Control System Design of a Continuous-Flow Stirred Tank Reactor (CSTR)

Important note: The due date is 17/11/2024. Late submission is absolutely not allowed as the grades have to be submitted to the department very soon after the final exam. You may work together with your classmates. But do write your report independently. And the results are supposed to be different from each other as the parameters are based upon your matriculation numbers.

1 Background

This is a real control problem in chemical engineering, where we are trying to control the reaction process that takes place in a tank shown in the following figure [1]. The chemical reaction is described by $A \rightarrow B$. A and B stand for the reactant and product. The reaction is conducted in the large container located at the central of the tank, meanwhile there is water flow inside the surrounded container wall (we call it cooling jacket) to control the reaction temperature.

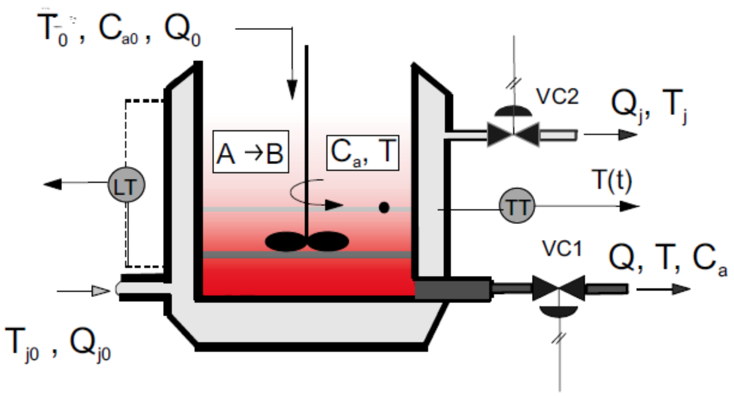

Figure 1. CSTR reactor [1]

The real plant is a non-linear, time delayed, time-varying system associated with variety of inputs and outputs. However, for rough analysis, the system could be simplified so that it becomes a 3rd order LTI state-space model with 2 inputs and 3 outputs, as is presented in [1]. Now we are making further assumption that the component concentration sensor is missing, which often happens in some small factories since the component concentration sensor is very expensive and hard to install, the plant becomes a 2-input-2-output LTI system.

This is an interesting application. You are supposed to control the component concentration of the reactant. But you cannot measure it directly because the sensor is too expensive and you want to keep the cost as low as possible. Can you solve the control problem with the cheap temperature sensors? If you are confident that the model is accurate enough, then you can do it with the knowledge you have learned from this course.

2 Modeling

For model-based control, the first step is to build an effective dynamic model for our target plant, i.e., the continuous-flow stirred tank reactor (CSTR) in this project. Usually, there are two classes of methods to build a dynamic model, either by first principles or by system identification. For many complex process control applications, it is typically difficult, laborious, and time-consuming to derive a dynamic model from ~~first principle equations~~. Besides, the acquired non-linear system also requires a significant time for calibration and verification, as well as more advanced control technologies.

In this project, the most important factors that we want to control is the component concentration of the reactant during reaction and the reaction temperature. However, as aforementioned, only temperature sensors are available. There are 2 sensors, one installed in the reaction container to detect the reaction temperature, the other installed in the cooling jacket outlet pipe to detect the outflow water temperature.

Notice that the input amount of the reactant and the temperature of the input water to the cooling jacket are fixed. Our objective is to use the 2 valves VC1 and VC2, shown in Figure 1 to control the outlet flow rate of the reaction and the flow rate of the water coming out from the cooling jacket in order to maintain the whole reaction on the given operating point.

