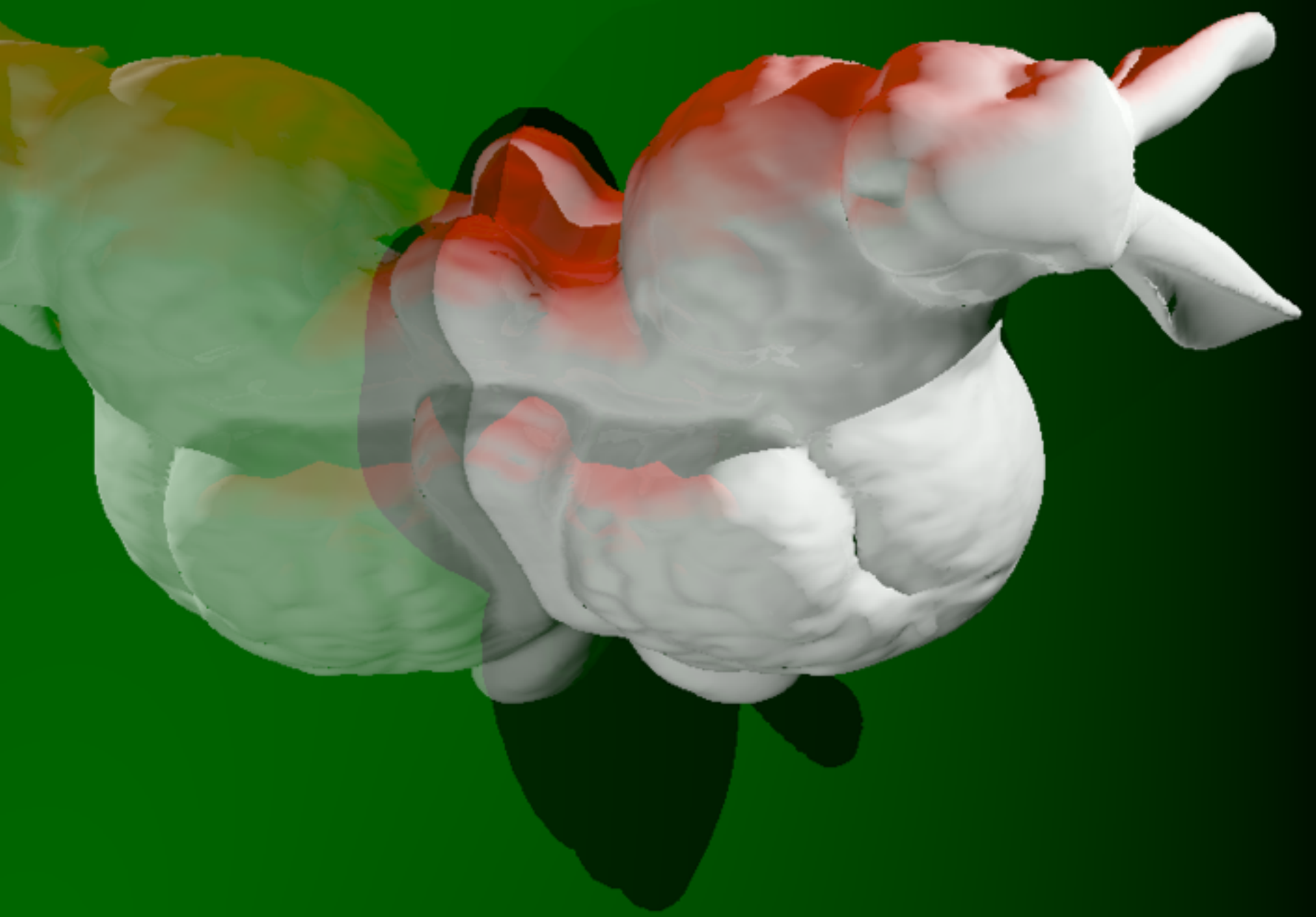
Texture Mapping



Overview

1. Simple Shapes
   - rectangle, disc, sphere, cylinder
2. Triangle Meshes
3. Ray Tracer API

Texture

- 2D bitmap mapped onto a 3D object
- function $[0,1] \times [0,1] \rightarrow$ Colour

- (0,0) is the bottom left corner
- (1,1) is the top right corner

where [0,1] is the interval from 0 to 1

Texture

- 2D bitmap mapped onto a 3D object
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  normalised texture coordinates

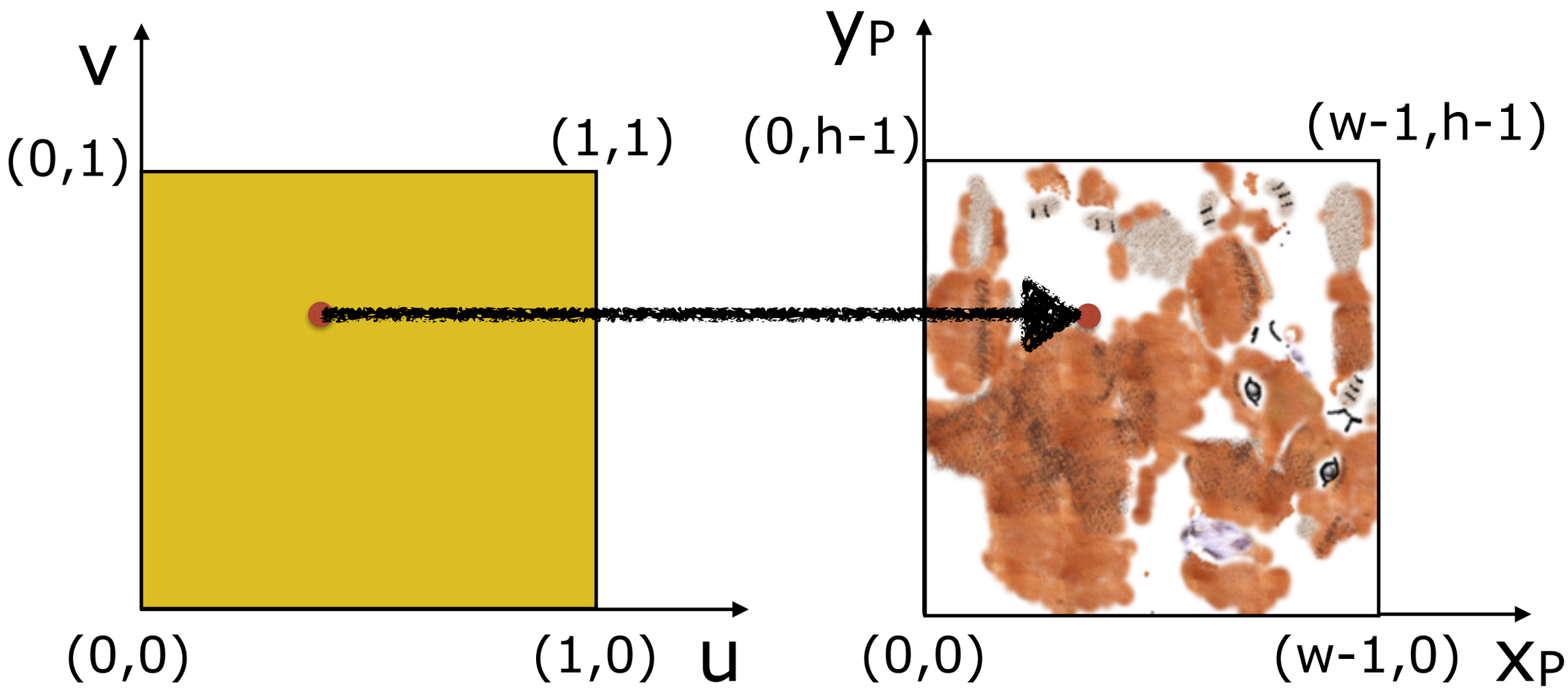
- (0,0) is the bottom left corner
- (1,1) is the top right corner

where [0,1] is the interval from 0 to 1

Texture Coordinates

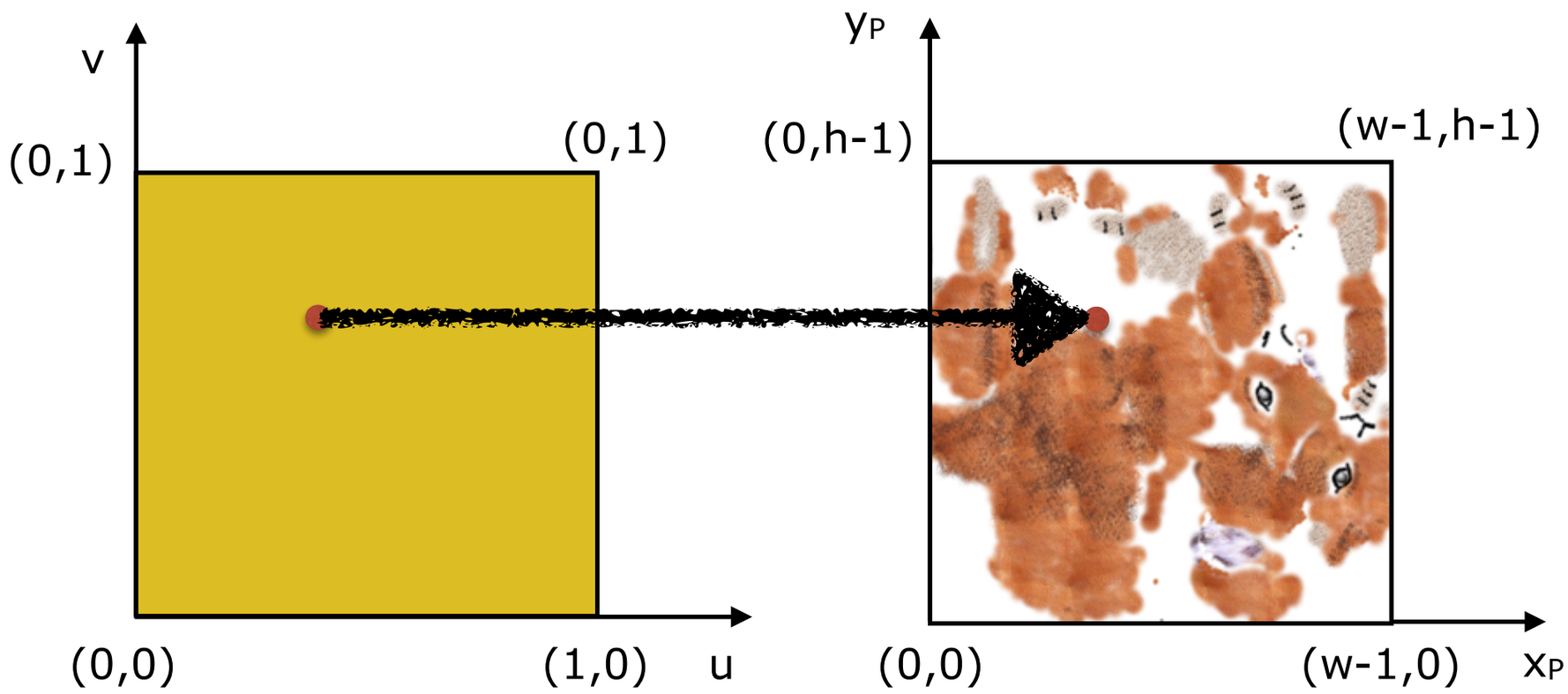
Texture coordinates
$[0,1] \times [0,1]$

Bitmap picture
width w, height h



Convert Texture Coordinates

$$(u,v) \longrightarrow \Big(u(w-1), v(h-1)\Big)$$



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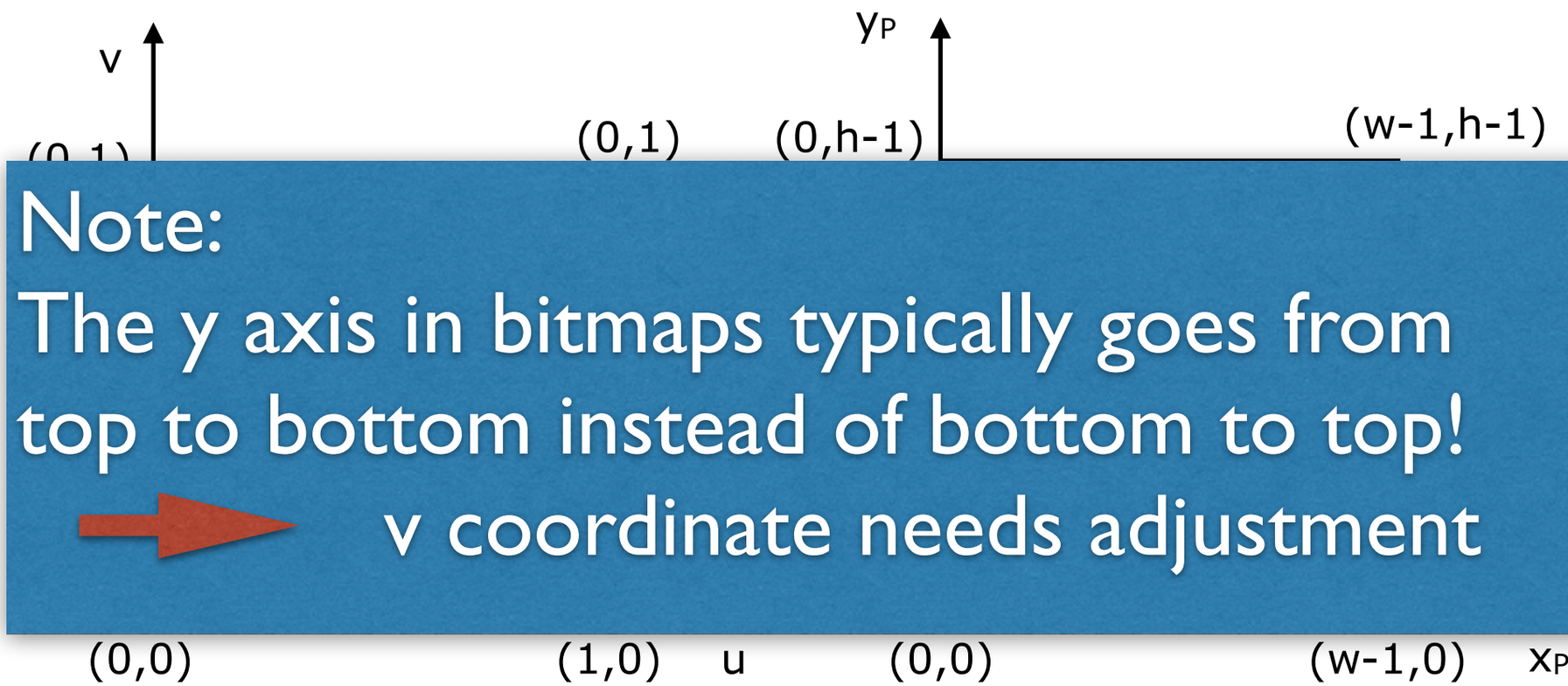
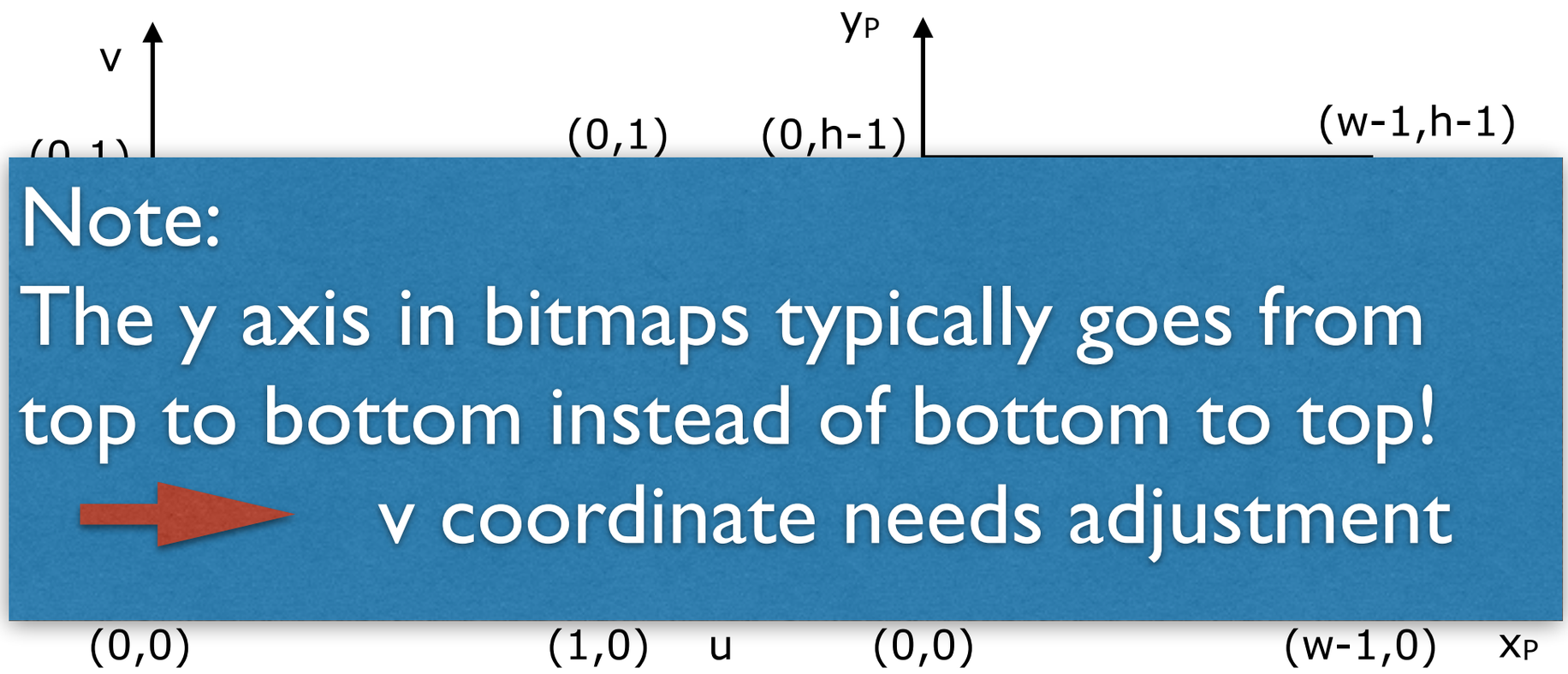


Note:
The y axis in bitmaps typically goes from top to bottom instead of bottom to top!

