Factor_Analysis

```
expect <- read.csv("C:/Users/dabre/OneDrive/Desktop/lfe.csv")

sapply(expect, function(x) sum(is.na(x)))
```

```
##                          Country                             Year
##                                0                                0
##                           Status                  Life.expectancy
##                                0                               10
##                  Adult.Mortality                     infant.deaths
##                               10                                0
##                          Alcohol           percentage.expenditure
##                              194                                0
##                      Hepatitis.B                          Measles
##                              553                                0
##                              BMI                under.five.deaths
##                               34                                0
##                            Polio                Total.expenditure
##                               19                              226
##                       Diphtheria                          HIV.AIDS
##                               19                                0
##                              GDP                       Population
##                              448                              652
##             thinness..1.19.years               thinness.5.9.years
##                               34                               34
## Income.composition.of.resources                         Schooling
##                              167                              163
```

```
expect <- expect[complete.cases(expect),]  ## to remove which has null values
sapply(expect, function(x) sum(is.na(x)))
```

```
##                          Country                             Year
##                                0                                0
##                           Status                  Life.expectancy
##                                0                                0
##                  Adult.Mortality                     infant.deaths
##                                0                                0
##                          Alcohol           percentage.expenditure
##                                0                                0
##                      Hepatitis.B                          Measles
##                                0                                0
##                              BMI                under.five.deaths
##                                0                                0
##                            Polio                Total.expenditure
##                                0                                0
##                       Diphtheria                          HIV.AIDS
##                                0                                0
```

```
##                                GDP                         Population
##                                  0                                  0
##            thinness..1.19.years               thinness.5.9.years
##                                  0                                  0
## Income.composition.of.resources                         Schooling
##                                  0                                  0
```

```
View(expect)
attach(expect)
expect[1]
```

```
##                       Country
## 1                 Afghanistan
## 2                 Afghanistan
## 3                 Afghanistan
## 4                 Afghanistan
## 5                 Afghanistan
## 6                 Afghanistan
## 7                 Afghanistan
## 8                 Afghanistan
## 9                 Afghanistan
## 10                Afghanistan
## 11                Afghanistan
## 12                Afghanistan
## 13                Afghanistan
## 14                Afghanistan
## 15                Afghanistan
## 16                Afghanistan
## 17                    Albania
## 18                    Albania
## 19                    Albania
## 20                    Albania
## 21                    Albania
## 22                    Albania
## 23                    Albania
## 24                    Albania
## 25                    Albania
## 26                    Albania
## 27                    Albania
## 28                    Albania
## 29                    Albania
## 30                    Albania
## 31                    Albania
## 32                    Albania
## 34                    Algeria
## 35                    Algeria
## 36                    Algeria
## 37                    Algeria
## 38                    Algeria
## 39                    Algeria
```

```
## 2930                  Zimbabwe
## 2931                  Zimbabwe
## 2932                  Zimbabwe
## 2933                  Zimbabwe
## 2934                  Zimbabwe
## 2935                  Zimbabwe
## 2936                  Zimbabwe
## 2937                  Zimbabwe
## 2938                  Zimbabwe
```

```
corrm.expect <- cor(expect[,5:22])
corrm.expect
```

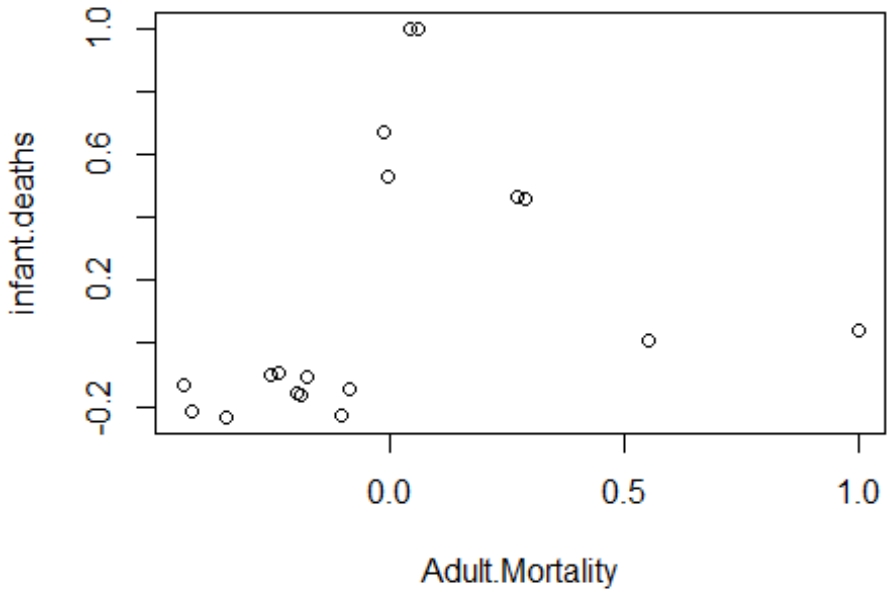
```
##                                 Adult.Mortality infant.deaths     Alcohol
## Adult.Mortality                     1.000000000   0.042450237 -0.17553509
## infant.deaths                       0.042450237   1.000000000 -0.10621692
## Alcohol                            -0.175535086  -0.106216917  1.00000000
## percentage.expenditure             -0.237609890  -0.090764632  0.41704736
## Hepatitis.B                        -0.105225443  -0.231768937  0.10988939
## Measles                            -0.003966685   0.532679832 -0.05011023
## BMI                                -0.351542478  -0.234425154  0.35339621
## under.five.deaths                   0.060365026   0.996905622 -0.10108216
## Polio                              -0.199853000  -0.156928805  0.24031453
## Total.expenditure                  -0.085226535  -0.146951117  0.21488509
## Diphtheria                         -0.191428759  -0.161871004  0.24295143
## HIV.AIDS                            0.550690745   0.007711547 -0.02711264
## GDP                                -0.255034733  -0.098092020  0.44343279
## Population                         -0.015011838   0.671758310 -0.02888023
## thinness..1.19.years                0.272230044   0.463415256 -0.40375499
## thinness.5.9.years                  0.286722882   0.461907925 -0.38620819
## Income.composition.of.resources    -0.442203288  -0.134753863  0.56107433
## Schooling                          -0.421170523  -0.214371900  0.61697481
##                                 percentage.expenditure Hepatitis.B      Me
asles
## Adult.Mortality                            -0.23760989 -0.10522544 -0.0039
66685
## infant.deaths                              -0.09076463 -0.23176894  0.5326
79832
## Alcohol                                     0.41704736  0.10988939 -0.0501
10235
## percentage.expenditure                      1.00000000  0.01676017 -0.0630
70789
## Hepatitis.B                                 0.01676017  1.00000000 -0.1247
99993
## Measles                                    -0.06307079 -0.12479999  1.0000
00000
## BMI                                         0.24273824  0.14330179 -0.1532
45464
## under.five.deaths                          -0.09215806 -0.24076603  0.5175
```

```
05563
## Polio                                0.12862605  0.46333080 -0.0578
50133
## Total.expenditure                    0.18387236  0.11332668 -0.1135
82738
## Diphtheria                           0.13481324  0.58898993 -0.0586
05907
## HIV.AIDS                            -0.09508499 -0.09480197 -0.0035
21854
## GDP                                  0.95929886  0.04184950 -0.0647
67590
## Population                          -0.01679214 -0.12972265  0.3219
46377
## thinness..1.19.years                -0.25503460 -0.12940595  0.1806
41506
## thinness.5.9.years                  -0.25563544 -0.13325099  0.1749
46217
## Income.composition.of.resources      0.40216974  0.18492097 -0.0582
77256
## Schooling                            0.42208845  0.21518159 -0.1156
60481
##                                           BMI under.five.deaths       Polio
## Adult.Mortality                 -0.35154248        0.06036503 -0.19985300
## infant.deaths                   -0.23442515        0.99690562 -0.15692881
## Alcohol                          0.35339621       -0.10108216  0.24031453
## percentage.expenditure           0.24273824       -0.09215806  0.12862605
## Hepatitis.B                      0.14330179       -0.24076603  0.46333080
## Measles                         -0.15324546        0.51750556 -0.05785013
## BMI                              1.00000000       -0.24213740  0.18626797
## under.five.deaths               -0.24213740        1.00000000 -0.17116419
## Polio                            0.18626797       -0.17116419  1.00000000
## Total.expenditure                0.18946896       -0.14580310  0.11976798
## Diphtheria                       0.17629450       -0.17844819  0.60924547
## HIV.AIDS                        -0.21089675        0.01947593 -0.10788547
## GDP                              0.26611397       -0.10033126  0.15680869
## Population                      -0.08141598        0.65867969 -0.04538657
## thinness..1.19.years            -0.54701751        0.46478470 -0.16406959
## thinness.5.9.years              -0.55409398        0.46228938 -0.17448925
## Income.composition.of.resources  0.51050483       -0.14809728  0.31468159
## Schooling                        0.55484390       -0.22601262  0.35014660
##                                 Total.expenditure  Diphtheria     HIV.AIDS
## Adult.Mortality                       -0.08522653 -0.19142876  0.550690745
## infant.deaths                         -0.14695112 -0.16187100  0.007711547
## Alcohol                                0.21488509  0.24295143 -0.027112636
## percentage.expenditure                 0.18387236  0.13481324 -0.095084991
## Hepatitis.B                            0.11332668  0.58898993 -0.094801971
## Measles                               -0.11358274 -0.05860591 -0.003521854
## BMI                                    0.18946896  0.17629450 -0.210896746
## under.five.deaths                     -0.14580310 -0.17844819  0.019475927
## Polio                                  0.11976798  0.60924547 -0.107885468
```

