Discrete Mathematics

Basic Mathematics

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Today's lecture

1. Integers
   - Arithmetic properties
   - Powers
   - Divisibility
   - Primes and composite numbers
2. Rational numbers
   - Equivalent fractions
   - Operating with fractions
   - Decimals
3. Irrational numbers
4. Real numbers
   - Square roots
   - n-th roots
   - Logarithms
   - Inequalities
5. Order of operations

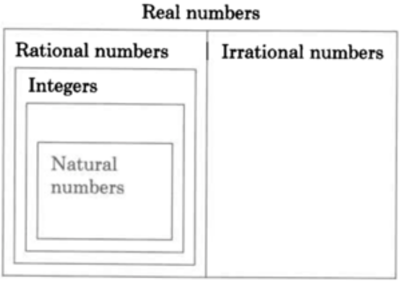
Numbers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, the set of natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of integers

$\mathbb{Q} = \{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z},$ and $q \neq 0\}$, the set of rational numbers

$\mathbb{R}$, the set of real numbers



Outline

Simple rules of addition

For any integer a,

$$\boxed{0 + a = a + 0 = a,}$$

$$\boxed{a + (-a) = 0} \text{ and also } \boxed{-a + a = 0.}$$

We call $-a$ the additive inverse of a. We use the name minus a for $-a$, rather than "negative a".

Example The additive inverse of -3 is 3, because $3 + (-3) = 3 - 3 = 0$.

Rules of addition (I)

Commutativity

If a and b are integers, then

$$a + b = b + a.$$

Examples

- $3 + 5 = 5 + 3 = 8,$
- $-2 + 5 = 5 + (-2) = 3.$

Associativity

If a, b and c are integers, then

$$(a + b) + c = a + (b + c).$$

Examples

- $(3 + 5) + 9 = 8 + 9 = 17,$
  $3 + (5 + 9) = 3 + 14 = 17.$
- $(-2 + 5) + 4 = 3 + 4 = 7,$
  $-2 + (5 + 4) = -2 + 9 = 7.$

Rules of addition (II)

$$\boxed{\text{If } a+b=0, \text{ then } b=-a \text{ and } a=-b.}$$

Proof: Add $-a$ to both sides of the equation $a+b=0$. We get

$$-a+a+b=-a+0=-a.$$

Since $-a+a+b=0+b=b$, we find

$$b=-a$$

as desired. Similarly, we can find $a=-b$.

□

A special case of the above rule is

$$\boxed{a=-(-a)}$$

Rules of addition (III)

$\boxed{\text{If } a, b \text{ are positive integers, then } a+b \text{ is also a positive integer.}}$

$\boxed{\text{If } a, b \text{ are negative integers, then } a+b \text{ is also negative.}}$

If we have the relationship between three integers

$$\boxed{a+b=c}$$

then we can derive other relationships between them

$$\boxed{a=c-b}\ \boxed{b=c-a}$$

Example Solve for x.

$$x+3=5$$
$$x=5-3=2$$

Rules of addition (IV)

Cancellation rule for addition

$$\boxed{\text{If } a+b=a+c\text{, then } b=c.}$$

Exercise Prove that if $a+b=a$, then $b=0$.

Rules for multiplication (I)

Commutativity

If a and b are integers, then $ab = ba$.

Remember that multiplication ab is also denoted by a dot $a \cdot b$.

Associativity

If a, b and c are integers, then $(ab)c = a(bc)$.

For any integer a, the rules of multiplication by 1 and 0 are:

$$\boxed{1a = a} \text{ and } \boxed{0a = 0.}$$

Examples

- $(2a)(3b) = 2(a(3b)) = \ 2(3a)b = \ (2 \cdot 3)ab = 6ab.$
- $(5x)(7y) = 35xy.$
- $(2a)(3b)(5x) = 30abx.$

Rules for multiplication (II)

Distributivity

$$a(b+c) = ab + ac$$

and also on the other side,

$$(b+c)a = ba + ca.$$

Using all these properties, we have

$$\boxed{(-1)a = -a}$$

and also

$$\boxed{-(ab) = (-a)b,} \text{ or similarly, } \boxed{-(ab) = a(-b)}$$

Examples

- $(-2a)(3b)(4c) = \ (-2) \cdot 3 \cdot 4abc = \ -24abc.$
- $(-4x)(y - 3z) = \ -4xy - (-4x)3z = \ \ -4xy + 12xz.$

Note also that

$$\boxed{(-a)(-b) = ab.}$$

Powers (I)

An exponent is used to indicate repeated multiplication. It tells how many times the base is used as a factor.

