Devansh Shah
1914078
B2

039

Exponential distribution

03/02

Q) The mileage which car owners get with a certain kind of radial tyre is a R.V. having an exponential distribution with mean 40000 km. Find the probability that one of these tyres will last
(i) at least 20000 (ii) at most 40000 km

Soln) ∵ mean of exponential distribution is $1/\lambda$

$$\therefore \frac{1}{\lambda} = 40000$$

$$\lambda = \frac{1}{40000}$$

The distribution of fn of x is :-

$$f(x) = \lambda e^{-\lambda x}$$

$$= \frac{1}{40000} \times e^{-x/40000}$$

$$f(x) = 1 - e^{-\lambda x}$$

$$= 1 - e^{-x/40000}, \quad x > 0$$

(i) $P(x \geq 20000) = 1 - P(x < 20000)$

$= 1 - f(x = 20000)$

$= 1 - \left(1 - e^{-20000/40000}\right)$

$= e^{-1/2}$

$= \underline{0.607}$

(ii) $P(x \leq 20000) = f(x = 20000) = 1 - e^{-20000/40000}$

$= 1 - e^{-1/2}$

$= 1 - 0.607$

$= \underline{0.393}$

Devansh Shah
191078 B2

DATE
0 40

Q) The daily consumption of milk in excess of 20L in a town is approximately exponentially distributed with parameter $\frac{1}{3000}$. The town has daily stock of 30kL. Find the probability that of two days selected at a random is sufficient for both days.

Soln) Given, $\lambda = \frac{1}{3000}$

we have $P(x) = \frac{1}{3000} \cdot e^{-x/3000}$

$\therefore f(x) = 1 - \frac{1e^{-x/3000}}{3000}$

let daily consumption = y
& excess consumption = x

P(stock insufficient for 1 day) = $P(x > 35000)$
$= P(x + 20000 > 35000)$
$= P(x > 15000)$

P(stock sufficient for 1 day) = $1 - P(x > 15000)$
$= 1 - (1 - P(x \leq 15000))$
$= 1 - (1 - (1 - e^{-15000/3000}))$
$= 1 - e^{5}$
$= 0.3935$

P(stock for 2 days) = $(1 - e^{5})^2$
$= (0.3935)^2$
$= 0.1548$

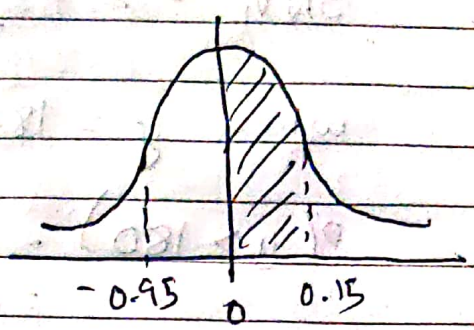
Devansh Shah
1914072 B2

14/02

Normal distribution

1) If $z \sim N(0,1)$
Find :- $P(0<z<0.95)$, $P(z>0.95)$, $P(z<0.95)$
$P(|z|<0.95)$

Ans) $P(0<z<0.95) = 0.3289$
$P(z<0.95) = 0.5 + 0.3789$
$= 0.8289$
$P(z>0.95) = 0.5 - 0.1289$
$= 0.1711$
$P(z<-0.95) = 0.5 - 0.3289 = 0.1711$
$P(|z| \leq 0.95) = 2(0.3289) = 0.6578$
(All values calculated from table)



2) What is the value of const, if
i) $P(0<z<c) = 0.2291 \longrightarrow \boxed{c = 0.61}$

ii) $P(z<c) = 0.7291 = 0.5 + 0.2291 \longrightarrow \boxed{c = 0.61}$

05/02

lec 12

1) In an exam, marks obtained by students in math, physics and chemistry are normally distributed with means 51, 53 & 46 with std as 15, 12, 16 resp. Find probability of securing total marks
i) 180 or above ii) 90 or below

Sol) let x_1, x_2, x_3 denote the marks obtained in 3 subjects then x_1, x_2, x_3 are normal variables with mean 51, 53, 46 and variance $(15)^2$, $(12)^2$, $(16)^2$

Devansh Shah
19140078 B2

DATE 04 2

$y = x_1 + x_2 + x_3$ is distributed normally with mean $m = 51 + 53 + 46 = 150$ &
$\sigma^2 = 15^2 + 12^2 + 16^2 = 625 = (25)^2$