```
## Total.expenditure                      1.00000000  0.12991481  0.043100657
## Diphtheria                             0.12991481  1.00000000 -0.117601074
## HIV.AIDS                               0.04310066 -0.11760107  1.000000000
## GDP                                    0.18037347  0.15843774 -0.108080600
## Population                            -0.07996224 -0.03989754 -0.027800562
## thinness..1.19.years                  -0.20987232 -0.18724165  0.172591767
## thinness.5.9.years                    -0.21786479 -0.18095238  0.183146727
## Income.composition.of.resources        0.18365319  0.34326177 -0.248589855
## Schooling                              0.24378345  0.35039793 -0.211840201
##                                             GDP   Population thinness..1.19.ye
ars
## Adult.Mortality                  -0.25503473 -0.015011838            0.2722
300
## infant.deaths                    -0.09809202  0.671758310            0.4634
153
## Alcohol                           0.44343279 -0.028880232           -0.4037
550
## percentage.expenditure            0.95929886 -0.016792141           -0.2550
346
## Hepatitis.B                       0.04184950 -0.129722655           -0.1294
060
## Measles                          -0.06476759  0.321946377            0.1806
415
## BMI                               0.26611397 -0.081415982           -0.5470
175
## under.five.deaths                -0.10033126  0.658679691            0.4647
847
## Polio                             0.15680869 -0.045386572           -0.1640
696
## Total.expenditure                 0.18037347 -0.079962237           -0.2098
723
## Diphtheria                        0.15843774 -0.039897537           -0.1872
416
## HIV.AIDS                         -0.10808060 -0.027800562            0.1725
918
## GDP                               1.00000000 -0.020368964           -0.2774
983
## Population                       -0.02036896  1.000000000            0.2825
293
## thinness..1.19.years             -0.27749835  0.282529280            1.0000
000
## thinness.5.9.years               -0.27795855  0.277913374            0.9279
134
## Income.composition.of.resources   0.44685551 -0.008132466           -0.4536
789
## Schooling                         0.46794697 -0.040312419           -0.4911
992
##                                  thinness.5.9.years
## Adult.Mortality                           0.2867229
## infant.deaths                             0.4619079
```

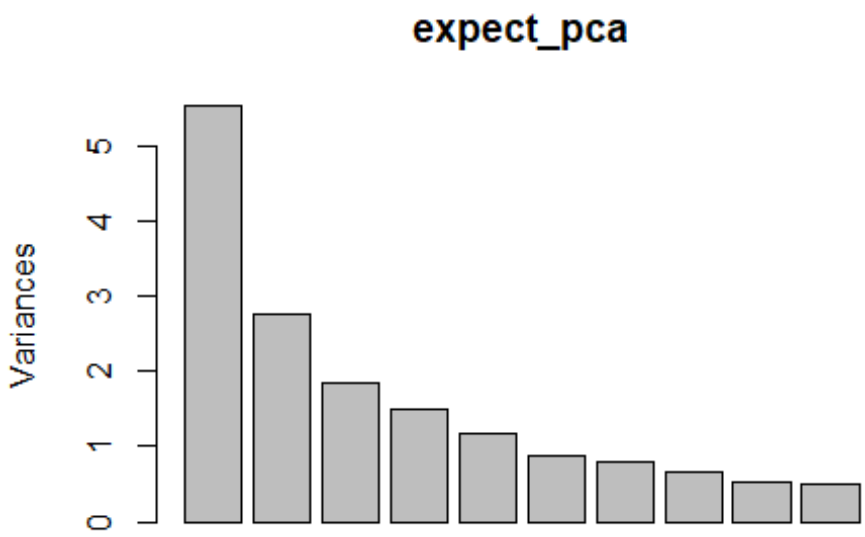
```
## Alcohol                                  -0.3862082
## percentage.expenditure                   -0.2556354
## Hepatitis.B                              -0.1332510
## Measles                                   0.1749462
## BMI                                      -0.5540940
## under.five.deaths                         0.4622894
## Polio                                    -0.1744893
## Total.expenditure                        -0.2178648
## Diphtheria                               -0.1809524
## HIV.AIDS                                  0.1831467
## GDP                                      -0.2779586
## Population                                0.2779134
## thinness..1.19.years                      0.9279134
## thinness.5.9.years                        1.0000000
## Income.composition.of.resources          -0.4384837
## Schooling                                -0.4724820
##                                Income.composition.of.resources   Schoolin
g
## Adult.Mortality                                    -0.442203288 -0.4211705
2
## infant.deaths                                      -0.134753863 -0.2143719
0
## Alcohol                                             0.561074332  0.6169748
1
## percentage.expenditure                              0.402169736  0.4220884
5
## Hepatitis.B                                         0.184920970  0.2151815
9
## Measles                                            -0.058277256 -0.1156604
8
## BMI                                                 0.510504831  0.5548439
0
## under.five.deaths                                  -0.148097276 -0.2260126
2
## Polio                                               0.314681594  0.3501466
0
## Total.expenditure                                   0.183653190  0.2437834
5
## Diphtheria                                          0.343261772  0.3503979
3
## HIV.AIDS                                           -0.248589855 -0.2118402
0
## GDP                                                 0.446855511  0.4679469
7
## Population                                         -0.008132466 -0.0403124
2
## thinness..1.19.years                               -0.453678854 -0.4911992
1
## thinness.5.9.years                                 -0.438483721 -0.4724820
3
```

```
## Income.composition.of.resources                      1.000000000  0.7847405
8
## Schooling                                            0.784740581  1.0000000
0

plot(corrm.expect)
```




```
expect_pca <- prcomp(expect[,5:22], scale=TRUE)
summary(expect_pca)

## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6    PC7
## Standard deviation     2.3541 1.6672 1.3567 1.22319 1.08768 0.93668 0.8910
## Proportion of Variance 0.3079 0.1544 0.1023 0.08312 0.06573 0.04874 0.0441
## Cumulative Proportion  0.3079 0.4623 0.5646 0.64769 0.71341 0.76216 0.8063
##                           PC8    PC9    PC10    PC11    PC12    PC13   PC1
4
## Standard deviation     0.8116 0.7237 0.71568 0.64688 0.61936 0.58951 0.575
5
## Proportion of Variance 0.0366 0.0291 0.02846 0.02325 0.02131 0.01931 0.018
4
## Cumulative Proportion  0.8428 0.8720 0.90040 0.92365 0.94496 0.96427 0.982
7
##                           PC15    PC16    PC17    PC18
## Standard deviation     0.4470 0.26619 0.19720 0.04920
## Proportion of Variance 0.0111 0.00394 0.00216 0.00013
## Cumulative Proportion  0.9938 0.99771 0.99987 1.00000
```

```r
plot(expect_pca)
```




```r
# A table containing eigenvalues and %'s accounted, follows. Eigenvalues are the sdev^2
(eigen_expect <- round(expect_pca$sdev^2,2))
```

```
##  [1] 5.54 2.78 1.84 1.50 1.18 0.88 0.79 0.66 0.52 0.51 0.42 0.38 0.35 0.33
0.20
## [16] 0.07 0.04 0.00
```

```r
names(eigen_expect) <- paste("PC",1:18,sep="")
eigen_expect
```

```
##  PC1  PC2  PC3  PC4  PC5  PC6  PC7  PC8  PC9 PC10 PC11 PC12 PC13 PC14 PC15
PC16
## 5.54 2.78 1.84 1.50 1.18 0.88 0.79 0.66 0.52 0.51 0.42 0.38 0.35 0.33 0.20
0.07
## PC17 PC18
## 0.04 0.00
```

```r
sumlambdas <- sum(eigen_expect)
sumlambdas
```

```
## [1] 17.99
```

```r
propvar <- round(eigen_expect/sumlambdas,2)
propvar
```

```
##  PC1  PC2  PC3  PC4  PC5  PC6  PC7  PC8  PC9 PC10 PC11 PC12 PC13 PC14 PC15
PC16
## 0.31 0.15 0.10 0.08 0.07 0.05 0.04 0.04 0.03 0.03 0.02 0.02 0.02 0.02 0.01
0.00
## PC17 PC18
## 0.00 0.00
```

```
cumvar_expect <- cumsum(propvar)
cumvar_expect
```

```
##  PC1  PC2  PC3  PC4  PC5  PC6  PC7  PC8  PC9 PC10 PC11 PC12 PC13 PC14 PC15
PC16
## 0.31 0.46 0.56 0.64 0.71 0.76 0.80 0.84 0.87 0.90 0.92 0.94 0.96 0.98 0.99
0.99
## PC17 PC18
## 0.99 0.99
```

```
matlambdas <- rbind(eigen_expect,propvar,cumvar_expect)
matlambdas
```

```
##                PC1  PC2  PC3  PC4  PC5  PC6  PC7  PC8  PC9 PC10 PC11 PC12
PC13
## eigen_expect  5.54 2.78 1.84 1.50 1.18 0.88 0.79 0.66 0.52 0.51 0.42 0.38
0.35
## propvar       0.31 0.15 0.10 0.08 0.07 0.05 0.04 0.04 0.03 0.03 0.02 0.02
0.02
## cumvar_expect 0.31 0.46 0.56 0.64 0.71 0.76 0.80 0.84 0.87 0.90 0.92 0.94
0.96
##               PC14 PC15 PC16 PC17 PC18
## eigen_expect  0.33 0.20 0.07 0.04 0.00
## propvar       0.02 0.01 0.00 0.00 0.00
## cumvar_expect 0.98 0.99 0.99 0.99 0.99
```

```
rownames(matlambdas) <- c("Eigenvalues","Prop. variance","Cum. prop. variance
")
rownames(matlambdas)
```

```
## [1] "Eigenvalues"         "Prop. variance"      "Cum. prop. variance"
```

```
eigvec.expect <- expect_pca$rotation
print(expect_pca)
```

```
## Standard deviations (1, .., p=18):
##  [1] 2.35411073 1.66723680 1.35671675 1.22318556 1.08768196 0.93667591
##  [7] 0.89095477 0.81162525 0.72368020 0.71568115 0.64687520 0.61936258
## [13] 0.58950583 0.57546855 0.44704909 0.26619070 0.19720320 0.04919734
##
## Rotation (n x k) = (18 x 18):
##                                        PC1          PC2          PC3
## Adult.Mortality                -0.2004584 -0.197310307  0.047004825
## infant.deaths                  -0.2389833  0.457782933 -0.032563118
## Alcohol                         0.2592069  0.186360057  0.083862992
```

```
