AE8803: Machine Learning with Gaussian Processes

Discrete distributions

Discrete distributions

Code

Scope

This notebook is has useful boiler plate code for generating distributions and visualizing them.

▼ Code

```
import numpy as np
from scipy.stats import bernoulli, binom
import matplotlib.pyplot as plt
import seaborn as sns
from scipy.special import comb
sns.set(font_scale=1.0)
sns.set_style("white")
sns.set_style("ticks")
palette = sns.color_palette('deep')
#plt.style.use('dark_background') # cosmetic!
```

Bernoulli

The probability mass function for a Bernoulli distribution is given by

$$p\left(x\right)=\begin{cases}1-p & \text{if } x=0\\ p & \text{if } x=1\end{cases}$$

for $x \in \{0,1\}$ and where $0 \leq p \leq 1$.

▼ Code

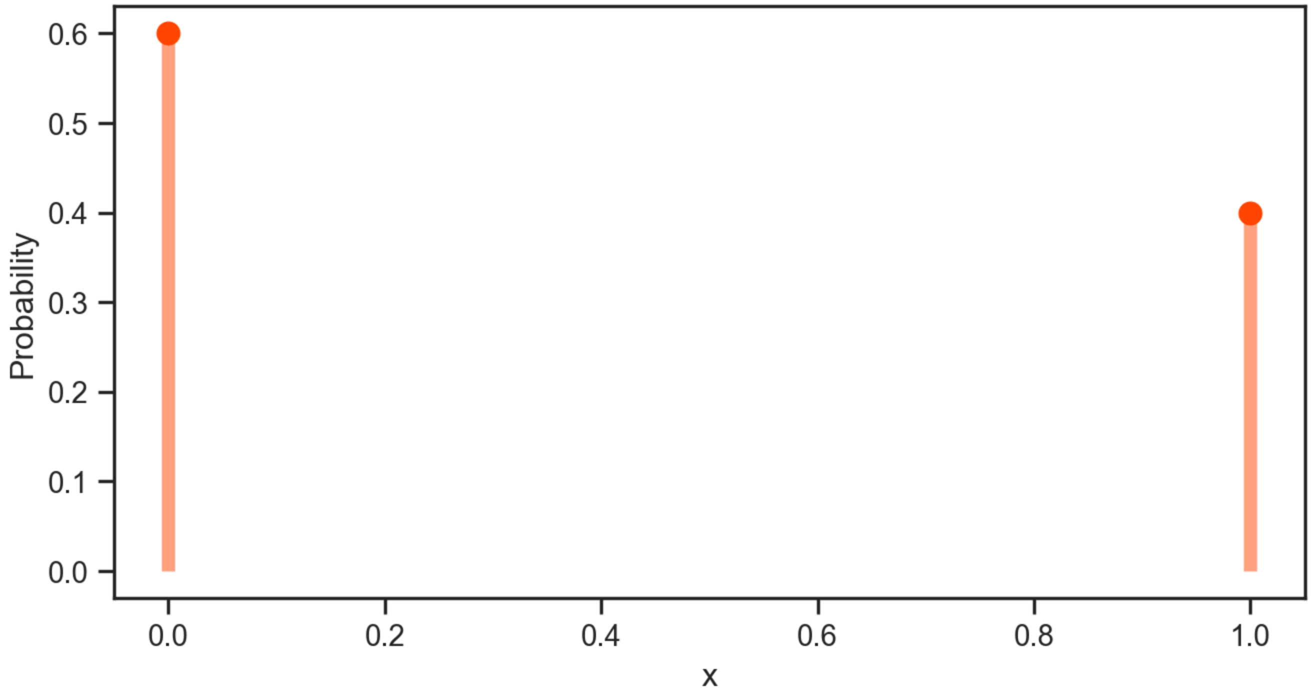
```
p = 0.4 # Bernoulli parameter
x = np.linspace(0, 1, 2)
probabilities = bernoulli.pmf(x, p)

fig = plt.figure(figsize=(8,4))

plt.plot(x, probabilities, 'o', ms=8, color='orangered')
plt.vlines(x, 0, probabilities, colors='orangered', lw=5, alpha=0.5)
```

```
plt.xlabel('x')
plt.ylabel('Probability')
plt.savefig('pdf.png', dpi=150, bbox_inches='tight', transparent=True)

plt.show()
```



