Fracciones ...

$$X_{(s)} = \frac{2s^3+8s^2+4s+8}{s(s+1)(s^2+4s+8)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{A}{s+2+j2} + \frac{A^*}{s+2-j2}$$

$$K_1 = sX_{(s)}\Big|_{s=0} = \frac{s(2s^3+8s^2+4s+8)}{s(s+1)(s^2+4s+8)} = \frac{8}{8} = 1$$

$$K_2 = -2$$

$$A = \frac{3}{2} + j\frac{1}{2}$$

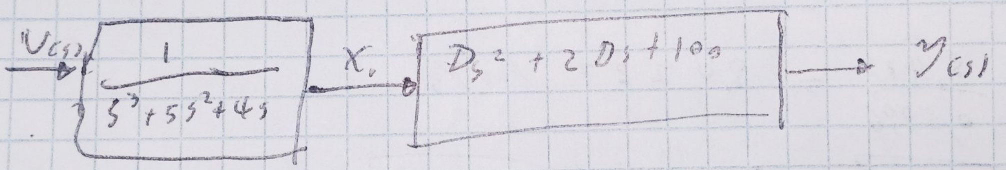
$$A^* = \frac{3}{2} - j\frac{1}{2}$$

$$X_{(s)} = \frac{1}{s} - \frac{2}{s+1} + \frac{3/2 + j\,1/2}{s+2+j2} + \frac{3/2 - j\,1/2}{s+2-j2}$$

Ejemplo diseño

Diseñar sistema de control para

$$G_{(s)} = \frac{20(s+5)}{s(s+1)(s+4)}, \quad \text{con } D_s = 9.3\% \text{ y } t_s = 0.74 \text{ seg}$$



$$\frac{X_{1(s)}}{U_{(s)}} = \frac{1}{s^3+5s^2+4s} \qquad \frac{y_{(s)}}{X_{1(s)}} = 20s+100$$

$$[s^3+5s^2+4s]\,X_{1(s)} = U_{(s)} \qquad y_{(s)} = X_{1(s)}[20s+100]$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u \qquad y = 20\dot{X}_1 + 100X_1$$

$$q_1 = x_1$$

$$q_2 = \dot{q}_1$$

$$q_3 = \dot{q}_2$$

$$\dot{q}_3 = \dddot{x}_1$$

$$\dot{q}_3 = u - 5q_3 - 4q_2$$

$$y = 20q_2 + 100q_1$$

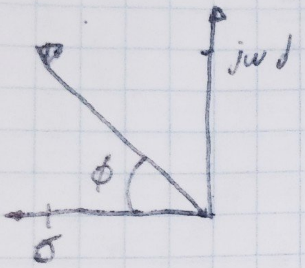
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$D_S\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 = 9.5$$

$$\zeta \cong 0{,}5996$$

$s = \sigma + j w_d$ donde $\sigma = \zeta w_n$



$$\zeta = \cos\phi$$

$$\phi = \cos^{-1}(0{,}5996) = 53{,}16^\circ$$

$$t_s = \frac{4}{\sigma} = 0{,}74 \rightarrow \sigma = 5{,}41$$

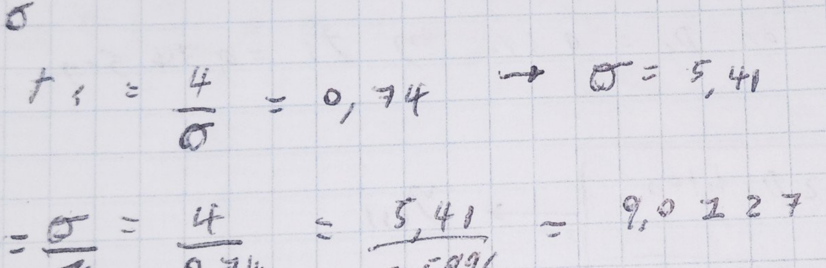
$$w_n = \frac{\sigma}{\zeta} = \frac{4}{0{,}74} = \frac{5{,}41}{0{,}5996} = 9{,}0227 \text{ rad/s}$$

$$w_d = w_n\sqrt{1-\zeta^2} = 9{,}0227\sqrt{1-0{,}5996^2} = 7{,}2207 \text{ rad/s}$$