## percentage.expenditure           0.2274513  0.229337318  0.262332157
## Hepatitis.B                      0.1537582 -0.063165953 -0.525765366
## Measles                         -0.1347974  0.314865840 -0.039859447
## BMI                              0.2801695  0.065428166  0.081492856
## under.five.deaths               -0.2423420  0.449882910 -0.020390046
## Polio                            0.1895116  0.057560556 -0.483930270
## Total.expenditure                0.1421700  0.001086825  0.028229062
## Diphtheria                       0.1974238  0.054925544 -0.525407427
## HIV.AIDS                        -0.1192477 -0.137988286  0.079373493
## GDP                              0.2419891  0.236129897  0.245788557
## Population                      -0.1410780  0.413995042 -0.062351234
## thinness..1.19.years            -0.3231604  0.089683003 -0.166027518
## thinness.5.9.years              -0.3215008  0.089549165 -0.162626860
## Income.composition.of.resources  0.3116863  0.224813986 -0.010804649
## Schooling                        0.3346879  0.187742412 -0.006051922
##                                         PC4          PC5          PC6
## Adult.Mortality                 -0.39963169  0.356355603 -0.198941188
## infant.deaths                    0.04439031  0.123407823  0.032286617
## Alcohol                         -0.18608774  0.257266938 -0.131825729
## percentage.expenditure          -0.39473989 -0.341375383 -0.052215568
## Hepatitis.B                     -0.10862464 -0.024051358  0.001877485
## Measles                          0.10378908  0.197022639 -0.230259219
## BMI                              0.23547390  0.203885315  0.039099624
## under.five.deaths                0.03586327  0.128070858  0.033389976
## Polio                           -0.13051481  0.003077132 -0.077211548
## Total.expenditure               -0.18582291  0.250677647  0.896479864
## Diphtheria                      -0.14162908  0.007339607 -0.072502333
## HIV.AIDS                        -0.49674787  0.471237694 -0.134585447
## GDP                             -0.38340984 -0.327569717 -0.068509119
## Population                       0.04642967  0.148235930  0.077711052
## thinness..1.19.years            -0.21689535 -0.275116332  0.144132072
## thinness.5.9.years              -0.22618967 -0.263775921  0.126960396
## Income.composition.of.resources  0.05285729  0.077094967 -0.053716898
## Schooling                        0.00553038  0.110617567 -0.009506680
##                                         PC7          PC8          PC9
## Adult.Mortality                  0.006331094  0.16898225 -0.02540600
## infant.deaths                   -0.019102876  0.09911247 -0.01011910
## Alcohol                          0.244533384 -0.31648876 -0.26489435
## percentage.expenditure          -0.197067690  0.15168137  0.04919738
## Hepatitis.B                     -0.135020597  0.20895659  0.43664302
## Measles                         -0.593168221 -0.46699758  0.27347439
## BMI                              0.160932301  0.32331209  0.53938947
## under.five.deaths               -0.012852856  0.10170343 -0.01894610
## Polio                           -0.051633910 -0.02388779 -0.39959824
## Total.expenditure               -0.228965700 -0.12177321 -0.03588565
## Diphtheria                      -0.103664095  0.06676123 -0.07714284
## HIV.AIDS                         0.079506564  0.02039929  0.20317354
## GDP                             -0.173233252  0.13458443  0.05543654
## Population                       0.146450803  0.49001905 -0.21695920
## thinness..1.19.years             0.290904942 -0.19695916  0.20783199
```

```
## thinness.5.9.years                0.318463589 -0.21723302  0.20534341
## Income.composition.of.resources  0.307181341 -0.23731677  0.12481330
## Schooling                         0.317486439 -0.20253133  0.10313809
##                                          PC10        PC11         PC12
## Adult.Mortality                  -0.185501837  0.35230797 -0.586025735
## infant.deaths                    -0.060953676  0.12901175  0.235435088
## Alcohol                          -0.415649327  0.32915190  0.257335612
## percentage.expenditure            0.034417162  0.02227256 -0.007753590
## Hepatitis.B                      -0.483253664 -0.02094629  0.153186592
## Measles                           0.108599109 -0.02088114 -0.240705976
## BMI                               0.328154109  0.44960979 -0.001119666
## under.five.deaths                -0.078869249  0.15083484  0.256505396
## Polio                             0.555065814  0.31527393  0.031924419
## Total.expenditure                -0.007401458  0.02758396 -0.081999570
## Diphtheria                       -0.031283924 -0.18274533 -0.062760796
## HIV.AIDS                          0.324298881 -0.39287465  0.403291378
## GDP                               0.028699293  0.01454476 -0.018616172
## Population                       -0.005681116 -0.33718660 -0.315231910
## thinness..1.19.years              0.093658014  0.10999823 -0.068770060
## thinness.5.9.years                0.064720514  0.08952949 -0.088157318
## Income.composition.of.resources  0.008192825 -0.28103714 -0.245956370
## Schooling                         0.022506954 -0.15399644 -0.201547541
##                                          PC13         PC14         PC15
## Adult.Mortality                   0.179814157  0.167451005  0.021061480
## infant.deaths                     0.267878792  0.198553817  0.069085595
## Alcohol                          -0.252915049 -0.337122441 -0.132642090
## percentage.expenditure            0.004236727 -0.007377541 -0.011011432
## Hepatitis.B                      -0.302588997  0.285009234 -0.024469525
## Measles                          -0.208106227 -0.141005693 -0.011882285
## BMI                               0.010791222 -0.272250252 -0.079048523
## under.five.deaths                 0.279018870  0.209595830  0.070627613
## Polio                            -0.214644555  0.288310548 -0.048945600
## Total.expenditure                 0.020383178  0.008269310 -0.034273006
## Diphtheria                        0.552275950 -0.537325140  0.054858450
## HIV.AIDS                         -0.054019911 -0.011482037 -0.016334922
## GDP                               0.005472687  0.031148296  0.006516814
## Population                       -0.445019000 -0.219255401 -0.080591955
## thinness..1.19.years             -0.081206535 -0.124530343 -0.046132712
## thinness.5.9.years               -0.046912838 -0.123768945 -0.006794498
## Income.composition.of.resources  0.238644739  0.343756318 -0.599288278
## Schooling                        -0.029230874  0.172010129  0.768366182
##                                          PC16         PC17          PC18
## Adult.Mortality                  -0.017162250  0.005586258  0.0078961967
## infant.deaths                     0.004350235  0.002979654  0.7155110383
## Alcohol                          -0.005257134 -0.001165953  0.0125548086
## percentage.expenditure            0.018025462  0.696595039  0.0013079706
## Hepatitis.B                       0.006967574  0.011395633  0.0009422315
## Measles                           0.005869773  0.000119301 -0.0152264020
## BMI                               0.017512881 -0.005642224 -0.0026586162
## under.five.deaths                -0.009945064  0.004975588 -0.6979460160
```

```
## Polio                             0.021998197  0.007864963 -0.0033752265
## Total.expenditure                 0.009490640 -0.010642769  0.0009096593
## Diphtheria                       -0.016976211 -0.006230969 -0.0073159670
## HIV.AIDS                         -0.001413983 -0.001456862  0.0006191748
## GDP                              -0.008438620 -0.715948052 -0.0005133846
## Population                        0.007811473 -0.005429426 -0.0145732679
## thinness..1.19.years             -0.707032132  0.009918310  0.0039651114
## thinness.5.9.years                0.704754206 -0.014686006 -0.0100607285
## Income.composition.of.resources  -0.006699827  0.020496559 -0.0061530136
## Schooling                        -0.035087667  0.031082395 -0.0045010519
```

```
# Taking the first four PCs to generate linear combinations for all the varia
bles with four factors
pcafactors.expect <- eigvec.expect[,1:8]
pcafactors.expect
```

```
##                                        PC1          PC2          PC3
## Adult.Mortality                  -0.2004584 -0.197310307  0.047004825
## infant.deaths                    -0.2389833  0.457782933 -0.032563118
## Alcohol                           0.2592069  0.186360057  0.083862992
## percentage.expenditure            0.2274513  0.229337318  0.262332157
## Hepatitis.B                       0.1537582 -0.063165953 -0.525765366
## Measles                          -0.1347974  0.314865840 -0.039859447
## BMI                               0.2801695  0.065428166  0.081492856
## under.five.deaths                -0.2423420  0.449882910 -0.020390046
## Polio                             0.1895116  0.057560556 -0.483930270
## Total.expenditure                 0.1421700  0.001086825  0.028229062
## Diphtheria                        0.1974238  0.054925544 -0.525407427
## HIV.AIDS                         -0.1192477 -0.137988286  0.079373493
## GDP                               0.2419891  0.236129897  0.245788557
## Population                       -0.1410780  0.413995042 -0.062351234
## thinness..1.19.years             -0.3231604  0.089683003 -0.166027518
## thinness.5.9.years               -0.3215008  0.089549165 -0.162626860
## Income.composition.of.resources   0.3116863  0.224813986 -0.010804649
## Schooling                         0.3346879  0.187742412 -0.006051922
##                                        PC4          PC5          PC6
## Adult.Mortality                  -0.39963169  0.356355603 -0.198941188
## infant.deaths                     0.04439031  0.123407823  0.032286617
## Alcohol                          -0.18608774  0.257266938 -0.131825729
## percentage.expenditure           -0.39473989 -0.341375383 -0.052215568
## Hepatitis.B                      -0.10862464 -0.024051358  0.001877485
## Measles                           0.10378908  0.197022639 -0.230259219
## BMI                               0.23547390  0.203885315  0.039099624
## under.five.deaths                 0.03586327  0.128070858  0.033389976
## Polio                            -0.13051481  0.003077132 -0.077211548
## Total.expenditure                -0.18582291  0.250677647  0.896479864
## Diphtheria                       -0.14162908  0.007339607 -0.072502333
## HIV.AIDS                         -0.49674787  0.471237694 -0.134585447
## GDP                              -0.38340984 -0.327569717 -0.068509119
## Population                        0.04642967  0.148235930  0.077711052
```

