B, B2
IT

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PSOT ESE

(1)

Q1) A

1) b

2) a

3) a

4) c

5) b

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Q1) (B)

1) Given,

$$F_{x,y}(x,y) = \begin{cases} cx+1 & x,y \geq 0 ,\ x+y<1 \\ 0 & \text{otherwise} \end{cases}$$

To find c, we use

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{x,y}(x,y)\,dx\,dy = 1$$

$$\therefore 1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{x,y}(x,y)\,dx\,dy$$

$$= \int_{0}^{1}\int_{0}^{1-x} (cx+1)\,dy\,dx$$

$$= \int_{0}^{1} \Big[(cx+1)y\Big]_{0}^{1-x} dx$$

$$= \int_{0}^{1} (cx+1)(1-x)\,dx$$

$$= \int_{0}^{1} (cx - cx^2 + 1 - x)\,dx$$

$$= \int_{0}^{1} \left((c-1)x - cx^2 + 1\right) dx$$

$$= \left[(c-1)\frac{x^2}{2} - \frac{cx^3}{3} + x\right]_{0}^{1}$$

$$1 = \frac{c}{2} - \frac{1}{2} - \frac{c}{3} + 1$$

$$\Rightarrow 1 = \frac{c}{6} + \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{c}{6} \qquad \therefore \underline{c = 3}$$

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Deshath (3)

Q1) (B)

3) $n = 100$

$\bar{x} = 1570$

$\mu = 1600$

$\sigma = 120$

$H_0 = \bar{x} = \mu$

$H_1 = \bar{x} \neq \mu$

Z_{table} at 5% LOS $= 1.96$

at 1% LOS $= 2.58$

$$Z_{calculate} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{1570 - 1600}{120/\sqrt{100}}$$

$$= \frac{30}{120/10}$$

$$= 2.5$$

$\therefore$ Answer at 5% LOS $Z_{cal} > Z_{table}$

$\therefore$ H_0 is rejected

but

at 1% LOS $Z_{cal} < Z_{table}$

$\therefore$ H_0 is accepted

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4) Introducing artificial variable A_2

Maximise $Z = 3x_1 - x_2 - 0S_1 - 0S_2 - 0S_3 - mA_2$ — (1)

subject to $2x_1 + x_2 + S_1 + 0S_2 + 0S_3 + 0A_2 = 2$

(2) — $x_1 + 3x_2 + 0S_1 - S_2 + 0S_3 + A_2 = 3$

$0x_1 + x_2 + 0S_1 + 0S_2 + S_3 + 0A_2 = 4$

where $x_1, x_2, S_1, S_2, S_3, A_2 \geq 0$

Eliminating mA_2 from (1) by adding m time (2)

$\therefore Z = (3+M)x_1 - (1-3M)x_2 - 0S_1 - MS_2 - 0S_3 - 0A_2 - 3M$

$\therefore Z - (3+M)x_1 + (1-3M)x_2 + 0S_1 + MS_2 + 0S_3 + 0A_2 = -3M$

Simplex table

Interation no.	Basic var	x_1	x_2	S_1	S_2	S_3	A_2	RHS
0	Z	-3-M	1-3M	0	M	0	0	-3M
A_2 leaves	S_1	2	1	1	0	0	0	2
x_2 enters	A_2	1	3	0	-1	0	1	3
	S_3	0	1	0	0	1	0	4

Hence the pivot element in the first iteration = 3

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Q1) B

5) Maximize $z = 3x_1 + 4x_2 - 2x_3$

Subject to

$6x_1 \cancel{x} 4x_2 \leq 5$

$3x_1 + x_2 + 4x_3 \geq 11$

$4x_1 + 3x_2 \leq 2$

$x_1, x_2, x_3 > 0$

Introducing slack variables,

maximize $z = 3x_1 + 4x_2 - 2x_3 + 0s_1 + 0S_2 + 0S_3$

subject to

$6x_1 - 4x_2 + S_1 + 0S_2 + 0S_3 + 0X_3 = 5$

$3x_1 + x_2 + 4x_3 + 0S_1 - S_2 + 0S_3 = 11$

$4x_1 + 3x_2 + 0x_3 + 0S_1 + 0S_2 + S_3 = 2$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

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Q1 B

6) We have $f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$

$\frac{\partial f}{\partial x_1} = 1 - 2x_1$ $\frac{\partial^2 f}{\partial x_1^2} = -2$

$\frac{\partial f}{\partial x_2} = x_3 - 2x_2$ $\frac{\partial^2 f}{\partial x_2^2} = -2$

$\frac{\partial f}{\partial x_3} = 2 + x_2 - 2x_3$ $\frac{\partial^2 f}{\partial x_3^2} = -2$

remaining second order partial derivative are

$\therefore$ ~~Hesia~~ Hessia matrix H =

$$H = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{vmatrix} = \begin{vmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$D_1 = [-2] = -2$