One can generate random values from this distribution, i.e.,

▼ Code

```
X = bernoulli.rvs(p, size=500)
print(X)
```

```
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0 0
 0 1 1 0 1 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 0
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 0 1 0 0 0 1 0 0 1 0 1 0 1 1 1 1 1 0 0 1 0 0 1 1 0 1 0 0 1 0 0 0 1 0 0 0 1
 0 0 0 1 1 0 0 0 0 1 1 0 1 1 0 1 0 0 1]
```

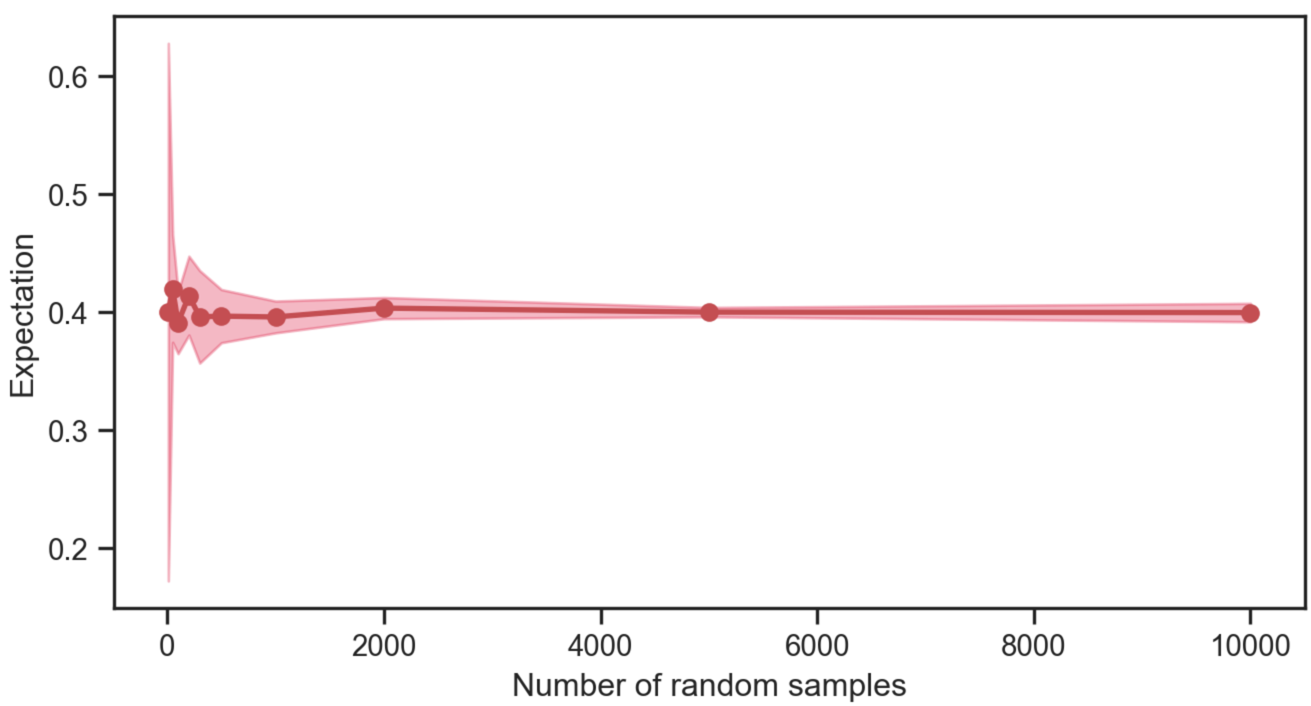
Thus, random values from a Bernoulli distribution are inherently binary, and the number of 0s vs 1s will vary depending on the choice of the parameter, p. We will see later on (in another notebook) how this relatively simple idea can be used to train a *Naive Bayes Classifier*. For now, we will plot the expected value of the Bernoulli random variable with increasing number of samples.

