ATTEMPTS TO EXPLAIN LEPTON MASSES AND MIXING PARAMETERS USING FLAVOR SYMMETRY

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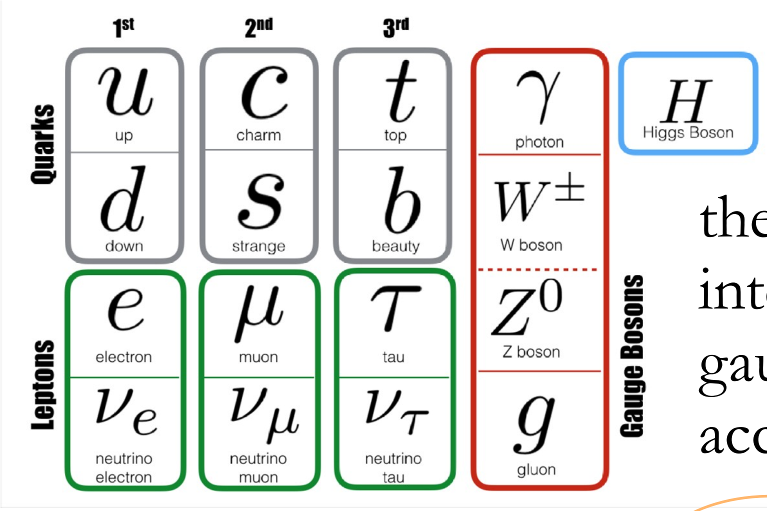
a short remainder



Standard Model

theory describing elementary particles and their interactions based on quantum field theory and gauge symmetry that is broken spontaneously according to the idea of Higgs mechanism

$$SU(3)\times SU(2)\times U(1)$$

strong interaction QCD	electroweak interaction EW	electromagnetic interaction QED

a large number of free parameters that are not predicted by the theory
PROBLEM

DIRAC NEUTRINOS

there are **25** input parameters in the **minimal extension of the SM**

there are **10** input parameters in the lepton sectors: 6 lepton's masses + 4 mixing parameters

the lepton mass matrices depend on Yukawa couplings that cannot be theoretically predicted
REASON

the extension of the gauge SM symmetry by adding some extra discrete symmetry and in the next step the imposition on Lagrangian to be invariant under transformations of this group
SOLUTION

the trivial mass matrices and mixing matrix, the minimal extension of the SM is not enough

SCHUR'S FIRST LEMMA

the Higgs-like particles called flavons must be introduced

broken

the bigger number of normal Higgs doublet must be introduced

not broken

NATURAL SOLUTIONS

after imposing flavor symmetry on Lagrangian of **multi-Higgs doublet model** and some induces manipulation we get equations restricting Yukawa couplings

$$(A^{\Phi\dagger} \otimes A^{L\dagger} \otimes A^{lT})_{k\alpha\delta;i\beta\gamma}(h_i^l)_{\beta\gamma} = (h_k^l)_{\alpha\delta}$$

$$(A^{\Phi T} \otimes A^{L\dagger} \otimes A^{\nu T})_{k\alpha\delta;i\beta\gamma}(h_i^\nu)_{\beta\gamma} = (h_k^\nu)_{\alpha\delta}$$

invariant equations
eigenequation to the eigenvalue 1

$$SU(3) \times SU(2) \times U(1) \times G$$

$$A^L, A^l, A^\nu, A^\phi$$

$$(A^{\Phi\dagger} \otimes A^{L\dagger} \otimes A^{lT})_{k\alpha\delta;i\beta\gamma}(h_i^l)_{\beta\gamma} = (h_k^l)_{\alpha\delta}$$
$$(A^{\Phi T} \otimes A^{L\dagger} \otimes A^{\nu T})_{k\alpha\delta;i\beta\gamma}(h_i^\nu)_{\beta\gamma} = (h_k^\nu)_{\alpha\delta}$$

$$h_i^l, h_i^\nu$$

$$M^l = -\frac{1}{\sqrt{2}}\sum_{i=1}^{N_d} v_i^* h_i^l, \;\; M^\nu = \frac{1}{\sqrt{2}}\sum_{i=1}^{N_d} v_i h_i^\nu$$

$$M^l, M^\nu$$

$$U^{l\dagger} M^l M^{l\dagger} U^l, \; U^{\nu\dagger} M^\nu M^{\nu\dagger} U^\nu$$

$$\{m_e^2, m_\mu^2, m_\tau^2, m_{\nu_e}^2, m_{\nu_\mu}^2, m_{\nu_\tau}^2\}$$

for one group

$$U^l, U^\nu$$

$$U_{PMNS} = U^{l\dagger} U^\nu$$

search for desired group

success of tri-bimaximal mixing pattern based on A_4 group

should be finite and non-abelian

G

to significantly reduce time of calculations (not necessary)

