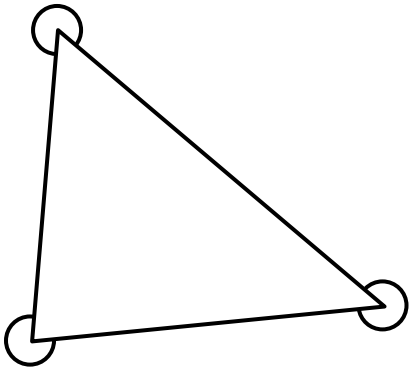
Hong Kong Mathematics Olympiad (1983 – 1984)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 附圖所示三角之和為 $a°$，求 a 的值。
In the given diagram, the sum of the three marked angles is $a°$.
Find the value of a.



$a =$

(ii) 一正 b 邊形之內角和為 $a°$，求 b 的值。
The sum of the interior angles of a regular b-sided polygon is $a°$.
Find the value of b.

$b =$

(iii) 若 $8^b = c^{21}$，求 c 的值。
If $8^b = c^{21}$, find the value of c.

$c =$

(iv) 若 $c = \log_d 81$，求 d 的值。
If $c = \log_d 81$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $100a = 35^2 - 15^2$，求 a 的值。
If $100a = 35^2 - 15^2$, find the value of a.

$a =$

(ii) 若 $(a-1)^2 = 3^{4b}$，求 b 的值。
If $(a-1)^2 = 3^{4b}$, find the value of b.

$b =$

(iii) 若 b 為 $x^2 + cx - 5 = 0$ 之一根，求 c 的值。
If b is a root of $x^2 + cx - 5 = 0$, find the value of c.

$c =$

(iv) 若 $x + c$ 為 $2x^2 + 3x + 4d$ 之因式，求 d 的值。
If $x + c$ is a factor of $2x^2 + 3x + 4d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 α、β為 $x^2 - 10x + 20 = 0$ 之根，且 $a = \frac{1}{\alpha} + \frac{1}{\beta}$，求 a 的值。

If α, β are roots of $x^2 - 10x + 20 = 0$, find the value of a, where $a = \frac{1}{\alpha} + \frac{1}{\beta}$.

$a =$

(ii) 若 $\sin\theta = a$ $(0° < \theta < 90°)$，且 $10\cos 2\theta = b$，求 b 的值。

If $\sin\theta = a$ $(0° < \theta < 90°)$, and $10\cos 2\theta = b$, find the value of b.

$b =$

(iii) 點 $A(b, c)$ 在直線 $2y = x + 15$ 上，求 c 的值。

The point $A(b, c)$ lies on the line $2y = x + 15$. Find the value of c.

$c =$

(iv) 若 $x^2 - cx + 40 \equiv (x + k)^2 + d$，求 d 的值。

If $x^2 - cx + 40 \equiv (x + k)^2 + d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 為 $2x^3 - 3x^2 + x - 1$ 被 $x + 1$ 除所得之餘數，求 a 的值。
If a is the remainder when $2x^3 - 3x^2 + x - 1$ is divided by $x + 1$, find the value of a.

$a =$

(ii) If b cm^2 is the total surface area of a cube of side $(8 + a)$ cm, find the value of b.
若 b cm^2 為一邊長$(8 + a)$ cm 的立方體之總表面積，求 b 的值。

$b =$

(iii) 一袋內有紅球 $b + 4$ 個，白球 $2b - 2$ 個。若隨意於袋內取球一個，而該球為白色之機會為 x，求 x 的值。
One ball is taken at random from a bag containing $b + 4$ red balls and $2b - 2$ white balls. If x is the probability that the ball is white, find the value of x.

$x =$

(iv) 若 $\sin\theta = x$ $(90° < \theta < 180°)$及 $\tan(\theta - 15°) = y$，求 y 的值。
If $\sin\theta = x$ $(90° < \theta < 180°)$ and $\tan(\theta - 15°) = y$, find the value of y.

