ITC Tutorial

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Q) Find the std array corresponding to (6,3) block of code if:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

No. of code words $= 2^k$
$= 2^6 = 64$

No. of rows $= 2^{n-k}$
$= 2^3 = 8$

$\therefore k = 3$

as dimension of syndrome is 3 bits, we have 8 possibilities,

i.e. 000
010
011
100
101
110
111

Now, $G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$ (I | P)

$\therefore H = \left[\begin{array}{ccc:ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$ (P | I)

P^T

$\therefore H^T = \left[\begin{array}{c:l} 0\ 1\ 1 & \rightarrow \text{1st bit error} \\ 1\ 0\ 1 & \rightarrow \text{2nd bit error} \\ 1\ 1\ 0 & \\ 1\ 0\ 0 & \\ 0\ 1\ 0 & \\ 0\ 0\ 1 & \rightarrow \text{6th bit error} \end{array}\right]$

We are left with syndrome 111 (0 0 0 is for 0 bit error)

Also, $C = [i][G]$

$= \{ 000000, 001110, 010101, 011011, 100011, 101101, 110110, 111000 \}$

Standard Array

000	000000	001110	010101	011011	100011	101101	110110	111000
011	100000	101110	110101	111011	000011	001101	010110	011000
101	010000	011110	000101	001011	110011	111101	100110	101000
110	001000	000110	011101	010011	101011	100101	111110	110000
100	000100	001010	010001	011111	100111	101001	110010	111100
010	000010	001100	010111	011001	100001	101111	110100	111010
001	000001	001111	010100	011010	100010	101100	110111	111001
111	100100	100100	100100	100100	100100	100100	100100	100100

Q) Decode the correct codeword if

$r_1 = 100101$

$r_2 = 111110$

(Ans) $H^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

i) $r_1 = 100101$

$S_1 = r_1 H^T$;

$$= [100101] \begin{bmatrix} 011 \\ 101 \\ 110 \\ 100 \\ 010 \\ 001 \end{bmatrix}$$

$= 110$

This syndrome is 3rd row of H^T

$\therefore e = 001000$

$C_1 = r_1 \oplus e$

$\therefore C_1 = 100101 \oplus 001000$

$\therefore C_1 = 101101$

ii) $r_2 = 111110$

$S_2 = r_2 H^T$

$$S_2 = [111110] \begin{bmatrix} 011 \\ 101 \\ 110 \\ 100 \\ 010 \\ 001 \end{bmatrix}$$

$\therefore S_2 = [110]$

This syndrome is 3rd row of H^T

$\therefore e = 001000$

$C_2 = r_2 \oplus e$

$= 111110 \oplus 001000$

$\therefore C_2 = 110110$