Programming Exercises Week 1

Machine Learning/Advanced Machine Learning
IT University of Copenhagen

Fall 2019

Programming Exercise 1.1

Consider the problem of fitting a polynomial $y(x, \mathbf{w}) = \sum_{j=0}^{M} w_j x^j$ of the order M to data points (x_n, y_n), $n = 1, 2, \ldots, N$. It can be shown that the coefficients $\mathbf{w} = (w_0, w_1, \ldots, w_M)$ that minimise the sum-of-squares error are given by the solution to the following set of $M + 1$ linear equations

$$\sum_{j=0}^{M} a_{ij} w_j = b_i \tag{PE1.1}$$

where

$$a_{ij} = \sum_{n=1}^{N} (x_n)^{i+j}, \quad b_i = \sum_{n=1}^{N} (x_n)^i y_n, , \tag{PE1.2}$$

and $i = 0, 1, \ldots, M,\ j = 0, 1, \ldots, M$.

(a) Use Python to find the polynomials of order $M \in \{1, 2, 5, 10\}$ that minimise the sum-of-squares error for the following data

$\mathbf{x} = (x_1, \ldots, x_{11}) = (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)$,
$\mathbf{y} = (y_1, \ldots, y_{11}) = (0.15, -0.16, -0.61, -0.86, -1.02, -0.44, 0.16, 0.05, 0.45, 1.39, 0.86)$.

Plot the training set and the found polynomials.

(b) The target values above are obtained by computing the values of the function $y_{\text{true}}(x) = \sin\left(-\frac{3}{2}\pi x\right) + \frac{1}{3}\sin(5\pi x)$ and then adding Gaussian noise. Discuss for which M the fitted polynomial is a good representation of the underlying function and for which Ms we observe *over-fitting*.

Hint: You can solve linear equations in Python using function `numpy.linalg.solve` from NumPy.