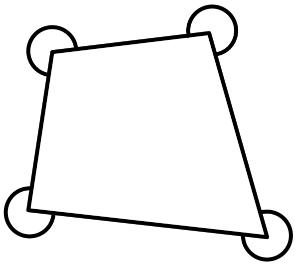
Hong Kong Mathematics Olympiad (1985 – 1986)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 附圖所示四角之和為 $a°$，求 a 的值。
In the given figure, the sum of the four marked angles is $a°$. Find the value of a.



$a =$

(ii) 一正 b 邊形之內角和為 $a°$，求 b 的值。
The sum of the interior angles of a regular b-sided polygon is $a°$.
Find the value of b.

$b =$

(iii) 若 $b^5 = 32^c$，求 c 的值。
If $b^5 = 32^c$, find the value of c.

$c =$

(iv) 若 $c = \log_4 d$，求 d 的值。
If $c = \log_4 d$, find the value of d.

$d =$

FOR OFFICIAL USE

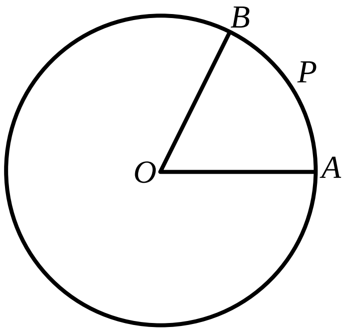
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 附圖所示的圓之半徑為 18 cm，圓心為 O。
若 $\angle AOB = \frac{\pi}{3}$，且弧 APB 之長為 $a\pi$ cm，求 a 的值。
The given figure shows a circle of radius 18 cm, centre O.
If $\angle AOB = \frac{\pi}{3}$ and the length of arc APB is $a\pi$ cm, find the value of a.

a =



(ii) 若不等式 $2x^2 - ax + 4 < 0$ 之解為 $1 < x < b$，求 b 的值。
If the solution of the inequality $2x^2 - ax + 4 < 0$ is $1 < x < b$, find the value of b.

b =

(iii) 若 $b(2x - 5) + x + 3 \equiv 5x - c$，求 c 的值。
If $b(2x - 5) + x + 3 \equiv 5x - c$, find the value of c.

c =

(iv) 過 (2, 6) 及 (5, c) 之直綫與 x-軸相交於(d, 0)。求 d 的值。
The line through (2, 6) and (5, c) cuts the x-axis at (d, 0). Find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐ Team No. ☐

\+ Bonus score ☐ Time ☐ ☐

Total score ☐ Min. Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若方程 $3x^2 - 4x + \frac{h}{3} = 0$ 有等根，求 h 的值。

If the equation $3x^2 - 4x + \frac{h}{3} = 0$ has equal roots, find the value of h.

$h =$

(ii) 若一圓柱體之高增加一倍，且新半徑為原來之 h 倍，則新體積為原來之 k 倍，求 k 的值。

If the height of a cylinder is doubled and the new radius is h times the original, then the new volume is k times the original. Find the value of k.

$k =$

(iii) 若 $\log_{10}210 + \log_{10}k - \log_{10}56 + \log_{10}40 - \log_{10}120 + \log_{10}25 = p$，求 p 的值。

If $\log_{10}210 + \log_{10}k - \log_{10}56 + \log_{10}40 - \log_{10}120 + \log_{10}25 = p$, find the value of p.

$p =$

(iv) 若 $\sin A = \frac{p}{5}$ 且 $\frac{\cos A}{\tan A} = \frac{q}{15}$，求 q 的值。

If $\sin A = \frac{p}{5}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find the value of q.

$q =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某公司的一百個員工之月薪如附表所示。若平均月薪為\$$m$，求 m 的值。

月薪(\$)	6000	4000	2500
員工人數	5	15	80

The monthly salaries of 100 employees in a company are as shown:

Salaries (\$)	6000	4000	2500
No. of employees	5	15	80

If the mean salary is \$$m$, find the value of m.

m =

(ii) 若 $8\sin^2(m+10)° + 12\cos^2(m+25)° = x$，求 x 的值。

If $8\sin^2(m+10)° + 12\cos^2(m+25)° = x$, find the value of x.

x =

(iii) 如圖所示，AP // CR // BQ，$AC = x$，$CB = 12$，$AP = 10$，$BQ = 15$ 及 $CR = y$。求 y 的值。

In the figure, AP // CR // BQ, $AC = x$, $CB = 12$, $AP = 10$, $BQ = 15$ and $CR = y$. Find the value of y.

