Elissa

The matrix $A - \lambda I$ for $\lambda_1 = -21.25$ is

$$A - (-21.125)I = \begin{bmatrix} 4-(-21.25) & 8 & -1 & -2 \\ -2 & -9-(-21.125) & -2 & -4 \\ 0 & 10 & 5-(-21.125)-10 & \\ -1 & -13 & -14 & -13-(-21.125) \end{bmatrix}$$

$$= \begin{bmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{bmatrix}$$

Solve $(A - (-21.125)I)v = 0$:

$$\begin{pmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

Row reduction leads to

$v_1 \approx 0.0248$, $v_2 \approx -0.3345$, $v_3 \approx -0.2223$, $v_4 \approx -0.9154$

Thus eigen vector is:

$$\boxed{v_1 \approx \begin{pmatrix} 0.0248 \\ -0.3345 \\ -0.2223 \\ -0.9154 \end{pmatrix}}$$

Liliane

The matrix $A - \lambda I$ for $\lambda_4 = 11.054$ is:

$$A - 11.054I = \begin{pmatrix} 4 - 11.054 & 8 & -1 & -2 \\ -2 & -9 - 11.054 & -2 & -4 \\ 0 & 10 & 5 - 11.054 & -10 \\ -1 & -13 & -14 & -13 - 11.054 \end{pmatrix}$$

$$= \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix}$$

$$= \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

* Row reduction leads to:

$v_1 \approx -0.0360$, $v_2 \approx -0.0121$, $v_3 \approx -0.8601$, $v_4 \approx 0.5086$

* Thus, the eigenvector is:

$$\boxed{v_4 = \begin{pmatrix} -0.0360 \\ -0.0121 \\ -0.8601 \\ 0.5086 \end{pmatrix}}$$

* Importance of eigenvalue λ_4 in percentage form

Absolute value $|\lambda_4| = 11.054$

Total Absolute value $= 40.458$

$\%$ of $\lambda_4 = \left(\frac{11.054}{40.458}\right) \times 100 \approx 27.32\%$

$$\boxed{\% \text{ of } \lambda_4 \approx 27.32\%}$$

Bihibindi Aubert Gloire

for $\boxed{\lambda_2 = -5.604}$:

$$A-(-5.604)I = \begin{pmatrix} 4-(-5.604) & 8 & -1 & -2 \\ -2 & -9-(-5.604) & -2 & -4 \\ 0 & 10 & 5-(-5.604) & -10 \\ 1 & -13 & -14 & -13-(-5.604) \end{pmatrix}$$

$$= \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$$

Solve $(A+5.604I)v = 0$:

$$\begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ 1 & -13 & -14 & -7.396 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = 0$$

Row Reduction

$V_1 \approx 0.5634$, $V_2 \approx -0.6161$, $V_3 \approx 0.5495$, $V_4 = -0.0334$

Eigenvector = $\boxed{v_2 \approx \begin{pmatrix} 0.5634 \\ -0.6161 \\ 0.5495 \\ -0.0334 \end{pmatrix}}$

Caline Uhlingabirt

Given matrix:

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

with Eigenvalue for $\lambda_3 = 2.675$

matrix $A - \lambda I$ for $\lambda_3 = 2.675$ is:

$$A - 2.675 = \begin{pmatrix} 4-2.675 & 8 & -1 & -2 \\ -2 & -9-2.675 & -2 & -4 \\ 0 & 10 & 5-2.675 & -10 \\ -1 & -13 & -14 & -13-2.675 \end{pmatrix} = \begin{pmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{pmatrix}$$

Solve $(A - 2.675I)v = 0$:

$$\begin{pmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

Row reduction leads to

$v_1 \approx 0.9583$ $\quad v_2 \approx -0.1610$ $\quad v_3 \approx 0.2074$ $\quad v_4 \approx -0.1128$

Thus, the eigenvector is:

$$v_3 = \begin{pmatrix} 0.9583 \\ -0.1610 \\ 0.2074 \\ -0.1128 \end{pmatrix}$$

In summary

Eigenvalues $\boxed{-21.125}$, $\boxed{-5.604}$, $\boxed{2.675}$, $\boxed{11.054}$