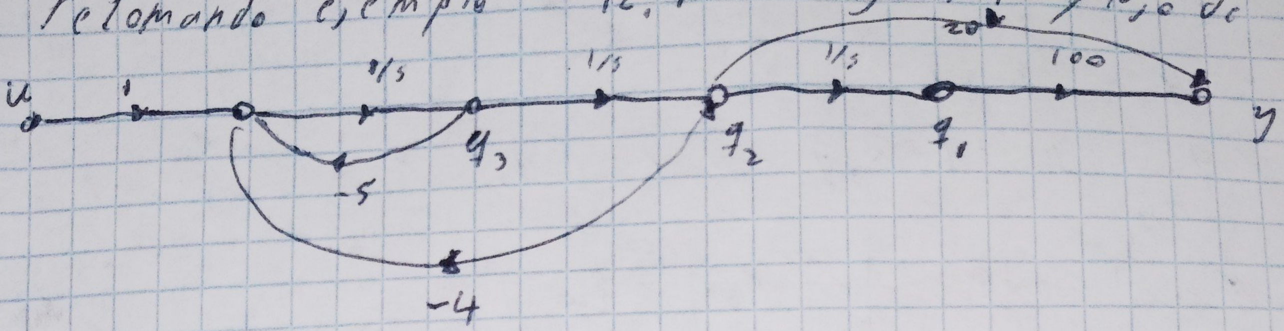
$$w_d = \sigma \tan\phi = 5{,}41 \tan(53{,}16) = 7{,}2212$$