Examples

- $aa = a^2$,
- $aaa = a^3$,
- $aaaa = a^4$,
- In general, if n is a positive integer, $a^n = aa \cdots a$ (the product is taken n times).

We say that a^n is the n-th power of a.

If m, n are positive integers, then

$$\boxed{a^{m+n} = a^m a^n.}$$

Examples

- $a^2 a^3 = (aa)(aaa) = a^{2+3} = a^5.$
- $(4x)^2 = 4x \cdot 4x = 4 \cdot 4xx = 16x^2.$
- $(7x)(2x)(5x) = 7 \cdot 2 \cdot 5xxx = 70x^3.$

Powers (II)

$$\boxed{(a^m)^n = a^{mn}}$$

Examples

- $(a^3)^4 = \; a^{12}$.
- $(ab)^n = \; a^n b^n$.
- $(2a^3)^5 = \; 2^5(a^3)^5 = \; 32a^{15}$.

Some other important formulas worth to remember: (notable products)

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2,} \; \boxed{(a-b)^2 = a^2 - 2ab + b^2,}$$

$$\boxed{(a+b)(a-b) = a^2 - b^2.}$$

Proof Applying repeatedly the rules for multiplication. For example, for the first formula,

$$\begin{aligned} (a+b)^2 = (a+b)(a+b) &= a(a+b) + b(a+b) \\ &= aa + ab + ba + bb \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2. \end{aligned}$$

Powers (III)

Examples

$$(2+3x)^2 = 2^2 + 2 \cdot 2 \cdot 3x + (3x)^2$$
$$= 4 + 12x + 9x^2.$$

$$(3-4x)^2 = 3^2 - 2 \cdot 3 \cdot 4x + (4x)^2$$
$$= 9 - 24x + 16x^2.$$

Exercise Expand the expression $(4a-6)(4a+6)$. $\quad = (4a)^2 - 36 = 16a^2 - 36.$

Even and odd integers (I)

The usual way of describing an even integer is to say that it is an integer which can be written in the form $2n$ for some integer n. For instance, we can write

$$\begin{aligned} 2 &= 2 \cdot 1, \\ 4 &= 2 \cdot 2, \\ 6 &= 2 \cdot 3, \\ 8 &= 2 \cdot 4. \end{aligned}$$

and so on.

Similarly, an odd integer is an integer which differs from an even integer by 1. It can be written in the form $2m - 1$ for some integer m. For instance,

$$\begin{aligned} 1 &= 2 \cdot 1 - 1, \\ 3 &= 2 \cdot 2 - 1, \\ 5 &= 2 \cdot 3 - 1, \\ 7 &= 2 \cdot 4 - 1. \end{aligned}$$

We can also write an odd integer in the form $2m + 1$.

Even and odd integers (II)

Theorem

Let a, b be integers.
If a is even and b is even, then $a + b$ is even.
If a is even and b is odd, then $a + b$ is odd.
If a is odd and b is even, then $a + b$ is odd.
If a is odd and b is odd, then $a + b$ is even.

Proof We prove the second statement. Assume a is even and b is odd. Then, we can write $a = 2n$ and $b = 2k + 1$ for some integers n and k. Then

$$\begin{aligned} a + b &= 2n + 2k + 1 \\ &= 2(n + k) + 1 \\ &= 2m + 1. \text{ (letting } m = n + k) \end{aligned}$$

This proves that $a + b$ is odd. □

Divisibility

Given two integers a and b, with $a \neq 0$, we say that a divides b, or that b is divisible by a if there is an integer c such that $b = ac$.

