ITC Sem IV Tut 2

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Q1) A black and white TV picture consists of 526 lines of picture information. Assume that each line columns 2048 picture elements and each pixel has 255 brigtness levels. Each picture is represented at the rate of 30 frames/sec. Calculate the average rate of information seen by the viewer.

Ans) Let us assume uniform probability for each brigtness level then entropy:

$$H(s) = \sum_{i=1}^{255} P_i \times \log_2(1/P_i)$$

$$= 255 \times \frac{1}{255} \times \log_2\left(1/\left(\frac{1}{255}\right)\right)$$

$$= 255 \times ((1/255) \times \log_2(255))$$

$$\therefore H(s) = 7.99 \text{ bits/symbol}$$

In this case,
Information rate = Entropy $(H(s)) \times r_s$ (symbol rate)

But r_s = (symbol/line) x (lines/frame) x (frames/sec)

$$r_s = 2048 \times 526 \times 30$$
$$= 32317440$$
$$= 32.31 \times 10^6 \text{ Megapixels/sec}$$

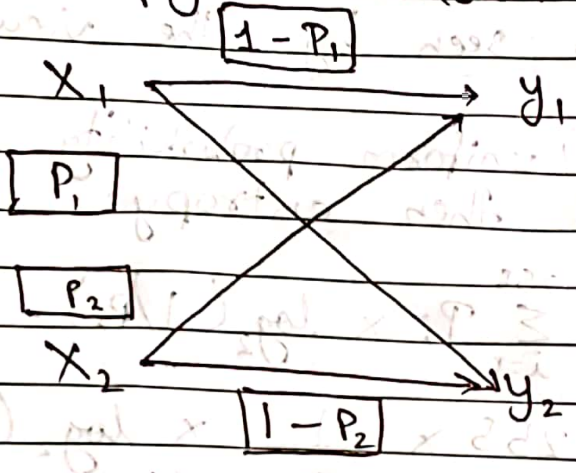
Scanned with CamScanner

Thus,

Information rate $= 7.99 \times 32.31 \times 10^6$
$= 258.16 \times 10^6$ Mbits/sec

$\therefore$ The average rate of information seen by the viewer $= 258.16 \times 10^6$ Mbits/sec

Q.2) A non symmetic binary channel is given in the figure below:



a) Given that $p1 = 0.97$, $p2 = 0.90$, $P(x_1) = 1/4$ and $P(X_2) = 3/4$. Calculate $H(X)$, $X(X/Y)$ and $H(Y/X)$

b) Find the capacity of the channel if $p_1 = p_2 = 0.75$

Ans) a)

Channel Matrix :-

$$P(Y/X) = \begin{array}{c|cc} & Y_1 & Y_2 \\ \hline X_1 & 1-P_1 & P_1 \\ X_2 & P_2 & 1-P_2 \end{array} = \begin{array}{cc} 0.03 & 0.97 \\ 0.90 & 0.10 \end{array}$$

$$P(X) = [P(X_1) \; P(X_2)]$$
$$= [1/4 \quad 3/4]$$

Hence, $P(X,Y) = P(X) \times P(Y/X)$

$$= \begin{bmatrix} 0.03 \times 1/4 & 0.97 \times 1/4 \\ 0.90 \times 3/4 & 0.10 \times 3/4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0075 & 0.2425 \\ 0.675 & 0.075 \end{bmatrix}$$

$\therefore$ $P(Y_1) = 0.0075 + 0.675 = 0.6825$
$P(Y_2) = 0.2425 + 0.075 = 0.3175$

Now,
$$H(Y) = \sum P(y_i) \times \log_2 (1/P(y_i))$$
$$= 0.6825 \times \log_2 (1/0.6825) + 0.3175 \times \log_2 (1/0.3175)$$
$$= 0.3761 + 0.5255$$
$$= \underline{0.901 \text{ bits/message}}$$

$$H(X) = \sum P(x_i) \times \log_2 (1/P(x_i))$$
$$= \frac{1}{4} \log_2 (1/(1/4)) + \frac{3}{4} \log_2 (1/(3/4))$$
$$= 0.5 + 0.31$$
$$= \underline{0.81 \text{ bits/message}}$$

$$H(Y/X) = \sum i=1,2 \; \sum j=1,2 \; p(x_i, y_i) \times \log_2 (1/p(x_i, y_i))$$

$$= \sum i=1,2 \; \sum j=1,2 \; p(x_i, y_i) \times \log(1/p(y/x))$$

$$= 0.0075 \times \log_2 (1/0.03) +$$
$$0.2425 \times \log_2 (1/0.97) +$$
$$0.675 \times \log_2 (1/0.90) +$$
$$0.75 \times \log_2 (1/0.10)$$

$$= 0.0379 + 0.0106 + 0.1026 + 0.2491$$
$$= \underline{0.4 \text{ bits/message}}$$

$$H(X/Y) = \sum i=1,2 \; \sum y=1,2 \; P(y_i, x_i) \times \log_2 (1/P(y_i x_i))$$

$$= \sum i=1,2 \; \sum y=1,2 \; P(y_i x_i) \times \log_2 (1/P(x/y))$$

$$= \sum i=1,2 \; \sum y=1,2 \; P(x_i, y_i) \times P(y_i) \times \log_2 \left(\frac{1}{P(x,y)}\right)$$

$$= 0.01 \times 0.6825 \times \log_2 \left(\frac{1}{0.98}\right) +$$

$$0.236 \times 0.3125 \times \log_2 \left(\frac{1}{0.236}\right)$$

$$= \underline{0.3112 \text{ bits/message}}$$

(b)

Given $p_1 = p_2 = 0.75$

Channel capacity (C) $= 1 - h$

$$= 1 - [p \log_2 (1/p) + (1-p) \log_2 (1/(1-p))]$$

$$= 0.1887 \text{ bits/symbol}$$

$\therefore$ Channel capacity = 0.1887 bits/symbol

Q3) Write the equation for mutual information $I(x;y)$ in terms of marginal entropies $H(X)$, $H(Y)$, conditional entropies $H(X/Y)$, $H(Y/X)$ are joint entropy $H(X,Y)$?

Ans) Mutual information can be equivalently expressed as:-

$$
\begin{aligned}
I(X;Y) &= H(X) - H(X/Y) \\
&= H(Y) - H(Y/X) \\
&= H(X) + H(Y) - H(X,Y) \\
&= H(X,Y) - H(X/Y) - H(Y/X)
\end{aligned}
$$

where $H(X)$ and $H(Y)$ are marginal entropies $H(X/Y)$ and $H(Y/X)$ are the conditional entropies and $H(X,Y)$ is the joint entropy of X and Y.