```
## thinness..1.19.years             -0.21689535 -0.275116332  0.144132072
## thinness.5.9.years               -0.22618967 -0.263775921  0.126960396
## Income.composition.of.resources   0.05285729  0.077094967 -0.053716898
## Schooling                         0.00553038  0.110617567 -0.009506680
##                                          PC7          PC8
## Adult.Mortality                   0.006331094  0.16898225
## infant.deaths                    -0.019102876  0.09911247
## Alcohol                           0.244533384 -0.31648876
## percentage.expenditure           -0.197067690  0.15168137
## Hepatitis.B                      -0.135020597  0.20895659
## Measles                          -0.593168221 -0.46699758
## BMI                               0.160932301  0.32331209
## under.five.deaths                -0.012852856  0.10170343
## Polio                            -0.051633910 -0.02388779
## Total.expenditure                -0.228965700 -0.12177321
## Diphtheria                       -0.103664095  0.06676123
## HIV.AIDS                          0.079506564  0.02039929
## GDP                              -0.173233252  0.13458443
## Population                        0.146450803  0.49001905
## thinness..1.19.years              0.290904942 -0.19695916
## thinness.5.9.years                0.318463589 -0.21723302
## Income.composition.of.resources   0.307181341 -0.23731677
## Schooling                         0.317486439 -0.20253133
```

```
# Multiplying each column of the eigenvectorâ□□s matrix by the square-root of
the corresponding eigenvalue in order to get the factor loadings
unrot.fact.expect <- sweep(pcafactors.expect,MARGIN=2,expect_pca$sdev[1:8],`*
`)
unrot.fact.expect
```

```
##                                         PC1          PC2          PC3
## Adult.Mortality                  -0.4719013 -0.328963006  0.063772234
## infant.deaths                    -0.5625932  0.763232554 -0.044178928
## Alcohol                           0.6102016  0.310706345  0.113778326
## percentage.expenditure            0.5354456  0.382359617  0.355910432
## Hepatitis.B                       0.3619638 -0.105312602 -0.713314680
## Measles                          -0.3173281  0.524955917 -0.054077979
## BMI                               0.6595500  0.109084246  0.110562723
## under.five.deaths                -0.5704999  0.750061345 -0.027663517
## Polio                             0.4461313  0.095967078 -0.656556304
## Total.expenditure                 0.3346840  0.001811995  0.038298842
## Diphtheria                        0.4647575  0.091573889 -0.712829059
## HIV.AIDS                         -0.2807222 -0.230059150  0.107687347
## GDP                               0.5696691  0.393684455  0.333465454
## Population                       -0.3321132  0.690227771 -0.084592963
## thinness..1.19.years             -0.7607553  0.149522804 -0.225252316
## thinness.5.9.years               -0.7568484  0.149299664 -0.220638585
## Income.composition.of.resources   0.7337441  0.374818152 -0.014658848
## Schooling                         0.7878924  0.313011058 -0.008210744
##                                         PC4          PC5          PC6
```

```
## Adult.Mortality                  -0.48882371  0.387601560 -0.186343419
## infant.deaths                     0.05429759  0.134228462  0.030242096
## Alcohol                          -0.22761983  0.279824606 -0.123477985
## percentage.expenditure           -0.48284013 -0.371307845 -0.048909065
## Hepatitis.B                      -0.13286810 -0.026160229  0.001758595
## Measles                           0.12695331  0.214297969 -0.215678264
## BMI                               0.28802828  0.221762379  0.036623676
## under.five.deaths                 0.04386743  0.139300361  0.031275586
## Polio                            -0.15964383  0.003346941 -0.072322197
## Total.expenditure                -0.22729590  0.272657554  0.839711096
## Diphtheria                       -0.17323864  0.007983158 -0.067911189
## HIV.AIDS                         -0.60761482  0.512556738 -0.126062947
## GDP                              -0.46898138 -0.356291671 -0.064170842
## Population                        0.05679210  0.161233546  0.072790071
## thinness..1.19.years             -0.26530326 -0.299239071  0.135005040
## thinness.5.9.years               -0.27667193 -0.286904310  0.118920745
## Income.composition.of.resources   0.06465427  0.083854805 -0.050315324
## Schooling                         0.00676468  0.120316732 -0.008904678
##                                          PC7         PC8
## Adult.Mortality                   0.005640719  0.13715026
## infant.deaths                    -0.017019799  0.08044218
## Alcohol                           0.217868185 -0.25687027
## percentage.expenditure           -0.175578398  0.12310843
## Hepatitis.B                      -0.120297245  0.16959445
## Measles                          -0.528486056 -0.37902703
## BMI                               0.143383401  0.26240826
## under.five.deaths                -0.011451314  0.08254507
## Polio                            -0.046003479 -0.01938793
## Total.expenditure                -0.203998083 -0.09883421
## Diphtheria                       -0.092360019  0.05418510
## HIV.AIDS                          0.070836753  0.01655658
## GDP                              -0.154342992  0.10923212
## Population                        0.130481042  0.39771183
## thinness..1.19.years              0.259183145 -0.15985702
## thinness.5.9.years                0.283736653 -0.17631181
## Income.composition.of.resources   0.273684681 -0.19261228
## Schooling                         0.282866057 -0.16437954
```

```
# Computing communalities
communalities.expect <- rowSums(unrot.fact.expect^2)
communalities.expect
```

```
##                  Adult.Mortality                      infant.deaths
##                        0.7777239                         0.9296275
##                          Alcohol           percentage.expenditure
##                        0.7406383                         0.9789528
##                      Hepatitis.B                           Measles
##                        0.7125015                         0.9107171
##                              BMI                 under.five.deaths
##                        0.6820267                         0.9180793
```

```
##                            Polio                Total.expenditure
##                        0.6725291                        0.9959872
##                      Diphtheria                         HIV.AIDS
##                        0.7786643                        0.7964228
##                              GDP                       Population
##                        0.9774681                        0.8035895
##            thinness..1.19.years              thinness.5.9.years
##                        0.9227307                        0.9283872
## Income.composition.of.resources                         Schooling
##                        0.8048302                        0.8404528
```

```
# Performing the varimax rotation. The default in the varimax function is nor
m=TRUE thus, Kaiser normalization is carried out
rot.fact.expect <- varimax(unrot.fact.expect)
View(unrot.fact.expect)
rot.fact.expect
```

```
## $loadings
##
## Loadings:
##                                 PC1    PC2    PC3    PC4    PC5    PC6
## Adult.Mortality                -0.255                -0.820  0.109
## infant.deaths                         0.874  0.132
## Alcohol                         0.781                -0.110 -0.251
## percentage.expenditure          0.221                       -0.954
## Hepatitis.B                           -0.106 -0.825
## Measles                               0.365
## BMI                             0.396                0.212
## under.five.deaths                     0.867  0.148
## Polio                           0.230        -0.779
## Total.expenditure               0.119                              0.975
## Diphtheria                      0.179        -0.856
## HIV.AIDS                                            -0.880
## GDP                             0.264                       -0.939
## Population                            0.893
## thinness..1.19.years           -0.233  0.289                0.115
## thinness.5.9.years             -0.204  0.283        -0.103  0.120
## Income.composition.of.resources  0.779        -0.188  0.267 -0.185
## Schooling                       0.788        -0.210  0.210 -0.200
##                                 PC7    PC8
## Adult.Mortality                       -0.128
## infant.deaths                  -0.286 -0.251
## Alcohol                               0.184
## percentage.expenditure
## Hepatitis.B
## Measles                        -0.877
## BMI                             0.265  0.629
## under.five.deaths              -0.278 -0.253
## Polio
## Total.expenditure                     0.109
```

```
## Diphtheria
## HIV.AIDS                                  -0.102
## GDP                                        0.111
## Population
## thinness..1.19.years                      -0.869
## thinness.5.9.years                        -0.878
## Income.composition.of.resources            0.236
## Schooling                                  0.268
##
##                  PC1   PC2   PC3   PC4   PC5   PC6   PC7   PC8
## SS loadings    2.378 2.636 2.187 1.662 1.990 0.999 1.033 2.286
## Proportion Var 0.132 0.146 0.122 0.092 0.111 0.055 0.057 0.127
## Cumulative Var 0.132 0.279 0.400 0.492 0.603 0.658 0.716 0.843
##
## $rotmat
##             [,1]        [,2]         [,3]        [,4]        [,5]
[,6]