Example

- 12 is divisible by 3 because $12 = 3 \cdot 4$. We say that 3 is a factor (or divisor) of 12.
- 12 is not divisible by 5.

Remember that every integer is divisible by 1, because we can always write

$$n = 1 \cdot n.$$

Also, every positive integer is divisible by itself.

Primes

A prime p is an integer greater than 1 if its only positive factors are 1 and p. Any other integer greater than 1 but not being prime, is called a composite.

Fundamental Theorem of Arithmetic

Every integer greater than 1 can be written uniquely as a prime or as the product of two or more primes where the prime factors are written in increasing order.

Example The prime factors of 9780 and 330 are given by

$9780 = 2 \cdot 2 \cdot 5 \cdot 163 = 2^2 \cdot 3 \cdot 5 \cdot 163$
$330 = 2 \cdot 3 \cdot 5 \cdot 11$

Outline

Rational numbers

By a rational number we mean a fraction, that is a quotient

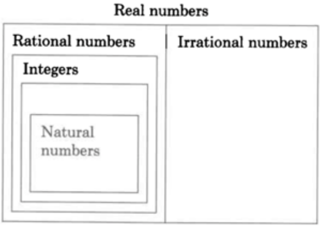
$$\frac{m}{n},$$

where both m, n are integers, $n \neq 0$, m is called numerator and n is the denominator.

The numerator tells how many fractional pieces there are.

An improper fraction is a fraction that has a numerator larger than or equal to its denominator.

A proper fraction is a fraction with the numerator smaller than the denominator.



Equivalent fractions

Two fractions are equivalent if they represent the same value. Equivalent fractions represent the same portion of a whole.

For instance, we have

$$\frac{1}{2} = \frac{2}{4}.$$

How can we know whether two fractions are equivalent?

Rule for cross-multiplying

Let m, n, r, s be integers and assume that $n \neq 0$ and $s \neq 0$. Then

$$\frac{m}{n} = \frac{r}{s} \text{ if and only if } ms = rn.$$

Examples

- $\frac{1}{2} = \frac{2}{4}$ because $1 \cdot 4 = 2 \cdot 2$.
- $\frac{3}{7} = \frac{9}{21}$ because $3 \cdot 21 = 9 \cdot 7$.

Simplifying fractions (I)

We can simplify four special fraction forms.

- ***Fractions that have the same numerator and denominator:*** In this case, we have a number divided by itself. The result is 1 (provided the numerator and denominator are not 0). We call each of the following fractions a **form of 1.**

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \dots$$

- ***Fractions that have a denominator of 1:*** In this case, we have a number divided by 1. The result is simply the numerator.

$$\frac{5}{1} = 5 \qquad \frac{24}{1} = 24 \qquad \frac{-7}{1} = -7$$

- ***Fractions that have a numerator of 0:*** In this case, we have division of 0. The result is 0 (provided the denominator is not 0).

$$\frac{0}{8} = 0 \qquad \frac{0}{56} = 0 \qquad \frac{0}{-11} = 0$$

- ***Fractions that have a denominator of 0:*** In this case, we have division by 0. The division is undefined.

$$\frac{7}{0} \text{ is undefined} \qquad \frac{-18}{0} \text{ is undefined}$$

Simplifying fractions (II)

Cancellation rule for fractions

Let a be a non-zero integer. Let m, n be integers, $n \neq 0$. Then

$$\frac{am}{an} = \frac{m}{n}.$$

Proof: Applying the rule for cross-multiplying and using the associativity and commutativity laws. $\square$

Example $\frac{8}{-10} = \frac{(-2)(-4)}{(-2)5} = \frac{-4}{5}$.

It is useful to observe

$$\boxed{\frac{-m}{n} = \frac{m}{-n} = -\frac{m}{n}.}$$

The cancellation rule leads to the notion of divisibility we have already seen.

Example We have $\frac{10}{15} = \frac{2\cdot 5}{3\cdot 5} = \frac{2}{3}$ because 10 and 15 are divisible by 5.

Simplifying fractions (III)

A fraction is in simplest form when the numerator and denominator have no common factors (or divisors) other than 1.

Theorem

Any positive rational number has an expression as a fraction in lowest form.