$D_2 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = 4$

$D_3 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} = -6$

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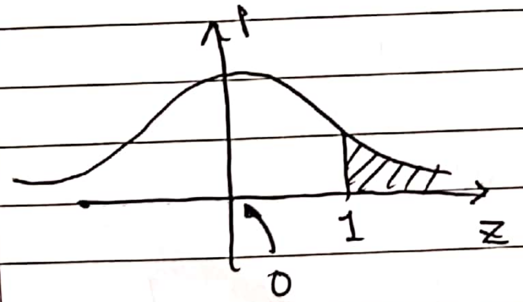
Q2) (A)

i) $n = 500$ students
$\mu = 68$ inches
$\sigma = 4$ inches

Normal distribution,
$$z(x) = \frac{x - \mu}{\sigma} = z(0 < x < X)$$
$$= \frac{x - 68}{4}$$

(i) greater than 72 inches,
Now to calculate z for 72 inches
$$Z = \frac{72 - 68}{4} = \underline{\underline{1}}$$

$z(x > 72) = 1$
$$P(x > 72) = 0.5 - P(0 < x < 1)$$
$$= 0.5 - P(z(1))$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$



No. of student with height > 72 inches
$= 79.33 \approx \underline{79 \text{ students}}$

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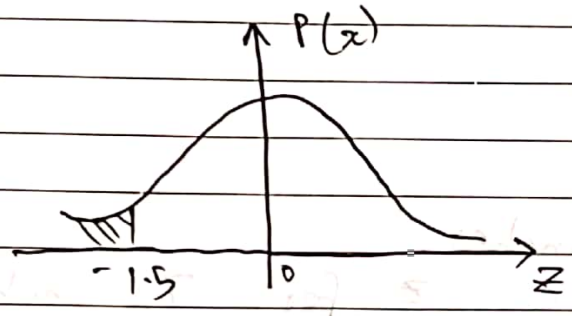
(ii) less than 62 inches

$$z(x<62) = \frac{62-68}{4} = \frac{-6}{4} = -1.5$$

$$\begin{aligned} P(x<62) &= 0.5 - P(0<z<1.5) \\ &= 0.5 - P(z=1.5) \\ &= 0.5 - 0.4332 \\ &= 0.0668 \end{aligned}$$



No. of students with height < 62 inches
= 33.4 ≈ 33 students

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Q2) B

Given : $R = 0.5$, $n = 10$

But $R = 1 - \frac{6 \Sigma d_i^2}{n^3 - n}$

$\therefore \quad 0.5 = 1 - \frac{6 \Sigma d_i^2}{1000 - 10}$

$\Sigma d_i^2 = \frac{495}{6}$

Now,
Correct Σd_i^2 = Incorrect Σd_i^2 - (Incorrect rank diff)^2 + (correct rank diff)^2

$= \frac{495}{6} - 3^2 + 7^2$

$= \frac{735}{6}$

Correct $R = 1 - \frac{\cancel{6} \times 735}{990 \times \cancel{6}}$

$= 1 - \frac{735}{990}$

$= 0.26$

Correct coefficient of rank correlation $= 0.26$

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Q3) (A)

ii) Null hypothesis H_0 would be the proportion of ~~not~~ the beans in the four groups A, B, C, D is the given proportion 9 : 3 : 3 : 1

Alternative hypothesis H_a : The proportion is not as given above

→ Calculation of test statistic :

∵ given hypothesis is 9:3:3:1
∴ 9 + 3 + 3 + 1 = 16 & no. of beans is four groups will be :-

$A = \frac{9}{16} \times 1600 = 900$, $B = \frac{3}{16} \times 1600 = 300$

$C = \frac{3}{16} \times 1600 = 300$, $D = \frac{1}{16} \times 1600 = 100$

$$\therefore \chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{(882-900)^2}{900} + \frac{(313-300)^2}{300} + \frac{(287-300)^2}{300} + \frac{(118-100)^2}{100}$$

$$= 0.36 + 0.56 + 0.56 + 3.24$$
$$= 4.72$$

→ level of significance $\alpha = 0.05$

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→ Degrees of freedom $= n - 1$
$= 4 - 1$
$= 3$.

→ Critical value : for 3 degrees of freedom at 5% level of significance
value = 7.81

→ Decision : Since the calculated value of $x^2 = 4.72$ is less than table value
∴ Null hypothesis is accepted

∴ proportion 9 : 3 : 3 : 1 is correct.