▼ Code

```
numbers = [10, 50, 100, 200, 300, 500, 1000, 2000, 5000, 10000]
means = []
stds = []
for j in numbers:
    X_val = []
    for q in range(0, 10):
        X = bernoulli.rvs(p, size=j)
        X_val.append(np.mean(X))
    means.append(np.mean(X_val))
    stds.append(np.std(X_val))

means = np.array(means)
stds = np.array(stds)
numbers = np.array(numbers)

fig = plt.figure(figsize=(8,4))
plt.plot(numbers, means, 'ro-', lw=2)
plt.fill_between(numbers, means + stds, means - stds, color='crimson', alpha=0.3)
plt.xlabel('Number of random samples')
plt.ylabel('Expectation')
plt.savefig('convergence.png', dpi=150, bbox_inches='tight', transparent=True)
plt.show()
```



Binomial

Next, we consider the Binomial distribution. It has a probability mass function

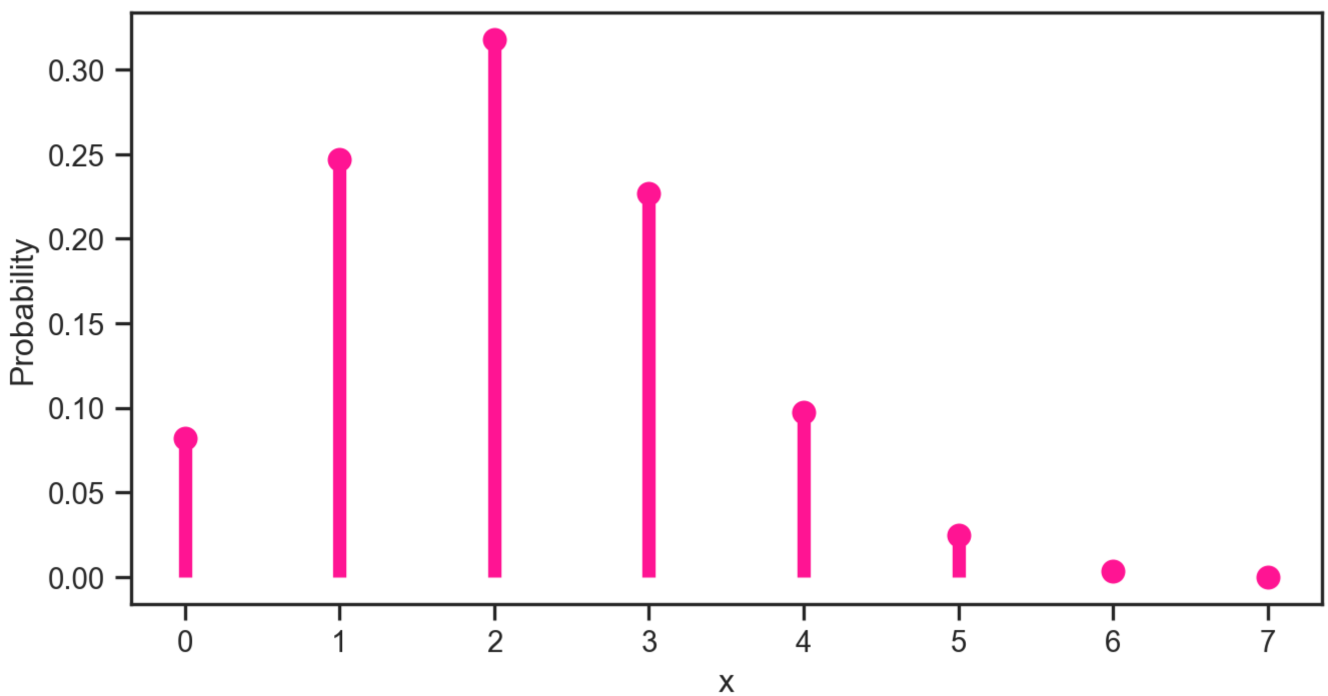
$$p\left(x\right) = \binom{n}{x} p^{x} \left(1 - p\right)^{n-x}$$

for $x \in \{0, 1, \ldots, n\}$ and where $0 \leq p \leq 1$.

▼ Code

```
p = 0.3 # Bernoulli parameter
n = 7
x = np.arange(0, n+1)
probabilities = binom(n, p)

fig = plt.figure(figsize=(8,4))
plt.plot(x, probabilities.pmf(x), 'o', ms=8, color='deeppink')
plt.vlines(x, 0, probabilities.pmf(x), colors='deeppink', lw=5 )
plt.xlabel('x')
plt.ylabel('Probability')
plt.savefig('pdf_2.png', dpi=150, bbox_inches='tight', transparent=True)
plt.show()
```



To work out the probability at $x = 3$, we can compute:

▼ Code

```
prob = comb(N=n, k=3) * p**3 * (1 - p)**(n - 3)
print(prob)
```

```
0.22689449999999992
```