Number inequality (HKMO Classified Questions by topics)

1982 FG10.3

以下兩個數,哪一個較大?
$$2^{\frac{1}{2}} \times 9^{\frac{1}{9}}$$
, $3^{\frac{1}{3}} \times 8^{\frac{1}{8}}$ 。

What is the greatest number? $2^{\frac{1}{2}} \times 9^{\frac{1}{9}}$, $3^{\frac{1}{3}} \times 8^{\frac{1}{8}}$.

1983 FG6.4

已知
$$\sqrt{90} = 9.49$$
 (準至兩位小數)

若
$$d < 7\sqrt{0.9} < d+1$$
,其中 d 為整數,求 d 。

It is given that $\sqrt{90} = 9.49$, to 2 decimal places.

If $d < 7\sqrt{0.9} < d + 1$, where d is an integer, what is d?

1983 FG9.2

考慮以下步驟,用以證明 $\frac{1}{8} > \frac{1}{4}$ 。

Examine the following proof carefully: To prove $\frac{1}{8} > \frac{1}{4}$.

Steps 步驟

~ •	2000 7 41					
1	3 > 2	3 > 2				
12	兩邊乘以 $\log\left(\frac{1}{2}\right)$,	Multiply both sides by $\log\left(\frac{1}{2}\right)$,				
	使得 $3\log\left(\frac{1}{2}\right) > 2\log\left(\frac{1}{2}\right)$	then $3\log\left(\frac{1}{2}\right) > 2\log\left(\frac{1}{2}\right)$				
3	$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$	$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$				
4	$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2$				

$$\therefore \frac{1}{8} > \frac{1}{4}$$

Which step is incorrect? 以上哪一步是錯的?

1991 HG7

求滿足不等式
$$\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$$
 的 n 的最小正整數值。

Find the least positive integral value of *n* such that $\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$.

1996 HI4

能滿足不等式 $n^{200} < 5^{300}$ 的最大整數 n 是多少?

What is the largest integral value *n* that satisfies the inequality $n^{200} < 5^{300}$?

1999 FG5.3

若 $R^{2000} < 5^{3000}$, 其中 R 為正整數, 求 R 之最大值。

If $R^{2000} < 5^{3000}$, where R is a positive integer, find the largest value of R.

2002 FI4.3

已知x和y為兩正整數。若不等式 $x^2 + y^2 \le 8$ 的解(x, y)的數目為R,求R的值。

It is given that x and y are positive integers. If the number of solutions (x, y) of the inequality $x^2 + y^2 \le 8$ is R, find the value of R.

2007 FG1.2

在坐標平面上滿足 $x^2 + y^2 < 10$, 其中 x 及 y 為整數的點 (x, y) 共有 T 個,求 T 的值。

On the coordinate plane, there are T points (x, y), where x, y are integers, satisfying $x^2 + y^2 < 10$, find the value of T.

2008 FI4.3

設R為最大的整數使得 $R^{300} < 5^{200}$ 成立,求R的值。

Let R be the largest integer such that $R^{300} < 5^{200}$, find the value of R.

2018 FG2.4

求最小正整數 m,使得 $m^{200} > 6^{300}$ 。

Determine the least positive integer m such that $m^{200} > 6^{300}$.

Answers

$ \begin{array}{c} 1982 \text{ FG10.3} \\ \frac{1}{3^3} \times 8^{\frac{1}{8}} \end{array} $	1983 FG6.4	1983 FG9.2	1991 HG7	1996 HI4
	6	step 2	1601	11
1999 FG5.3	2002 FI4.3	2007 FG1.2	2008 FI4.3	2018 FG2.4
11	4	29	2	15