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B2

ITC Tut 7

Q) A linear (7, 4) Hamming code is defined by generator polynomial $G(d) = 1 + d + d^3$

a) Determine the generator matrix G & H

b) Design the encoder circuit.

Ans) a) $n = 7,\ k = 4$

$G(d) = 1 + d + d^3$

we know $G = [I_k / P]$

$$P(d) = \text{Remainder}\left(\frac{d^{n-k}\, d(n)}{G(d)}\right)$$

1st row, $n = 7,\ k = 1$

$$\frac{d^{7-1}}{G(d)} = \frac{d^6}{d^3 + d + 1}$$

$$\begin{array}{r|l} & d^3 + d + 1 \\ \hline d^3 + d + 1 & d^6 \\ & -\ d^6 + d^4 + d^3 \\ \hline & d^4 + d^3 \\ & -\ d^4 + 0d^3 + d^2 + d \\ \hline & d^3 + d^2 + d \\ & -\ d^3 + 0d^2 + d + 1 \\ \hline & d^2 + 1 \end{array}$$

$= [1 0 1]$

2nd row: $n = 7,\ k = 2$

$$\frac{d^{7-2}}{G(d)} = \frac{d^5}{d^3 + d + 1}$$

$$
\begin{array}{r|l}
 & d^2+1 \\
\hline
d^3+d+1 & d^5 \\
 & -\underline{d^5+d^3+d^2} \\
 & \quad d^3+d^2 \\
 & \quad \underline{d^3+0+d+1} \\
 & \qquad d^2+d+1
\end{array}
$$

$= [1\ 1\ 1]$

3rd row: $n=7$, $k=3$

$$\frac{d^{7-3}}{G(d)} = \frac{d^4}{d^3+d+1}$$

$$
\begin{array}{r|l}
 & d \\
\hline
d^3+d+1 & d^4 \\
 & \underline{d^4+d^2+d} \\
 & \quad d^2+d
\end{array}
$$

$= [1\ 1\ 0]$

4th row: $n=7$, $k=4$

$$\frac{d^{7-4}}{G(d)} = \frac{d^3}{d^3+d+1}$$

$$
\begin{array}{r|l}
 & 1 \\
\hline
d^3+d+1 & d^3 \\
 & \underline{d^3+d+1} \\
 & \quad d+1
\end{array}
$$

$= [0\ 1\ 1]$

$G = [I_k / P]$

$$
= \left[\begin{array}{cccc|ccc}
1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right]
$$

Scanned with CamScanner

Now parity check matrix :

$$H = [P^T \;\vdots\; I_{n-10}]$$

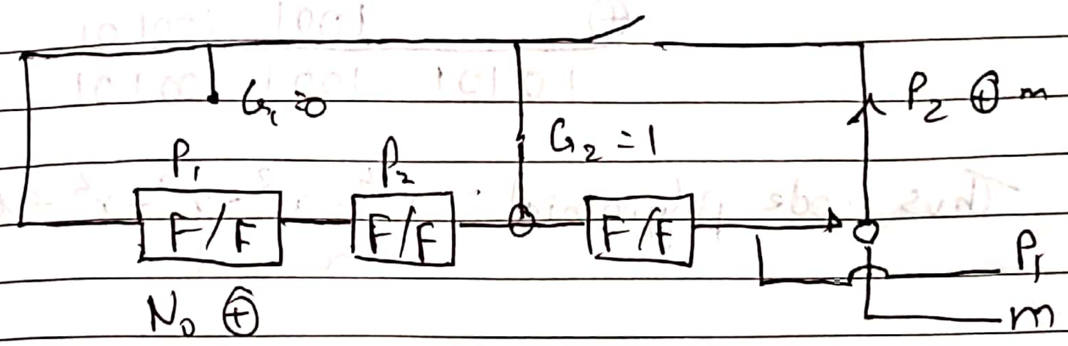
where P^T is transpose of P.

Here $H = [P^T : I_{3\times3}]$

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}_{3\times4}$$

$$\therefore H = \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]_{3\times2}$$

b)



No $\oplus$
as $G_1 = 0$

Q) A (15, 5) LBC is defined by the generator polynomial. $G(x) = 1 + x^2 + x^4 + x^5 + x^8 + x^{10}$
Find the code polynomial to the message polynomial $m(x) = 1 + x^2 + x^4$ in systematic form.

Ans) Polynomial

$$G(x) = 1 + x^2 + x^4 + x^5 + x^8 + x^{10}$$
$$= 10100110101$$

Message polynomial

$m(x) = 1 + x^2 + x^4$

$= 10101$

Now length of $g(x) = 11$ bits

Add $(11-1)$ 0 bits to message bits

$m(x) = 10101\,0000000000$

Now,

$$
\begin{array}{rl}
 & 10101\ 0000\ 0000\ 00 \\
\oplus & 10100\ 1101\ 01 \\
\hline
 & 0000\,1\ 1101\ 0100\ 00 \\
\oplus & 1\,0100\ 1101\ 01 \\
\hline
 & 0\,1\,00\ 1001\ 01 \leftarrow \text{remainder}
\end{array}
$$

Code vector = Data $\oplus$ remainder

$$
\begin{array}{rl}
= & 10101\ 0000\ 000000 \\
\oplus & 1001\ 100101 \\
\hline
 & 10101\ 1001\ 100101
\end{array}
$$

Thus code polynomial $= x^{14} + x^{12} + x^{10} + x^{9} + x^{6} + x^{5} + x^{2} + 1$