Mandatory 1

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Mandatory exercises

This is the first mandatory exercise of the course. **The deadline is Tuesday February 12 at 23:55**, and solutions **in the form of a single .pdf file** should be submitted through learnit. You do not have to type up your solution in LaTeX, it is fine to submit a scan of a *clearly readable* hand-written solution. All ITU printers can scan a document.

Submit the following exercises: Chapter 1, review exercise 34. Exercises 1.1.68 (translate the prose of the exercise to a system of linear equations and solve them) and 2.3.14.

Review exercise 34

$2x + y + 2z = 4$
$2x + 2y = 5$
$2x - y + 6z = 2$

Setting up argumented matrix

$$\begin{matrix} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{matrix}$$

$(\frac{1}{2})R^1 \Rightarrow R^1$

$$\begin{matrix} 1 & \frac{1}{2} & 1 & 2 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{matrix}$$

$R^2 + (-2)R^1 \Rightarrow R^2$

$$\begin{matrix} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 2 & -1 & 6 & 2 \end{matrix}$$

$R^3 + (-2)R^1 \Rightarrow R^3$

$$\begin{matrix} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{matrix}$$

$R^3 + (2)R^2 \Rightarrow R^3$

$$\begin{matrix} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$R^1 + (-\frac{1}{2})R^2 \Rightarrow R^1$$

$$\begin{matrix} 1 & 0 & 2 & \frac{3}{2} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$let\ t = z\ where\ t\ is\ a\ real\ number$
$x = -2t + \frac{3}{2}$
$y = 2t + 1$
$z = t$

The solution set has an infinite number of solutions

Exercise 1.1.68

- Two planes start from LA
- Both planes fly in opposite direction of each other
- Plane A flies 1/2 before Plane B
- Plane B flies 1/2 after Plane A
- Plane B is 80 km/h faster than Plane A
- Two hours after first plane departs the planes are 3200 km apart
- **GOAL: Find airplane speeds**

We have the following variables to consider
- Time in hours
- Distance between planes
- Plane Speeds

$let\ x = first\ plane\ speed\ \ in \frac{km}{hr}$
$let\ y = \ second\ plane\ speed\ in \frac{km}{hr}$

We define
$-x + y = 80$
$2x + 1.5y = 3200$

Setup argumented matrix

$$\begin{matrix} -1 & 1 & 80 \\ 2 & 1.5 & 3200 \end{matrix}$$

Solve with gauss Jordan elimination

$(\frac{1}{2})R^1 = R^1$

$$\begin{matrix} 1 & 0.75 & 1600 \\ -1 & 1 & 80 \end{matrix}$$

$R^2 + R^1 = R^2$

$$\begin{matrix} 1 & 0.75 & 1600 \\ 0 & 1.75 & 1680 \end{matrix}$$

$\frac{R^2}{1.75} = R^2$

$$\begin{matrix} 1 & 0.75 & 1600 \\ 0 & 1 & 960 \end{matrix}$$

$R^1 + (-0.75)R^2 = R^1$

$$\begin{matrix} 1 & 0 & 880 \\ 0 & 1 & 960 \end{matrix}$$

And so
$x = 880$
$y = 960$

Testing
$-x + y = 80$
$-880 + 960 = 80$
$80 = 80$

The system has exactly one solution based on the given conditions.
The first plane's airspeed is 880 km/hr and the second plane's airspeed is 960

Exercise 2.3.14

Finding inverse of

$$A = \begin{matrix} 1 & 12 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{matrix}$$

Adjoining A with identity matrix

$$[A \;\; I] = \begin{matrix} 1 & 12 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{matrix}$$

Use gauss jordan elimination to get $[I \quad A^{-1}]$

$R^2 + (-3)R^1 = R^2$

$$\begin{matrix} 1 & 12 & 2 & 1 & 0 & 0 \\ 0 & -29 & 3 & -3 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{matrix}$$

$R^3 + R^1 = R^3$

$$\begin{matrix} 1 & 12 & 2 & 1 & 0 & 0 \\ 0 & -29 & 3 & -3 & 1 & 0 \\ 0 & 8 & -5 & 1 & 0 & 1 \end{matrix}$$

$\frac{R^2}{-29} = R^2$

$$\begin{matrix} 1 & 12 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{29} & \frac{3}{29} & -\frac{1}{29} & 0 \\ 0 & 8 & -5 & 1 & 0 & 1 \end{matrix}$$

$R^1 + (-12)R^2 = R^1$

$$\begin{matrix} 1 & 0 & \frac{94}{29} & \frac{32}{29} & -\frac{12}{29} & 0 \\ 0 & 1 & -\frac{3}{29} & \frac{3}{29} & -\frac{1}{29} & 0 \\ 0 & 8 & -5 & 1 & 0 & 1 \end{matrix}$$

$R^3 + (-8)R^2 = R^3$

$$\begin{matrix} 1 & 0 & \frac{94}{29} & \frac{32}{29} & -\frac{12}{29} & 0 \\ 0 & 1 & -\frac{3}{29} & \frac{3}{29} & -\frac{1}{29} & 0 \\ 0 & 0 & -\frac{121}{29} & \frac{5}{29} & \frac{8}{29} & 1 \end{matrix}$$

$\frac{R^3}{-\frac{121}{29}} = R^3$

$$\begin{matrix} 1 & 0 & \frac{94}{29} & \frac{32}{29} & -\frac{12}{29} & 0 \\ 0 & 1 & -\frac{3}{29} & \frac{3}{29} & -\frac{1}{29} & 0 \\ 0 & 0 & 1 & -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$

$R^1 + (-\frac{94}{29})R^3 = R^1$

$$\begin{matrix} 1 & 0 & 0 & -\frac{13}{121} & \frac{76}{121} & \frac{94}{121} \\ 0 & 1 & -\frac{3}{29} & \frac{3}{29} & -\frac{1}{29} & 0 \\ 0 & 0 & 1 & -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$

$R^2 + (\frac{3}{29})R^3 = R^2$

$$\begin{matrix} 1 & 0 & 0 & -\frac{13}{121} & \frac{76}{121} & \frac{94}{121} \\ 0 & 1 & 0 & \frac{12}{121} & -\frac{5}{121} & -\frac{3}{121} \\ 0 & 0 & 1 & -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$

$$A^{-1} = \begin{matrix} -\frac{13}{121} & \frac{76}{121} & \frac{94}{121} \\ \frac{12}{121} & -\frac{5}{121} & -\frac{3}{121} \\ -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$

Testing

$$AA^{-1} = \begin{matrix} 1 & 12 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{matrix} * \begin{matrix} -\frac{13}{121} & \frac{76}{121} & \frac{94}{121} \\ \frac{12}{121} & -\frac{5}{121} & -\frac{3}{121} \\ -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$

$$AA^{-1} = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

And so the inverse matrix of A is

$$A^{-1} = \begin{matrix} -\frac{13}{121} & \frac{76}{121} & \frac{94}{121} \\ \frac{12}{121} & -\frac{5}{121} & -\frac{3}{121} \\ -\frac{5}{121} & -\frac{8}{121} & -\frac{29}{121} \end{matrix}$$