Exercises Week 9

Machine Learning/Advanced Machine Learning
IT University of Copenhagen

Fall 2019

Theoretical Exercises 9.1: from the Book

As stated on learnit, solve the following exercises from the book:

- (15.12.1)
- (15.12.2)

Theoretical Exercises 9.2.

What are the fundamental differences between the Maximum-Likelihood and Bayesian approach of estimation?

Programming and Principles Exercise 9.3

The linear regression model of degree one is defined by two parameters w_0, w_1 as:

$$r_i = f(x_i|w_0, w_1) = w_0 + w_1 x_i + \epsilon_i,\ i = 1, \ldots, N = 20,\ \epsilon \sim \mathcal{N}(0, 1/\beta). \tag{1}$$

The file `points.txt` contains 20 points, which define the training data: $\mathcal{X} = (x_i, r_i),\ i = 1, \ldots N = 20$

(a) Estimate w_0, w_1 by the ML-estimate using Eq. (16.21). Use them to estimate r_i by Eq. (16.22): $\widehat{r_i} = \widehat{w}_0 + \widehat{w}_1 x_i$.

(b) Estimate w_0, w_1 by Bayesian estimation as follows. Assume $p(\boldsymbol{w}) \sim \mathcal{N}(\mathbf{0}, (1/\alpha)\boldsymbol{I})$, $p(\boldsymbol{w}|\mathcal{X}) \sim \mathcal{N}(\boldsymbol{\mu}_N, \beta\Sigma_N)$, $\alpha = 2$, $\beta = 25$. Use Eq. (16.23) to determine $\boldsymbol{\mu}_N$, Σ_N. To receive one estimate $\widehat{\boldsymbol{w}} = (\widehat{w}_0,\ \widehat{w}_1)^{\mathrm{T}}$, get one sample from the posterior $p(\boldsymbol{w}|\mathcal{X})$.
Hint: `w = np.random.multivariate_normal(m_N.ravel(), S_N, num_w_samples).T`

(c) Create 10 different estimates for lines from (b) by sampling $\widehat{\boldsymbol{w}}$ to predict r_i. From these different lines, estimate the average line.

(d) Revisit the programming exercise 3.2., which was the linear regression of bodyfat data. Describe the necessary changes to use Bayesian estimation.