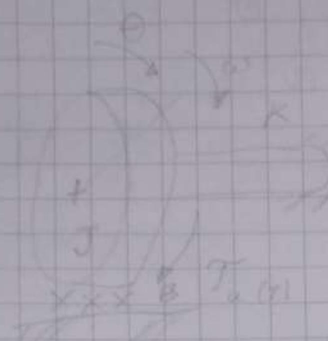
Alumno: Miguel Angel Henao Prada				Código	Materia
Curso	Bimestre	Semestre	Salón	Hoja No ___ de ___	CALIFICACIÓN
Profesor: cod. 20191005163					

Parcial #2
Sistemas Dinamicos

(1)



$$J\ddot{\theta} + b\dot{\theta} + K\theta = \tau$$

$$q_1 = \theta$$
$$q_2 = \dot{q}_1 = \dot{\theta}$$
$$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$$

$$J\dot{q}_2 + B\dot{q}_1 + Kq_1 = \tau \rightarrow \dot{q}_2 = \frac{\tau}{J} - \frac{K \cdot q_1}{J} - \frac{B\dot{q}_1}{J} \quad (1)$$

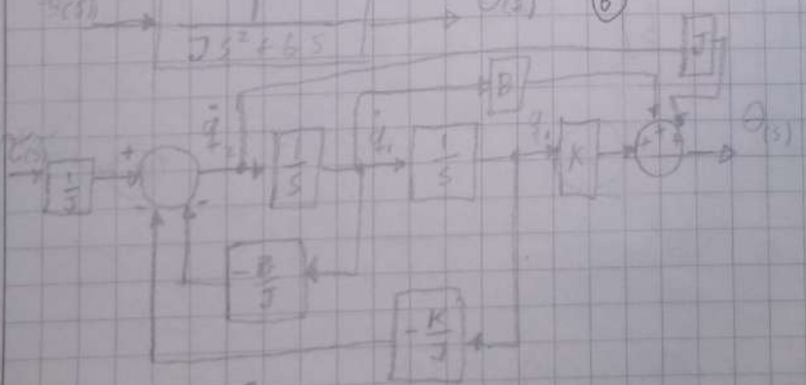
$$\left.\begin{aligned} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix}\tau \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \end{aligned}\right\} a)$$

$$Js^2\theta + bs\theta = \tau \rightarrow \theta[Js^2 + bs] = \tau$$

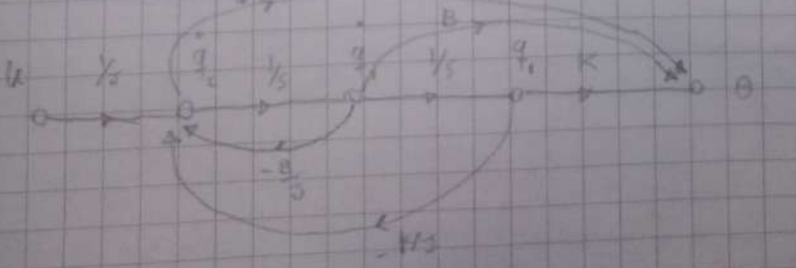
$$\left.\frac{\theta}{\tau} = H_{(s)} = \frac{1}{Js^2 + bs}\right\} b)$$

(b)

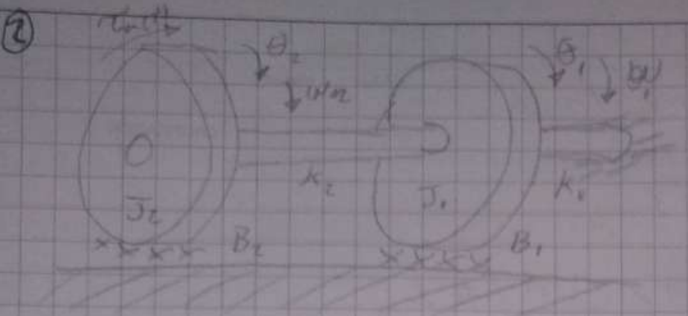
$$\tau_{(s)} \rightarrow \boxed{\frac{1}{Js^2+bs}} \rightarrow \theta_{(s)}$$



(c)