Example The rational number $\frac{2}{4}$ is not in its simplest form. But $\frac{1}{2}$ is.

Operating with fractions: addition and subtraction (I)

- Addition (or subtraction) of fractions with the same denominator.

$$\boxed{\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d} \text{ or } \frac{a}{d}-\frac{b}{d}=\frac{a-b}{d}.}$$

Example: $\frac{-5}{8}+\frac{2}{8}=\frac{-3}{8}$.

- Addition (or subtraction) of fractions with different denominator.

$$\boxed{\frac{m}{n}+\frac{r}{s}=\frac{ms+rn}{ns} \text{ or } \frac{m}{n}-\frac{r}{s}=\frac{ms-rn}{ns}.}$$

Example: $\frac{3}{5}+\frac{4}{7}= \frac{3\cdot 7+4\cdot 5}{35}= \frac{21+20}{35}= \frac{41}{35}$.

Exercise Compute $\frac{-5}{2}+\frac{3}{14}$.
$=\frac{(-5)14+3\cdot 2}{2\cdot 14}=\frac{-64}{28}=\frac{-(2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2)}{2\cdot 2\cdot 7}=\frac{-16}{7}$.

Rember to simplify the result, if possible!

Operating with fractions: addition and subtraction (II)

$\boxed{\text{The sum of positive rational numbers is also positive.}}$

Addition of rational numbers satifies the same basic rules as addition of integers (commutativity and associativity), and also

$$\boxed{0 + a = a + 0 = a,}$$

for any rational number a.

Operating with fractions: multiplication (I)

To multiply two rational numbers, multiply the numerators, multiply the denominators, and simplify the result, if possible.

$$\boxed{\frac{m}{n} \cdot \frac{r}{s} = \frac{mr}{ns}.}$$

Examples

- $\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$.
- $\frac{2}{7} \cdot \frac{11}{16} = \frac{2 \cdot 11}{7 \cdot 2 \cdot 8} = \frac{11}{7 \cdot 8} = \frac{11}{56}$.

Exercise Compute $\frac{-4}{5} \cdot \frac{7}{-3}$. $= \frac{(-4)7}{5(-3)} = \frac{-28}{-15} = \frac{28}{15}$

Operating with fractions: multiplication (II)

Example Let $a = m/n$ be a rational number expressed as a quotient of integers. Then,

$$a^2 = \left(\frac{m}{n}\right)^2 = \frac{m}{n} \cdot \frac{m}{n} = \frac{m^2}{n^2}.$$

Similarly,

$$a^3 = \left(\frac{m}{n}\right)^3 = \frac{m}{n} \cdot \frac{m}{n} \cdot \frac{m}{n} = \frac{m^3}{n^3}.$$

In general, for any positive integer k, we have

$$\boxed{a^k = \left(\tfrac{m}{n}\right)^k = \tfrac{m^k}{n^k}.}$$

Multiplication of rational numbers satisfies the same basic rules as multiplication of integers.
For any rational number a we have

$$\boxed{1a = a}\text{, and also } \boxed{0a = 0}$$

Furthermore, multiplication is associative, commutative, and distributive with respect to addition.

Formulas like

$$(a+b)^2 = a^2 + 2ab + b^2$$

are also valid for rational numbers.

Operating with fractions: multiplication (III)

Example Solve a in the equation

$$3a - 1 = 7.$$

We add 1 to both sides of the equation, and thus obtain

$$3a = 8.$$

We then divide by 3 and get

$$a = \frac{8}{3}.$$

Cancellation rule for multiplication

$$\boxed{\text{If } ab = ac\text{, then } b = c.}$$

Operating with fractions: multiplication (IV)

Exercise Solve for x in the equation

$$2(x-3)=7.$$

To do this, we use distributivity first, and get the equivalent equation

$$2x-6=7.$$

Next we find

$$2x=7+6=13,$$

and then

$$x=\frac{13}{2}.$$

We could have given other arguments to find the answer. For instance, we could first get

$$x-3=\frac{7}{2},$$

and then

$$x=\frac{7}{2}+3.$$

We can also give the answer in fraction form.

