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B.Sc. (H) Electronics
V semester

Subject : Electromagnetics
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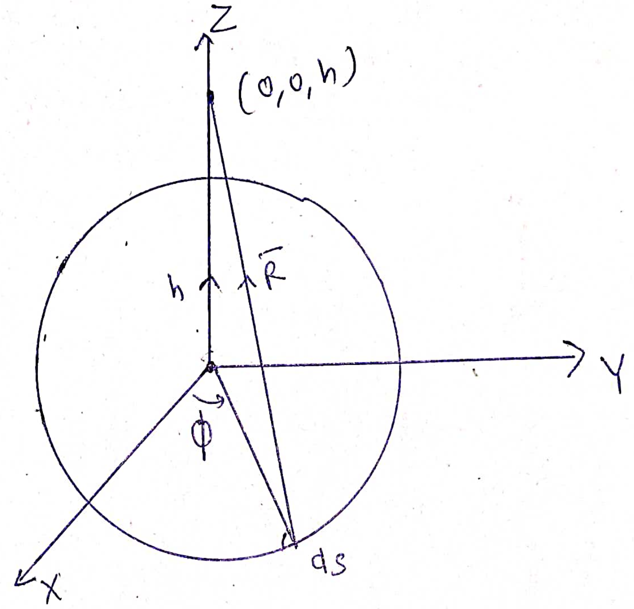
B.sc. (H) Electronics (CBCS)
Year - III Sem-V
Paper Nam → Electromagnetics (Core Course-XII)
Assignment

Assignment

Q-1 A circular disc of radius 'a' is uniformly charged with ρ_s C/m². The disc lies on the z=0 plane with its axis along the z-axis. Show that at point (0,0,h)

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\left\{1 - \frac{h}{\sqrt{h^2+a^2}}\right\}\hat{a}_z$$

Sol :-



$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\bar{a}_r$$

$$dQ = \rho_s\, ds \;;\; ds = d\rho \cdot \rho\, d\phi,$$
$$= \rho_s\, \rho\, d\rho\, d\phi$$

$$\rho\bar{a}_\rho + \bar{R} = h\bar{a}_z$$

$$\bar{R} = h\bar{a}_z - \rho\bar{a}_\rho$$

$$E = \int_S \frac{\rho_s\, \rho\, d\rho\, d\phi\,(h\bar{a}_z - \rho\bar{a}_\rho)}{4\pi\varepsilon_0 (h^2+\rho^2)^{3/2}}$$

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$$\bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\,\bar{a}_z \int_0^{2\pi} d\phi \int_0^a \frac{\rho h}{(h^2+\rho^2)^{3/2}}\, d\rho$$

$$= \frac{\rho_s}{4\pi\epsilon_0}\,\bar{a}_z\, 2\pi h \int_0^a \frac{1}{2}(h^2+\rho^2)^{-3/2}\, d(\rho^2)$$

$$= \frac{\rho_s h}{2\epsilon_0}\,\bar{a}_z\,\frac{1}{2}\left[\frac{(h^2+\rho^2)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right]_0^a$$

$$= \frac{\rho_s h}{4\epsilon_0}\,\bar{a}_z \left\{-2\left[(h^2+a^2)^{-1/2} - (h^2)^{-1/2}\right]\right\}$$

$$= \frac{-\rho h\,\bar{a}_z}{2\epsilon_0}\left[\frac{1}{\sqrt{(h^2+a^2)}} - \frac{1}{h}\right]$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{h}{(h^2+a^2)^{1/2}}\right]\bar{a}_z}$$

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Q.2 A charged particle of mass 3 kg and charge 4 C starts at point (2, −5, 0) with velocity $6\hat{a}_x + 5\hat{a}_z$ m/s in an electric field $15\hat{a}_x + 12\hat{a}_y$ V/m. At time $t = 1$ sec, determine

(a) The acceleration of the particle.
(b) Its velocity
(c) Its Kinetic energy
(d) Its position.

