The University of Southern Mississippi

AN INVESTIGATION INTO

THE DOUBLE SLIT EXPERIMENT

AND ITS IMPLICATIONS ON REALITY

by

Matthew A. Bennett

Abstract of a Dissertation
Submitted to the Graduate Studies Office
of The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

December 2006

ABSTRACT

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In this we examine the outcomes of the famous double slit experiment of quantum mechanics and its fundamental implications on reality. This is not a real dissertation. It is in fulfillment of the requirements of COS701, the point being to practice creating a dissertation in LaTeX.

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University Coordinator, Graduate Studies

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I am forever indebted to my advisor, Ray Seyfarth, who is the source of all the best time-consuming assignments. I will always persevere in his tepid, moldy shadow. I would like to thank Richard Feynman for making these ideas accessible to Mere Mortals as in [3]. The majority of this text follows the general gist of the article by Feynman, part of the Feynman Lectures on Physics [2]. Also, I would like to thank coffee for getting me here slightly on time.

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LIST OF ABBREVIATIONS

e^-	-	An Electron, and its quantized charge
QED	-	Quantum Electrodynamics
A^+	-	My grade for this assignment
$I(x,t)$	-	Wave Intensity
QCD	-	Quantum Chromodynamics

NOTATION AND GLOSSARY

General Usage and Terminology

The notation used in this text represents fairly standard mathematical and computational usage. In many cases these fields tend to use different preferred notation to indicate the same concept, and these have been reconciled to the extent possible, given the interdisciplinary nature of the material. In particular, the notation for partial derivatives varies extensively, and the notation used is chosen for stylistic convenience based on the application. While it would be convenient to utilize a standard nomenclature for this important symbol, the many alternatives currently in the published literature will continue to be utilized.

The blackboard fonts are used to denote standard sets of numbers: $\mathbb{R}$ for the field of real numbers, $\mathbb{C}$ for the complex field, $\mathbb{Z}$ for the integers, and $\mathbb{Q}$ for the rationals. The capital letters, $A, B, \cdots$ are used to denote matrices, including capital greek letters, e.g., Λ for a diagnonal matrix. Functions which are denoted in boldface type typically represent vector valued functions, and real valued functions usually are set in lower case roman or greek letters. Caligraphic letters, e.g., $\mathcal{V}$, are used to denote spaces such as $\mathcal{V}$ denoting a vector space, $\mathcal{H}$ denoting a Hilbert space, or $\mathcal{F}$ denoting a general function space. Lower case letters such as i, j, k, l, m, n and sometimes p and d are used to denote indices.

Vectors are typset in square brackets, e.g., $[\cdot]$, and matrices are typeset in parenthesese, e.g., $(\cdot)$. In general the norms are typeset using double pairs of lines, e.g., $||\cdot||$, and the abolute value of numbers is denoted using a single pairs of lines, e.g., $|\cdot|$. Single pairs of lines around matrices indicates the determinant of the matrix.

Chapter 1

Motivation from Basic Mechanics

At the quantum level, energy is transmitted in indivisible units. For radiation transfer, such a unit of energy is called a “photon.” We know from Einstein that photons travel at the maximum speed that anything in the universe may travel.

Matter is made of particles so tiny and numerous to startle the imagination. These particles are atoms, and are in turn made of even smaller particles, known as *elementary particles* or *subatomic particles*. There are about 38 classes of known subatomic particles, with many more predicted by theory but not yet detected. Quarks (Appendix A table figure A.1.1), Leptons (Appendix A Table Figure A.1.2) and Bosons (Appendix A Figure Table A.2) represent three major families of particles in the standard model.

1.1 Human Scales

Newton’s laws work well at describing the universe and everything in it on a certain scale. That scale is the same scale that life forms exist on. The reason that classical mechanics has dominated the world of physics for over 300 years (until the turn of the 20th century) was the lack of our ability to observe occurrences at scales other than those very close to human scale. These subatomic particles are so small that they boggle the imagination. [1]

Atomic (meaning indivisible) particles do not behave like anything we’re used to on the personal scale. Take photons and the theory of light. Newton thought they acted like particles (a.k.a. perfectly rigid billiard balls), but they do not. Huygens, the Dutch astronomer and Newton’s contemporary, believed them to act like waves, which they also do not. In a sense they do act like both, but in another sense they act like neither. In fact the photons do not act like anything we have ever experienced. They do not act like clouds, swarms, or billiard balls, or beanbags, or anything else that we can describe using the languages of everyday thought. They can only be described in an operational manner, and that holds some very fundamental implications on the way the world works and the way that we are able to consider and perceive it.