should be a subgroup of U(3)

should posses 3-dimensional and n-dimensional irreducible representations

the existence of 3 families of fermions and n Higgs doublet

to find such a group we use GAP – the program for discrete algebra computation, that gives us the matrix representation for all elements of the given group, $\{A^{\Phi}, A^{L}, A^{l}, A^{\nu}\}$

search for Yukawa matrices

$$(A^{\Phi\dagger} \otimes A^{L\dagger} \otimes A^{lT})_{k\alpha\delta;i\beta\gamma}(h^l_i)_{\beta\gamma} = (h^l_k)_{\alpha\delta} \quad (A^{\Phi T} \otimes A^{L\dagger} \otimes A^{\nu T})_{k\alpha\delta;i\beta\gamma}(h^\nu_i)_{\beta\gamma} = (h^\nu_k)_{\alpha\delta}$$

If these relations hold for the generator's representation of some group then they hold for the representations of all group's elements

To solve the invariant equations we need to:

construct matrices for each generator separately → look for the eigenvectors correspond to eigenvalue 1 for these generators → determine the common eigenspace of the individual eigensubspace and establish the base vector

for each possible combination of irreducible representations

For this purpose we can use Wolfram Mathematica, python with numpy library, c plus plus with eigen library.

useful theories

The order of finite group is divisible by dimension of its irreducible representation

no need to look for group of any order

If the coefficients h constitute the solution to the equation

$$(D^* \otimes A \otimes B)h = h$$

then they can be interpreted as the Clebsch-Gordan coefficients for the decomposition

$$A \otimes B = \oplus_D D$$

After applying this theorem to our equations

$$A^L \otimes A^l = \oplus_D D,\ D = A^\Phi$$

$$A^L \otimes A^\nu = \oplus_D D,\ D = (A^\Phi)^*$$

no need to solve invariance equation for all possible combinations of irr representations

3-Higgs doublet model

$$(A^{\Phi\dagger} \otimes A^{L\dagger} \otimes A^{lT})_{k\alpha\delta;i\beta\gamma}(h_i^l)_{\beta\gamma} = (h_k^l)_{\alpha\delta} \quad (A^{\Phi T} \otimes A^{L\dagger} \otimes A^{\nu T})_{k\alpha\delta;i\beta\gamma}(h_i^\nu)_{\beta\gamma} = (h_k^\nu)_{\alpha\delta}$$

$$N^{l,\nu}\Gamma^{l,\nu} = \Gamma^{l,\nu} \qquad \Gamma^{l,\nu} = [(h_1^{l,\nu})_{11}, (h_1^{l,\nu})_{12}, \ldots, (h_1^{l,\nu})_{33}, \ldots, (h_{N_d}^{l,\nu})_{11}, \ldots, (h_{N_d}^{l,\nu})_{33}]$$

3-Higgs → $N_d = 3$ → each matrix representation is 3-dimensional → $N_{27\times 27}, \Gamma_{27}$

$$N^{l,\nu}\Gamma^{l,\nu} = c^{l,\nu}\Gamma^{l,\nu}$$

$$\nu_1 = |\nu_1|e^{i\varphi_1} \qquad \nu_3 = |\nu_3|e^{i\varphi_3}$$

$$\nu_2 = |\nu_2|e^{i\varphi_2} \qquad c^{l,\nu} = |c^{l,\nu}|e^{i\varphi_{l,\nu}}$$

$$M^l = -\frac{1}{\sqrt{2}}\sum_{i=1}^{3}\nu_i^* h_i^l \qquad M^\nu = \frac{1}{\sqrt{2}}\sum_{i=1}^{3}\nu_i h_i^\nu$$

$$M^l = -\frac{|c^l|}{\sqrt{2}}e^{i\xi_l}(|v_1|h_1^l + |v_2|e^{-i\xi_2}h_2^l + |v_3|e^{-i\xi_3}h_3^l) \qquad M^\nu = \frac{|c^\nu|}{\sqrt{2}}e^{i\xi_\nu}(|v_1|h_1^\nu + |v_2|e^{i\xi_2}h_2^\nu + |v_3|e^{i\xi_3}h_3^\nu)$$

$$\xi_3 = \varphi_3 - \varphi_1 \qquad \xi_2 = \varphi_2 - \varphi_1 \qquad \xi_l = \varphi_l - \varphi_1 \qquad \xi_\nu = \varphi_\nu + \varphi_1$$

$$|\nu_1|^2 + |\nu_2|^2 + |\nu_3|^2 = 246^2 \qquad \{|c^l|, |c^\nu|, |\nu_2|, |\nu_3|, \xi_2, \xi_3\}$$