Corresponding Eigenvectors

$$V_1 = \begin{pmatrix} 0.0248 \\ -0.3345 \\ -0.2223 \\ -0.9154 \end{pmatrix} \quad V_2 = \begin{pmatrix} 0.5634 \\ -0.6161 \\ 0.5495 \\ -0.0334 \end{pmatrix} \quad V_3 = \begin{pmatrix} 0.9583 \\ -0.1610 \\ 0.2074 \\ -0.1128 \end{pmatrix} \quad V_4 = \begin{pmatrix} -0.0360 \\ -0.0121 \\ -0.8601 \\ 0.5086 \end{pmatrix}$$

Absolute Values

$|\lambda_1| = 21.125$, $|\lambda_2| = 5.604$, $|\lambda_3| = 2.675$, $|\lambda_4| = 11.054$

Sum of Absolute Values

Total $= 21.125 + 5.604 + 2.675 + 11.054 = 40.458$

Percentages

$$\% \text{ of } \lambda_1 = \left(\frac{21.125}{40.458}\right) \times 100 = 52.21\%$$

$$\% \text{ of } \lambda_2 = \left(\frac{5.604}{40.458}\right) \times 100 = 13.85\%$$

$$\% \text{ of } \lambda_3 = \left(\frac{2.675}{40.458}\right) \times 100 = 6.61\%$$

$$\% \text{ of } \lambda_4 = \left(\frac{11.054}{40.458}\right) \times 100 = 27.32\%$$

Final Importance in percentage Form:

$$\boxed{\begin{aligned} \lambda_1 &: 52.21\% \\ \lambda_2 &: 13.85\% \\ \lambda_3 &: 6.61\% \\ \lambda_4 &: 27.32\% \end{aligned}}$$

$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

* $M_1 = \begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$

$M_1 = (-9-\lambda)(\lambda^2 + 8\lambda - 205) + 2(-130 - 10\lambda - 130) - 4(-140 + 65 - 13\lambda)$

$$\boxed{M_1 = -\lambda^3 - 17\lambda^2 + 165\lambda + 1625}$$

* $M_2 = \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$

$M_2 = -2(\lambda^2 + 8\lambda - 205) - 20 - 4(5-\lambda)$

$$\boxed{M_2 = -2\lambda^2 - 12\lambda + 370}$$

* $M_3 = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$

$M_3 = -2(-130 - 10\lambda - 130) - 10(9+\lambda) - 40$

$$\boxed{M_3 = 390 + 10\lambda}$$

* $M_4 = \begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$

$M_4 = 150 + 26\lambda + 45 - 9\lambda + 5\lambda - \lambda^2 - 20$

$$\boxed{M_4 = -\lambda^2 + 22\lambda + 175}$$

$\det(A - \lambda I) = (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(-2\lambda^2 - 12\lambda + 370) - (390 + 10\lambda) + 2(-\lambda^2 + 22\lambda + 175)$

$$\boxed{= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500}$$

* For $\lambda \approx -21.125$: $P(-21.125)$

$= (-21.125)^4 + 13(-21.125)^3 - 219(-21.125)^2 - 835(-21.125) + 3500 \approx 0$

* For $\lambda \approx -5.604$: $P(-5.604)$

$= (-5.604)^4 + 13(-5.604)^3 - 219(-5.604)^2 - 835(-5.604) + 3500 \approx 0$

* For $\lambda \approx 2.675$: $P(2.675)$

$= (2.675)^4 + 13(2.675)^3 - 219(2.675)^2 - 835(2.675) + 3500 \approx 0$

* For $\lambda \approx 11.054$: $P(11.054)$

$= (11.054)^4 + 13(11.054)^3 - 219(11.054)^2 - 835(11.054) + 3500 \approx 0$

Thus, The eigenvalue are

$$\boxed{\lambda_1 = -21.125}$$

$$\boxed{\lambda_2 = -5.604}$$

$$\boxed{\lambda_3 = 2.675}$$

$$\boxed{\lambda_4 = 11.054}$$