Lu Xin (lx108) - Comp Finance Coursework 1

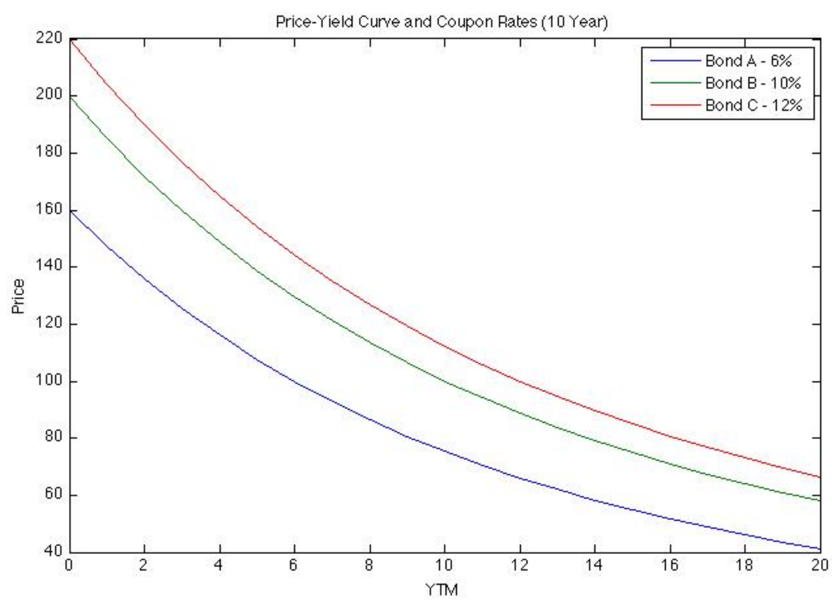
Friday 24th February 2012

Question 1: Price-Yield Curve

a)

Bond	**A**	**B**	**C**
Coupon Rate	6%	10%	12%
Maturity (Years)	10	10	10
Face Value (£)	100	100	100

The price-yield curve of each bond with annual compound and range of yields from 0≤λ≤20 are as follows:



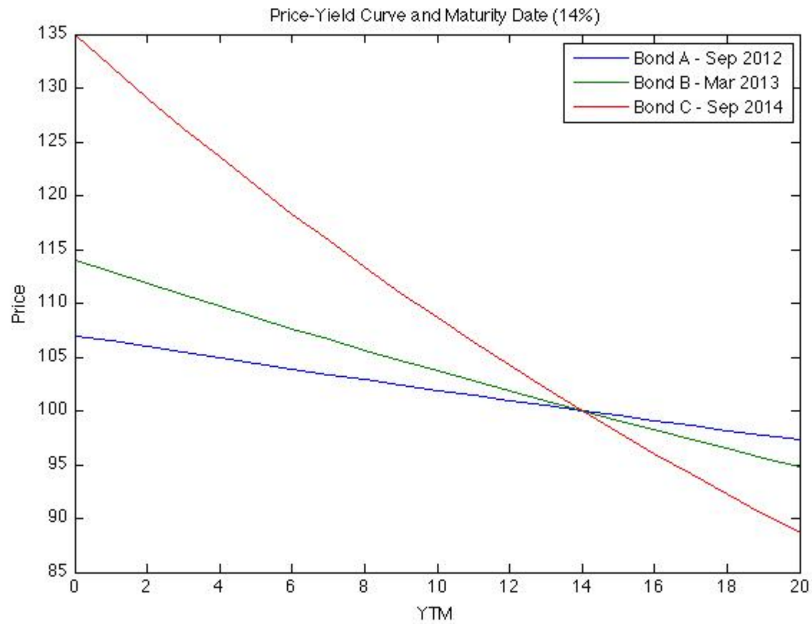
We notice that the higher the YTM, the lower the price. When YTM = 0, there is no discounting, thus price is the undiscounted sum of all coupon payments and face value.

b)

Current Date: Mar 2012

Bond	**A**	**B**	**C**
Coupon Rate	6%	10%	12%
Maturity Date	Sep 2012	Mar 2013	Sep 2014
Face Value (£)	100	100	100

The price-yield curve of each bond with semi-annual compound and range of yields from 0≤λ≤20 are as follows



The price-yield relation in tabular form is as follows:

λ	Bond A	Bond B	Bond C
0	107	114	135
1	106.4677	112.9031	132.0181
2	105.9406	111.8224	129.1206
3	105.4187	110.7574	126.3045
4	104.902	109.7078	123.5673
5	104.3902	108.6734	120.9062
6	103.8835	107.6539	118.3188
7	103.3816	106.6489	115.8027
8	102.8846	105.6583	113.3555
9	102.3923	104.6817	110.9749
10	101.9048	103.7188	108.659
11	101.4218	102.7695	106.4054
12	100.9434	101.8334	104.2124
13	100.4695	100.9103	102.0778
14	100	100	100
15	99.5349	99.1022	97.9771
16	99.0741	98.2167	96.0073
17	98.6175	97.3433	94.089
18	98.1651	96.4818	92.2207
19	97.7169	95.6319	90.4007
20	97.2727	94.7934	88.6276

We notice that when the YTM value is 14%, equal to the coupon rate, the price is equal to the face value as expected.

Question 2: Portfolio Optimization with a Twist

a)

i. The Markowitz model for an investment horizon of one month in optimizing the portfolio with the highest expected return is the following:

minimize: $-\sum_{i=1}^{n} w_i \overline{r_i}$

subject to: $\sum_{i=1}^{n} w_i = 1$

$w_i \geq 0 \quad for\ i = 1, 2, \ldots, n$

ii. The Markowitz model for an investment horizon of one month in optimizing the portfolio with the smallest variance is the following:

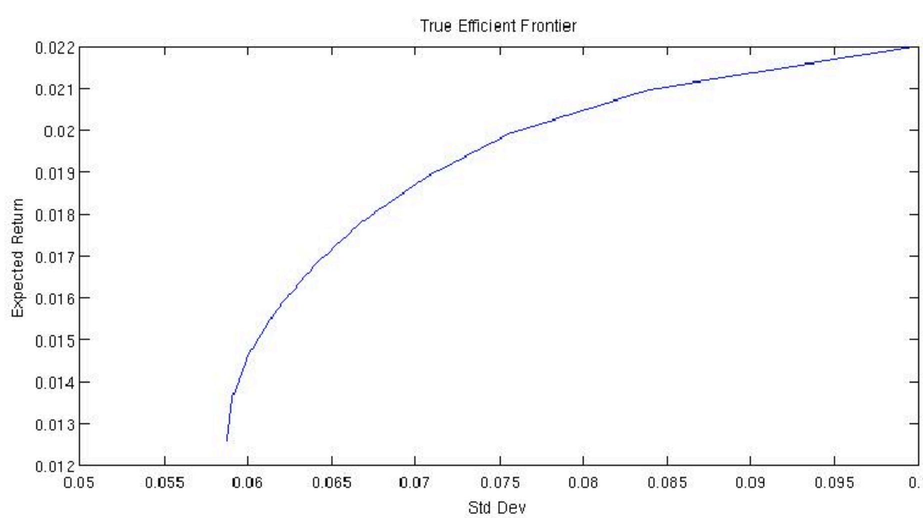
minimize: $\frac{1}{2}\sum_{i,j=1}^{n} w_i \sigma_{ij} w_j$

subject to: $\sum_{i=1}^{n} w_i = 1$

$w_i \geq 0 \quad for\ i = 1, 2, \ldots, n$

b) The true frontier for the given set of values is the following:

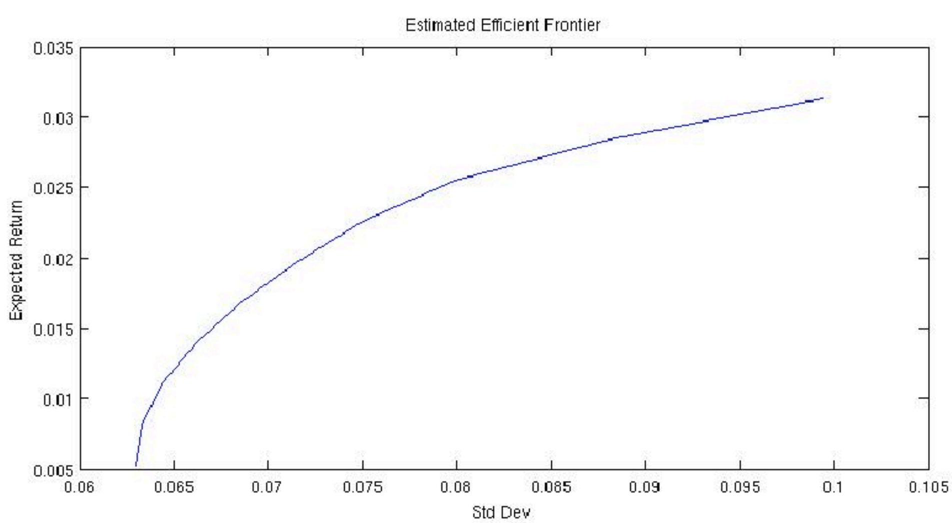
Std Dev	Return	w1	w2	w3	w4	w5
0.0588	0.0126	**0.2403**	**0.3006**	**0.1507**	**0.1911**	**0.1172**
0.0591	**0.0136**	0.1856	0.2732	0.1546	0.2226	0.164
0.0601	**0.0147**	0.1309	0.2458	0.1585	0.254	0.2107
0.0617	**0.0157**	0.0763	0.2184	0.1624	0.2855	0.2575
0.0639	**0.0168**	0.0216	0.1909	0.1663	0.3169	0.3043
0.0668	**0.0178**	0	0.127	0.1579	0.3502	0.3649
0.0706	**0.0189**	0	0.0391	0.1416	0.3847	0.4346
0.0755	**0.0199**	0	0	0.0603	0.4029	0.5368
0.084	**0.021**	0	0	0	0.2617	0.7383
0.1	0.022	**0**	**0**	**0**	**0**	**1**



c) Simulate a situation experimentally by using random number generator for multivariate normal distributions provided by matlab for a 24 month period.

- The **random returns** are generated via matlab as such:
    - `randomReturns = mvnrnd(mu, covMatrix, months);`
        - Where mu is the true returns, covMatrix is the covariance matrix, and months = 24.
- The **mean** returns are the mean of the random returns that we generated from before:
    - estMu = mean(randomReturns);
- The **covariance** matrix is calculated using the set of generated random returns
    - `covMatrix = cov(randomReturns);`
- The **rmin** and **rmax** are the calculated using the same method mentioned above in part 2a)
- The **weights** for the efficient frontier of the estimated random returns are then stored and used later

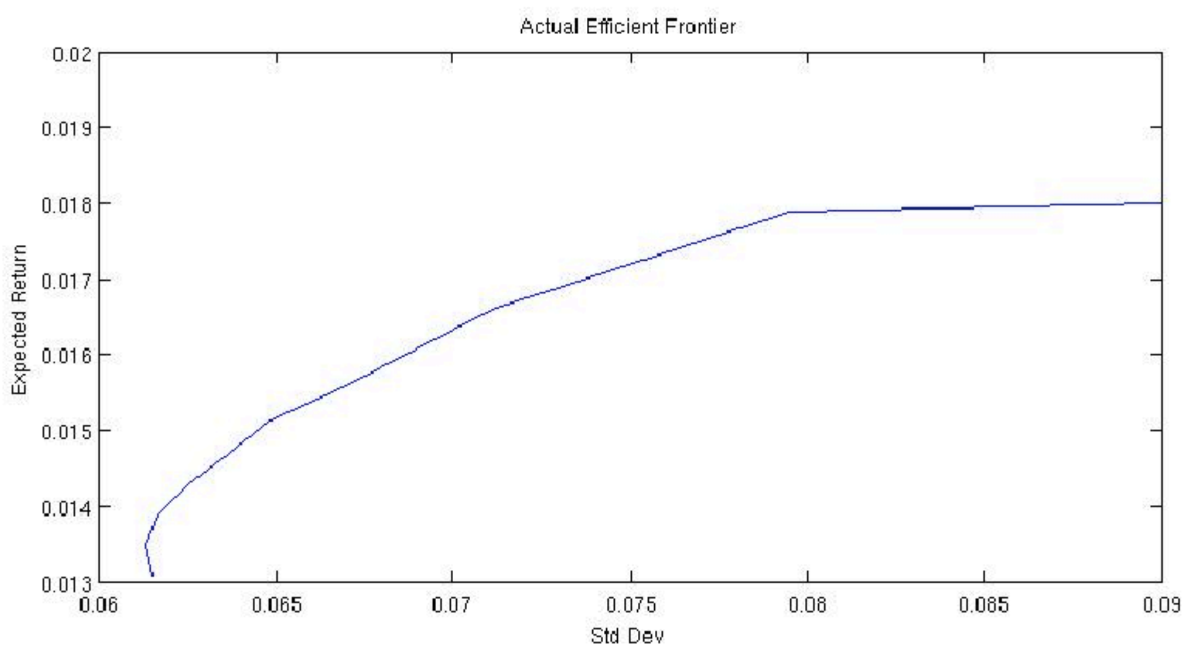
Estimated Frontier over 24 months						
Std Dev	**Return**	**w1**	**w2**	**w3**	**w4**	**w5**
0.0477	-0.0006	0.2356	0.522	0.2388	0.0036	0
0.0492	0.003	0.1963	0.4546	0.2453	0	0.1038
0.0521	0.0066	0.1633	0.3812	0.2446	0	0.2109
0.0563	0.0101	0.1303	0.3078	0.244	0	0.3179
0.0615	0.0137	0.0973	0.2345	0.2433	0	0.4249
0.0674	0.0172	0.0643	0.1611	0.2426	0	0.532
0.074	0.0208	0.0313	0.0877	0.242	0	0.639
0.0809	0.0243	0	0.0128	0.2406	0	0.7466
0.089	0.0279	0	0	0.1274	0	0.8726
0.0989	0.0314	0	0	0	0	1