$$x=\frac{7}{2}+\frac{6}{2}=\frac{13}{2}.$$

Operating with fractions: multiplication (V)

Exercise Solve for x in the equation

$$\frac{3x-7}{2} + 4 = 2x.$$

We multiply both sides of the equation by 2 and obtain

$$3x - 7 + 8 = 4x.$$

We then add $-3x$ to both sides, to get

$$1 = 4x - 3x = x.$$

Operating with fractions: division (I)

If a is a rational number and $a \neq 0$, then there exists a rational number, denoted by a^{-1}, such that

$$a^{-1}a = aa^{-1} = 1.$$

Note that if $a = \frac{m}{n}$, then $a^{-1} = \frac{n}{m}$.

We call a^{-1} the multiplicative inverse (or reciprocal) of a.

Example The multiplicative inverse of $\frac{1}{2}$ is $\frac{2}{1}$, or simply 2, because

$$2 \cdot \frac{1}{2} = 1.$$

Operating with fractions: division (II)

We write $\frac{a}{b}$ or a/b instead of $b^{-1}a$ or ab^{-1}.

Example Let $a = \frac{3}{4}$ and $b = \frac{5}{7}$. Then

$$\frac{a}{b} = \frac{3/4}{5/7} = \frac{3}{4}\left(\frac{5}{7}\right)^{-1} = \frac{3}{4}\cdot\frac{7}{5} = \frac{21}{20}.$$

Exercise Compute a/b for $a = 1 + \frac{1}{2}$ and $b = 2 - \frac{4}{3}$.

$$\frac{1+\frac{1}{2}}{2-\frac{4}{3}} = \left(1+\frac{1}{2}\right)\cdot\left(2-\frac{4}{3}\right)^{-1} = \frac{2+1}{2}\cdot\left(\frac{6-4}{3}\right)^{-1}$$

$$= \frac{3}{2}\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}\cdot\frac{3}{2} = \frac{9}{4}.$$

Operating with fractions: division (III)

Example Solve for x in the equation

$$\frac{3}{x-1} = 2.$$

By cross-multiplying,

$$3 = 2(x-1) = 2x - 2,$$

which is equivalent to

$$3 + 2 = 2x.$$

Thus

$$x = \frac{5}{2}.$$

Decimals

Finite decimals give us examples of rational numbers.

Examples

- $1.4 = \frac{14}{10}$
- $1.41 = \frac{141}{100}$
- $0.2 = \frac{1}{5}$
- $0.75 = \frac{3}{4}$
- $0.3333\ldots = 0.\overline{3} = \frac{1}{3}$

Outline

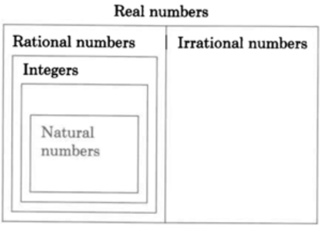
Irrational numbers (I)

An irrational number is a number that cannot be expressed as a fraction p/q for any integers p and q.

Irrational numbers have decimal expansions that neither terminate nor become periodic.

Some examples of irrational numbers:

- $\sqrt{2} = 1.41421356237\ldots$
- $\sqrt{3} = 1.73205080757\ldots$
- $\sqrt{5} = 2.2360679775\ldots$
- $\pi = 3.14159265359\ldots$



Irrational numbers (II)

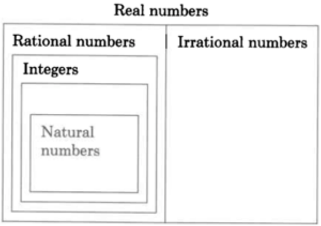
Exercise Is $\sqrt{25}$ an irrational number? And what about $\sqrt{-1}$?

$\sqrt{25}$ is not an irrational number because $\sqrt{25} = \pm 5$.
$\sqrt{-1}$ is not an irrational number either because it is not a real number (it is an imaginary number).

Outline

Real numbers

Integers, rational and irrational numbers are part of a larger system of numbers.



Real numbers can be described as all numbers that consist of a decimal expansion, possibly infinite.