→ v coordinate needs adjustment

Convert Texture Coordinates

$$(u,v) \longrightarrow \Big(u(w-1),(1-v)(h-1)\Big)$$



Note:
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More Than Just Colour

- Simple texture

$$[0,1] \times [0,1] \rightarrow \text{Colour}$$

- We generalise this

$$[0,1] \times [0,1] \rightarrow \text{Material}$$

More Than Just Colour

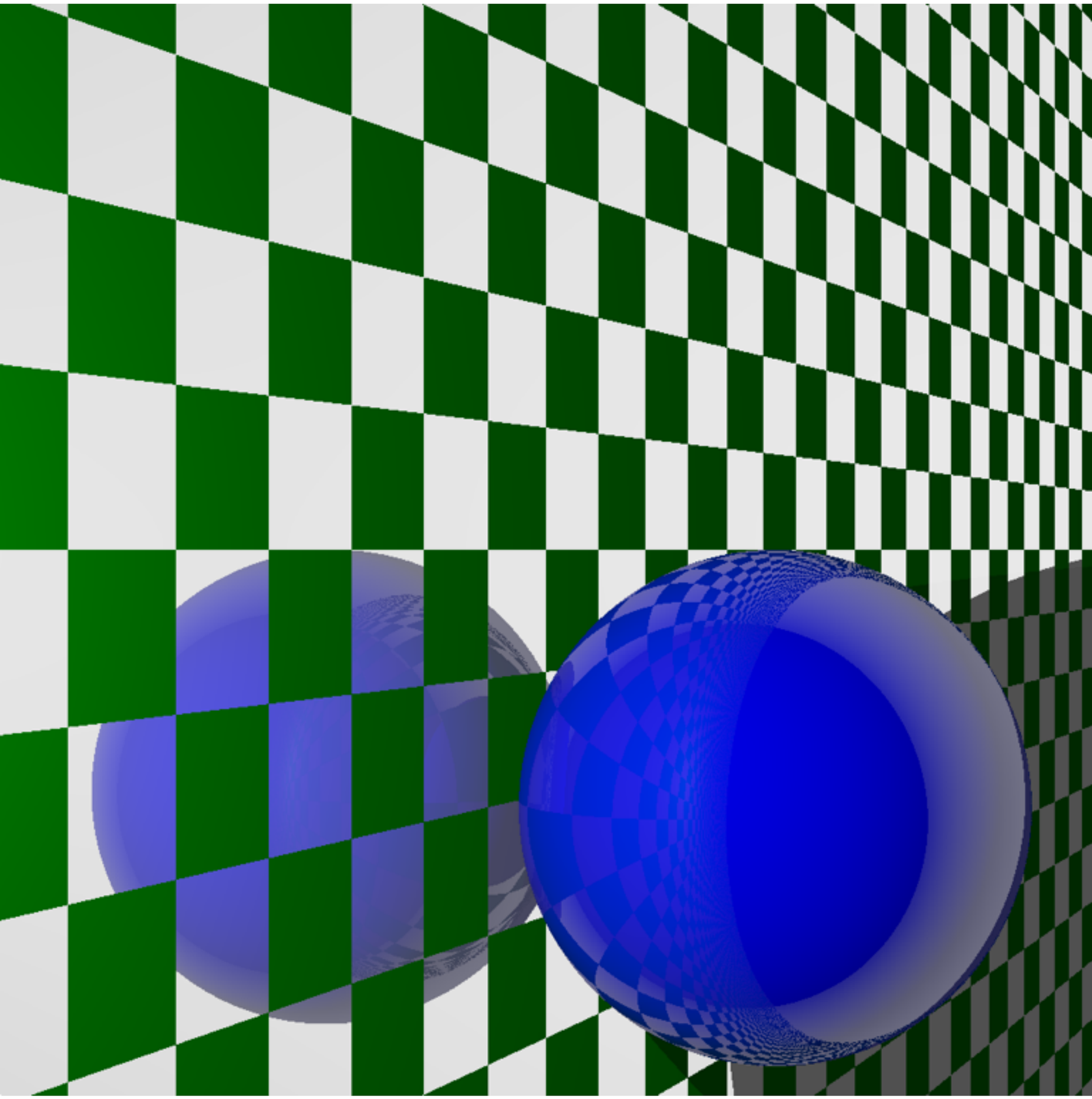
- Simple texture

$$[0,1] \times [0,1] \rightarrow \text{Colour}$$

- We generalise this

$$[0,1] \times [0,1] \rightarrow \text{Material}$$

contains colour and reflection factor



Load Texture from File

```
let loadTexture (file : string)
        : float -> float -> material =
  let img = new Bitmap(file)
  let widthf  = float (img.Width - 1)
  let heightf = float (img.Height - 1)
  let texture (u : float) (v : float) =
    let color = img.GetPixel
                  (int (widthf * u),
                   int (heightf * (1.0 - v)))
    mkMaterial (fromColor color) 0.0
  texture
```

Ray Tracing With Textures

Without textures:

1. find intersection of ray with shape
2. determine the colour of the pixel based on the **(single) colour of the shape** + lighting and reflection

With textures:

1. find intersection of ray with shape
2. determine (u,v) coordinates of intersection point
3. determine the colour of the pixel based on the **colour of the texture at (u,v)** + lighting and reflection

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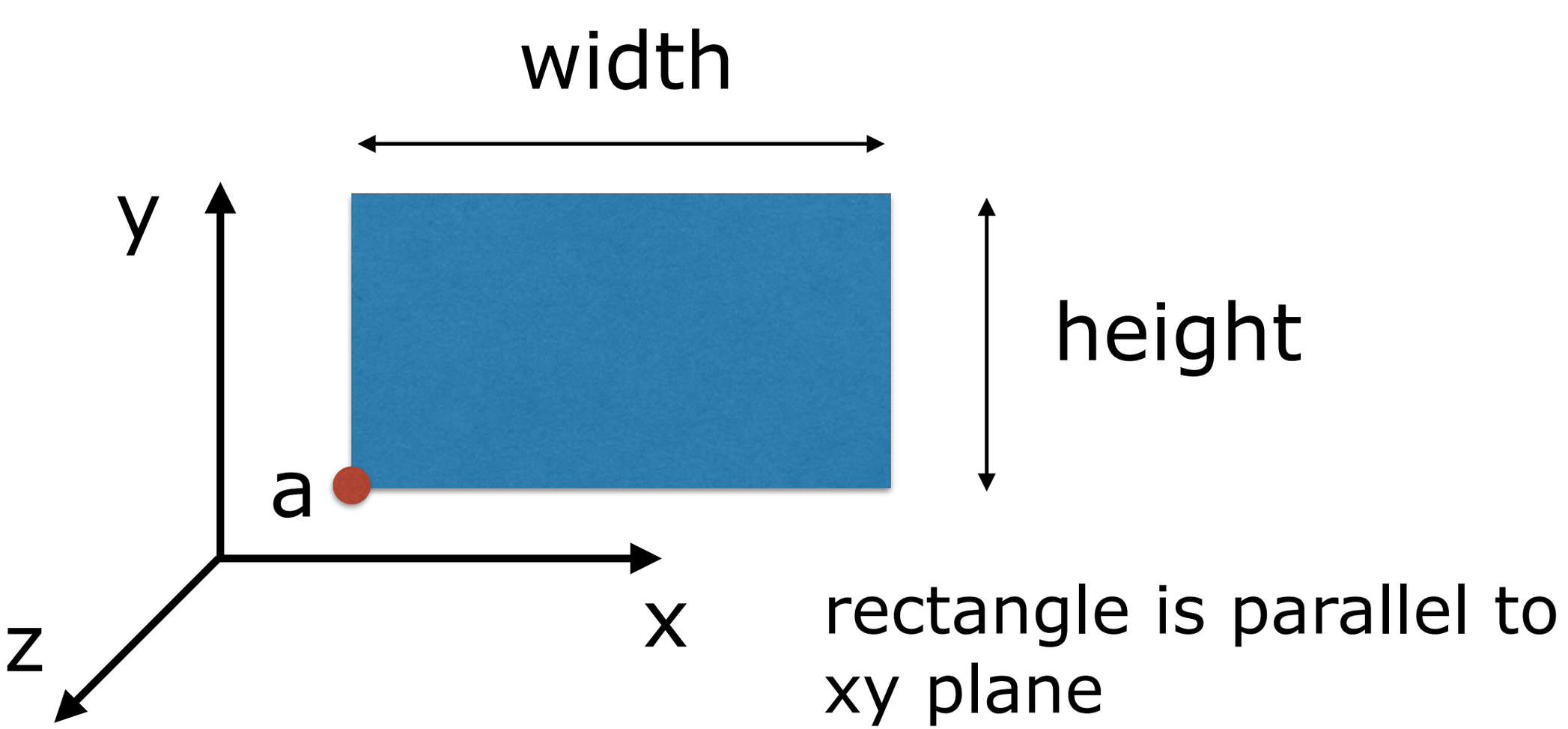
Questions

Rectangle

- We only consider rectangles that are axis aligned
- They can be transformed later

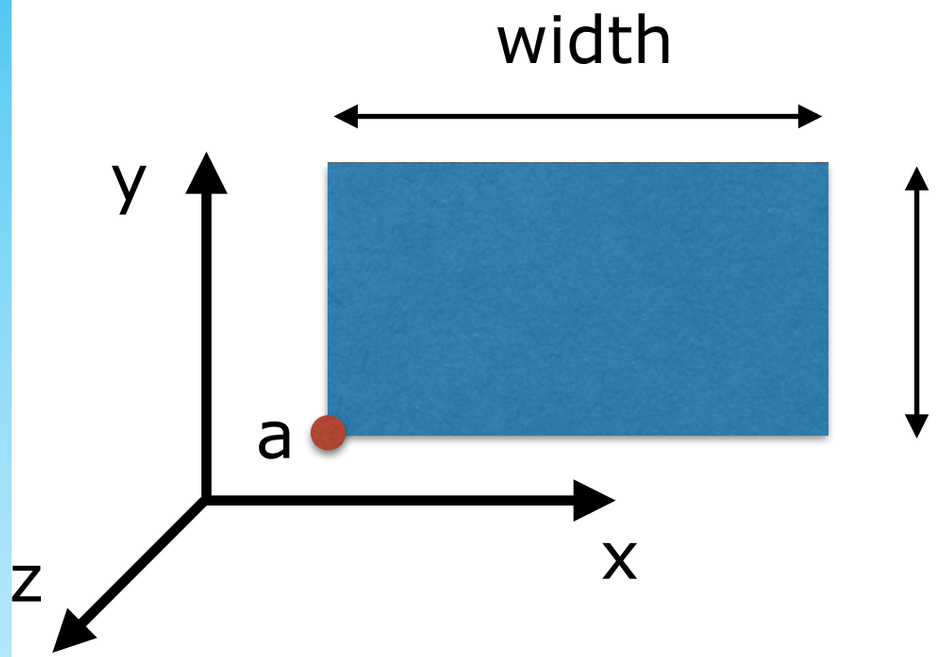


rectangle is parallel to xy plane

Hit function

- check intersection with the plane the rectangle is in (equation: $p_z = a_z$)
- check that the intersection point p (if any) is in the rectangle:

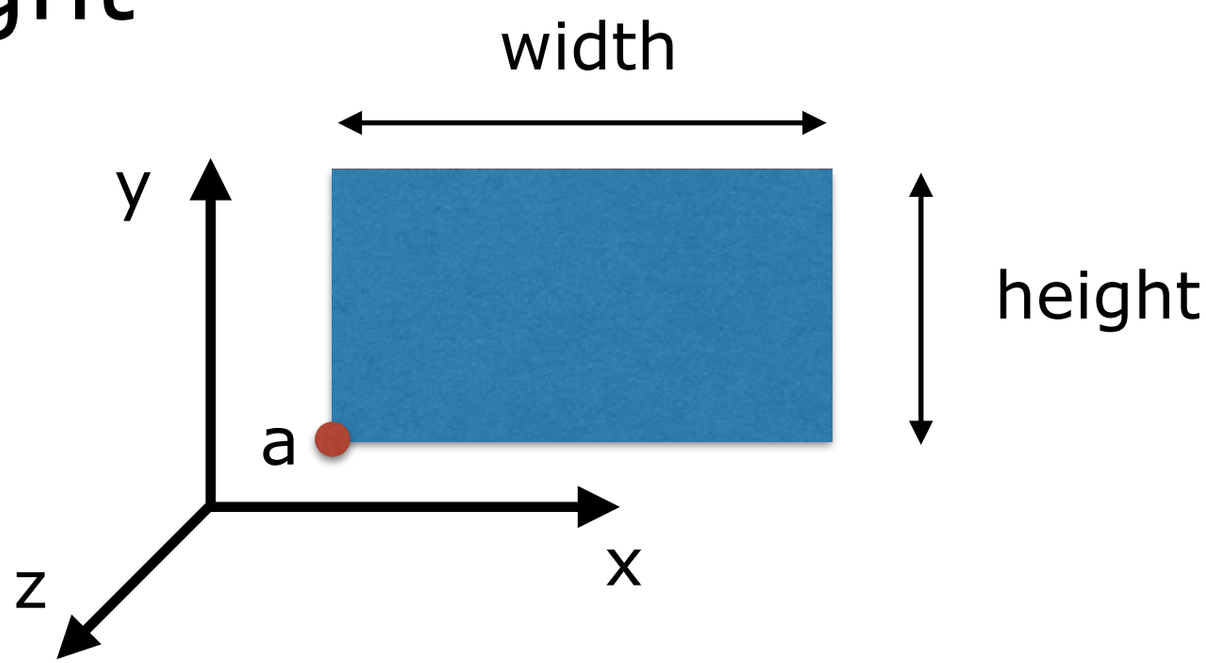
$$a_x \leq p_x \leq a_x + \text{width}$$
$$a_y \leq p_y \leq a_y + \text{height}$$



normal vector: [0,0,1]

