



pendulo:

$$x_g = x + l \sin \theta$$

$$y_g = l \cos(\theta)$$

para la varilla:

$$I \ddot{\theta} = V l \sin \theta - H l \cos \theta$$

$$H = M \ddot{x} + m l \frac{d(\dot{\theta} \cos \theta)}{dt}$$

$$H = m \ddot{x} + m l [\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta]$$

$$V - mg = m \frac{d^2[l \cos \theta]}{dt^2}$$

$$V - mg = m l \frac{d[\dot{\theta} \sin \theta]}{dt} = -m l \frac{d[\dot{\theta} \sin(\theta)]}{dt}$$

$$V - mg = -m l [\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta]$$

carta:

$$M a = M \ddot{x} = u - H$$

linearization si  $\theta \ll 1$  radian =  $\begin{cases} \sin \theta = \theta \\ \cos \theta = 1 \\ \dot{\theta} \dot{\theta} = 0 \end{cases}$

Ecuaciones

$$(1) I \ddot{\theta} = V l \theta - H l$$

$$(2) H = m \ddot{x} + m l \ddot{\theta}$$

$$(3) V = mg$$

despejando:

$$M \ddot{x} = u - H = u - m \ddot{x} - m l \ddot{\theta}$$

$$u = \ddot{x} (M + m) + m l \ddot{\theta} \quad (4)$$



$$I \ddot{\theta} = V_L \theta - H_L$$

$$I \ddot{\theta} = m l g \theta - l (m \ddot{x} + m l \ddot{\theta})$$

$$m l g \theta = \ddot{\theta} (I + m l^2) + m l \ddot{x}$$

$$x_1 = x$$

$$x_3 = \theta$$

$$x_2 = \dot{x}_1$$

$$x_4 = \dot{x}_3$$

$$\dot{x}_2 = \ddot{x}_1$$

$$\dot{x}_4 = \ddot{x}_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+ml)g}{ml} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{ml} \end{bmatrix} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$