Properties of real numbers (I)

Properties of addition

Addition is commutative and associative.

$$a + b = b + a \text{ and } a + (b + c) = (a + b) + c,$$

for all real numbers a, b, c.
Furthermore, we have

$$0 + a = a \text{ and } a + (-a) = 0.$$

Properties of multiplication

Multiplication is commutative and associative.

$$ab = ba \text{ and } a(bc) = (ab)c,$$

for all real numbers a, b, c.
Furthermore, we have

$$1a = a \text{ and } 0a = 0.$$

Multiplication is distributive with respect to addition.

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca.$$

Properties of real numbers (II)

We also have again

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2,} \quad \boxed{(a-b)^2 = a^2 - 2ab + b^2,}$$

$$\boxed{(a+b)(a-b) = a^2 - b^2.}$$

Existence of multiplicative inverse

If a is a real number and $a \neq 0$, then there exists a real number denoted by a^{-1} such that

$$a^{-1}a = aa^{-1} = 1.$$

Absolute value

The absolute value $|a|$ of a real number a is the non-negative value of a without regard to its sign.

Namely, $|a| = a$ for a positive a, $|a| = -a$ for a negative a (in which case $-a$ is positive), and $|0| = 0$.

Example The absolute value of 3 is 3, and the absolute value of -3 is also 3.

Example Find all values of x such that $|x + 5| = 2$.
To do this, we note that

$$|x + 5| = 2$$

if and only if

$$x + 5 = 2 \text{ or } x + 5 = -2.$$

Thus we have two possibilities, namely

$$x = 2 - 5 = -3 \text{ and } x = -5 - 2 = -7.$$

This solves our problem.

Square roots

If $a > 0$, then there exists a number b such that

$$b^2 = a.$$

Example What are all the real numbers x such that $x^2 = 2$?
There are too such numbers: $x = \sqrt{2}$ and $x = -\sqrt{2}$.

In general, the solutions of the equation $x^2 = a$ are

$$x = \pm\sqrt{a}.$$

n-th roots (I)

There exists a unique positive real number r such that

$$r^n = a.$$

This number r is called the n-th root of a, and is denoted by

$$a^{1/n} \text{ or } \sqrt[n]{a}.$$

The same properties as when doing exponentiation, but the exponent is always a rational number.

$$\boxed{(ab)^{1/n} = a^{1/n} b^{1/n}.}$$

$$\boxed{a^{x+y} = a^x a^y.}$$

$$\boxed{(a^x)^y = a^{xy}.}$$

$$\boxed{(ab)^x = a^x b^x.}$$

$$\boxed{a^0 = 1.}$$

$$\boxed{a^{-x} = \tfrac{1}{a^x}.}$$

$$\boxed{a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m.}$$

n-th roots (II)

Examples

- $a^{-3} = (a^3)^{-1} = \frac{1}{a^3}$.
- $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$.
- $(\sqrt{2})^{3/4} = (\sqrt{2^{1/4}})^3 = (2^{1/8})^3 = 2^{3/8}$.
- $(\sqrt{2})^3 = \sqrt{2}\sqrt{2}\sqrt{2} = 2\sqrt{2} = 2^{3/2}$.

Logarithms

Logarithms can be seen as the reverse operation of the exponentiation.

The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number.

Examples

- $\log_{10}(10000) = 4$ because $10^4 = 10000$.
- $\log_2(16) = 4$ because $2^4 = 16$.
- $\log_3(\frac{1}{3}) = -1$ because $3^{-1} = \frac{1}{3}$.

For any two real numbers b and x, where b is positive and $b \neq 1$, we have,

$$\boxed{y = b^x \text{ if and only if } x = \log_b(y)}$$

Properties of logarithms

Product of logarithms

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

Quotient of logarithms

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

Power

$$log_b(x^p) = p\log_b(x)$$

Change of base

The logarithm $\log_b(x)$ can be computed from the logarithms of x and b with respect to another base k by

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$$

Inequalities

Symbol	Meaning	Example
$>$	greater than	$x+3>2$
$<$	less than	$7x<28$
$\geq$	greater than or equal	$5\geq x-1$
$\leq$	less than or equal	$2y+1\leq 7$

Some rules of inequalities (I)

Let a, b, c be real numbers.