1.2 Correspondence

"Quantum mechanics" is the description of matter on all scales, because of the "correspondence principle." This principle states that for any theory that deals with Statistically, quantum mechanics produces all of Newton's laws of mechanics to an incredible precision, as well as Maxwell's laws of electrodynamics and many other discoveries of what is now known as the field of "classical" mechanics.

It turns out that photons and electrons behave in exactly the same way, which is useful. We can design an experiment using a cathode / anode pair to fire electrons at a phosphorescent screen. This experiment will be the main focus of this paper, but first a bit of background

Chapter 2

Interference

2.1 Non-interfering systems

First we will consider a classical scenario in which the principle of interference has no bearing upon the experimental outcome.

2.1.1 A Bullet Experiment

Consider a machine gun that shoots bullets towards a bulletproof shield. This shield is not a perfect shield. It has two holes in it just wide enough for a single bullet to get through them. Now, this gun is extremely imprecise, so when we fire the gun, the bullet is likely to go one of many angles off of where it is aimed. Now, behind the shield is a fence. Whenever a bullet makes it through one of the two bullet-sized holes in the shield, it will be stopped by the fence.

Additionally, we have a bullet detector. We can place this detector at any distance x from the point on the fence which is center between the two holes in the shield. The detector can tell us only if a bullet strikes that location on the fence or not. This is illustrated in the figure below.

It is difficult to know exactly where a single bullet will go, because the bullets may leave the gun at any number of angles and may ricochet off the side of the holes. We can speak of the probability of a bullet ending up at a distance x from the center of the screen. To measure this probability, we simply move the detector along the x value, taking samples at various locations. For a single hole in the shield, we have the following type of nonanalytic probability distribution:

$$P(X) = \exp(x^2)$$

You may recognize this as the normal distribution, shown below in the figure 2.1.1. The probability of a bullet striking any single point can be measured simply by observing the ratio that bullets hit that point experimentally to the total number of bullets that hit the

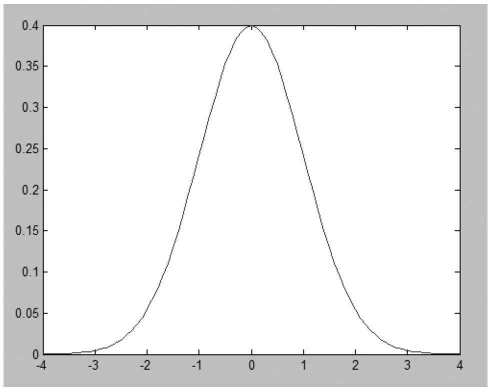


Figure 2.1: A Normal Distribution

fence in the same amount of time. These probabilities are normally distributed along the fence (when there are two holes) as in the figure 2.1.1. If we close up the hole on the left, the total distribution will look like the one on the right. Similarly, if we close up the hole on the right, the distribution will look like the one on the left.

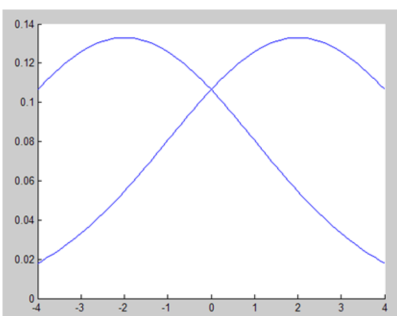


Figure 2.2: Distributions of bullets hitting the fence behind a two-hole screen

Now if we take the two holes together, we should expect to see the cumulative probability distribution over both holes to be the sum of the probabilities over each hole.

$$P_{12} = P_1 + P_2 \tag{2.1}$$

This is exactly what we get as seen in the figure 2.1.1. This is called the "no interference scenario" for reasons we will now cover, and is a property of discrete particles (bullets) but not of waves.

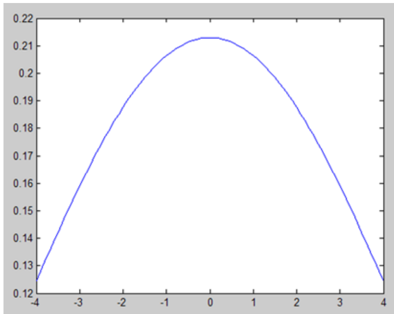


Figure 2.3: Classical Sum of Probability P_{12}

2.2 Interference patterns

We will now see the principle of interference of waves. Whenever two waves collide, as in the ocean, they will either be in phase or out of phase with one another. In phase means that the troughs of the first wave meet the troughs of the second wave, and the peaks of the first meet the peaks of the second, increasing the total amplitude of the two combined. Out of phase means that the troughs of the first wave meet the peaks of the second wave, and they "cancel," decreasing or annihilating the amplitude of the combined wave.