$$\dot{x} = Ax + Bu$$

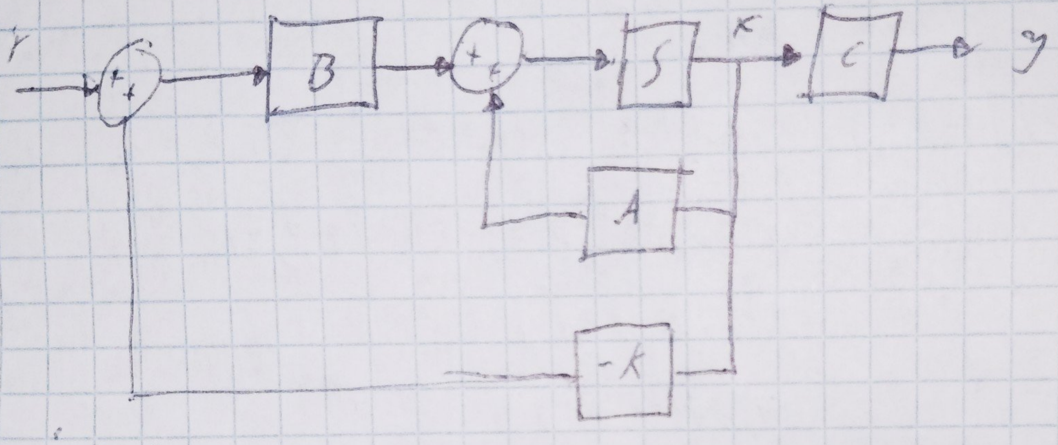
$$y = Cx$$



retomando ejemplo 12.1 diagrama flujo de señal



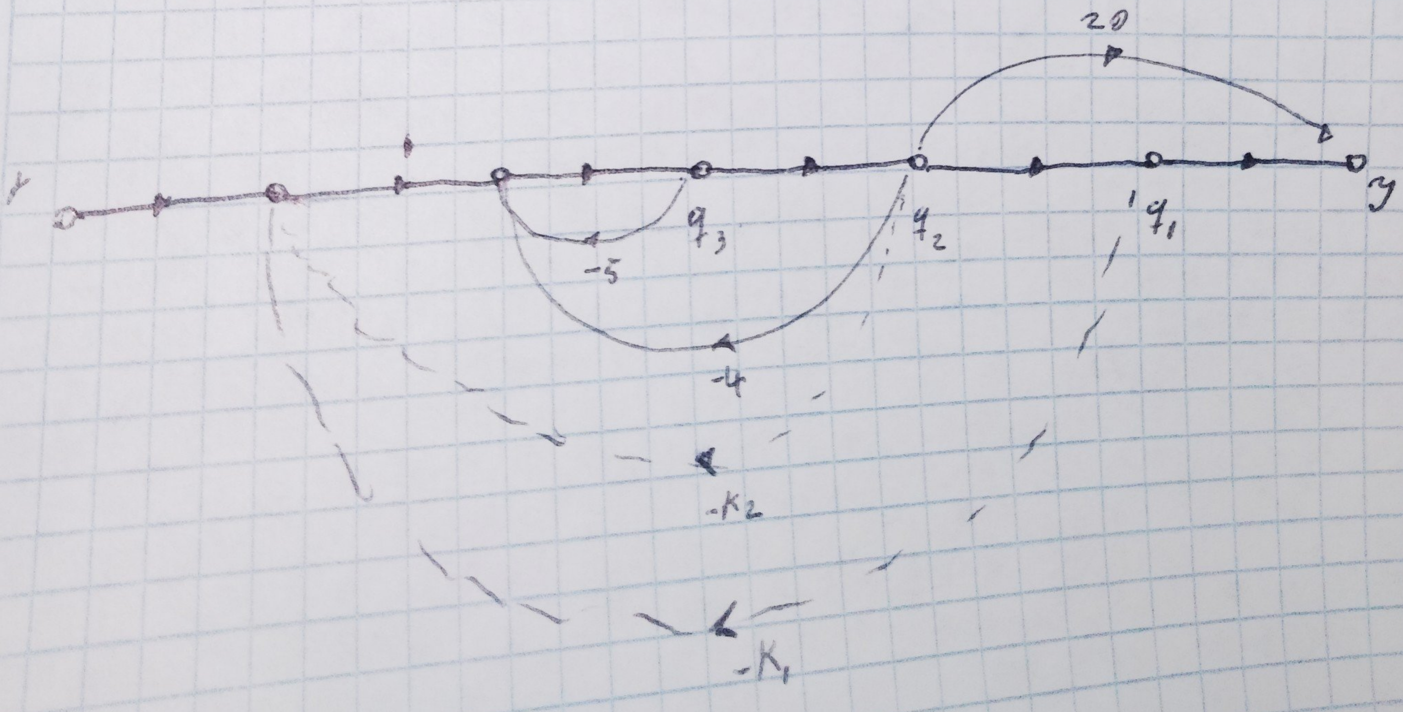
Redibujando diagrama de bloques



$\dot{x} = Ax + Bu$

$\dot{x} = Ax + B(r - Kx)$

$\dot{x} = (A - BK)x + Br$



se reescribe el espacio de estados

- $\dot{q}_3 = -5 \cdot q_3 - 4 q_2 + u$

$\dot{q}_3 = -5 q_3 - 4 q_2 + (-k_3 q_3 - k_2 q_2 - k_1 q_1) + r$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -k_1 & -4-k_2 & -5-k_3 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

Ecuación característica del sistema:

$Det\,[sI - (A - Bk)] = s^3 + (5 + k_3)\, s^2 + (4 + k_2\, s^2 + k_1 s = \emptyset$

Polo dominante en lazo cerrado: $s = -5,4 + j\,7,21$

Cero de la planta en $s\,5$; con $s = 5,1$

$(s + 5,4 - j\,7,2)(s + 5,4 + j\,7,2)(s + 5,1)$

$s^3 + 15,9 s^2 + 136,22 s + 413,83 = \emptyset$

Se compara:

$k_3 = 15,9 - 5 = 10,9$

$k_2 = 136,22 - 4 = 132,22$

$k_1 = 413,83$