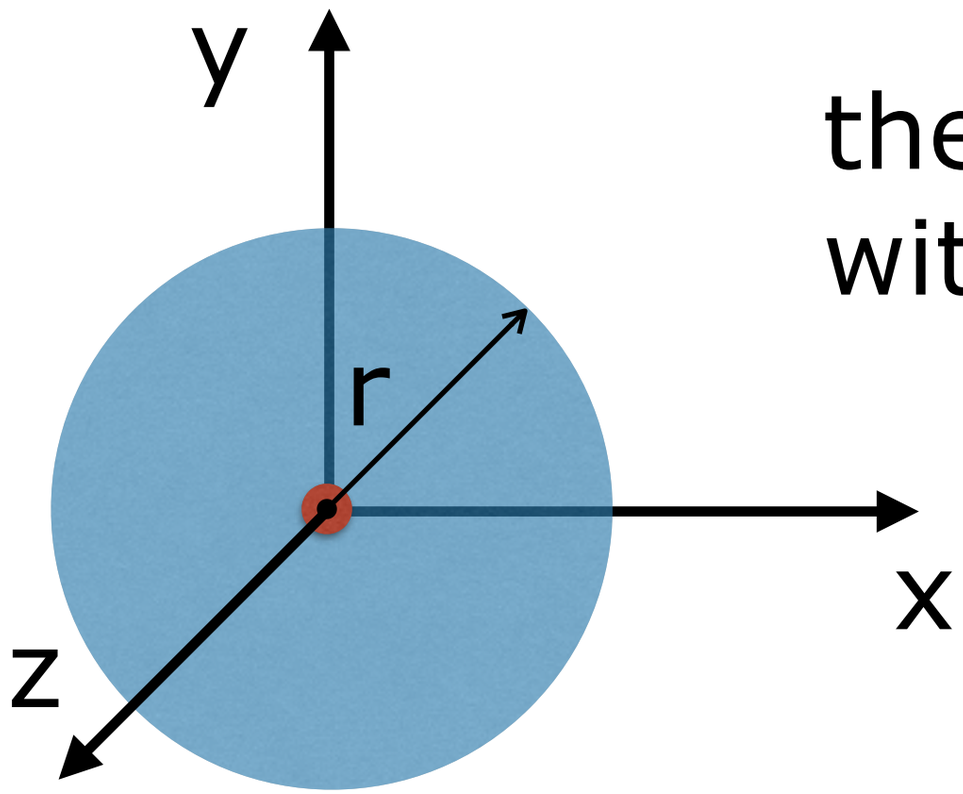
Texture Mapping

- recall: $[0,1] \times [0,1] \rightarrow$ Colour
- we need to map the intersection point p to u-v coordinates in $[0,1] \times [0,1]$
- $u = (p_x - a_x)/\text{width}$
- $v = (p_y - a_y)/\text{height}$



Discs

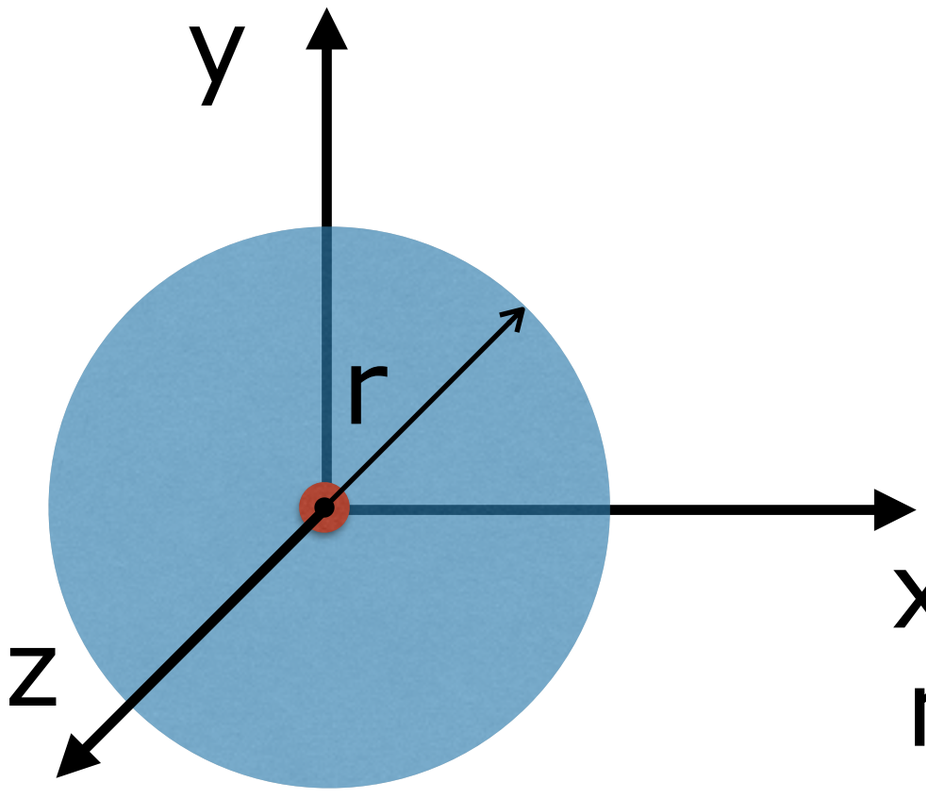
- We only consider discs that are axis aligned
- They can be transformed later



the disc is in xy plane with center (0,0,0)

Hit function

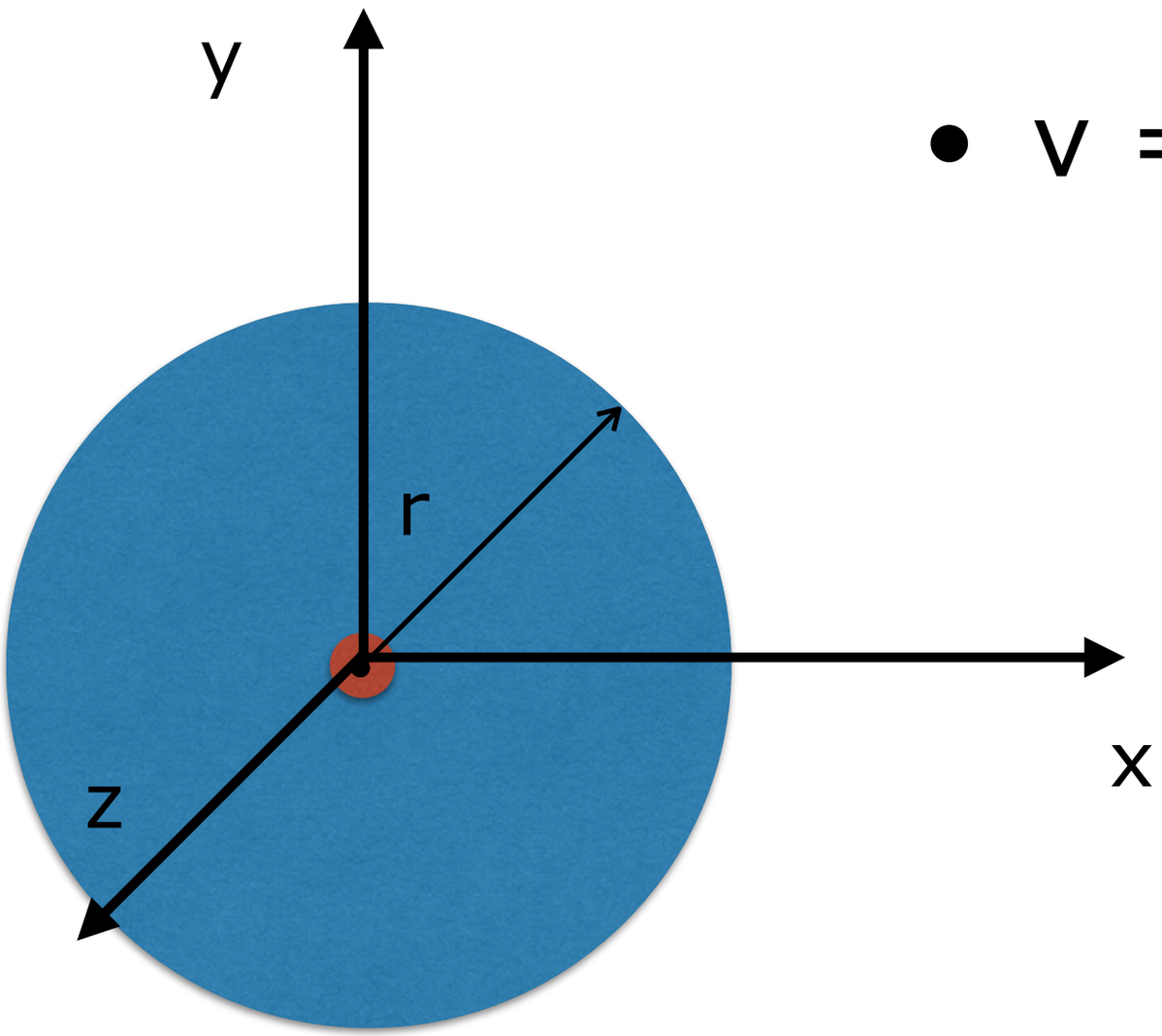
- check intersection with the plane the disc is in: $p_z = 0$
- check that the intersection point p (if any) is in the disc:



$$p_x^2 + p_y^2 \leq r^2$$

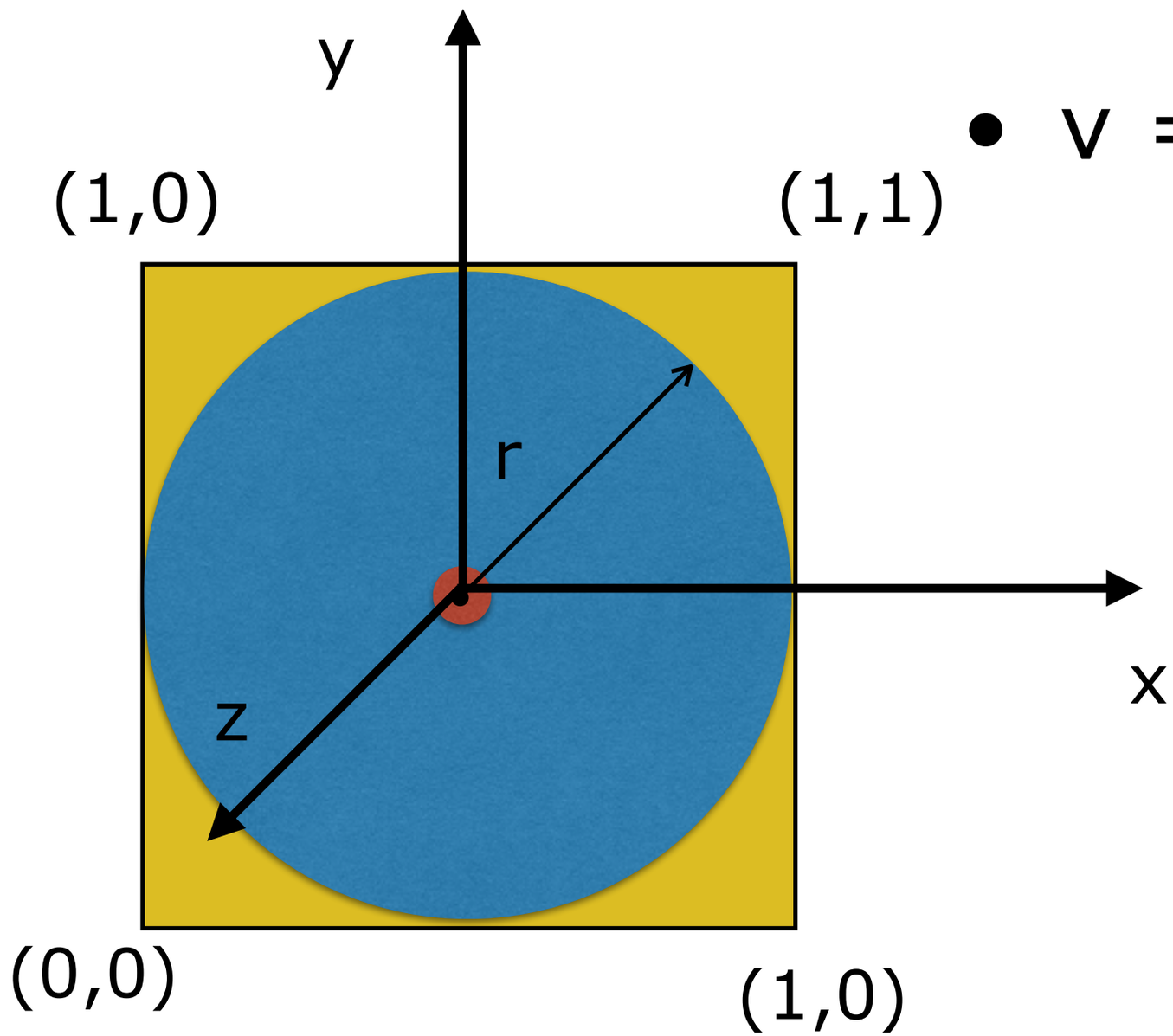
normal vector: (0,0,1)