Sol) (a) The acceleration of the particle

We know that,

$$F = ma = qE$$

$$a = \frac{qE}{m}$$

$$\therefore\ a = \frac{4}{3}(15\hat{a}_x + 12\hat{a}_y)$$

$$= \frac{4}{3} \times 15\hat{a}_x + \frac{4}{3} \times 12\hat{a}_y$$

$$\boxed{a = 20\hat{a}_x + 16\hat{a}_y \ m/s^2}$$

(b) Its velocity

Since $a = 20\hat{a}_x + 16\hat{a}_y$

, $\frac{dv}{dt} = \frac{d}{dt}(\hat{u}_x, \hat{u}_y, \hat{u}_z) = 20\hat{a}_x + 16\hat{a}_y$

where, u is the initial velocity of the particle.

$$\frac{d\hat{u}_x}{dt} = 20 \Rightarrow \int d\hat{u}_x = \int 20\,dt$$

$$\Rightarrow \hat{u}_x = 20t + A \quad —(1)$$

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(9)

$$\frac{d\hat{u}_y}{dt} = 16 \Rightarrow \int d\hat{u}_y = \int 16\,dt$$

$$\Rightarrow \hat{u}_y = 16t + B \quad \text{— (ii)}$$

$$\frac{d\hat{u}_z}{dt} = 0 \Rightarrow \int d\hat{u}_z = \int 0\,dt$$

$$\hat{u}_z = C \quad \text{— (iii)}$$

Where A, B & C are integration constants

$$\text{At } t=0, \quad u = 6\hat{a}_x + 5\hat{a}_z$$

Hence,

$$\hat{u}_x(t=0) = 6$$
$$\hat{u}_y(t=0) = 0$$
$$\hat{u}_z(t=0) = 5$$

Putting these value of $\hat{u}_x$, $\hat{u}_y$ and $\hat{u}_z$ in equation (i), (ii) & (iii)

we get,

$$6 = 20t + A \Rightarrow 6 = 20\times 0 + A$$
$$0 = 16t + B \Rightarrow 0 = 16\times 0 + B$$
$$5 = C \Rightarrow 5 = C$$

$$\therefore A = 6$$
$$B = 0$$
$$C = 5$$

Putting the values of A, B & C in (i), (ii) & (iii)

$$u(t=1s) = 20\times 1 + 6 + 16\times 1 + 5$$

$$\boxed{u(t=1s) = 26\hat{a}_x + 16\hat{a}_y + 5\hat{a}_z \text{ m/s}}$$

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(c) Its Kinetic energy

$$K.E. = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 3 \times ((26)^2 + (16)^2 + (5)^2)$$

$$\boxed{K.E. = 1435.5 \, J}$$

Hence, kinetic energy at $t = 15$ is $1435.5 \, J$

d) Its position

$$u(t) = (20t + 6, 16t, 5)$$

$$\frac{d}{dt}(\hat{x}, \hat{y}, \hat{z}) = (20t + 6, 16t, 5)$$

$$\therefore \frac{d\hat{x}}{dt} = 20t + 6$$

$$\frac{d\hat{y}}{dt} = 16t$$

$$\frac{d\hat{z}}{dt} = 5$$

Now, $\int d\hat{x} = \int (20t + 6) dt$

$$\hat{x} = \frac{20t^2}{2} + 16t + M \quad \text{— (iv)}$$

and $\int d\hat{y} = \int 16t \, dt$

$$\hat{y} = \frac{16t^2}{2} + N$$

$$\hat{y} = 8t^2 + N \quad \text{— (V)}$$

also $\int d\hat{z} = \int 5 \, dt$

$$\hat{z} = 5t + 0 \quad \text{— (VI)}$$

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6

Hence, $\hat{x} = 10t^2 + 6t + M$

$\hat{y} = 8t^2 + N$

$\hat{z} = 5t + O$

At $t = 0$ sec.

$(\hat{x}, \hat{y}, \hat{z}) = (2, -5, 0)$

$\therefore \hat{x}(t=0) = 2 = 10t^2 + 6t + M$

$10 \times 0^2 + 6 \times 0 + M = 2$

$M = 2$

$\hat{y}(t=0) = -5 = 8t^2 + N$

$8 \times 0^2 + N = -5$

$N = -5$

$\hat{z}(t=0) = 5t + O$

$O = 0$

$\therefore M = 2, \quad N = -5, \quad O = 0$

Putting these value of M, N & O in eq. (iv), (v) & (vi)
we get, $(\hat{x}, \hat{y}, \hat{z}) = (10t^2 + 6t + 2,\ 8t^2 - 5,\ 5t)$

at $t = 1$ sec.