$y =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

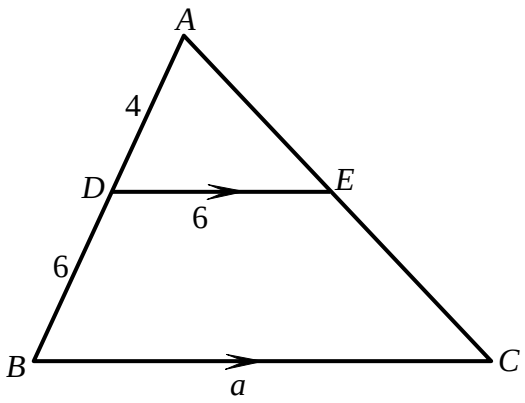
Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在圖一中，$DE//BC$，若 $AD = 4$，$DB = 6$，$DE = 6$，且 $BC = a$，求 a 的值。
In figure 1, DE // BC. If $AD = 4$, $DB = 6$, $DE = 6$ and $BC = a$, find the value of a.

$a =$



圖一 Figure 1

(ii) θ為銳角，$\cos\theta = \frac{a}{17}$。若 $\tan\theta = \frac{b}{15}$，求 b 的值。
θ is an acute angle such that $\cos\theta = \frac{a}{17}$. If $\tan\theta = \frac{b}{15}$, find the value of b.

$b =$

(iii) 若 $c^3 = b^2$，求 c 的值。
If $c^3 = b^2$, find the value of c.

$c =$

(iv) 一等邊三角形之面積為 $c\sqrt{3}$ cm^2。若其周界長 d cm，求 d 的值。
The area of an equilateral triangle is $c\sqrt{3}$ cm^2.
If its perimeter is d cm, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

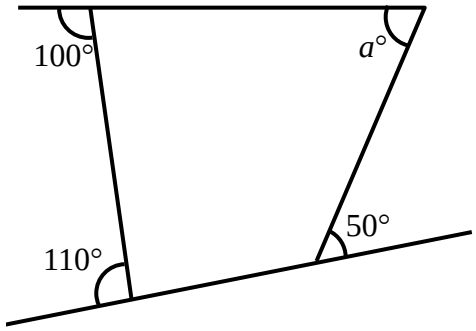
Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在圖二，求 a 的值。
In Figure 2, find the value of a .



圖二 Figure 2

$a =$

(ii) 若 $b = \log_2\left(\frac{a}{5}\right)$，求 b 的值。
If $b = \log_2\left(\frac{a}{5}\right)$, find the value of b .

$b =$

(iii) 一繩長 20 m，依 $b - 2 : b : b + 2$ 之比例分成三段。
若最長一段為 N m，求 N 的值。
A piece of string, 20 m long, is divided into 3 parts in the ratio of $b - 2 : b : b + 2$.
If N m is the length of the longest portion, find the value of N .

$N =$

(iv) 正 N 邊形之每一內角為 $x°$。求 x 的值。
Each interior angle of an N-sided regular polygon is $x°$. Find the value of x .

$x =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]
Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某兩數之和為 20，其積為 10，若該兩數倒數之和為 a，求 a 的值。
The sum of 2 numbers is 20 , their product is 10 .
If the sum of their reciprocals is a , find the value of a .

$a =$

(ii) $1^2 - 1 = 0\times2$，$2^2 - 1 = 1\times3$，$3^2 - 1 = 2\times4$，…，$b^2 - 1 = 135\times137$。
若 $b > 0$，求 b 的值。
$1^2 - 1 = 0\times2$, $2^2 - 1 = 1\times3$, $3^2 - 1 = 2\times4$, … , $b^2 - 1 = 135\times137$.
If $b > 0$, find the value of b .

$b =$

(iii) 若兩直綫 $x + 2y + 1 = 0$ 及 $cx + 3y + 1 = 0$ 互相垂直，求 c 的值。
If the lines $x + 2y + 1 = 0$ and $cx + 3y + 1 = 0$ are perpendicular, find the value of c .

$c =$

(iv) (2, –1)、(0, 1)、(c, d)三點共線。求 d 的值。
The points (2, –1), (0, 1), (c, d) are collinear. Find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $p=\dfrac{21^3-11^3}{21^2+21\times11+11^2}$，求 p 的值。
If $p=\dfrac{21^3-11^3}{21^2+21\times11+11^2}$, find the value of p.

