



Recursion uses Stack STACK

↳ LIFO: Last In First Out

It is a collection of elements that follow LIFO for insertion and deletion.

ADT Stack (Abstract data type)

Data : 1. Space for storing elements
2. Top pointer

Implementation of Stack using

1. Array
2. Linked List

Operations: 1. push(x)
2. pop()
3. peek(index)
4. StackTop()
5. isEmpty()
6. isFull()

IMPLEMENTATION OF STACK USING ARRAY

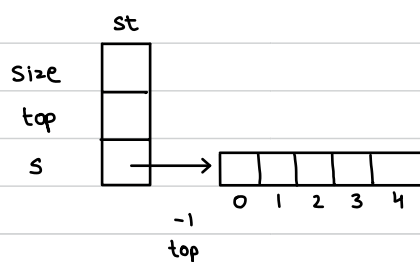
```
struct stack
```

```
{  
    int size;  
    int top;  
    int *s;  
};
```

```
int main()  
{
```

```
    struct stack st;  
    printf("Enter size of stack");  
    scanf("%d", &st.size);  
    st.s = new int[st.size];  
    st.top = -1;
```

```
}
```



Stack empty - if (top == -1)
 Stack full - if (top == size-1)

push() $O(1)$

```
void push (Stack *st, int x)
{
    if (st -> top == st -> size - 1)
        printf("Stack Overflow");
    else
    {
        st -> top++;
        st -> s[st -> top] = x;
    }
}
```

peek()

```
int peek (Stack st, int pos)
{
    int x = -1;

    if (st.top - pos + 1 < 0)
        printf("Invalid Position");
    else
        x = st.s[st.top - pos + 1];

    return x;
}
```

Stacktop()

```
int Stacktop (Stack st)
{
    if (st.top == -1)
        return -1;
    else
        return st.s[st.top];
}
```

pop() $O(1)$

```
int pop (Stack *st)
{
    int x = -1;

    if (st -> top == -1)
        printf("Stack Underflow");
    else
    {
        x = st -> s[st -> top];
        st -> top--;
    }
    return x;
}
```

pos	Index = $\text{Top} - \text{pos} + 1$
1	3
2	2
3	1
4	0

isEmpty()

```
int isEmpty (Stack st)
{
    if (st.top == -1)
        return 1;
    else
        return 0;
}
```

isFull()

```
int isFull (Stack st)
{
    if (st.top == -1)
        return 1;
    else
        return 0;
}
```

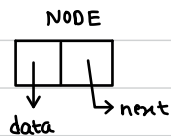
STACK USING LINKED LIST

```
struct Node  
{
```

```
    int data;
```

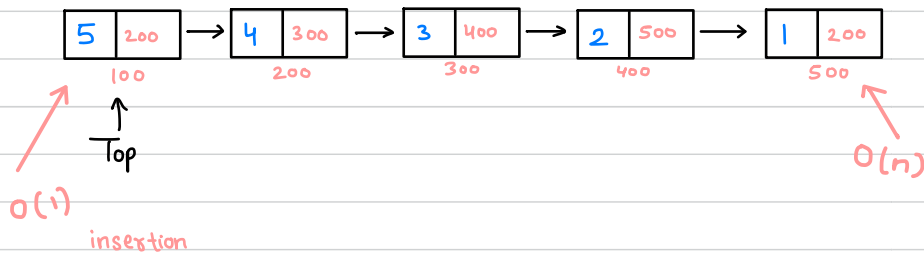
```
    struct Node *next;
```

```
}
```



Empty : if (top == NULL)

Full : Node *t = new Node;
if (t == NULL)



push()

```
void push(int x)  
{
```

```
    Node *t = new Node;
```

```
    if (t == NULL)
```

```
        printf("Stack Overflow");
```

```
    else  
    {
```

```
        t->data = x;
```

```
        t->next = top;
```

```
        top = t;
```

```
    }
```

```
}
```

peek()

```
int Peek(int pos)  
{
```

```
    int i;
```

```
    Node *p = top;
```

```
    for (i = 0; p != NULL && i < pos - 1; i++)
```

```
        p = p->next;
```

pop()

```
int pop()  
{
```

```
    Node *p;
```

```
    int x = -1;
```

```
    if (top == NULL)
```

```
        printf("Stack is empty");
```

```
    else  
    {
```

```
        p = top;
```

```
        top = top->next;
```

```
        x = p->data;
```

```
        free(p);
```

```
    }
```

```
    return x;
```

```
}
```

```
if (p != NULL)
```

```
    return p->data;
```

```
else
```

```
    return -1;
```

```
}
```

Stack top()

```
int Stacktop()
{
    if (top)
        return top->data;
    return -1;
}
```

isFull

```
int isFull()
{
    Node *t = new Node;
    int r = t ? 1 : 0;
    free(t);
    return r;
}
```

isEmpty

```
int isEmpty()
{
    return Top ? 0 : 1;
}
```

False
→ True

Paranthesis Matching

exp

((a	+	b)	*	((-	d))	\0
0	1	2	3	4	5	6	7	8	9	10	11	12	13

```
int isBalance(char *exp)
{
    struct stack st;
    st.size = strlen(exp);
    st.top = -1;
    st.s = new char[st.size];
}
```

} initializing of stack

```
for (i=0; exp[i] != '\0'; i++)
{
    if (exp[i] == '(')
        push(&st, exp[i]);
    else if (exp[i] == ')')
        if (isEmpty(st))
            return false;
        else
            pop(&st);
}
return isEmpty(st) ? true : false;
}
```

ASCII

(40
)	41
[91
]	93
{	123
}	125

INFIX TO POSTFIX CONVERSION

1. What is postfix
2. Why postfix
3. Precedence
4. Manual Conversion

1. Infix: Operand Operator Operand
 $a + b$

2. Prefix: Operator Operand Operand
 $+ab$

3. Postfix: Operand Operand Operator
 $ab +$

SYMBOL	PRECEDENCE	ASSOCIATIVITY
$+, -$	1	L-R
$*, /$	2	L-R
\wedge	3	R-L
$—$	4	R-L
$()$	5	L-R

Unary minus 

Eg

$$a + b * c$$

$(a + (b * c))$ First fully parenthesise it.

prefix

postfix

$$(a + [*bc])$$
$$[+a*bc]$$

$$(a + [bc*])$$
$$[abc*+]$$

ASSOCIATIVITY

Left to Right

Right to Left

$$a + b + c - d$$

$$a = b = c = 5$$

$$(((a+b)+c)-d)$$

$$(a = (b = (c = 5)))$$

Power operator Example

$$a^b c$$

$$(a^{(b^c)})$$

Postfix: $(a^b [c^a])$
 $abc^{^^}$

Unary Operators Example

(1) $-a$ negation of a

pre: $-a$

post: $a-$

$$(-(-a))$$

(2) $*p$

pre: $*p$

post: p^*

$$(*(*p))$$

(3) $n!$

pre: $!n$

post: $n!$

(4) $\log x$

pre: $\log x$

post: $x \log$

Example: $-a + b * \log n!$

$$-a + b * \log [n!]$$

$$-a + b * [n! \log]$$

$$[a-] + b * [n! \log]$$

$$[a-] + [bn! \log^*]$$

$$a - bn! \log^* +$$

INFIX TO POSTFIX CONVERSION

$$a + b * c - d / e$$

			SYMBOL	PRECEDENCE	ASSOCIATIVITY
			$+, -$	1	L-R
			$*, /$	2	L-R
Symbol	Stack	Postfix			
a		a			
+	+	a			
b	+	ab			
*	*, +	ab			
c	*, +	abc			
-	-	abc*+			
d	-	abc*+d			
/	/, -	abc*+d			
e	/, -	abc*+de			

$abc*+de/-$ Ans

PROGRAM

infix	a	+	b	*	c	-	d	/	e	\0
	0	1	2	3	4	5	6	7	8	9

```
char *convert (char *infix)
{
```

```
    struct stack st; // Initialized
```

```
    char *postfix = new char[strlen(infix)+1];
```

```
    int i=0; j=0;
```

↳ for null string

```
    while (infix[i] != '\0')
```

```
    {
```

```
        if (isOperand(infix[i]))
```

```
            postfix[j++] = infix[i++];
```

```
        else
```

```
        {
```

```
            if (pre(infix[i]) > pre(stacktop(st)))
```

```
                push(&st, infix[i++]);
```

```
            else
```

```
                postfix[j++] = pop(&st);
```

```
        }
```

```
    }
```

```
    while (!isEmpty(st))
```

```
        postfix[j++] = pop(&st);
```

```
    postfix[j] = '\0';
```

```
    return postfix;
```

```
}
```

```
int pre(char x)
```

```
{
```

```
    if (x == '+' || x == '-')
```

```
        return 1;
```

```
    else if (x == '*' || x == '/')
```

```
        return 2;
```

```
    return 0;
```

```
}
```

```
int isOperand(char x)
```

```
{
```

```
    if (x == '+' || x == '-' || x == '*' || x == '/')
```

```
        return 0;
```

```
    else
```

```
        return 1;
```

```
}
```


Q

$((a+b)*c) - d^e f$
 $([ab+]^*c) - d^e f$
 $[ab+c^*] - d^e f$
 $[ab+c^*] - d^e [ef^*]$
 $[ab+c^*] - [def^{**}]$
 $ab+c^*def^{**} -$

SYMBOL	OUT STACK PRE	IN STACK PRE
	1	2
+ , -	3	4
* , /	6	5
^	7	0
(0	?

closing bracket
cannot be pushed into
Stack

because of
R-L associativity

EVALUATION OF POSTFIX

$35 * 62 / + 4 -$

SYMBOL	STACK	OPERATION
3	3	
5	5, 3	
*		$5 * 3$
	15	
6	6, 15	
2	2, 6, 15	
/		$6 / 2$
	3, 15	
+		$15 + 3$
	18	
4	4, 18	
-		$18 - 4$
	14	

$x = 6 + 5 + 3 * 4$
 $x = 65 + 34 * +$

* Here + gets executed first instead of * because precedence and associativity are meant for parenthesisation, they don't decide which operator gets executed first.

PROGRAM FOR EVALUATION OF POSTFIX

postfix	3	5	*	6	2	/	+	4	-	\0
	0	1	2	3	4	5	6	7	8	9

```
int Eval(char *postfix)
{
```

```
    struct stack st;
```

```
    int i, x1, x2, r;
```

```
    for (i=0; postfix[i] != '\0'; i++)
    {
```

```
        if (isOperand(postfix[i]))
```

```
            push(&st, postfix[i] - '0');
```

```
        else
```

```
        {
```

```
            x2 = pop(&st);
```

```
            x1 = pop(&st);
```

```
            switch (postfix[i])
            {
```

```
                case '+': r = x1 + x2; push(&st, r); break;
```

```
                case '-': r = x1 - x2; push(&st, r); break;
```

```
                case '*': r = x1 * x2; push(&st, r); break;
```

```
                case '/': r = x1 / x2; push(&st, r); break;
```

```
            }
```

```
        }
```

```
    return pop(&st);
```

```
}
```

because operand will be pushed into the stack in its ASCII value because postfix expression is in char.

Foreg : 3

'51' - '48' = 3