Texture Mapping



- $u = (p_x+r)/2r$
- $v = (p_y+r)/2r$

Texture Mapping



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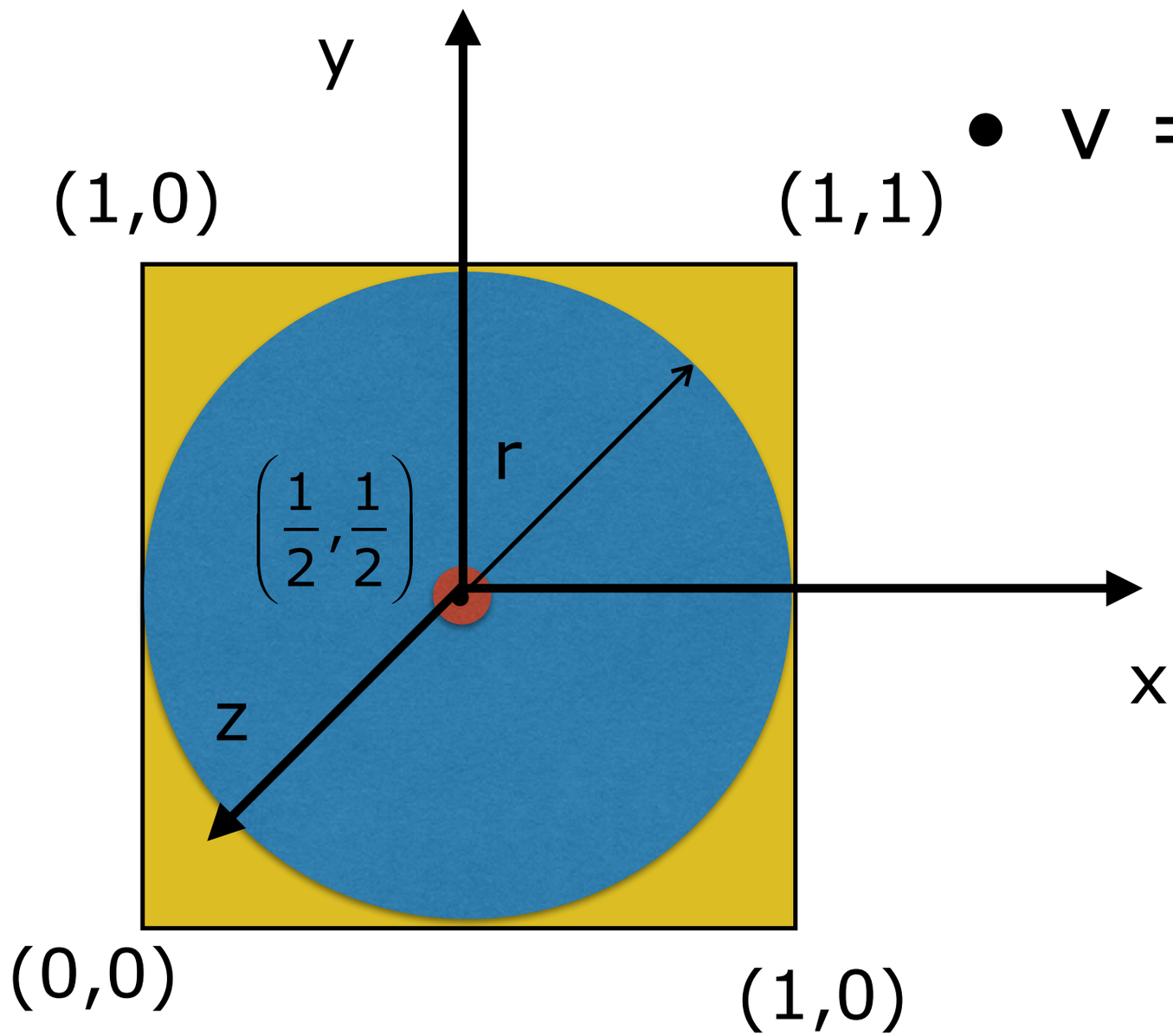
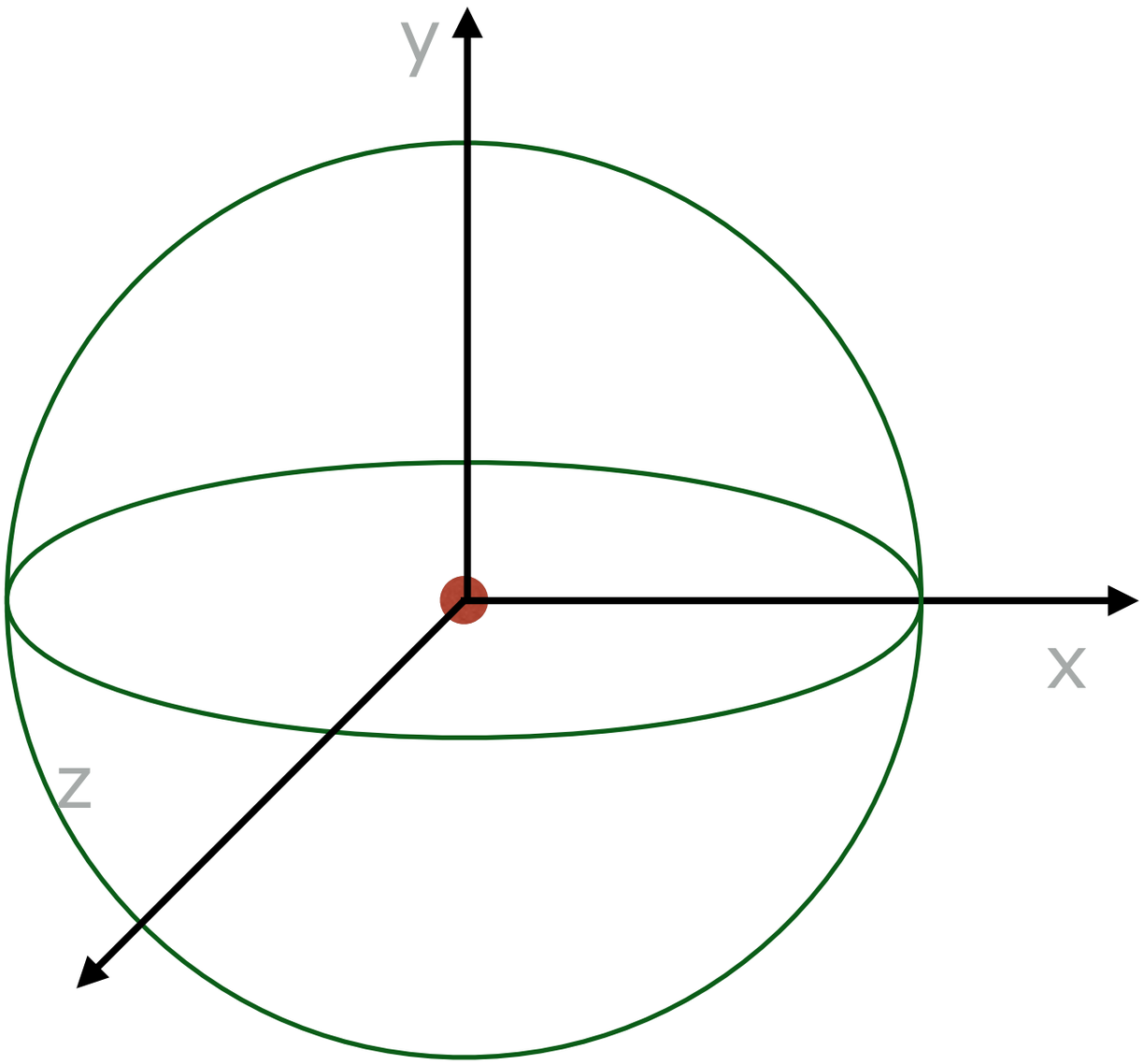


- $u = (p_x+r)/2r$
- $v = (p_y+r)/2r$

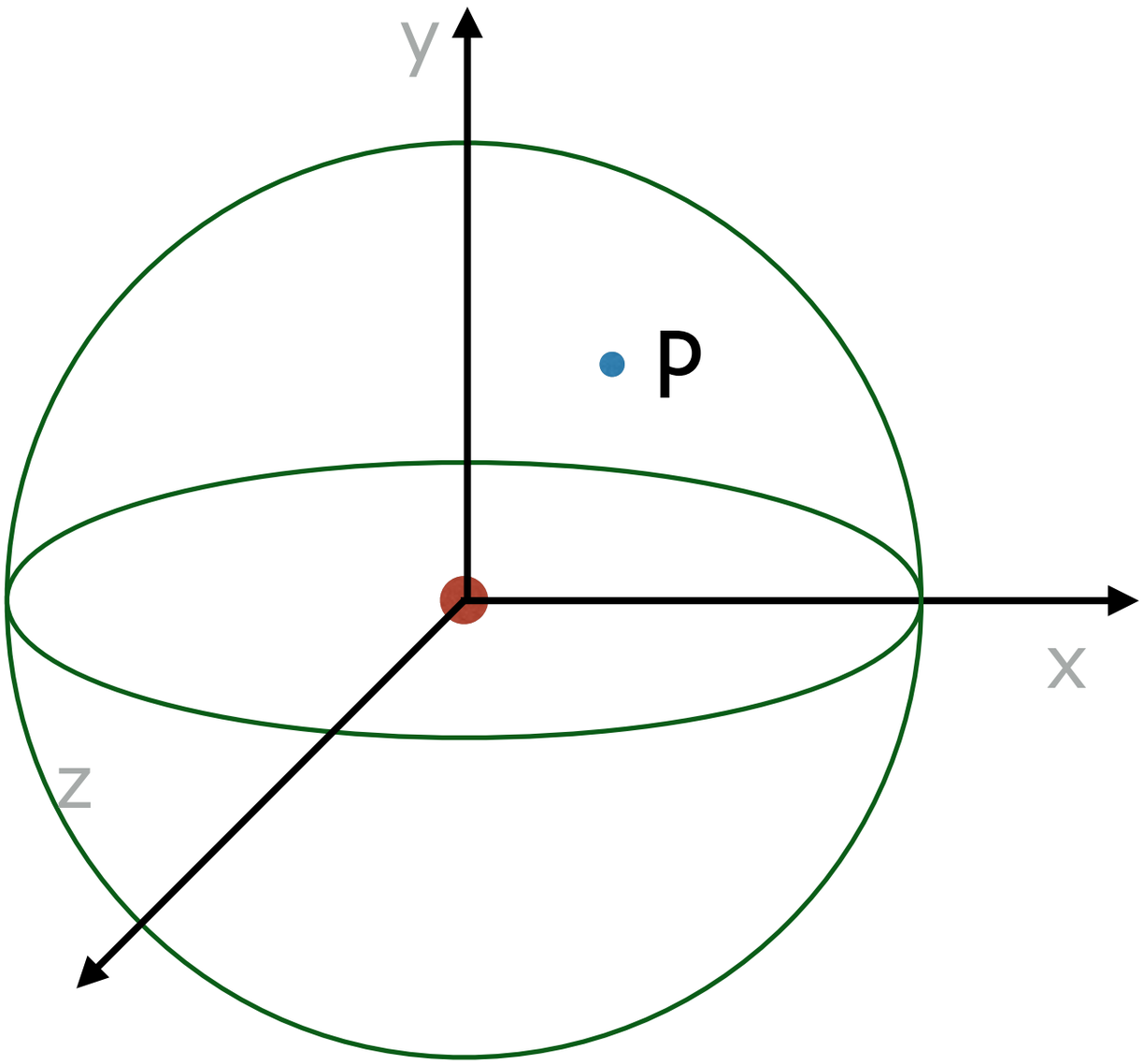
Texturing a Sphere

Sphere with radius r



Texturing a Sphere

Sphere with radius r



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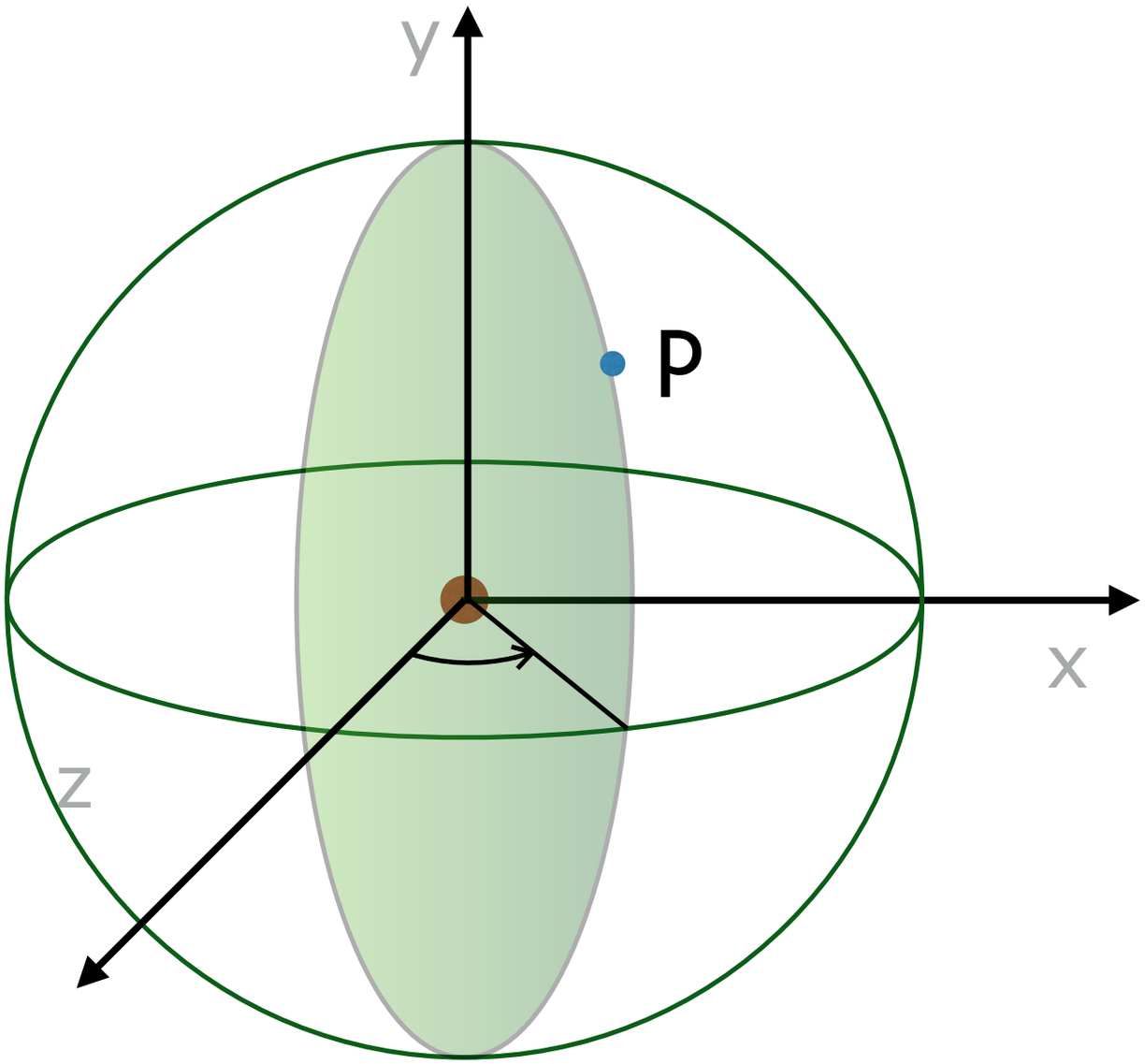

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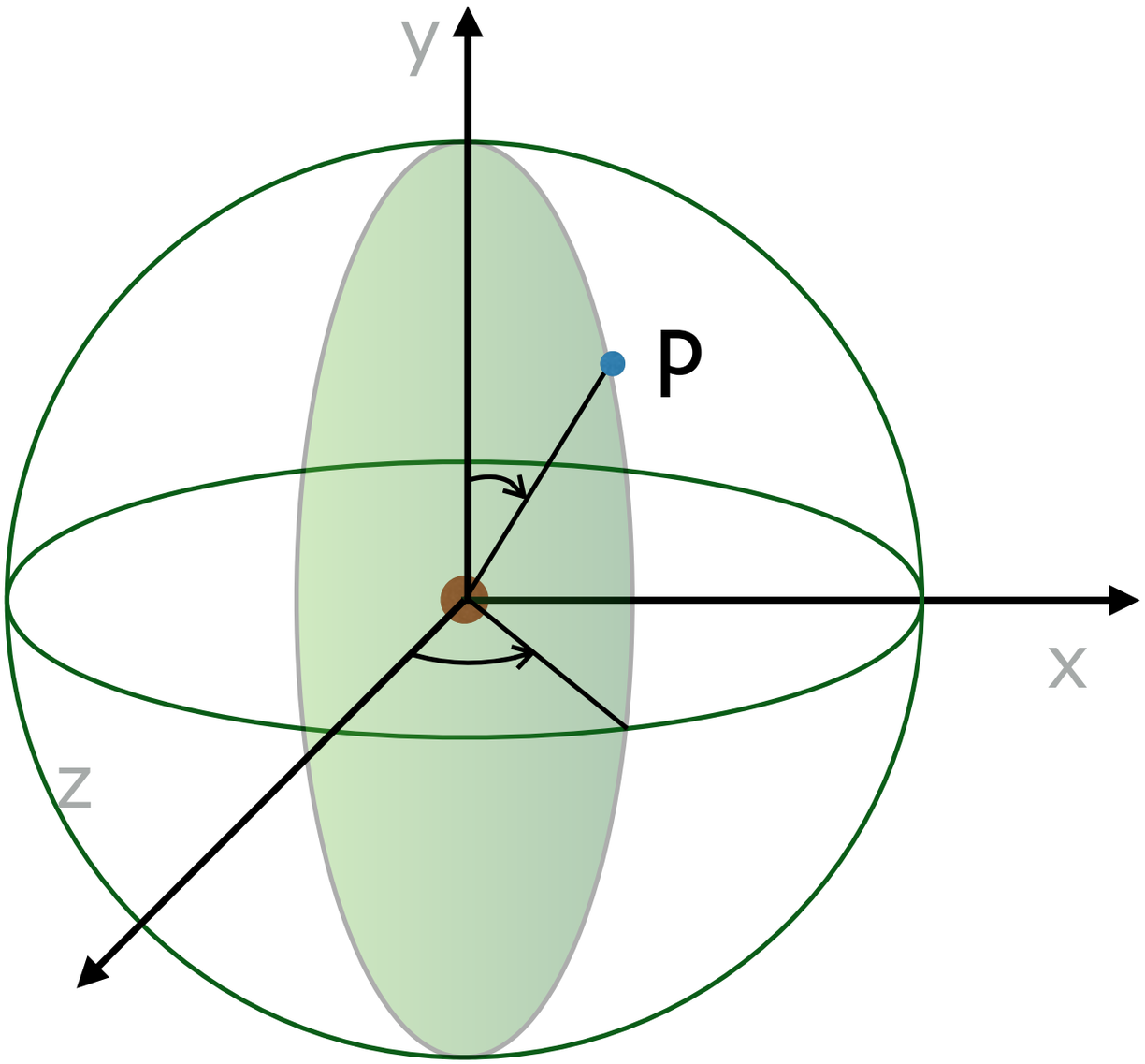