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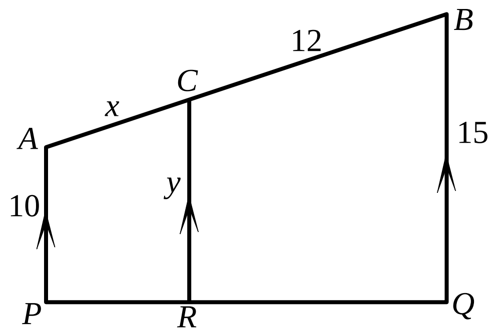


(iv) 定義$(a, b, c)\cdot(p, q, r) = ap + bq + cr$，其中 a、b、c、p、q、r 為實數。若$(3, 4, 5)\cdot(y, -2, 1) = n$，求 n 的值。

Define $(a, b, c)\cdot(p, q, r) = ap + bq + cr$, where a, b, c, p, q, r are real numbers.
If $(3, 4, 5)\cdot(y, -2, 1) = n$, find the value of n.

n =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

\+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\begin{cases} 1 = 1^2 \\ 1+3 = 2^2 \\ 1+3+5 = 3^2 \\ 1+3+5+7 = 4^2 \end{cases}$ It is known that $\begin{cases} 1 = 1^2 \\ 1+3 = 2^2 \\ 1+3+5 = 3^2 \\ 1+3+5+7 = 4^2 \end{cases}$

若 $1+3+5+\cdots+n = 20^2$，求 n 的值。 If $1+3+5+\cdots+n = 20^2$, find the value of n.

$n =$

(ii) 若直綫 $x + 2y = 3$ 及 $nx + my = 4$ 平行，求 m 的值。
If the lines $x + 2y = 3$ and $nx + my = 4$ are parallel, find the value of m.

$m =$

(iii) 若由整數 1 至 m 抽出一個數字，而每一數字被抽出之機會均等，被抽出數字為 m 之因數的或然率為 $\frac{p}{39}$，求 p 的值。
If a number is selected from the whole numbers 1 to m, and if each number has an equal chance of being selected, the probability that the number is a factor of m is $\frac{p}{39}$, find the value of p.

$p =$

(iv) 某小童以速率 p km/h 由家步行上學，並依照原來路線以速率 3 km/h 步行回家。若來回兩程之平均速率為 $\frac{24}{q}$ km/h，求 q 的值。
A boy walks from home to school at a speed of p km/h and returns home along the same route at a speed of 3 km/h.
If the average speed for the double journey is $\frac{24}{q}$ km/h, find the value of q.

$q =$

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐ Team No. ☐

\+ Bonus score ☐ Time ☐ ☐

Total score ☐ Min. Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

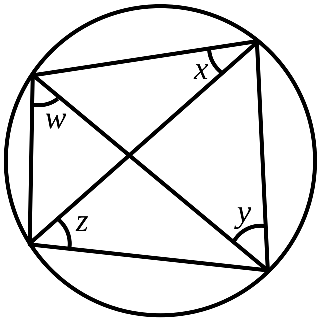
(i) 投擲一骰子，若擲出質數之或然率為 $\frac{a}{72}$，求 a 的值。

A die is rolled. If the probability of getting a prime number is $\frac{a}{72}$, find the value of a.

a =

(ii) 如圖所示，$x = a°$，$y = 44°$，$z = 52°$及 $w = b°$。求 b 的值。
In the figure, $x = a°$, $y = 44°$, $z = 52°$ and $w = b°$. Find the value of b.

b =



(iii) A，B 兩城相距 b km。彼得從 A 城以速率 7 km/h 踏單車往 B 城，與此同時，約翰從 B 城以速率 5 km/h 踏單車往 A 城。若兩人於 p 小時後相遇，求 p 的值。
A, B are two towns b km apart. Peter cycles at a speed of 7 km/h from A to B and at the same time John cycles from B to A at a speed of 5 km/h.
If they meet after p hours, find the value of p.

