Examples on Mathematical Induction: divisibility 5

Created by Mr. Francis Hung Last updated: September 1, 2021

1. Prove by mathematical induction $8^n - 3^n$ is divisible by 5 for all non-negative integer n.
2. Prove by mathematical induction $2^{2n+1} + 3^{2n+1}$ is divisible by 5 for all non-negative integer n.
3. Prove by mathematical induction $4^n - 3^{2n}$ is divisible by 5 for all non-negative integer n.
4. Prove by mathematical induction $81\cdot 3^{2n} - 2^{2n}$ is divisible by 5 for all non-negative integer n.

   Let $P(n) \equiv$ "$81\cdot 3^{2n} - 2^{2n}$ is divisible by 5 for all non-negative integer n."

   $n = 0$, $81 - 1 = 80 = 5\times 16$, which is divisible by 5, $P(0)$ is true.

   Suppose $P(k)$ is true. i.e. $81\cdot 3^{2k} - 2^{2k} = 5m$, where m is an integer.

   When $n = k + 1$, $81\cdot 3^{2(k+1)} - 2^{2(k+1)} = 729\cdot 3^{2k} - 4\cdot 2^{2k}$

   $= 9\cdot(5m + 2^{2k}) - 4\cdot 2^{2k}$, by induction assumption.

   $= 9\cdot 5m + 5\cdot 2^{2k}$

   $= 5\cdot(9m + 2^{2k})$, which is divisible by 5.

   If $P(k)$ is true then $P(k + 1)$ is also true.

   By the principle of mathematical induction, $P(n)$ is true for all non-negative integer n.