Machine Learning/Advanced Machine Learning

Lecture 8.2:
Reinforcement Learning

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Based on Slides by D. Silver, UCL and F.F. Li, Stanford, and Ch 18 in Alpaydin, 2014
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Learning Objectives for Thursday

- Reflect the characteristics of reinforcement learning
- Explain the concepts of model, policy and value in relation to RL
- Explain the concepts of environment, agent, state, and action in relation to RL
- Implement simple RL agents based on value iteration or policy iteration
- Explain and apply Bellman equation in Q-learning.

Outline of lecture

Introduction to Reinforcement Learning

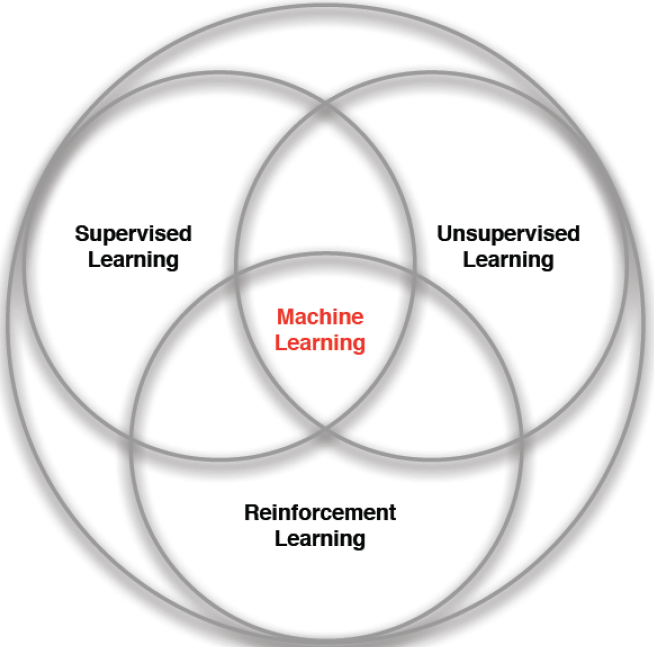
Elements of the Reinforcement Learning

Categorisation of the RL agents

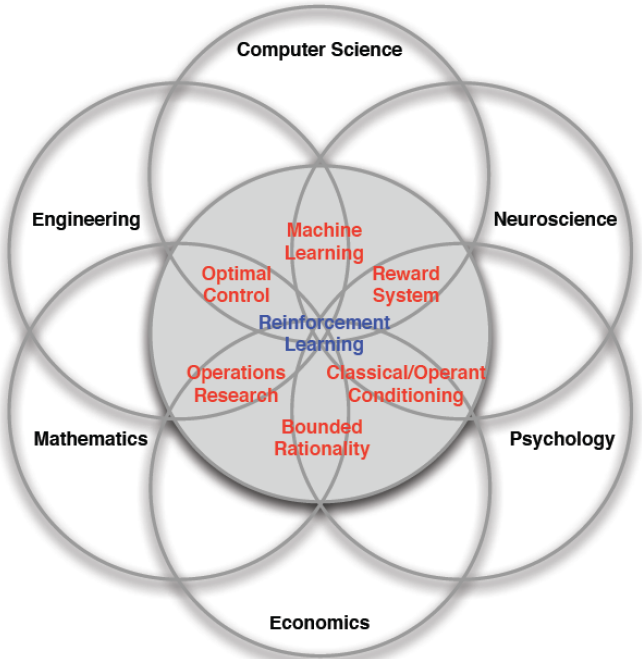
Model-Based Learning

Model Free Learning

RL as part of Machine Learning



Many Faces of Reinforcement Learning



Characteristics of Reinforcement Learning

What makes reinforcement learning different from other machine learning paradigms?

- There is **no supervisor**, only a **reward** signal
- Feedback is delayed, not instantaneous
- Time really matters (sequential, non i.i.d data)
- Agent's actions affect the subsequent data it receives

Examples of Reinforcement Learning

- Fly stunt manoeuvres in a helicopter
- Defeat the world champion at Backgammon
- Manage an investment portfolio
- Control a power station
- Make a humanoid robot walk
- Play many different Atari games better than humans

Stanford Autonomous Helicopter

https://www.youtube.com/watch?v=0JL04JJjocc

Google’s DeepMind AI Taught How to Walk

https://www.youtube.com/watch?v=gn4nRCC9TwQ

Google Deep Mind's Deep Q Learning Playing Atari Breakout

https://www.youtube.com/watch?v=V1eYniJ0Rnk

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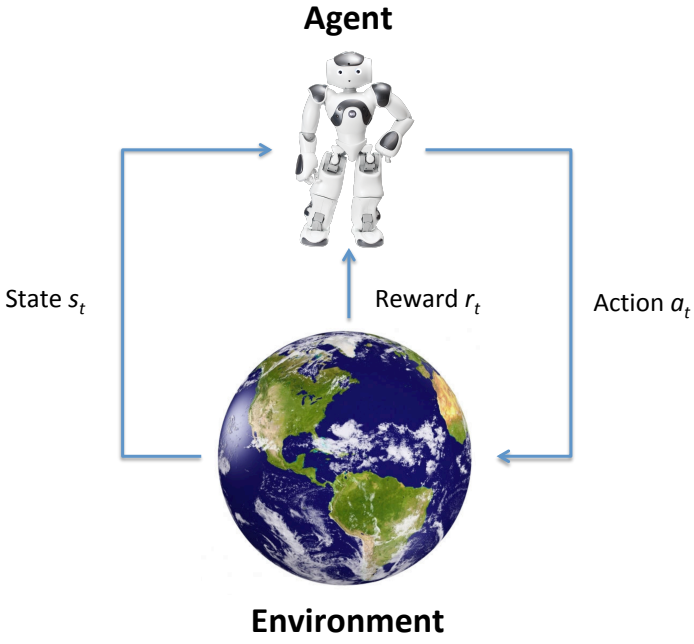
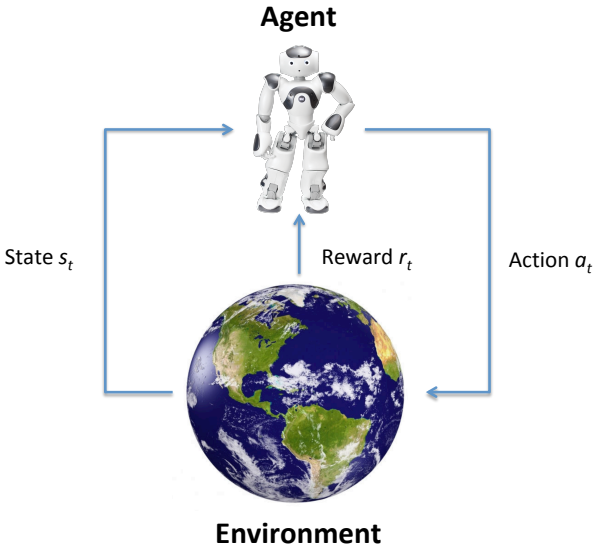
Agent and Environment



Agent and Environment



At each time instant t

- There is the **state** s_t of the agent/environment
- Agent makes and **action** a_t
- The agent receives the **reward** r_t

Reward Hypothesis

A **reward** r_t is a scalar feedback signal of the action

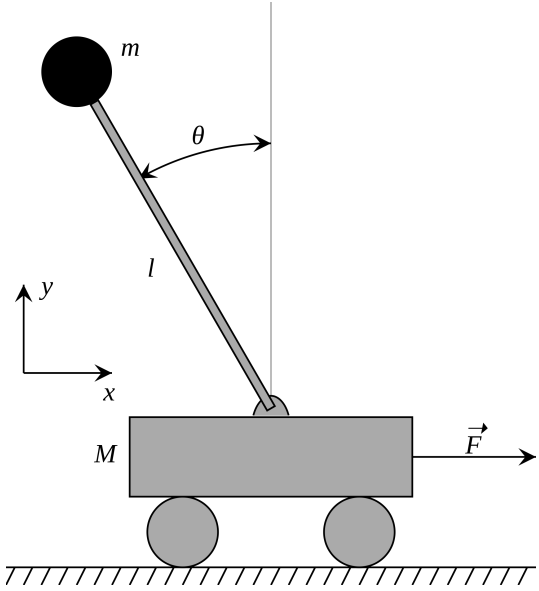
- Indicates how well agent is doing at step t
- The agent's job is to maximise **cumulative reward**

Reinforcement learning is based on the **reward hypothesis**

Definition

Reward Hypothesis is the assumption that all goals can be described by the maximisation of expected cumulative reward

Example: Cart Pole Problem



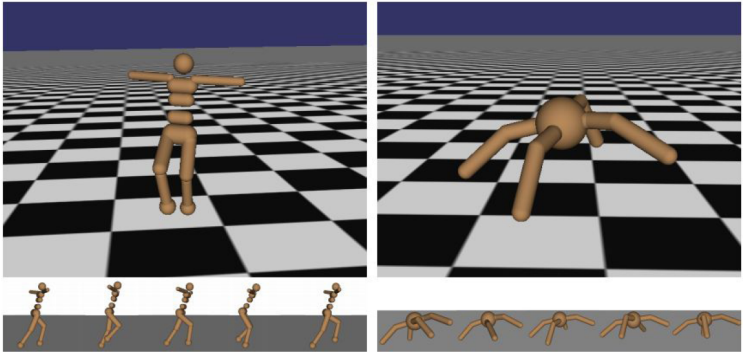
Objective: Balance a pole on top of a movable cart

Action: Horizontal force applied on the cart

State: Angle, angular speed, position, horizontal velocity

Reward: 1 at each time step if the pole is upright

Example: Robot Locomotion



Objective: Make the robot move forward

Action: Torques applied on joints

State: Angle and position of the joints

Reward: 1 at each time step upright + forward movement

Example: Atari Games

Objective: Complete the game with the highest score

Action: Game controls e.g. Left, Right, Up, Down

State: Raw pixel inputs of the game state

Reward: Score increase/decrease at each time step

Sequential Decision Making

Goal: select actions to maximise total **future** reward

- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward

Examples:

- A financial investment (may take months to mature)
- Refuelling a helicopter (might prevent a crash in several hours)
- Blocking opponent moves (might help winning chances many moves from now)

Markov System

Definition

A variable s_t follows a Markov system if and only if

$$P(s_{t+1}|s_t) = P(s_{t+1}|s_1, \ldots, s_t). \tag{1}$$

"The future is independent of the past given the present"

- Once the state is known, the history may be thrown away
- The state is a sufficient statistic of the future

Mathematical formulation of the RL problem

Markov Decision Process

1. At time step $t = 0$, environment samples the initial state s_0 from $p(s_0)$
2. Repeat until done:
   - Select action a_t. **(Agent)**
   - Sample reward r_t from $P(r_t|s_t, a_t)$. **(Environment)**
   - Sample next state s_{t+1} from $P(s_{t+1}|s_t, a_t)$. **(Environment)**
   - Increment t.

The **model** predicts what the environment will do next

- $P(s_t|s_t, a_t)$ predicts the next state
- $P(r_t|s_t, a_t)$ predicts the next (immediate) reward

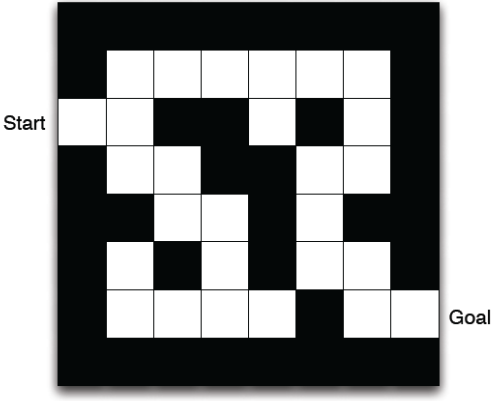
The **policy** $\pi : \mathcal{S} \to \mathcal{A}$, a mapping from the space of states to the space of actions

- It *defines the action* to be taken at state s_t, i.e., $a_t = \pi(s_t)$.

The **value** $V^\pi(s_t)$ of a policy is the *expected cumulative reward* achievable from state s_t.

Objective: find the policy π^* that maximises the value.

Example: Maze

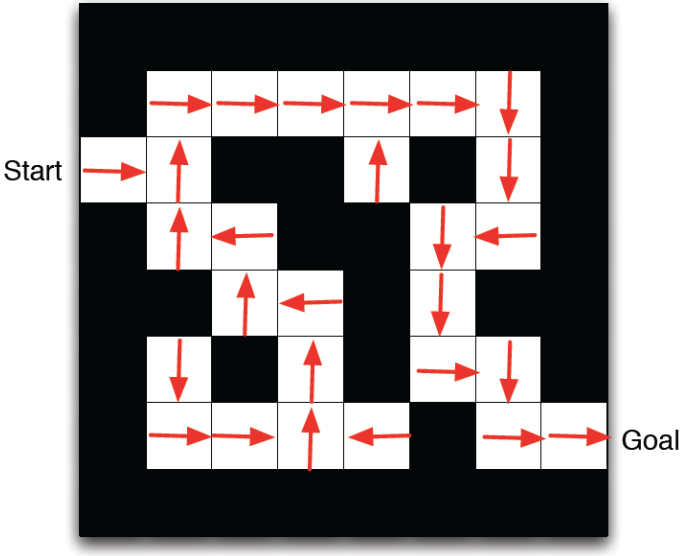


State: Agent's location

Actions: N, E, S, W

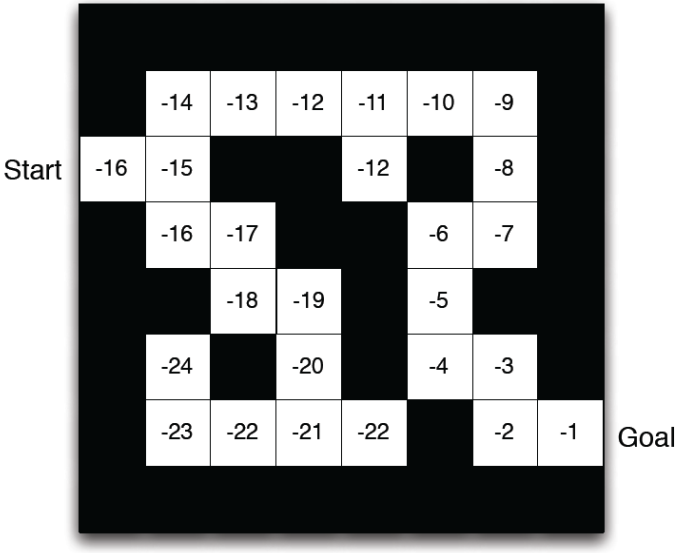
Reward: -1 for each time step

Example: Maze - Policy



Arrows represent the (optimal) policy π for each state s.

Example: Maze - Value Function



Numbers represent the value $V^{\pi}(s_t) = \mathrm{E}[\sum_{i=1}^{T} r_{t+T}]$ for each state s (**finite horizon model**).

Value Function and Q-Value Function

The **value function**, in an **infinite horizon model**, is the expected sum of rewards

$$V^{\pi}(s_t) = \mathrm{E}[\sum_{i=1}^{T} \gamma^{i-1} r_{t+i}]$$

where γ is the **discount rate**.

- Quantifies each state.

The **Q-value function** $Q^*(s_t, a_t)$ is the expected cumulative reward of state–action pair when optimal policy is followed thereafter.

The **optimal policy** π^* that maximises the expected sum of rewards, or,

$$\begin{aligned} V^*(s_t) &= \max_{\pi} V^{\pi}(s_t), \forall s_t \\ &= \max_{a_t} Q^*(s_t, a_t) \end{aligned}$$

Optimal Policy

The expected sum of rewards by the optimal policy hence is

$$
\begin{aligned}
V^*(s_t) &= \max_{a_t} Q^*(s_t, a_t) \\
&= \max_{a_t} \mathrm{E}\left[\sum_{i=1}^{T} \gamma^{i-1} r_{t+i}\right] \\
&= \max_{a_t} \mathrm{E}\left[r_{t+1} + \sum_{i=1}^{T} \gamma^{i+1-1} r_{t+i+1}\right] \\
&= \max_{a_t} \mathrm{E}\left[r_{t+1} + \gamma V^*(s_{t+1})\right]
\end{aligned}
$$

Bellman Equation

$$
V^*(s_t) = \max_{a_t} \left(\mathrm{E}\left[r_{t+1}\right] + \gamma \sum_{s_{t+1}} P(s_{t+1}|s_t, a_t) V^*(s_{t+1}) \right)
$$

Optimal Policy with the Q-Value Function

From the Bellman equation

$$Q^*(s_t, a_t) = \mathrm{E}\left[r_{t+1}\right] + \gamma \sum_{s_{t+1}} P(s_{t+1}|s_t, a_t) \max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1}).$$

Optimal Policy

$$\pi^*(s_t) : \text{Choose } a_t^* = \arg\max_{a_t} Q^*(s_t, a_t) \tag{2}$$

The optimal policy π^* corresponds to taking the best action in any state as specified by Q^*.

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Model Free Learning

Categorisation of the RL agents

Value-based

- *No policy (implicit)*
- Value function

Policy-based

- Policy
- *No Value function*

Actor Critic

- Policy
- Value function

Another Categorisation of the RL agents

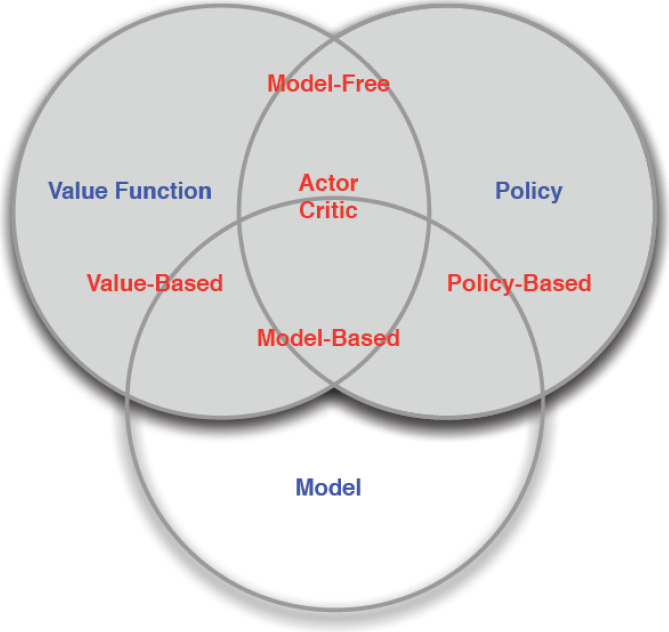
Model-based

- Policy and/or Value Function
- Model

Model free

- Policy and/or Value Function
- *No Model*

RL Agent Taxonomy



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Model-Based Learning: Value Iteration

Assume that we **know** the model parameters $P(s_t|s_t, a_t)$ and $P(r_t|s_t, a_t)$.

The optimal policy can be found by **Value Iteration**

- Shown to converge to the correct values V^*.

Initialize $V(s)$ to arbitrary values
Repeat
  For all $s \in S$
    For all $a \in \mathcal{A}$
      $Q(s,a) \leftarrow E[r|s,a] + \gamma \sum_{s' \in S} P(s'|s,a)V(s')$
    $V(s) \leftarrow \max_a Q(s,a)$
Until $V(s)$ converge

Figure 18.2 Value iteration algorithm for model-based learning.

Model-Based Learning: Policy Iteration

Assume that we **know** the model parameters $P(s_t|s_t, a_t)$ and $P(r_t|s_t, a_t)$.

In **Policy Iteration** we store and update the policy rather than the values

- Converges faster than value iteration.

Initialize a policy π' arbitrarily
Repeat
  $\pi \leftarrow \pi'$
  Compute the values using π by
    solving the linear equations
      $V^{\pi}(s) = E[r|s, \pi(s)] + \gamma \sum_{s' \in S} P(s'|s, \pi(s)) V^{\pi}(s')$
  Improve the policy at each state
    $\pi'(s) \leftarrow \arg\max_a (E[r|s, a] + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi}(s'))$
Until $\pi = \pi'$

Figure 18.3 Policy iteration algorithm for model-based learning.

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Model-Free Learning

Model is defined by the **reward** and **the next state probability distributions**

We seldom have such knowledge of the environment

Exploration of the environment is required to query the model

Option: ϵ-greedy search

- With probability ϵ, choose one action randomly;
- With probability $1 - \epsilon$, choose the best action.

Q-learning

We may learn Q by querying the action state pairs.

In principle, we use the Bellman equation

$$Q_{i+1}(s,a) = \mathrm{E}\left[r + \gamma \max_{a'} Q_i(s',a')|s,a\right] \tag{3}$$

to compute the update for Q.

Note: in Alpaydin, 2014 Q-learning defined as computing the running average in the non-deterministic case

Deep Q-learning

Updating Q-directly is not **scalable**

- We would need to compute $Q(s, a)$ for every state–action pair!
- E.g. in Atari games the current state is a bitmap image (huge state space)

Solution: Use a **function approximator** for $Q^*(s, a) \approx Q(s, a; \theta)$

If the function approximator is a *deep neural network*, this is referred to as **deep Q-learning**.

Case Example: Atari Games

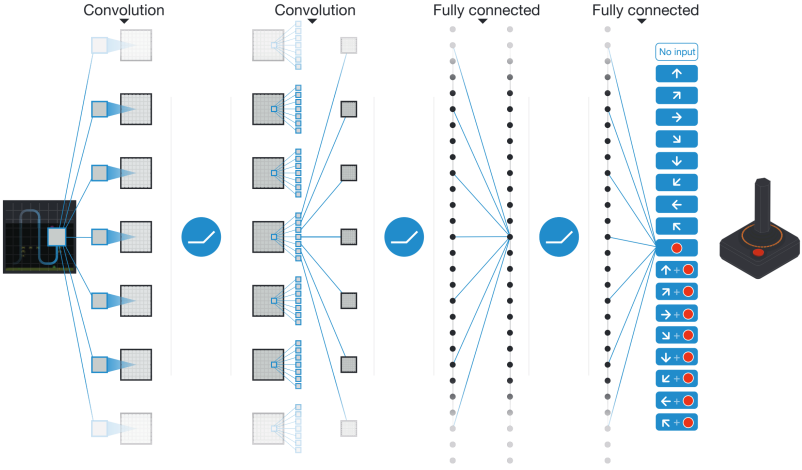
Objective: Complete the game with the highest score

Action: Game controls e.g. Left, Right, Up, Down

State: Raw pixel inputs of the game state

Reward: Score increase/decrease at each time step

Atari Games Case: Q-Network Architecture



Deep Neural Network $Q(s, a; \theta)$. Current state s is the $84 \times 84 \times 4$ stack of last 4 frames. The output is the Q-values $Q(s, a, \theta)$ for all the actions $a \in \{a_1, \ldots, a_2\}$.

From. Mnih et al., Nature 2015

Atari Games Case: Training the Q-network

Loss function

$$L_i(\theta_i) = \mathrm{E}_{s',a\sim\rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right] \tag{4}$$

where $y_i = \mathrm{E}_{s'\sim\mathcal{E}}\left[r + \gamma \max_{a'} Q_i(s',a';\theta_{i-1})|s,a\right]$.

Gradient update

$$\nabla_{\theta_i} L_i(\theta_i) = \mathrm{E}_{s,a\sim\rho(\cdot);s'\sim\mathcal{E}}\left[\left(r + \gamma \max_{a'} Q_i(s',a';\theta_{i-1}) - Q_i(s',a';\theta_i)\right)\right]. \tag{5}$$

Atari Games Case: Experience Replay

Learning from batches of **consecutive** samples is problematic

- Correlated samples (inefficient learning)
- Current Q-network determine next training samples (bad feedback loops)

Solution

- Keep memory of table of transitions (s_t, a_t, r_t, s_{t+1}) as games is played
- Train the Q-network on random minibatches of transitions from the replay memory instead.

Atari Games Case: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Summary

- RL can be seen as the third major branch of machine learning
  - No supervisor, only a reward
  - Sequential learning
- Based on a Markov System
- Different approaches based on
  - Model
  - Policy
  - Value
- Various applications from autonomous vehicles and control systems and investment portfolio management to computer games.

References I


Alpaydin, E. (2014). *Introduction to Machine Learning*. Third Edition. The MIT Press.