Define the state vector $x = [C_a \; T \; T_j]^T$, where C_a is the component concentration of the reactant, T is the reaction temperature in the tank, and T_j is the temperature of the outflow water of the cooling jacket. Define the control signal $u = [F \quad F_j]^T$, where F is the outlet flow rate of the reaction; F_j is the flow rate of the water in the cooling jacket. Define the measurement vector $y = [T \quad T_j]^T$, because we only have two temperature sensors. The system is described by

$$\dot{x} = Ax + Bu + Bw,$$

$$y = Cx$$

where

$$A = \begin{bmatrix} -1.7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -200 - ab0 & -220 - ba0 \end{bmatrix}, \tag{1}$$

$$B = \begin{bmatrix} 3 + a & 0 \\ -30 - dc & 0 \\ 0 & -420 - cd0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where the vector w describes some possible load disturbances. The initial condition of the system is assumed to be

$$x_0 = [1 \quad 100 \quad 200]^T. \tag{2}$$

The parameter values of a, b, c, d in equation (1) should be chosen to be the last four digits of your matriculation number. For example, if your matriculation number is A0235004X, then $a = 5$, $b = 0$, $c = 0$, $d = 4$. In this case, we have

$$\mathrm{A} = \begin{bmatrix} -1.7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -200 - 500 & -220 - 50 \end{bmatrix}, \quad B = \begin{bmatrix} 3 + 5 & 0 \\ -30 - 40 & 0 \\ 0 & -420 - 40 \end{bmatrix}. \tag{3}$$

3 Control System Design

After all, we get a linear state space model (1) for the CSTR system. In the following, different control strategies will be explored to achieve control of this system. We will target both the regulation and set point tracking problems.

3.1 Design specifications

The transient response performance specifications for all the outputs y in state space model (1) are as follows:

1) The overshoot is less than 10%.
2) The 2% settling time is less than 30 seconds.

Note: (a) This transient response is checked by giving a step reference signal for each input channel, i.e., [1, 0] and [0, 1], with ZERO initial conditions; (b) For all the following task 1) to 5), your control system should first satisfy this performance specification and then you are supposed to finish the required investigation for each task as well.

3.2 Tasks

Your study should include, but not limited to

1) Assume that you can measure all the three state variables, design a state feedback controller using the pole place method, simulate the designed system and show all the three state responses to non-zero initial state x_0 with zero external inputs. Discuss effects of the positions of the poles on system performance and monitor control signal size. In this step, both the disturbance and set point can be assumed to be zero. (10 points)
2) Assume that you can measure all the three state variables, design a state feedback controller using the LQR method, simulate the designed system and show all the state responses to non-zero initial state with zero external inputs. Discuss effects of weightings Q and R on system performance and monitor control signal size. In this step, both the disturbance and set point can be assumed to be zero. (10 points)
3) Assume you can only measure the two outputs in y. Design a state observer, simulate the resultant observer-based LQR control system, monitor the state estimation error, investigate effects of observer poles on state estimation error and closed-loop control performance. In this step, both the disturbance and set point can be assumed to be zero. (10 points)
4) Assume that you can measure all the three state variables, design a decoupling controller with closed-loop stability and simulate the step response of the resultant control system to verify decoupling performance with stability. In this question, the disturbance can be assumed to be zero. Is the decoupled system internally stable? Please provide both the step (transient) response with zero initial states and the initial response with respect to x_0 of the decoupled system to support your conclusion. (10 points)
5) In an application, the operating set point for the two outputs is

$$y_{sp} = [100, 150]^T.$$

Assume that you only have two cheap sensors to measure the output. Design a controller such that the plant can operate around the set point as close as possible at steady state even when step disturbances are present at the plant input. Plot out both the control and output signals. In your simulation, you may assume the step disturbance $w = [-2,\ 5]^T$ takes effect from time $t_d = 10s$ afterwards. (10 points)

6) To make this task more interesting, suppose we intend to manipulate the three state variables directly instead of the two outputs. Our target is to maintain the states x around a given set point $x_{sp} = [5,\ 250,\ 300]^T$ at steady state starting from the initial state x_0. Is it possible? In this question, you may assume that all the state variables can be measured directly and there are no disturbances. (10 points)

   (a) If your answer is YES, please detail your control system design strategy to ensure x to be x_{sp} at steady state and demonstrate its effectiveness through simulation.

