Advanced Algorithms Assignment III

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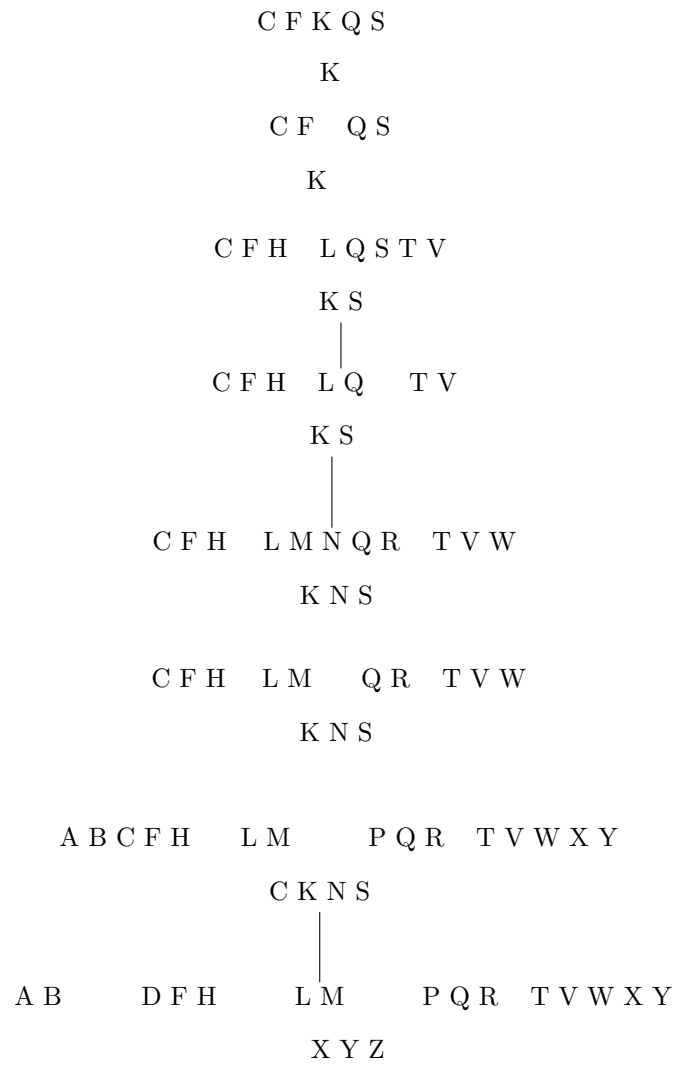
B-Trees and Binomial Heaps HW Draft date February 15, 2006

Exercises 18.2-1 Show the results of inserting the keys F, S, Q, K, C, L, H, T, V, W, M, R, N, P, A, B, X, Y, D, Z, E in order into an empty B-tree with minimum degree 3. Only draw the configurations of the tree just before some node must split, and also draw the final configuration.



(Step skipped intentionally due to LaTeXrestrictions)

Figure 1: Final Result
N

Exercises 18.2-3 Explain how to find the minimum key stored in a B-tree and how to find the predecessor of a given key stored in a B-tree.

To find the minimum key, use the following method: starting at the root, if the current node is a leaf, return the leftmost element. Otherwise, recurse into the leftmost child pointer.
To find the predecessor of a key, start at the node of that key.
CASE 1: If there is a child node between the previous key in that node (or null) and the key. Visit it and return the rightmost key if that node is a leaf. Otherwise, visit the rightmost child node and recurse.
CASE 2: If the key is the leftmost member of a node with no children left of the key. Visit its parent. The last member of member of that node which precedes the key will be the predecessor.
CASE 3: If the key is the only member of a root node with no children. Then its predecessor does not exist. Return null.

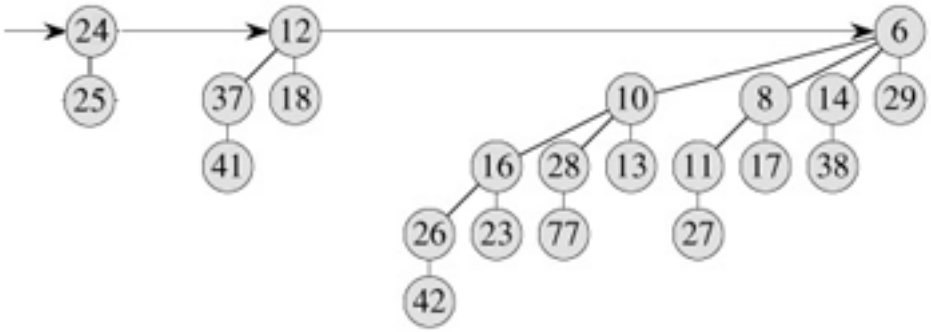
Exercises 19.1-1 Suppose that x is a node in a binomial tree within a binomial heap, and assume that sibling[x] $\neq$ NIL. If x is not a root, how does degree[sibling[x]] compare to degree[x]? How about if x is a root?

Assume x is the root of a subtree B_k with degree k and x has a sibling. If x is not a root then the sibling must be root of a the next subtree, B_{k-1}, by the binomial heap definition.
If x is a root then the next tree occurring in the root list must be the tree B_i, where i is the position of the next 1-bit in the binary representation of the number of nodes.

Exercises 19.1-2 If x is a non-root node in a binomial tree within a binomial heap, how does degree[x] compare to degree[parent[x]]?

$degree[parent[x]] = 1 + degree[x]$, since: the root has degree k, which is greater than that of any other node; moreover if i the children of the root are numbered from left to right by $k-1, k-2, \ldots, 0$, child i is the root of a subtree B_i

Exercises 19.2-2 Show the binomial heap that results when a node with key 24 is inserted into the binomial heap shown in Figure 19.7(d).



Exercises 19.2-3 Show the binomial heap that results when the node with key 28 is deleted from the binomial heap shown in Figure 19.8(c).