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Q3) B

λ, the rate of arrival = 1/8 per minute

μ, the rate of service = 1/4 per minute

$\therefore \rho = \frac{\lambda}{\mu}$ the traffic intensity,

$= \frac{1/8}{1/4} = \frac{1}{2} \neq 1$

N=3,
Time a car spends at a petrol station $W_s = \frac{L_s}{\lambda(1-P_N)}$

$L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}$

$= \frac{1/2}{1-1/2} - \frac{4(1/2)^4}{1-(1/2)^4}$

$= 0.733$

$W_s = \frac{0.733}{1/8(1-P_N)}$

where $P_N = \rho^N P_0$

$= \left(\frac{1}{2}\right)^3 \left(\frac{1-1/2}{1-(1/2)^4}\right)$

$= 0.0667$

$\therefore W_s = \frac{0.733 \times 8}{1-0.0667} = \underline{6.283 \text{ minutes}}$

Time a car spends at petrol station = 6.283 mins

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Q4) A

We express this problem using $\leq$ in first constraint

$\therefore$ to minimize: $z = 2x_1 + 2x_2 + 4x_3$
subject to : $-2x_1 - 3x_2 - 5x_3 \leq -2$
$3x_1 + x_2 + 7x_3 \leq 3$

Introducing slack variables s_1, s_2 we have :-

Minimize : $z = 2x_1 + 2x_2 + 4x_3 - 0s_1 - 0s_2$
that is $z - 2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 = 0$

subject to $-2x_1 - 3x_2 - 5x_3 + s_1 + 0s_2 = -2$
$3x_1 + x_2 + 7x_3 + 0s_1 + s_2 = 3$

Simplex table

Iteration Number	Basic var	x_1	x_2	x_3	s_1	s_2	R.H.S. soln
0	z	-2	-2	-4	0	0	0
s_1 leaves	s_1	-2	-3	-5	1	0	-2
x_2 enters	s_2	3	1	7	0	1	3
Ratio		1	$2/3$	$4/5$			
			$\uparrow$				
1	z	$-2/3$	0	$-2/3$	$-2/3$	0	$4/3$
	x_2	$2/3$	1	$5/3$	$-1/3$	0	$2/3$
	s_2	$7/3$	0	$16/3$	$1/3$	1	$7/3$

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$\therefore$ $x_1 = 0$, $x_2 = \frac{2}{3}$, $x_3 = 0$

$Z_{min} = \frac{4}{3}$

We stop here as all values in RHS column are positive and all values from z row are negative or 0.

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Q4) (B)

The lagrangian function is

$L(x_1, x_2, \lambda) = (6x_1 + 8x_2 - x_1^2 - x_2^2) - \lambda(4x_1 + 3x_2 - 16)$

$\therefore \frac{\partial L}{\partial x_1} = 6 - 2x_1 - 4\lambda$

$\frac{\partial L}{\partial x_2} = 8 - 2x_2 - 3\lambda$

$\frac{\partial L}{\partial \lambda} = -4x_1 - 3x_2 + 16$

Solving the equations $\frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0, \frac{\partial L}{\partial \lambda} = 0$

$\therefore 6 - 2x_1 - 4\lambda = 0$
$8 - 2x_2 - 3\lambda = 0$
$-4x_1 - 3x_2 + 16 = 0$

Solving these eqns, we get,

$x_1 = \frac{43}{25}, \quad x_2 = \frac{76}{25}, \quad \lambda = \frac{16}{25}$

Hence X_0 is $\left(\frac{43}{25}, \frac{76}{25}\right)$

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Hence, $h(x_1, x_2) = 4x_1 + 3x_2 - 16 = 0$

$\therefore \frac{\partial h}{\partial x_1} = 4$ and $\frac{\partial h}{\partial x_2} = 3$

all other partial derivatives are $= 0$

$f(x_1, x_2) = 6x_1 + 8x_2 - x_1^2 - x_2^2$

$\frac{\partial f}{\partial x_1} = 6 - 2x_1$, $\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$,

$\frac{\partial^2 f}{\partial x_1^2} = -2$, $\frac{\partial f}{\partial x_2} = 8 - 2x_2$, $\frac{\partial^2 f}{\partial x_2 \partial x_1} = 0$

$\frac{\partial^2 f}{\partial x_2^2} = -2$

$$\Delta_3 = \begin{vmatrix} 0 & \partial h/\partial x_1 & \partial h/\partial x_2 \\ \partial h/\partial x_1 & \partial^2 f/\partial x_1^2 - \lambda \partial^2 h/\partial x_1^2 & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \frac{\lambda \partial^2 h}{\partial x_1 \partial x_2} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \frac{\lambda \partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \frac{\lambda \partial^2 h}{\partial x_2^2} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 4 & 3 \\ 4 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 3 & 0 \end{vmatrix}$$

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$= 32 + 18$
$= 50$

$\because$ Δ_3 is positive, X_0 is maxima

$$Z_{max} = 6\left(\frac{43}{25}\right) + 8\left(\frac{76}{25}\right) - \left(\frac{43}{25}\right)^2 - \left(\frac{76}{25}\right)^2$$
$$= \frac{866}{25} - \frac{7625}{625}$$
$$= \frac{561}{25}$$

Hence,
$x_1 = \frac{43}{25}$, $x_2 = \frac{76}{25}$, $Z_{max} = \frac{561}{25}$

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