## [1,]  0.52640103 -0.36034985 -0.336277687  0.24353330 -0.33872463  0.13768
0658
## [2,]  0.35765131  0.78244415 -0.024511924  0.25014789 -0.32756010  0.00341
4743
## [3,]  0.06380306 -0.08762933  0.887940126 -0.09089856 -0.36302737  0.03384
2927
## [4,] -0.06676071  0.07791255  0.215014154  0.66692033  0.56079234 -0.17985
3511
## [5,]  0.29076340  0.23150227  0.007120906 -0.59495208  0.46359333  0.25130
2679
## [6,] -0.10516989  0.08627955  0.075906392  0.23854362  0.08904503  0.90476
1912
## [7,]  0.54219628  0.08881104  0.152932841 -0.03621560  0.26870550 -0.22383
2193
## [8,] -0.44371026  0.41863471 -0.149822522 -0.11353274 -0.19142269 -0.12526
6689
##             [,7]        [,8]
## [1,]  0.14535751  0.51559639
## [2,] -0.28573205 -0.08798755
## [3,]  0.02662144  0.24063901
## [4,] -0.06789705  0.38342338
## [5,] -0.18783688  0.44101038
## [6,]  0.26268856 -0.15264954
## [7,]  0.66681461 -0.32632891
## [8,]  0.58564493  0.44408934

# The print method of varimax omits loadings less than abs(0.1). In order to
display all the loadings, it is necessary to ask explicitly the contents of t
he object $loadings
fact.load.expect <- rot.fact.expect$loadings[,1:8]
fact.load.expect
```

```
##                                         PC1           PC2          PC3
## Adult.Mortality                 -0.254858862  0.0005504121  0.087205188
## infant.deaths                   -0.038695120  0.8738668487  0.131522194
## Alcohol                          0.781242992 -0.0385397747 -0.096302365
## percentage.expenditure           0.220912080 -0.0168145109 -0.028873775
## Hepatitis.B                     -0.032035249 -0.1062696737 -0.824947460
## Measles                         -0.024588613  0.3651204582  0.034239970
## BMI                              0.395964579  0.0375216708 -0.077390094
## under.five.deaths               -0.042365695  0.8667886314  0.147577071
## Polio                            0.230173354 -0.0582463185 -0.779281287
## Total.expenditure                0.118657358 -0.0641728315 -0.078165346
## Diphtheria                       0.178828897 -0.0363866376 -0.856071596
## HIV.AIDS                         0.010734018 -0.0146239754  0.067447547
## GDP                              0.264262442 -0.0190029209 -0.053334876
## Population                      -0.003649629  0.8932696200 -0.001096928
## thinness..1.19.years            -0.233393207  0.2886698352  0.066810109
## thinness.5.9.years              -0.204471008  0.2825571882  0.072241333
## Income.composition.of.resources  0.778574654 -0.0060649635 -0.187552987
## Schooling                        0.787946721 -0.0543647282 -0.210391173
##                                         PC4         PC5         PC6
## Adult.Mortality                 -0.819847086  0.10867867 -0.06565416
## infant.deaths                    0.012976927  0.03199601 -0.03128599
## Alcohol                         -0.110483781 -0.25097454  0.07187663
## percentage.expenditure           0.073302922 -0.95382823  0.06022871
## Hepatitis.B                      0.039119070  0.01957161  0.04993078
## Measles                          0.026845735  0.03704775 -0.04207268
## BMI                              0.211765212 -0.04338500  0.06701899
## under.five.deaths               -0.003912099  0.03068094 -0.02928492
## Polio                            0.070488037 -0.04726818  0.01637652
## Total.expenditure               -0.016411216 -0.09005056  0.97456198
## Diphtheria                       0.061593428 -0.06333134  0.02578247
## HIV.AIDS                        -0.880396640  0.03286407  0.07031089
## GDP                              0.083985106 -0.93924537  0.04867754
## Population                       0.008902193 -0.01087973 -0.02909554
## thinness..1.19.years            -0.085327429  0.11524207 -0.05517912
## thinness.5.9.years              -0.102629631  0.11997499 -0.06732847
## Income.composition.of.resources  0.266966972 -0.18492836  0.02859589
## Schooling                        0.210146391 -0.20017544  0.08750792
##                                         PC7         PC8
## Adult.Mortality                  0.022614789 -0.12799855
## infant.deaths                   -0.286227194 -0.25118091
## Alcohol                         -0.071752428  0.18446859
## percentage.expenditure           0.022753959  0.09862747
## Hepatitis.B                      0.097219221  0.07606355
## Measles                         -0.877468893 -0.04256997
## BMI                              0.265342429  0.62945751
## under.five.deaths               -0.278201728 -0.25292788
## Polio                           -0.030868596  0.02129381
## Total.expenditure                0.040040078  0.10921062
## Diphtheria                      -0.015016550  0.06170175
```

```
## HIV.AIDS                          -0.003408854 -0.10203417
## GDP                                0.022158040  0.11104907
## Population                         0.057156749 -0.03651462
## thinness..1.19.years               0.029591876 -0.86947255
## thinness.5.9.years                 0.031312151 -0.87811220
## Income.composition.of.resources    0.035503837  0.23641213
## Schooling                          0.091822996  0.26842496
```

```
# Computing the rotated factor scores for the 30 European Countries. Notice t
hat signs are reversed for factors F2 (PC2), F3 (PC3) and F4 (PC4)
scale.expect <- scale(expect[,5:22])
scale.expect
```

```
##      Adult.Mortality infant.deaths      Alcohol percentage.expenditure
## 1         0.756399351   0.243670849 -1.122607025          -0.3568005157
## 2         0.820240811   0.260220675 -1.122607025          -0.3555249810
## 3         0.796300263   0.276770502 -1.122607025          -0.3556979766
## 4         0.828220994   0.301595241 -1.122607025          -0.3528757337
## 5         0.852161541   0.318145067 -1.122607025          -0.3932838289
## 6         0.884082271   0.342969806 -1.122607025          -0.3520258433
## 7         0.900042636   0.367794546 -1.122607025          -0.3650526557
## 8         0.947923731   0.392619285 -1.117643247          -0.3826105095
## 9         1.011765192   0.409169111 -1.120125136          -0.3911163757
## 10        1.011765192   0.425718937 -1.117643247          -0.3875572255
## 11        0.979844462   0.433993851 -1.120125136          -0.3965286935
## 12        0.995804827   0.450543677 -1.120125136          -0.3886232895
## 13        1.011765192   0.450543677 -1.122607025          -0.3910146853
## 14       -1.318448107   0.458818590 -1.122607025          -0.3877187545
## 15        1.179349025   0.458818590 -1.122607025          -0.3913070432
## 16        1.219249937   0.458818590 -1.122607025          -0.3913921760
## 17       -0.751855147  -0.269373763  0.016580042          -0.1898549112
## 18       -1.278547194  -0.269373763 -0.005756959          -0.1536039024
## 19       -0.672053322  -0.269373763  0.056290267          -0.1523943320
## 20       -0.656092957  -0.269373763  0.150602050          -0.1628725692
## 21       -0.640132592  -0.269373763  0.207685498          -0.1488785189
## 22       -0.616192044  -0.261098850  0.185348497          -0.3735447035
## 23       -0.616192044  -0.261098850  0.311924837          -0.1994723480
## 24       -1.334408472  -0.261098850  0.267250835          -0.3765009348
## 25       -1.270567012  -0.261098850  0.259805168          -0.3789881126
## 26       -0.552350584  -0.261098850  0.192794164          -0.3954409976
## 27       -1.222685917  -0.261098850  0.155565828          -0.3819743241
## 28       -1.206725552  -0.261098850  0.001688708          -0.2712157808
## 29       -1.198745369  -0.261098850 -0.060358518          -0.3889511476
## 30       -1.222685917  -0.261098850 -0.199344304          -0.3379074178
## 31       -1.230666099  -0.261098850 -0.070286074          -0.3426318416
## 32       -1.254606647  -0.261098850 -0.216717527          -0.3451863864
## 34       -1.254606647  -0.095600588 -1.122607025          -0.3664878856
## 35       -0.448608211  -0.095600588 -0.993548795          -0.0878355151
## 36       -0.440628029  -0.095600588 -0.961284238          -0.0813125791
## 37       -0.416687481  -0.095600588 -0.986103128          -0.1079856467
```

```
##       Hepatitis.B      Measles          BMI under.five.deaths       Polio
## 1    -0.555278045 -0.106138732 -0.963267347       0.238062265 -3.45490684
## 2    -0.672444199 -0.171775555 -0.988578356       0.256478698 -1.13870600
## 3    -0.594333430 -0.177922810 -1.013889366       0.274895131 -0.96053671
## 4    -0.477167275  0.055772041 -1.039200376       0.299450374 -0.73782509
## 5    -0.438111891  0.078179778 -1.059449184       0.324005617 -0.69328277
## 6    -0.516222660 -0.023349084 -1.084760194       0.354699671 -0.78236741
## 7    -0.633388815  0.063109087 -1.110071203       0.379254915 -0.91599439
## 8    -0.594333430 -0.062017303 -1.135382213       0.403810158 -0.87145206
## 9    -0.633388815 -0.107427673 -1.160693223       0.422226591 -0.91599439
## 10   -0.594333430 -0.023249935 -1.186004233       0.440643023 -1.13870600
## 11   -0.516222660 -0.092059535 -1.211315243       0.452920645 -1.13870600
## 12   -0.477167275 -0.174353436 -1.231564050       0.465198267 -3.49944916
## 13   -0.555278045 -0.141435876 -1.251812858       0.477475888 -1.89592551
## 14   -0.594333430  0.025928108 -1.272061666       0.477475888 -2.11863713
## 15   -0.633388815  0.648188984 -1.292310474       0.477475888 -2.16317945
## 16   -0.672444199  0.427086091 -1.312559282       0.477475888 -2.65314501
## 17    0.772605038 -0.220557000  1.005929215      -0.271459034  0.68752927
## 18    0.733549653 -0.220557000  0.965431599      -0.265320224  0.64298695
## 19    0.772605038 -0.220557000  0.929996185      -0.265320224  0.68752927
## 20    0.772605038 -0.219664657  0.894560772      -0.265320224  0.68752927
## 21    0.772605038 -0.217780820  0.859125358      -0.265320224  0.68752927
## 22    0.772605038 -0.219565508  0.818627742      -0.265320224  0.68752927
## 23    0.733549653 -0.220557000  0.778130127      -0.265320224  0.64298695
## 24    0.772605038 -0.220557000  0.732570309      -0.265320224  0.68752927
## 25    0.733549653 -0.218375716  0.687010491      -0.265320224  0.68752927
## 26    0.733549653 -0.213814849 -1.636540207      -0.265320224  0.59844462
## 27    0.733549653 -0.219962105  0.595890856      -0.265320224  0.59844462
## 28    0.772605038 -0.219862955  0.545268837      -0.265320224  0.64298695
## 29    0.694494268 -0.219763806  0.494646817      -0.265320224  0.59844462
## 30    0.655438883 -0.218970612  0.444024798      -0.265320224  0.64298695
## 31    0.655438883 -0.218772313  0.398464980      -0.265320224  0.59844462
## 32    0.655438883 -0.154920177  0.347842960      -0.265320224  0.59844462
## 34    0.616383498 -0.220557000  1.026178023      -0.124127574  0.50935998
## 35    0.616383498 -0.218078268  0.965431599      -0.124127574  0.50935998
## 36    0.616383498 -0.218772313  0.909747378      -0.124127574  0.50935998
## 37    0.616383498 -0.209452281  0.854063156      -0.124127574  0.50935998
## 38    0.616383498 -0.210344625  0.798378935      -0.124127574  0.50935998
## 39    0.577328114 -0.209948027  0.742694713      -0.130266385  0.46481765
## 40    0.460161959 -0.199041607  0.692072693      -0.130266385  0.37573300
## 41   -2.742379593 -0.220557000 -1.636540207      -0.130266385  0.50935998
## 42   -2.781434977 -0.126960081  0.590828654      -0.130266385  0.50935998
## 43    0.147718881  0.007684640  0.545268837      -0.136405196  0.19756371
## 44    0.069608112  0.105544979  0.494646817      -0.130266385  0.10847906
## 50   -0.594333430  0.939390417 -0.781028076       0.348560861 -0.69328277
## 51   -0.086613428  0.624492306 -0.811401288       0.373116104 -0.73782509
## 52   -0.164724197  0.221450486 -0.841774500       0.403810158 -0.38148650
## 53   -0.281890352 -0.076889695 -0.867085509       0.434504212 -0.47057115
## 54   -0.086613428 -0.102569358 -1.808655074       0.471337078 -0.11423256
```

```
##      Total.expenditure   Diphtheria     HIV.AIDS           GDP    Populati
on
## 1          0.958549697 -0.887672015 -0.312293880 -0.4341073577  0.27083113
21
## 2          0.967247673 -1.026694824 -0.312293880 -0.4316293556 -0.20332052
09
## 3          0.945502732 -0.934012952 -0.312293880 -0.4299694891  0.24237818
28
## 4          1.115113271 -0.794990143 -0.312293880 -0.4266395522 -0.15550108
88
## 5          0.832429040 -0.748649207 -0.312293880 -0.4794826201 -0.16569630
58
## 6          1.410844467 -0.841331079 -0.312293880 -0.4368025946 -0.16705071
21
## 7          1.506522207 -0.980353888 -0.312293880 -0.4461644435 -0.20393435
51
## 8          1.032482496 -0.934012952 -0.312293880 -0.4524848350 -0.16923259
03
## 9          0.336644388 -0.980353888 -0.312293880 -0.4527920283  0.16978568
43
## 10         0.641073560 -1.212058569 -0.312293880 -0.4612682284 -0.17122074
27
## 11         1.193395058 -1.212058569 -0.312293880 -0.4828150908 -0.20431092
13
## 12         1.232535952 -3.668128191 -0.312293880 -0.4659234116  0.13433579
71
## 13         1.245582916 -1.999854486 -0.312293880 -0.4677021661 -0.17440684
47
## 14         0.784590170 -2.231559167 -0.312293880 -0.4686504659  0.10397752
21
## 15         0.801986122 -2.370581976 -0.312293880 -0.4747806143 -0.16586854
44
## 16         0.975945649 -2.787650402 -0.312293880 -0.4750365402 -0.20380059
20
## 17         0.019168251  0.687919817 -0.312293880 -0.1404512100 -0.20755990
97
## 18        -0.033019607  0.641578881 -0.312293880 -0.0862911048 -0.20386931
15
## 19        -0.128697347  0.687919817 -0.312293880 -0.1003240474 -0.20385968
90
## 20        -0.159140264  0.687919817 -0.312293880 -0.1148857600 -0.20792794
62
## 21        -0.106952406  0.687919817 -0.312293880 -0.0983672910 -0.20378016
92
## 22        -0.267864968  0.687919817 -0.312293880 -0.4419411988 -0.20383515
04
## 23        -0.072160501  0.641578881 -0.312293880 -0.1265169030 -0.16642125
20
## 24        -0.037368595  0.687919817 -0.312293880 -0.4468923734 -
```

```
##         Schooling
## 1    -0.722579898
## 2    -0.758353113
## 3    -0.794126328
## 4    -0.829899544
## 5    -0.937219189
## 6    -1.044538835
## 7    -1.151858480
## 8    -1.223404911
## 9    -1.330724556
## 10   -1.438044202
## 11   -1.509590632
## 12   -1.903096000
## 13   -2.010415645
## 14   -2.117735291
## 15   -2.225054936
## 16   -2.368147797
## 17    0.744121925
## 18    0.744121925
## 19    0.744121925
## 20    0.744121925
## 21    0.422162988
## 22    0.135977267
## 23    0.028657621
## 24   -0.042888809
## 25   -0.185981670
## 26   -0.257528100
## 27   -0.472167392
## 28   -0.436394176
## 29   -0.507940607
## 30   -0.507940607
## 31   -0.543713822
## 32   -0.507940607
## 34    0.815668356
## 35    0.815668356
## 36    0.815668356
## 37    0.672575495
## 38    0.529482634
## 39    0.350616558
## 40    0.171750482
## 41    0.064430836
## 42    0.064430836
## 43   -0.042888809
## 44   -0.150208455
## 50   -0.257528100
## 51   -0.257528100
## 52   -0.651033468
## 53   -0.972992404
## 54   -1.116085265
## 55   -1.294951341
```

```
## 56   -1.438044202
## attr(,"scaled:center")
##                 Adult.Mortality                    infant.deaths
##                    1.682153e+02                     3.255306e+01
##                         Alcohol           percentage.expenditure
##                    4.533196e+00                     6.989736e+02
##                     Hepatitis.B                          Measles
##                    7.921771e+01                     2.224494e+03
##                             BMI                  under.five.deaths
##                    3.812862e+01                     4.422013e+01
##                           Polio                 Total.expenditure
##                    8.356458e+01                     5.955925e+00
##                      Diphtheria                         HIV.AIDS
##                    8.415525e+01                     1.983869e+00
##                             GDP                       Population
##                    5.566032e+03                     1.465363e+07
##             thinness..1.19.years                 thinness.5.9.years
##                    4.850637e+00                     4.907762e+00
## Income.composition.of.resources                        Schooling
##                    6.315512e-01                     1.211989e+01
## attr(,"scaled:scale")
##                 Adult.Mortality                    infant.deaths
##                    1.253104e+02                     1.208472e+02
##                         Alcohol           percentage.expenditure
##                    4.029189e+00                     1.759229e+03
##                     Hepatitis.B                          Measles
##                    2.560466e+01                     1.008580e+04
##                             BMI                  under.five.deaths
##                    1.975425e+01                     1.628980e+02
##                           Polio                 Total.expenditure
##                    2.245056e+01                     2.299385e+00
##                      Diphtheria                         HIV.AIDS
##                    2.157919e+01                     6.032360e+00
##                             GDP                       Population
##                    1.147590e+04                     7.046039e+07
##             thinness..1.19.years                 thinness.5.9.years
##                    4.599228e+00                     4.653757e+00
## Income.composition.of.resources                        Schooling
##                    1.830887e-01                     2.795388e+00
```

```
as.matrix(scale.expect)%*%fact.load.expect%*%solve(t(fact.load.expect)%*%fact
.load.expect)
```

```
##                PC1           PC2          PC3           PC4           PC5
## 1   -0.4512141098 -0.0035488089  1.726400437  0.4544201925  1.340602e-01
## 2   -0.3156043928 -0.2606289987  0.994612228  0.4001947843  1.728338e-01
## 3   -0.3892192342 -0.0137389720  0.815163846  0.3783699080  1.519083e-01
## 4   -0.4031564817 -0.2230888512  0.640954348  0.3819238992  1.551339e-01
## 5   -0.4353406385 -0.2227666207  0.572251437  0.3610355131  1.773233e-01
## 6   -0.5166348387 -0.1980872490  0.688512787  0.3675528388  1.346304e-01
```

```