p =

(iv) 一角錐體之底為三角形，其邊長分別為 3 cm，p cm 及 5 cm。若該角錐體之高及體積依次為 q cm 及 12 cm^3，求 q 的值。
The base of a pyramid is a triangle with sides 3 cm, p cm and 5 cm. If the height and volume of the pyramid are q cm and 12 cm^3 respectively, find the value of q.

q =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+ Bonus score				Time
			Total score				Min. Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某兩數之和為 50，其積為 25。若該兩數倒數之和為 a，求 a 的值。
The sum of two numbers is 50, and their product is 25.
If the sum of their reciprocals is a, find the value of a.

$a =$

(ii) 若直綫 $ax + 2y + 1 = 0$ 及 $3x + by + 5 = 0$ 互相垂直，求 b 的值。
If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular,
find the value of b.

$b =$

(iii) 一正三角形之面積為 $100\sqrt{3}$ cm^2。若其周界為 p cm，求 p 的值。
The area of an equilateral triangle is $100\sqrt{3}$ cm^2. If its perimeter is p cm,
find the value of p.

$p =$

(iv) 若 $x^3 - 2x^2 + px + q$ 可被 $x + 2$ 整除，求 q 的值。
If $x^3 - 2x^2 + px + q$ is divisible by $x + 2$, find the value of q.

$q =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $12345\times6789 = a\times10^p$，其中 p 為正整數，且 $1 \le a < 10$，求 p 的值。
If $12345\times6789 = a\times10^p$ where p is a positive integer and $1 \le a < 10$, find the value of p.

$p =$

(ii) 若 (p, q)、$(5, 3)$ 及 $(1, -1)$ 共綫，求 q 的值。
If (p, q), $(5, 3)$ and $(1, -1)$ are collinear, find the value of q.

$q =$

(iii) 若 $\tan\theta = \frac{-7}{24}$，$90° < \theta < 180°$ 及 $100\cos\theta = r$，求 r 的值。
If $\tan\theta = \frac{-7}{24}$, $90° < \theta < 180°$ and $100\cos\theta = r$, find the value of r.

$r =$

(iv) x、y、z 之平均數為 10。x、y、z、t 之平均數為 12。求 t 的值。
The average of x, y, z is 10 . The average of x, y, z, t is 12 . Find the value of t.

$t =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]   Team No. [ ]

\+ Bonus score [ ]   Time [ ] Min. [ ] Sec.

Total score [ ]

Hong Kong Mathematics Olympiad (1985 – 1986)

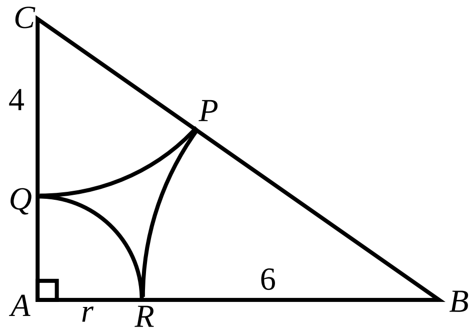
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖所示，依次以 A、B、C 為圓心之弧 QR、RP、PQ 相切於 R、P、Q。若 $AR = r$，$RB = 6$，$QC = 4$，$\angle A = 90°$，求 r 的值。

In the figure, QR, RP, PQ are 3 arcs, centres at A, B, C respectively, touching one another at R, P, Q. If $AR = r$, $RB = 6$, $QC = 4$, $\angle A = 90°$, find the value of r.

$r =$



(ii) M、N 依次為 $(3, 2)$ 及 $(9, 5)$。若 $P(s, t)$ 為 MN 上一點使 $MP : PN = 4 : r$，求 s 的值。

M, N are the points $(3, 2)$ and $(9, 5)$ respectively. If $P(s, t)$ is a point on MN such that $MP : PN = 4 : r$, find the value of s.

$s =$

(iii) $x^2 + 10x + t \equiv (x + a)^2 + k$，其中 t、a、k 為常數，求 a 的值。

$x^2 + 10x + t \equiv (x + a)^2 + k$, where t, a, k are constants. Find the value of a.