2



$$J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 + K_1 \theta_1 = K_2 (\theta_2 - \theta_1) \quad ①$$

$$J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 + K_2 (\theta_2 - \theta_1) = \tau \quad ②$$

$$q_1 = \theta_1 , \quad q_2 = \dot{q}_1 , \quad q_3 = \theta_2 , \quad q_4 = \dot{\theta}_2 = \dot{q}_3$$

$$J_1 \dot{q}_2 = K_2 q_3 - K_2 q_1 - B_1 q_2 - K_1 q_1$$

$$\dot{q}_2 = -\frac{q_1 (K_2 + K_1)}{J_1} - \frac{B_1 q_2}{J_1} + \frac{K_2 q_3}{J_1} \quad *$$

$$\dot{q}_4 = \frac{K_2}{J_2} q_1 - \frac{K_2}{J_2} q_3 - \frac{B_2}{J_2} q_4 + \frac{\tau}{J_2} \quad *$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1 + K_2}{J_1} & -\frac{B_1}{J_1} & \frac{K_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & 0 & -\frac{K_2}{J_2} & -\frac{B_2}{J_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} \tau$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad b)$$

$$J_2 s^2 \theta_2 + B_2 s \theta_2 + K_2 (\theta_2 - \theta_1) = \tau$$

$$J_1 s^2 \theta_1 + B_1 s \theta_1 + K_1 \theta_1 = K_2 \theta_2 - K_2 \theta_1$$

$$\Rightarrow \theta_1 = \frac{K_2 \theta_2}{J s^2 + B_1 s + K_1 + K_2}$$

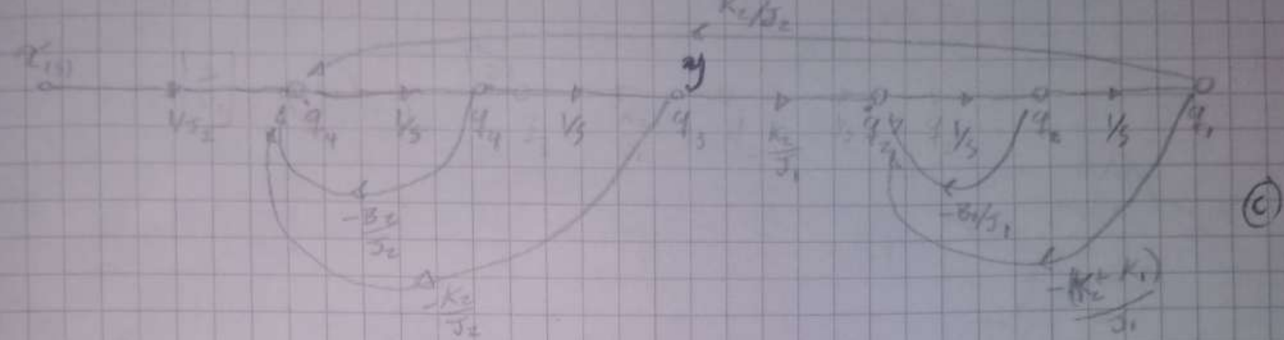
Reemplazando:

$$\theta_2 \left[J_2 s^2 + B_2 s + K_2 - \frac{K_2}{J_1 s^2 + B_1 s + K_1 + K_2} \right] = \tau$$

$$\Rightarrow H_{(s)} = \frac{1}{J_2 s^2 + B_2 s + K_2 - \frac{K_2^2}{J_1 s^2 + B_1 s + K_1 + K_2}} \quad a)$$

$$\tau_{(s)} \longrightarrow \boxed{\dfrac{1}{J_2 s^2 + B_2 s + K_2\left(1 - \dfrac{K_2^2}{J_1 s^2 + B_1 s + K_1 + K_2}\right)}} \longrightarrow \theta_{2(s)}$$

(b)



(c)

(3)

$$\dot{q}_2 = \frac{K_2}{J_1} q_3 - \frac{K_2}{J_1} q_1 - \frac{B_1}{J_1} q_2 \quad (1)$$

$$\dot{q}_4 = \frac{\tau}{J_2} + \frac{K_2}{J_2} q_1 - \frac{K_2}{J_2} q_3 - \frac{B_2}{J_2} q_4 \quad (2)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_2}{J_1} & -\frac{B_1}{J_1} & \frac{K_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & 0 & -\frac{K_2}{J_2} & -\frac{B_2}{J_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} \tau$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

