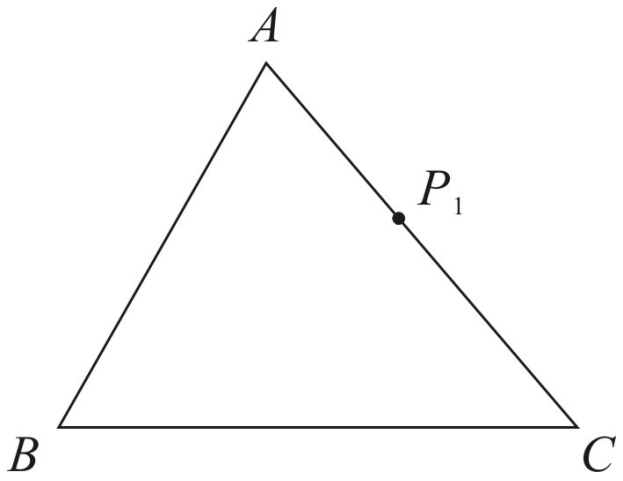
(3) _____

(4) _____

第三題 Question No. 3

如圖三，ΔABC 為一銳角三角形。 P_1 是 AC 上的一點。試構作三角形 P_1XY ，使得 X 及 Y 分別為 AB 及 BC 上的點，且 ΔP_1XY 的周界為最小。

In Figure 3, ΔABC is an acute angle triangle. P_1 is a point on AC. Construct a triangle P_1XY such that X is a point on AB, Y is a point on BC and the perimeter of ΔP_1XY is the least.



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