## 7    -0.5557188797 -0.2269722386  0.845100517  0.3781270384  1.325677e-01
## 8    -0.5552722924 -0.1881343944  0.774578544  0.3333742773  1.139923e-01
## 9    -0.6173749888 -0.0093080903  0.775127733  0.2602019280  5.778483e-02
## 10   -0.6645692473 -0.1943724585  0.966073278  0.3045146928  6.084375e-02
## 11   -0.7528268728 -0.1926777940  0.944130459  0.3371701741  8.355668e-02
## 12   -0.9477385070 -0.0822786799  2.841446015  0.4632628014  7.233900e-03
## 13   -0.9950303591 -0.1928691772  1.561429708  0.3666949792  1.109794e-03
## 14   -1.7070303213 -0.0648365747  1.828328351  1.3717149081 -3.711897e-02
## 15   -2.3595138544 -0.0322978192  1.815740593 -0.2383440357 -1.306260e-01
## 16   -2.4144508227 -0.0471718499  2.203278129 -0.2238396303 -1.451397e-01
## 17    0.3230471083  0.0116092315 -0.743122852  0.5007143969  4.455481e-01
## 18    0.3249821992 -0.0249691720 -0.666314351  0.7902017054  4.049337e-01
## 19    0.3357813490 -0.0024668456 -0.751177524  0.4455053256  3.872335e-01
## 20    0.3708504279 -0.0181161351 -0.748063351  0.4158716809  3.979624e-01
## 21    0.2180566236 -0.0219168520 -0.760926284  0.3891567235  3.390457e-01
## 22    0.1296862844 -0.0289571170 -0.780082728  0.3742613462  6.065298e-01
## 23    0.0701560026 -0.0223553685 -0.723491494  0.3495746128  3.298780e-01
## 24    0.0759826177 -0.0684723450 -0.746734773  0.7481867157  6.132300e-01
## 25   -0.0081700638 -0.0978125763 -0.716820894  0.7185604069  5.960265e-01
## 26   -0.1922282648 -0.5639484869 -0.776195686  0.3222843990  3.530146e-01
## 27   -0.2328912817 -0.1031589330 -0.711491535  0.7077746160  5.417996e-01
## 28   -0.3561456680 -0.0990700152 -0.737696853  0.7274765522  3.666213e-01
## 29   -0.3739778113 -0.1146182154 -0.696524660  0.7405816383  5.197116e-01
## 30   -0.4647952648 -0.1208594773 -0.727718487  0.7777543430  4.158488e-01
## 31   -0.4211825441 -0.1431650456 -0.688527649  0.7546923964  4.149856e-01
## 32   -0.4981332597 -0.1518987207 -0.698541398  0.7999187078  4.157157e-01
## 34    0.0713892906  0.3850318311 -0.618433643  1.1294694802  6.808512e-01
## 35    0.0035182731  0.4087852229 -0.656764150  0.6542231554  2.553805e-01
## 36    0.0311440719  0.3828140667 -0.666532252  0.6295614275  2.244089e-01
## 37   -0.0391293455  0.3630353994 -0.686387068  0.6061431417  2.039979e-01
## 38   -0.1227693611  0.3486324861 -0.699399952  0.5923411974  2.439346e-01
## 39   -0.1579294190  0.0874966351 -0.638320091  0.6028929320  2.831199e-01
## 40   -0.1459382284  0.0559789605 -0.533834844  0.5873392037  4.988012e-01
## 41    0.2916149903 -0.6905315068  0.733385366  0.6236811574  5.795788e-02
## 42    0.3484064001  0.0549637040  0.761905710  0.6044923525  2.889034e-01
## 43   -0.2173831541  0.1917128122 -0.293600605  0.5244433668  4.670666e-01
## 44   -0.2469055166 -0.1076864219 -0.111308872  1.0850499224  3.007191e-01
## 50    0.6321053696 -0.2700884339  0.832781098 -0.8278177613  3.067201e-01
## 51    0.4465411339 -0.1633672578  0.370051131 -0.9052817725  2.865733e-01
## 52    0.2630220260 -0.1349299492  0.291815222 -0.9906831324 -4.732400e-02
## 53    0.0688332528  0.0807360069  0.420236799 -1.0015134713 -8.219050e-02
## 54   -0.0990060879 -0.0895486663  0.041460799 -1.0381494454 -1.460475e-01
## 55   -0.0107078863 -0.0367493612  2.054899371 -0.8219303905 -3.216408e-02
## 56   -0.2566484539 -0.0170038896  0.629405587 -0.9701078052 -1.441624e-01
## 57   -0.4386930861  0.0203814300  0.279442825 -0.9838336448 -1.198840e-01
## 82    1.2673492150  0.3536706312 -0.394816778  0.1428158289  7.613679e-02
## 83    1.2891362355  0.3424974449 -0.506170637  0.1132363944 -2.392704e-04
## 84    1.3729824338 -0.0068248959 -0.306978195  0.6128811856 -2.760604e-04
## 85    1.0955601931 -0.3421115554 -0.319570294  0.5803687111 -3.913690e-01
## 86    1.2847628805  0.3223727125 -0.431865325  0.1474744297  8.092211e-01
```

```
## 87    1.0866837744  0.0320327849 -0.421551016  0.1385944336  9.722595e-02

##                PC6           PC7          PC8
## 1     1.4157116806  9.803565e-01 -2.497064991
## 2     1.3468217202  6.873352e-01 -2.818187237
## 3     1.3156435375  8.466048e-01 -2.778177621
## 4     1.4969930922  4.574028e-01 -2.917805415
## 5     1.2128836950  4.386059e-01 -2.944758387
## 6     1.8190852928  5.191189e-01 -3.010939416
## 7     1.9405098603  4.276567e-01 -3.054968767
## 8     1.4454794267  5.701484e-01 -3.070669230
## 9     0.7265826721  7.644820e-01 -2.985719585
## 10    1.0766482257  5.738172e-01 -3.082406045
## 11    1.6604313629  6.082536e-01 -3.146130954
## 12    1.8789681545  1.106402e+00 -2.890467573
## 13    1.7872108387  7.421654e-01 -3.114829552
## 14    1.4495277046  7.425624e-02 -1.227704520
## 15    1.1085537193 -1.012650e+00  1.048249966
## 16    1.3193649017 -7.697604e-01  1.034736610
## 17   -0.0991430786  2.363540e-01  0.816711333
## 18   -0.1041433170  2.041771e-01  0.748105033
## 19   -0.2593230557  2.229672e-01  0.781063039
## 20   -0.2914803261  1.981390e-01  0.764433653
## 21   -0.2305741363  1.650964e-01  0.769166139
## 22   -0.3693397270  1.417628e-01  0.822277961
## 23   -0.1812304227  1.266529e-01  0.774270353
## 24   -0.0585889035  7.028465e-02  0.690202180
## 25    0.0488605676  3.292069e-02  0.679874537
## 26   -0.0605832870 -6.473796e-01 -0.006311376
## 27    0.0754081161  2.084760e-02  0.687000561
## 28    0.1888239173  3.016355e-02  0.668143335
## 29    0.1587141065  2.126863e-02  0.656751667
## 30    0.1738390610  1.694016e-02  0.624715617
## 31    0.0437483546 -4.694854e-03  0.599876971
## 32    0.1735029603 -6.270932e-02  0.586428330
## 34    0.6339857429  8.292303e-01 -0.025156278
## 35    0.4936607942  8.322674e-01  0.032096686
## 36    0.0526734349  8.203374e-01  0.031766408
## 37   -0.3243410955  7.968147e-01  0.058509019
## 38   -0.3907613607  7.735565e-01  0.073704342
## 39   -0.2647590806  5.741508e-01 -0.040483816
## 40   -0.7608666015  5.553908e-01  0.006347187
## 41   -0.9353354708 -4.345508e-01 -0.990383265
## 42   -1.1942800409  3.913900e-01 -0.130755076
## 43   -1.1779909686  4.837750e-01  0.096183650
## 44   -0.9449974878  1.670608e-01 -0.133851007
## 50   -1.1240835367 -1.003509e+00 -0.794443577
## 51   -0.7124314602 -7.022370e-01 -0.827726569
## 52   -1.1697050953 -3.752700e-01 -0.861561490
## 53   -1.1243991840  2.831782e-02 -0.793534154
```

```r
#install.packages("psych", lib="/Library/Frameworks/R.framework/Versions/3.5/Resources/library")
library(psych)
fit.pc <- principal(expect[,5:22], nfactors=8, rotate="varimax")
fit.pc

## Principal Components Analysis
## Call: principal(r = expect[, 5:22], nfactors = 8, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                                   RC2   RC1   RC8   RC3   RC5   RC4   RC7
RC6
## Adult.Mortality                  0.00 -0.25  0.13 -0.09 -0.11  0.82 -0.02
-0.07
## infant.deaths                    0.87 -0.04  0.25 -0.13 -0.03 -0.01  0.29
-0.03
## Alcohol                         -0.04  0.78 -0.18  0.10  0.25  0.11  0.07
0.07
## percentage.expenditure          -0.02  0.22 -0.10  0.03  0.95 -0.07 -0.02
0.06
## Hepatitis.B                     -0.11 -0.03 -0.08  0.82 -0.02 -0.04 -0.10
0.05
## Measles                          0.37 -0.02  0.04 -0.03 -0.04 -0.03  0.88
-0.04
## BMI                              0.04  0.40 -0.63  0.08  0.04 -0.21 -0.27
0.07
## under.five.deaths                0.87 -0.04  0.25 -0.15 -0.03  0.00  0.28
-0.03
## Polio                           -0.06  0.23 -0.02  0.78  0.05 -0.07  0.03
0.02
## Total.expenditure               -0.06  0.12 -0.11  0.08  0.09  0.02 -0.04
0.97
## Diphtheria                      -0.04  0.18 -0.06  0.86  0.06 -0.06  0.02
0.03
## HIV.AIDS                        -0.01  0.01  0.10 -0.07 -0.03  0.88  0.00
0.07
## GDP                             -0.02  0.26 -0.11  0.05  0.94 -0.08 -0.02
0.05
## Population                       0.89  0.00  0.04  0.00  0.01 -0.01 -0.06
-0.03
## thinness..1.19.years             0.29 -0.23  0.87 -0.07 -0.12  0.09 -0.03
-0.06
## thinness.5.9.years               0.28 -0.20  0.88 -0.07 -0.12  0.10 -0.03
-0.07
## Income.composition.of.resources -0.01  0.78 -0.24  0.19  0.18 -0.27 -0.04
0.03
## Schooling                       -0.05  0.79 -0.27  0.21  0.20 -0.21 -0.09
0.09
##                                   h2    u2 com
## Adult.Mortality                 0.78 0.222 1.3
## infant.deaths                   0.93 0.070 1.5
```

```
## Alcohol                          0.74 0.259 1.5
## percentage.expenditure           0.98 0.021 1.2
## Hepatitis.B                      0.71 0.287 1.1
## Measles                          0.91 0.089 1.4
## BMI                              0.68 0.318 2.5
## under.five.deaths                0.92 0.082 1.5
## Polio                            0.67 0.327 1.2
## Total.expenditure                1.00 0.004 1.1
## Diphtheria                       0.78 0.221 1.1
## HIV.AIDS                         0.80 0.204 1.1
## GDP                              0.98 0.023 1.2
## Population                       0.80 0.196 1.0
## thinness..1.19.years             0.92 0.077 1.5
## thinness.5.9.years               0.93 0.072 1.4
## Income.composition.of.resources 0.80 0.195 1.7
## Schooling                        0.84 0.160 1.8
##
##                        RC2  RC1  RC8  RC3  RC5  RC4  RC7  RC6
## SS loadings           2.64 2.38 2.29 2.19 1.99 1.66 1.03 1.00
## Proportion Var        0.15 0.13 0.13 0.12 0.11 0.09 0.06 0.06
## Cumulative Var        0.15 0.28 0.41 0.53 0.64 0.73 0.79 0.84
## Proportion Explained  0.17 0.16 0.15 0.14 0.13 0.11 0.07 0.07
## Cumulative Proportion 0.17 0.33 0.48 0.63 0.76 0.87 0.93 1.00
##
## Mean item complexity =  1.4
## Test of the hypothesis that 8 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.04
##  with the empirical chi square  830.87  with prob <  1.8e-150
##
## Fit based upon off diagonal values = 0.98
```

```
round(fit.pc$values, 3)
```

```
##  [1] 5.542 2.780 1.841 1.496 1.183 0.877 0.794 0.659 0.524 0.512 0.418 0.3
84
## [13] 0.348 0.331 0.200 0.071 0.039 0.002
```

```
fit.pc$loadings
```

```
##
## Loadings:
##                                 RC2    RC1    RC8    RC3    RC5    RC4
## Adult.Mortality                        -0.255  0.128         -0.109  0.820
## infant.deaths                   0.874         0.251 -0.132
## Alcohol                                 0.781 -0.184         0.251  0.110
## percentage.expenditure                  0.221                0.954
## Hepatitis.B                    -0.106                 0.825
## Measles                         0.365
## BMI                                     0.396 -0.629               -0.212
## under.five.deaths               0.867         0.253 -0.148
```

```