$a =$

(iv) 若 $9^{p+2} = 240 + 9^p$，求 p 的值。

If $9^{p+2} = 240 + 9^p$, find the value of p.

$p =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在所示乘法中，不同字母代表可能為 2、4、5、6、7、8、9 之不同整數。

In the given multiplication, different letters represent different integers whose possible values are 2, 4, 5, 6, 7, 8, 9.

$$\begin{array}{rcccccc} & 1 & A & B & C & D & E \\ \times & & & & & & 3 \\ \hline & A & B & C & D & E & 1 \end{array}$$

(i) 求 A 的值。
Find the value of A. | A =

(ii) 求 B 的值。
Find the value of B. | B =

(iii) 求 C 的值。
Find the value of C. | C =

(iv) 求 D 的值。
Find the value of D. | D =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 7 個橙和 5 個蘋果值\$13。3 個橙和 4 個蘋果值\$8。37 個橙和 45 個蘋果值\$$C$。求 C 的值。
7 oranges and 5 apples cost \$13. 3 oranges and 4 apples cost \$8. 37 oranges and 45 apples cost \$$C$. Find the value of C.

$C =$

(ii) 方程$(\sin^2 \theta - 1)(2 \sin^2 \theta - 1) = 0$，其中 $0° \le \theta \le 360°$，共有 n 個根。求 n 的值。
There are exactly n values of θ satisfying the equation $(\sin^2 \theta - 1)(2 \sin^2 \theta - 1) = 0$, where $0° \le \theta \le 360°$. Find the value of n.

$n =$

(iii) 若 $S = ab + a - b - 1$ 及 $a = 101$，$b = 49$，求 S 的值。
If $S = ab + a - b - 1$ and $a = 101$, $b = 49$, find the value of S.

$S =$

(iv) 若 (13, 5) 與 (5, −10) 兩點之距離為 d，求 d 的值。
If d is the distance between the points (13, 5) and (5, −10), find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	Min.	Sec.
			Total score					

Hong Kong Mathematics Olympiad (1985 – 1986)
Final Event 10 (Group)

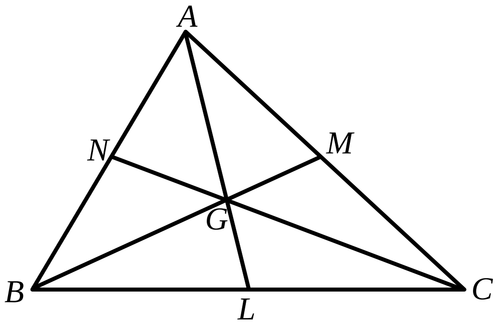
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $b + c = 3$，$c + a = 6$，$a + b = 7$，且 $P = abc$，求 P 的值。
If $b + c = 3$, $c + a = 6$, $a + b = 7$ and $P = abc$, find the value of P.

$P =$

(ii) ΔABC 之中綫 AL、BM、CN 相交於 G。
若 ΔABC 之面積為 54 cm^2，ΔANG 之面積為 x cm^2，求 x 的值。
The medians AL, BM, CN of ΔABC meet at G.
If the area of ΔABC is 54 cm^2 and the area of ΔANG is x cm^2. Find the value of x.

$x =$



(iii) 若 $k = \dfrac{3\sin\theta + 5\cos\theta}{2\sin\theta + \cos\theta}$ 及 $\tan\theta = 3$，求 k 的值。
If $k = \dfrac{3\sin\theta + 5\cos\theta}{2\sin\theta + \cos\theta}$ and $\tan\theta = 3$, find the value of k.

$k =$

(iv) 若 $S = \left(1 - \dfrac{1}{2^2}\right)\left(1 - \dfrac{1}{3^2}\right)\left(1 - \dfrac{1}{4^2}\right)\cdots\left(1 - \dfrac{1}{10^2}\right)$，求 S 的值。
If $S = \left(1 - \dfrac{1}{2^2}\right)\left(1 - \dfrac{1}{3^2}\right)\left(1 - \dfrac{1}{4^2}\right)\cdots\left(1 - \dfrac{1}{10^2}\right)$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ] Team No. [ ]
\+ Bonus score [ ] Time [ ] [ ]
Total score [ ] Min. Sec.