$(\hat{x}, \hat{y}, \hat{z}) = (10 \times 1^2 + 6 \times 1 + 2,\ 8 \times 1^2 - 5,\ 5 \times 1)$

$(\hat{x}, \hat{y}, \hat{z}) = (10 + 6 + 2,\ 8 - 5,\ 5)$

$(\hat{x}, \hat{y}, \hat{z}) = (18, 3, 5)$

Hence, at $t = 1$ sec, the position of the particle is
$(18, 3, 5)$.

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(7)

Q-3 - A Current distribution gives rise to the vector magnetic potential $\vec{A} = x^3y^2\hat{a}_x + xy^3\hat{a}_y - 6x^2y^2z^2\hat{a}_z$ Wb/m. Find the flux through the surface defined by

$z = 1, 0 \leq x \leq 2, -1 \leq y \leq 5$.

Sol ÷ As given $\vec{A} = x^3y^2\hat{a}_x + xy^3\hat{a}_y - 6x^2y^2z^2\hat{a}_z$ Wb/m

Surface → $z = 1$:

$0 \leq x \leq 2$

$-1 \leq y \leq 5$

As we know, $\psi = \int_C \vec{A} \cdot \vec{dl}$

$$\psi = \int_0^2 x^3(-1)^2 . dx + \int_{-1}^5 2(y)^3 . dy + \int_2^0 x^3(5)^2 dx + 0$$

$$\psi = \int_0^2 x^3 . dx + 2\int_{-1}^5 y^3 . dy + 25\int_2^0 x^3 . dx$$

$$\psi = \left[\frac{x^4}{4}\right]_0^2 + 2\left[\frac{y^4}{4}\right]_{-1}^5 + 25\left[\frac{x^4}{4}\right]_2^0$$

$$\psi = \left(\frac{16}{4} - \frac{0}{4}\right) + 2\left[\frac{625}{4} - \frac{1}{4}\right] + 25\left(\frac{0}{4} - \frac{16}{4}\right)$$

$$\psi = 4 + 2(156) + 25(-4)$$

$$\psi = 4 + 312 - 100$$

$$\boxed{\psi = 216 \text{ webers}}$$

Ans

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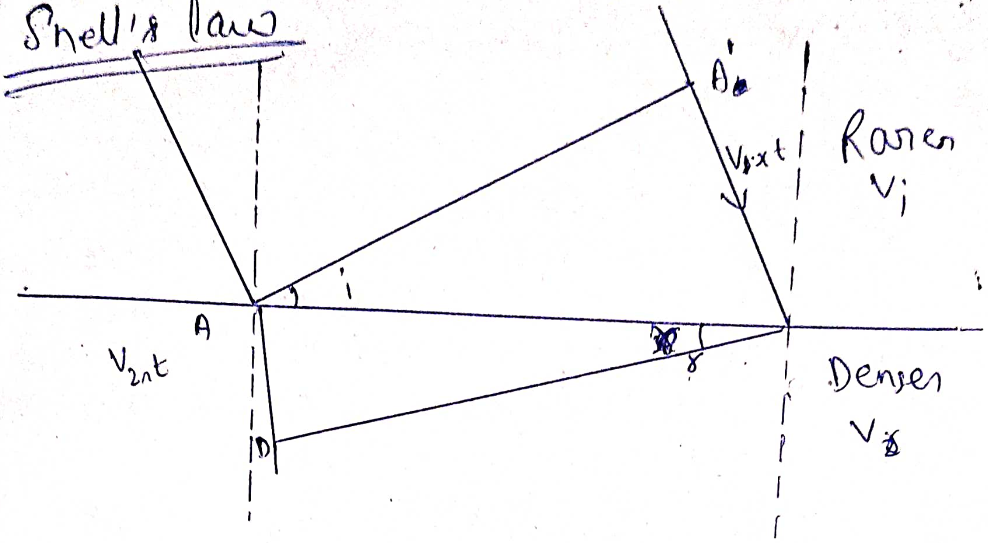
Q-4 : Prove Snell's law. Derive expression for Brewster angle of plane electromagnetic wave for parallel polarization.