$p =$

(ii) 若 p 人可在 6 日完成某一工程，且 4 人可在 q 日完成同一工程，求 q 的值。
If p men can do a job in 6 days and 4 men can do the same job in q days, find the value of q.

$b =$

(iii) 某年三月第 q 日為星期三，而同年三月第 r 日為星期五，且 $18 < r < 26$，求 r 的值。
If the q^{th} day of March in a year is Wednesday and the r^{th} day of March in the same year is Friday, where $18 < r < 26$, find the value of r.

$r =$

(iv) 若 $a*b = ab + 1$，且 $s = (3*4)*2$，求 s 的值。
If $a*b = ab + 1$, and $s = (3*4)*2$, find the value of s.

$s =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 凌晨三點卅分，時鐘兩針間之銳角為 $p°$，求 p 的值。
The acute angle between the 2 hands of a clock at 3:30 a.m. is $p°$.
Find the value of p.

$p =$

(ii) 在ΔABC 中，$\angle B = \angle C = p°$。若 $q = \sin A$，求 q 的值。
In ΔABC, $\angle B = \angle C = p°$. If $q = \sin A$, find the value of q.

$q =$

(iii) 三點(1, 3)、(a, 5)、(4, 9)共綫，求 a 的值。
The 3 points (1, 3), (a, 5), (4, 9) are collinear. Find the value of a.

$a =$

(iv) 7、9、x、y、17 之平均數為 10。若 m 為 $x + 3$、$x + 5$、$y + 2$、8、$y + 18$ 之平均數，求 m 的值。
The average of 7, 9, x, y, 17 is 10.
If m is the average of x +3, x +5, y +2,8, y +18, find the value of m.

$m =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

如圖所示加法中，每字母代表由零至九之不同整數。
已知 $S = 9$，$O =$零，$E = 5$。
求下列字母所代表之數字：

$$\begin{array}{cccccc} & S & E & N & D \\ + & M & O & R & E \\ \hline M & O & N & E & Y \end{array}$$

(i) M
(ii) N
(iii) R
(iv) Y

In the addition shown, each letter represents a different digit ranging from zero to nine. It is already known that $S = 9$, $O =$ zero, $E = 5$.
Find the numbers represented by

$$\begin{array}{cccccc} & S & E & N & D \\ + & M & O & R & E \\ \hline M & O & N & E & Y \end{array}$$

(i) M
(ii) N
(iii) R
(iv) Y

$M =$

$N =$

$R =$

$Y =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

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Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $x=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{100}\right)$，試以最簡單的分數表 x。

If $x=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{100}\right)$, find x in the simplest fractional form.

$x =$

(ii) 一長方體之長、闊、高依次為 2、3 及 4。若其總面積為 A，求 A 的值。
The length, width and height of a rectangular block are 2, 3 and 4 respectively. Its total surface area is A, find the value of A.

$A =$

(iii) 若 m 為 1、2、3、...、1001 之平均數，求 m 的值。
The average of the integers 1, 2, 3, ... , 1001 is m. Find the value of m .

$m =$

(iv) 一面積為 12π 之圓，內接於一周界為 P 之等邊三角形，求 P 的值。
The area of a circle inscribed in an equilateral triangle is 12π.
If P is the perimeter of this triangle, find the value of P .

$P =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1983 – 1984)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 一正方形內接於一直徑為 10 之圓。若 A 為正方形的面積，求 A 的值。
If A is the area of a square inscribed in a circle of diameter 10, find the value of A.

$A =$

(ii) 若 $a+\frac{1}{a}=2$，及 $S=a^3+\frac{1}{a^3}$，求 S 的值。
If $a+\frac{1}{a}=2$, and $S=a^3+\frac{1}{a^3}$, find the value of S.

$S =$

(iii) 一凸 n 邊形有 14 條對角線，求 n 的值。
An n-sided convex polygon has 14 diagonals. Find the value of n.

$n =$

(iv) 若 d 為兩點(2, 3)及(−1, 7)間之距離，求 d 的值。
If d is the distance between the 2 points (2, 3) and (−1, 7), find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]
Min. Sec.