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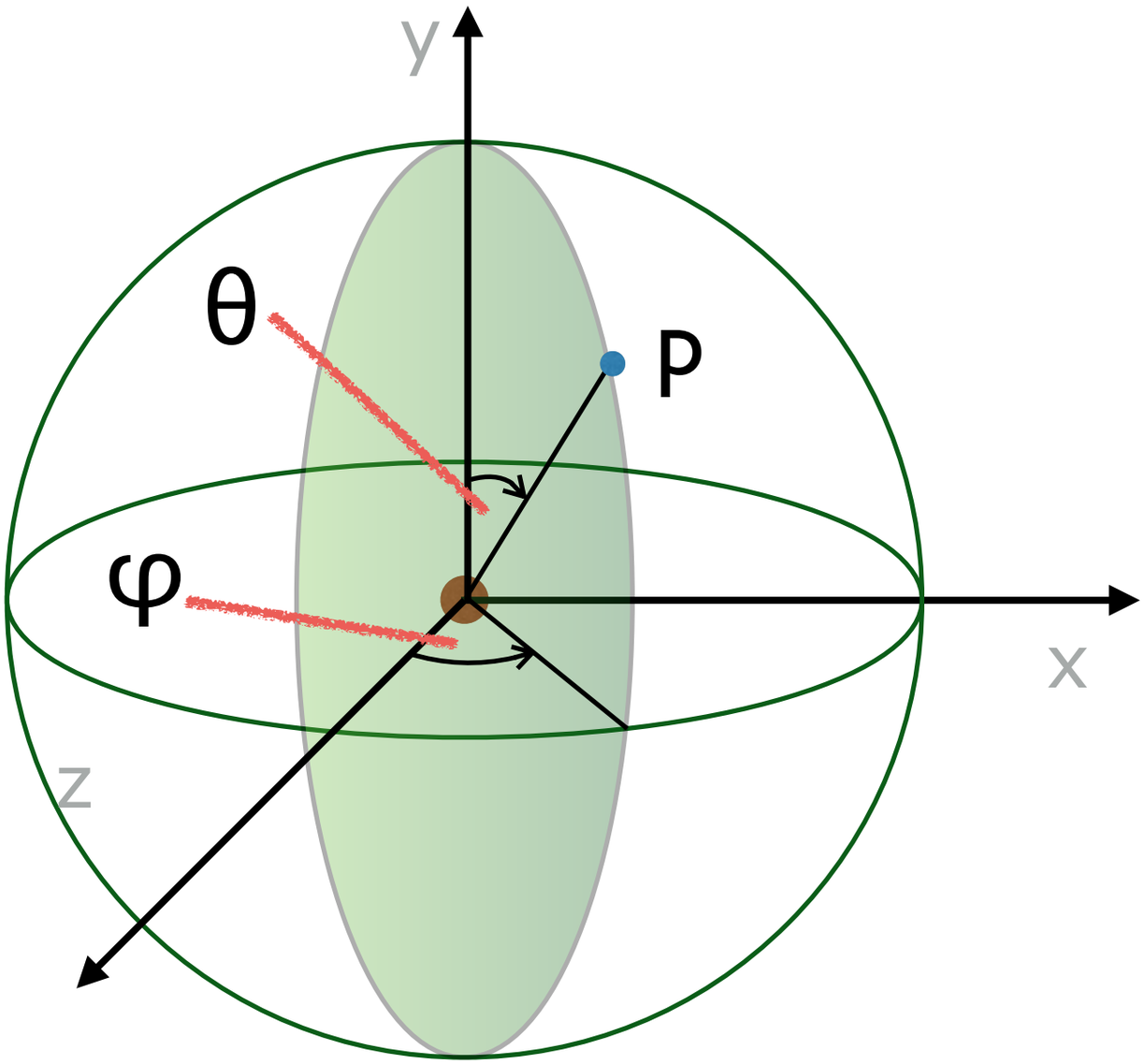

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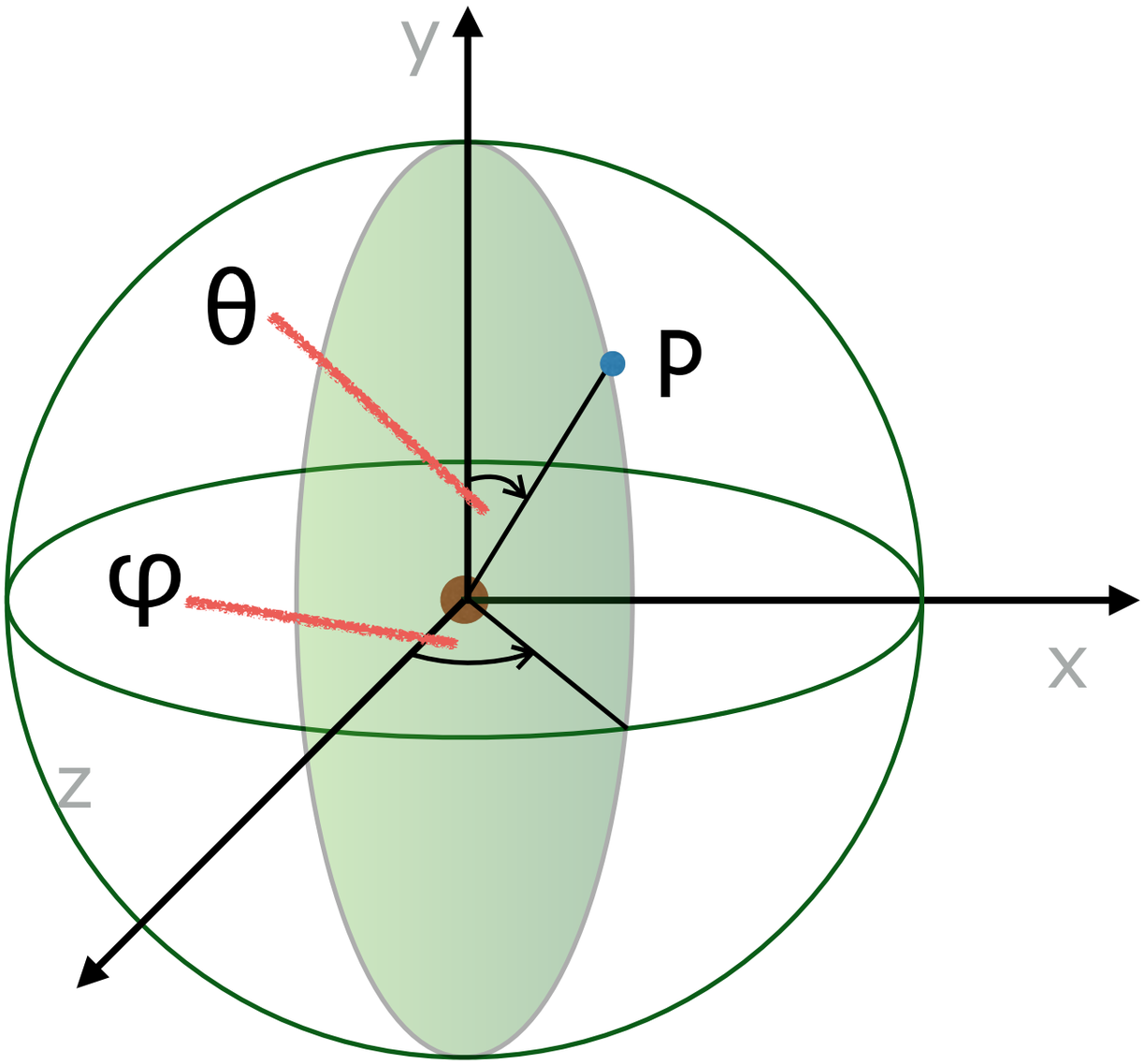
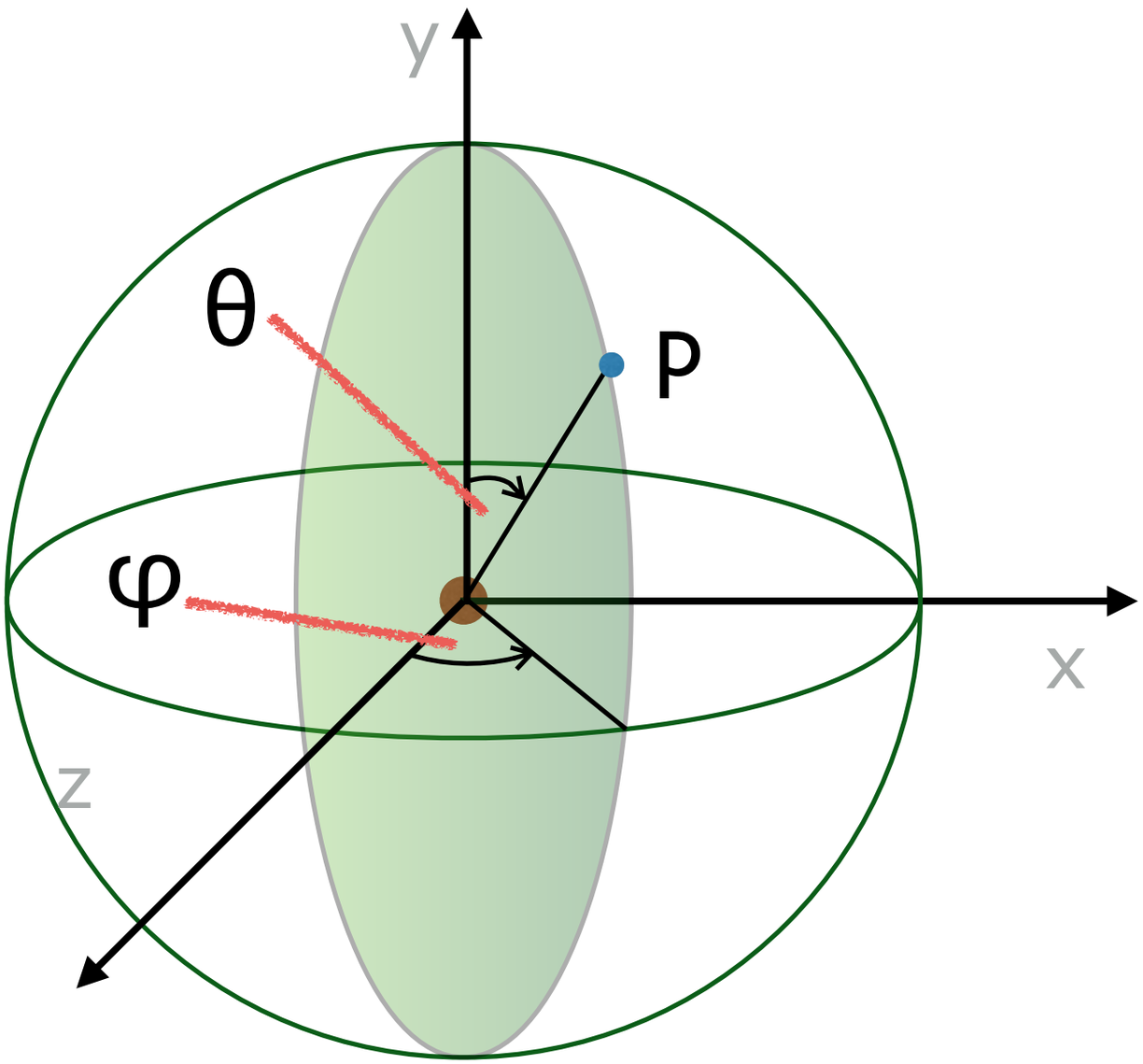


$u = \varphi/2\pi$

$v = 1-\theta/\pi$

Texturing a Sphere

Sphere with radius r



$n = p/r$

$\theta = \text{acos}(n_y)$

$\varphi = \tan^{-1}(n_x/n_z)$

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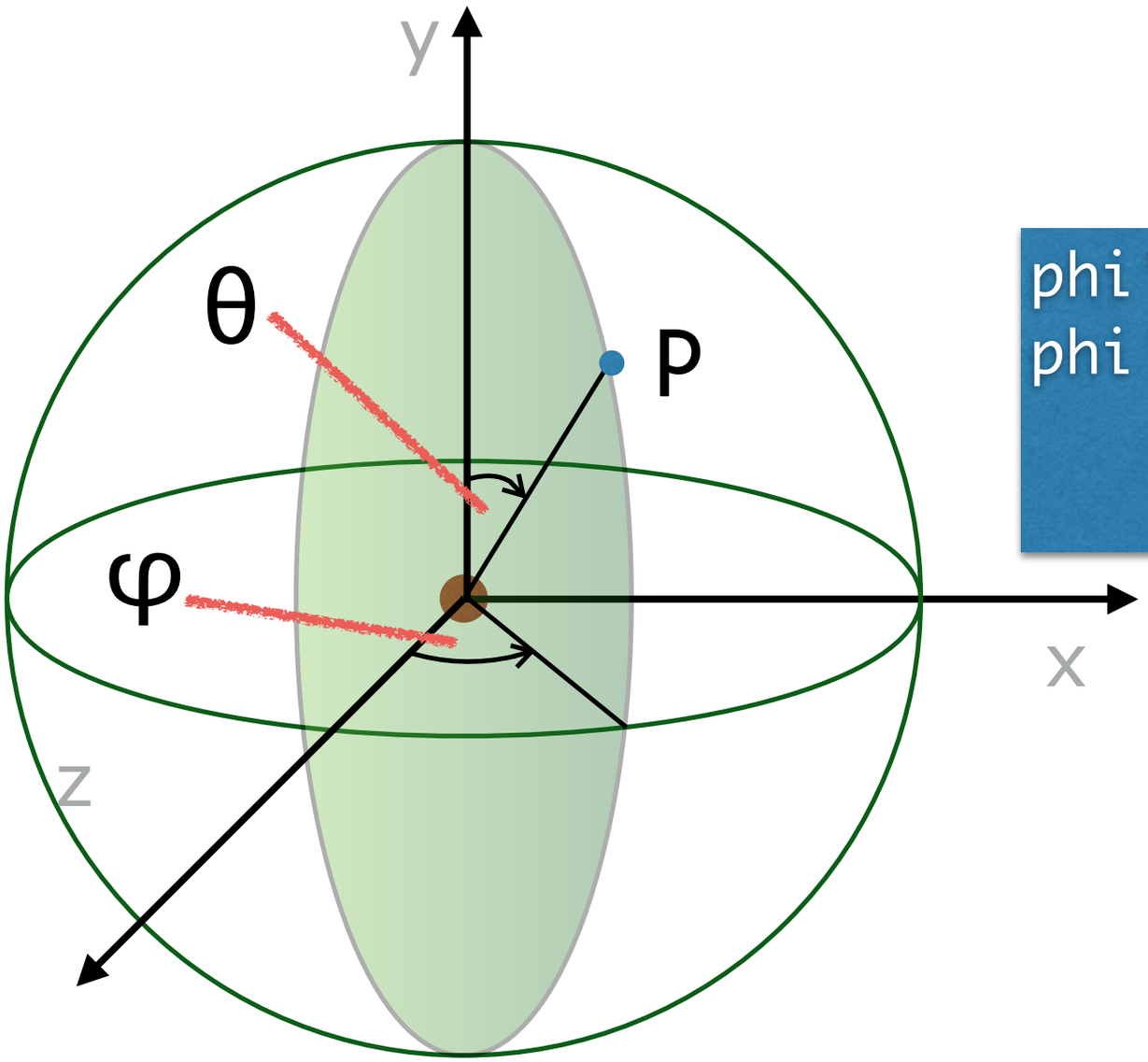

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```
phi' = atan2(nx,nz)
phi  = if phi' < 0
       then phi' + 2 * pi
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```

