Examples on Mathematical Induction: divisibility 9

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1. Prove by mathematical induction $(3n+1)\cdot 7^n - 1$ is divisible by 9 for all non-negative integer n.

   We first prove that $21\cdot 7^n + 6$ is divisible by 9 for all positive integer n $\cdots\cdots$ (*)

   $n = 0$, $21 + 6 = 27 = 9\cdot 3$ which is divisible by 9 .
   Suppose $21\cdot 7^k + 6$ is divisible by 9 for some positive integer k ,
   i.e. $21\cdot 7^k + 6 = 9m$, for some integer m
   $21\cdot 7^{k+1} + 6 = 7(21\cdot 7^k) + 6 = 7(9m - 6) + 6 = 63m - 36 = 9(7m - 4)$ which is divisible by 9 .
   Let $P(n) \equiv$ "$(3n+1)\cdot 7^n - 1$ is divisible by 9 for all non-negative integer n ."
   $n = 0$, $1\cdot 1 - 1 = 0$, which is divisible by 5, $P(0)$ is true.
   Suppose $P(k)$ is true. i.e. $(3k + 1)\cdot 7^k - 1 = 9m$, where m is an integer.
   When $n = k + 1$, $[3(k + 1) + 1]\cdot 7^{k+1} - 1 = (3k + 4)\cdot 7\cdot 7^k - 1$
   $= 7(3k + 1 + 3)7^k - 1$
   $= 7(3k + 1)7^k + 21\cdot 7^k - 1$
   $= 7\cdot(9m + 1) + 21\cdot 7^k - 1$, by induction assumption.
   $= 63m + 21\cdot 7^k + 6$
   $= 63m + 9q$, by (*), where m, q are integers.
   $= 9(7m + q)$
   If $P(k)$ is true then $P(k + 1)$ is also true.
   By the principle of mathematical induction, $P(n)$ is true for all non-negative integer n.

2. Prove by mathematical induction $4^n + 5^n$ is divisible by 9 for all odd positive integer n.

3. M2 PP Q3
   Prove by mathematical induction that $4^n + 15n - 1$ is divisible by 9 for all positive integers n.
   Let $P(n) \equiv$ "$4^n + 15n - 1$ is divisible by 9 for all positive integers n."
   $4^1 + 15 - 1 = 18 = 2\times 9$, which is divisible by 9.
   $P(1)$ is true.
   Suppose that $P(k)$ is true for some positive integer k.
   i.e. $4^k + 15k - 1$ is divisible by 9 for some positive integer k.
   $4^k + 15k - 1 = 9m$, where m is an integer $\cdots\cdots$ (*)
   When $n = k + 1$,
   $4^{k+1} + 15(k + 1) - 1 = 4\times 4^k + 15k + 14$
   $= 4(9m - 15k + 1) + 15k + 14$ by (*)
   $= 36m - 60k + 4 + 15k + 14$
   $= 36m - 45k + 18$
   $= 9(4m - 5k + 2)$
   $\because$ $4m - 5k + 2$ is an integer
   $\therefore$ $4^{k+1} + 15(k + 1) - 1$ is divisible by 9
   If $P(k)$ is true then $P(k + 1)$ is also true
   By the principle of mathematical induction, $P(n)$ is true for all positive integer n.