#7-Q4 cont'd

per Lec. 6 notes, the affine solution to a linear estimator (where the random variables do not have a 0-mean), we have the following scenario:

$$\hat{x} = \mu_x + R_{xy} R_y^{-1}(y - \mu_y)$$

So, compute it:

$$R_{xy} = \mathbb{E}((x-\mu_x)(y-\mu_y)) = \mathbb{E}((x-\mu_x)((x+v) - \mathbb{E}(x+v)))$$
$$= \mathbb{E}((x-\mu_x)((x+v) - \mu_x - \mu_v))$$
$$= \mathbb{E}((x-\mu_x)((x-\mu_x) + (v-\mu_v)))$$
$$= \mathbb{E}((x-\mu_x)(x-\mu_x) + (x-\mu_x)(v-\mu_v))$$
$$= R_x + \cancelto{0}{R_{xv}}$$

(b/c independent, $\mathbb{E}[xv] = \mathbb{E}[x]\mathbb{E}[v]$)

$R_{xv} = \mathbb{E}(xv) - \mathbb{E}(x)\mathbb{E}(v)$ by definition of covariance.

$$= \frac{1}{\lambda^2} + 0$$
$$= \frac{1}{\lambda^2}$$

$$R_y^{-1} = \left(\mathbb{E}((y-\mu_y)^2)\right)^{-1} = \left[\mathbb{E}((x-\mu_x+v-\mu_v)(x-\mu_x+v-\mu_v))\right]^{-1}$$
$$= \left[\mathbb{E}((x-\mu_x)^2 + 2(x-\mu_x)(v-\mu_v) + (v-\mu_v)^2)\right]^{-1}$$
$$= \left[R_x + 2\cancelto{0}{R_{xv}} + R_v\right]^{-1}$$
$$= \left[\frac{1}{\lambda^2} + \frac{1}{\mu^2}\right]^{-1} = \left(\frac{\mu^2+\lambda^2}{\mu^2\lambda^2}\right)^{-1} = \frac{\lambda^2\mu^2}{\lambda^2+\mu^2}$$

$$\Rightarrow K = R_{xy}R_y^{-1} = \frac{1}{\lambda^2}\left(\frac{\lambda^2\mu^2}{\lambda^2+\mu^2}\right) = \frac{\mu^2}{\lambda^2+\mu^2}$$

$$\Rightarrow \hat{x} = \frac{1}{\lambda} + \frac{\mu^2}{(\lambda^2+\mu^2)}(y-\mu_y) = \frac{1}{\lambda} + \frac{\mu^2}{\lambda^2+\mu^2}\left(y - \frac{\lambda+\mu}{\lambda\mu}\right)$$

Also, $\hat{x}_{LLMS}$ has no exponential terms with y

$\Rightarrow$ MATLAB $\Rightarrow$ $\boxed{= \frac{\lambda-\mu}{\mu^2+\lambda^2} + \left(\frac{\mu^2}{\mu^2+\lambda^2}\right)y = \hat{x}_{LLMS}}$

Comment:
both $\hat{x}_{non\text{-}LLMS}$ and $\hat{x}_{LLMS}$ have an offset term due to non-0 mean distributions. But the coefficient term for y in $\hat{x}_{LLMS}$ is constant, whereas it changes as a function of y in $\hat{x}_{nonLLMS}$.