Sol. • Snell's law



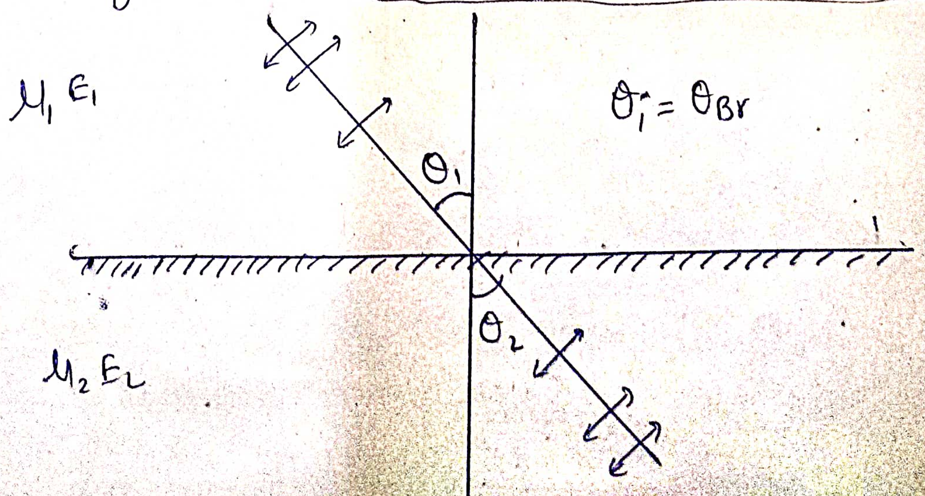
AA' = incident wave front
BD = Refracted wavefront

$$\sin i = \frac{A'B}{AB} \quad , \quad \sin r = \frac{AD}{AB}$$

$$\frac{\sin i}{\sin r} = \frac{A'D}{AB} \times \frac{AB}{AD} = \frac{A'B}{A} = \frac{V_i \times t}{V_r \times t}$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{V_i}{V_r} \Rightarrow \boxed{\frac{\sin i}{\sin r} = \mu}$$ Hence proved.

• Brewster angle of plane electromagnetic wave for parallel polarisation



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⑨

For parallel ~~plane~~ polarization the effective coefficient

$$\frac{E_r}{E_i} = \frac{\sqrt{E_2}\cos\theta_1 - \sqrt{E_1}\cos\theta_2}{\sqrt{E_2}\cos\theta_1 + \sqrt{E_1}\cos\theta_2}$$

At Brewster's angle $\frac{E_r}{E_i} = 0$

$$\Rightarrow \sqrt{E_2}\cos\theta_1 = \sqrt{E_r}\cos\theta_2$$

$$\frac{\sqrt{E_2}}{\sqrt{E_1}}\cos\theta_1 = \sqrt{1-\cos^2\theta_2} \quad \text{—— (I)}$$

According to Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{\sqrt{E_2}}{\sqrt{E_1}}$

$$\Rightarrow \sin\theta_2 = \frac{\sqrt{E_1}}{\sqrt{E_2}}\sin\theta_1 \quad \text{—— (II)}$$

Putting (II) in (I)

$$\sqrt{\frac{E_2}{E_1}}\cos\theta_1 = \sqrt{1-\frac{E_1}{E_2}\sin\theta_1}$$

$$\frac{E_2}{E_1}\cos^2\theta_1 = 1-\frac{E_1}{E_2}\sin^2\theta_1$$

$$\frac{E_2}{E_1}(1-\sin^2\theta_1) = 1-\frac{E_1}{E_2}\sin^2\theta_1$$

$$\frac{E_2}{E_1} - 1 = \sin^2\theta_1\left(\frac{E_2}{E_1} - \frac{E_1}{E_2}\right)$$

$$\frac{\cancel{(E_2-E_1)}}{\cancel{E_1}} = \sin^2\theta_1\,\frac{\cancel{(E_2-E_1)}(E_1+E_2)}{\cancel{E_1}E_2}$$

$$\sin^2\theta = \frac{E_2}{E_1+E_2}$$

$$\cos^2\theta_2 = \frac{E_1}{E_1+E_2}$$

$$\tan^2\theta_2 = \frac{E_2}{E_1}$$

$$\tan\theta_2 = \sqrt{\frac{E_2}{E_1}} \Rightarrow \boxed{\theta_2 = \tan^{-1}\sqrt{\frac{E_2}{E_1}}}$$

