TWK Literature Reviews for Sparse Supermod

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March 8, 2016

1 Boutsidis, et. al. 2015

What is the problem?

- Definition of *weak-α-supermodularity* for set functions
- Evaulate a greedy algorithm based on this definition in determining a minimal solution

Submodular function:

$$f(S \cup T) - f(S) \geq f(P \cup T) - f(P), \qquad \text{for } S \subseteq P$$

Supermodular beginnings:

$$f(S \cup T) \leq f(S) \Rightarrow f(S) - f(S \cup T) \text{ is the reduction in } f \text{ by adding } T$$

Then the average gain of adding elements of T *sequentially* is $\frac{[f(S)-f(S\cup T)]}{|T\setminus S|}$. One hopes that there exists an element $i \in T \setminus S$ such that $f(S) - f(S \cup \{i\}) \geq \frac{[f(S)-f(S\cup T)]}{|T\setminus S|}$. In general this is not possible since the elements of T are not independent. This is, however a feature of supermodular functions (Lemma 1).

Supermodular definition: *A set function $f(S) : 2^{[n]} \to \mathbb{R}_+$ is said to be supermodular if for any two sets $S, T \subseteq [n]$*

$$f(S \cap T) + f(S \cup T) \geq f(S) + f(T)$$

Extends to definition of *weak-α-supermodularity.* *A non-negative, non-increasing, set function $f(S) : 2^{[n]} \to \mathbb{R}_+$ is said to be weakly-α-supermodular if there exists $\alpha \geq 1$ such that for any two sets $S, T \in [n]$:*

$$f(S) - f(S \cup T) \leq \alpha|T \setminus S| \max_{i \in T\setminus S}[f(S) - f(S \cup \{i\})]$$

Why is it important?

- Provides a conversant analog to the submodular formulation in a few select problems. (Most particularly that of Sparse Multiple Linear Regression (SMLR))
- (In general, I have no idea why supermodularity is an improvement over submodularity...)

Why is it hard?

- Sparse combinatorial optimization based on cardinality constraints, hard to minimize in general

Why existing solutions do not work?

- Not all problems that are related through submodularity can be redefined as supermodular
- The average gain of adding elements of T *sequentially* is $\frac{[f(S)-f(S\cup T)]}{|T\setminus S|}$. One hopes that there exists an element $i \in T \setminus S$ such that $f(S) - f(S \cup \{i\}) \geq \frac{[f(S)-f(S\cup T)]}{|T\setminus S|}$. In general this is not possible since the elements of T are not independent. This is, however a feature of supermodular functions (Lemma 1).

What is the core intuition for the solution?

- That a non-increasing supermodular function is *weakly-α-supermodular* with $\alpha = 1$
- Many of the underlying results depend on those of referenced papers, intuition is built through the literature

Solution step-by-step?

- Iteratively add the element that minimizes f for each step up to the size of $\alpha k \ln(f(S_0)/E)$ (see Thm 1)
- To show the bounds on supermodular minimization, there is some construction toward Sparse Regression and Column Subset Selection where the intermediate results build toward the claim set out in the abstract.

Does the paper prove its claims?

- It does walk through the construction of

Exact setup of analysis/experiments

- Evaluates Clustering as a supermodular problem as introduction of methodology of general analysis
- Builds further evidence for its results via SMLR, Sparse Regression, and Column Subset Selection (relying heavily on literature).
- Key result is in improvement of Natarajan

Are there any gaps in the logic/proof?

- Jumps between full clustering objective being supermod. to constrained objective being supermod. (Lemma 2)
- No proof for Lemma 6

Possible next steps

- Determine exact or strong supermodularity expressions for a general (or as general as we can go) set function
- Applying results of this paper empirically.