2.2.1 Young's Experiment

Imagine taking the apparatus from the bullet experiment and dropping it into a perfectly unclouded lake. The gun is replaced by a wave generator. Think of a kid in a pool pushing the water away from himself. Instead of bullets, we will be using the peaks of these waves.

Each wave emanates from the child in a perfect circle until it meets the screen (two holes as before). As it contacts the holes, two perfectly circular waves are generated as illustrated in the figure.

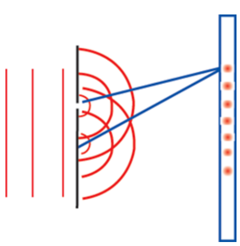


Figure 2.4: Wave interference in Action

If we measure the average amplitude (intensity) of these waves as they hit the fence, we get a clear picture of wave interference. If the instantaneous height of the water wave coming only from the first hole is

$$\mathrm{Re} h_1 e^{i\omega t}$$

then we have $|h_1|^2$ for the intensity of that wave at the detector at instant t in time. When both holes are open, we have the following relations 2.2.

$$I_1 = |h_1|^2 \tag{2.2}$$
$$I_2 = |h_2|^2 \tag{2.3}$$
$$I_{12} = |h_1 + h_2|^2 \tag{2.4}$$
$$\tag{2.5}$$

Chapter 3

Quantum interference

3.1 Electron behavior

We want to repeat the experiment used in the last chapter, but again change the medium. This time, think of the machine gun, only this one fires electrons. There exists such a device, known as a cathode-ray tube, that we can use to perform this experiment. The fence is replaced by a phosphorescent membrane, which lights a single discrete point when hit by an electron at that point. The shield is also available in a device known as an anode.

A detector of electrons can be built to issue a click whenever it is hit by an electron. By moving this detector along the back wall, we can detect the arrival of a single electron or the rate of multiple arrivals.

Once the experimental apparatus is built, we can perform the experiment with electrons and observe what happens. We should expect the electrons to act like bullets, as in 2.1. When we perform the experiment, though, we get something more like the wave interference pattern. But if electrons are discrete particles, like bullets, how can this be? The truth is that they are not like bullets. Whenever we substitute in the electrons, we get something that looks like Figure 3.1 on the back plate.

Ah-ha! an interference pattern. So perhaps electrons are actually waves, but they are not, because it is possible to register a single electron after it passes the screen. Additionally, these electrons will statistically establish an interference pattern over time no matter how much time is allotted to 'wait' between firings of the [1] cathode.

3.2 Consequences

It turns out that electrons act like waves and particles both depending on the context in which they are observed. We can use certain particles to determine their velocity (momentum), but then we cannot know their position. Similarly, we can determine their momentum, but we cannot know their velocity. This is an essential consequence of the uncertainty [2] principle.

[1] I intended to have more at this point, but I have fulfilled all of the requirements for this assignment

