Exercises Lecture 12
Intelligent System Programming (ISP)

Exercise 1 (adapted from C83 3.9)

Solve the following problem by the two phase simplex method:

Maximize $3x_1 + x_2$

Subject to

$x_1 - x_2 \leq -1$
$-x_1 - x_2 \leq -3$
$2x_1 + x_2 \leq 4$

$x_1 \geq 0,\ x_2 \geq 0$

Exercise 2

Write an LP problem in standard form with $n = 2$, where the feasible set only contains origo (0,0) and where the simplex algorithm performs at least one degenerate pivot. Write the sequence of dictionaries explored by simplex leading to and including the degenerate pivot.

Exercise 3 (adopted from C83 3.2)

Show that simplex cycles when you use the largest-coefficient pivot rule and break ties among variables to leave the basis by favoring the variable of the constraint row that lies highest in the dictionary (so do not swap the rows below before converting them to slack form).

Maximize $2x_1 + 3x_2 - x_3 - 12x_4$

Subject to

$-2x_1 - 9x_2 + x_3 + 9x_4 \leq 0$
$1/3x_1 + x_2 - 1/3x_3 - 2x_4 \leq 0$

$x_1, x_2, x_3, x_4 \geq 0$

Exercise 4

Consider maximizing a general objective function over a set of linear inequalities:

Maximize $f(x_1, x_2, \ldots, x_n)$

Subject to $\sum_{j=1}^{n} a_{ij}x_j \leq b_i \quad (i = 1,2,\ldots,m)$

$x_j \geq 0 \quad (j = 1,2,\ldots,n)$

Can the simplex algorithm be used to find optimal solutions to such problems (why / why not)?

Mandatory assignment

A *hypercube* is an n-dimensional analogue of a square (n = 2) and a cube (n = 3).

1) Argue that the number of corner points of a hypercube is 2^n
2) Argue that a hypercube can be represented by $2n$ inequality constraints. In the remainder, we consider hypercubes defined in this way.
3) Assume that you are given two arbitrary corner points $A = (a_1,a_2,\ldots,a_n)$ and $B = (b_1,b_2,\ldots,b_n)$ on an n-dimensional hypercube. What is the minimum number of simplex pivots needed to go from A to B?
4) Is it possible to define an objective, maximize $\sum_{j=1}^{n} c_j x_j$, such that the simplex algorithm from a feasible corner point of an n-dimensional hypercube uses more than n pivots to find an optimal solution? If "yes" show an example, if "no" argue why.

General hint: consider unit hypercubes with the lower-left corner in origo (e.g., for n=2 the corner points are (0,0),(0,1),(1,0),(1,1)).