   (b) If your answer is NO, explain why. In such a case, we may only want to keep the state variables at steady state close enough to the set point x_{sp}. However, in practice, we usually place different emphasis on the exactness of the three state variables. To address our purpose quantitively, we aim to minimize the following objective function

$$J(x_s) = \frac{1}{2}(x_s - x_{sp})^T W (x_s - x_{sp}), \tag{4}$$

   where $W = diag(a + b + 1,\ c + 4,\ d + 5)$ is a diagonal weight matrix and x_s is the state vector at steady state. Here, *a*, *b*, *c*, *d* are still the last four digits in your matriculation number, as defined above. Please detail your control system design strategy to minimize the objective $J(x_s)$ at steady state and demonstrate its effectiveness through simulation. Can you prove that $J(x_s)$ at steady state is indeed minimized theoretically in your design?

Note that there are no unique answers to the above design questions. For the tasks in our project, you can assume that the control input is unlimited. However, in practice all the physical actuators can only provide a limited drive capacity. You need to make your own judgement assuming you are the engineer responsible for the control system design in the real world. There are three major factors you should consider when you design and justify your controller:

- Speed --- Transient response
- Accuracy --- Steady state error
- Cost ---- Size of the control signals

Please do follow the design procedures you have learned in *linear systems* to solve all the above questions. List the necessary formulas and intermediate results in your report. If you only call the MATLAB built-in functions for control system design with no details, for example, simply use *place* to place poles or *lqr* to design the LQR regulator, you will get ZERO marks.

4 Reference

[1] P. Albertos, A.Sala, Multivariable Control Systems: An Engineering Approach, pp 19-27

5 Format of Reports

Your report should mainly contain the plant description, control and observer design method description, your design details, simulation results, possible comparison, comments and discussion, modification and refinements.

The report should include the following and be organized in the following sequence:

- A cover paper to indicate "Assignment for EE5101/ME5401 (or your specialization code if else) Linear Systems", a title of your report at your choice, your full name, your Matriculation number, email address and date;
- An abstract of 50-100 words on a separate page;
- A contents table on a separate page;
- Section 1 Introduction
- The major materials of your report organized nicely in a few sections each with specific focus. Label your equations, tables, and figures with number and caption for reference in the text. Your figure size and figure quality should be high enough to facilitate the verification of your results.
- The last section on conclusions.
- A list of reference books/papers if any;
- Appendices if any each on a separate page. Your MATLAB code should be in this appendix. If you use Simulink, a screenshot of your Simulink model should be inserted at proper position in the above major materials part as figures.

Pay attention to your presentation (English writing, organization, and layout et al). Make the report formal, complete and readable. It is also advisable to write your report with a word-processing software such as Word or LaTeX.

The final point to note about your report: it is the content that matters not the length. Keep in mind that there are only TWENTY SEVEN pages in John Nash's PhD thesis, which leaded to his Nobel Prize. Therefore, you will be penalized if you put too much "copy and paste" material in your report.

6 A Note on Access and Use of MATLAB

To complete the project, you are supposed to use SIMULINK and MATLAB. The easy way is to learn how to build various block diagrams in SIMULINK first, and then try to solve the control systems design for the mini-project. An excellent *Control Tutorial for MATLAB and Simulink* can be found at http://ctms.engin.umich.edu/CTMS/index.php?aux=Home. Besides, a Matlab manual is provided in CANVAS for the first timers.

If you don't have MATLAB on your PC currently, you can access MATLAB in either of the following two ways:

1) Go to PC clusters located at the third floor of E2: http://www.eng.nus.edu.sg/eitu/pc.html.

2) Download MATLAB from NUS information technology center: every NUS student can have a license. https://nusit.nus.edu.sg/services/software_and_os/software/software-student/#install-matlab.

Hint on MATLAB/SIMULINK:

A. You can use functions such as *step*, *initial* and *lsim* to simulate the system's corresponding response. Also, all these simulations can be done with SIMULINK.

B. In some cases, it may be easier to use SIMULINK for the simulation, for example, question 5.