[2] but not a direct statement of the principle itself

Figure 3.1: An Electron Interference Pattern

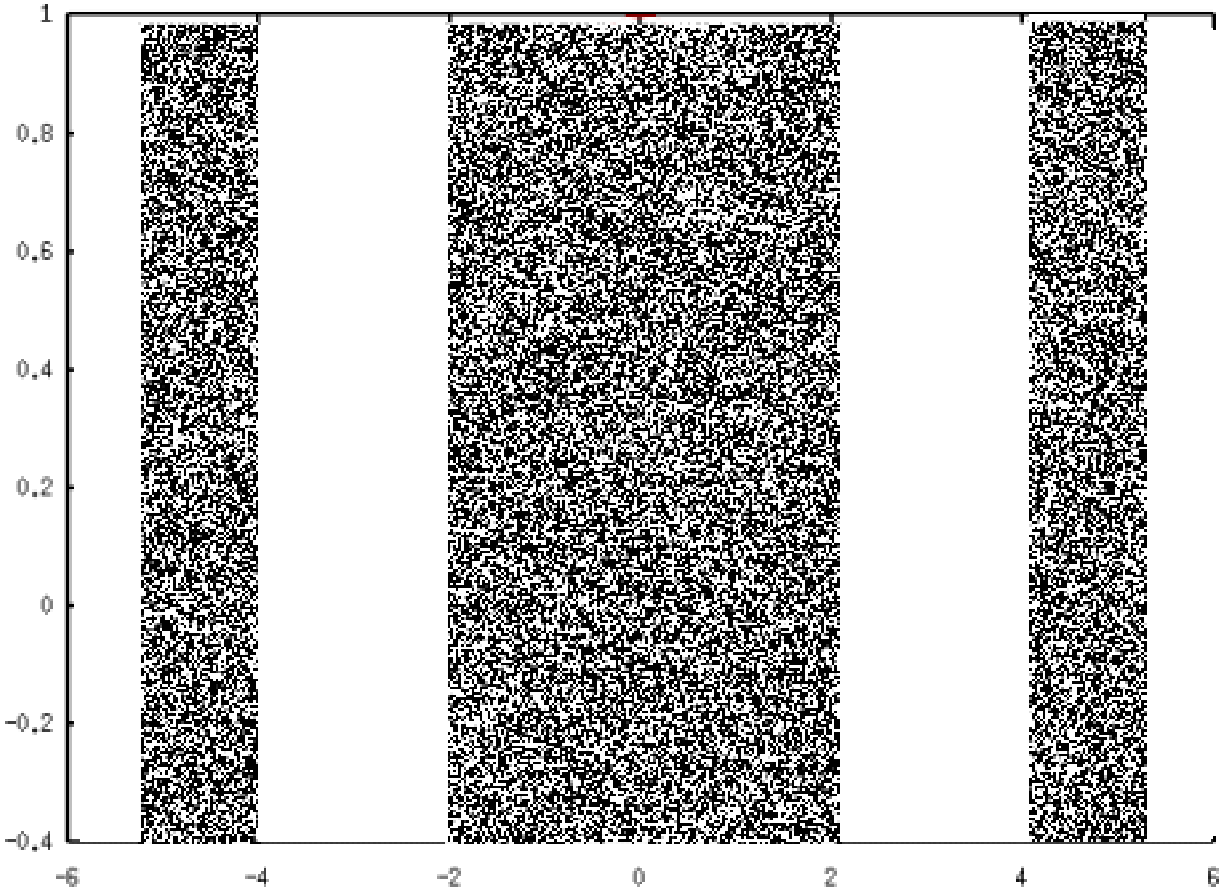
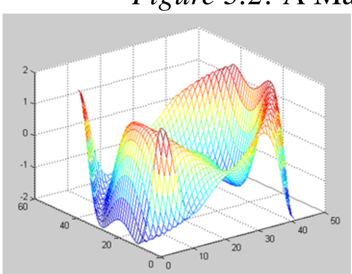
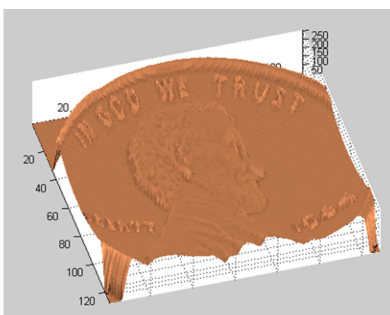


Figure 3.2: A Matter Wave / Energy Well propagating through space



Electrons can be seen to travel through space as matter, or as energy. Actually, because of relativity they can be seen as two sides of the same coin 3.2.

Figure 3.3: A coin



Appendix A

The Standard Model

This information was taken from Wikipedia [4].

A.1 Fermions (half-integer spin)

A.1.1 Quarks

Quarks interact via color, which means via strong nuclear influence.

A.1.2 Leptons

A.2 Bosons (integer spin)

The Higgs boson has been predicted, but not detected, in the field of quantum theory trying to unify the electromagnetic and weak nuclear forces.

Table A.1: List of Quarks

Name	Charge(e)	Mass(MeV)	Antiquark
Up (u)	+2/3	1.5 to 4	antiup quark ($\overline{u}$)
Down (d)	-1/3	4 to 8	antidown quark ($\overline{d}$)
Strange (s)	-1/3	80 to 130	antistrange quark ($\overline{s}$)
Charm (c)	+2/3	1,150 to 1,350	anticharm quark ($\overline{c}$)
Bottom (b)	-1/3	4,100 to 4,400	antibottom quark ($\overline{b}$)
Top (t)	+2/3	171,400 $\pm$ 2,100	antitop quark ($\overline{t}$)

Table A.2: List of Leptons

Name	Charge(e)	Mass(MeV)	Antiparticle
Electron (e^-)	-1	0.511	Positron (e^+)
Muon (μ^-)	-1	105.7	+Muon (μ^+)
Tau Lepton (τ^-)	-1	1,777	+ Tau(τ^+)

Table A.3: List of Bosons

Name	Charge (e)	Spin	Mass (GeV)	Force
Photon	0	1	0	Electromagnetism
$W\pm$	± 1	1	80.4	Weak nuclear
Z^0	0	1	91.2	Weak nuclear
Gluon	0	1	0	Strong nuclear
Higgs	0	0	¿ 112	???

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[4] Wikipedia "List of Particles".

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