Brewster's Angle

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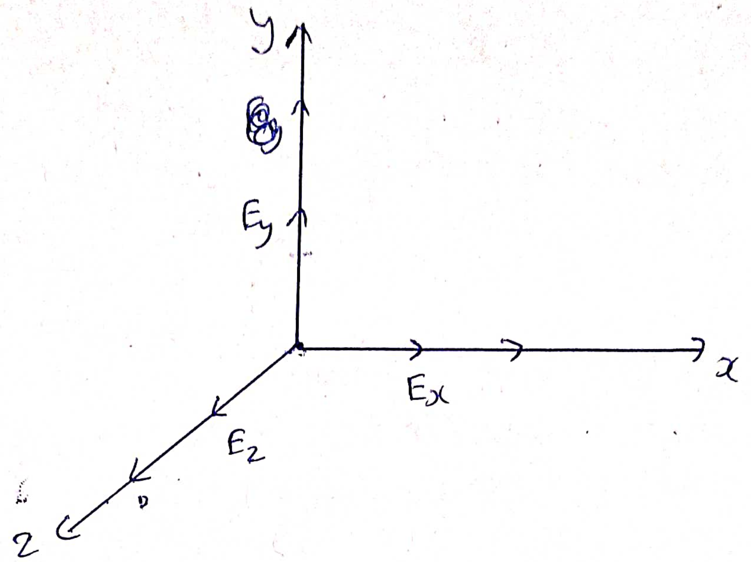
Q-5 ÷ Derive the field expressions for TM modes in a rectangular wave guide. Find the cut-off frequency, cut-off wavelength, intrinsic impedance.



As we know

$$E_{zx}(x,y,z) = (A_1 \cos K_x x + A_2 \sin K_x x)(A_3 \cos K_y y + A_4 \sin K_y y) e^{-\gamma z}$$

At the walls of the wave guide, the tangential components of the the Electric field must be continous

i.e.,
$$E_{zs} = 0 \text{ at } y=0 \quad \text{— (I)}$$
$$E_{zs} = 0 \text{ at } y=b \quad \text{— (II)}$$
$$E_{zs} = 0 \text{ at } x=0 \quad \text{— (III)}$$
$$E_{zs} = 0 \text{ at } x=a \quad \text{— (IV)}$$

Eqn (I) & (II) require that $A_1 = 0 = A_3$ in the eqn (A), So A because

$$E_{zs} = E_0 \sin K_x x . \sin K_y y \, e^{-\gamma z} \quad \text{— (V)}$$

where $E_0 = A_2 A_4$

Also, when (II) & (V) are applied to (V) it would simply that

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$$\sin k_x a = 0, \quad \sin k_y b = 0$$

$$\Rightarrow k_x a = m\pi, \quad m = 1,2,3 \ldots$$

$$k_y b = n\pi, \quad n = 1,2,3 \ldots$$

or $$\boxed{k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}} \quad \text{— (VI)}$$

Substituting (VI) in (V)

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{— (VII)}$$

Using (VII), we can say that

$$\boxed{E_{xs} = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$$\boxed{E_{ys} = \frac{-\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$$\boxed{H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

$$\boxed{H_{ys} = \frac{-j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$$

for Cutoff frequency

As $$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

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or $$\boxed{f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{a}\right)^2}}$$

for cutoff wavelength

As $$\lambda_c = \frac{u}{f_c} \Rightarrow \lambda_c = \frac{u'}{\frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

for intrinsic impedence

$$\eta_m = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$\frac{B}{\omega E} = \sqrt{\frac{u}{E}}\sqrt{1-\left(\frac{f_c}{F}\right)^2} = \eta_m = \eta\sqrt{1-\left[\frac{f_a}{F}\right]^2}$$