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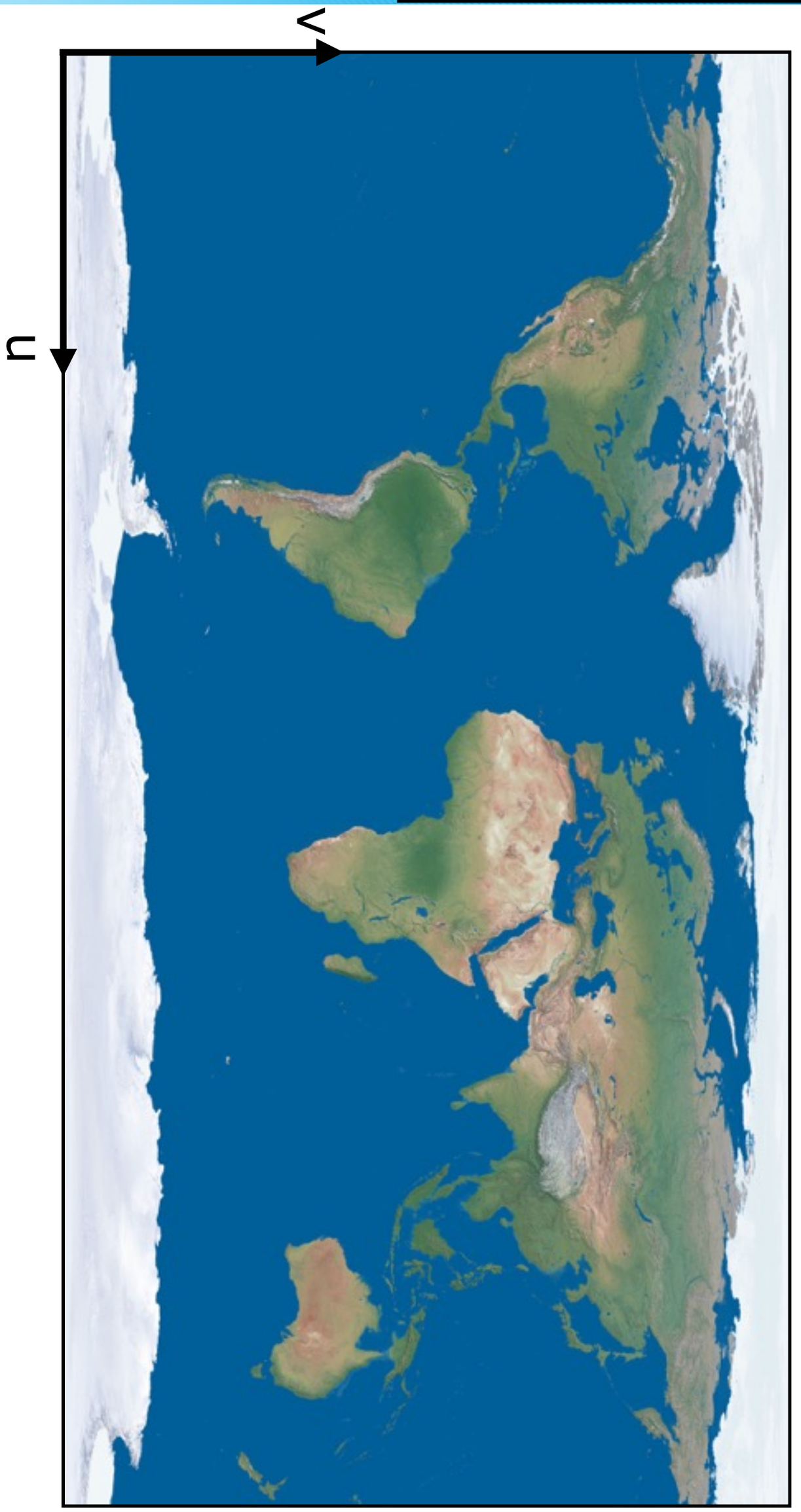
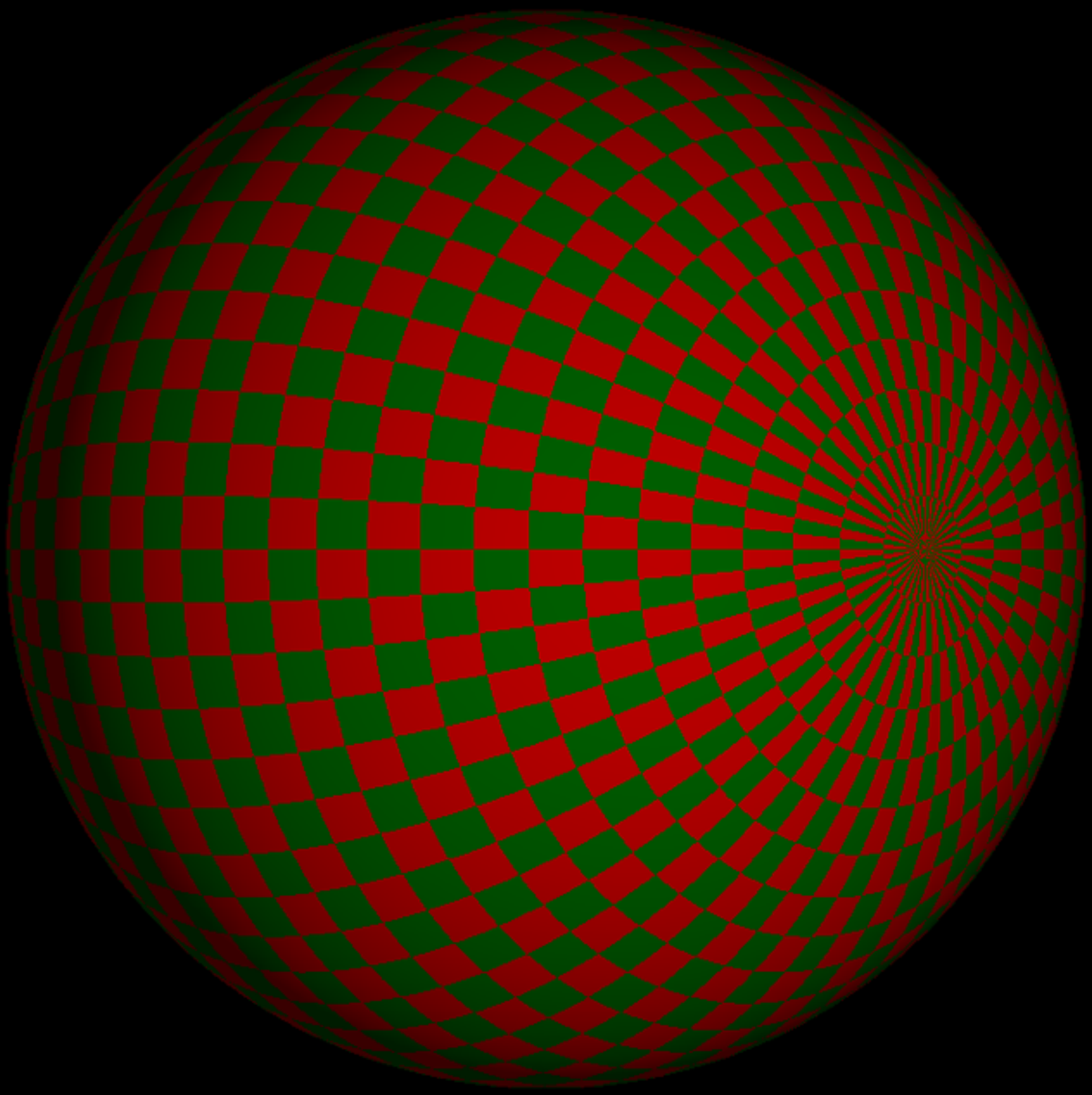
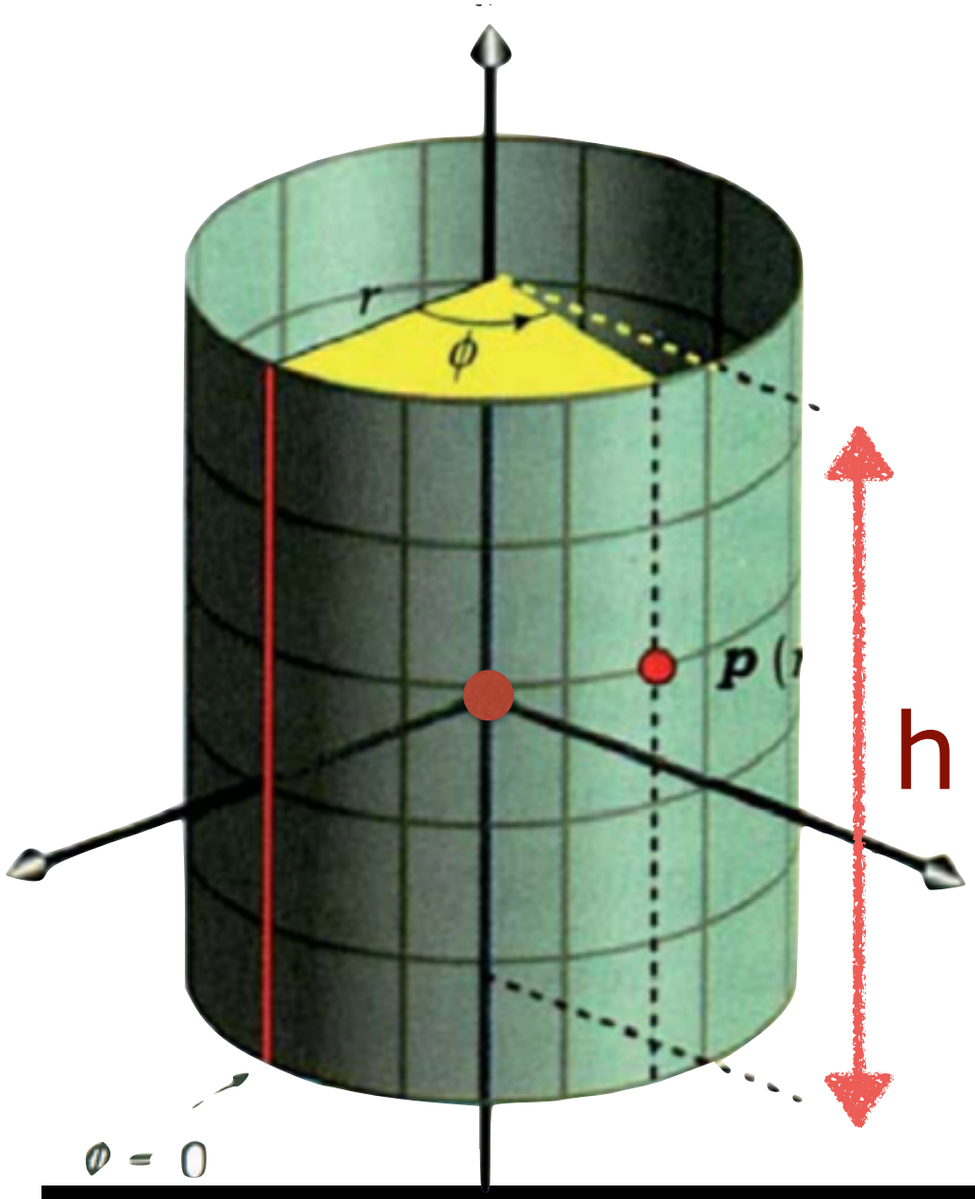

v
u



Open (hollow) Cylinder

- centre point (0,0,0)
- height h
- radius r



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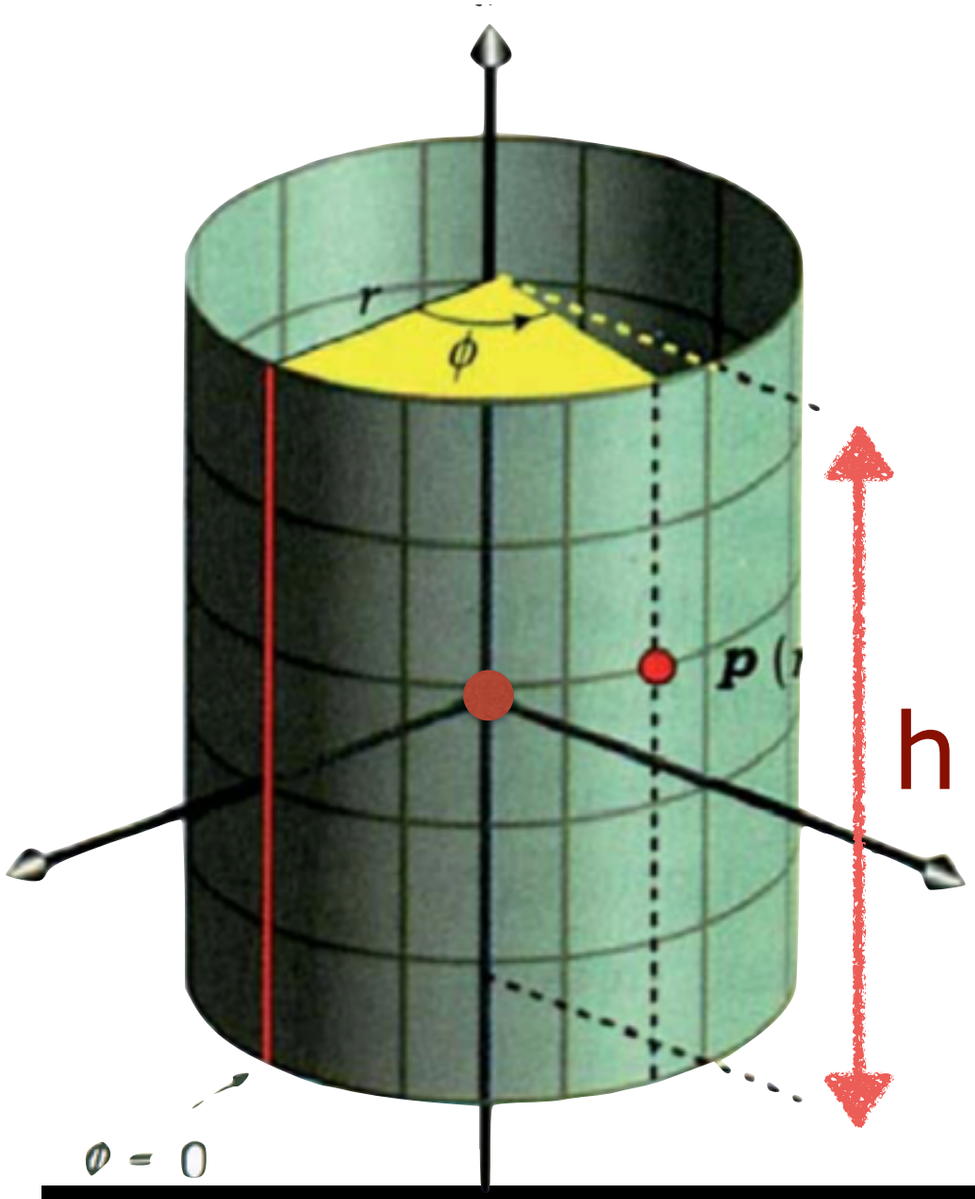

1. use equation for infinite cylinder

$$p_x^2 + p_z^2 = r^2$$

2. find intersections with ray

$$p = o + td$$

3. check that intersection point p satisfies

$$-h/2 \le p_y \le h/2$$

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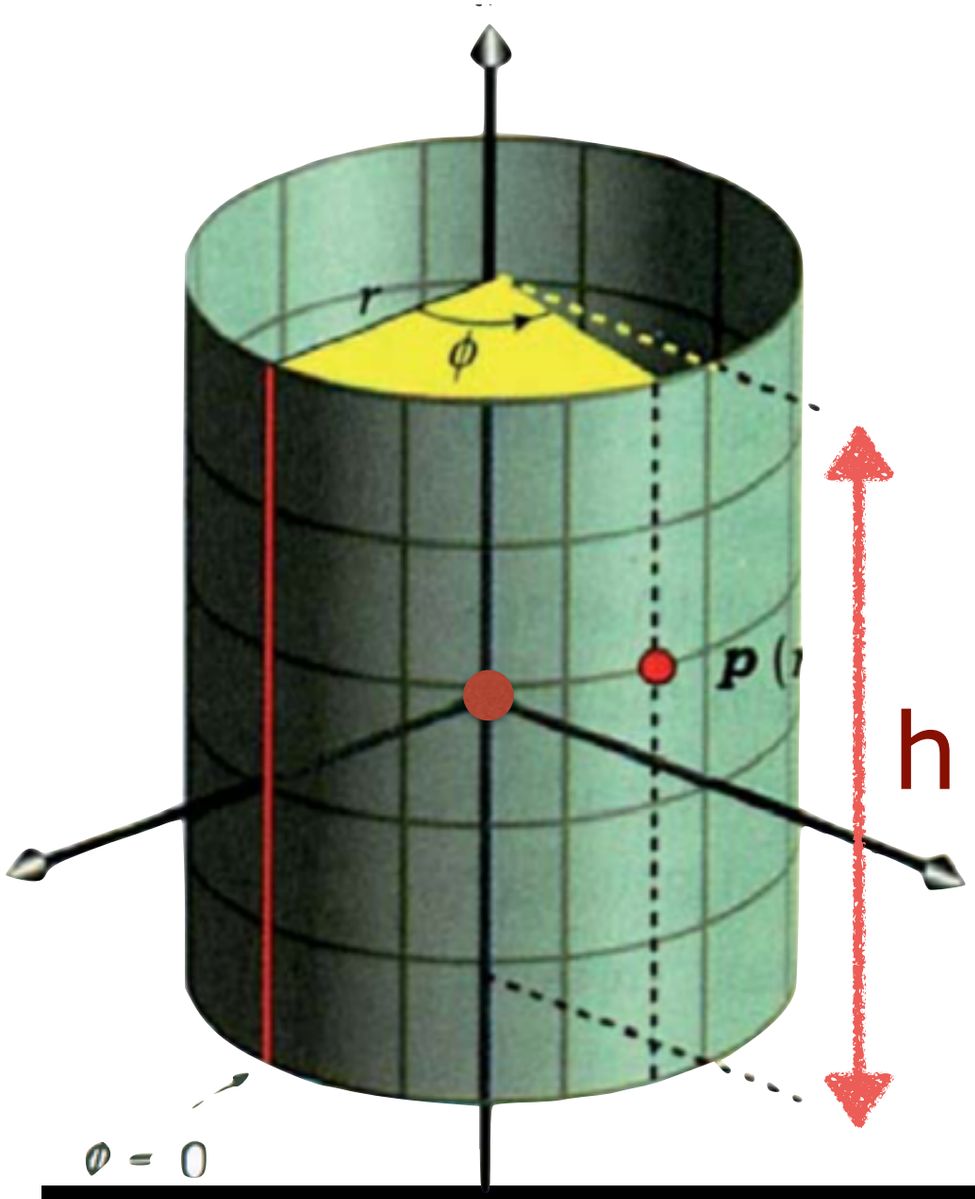