d) Using the estimated weights from before, we use them with the true mean values, variances, and covariances give before. Thus yielding the actual frontier.

- The values of the **actual returns** are calculated by using the efficient frontier's weights we obtained from our estimated frontier, and multiplying them with the true returns:
  - `actualReturns = estWeights * mu';`
- The **actual variance** is calculated using the estimated weights we obtained and the true covariance matrix:
  - $\sigma^2 = w^T \Sigma\, w$
  - We then square root this to obtain the **actual standard deviation**

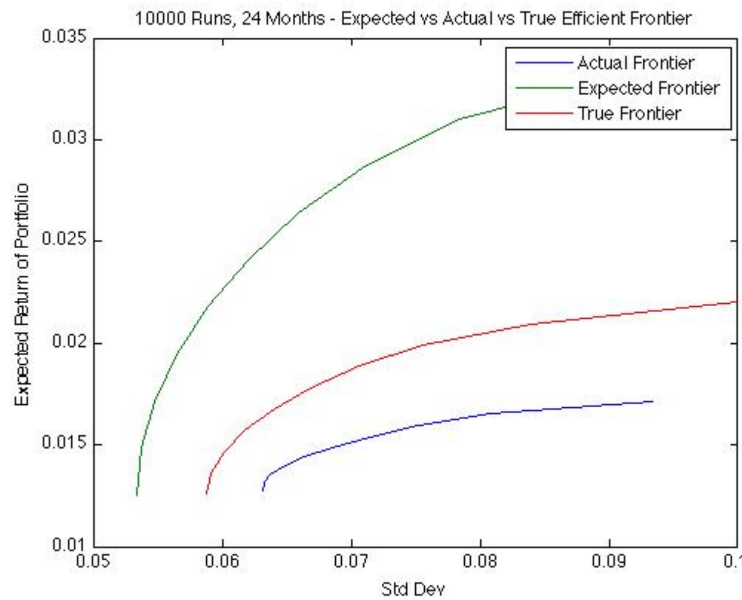
Actual Frontier over 24 months with Estimated Weights						
Std Dev	**Return**	**w1**	**w2**	**w3**	**w4**	**w5**
0.0634	0.01	0.2356	0.522	0.2388	0.0036	0
0.0619	0.0114	0.1963	0.4546	0.2453	0	0.1038
0.0619	0.0129	0.1633	0.3812	0.2446	0	0.2109
0.0637	0.0143	0.1303	0.3078	0.244	0	0.3179
0.0671	0.0157	0.0973	0.2345	0.2433	0	0.4249
0.0719	0.0171	0.0643	0.1611	0.2426	0	0.532
0.0779	0.0185	0.0313	0.0877	0.242	0	0.639
0.0847	0.0199	0	0.0128	0.2406	0	0.7466
0.0916	0.021	0	0	0.1274	0	0.8726
0.1	0.022	0	0	0	0	1



e, f) Simulating this process with 10,000 runs, and taking the average for horizons of 24 months, 360, and 1800 respectively, we get the following:

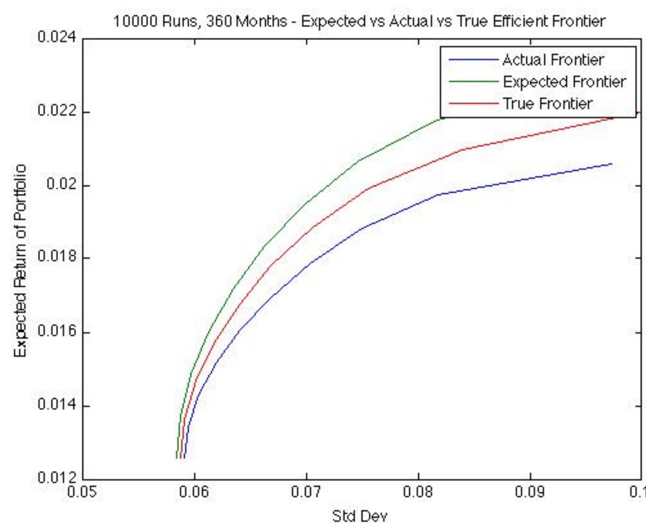
24 Months – Averaged Frontiers:

Std Dev	Returns
0.0631	0.0128
0.0631	0.0131
0.0637	0.0135
0.0648	0.014
0.0663	0.0144
0.0685	0.0149
0.0713	0.0154
0.0751	0.0159
0.0808	0.0165
0.0934	0.0171



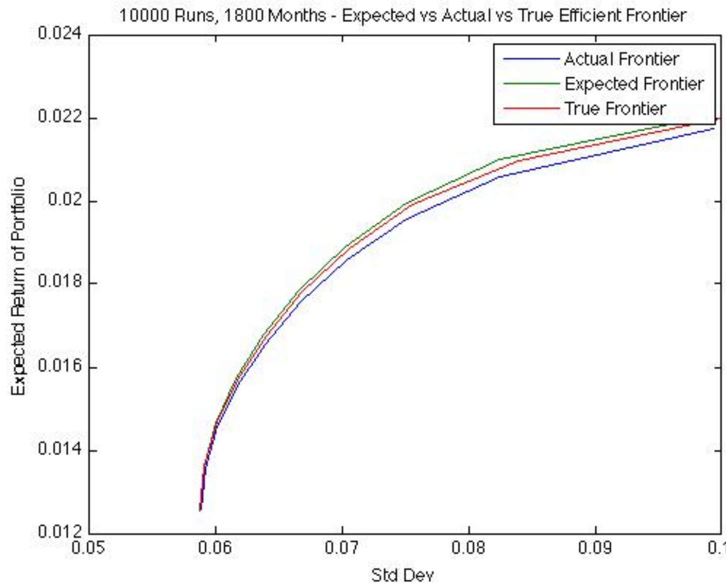
360 Months – Averaged Frontiers:

Std Dev	Returns
0.0591	0.0126
0.0594	0.0134
0.0603	0.0143
0.0619	0.0152
0.064	0.0161
0.0667	0.0169
0.0703	0.0179
0.075	0.0188
0.0818	0.0197
0.0973	0.0206



1800 Months – Averaged Frontiers:

Std Dev	Returns
0.0588	0.0126
0.0591	0.0136
0.0601	0.0146
0.0617	0.0156
0.0639	0.0166
0.0667	0.0176
0.0703	0.0186
0.0751	0.0196
0.0825	0.0206
0.0994	0.0217



As we can observe, the estimated and the actual frontiers get closer and closer to the true frontier as the time series data we have increases. As we can see, when we use data of 150 years, or 1800 months, the frontiers are almost identical. Even though this method seems to work well with large historical data, but in the real world, it is very hard to obtain 150 years of historical data for a given company, because we do not have this data and also because very few companies last for 150 years. Furthermore, this model works under the assumption that the returns for companies follow a normal distribution which is probably wrong. This is why big investment firms, investment banks, and other investors look at more than just historical data when making investment decisions. They take into account many other factors such as the finances of the company, the current CEO, the current economic outlook, world events, government regulations, people's confidence in the company, and many other factors. Thus basing our portfolio building strategies purely on technical analysis is insufficient and unwise in the long run.