## Polio                                    0.230         0.779
## Total.expenditure                        0.119 -0.109
## Diphtheria                               0.179         0.856
## HIV.AIDS                                       0.102                0.880
## GDP                                      0.264 -0.111         0.939
## Population                        0.893
## thinness..1.19.years              0.289 -0.233  0.869        -0.115
## thinness.5.9.years                0.283 -0.204  0.878        -0.120  0.103
## Income.composition.of.resources          0.779 -0.236  0.188  0.185 -0.267
## Schooling                                0.788 -0.268  0.210  0.200 -0.210
##                                   RC7    RC6
## Adult.Mortality
## infant.deaths                      0.286
## Alcohol
## percentage.expenditure
## Hepatitis.B
## Measles                            0.877
## BMI                               -0.265
## under.five.deaths                  0.278
## Polio
## Total.expenditure                         0.975
## Diphtheria
## HIV.AIDS
## GDP
## Population
## thinness..1.19.years
## thinness.5.9.years
## Income.composition.of.resources
## Schooling
##
##                  RC2   RC1   RC8   RC3   RC5   RC4   RC7   RC6
## SS loadings    2.636 2.378 2.286 2.187 1.990 1.662 1.033 0.999
## Proportion Var 0.146 0.132 0.127 0.122 0.111 0.092 0.057 0.055
## Cumulative Var 0.146 0.279 0.406 0.527 0.638 0.730 0.787 0.843

# Loadings with more digits
for (i in c(1,2,3,4,5,6,7,8)) { print(fit.pc$loadings[[1,i]])}

## [1] 0.0005504121
## [1] -0.2548589
## [1] 0.1279985
## [1] -0.08720519
## [1] -0.1086787
## [1] 0.8198471
## [1] -0.02261479
## [1] -0.06565416

# Communalities
fit.pc$communality
```

```
##                  Adult.Mortality                      infant.deaths
##                        0.7777239                         0.9296275
##                          Alcohol           percentage.expenditure
##                        0.7406383                         0.9789528
##                      Hepatitis.B                           Measles
##                        0.7125015                         0.9107171
##                              BMI                 under.five.deaths
##                        0.6820267                         0.9180793
##                            Polio                Total.expenditure
##                        0.6725291                         0.9959872
##                       Diphtheria                           HIV.AIDS
##                        0.7786643                         0.7964228
##                              GDP                        Population
##                        0.9774681                         0.8035895
##            thinness..1.19.years               thinness.5.9.years
##                        0.9227307                         0.9283872
## Income.composition.of.resources                          Schooling
##                        0.8048302                         0.8404528
```

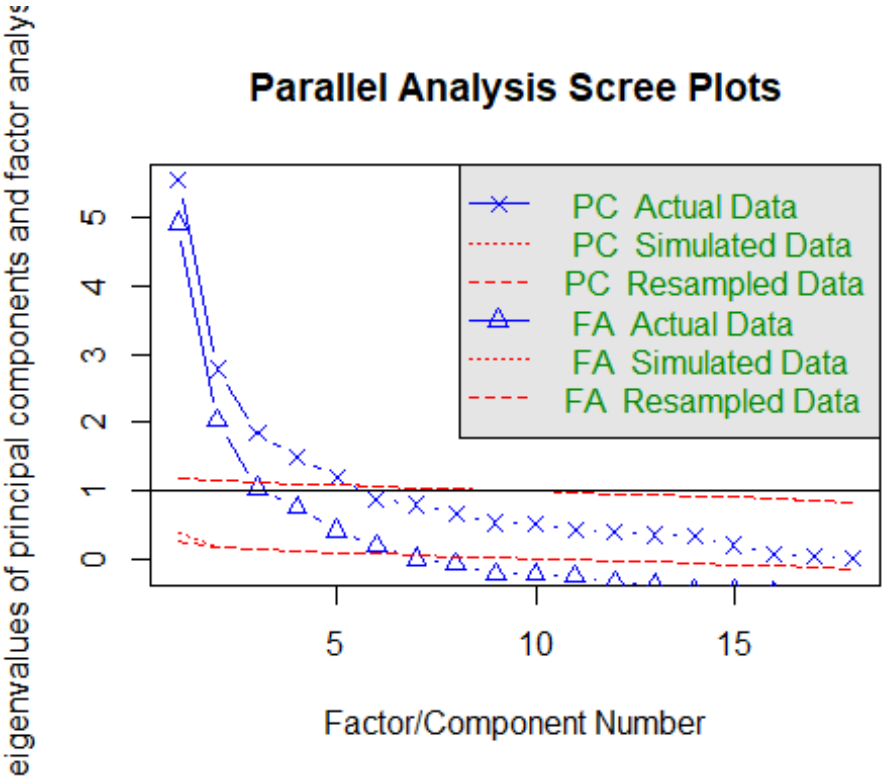
```
# Rotated factor scores, Notice the columns ordering: RC1, RC3, RC2 and RC4
fit.pc$scores
```

```
##                RC2           RC1          RC8          RC3           RC5
## 1    -0.0035488089 -0.4512141098  2.497064991 -1.726400437 -0.1340601844
## 2    -0.2606289987 -0.3156043928  2.818187237 -0.994612228 -0.1728338369
## 3    -0.0137389720 -0.3892192342  2.778177621 -0.815163846 -0.1519082875
## 4    -0.2230888512 -0.4031564817  2.917805415 -0.640954348 -0.1551339277
## 5    -0.2227666207 -0.4353406385  2.944758387 -0.572251437 -0.1773232775
## 6    -0.1980872490 -0.5166348387  3.010939416 -0.688512787 -0.1346303747
## 7    -0.2269722386 -0.5557188797  3.054968767 -0.845100517 -0.1325677162
## 8    -0.1881343944 -0.5552722924  3.070669230 -0.774578544 -0.1139923286
## 9    -0.0093080903 -0.6173749888  2.985719585 -0.775127733 -0.0577848261
## 10   -0.1943724585 -0.6645692473  3.082406045 -0.966073278 -0.0608437462
## 11   -0.1926777940 -0.7528268728  3.146130954 -0.944130459 -0.0835566811
## 12   -0.0822786799 -0.9477385070  2.890467573 -2.841446015 -0.0072338999
## 13   -0.1928691772 -0.9950303591  3.114829552 -1.561429708 -0.0011097936
## 14   -0.0648365747 -1.7070303213  1.227704520 -1.828328351  0.0371189689
## 15   -0.0322978192 -2.3595138544 -1.048249966 -1.815740593  0.1306260007
## 16   -0.0471718499 -2.4144508227 -1.034736610 -2.203278129  0.1451397064
## 17    0.0116092315  0.3230471083 -0.816711333  0.743122852 -0.4455480887
## 18   -0.0249691720  0.3249821992 -0.748105033  0.666314351 -0.4049337109
## 19   -0.0024668456  0.3357813490 -0.781063039  0.751177524 -0.3872335054
## 20   -0.0181161351  0.3708504279 -0.764433653  0.748063351 -0.3979623761
## 21   -0.0219168520  0.2180566236 -0.769166139  0.760926284 -0.3390457189
## 22   -0.0289571170  0.1296862844 -0.822277961  0.780082728 -0.6065297644
## 23   -0.0223553685  0.0701560026 -0.774270353  0.723491494 -0.3298780023
## 24   -0.0684723450  0.0759826177 -0.690202180  0.746734773 -0.6132300077
## 25   -0.0978125763 -0.0081700638 -0.679874537  0.716820894 -0.5960264508
## 26   -0.5639484869 -0.1922282648  0.006311376  0.776195686 -0.3530145593
## 27   -0.1031589330 -0.2328912817 -0.687000561  0.711491535 -0.5417996284
```

```
##                RC4           RC7           RC6
## 1    -0.4544201925 -9.803565e-01  1.4157116806
## 2    -0.4001947843 -6.873352e-01  1.3468217202
## 3    -0.3783699080 -8.466048e-01  1.3156435375
## 4    -0.3819238992 -4.574028e-01  1.4969930922
## 5    -0.3610355131 -4.386059e-01  1.2128836950
## 6    -0.3675528388 -5.191189e-01  1.8190852928
## 7    -0.3781270384 -4.276567e-01  1.9405098603
## 8    -0.3333742773 -5.701484e-01  1.4454794267
## 9    -0.2602019280 -7.644820e-01  0.7265826721
## 10   -0.3045146928 -5.738172e-01  1.0766482257
## 11   -0.3371701741 -6.082536e-01  1.6604313629
## 12   -0.4632628014 -1.106402e+00  1.8789681545
## 13   -0.3666949792 -7.421654e-01  1.7872108387
## 14   -1.3717149081 -7.425624e-02  1.4495277046
## 15    0.2383440357  1.012650e+00  1.1085537193
## 16    0.2238396303  7.697604e-01  1.3193649017
## 17   -0.5007143969 -2.363540e-01 -0.0991430786
## 18   -0.7902017054 -2.041771e-01 -0.1041433170
## 19   -0.4455053256 -2.229672e-01 -0.2593230557
## 20   -0.4158716809 -1.981390e-01 -0.2914803261
## 21   -0.3891567235 -1.650964e-01 -0.2305741363
## 22   -0.3742613462 -1.417628e-01 -0.3693397270
## 23   -0.3495746128 -1.266529e-01 -0.1812304227
## 24   -0.7481867157 -7.028465e-02 -0.0585889035
## 25   -0.7185604069 -3.292069e-02  0.0488605676
## 26   -0.3222843990  6.473796e-01 -0.0605832870
## 27   -0.7077746160 -2.084760e-02  0.0754081161
## 28   -0.7274765522 -3.016355e-02  0.1888239173
## 29   -0.7405816383 -2.126863e-02  0.1587141065
## 30   -0.7777543430 -1.694016e-02  0.1738390610
## 31   -0.7546923964  4.694854e-03  0.0437483546
## 32   -0.7999187078  6.270932e-02  0.1735029603
## 34   -1.1294694802 -8.292303e-01  0.6339857429
## 35   -0.6542231554 -8.322674e-01  0.4936607942
## 36   -0.6295614275 -8.203374e-01  0.0526734349
## 37   -0.6061431417 -7.968147e-01 -0.3243410955
## 38   -0.5923411974 -7.735565e-01 -0.3907613607
## 39   -0.6028929320 -5.741508e-01 -0.2647590806
## 40   -0.5873392037 -5.553908e-01 -0.7608666015
## 41   -0.6236811574  4.345508e-01 -0.9353354708
## 42   -0.6044923525 -3.913900e-01 -1.1942800409
## 43   -0.5244433668 -4.837750e-01 -1.1779909686
## 44   -1.0850499224 -1.670608e-01 -0.9449974878
## 50    0.8278177613  1.003509e+00 -1.1240835367
## 51    0.9052817725  7.022370e-01 -0.7124314602
## 52    0.9906831324  3.752700e-01 -1.1697050953
## 53    1.0015134713 -2.831782e-02 -1.1243991840
## 54    1.0381494454  2.755013e-01 -1.1091209326
```

```
## 55    0.8219303905 -5.139613e-02 -0.5339567450
```

```
# Play with FA utilities

fa.parallel(expect[,5:22]) # See factor recommendation
```




```
## Parallel analysis suggests that the number of factors =  6  and the number
of components =  5
```

```
fa.plot(fit.pc) # See Correlations within Factors
```

```
## Warning in plot.xy(xy.coords(x, y), type = type, ...): unimplemented pch v
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## '26'
```
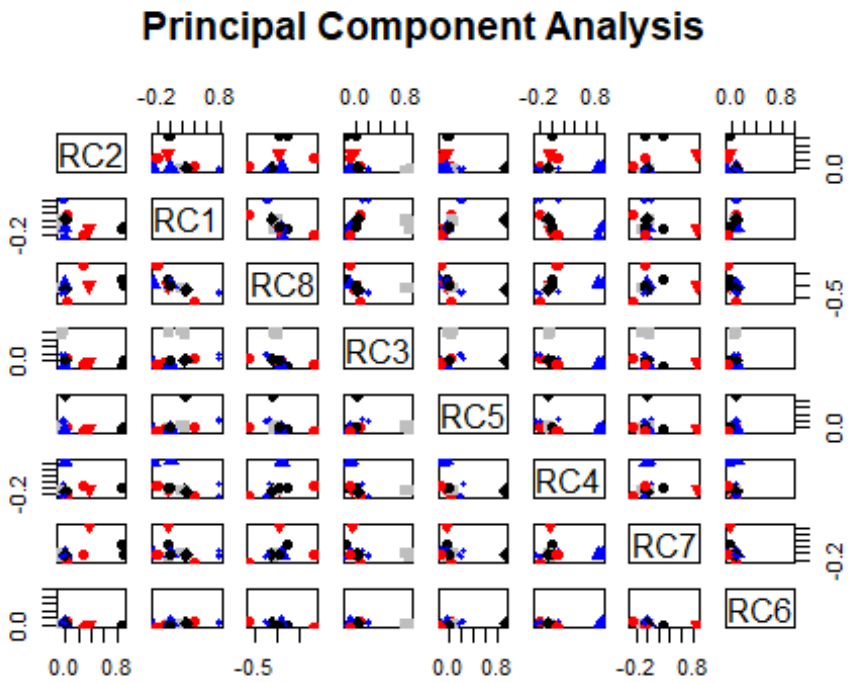
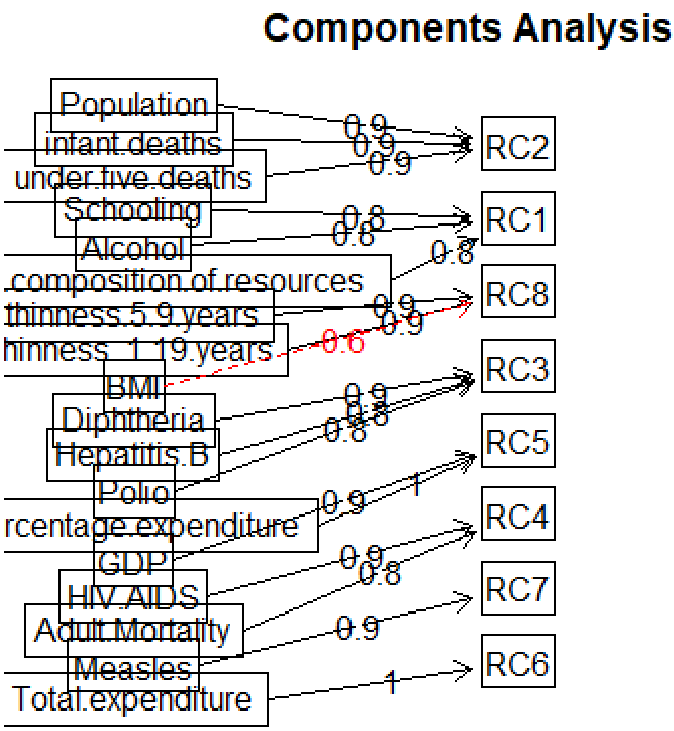




```
fa.diagram(fit.pc) # Visualize the relationship
```




```
vss(expect[,5:22]) # See Factor recommendations for a simple structure

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.ob
s, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.ob
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s, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate,
: An
## ultra-Heywood case was detected. Examine the results carefully

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.ob
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: An
## ultra-Heywood case was detected. Examine the results carefully
```

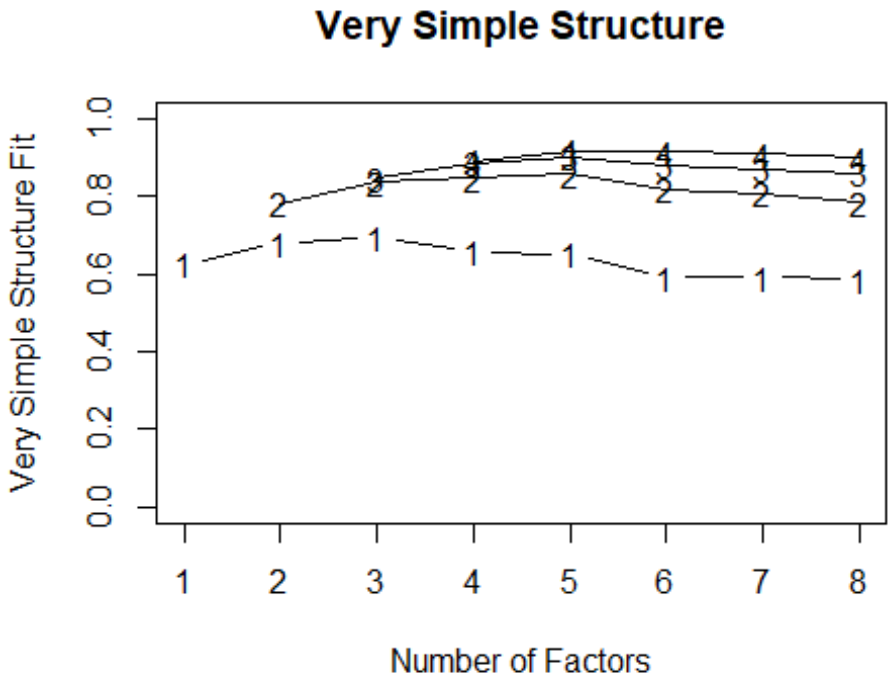


```
##
## Very Simple Structure
## Call: vss(x = expect[, 5:22])
## VSS complexity 1 achieves a maximimum of 0.7  with  3  factors
## VSS complexity 2 achieves a maximimum of 0.86  with  5  factors
##
```

```
## The Velicer MAP achieves a minimum of 0.05  with  2  factors
## BIC achieves a minimum of  -176.62  with  6  factors
## Sample Size adjusted BIC achieves a minimum of  13.99  with  6  factors
##
## Statistics by number of factors
##   vss1 vss2   map dof chisq    prob sqresid  fit RMSEA   BIC SABIC complex
## 1 0.62 0.00 0.063 135 18962 0.0e+00    18.2 0.62 0.291 17961 18390     1.0
## 2 0.68 0.78 0.051 118 11052 0.0e+00    10.6 0.78 0.237 10178 10553     1.2
## 3 0.70 0.84 0.055 102 10015 0.0e+00     7.3 0.85 0.243  9259  9583     1.3
## 4 0.66 0.85 0.064  87  5299 0.0e+00     5.3 0.89 0.191  4655  4931     1.4
## 5 0.65 0.86 0.062  73  3706 0.0e+00     3.8 0.92 0.174  3165  3397     1.4
## 6 0.59 0.82 0.077  60   268 4.7e-28     3.3 0.93 0.046  -177    14     1.5
## 7 0.59 0.81 0.082  48   377 1.1e-52     3.0 0.94 0.065    22   174     1.7
## 8 0.59 0.78 0.080  37   287 1.7e-40     2.8 0.94 0.064    13   131     1.5
##    eChisq   SRMR  eCRMS  eBIC
## 1 13381.5 0.1628 0.1734 12381
## 2  5283.1 0.1023 0.1165  4409
## 3  2913.4 0.0760 0.0931  2158
## 4  1169.6 0.0481 0.0638   525
## 5   453.9 0.0300 0.0434   -87
## 6    47.2 0.0097 0.0154  -397
## 7    21.0 0.0064 0.0115  -335
## 8     7.2 0.0038 0.0077  -267
```