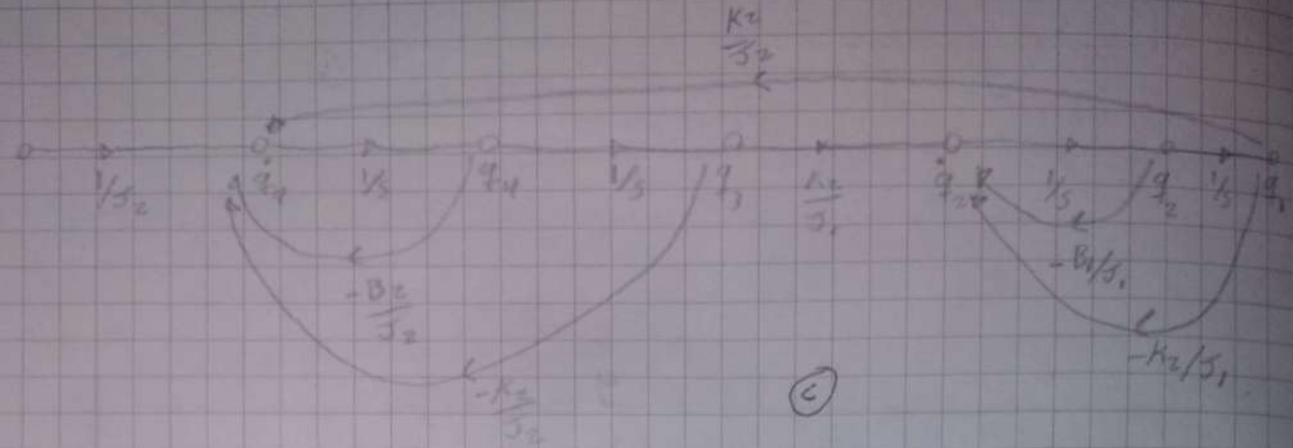
b)

$$H_{(s)} = \frac{1}{J_2 s^2 + B_2 s + K_2 - \dfrac{K_2^2}{J_1 s^2 + B_1 s + K_2}}$$

a)

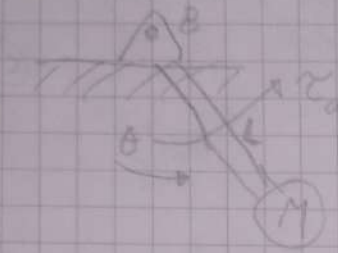
$$\tau_{(s)} \longrightarrow \boxed{\dfrac{1}{J_2 s^2 + B_2 s + K_2 - \dfrac{K_2^2}{J_1 s^2 + B_1 s + K_2}}} \longrightarrow \theta_{2(s)}$$

(b)



c)

4)



$$\ddot{\theta} = \frac{\tau}{ML^2} - \frac{B}{ML^2}\dot{\theta} + \frac{g}{L}\,\text{sen}\,\theta$$

Se asumen oscilaciones pequeñas para linealizar $\Rightarrow \text{sen}\,\theta = \theta$

$$\ddot{\theta} = \frac{\tau}{ML^2} - \frac{g}{L}\theta - \frac{B}{ML^2}\dot{\theta} \; *$$

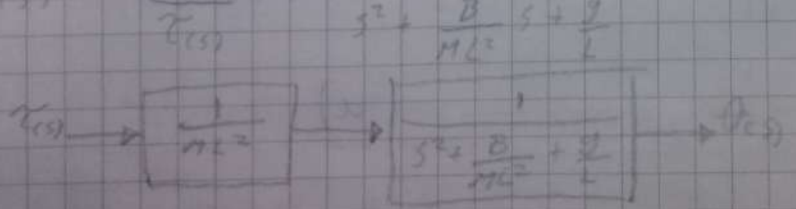
$q_1 = \theta$

$q_2 = \dot{q}_1$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & -\frac{B}{ML^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ML^2} \end{bmatrix} \tau \qquad b)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$H_{(s)} = \frac{\theta_{(s)}}{\tau_{(s)}} = \frac{\frac{1}{ML^2}}{s^2 + \frac{B}{ML^2}s + \frac{g}{L}} \qquad a)$$



c)