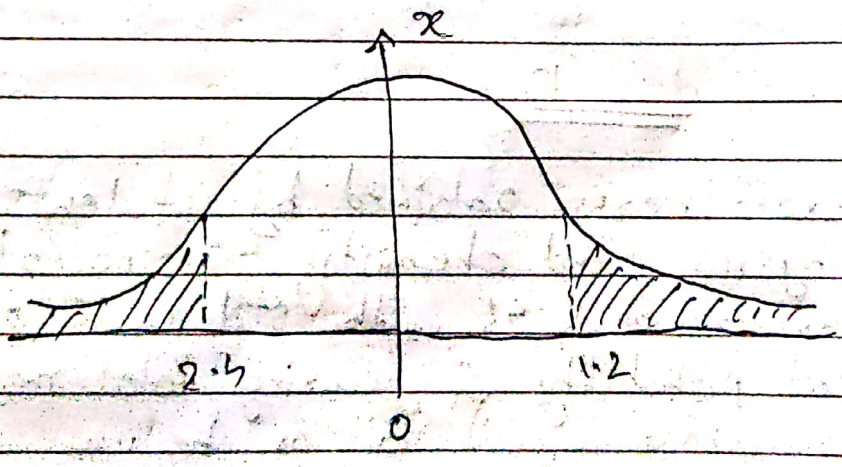
$$SNV_1 = z = \frac{y - M}{\sigma} = \frac{y - 150}{25}$$

when $y = 180$, $z = \frac{30}{25} = 1.2$

$P(y \geq 180) = p(z \geq 1.2)$
= Area on right of $z = 1.2$
$= 0.5 - 0.3849$
$= 0.1151$

when $y = 90$, $z = \frac{-60}{25} = -2.4$

$P(y < 90) = p(z \leq 2.4)$
$= 0.5 -$ area from $z = 0$ to $z = 2.4$
$= 0.5 - 0.4918$
$= 0.0082$



Devansh Sheth
19I4078 B2

DATE 06 6

26/02 **Logistic Regression**

Q) The relative risk for developing cardiovascular is for people with low and high salt diets is estimated as:

Developed CVD	Salt in diet		Total
	Yes	No	
Yes	88	112	200
No	1081	1134	2215
Total	1169	1246	2415

i) For each salt level, find the probability of developing CVD

Ans) For this we will see the data of row where Developed CVD = Yes

For low salt diet, $P_L = \frac{88}{1169} = 0.075$

For high salt diet, $P_H = \frac{112}{1246} = 0.0898$

(ii) Convert each of the probabilities that you found in (i) to odds

Ans) Odds (developing CVD in low salt diet)

$$= \frac{P_L}{1 - P_L}$$

$$= \frac{0.075}{1 - 0.075}$$

$$= 0.081$$

Devansh Shah
1914078 B2

DATE 06 7

Odds (Developing CVD in high salt diet)
$= \frac{P_h}{1 - P_h}$
$= \frac{0.0898}{1 - 0.0898}$
$= 0.098$

(iii) Find log of each found in (ii)

→ log (odds developing CVD in low salt diet)
$= \log_2 (0.081)$
$= -2.5133$

→ log (odds developing CVD in high salt diet)
$= \log_2 (0.098)$
$= -2.326$

Q) A survey was conducted for some student to find the number of hours each student spent in daily studying using logistic regression, it was found that

log (odds of passing exam) = 1.5046 hours - 4.07
Then Find :-

(i) Odds of passing exam

Ans) let $x = 1$ for passing & $x = 0$ for failing

$$\log\left(\frac{P_{pass}}{1 - P_{pass}}\right) = b_0 + b_1$$

$$\log\left(\frac{P_{failing}}{1 - P_{failing}}\right) = b_0$$

Devansh Shah
1914078 B2

068

For the entire model,

$$\log\left(\frac{P}{1-P}\right) = b_0 + b_1 x$$

$b_0 = -4.0777$
$b_1 = 1.5046$

$\therefore b_0 + b_1 = -2.5731$

$\therefore$ odds of passing exam $= b_0 + b_1 = -2.5731$

(ii) Probability of passing an exam

$$\log\left(\frac{P_{pass}}{1-P_{pass}}\right) = -2.5731$$

$$\frac{P_{pass}}{1-P_{pass}} = e^{-2.5731}$$

$$\frac{P_{pass}}{1-P_{pass}} = 0.0763$$

$$P_p = 0.0763 - 0.0763 P_p$$

$$P_p = 0.07089 \approx 0.0709$$

$\therefore$ Probability of passing $= 0.0709$

Devansh Shah
1914078 B2

DATE 06/9

(iii) Slope of the odds ratio

Ans) Slope of odds ratio is given as

Slope of odds ratio = $\frac{\text{odds for passing}}{\text{odds for failing}}$

$$= \frac{e^{b_0 + b_1}}{e^{b_0}}$$

$$= e^{b_0 + b_1 - b_0}$$

$$= e^{b_1}$$

$$= e^{1.5046}$$

$$= 4.5023$$

(iv) Probability of passing exam if student studies daily

Ans) To find probability of passing exam, if student studies daily,

$$\log\left(\frac{P_p}{1 - P_p}\right) = 1.5046 \times 2 - 4.0777$$

$$= -1.0685$$

$$\frac{P_p}{1 - P_p} = e^{-1.0685}$$

$$= 0.3435$$

$$\frac{P_p}{1 - P_p} = 0.3435$$

Devansh Shah
1914078 B2

DATE
0 7 0

$$P_f = \frac{0.3435}{1-0.3435}$$

$$= 0.2556$$

∴ Probability of passing the exam if student studies daily

$$= \underline{0.2556}$$