422 Computational Finance
Coursework 1

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10 February 2012

1. The deadline is Friday 24 February 2012, and the marked work will be returned by Friday 9 March 2012.
2. Please use MATLAB for the computational questions and submit your code together with the answers.

Question 1 (Price-Yield Curves) *Consider three coupon bonds A, B and C with the same face value of £100 and a maturity of 10 years.*

(a) The annual coupon rates are given in the following table. Assume that there is one coupon payment per year. Plot the price-yield curve of each bond using annual compounding and the range of yields $0 \leq \lambda \leq 20$.

Bonds	*A*	*B*	*C*
Coupon rates (%)	*6*	*10*	*12*

(b) Suppose that bonds A, B and C have the same annual coupon rates of 14% and that there are two coupon payments per year. The time to maturity for each bond is given in the following table:

Today's date : March 2012

Bonds	*A*	*B*	*C*
Maturity date	*Sep 2012*	*Mar 2013*	*Sep 2014*

Using semi-annual compounding and a range of yields $0 \leq \lambda \leq 20$, *show the price-yield relation in tabular form. Plot the price-yield curves.*

Question 2 (Portfolio Optimization with a Twist) *Consider a market consisting of five assets. It is assumed that the monthly returns of these assets are normally distributed with mean values and standard deviations as shown below:*

$$\begin{array}{lllll} \mu_1 = 0.006 & \mu_2 = 0.01 & \mu_3 = 0.014 & \mu_4 = 0.018 & \mu_5 = 0.022 \\ \sigma_1 = 0.085 & \sigma_2 = 0.08 & \sigma_3 = 0.095 & \sigma_4 = 0.09 & \sigma_5 = 0.1 \end{array}$$

The correlation matrix for the five assets is:

$$\begin{bmatrix} 1 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 1 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.3 & 1 \end{bmatrix}$$

It is further assumed that the returns corresponding to different months are mutually independent and that short selling is not allowed.

(a) *Formulate the usual Markowitz model for an investment horizon of one month as well as two related optimization problems to find the portfolios with the highest expected return and the smallest variance, respectively. Please state all of these problems mathematically.*

(b) *Find the optimal portfolio weights and the standard deviations for the 10 portfolios with expected returns*

$$\bar{r}_{\min} + i \times \frac{\bar{r}_{\max} - \bar{r}_{\min}}{9} \quad \text{for} \quad i = 0, \ldots, 9,$$

where

(i) $\bar{r}_{\max}$ *is the expected return of the risk-seeking portfolio from 2(a) (maximum expected return), and*

(ii) $\bar{r}_{\min}$ *is expected return of the risk-averse portfolio from 2(a) (minimum variance).*

Use these results to plot the efficient frontier. We refer to this as the "true frontier" as it is based on the true mean values, variances and covariances of the returns given above.

(c) *In reality, investors do not know the "true" mean values, variances and covariances. Instead, these parameters need to be estimated from historical data. We now simulate this situation experimentally: use Matlab's random number generator for multivariate normal distributions to simulate a random time series of 24 monthly returns. Assume that these are the historical returns available to the investor. Use this time series to compute the sample means, sample variances and sample covariances of the asset returns. Next compute the efficient frontier (including the portfolio weights) exactly as in Question 2(b) but use the estimated parameters instead of the true ones. We call this the "estimated frontier". Note that your result will change whenever you simulate a new time series of length 24! Plot the estimated frontier. The estimated frontier represents what an investor (who does not know the true parameters)* **believes** *to get.*

(d) *Consider the 10 portfolios on the estimated frontier (that is, their portfolio weights) computed in 2(c). Next, use the true mean values, variances and covariances given in the problem statement (instead of their estimates from 2(c)) to compute the mean returns and standard deviations of these portfolios. The curve joining these portfolios in the mean-standard deviation plane is called the "actual frontier". Again, the result will change if you simulate a new time series. Plot the actual frontier. The actual frontier represents what an investor (who does not know the true parameters)* **actually gets**.

(e) *As the results of (b) and (c) are random (changing with each new simulation run), we would like to compute* **averaged** *estimated and actual frontiers. This allows us to recognise if there is a systematic bias in the estimated and actual frontiers when compared to the true frontier. Please proceed as follows:*

   (i) *Compute the 10 portfolios on the estimated and actual frontiers from 2(c) and 2(d), respectively, 10,000 times. In each simulation run you should sample a new time series for estimating the assets' means and (co)variances.*

(ii) Average the expected returns and the standard deviations of the portfolios over the 10,000 simulation runs. This yields the "averaged estimated frontier" and the "averaged acutal frontier".

(iii) Plot the true, the averaged estimated and the averaged actual frontiers in the same diagram and compare them. Interpret your observations.

(f) Rerun the experiment from Question 2(e) by using time series of length 360 months and 1,800 months, respectively. How do the averaged estimated and actual frontiers change?

Remarks: *In questions 2(c) and 2(d) please state explicitly the formulas that you use to compute the estimated and actual frontiers; the values you obtain are not so important as they are highly random. In 2(e) and 2(f) please indicate explicitly the means and standard deviations of the portfolios on the averaged frontiers.*

Questions 1 and 2 carry 25% and 75% of the mark, respectively.