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$$-h/2 \le p_y \le h/2$$

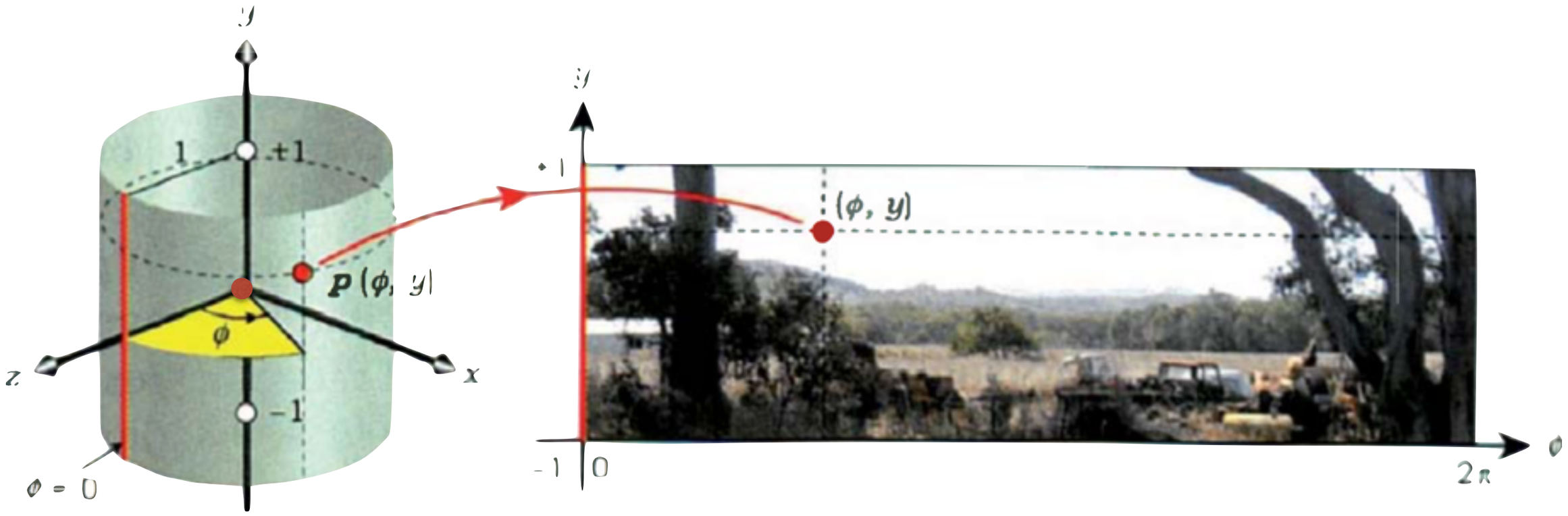
normal vector:

$$n = (p_x / r, 0, p_z / r)$$

Texture Mapping

$$u = \phi / 2\pi$$

$$v = \frac{p_y}{h} + \frac{1}{2}$$



Texture Mapping

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$$\phi = \tan^{-1}(n_x / n_z)$$

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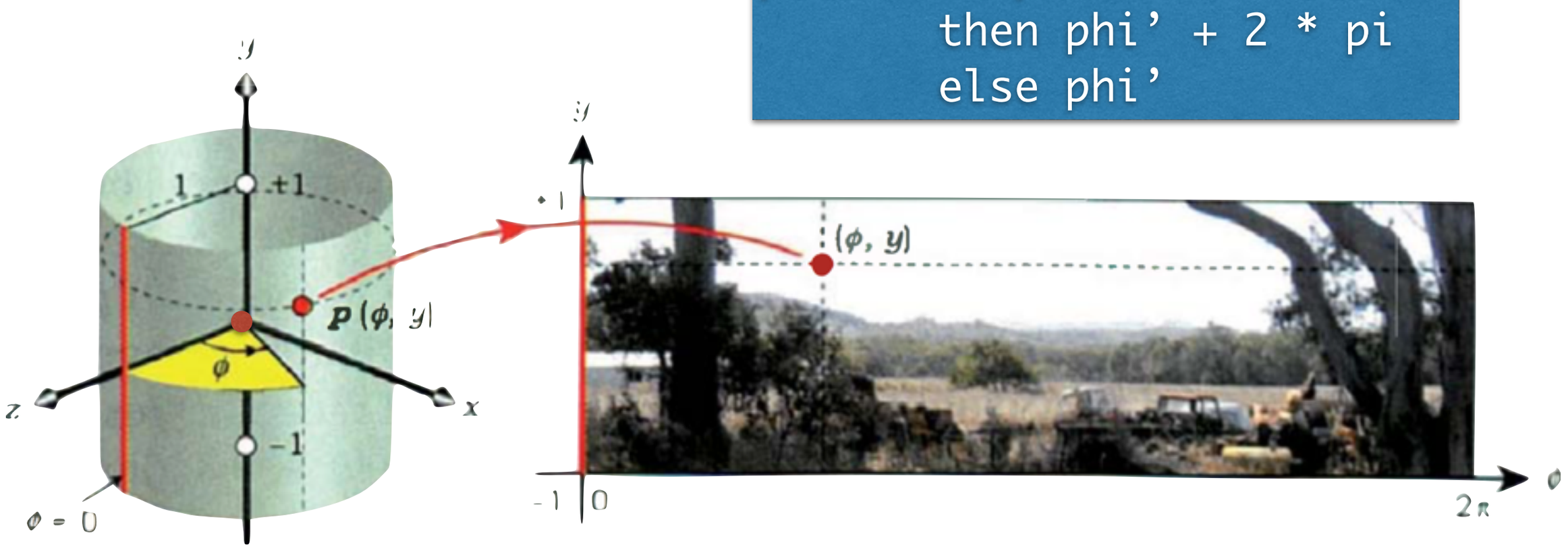
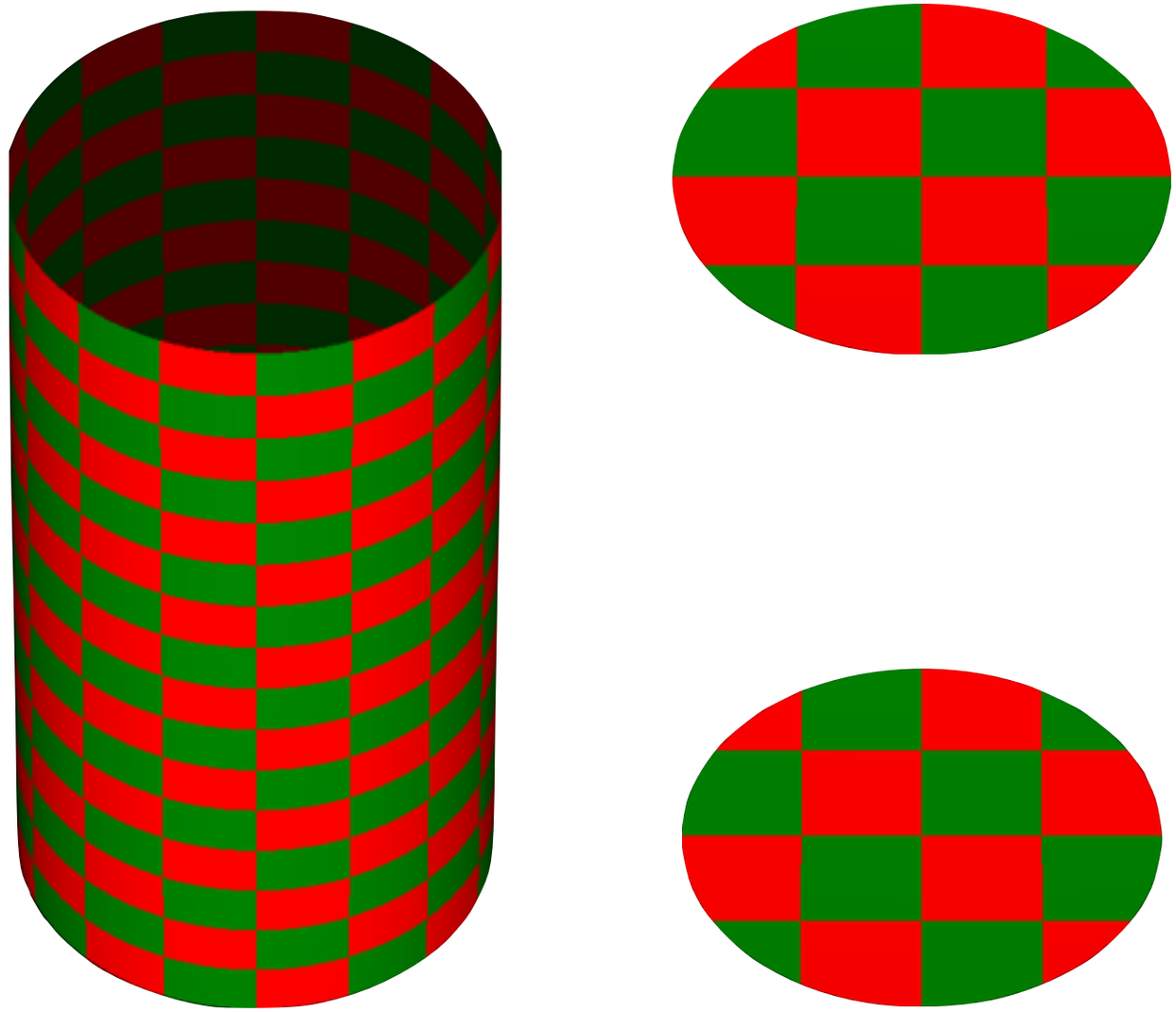


Solid Cylinder

open cylinder + 2 discs



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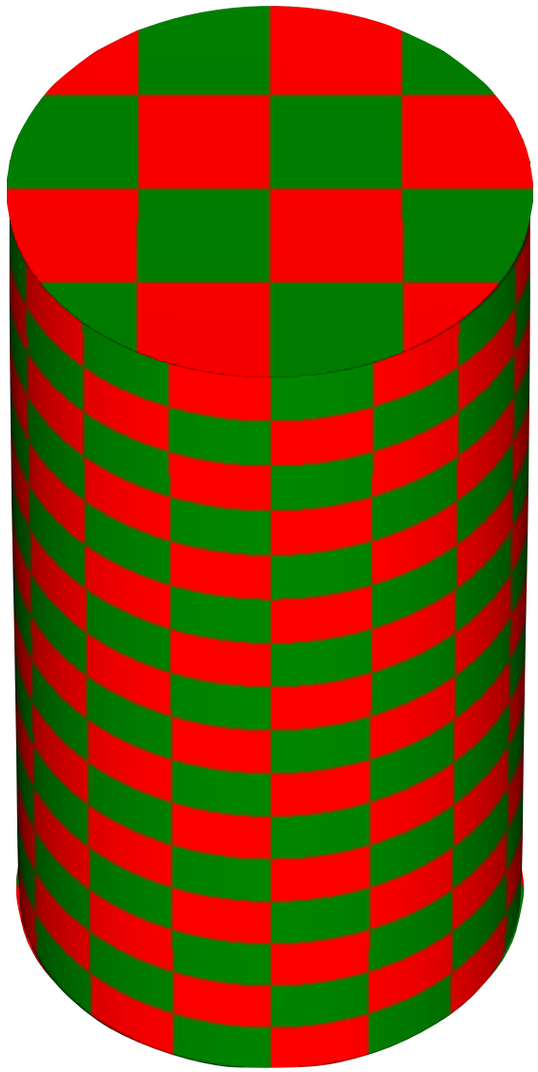

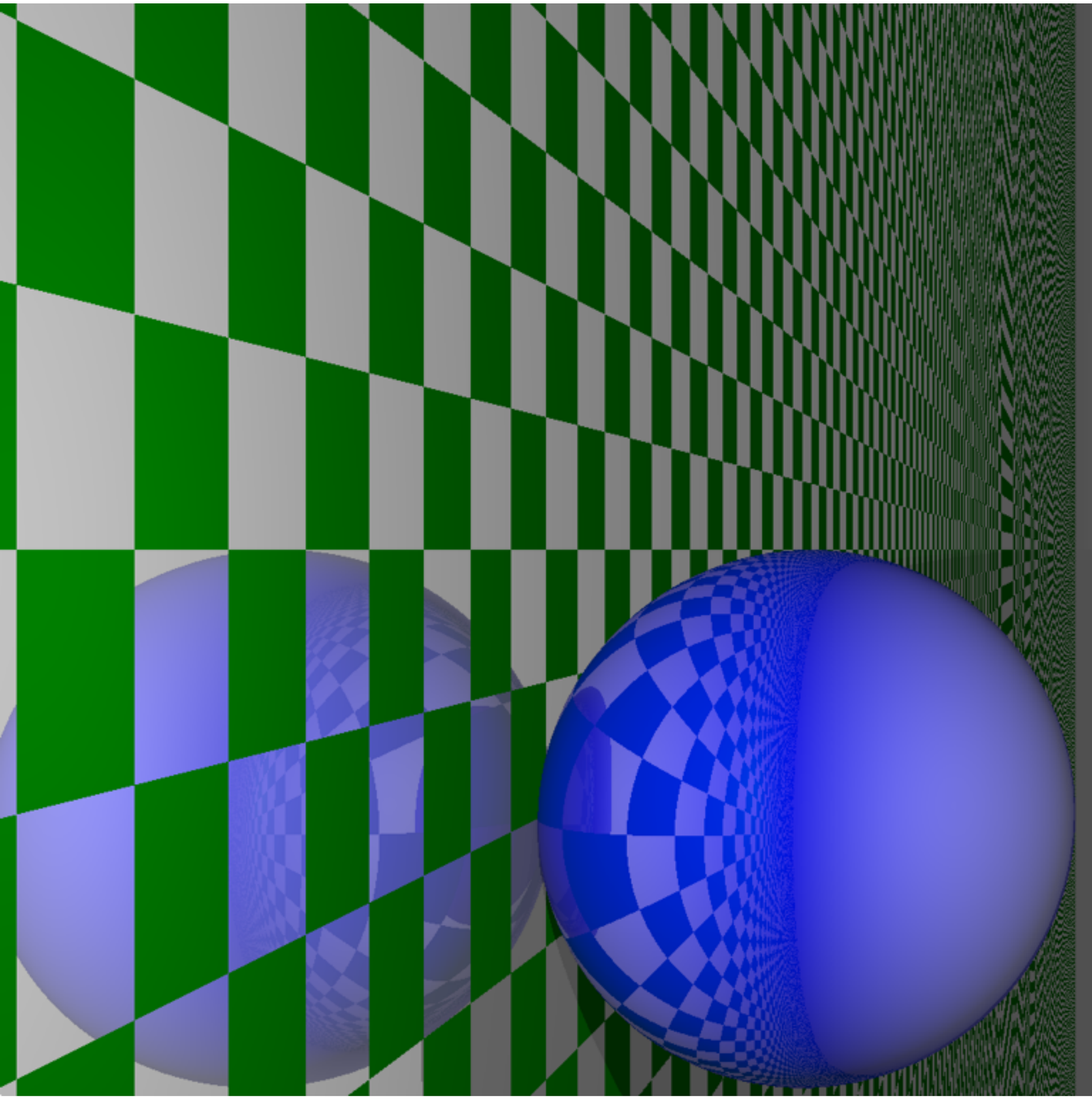

Infinite Plane

- We only consider simple case:

  the x-y plane
- simple equation: $p_z = 0$
- (u,v) coordinates may be arbitrary (i.e. not necessarily in [0,1])

$$u = p_x$$

$$v = p_y$$

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Questions

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Triangle Meshes

Texture Coords in PLY Files

```
ply
format ascii 1.0
comment this is a bunny!!
element vertex 35947
comment has texture coords and normals
property float x
property float y
property float z
property float nx
property float ny
property float nz
property float u
property float v
property float fooness
property float barness
element face 69451
property list uint uint vertex_indices
end_header
…
```