Transitivity of inequalities

If $a > b$ and $b > c$, then $a > c$.

An inequality which is multiplied by a positive number is preserved.

$$\boxed{\text{If } a > b \text{ and } c > 0, \text{ then } ac > bc.}$$

If we multiply both sides of an inequality by a negative number, then the inequality gets reversed.

$$\boxed{\text{If } a > b \text{ and } c < 0, \text{ then } ac < bc.}$$

Some rules of inequalities (II)

Example Solve for x the following inequality

$$2x - 4 > 5.$$

This is equivalent to show that

$$2x > 5 + 4 = 9,$$

which is equivalent to

$$x > \frac{2}{9}.$$

Some rules of inequalities (III)

Example Suppose that x is a number such that

$$\frac{3x+5}{x-4} < 2.$$

The quotient on the left makes no sense if $x = 4$, thus it is natural to consider two cases separately: $x > 4$ and $x < 4$.

- Suppose that $x > 4$. Then $x - 4 > 0$ and, in this case, the inequality is equivalent to

  $$3x + 5 < 2(x-4) = 2x - 8.$$

  This is equivalent to

  $$3x - 2x < -8 - 5$$

  or, in other words,

  $$x < -13.$$

  However, in our case $x > 4$, so that $x < -13$ is impossible. Hence there is no number $x > 4$ satisfying the inequality.

Some rules of inequalities (IV)

Example (cont.)

- Suppose $x < 4$. Then $x - 4 < 0$ and $x - 4$ is negative. We multiply both sides of the inequality by $x - 4$ to reverse the inequality. Thus we get

$$3x + 5 > 2(x - 4) = 2x - 8.$$

Furthermore, this inequality is equivalent to

$$3x - 2x > -8 - 5$$

or, in other words,

$$x > -13.$$

However, in our case, $x < 4$. Thus in this case, we find that the numbers x such that $x < 4$ and $x > -13$ are precisely those satisfying the inequality of the exercise. The solution can be written as

$$-13 < x < 4.$$

Outline

Order of operations (I)

1. Perform all calculations within parentheses and other grouping symbols following the order listed in Steps 2–4 below, working from the innermost pair of grouping symbols to the outermost pair.
2. Evaluate all exponential expressions.
3. Perform all multiplications and divisions as they occur from left to right.
4. Perform all additions and subtractions as they occur from left to right.
5. When grouping symbols have been removed, repeat Steps 2–4 to complete the calculation.
6. If a fraction is present, evaluate the expression in the numerator and the expression in the denominator separately. Then perform the division indicated by the fraction bar, if possible.

Order of operations (II)

Evaluate: $10 + 3[2^4 - 3(5 - 2)]$

Work within the *innermost* parentheses first and then within the *outermost* brackets.

$10 + 3[2^4 - 3(5 - 2)] = 10 + 3[2^4 - 3(3)]$	Do the subtraction within the parentheses.
$= 10 + 3[16 - 3(3)]$	Evaluate the exponential expression within the brackets: $2^4 = 16$.
$= 10 + 3[16 - 9]$	Do the multiplication within the brackets.
$= 10 + 3[7]$	Do the subtraction within the brackets.
$= 10 + 21$	Do the multiplication: $3[7] = 21$.
$= 31$	Do the addition.

Evaluate: $\dfrac{3^3 + 8}{7(15 - 14)}$

Evaluate the expressions above and below the fraction bar separately.

$\dfrac{3^3 + 8}{7(15 - 14)} = \dfrac{27 + 8}{7(1)}$	In the numerator, evaluate the exponential expression. In the denominator, subtract.
$= \dfrac{35}{7}$	In the numerator, add. In the denominator, multiply.
$= 5$	Divide.

Table of Greek letters

Α	α	alpha	Ι	ι	iota	Ρ	ρ	rho
Β	β	beta	Κ	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Ε	ε	epsilon	Μ	μ	mu	Υ	υ	upsilon
Δ	δ	delta	Ν	ν	nu	Φ	ϕ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	eta	Ο	ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega