MATRICES

SPECIAL MATRICES

↳ Only square matrix
nxn

1. Diagonal Matrix
2. Lower Triangular
3. Upper Triangular
4. Symmetric Matrix
5. Tridiagonal Matrix
6. Band Matrix
7. Toeplitz Matrix
8. Sparse Matrix

1. DIAGONAL MATRIX

	1	2	3	4	5
1	3	0	0	0	0
2	0	7	0	0	0
3	0	0	4	0	0
4	0	0	0	9	0
5	0	0	0	0	6

A

3	7	4	9	6
0	1	2	3	4

ROWS COLUMNS ELEMENT TO BE INSERTED

```
int A[5];
void set ( int A[], int i, int j, int x)
{
        if (i==j)
                A[i-1] = x;
}
void get ( int A[], int i, int j)
{
        if (i==j)
                return A[i-1];
        else
                return 0;
}
```

C++ Class For Diagonal Matrix

```
Class Diagonal()
{
    private:
        int n;
        int *A;

    public:
        Diagonal(int n)
        {
            This->n = n;
            A = new int(n);
        }

        void set(int i, int j, int k);
        void get(int i, int j);
        void Display();
        ~Diagonal()
        {
            delete []A;
        }
};
```

```
void Diagonal :: set(int i, int j, int x);
{
    if (i == j)
        A[i-1] = x;
}
```

```
int Diagonal :: get(int i, int j)
{
    if (i==j)
        return A[i-1];
    else
        return 0;
}
```

```
void Diagonal :: Display()
{
    for(i=0; i<n; i++)
    {
        for(j=0; j<n; j++)
        {
            if(i==j)
                cout << A[i-1];
            else
                cout << "0";
        }
        Cout << endl;
    }
}
```

LOWER TRIANGULAR MATRIX

$$M = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

$M[i,j] = 0$ if $i < j$

$M[i,j]$ = Non Zero if $i >= j$

$$\text{Non Zero} = 1+2+3+4+5$$
$$= 1+2+3+4 \ldots\ldots n$$
$$= \frac{n(n+1)}{2}$$

$$\text{Zero} = n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$$

ROW MAJOR

a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{44}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
row1	row2		row3			row 4				row 5				

Index (A[4][3]) = [1+2+3] +2 = 8
Index (A[5][4]) = [1+2+3+4] +3 = 13

$$\text{Index}(A[i][j]) = \left[\frac{i(i-1)}{2}\right] + j - 1$$

COLUMN MAJOR

a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{22}	a_{32}	a_{42}	a_{52}	a_{33}	a_{43}	a_{53}	a_{44}	a_{54}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
col 1					col2				col3			col 4		col5

Index (A[4][4]) = [5 + 4 + 3] + 0 = 12
Index (A[5][4]) = [5 + 4 + 3] + 1 = 13
Index (A[5][3]) = [5 + 4] + 2 = 11

$$\text{Index}(A[i][j]) = [n + n-1 + n-2 + \ldots. n - (j-2)] + (i-j)$$

$$= [n(j-1) - [1+2+3+\ldots. j-2]] + (i-j)$$
$$= \left[n(j-1) - \frac{(j-2)(j-1)}{2}\right] + (i-j)$$

UPPER TRIANGULAR MATRIX

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}$$

$M[i,j] = 0 \quad \text{if} \quad i > j$

$M[i,j] = \text{Non Zero} \quad \text{if} \quad i <= j$

$$\text{Non Zero} = 5+4+3+2+1 = \frac{n(n+1)}{2}$$

$$\text{Zero} = n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$$

ROW MAJOR

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{22}	a_{23}	a_{24}	a_{25}	a_{33}	a_{34}	a_{35}	a_{44}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		row1					row2				row3		row4	row5

INDEX (A[4][5]) = [5+4+3]+1 = 13

INDEX (A[i][j]) = [n+n-1+n-2+.... + n-(i-2)] + (j-i)

$$= \left[(i-1)n - \frac{(i-2)(i-1)}{2}\right] + (j-i)$$

COLUMN MAJOR

a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}	a_{15}	a_{25}	a_{35}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

INDEX (A[4][5]) = [1+2+3+4]+3 = 13

INDEX (A[i][j]) = [1+2+3+.... j-1] + i-1 = $\left[\frac{j(j-1)}{2}\right] + i - 1$

SYMMETRIC MATRIX

$$M = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 3 & 3 \\ 2 & 3 & 4 & 4 & 4 \\ 2 & 3 & 4 & 5 & 5 \\ 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

if $M[i,j] = M[j,i]$

Either we can store lower triangular matrix, or we can store upper triangular matrix

TRI DIAGONAL MATRIX

$$M = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix}$$

a_{21}	a_{32}	a_{43}	a_{54}	a_{11}	a_{22}	a_{33}	a_{44}	a_{55}	a_{12}	a_{23}	a_{34}	a_{45}
0	1	2	3	4	5	6	7	8	9	10	11	12

lower diagonal | main diagonal | upper diagonal

Main diagonal $i-j=0$

Lower diagonal $i-j=1$

Upper diagonal $i-j=-1$

$|i-j| \leq 1$

$M[i,j]$ = Non Zero if $|i-j| \leq 1$

$M[i,j]$ = Zero if $|i-j| > 1$

$5+4+4$

$n+n-1+n-1$

$3n-2$

Index (A[i][j])

case 1 : if $i-j=1$ index = i-1

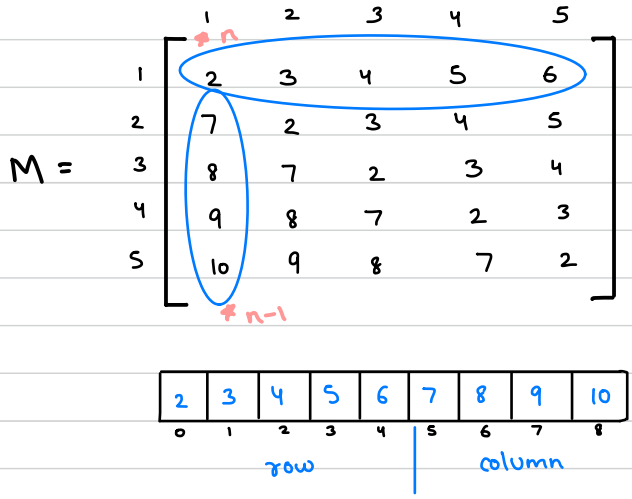
case 2 : if $i-j=0$ index = n-1 + i-1

case 3 : if $i-j=-1$ index = 2n-1 + i-1

SQUARE BAND MATRIX (Same as TRI DIAGONAL matrix)

When there are more than one diagonals below the main diagonal and the number of lower and upper diagonal is equal.

TOEPLITZ MATRIX



$M[i,j] = M[i-1, j-1]$

No of elements we want to store : $n + n-1$*

Index A[i][j]

case 1 : if $i <= j$ Index = $j-1$

case 2 : if $i > j$ Index = $n + i - j - 1$

CREATING A DYNAMICALLY ALLOCATED ARRAY

```
int *A, n;
printf("Enter dimension");
scanf("%d", &n);
A = (int*)malloc(n*sizeof(int));
A = new int[n]; C++
```