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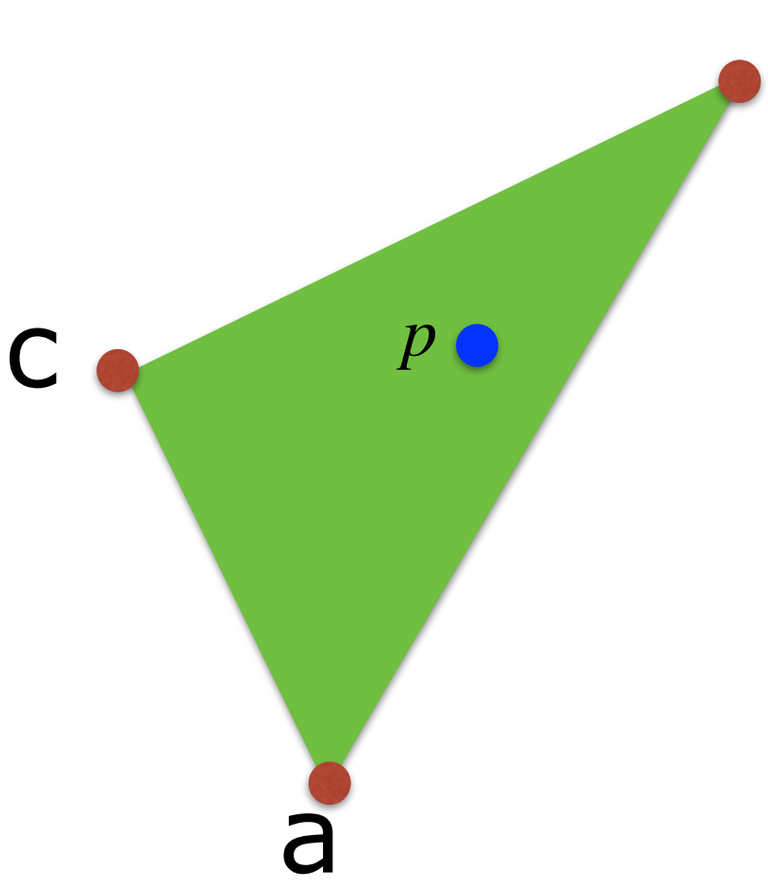
texture coordinates

Recall



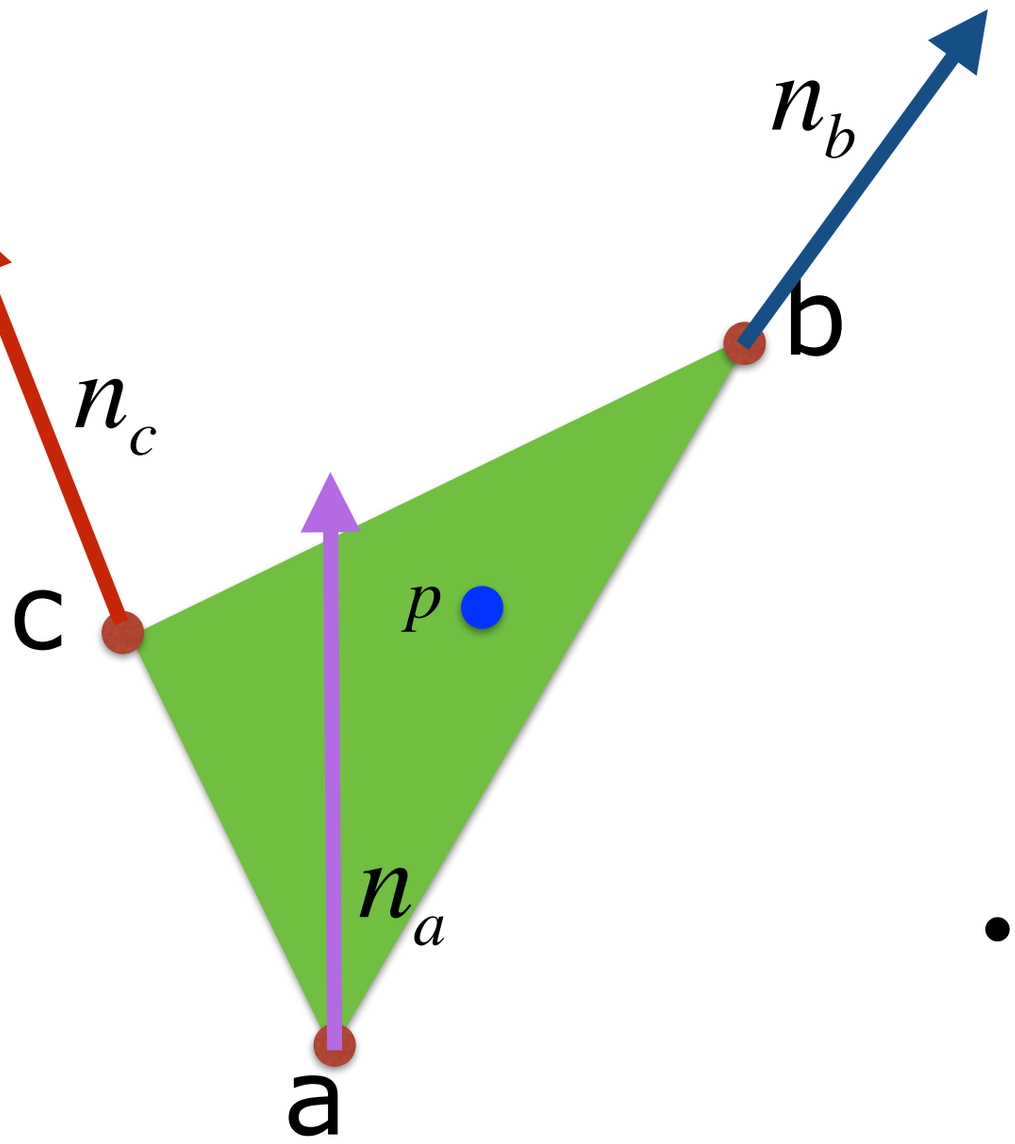
- Hit function calculates
  - distance t
  - coordinates β, γ

$$p = \alpha a + \beta b + \gamma c$$

$$\alpha = 1 - \beta - \gamma$$

- we used α,β,γ for interpolating normals
- texture coordinates are interpolated in the same way!

Recall



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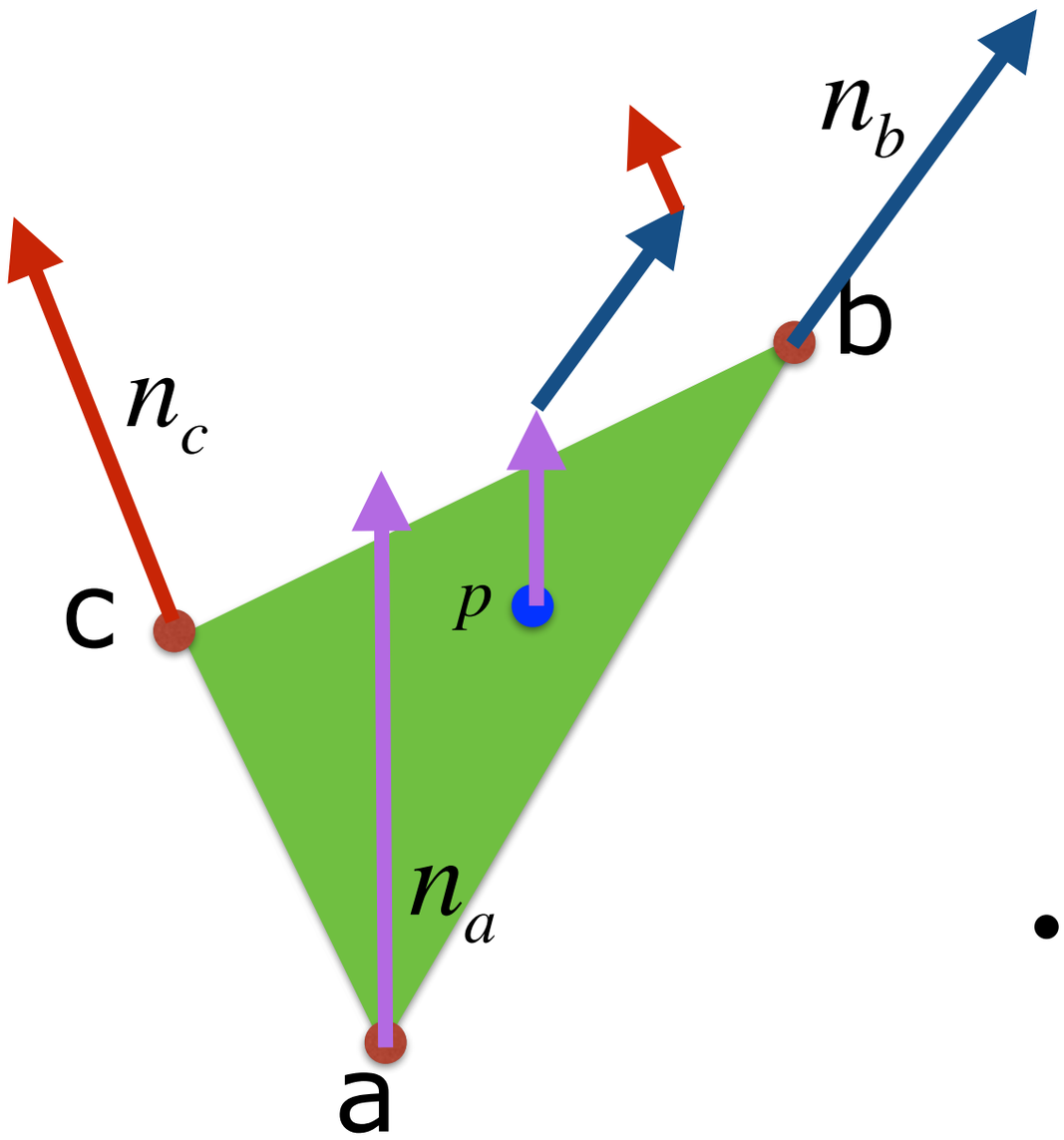


$$v = \alpha\, n_a + \beta\, n_b + \gamma\, n_c$$

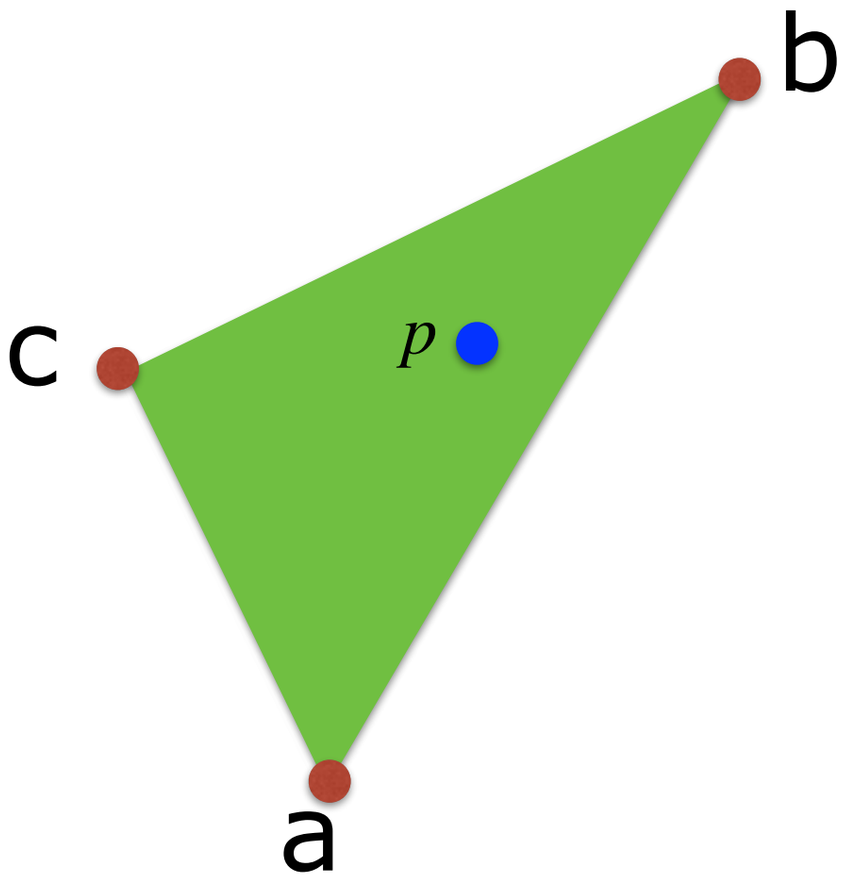
- Hit function calculates
  - distance t
  - coordinates β, γ

$$p = \alpha\, a + \beta\, b + \gamma\, c$$

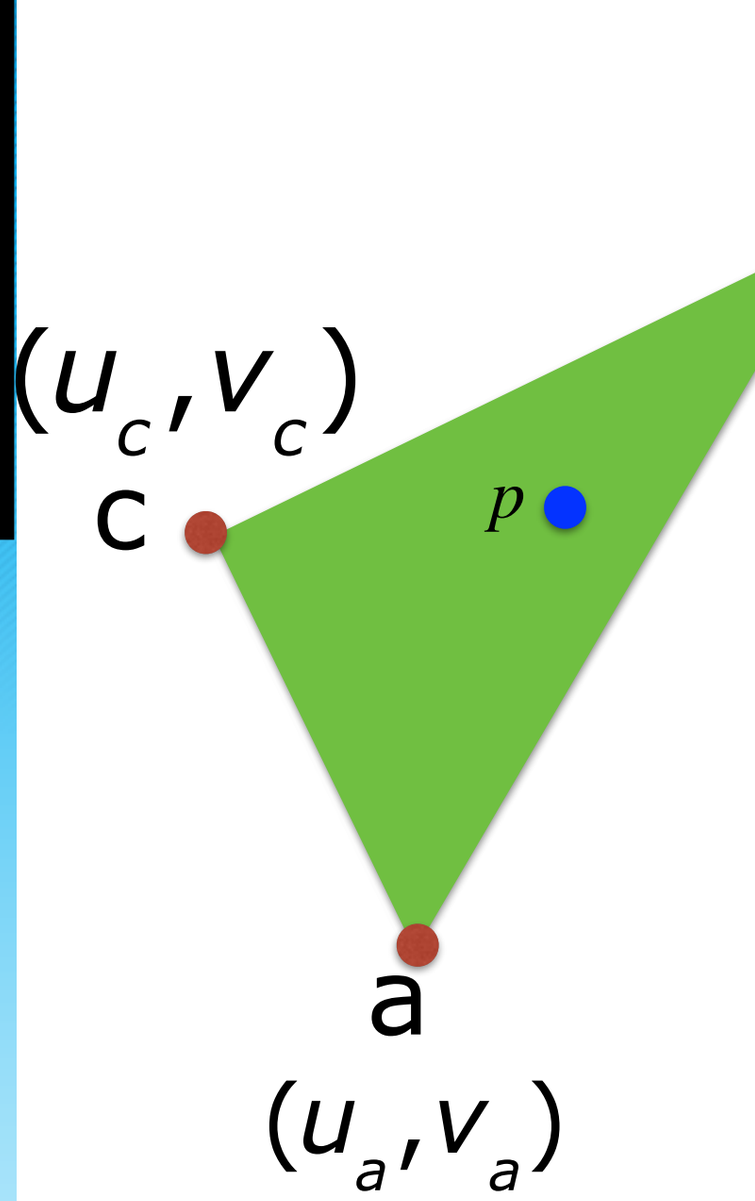
$$\alpha = 1 - \beta - \gamma$$

- we used α,β,γ for interpolating normals
- texture coordinates are interpolated in the same way!

Calculate Texture Coords



Calculate Texture Coords



$$v = \alpha v_a + \beta v_b + \gamma v_c$$

$$u = \alpha u_a + \beta u_b + \gamma u_c$$

Recap

1. find intersection of ray with a shape
2. calculate (u,v) coordinates of intersection point
3. lookup material at (u,v) coordinate
4. use the material to colour the pixel an to do calculate reflection

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