Experiment : 2

Name : Vishal Anand
Roll Code : 19/367
Subject : Electromagnetics
Course : B.Sc. Hons. Electronics
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Experiment : 2

Aim : Transformation of vectors into various coordinate systems.

Apparatus Required : A desktop with Scilab installed in it.

Theory :

Cartesian coordinates (x, y, z):

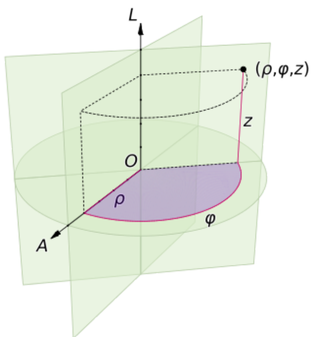
A three-dimensional Cartesian coordinate system is formed by a point called the origin (denoted by O) and a basis consisting of three mutually perpendicular vectors. These vectors define the three coordinate axes: the x − axis, y − axis , and z − axis. They are also known as the abscissa, ordinate and applicate axis, respectively. The coordinates of any point in space are determined by three real numbers: x , y , z .

a point P can be represented as (x, y, z) A vector A in Cartesian (otherwise known as rectangular) coordinates can be written as (A_x,A_y,A_z) or $A_xa_x + A_ya_y + A_za_z$ where a_x, a_y, and a_z are unit vectors along the x-, y-, and z-directions.

Cylindrical coordinates (ρ,ϕ,z):

ρ is the radius of the cylinder passing through P or the radial distance from the z-axis: ρ, called the azimuthal angle, is measured from the x-axis in the xy-plane; and z is the same as in the Cartesian system. The relationships between (x, y, z) and (ρ,ϕ,z).

The three coordinates (ρ, φ, z) of a point P are defined as: The axial distance or radial distance **ρ is the Euclidean distance from the z-axis to the point P**. The azimuth φ is the angle between the reference direction on the chosen plane and the line from the origin to the projection of P on the plane.



The relationships between (x, y, z) and (ρ,ϕ,z) :

$$\rho = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}, \qquad z = z$$

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi, \qquad z = z$$

The relationships between (Ax, Ay, Az) and (Aρ, Aϕ, Az) :

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Spherical coordinates (r,θ,ϕ) :

r is defined as the distance from the origin to point P or the radius of a sphere centered at the origin and passing through P; θ (called the colatitude) is the angle between the z-axis and the position vector of P; and ϕ is measured from the x-axis (the same azimuthal angle in cylindrical coordinates)

The relationships between (x, y, z) and (r,θ,ϕ) :

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi, \qquad y = r\sin\theta\sin\phi, \qquad z = r\cos\theta$$

The relationships between (Ax, Ay, Az) and (Ar,Aθ Aϕ) :

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Code :

1. Cartesian to cylindrical.
2. Cylindrical to cartesian.
3. Cartesian to spherical.
4. Spherical to cartesian.

```
clear;
clc;
disp("1 cartesian to cylindrical");
disp("2 cylindrical to cartesian");
disp("3 cartesian to spherical");
disp("4 spherical to cartesian");

ch=input("Enter your choice (1,2,3 or 4) : ");

//Cartesian to Cylindrical
if(ch==1) then
  disp("Enter a point P(x,y,z) in cartesian coordinates");
  x=input(" x = ");
  y=input(" y = ");
  z=input(" z = ");
  p=sqrt(x^2+y^2);
  if(x<0) then
    q=atand(y/x)+180;
  elseif(x>0 & y<0) then
    q=atand(y/x)+360;
  else
    q=atand(y/x);
  end
  z=z;
  mprintf(" point P(p,q,z) in cylindrical coordinates :  p=%f  q=%f  z=%f \n", p,q,z);

  disp("Enter the vector A(Ax,Ay,Az) in cartesian coordinates");
  Ax=input(" Ax = ");
  Ay=input(" Ay = ");
  Az=input(" Az = ");
  A=[cosd(q) sind(q) 0; -sind(q) cosd(q) 0; 0 0 1]*[Ax; Ay; Az];
  mprintf(" vector A(Ap,Aq,Az) in cylindrical coordinates :-  (%f)ap + (%f)aq + (%f)az \n",A(1,1),A(2,1),A(3,1));
end

//Cylindrical to Cartesian
if(ch==2) then
  disp("Enter a point P(p,q,z) in cylindrical coordinates");
  p=input(" p = ");
  q=input(" q = ");
  z=input(" z = ");
  x=p*cosd(q);
  y=p*sind(q);
  z=z;
  mprintf(" point P(x,y,z) in cartesian coordinates :  x=%f  y=%f  z=%f \n", x,y,z);

  disp("Enter the vector A(Ap,Aq,Az) in cylindrical coordinates");
```

```
   Ap=input(" Ap = ");
   Aq=input(" Aq = ");
   Az=input(" Az = ");
   A=[cosd(q) -sind(q) 0; sind(q) cosd(q) 0; 0 0 1]*[Ap; Aq; Az];
   mprintf(" vector A(Ax,Ay,Az) in cartesian coordinates :-  (%f)ax + (%f)ay + (%f)az \n",A(1,1),A(2,1),A(3,1));
end

//Cartesian to Spherical
if(ch==3) then
   disp("Enter a point P(x,y,z) in cartesian coordinates");
   x=input(" x = ");
   y=input(" y = ");
   z=input(" z = ");
   r=sqrt(x^2+y^2+z^2);
   if(z<0) then
      Q=atand(sqrt(x^2+y^2)/z)+180;
   else
      Q=atand(sqrt(x^2+y^2)/z);
   end
   if(x<0) then
      q=atand(y/x)+180;
   elseif(x>0 & y<0) then
      q=atand(y/x)+360;
   else
      q=atand(y/x);
   end
   mprintf(" point P(r,Q,q) in spherical coordinates :  r=%f  Q=%f  q=%f \n", r,Q,q);

   disp("Enter the vector A(Ax,Ay,Az) in cartesian coordinates");
   Ax=input(" Ax = ");
   Ay=input(" Ay = ");
   Az=input(" Az = ");
   A=[sind(Q)*cosd(q) sind(Q)*sind(q) cosd(Q); cosd(Q)*cosd(q) cosd(Q)*sind(q) -sind(Q); -sind(q) cosd(q)
0]*[Ax; Ay; Az];
   mprintf(" vector A(Ar,AQ,Aq) in cylindrical coordinates :-  (%f)ar + (%f)aQ + (%f)aq \n",A(1,1),A(2,1),A(3,1));
end

//Spherical to Cartesian
if(ch==4) then
   disp("Enter a point P(r,Q,q) in spherical coordinates");
   r=input(" r = ");
   Q=input(" Q = ");
   q=input(" q = ");
   x=r*sind(Q)*cosd(q);
   y=r*sind(Q)*sind(q);
   z=r*cosd(Q);
   mprintf(" point P(x,y,z) in cartesian coordinates :  x=%f  y=%f  z=%f \n", x,y,z);

   disp("Enter the vector A(Ar,AQ,Aq) in spherical coordinates");
   Ar=input(" Ar = ");
   AQ=input(" AQ = ");
   Aq=input(" Aq = ");
   A=[sind(Q)*cosd(q) cosd(Q)*cosd(q) -sind(q); sind(Q)*sind(q) cosd(Q)*sind(q) cosd(q); cosd(Q) -sind(Q)
0]*[Ar; AQ; Aq];
   mprintf(" vector A(Ax,Ay,Az) in cartesian coordinates :-  (%f)ax + (%f)ay + (%f)az \n",A(1,1),A(2,1),A(3,1));
end
```

Output :

- Cartesian to cylindrical & vice versa.

1.

```
 "1 cartesian to cylindrical"

 "2 cylindrical to cartesian"

 "3 cartesian to spherical"

 "4 spherical to cartesian"
Enter your choice (1,2,3 or 4) : 1


 "Enter a point P(x,y,z) in cartesian coordinates"
x = -4

y = 12

z = 6

point P(p,q,z) in cylindrical coordinates :  p=12.649111  q=108.434949  z=6.000000

 "Enter the vector A(Ax,Ay,Az) in cartesian coordinates"
Ax = 12

Ay = 2

Az = 0

vector A(Ap,Aq,Az) in cylindrical coordinates :-  (-1.897367)ap + (-12.016655)aq + (0.000000)az
```

2.

```
 "1 cartesian to cylindrical"

 "2 cylindrical to cartesian"

 "3 cartesian to spherical"

 "4 spherical to cartesian"
Enter your choice (1,2,3 or 4) : 2


 "Enter a point P(p,q,z) in cylindrical coordinates"
p = -2

q = -5

z = -9

point P(x,y,z) in cartesian coordinates :  x=-1.992389  y=0.174311  z=-9.000000

 "Enter the vector A(Ap,Aq,Az) in cylindrical coordinates"
Ap = 5

Aq = 3

Az = 0

vector A(Ax,Ay,Az) in cartesian coordinates :-  (5.242441)ax + (2.552805)ay + (0.000000)az
```

- Cartesian to spherical & vice versa.

3.

```
 "1 cartesian to cylindrical"

 "2 cylindrical to cartesian"

 "3 cartesian to spherical"

 "4 spherical to cartesian"
Enter your choice (1,2,3 or 4) : 3


 "Enter a point P(x,y,z) in cartesian coordinates"
x = 2

y = -4

z = 5

point P(r,Q,q) in spherical coordinates :  r=6.708204  Q=41.810315  q=296.565051

 "Enter the vector A(Ax,Ay,Az) in cartesian coordinates"
Ax = 200

Ay = -100

Az = -300

vector A(Ar,AQ,Aq) in cylindrical coordinates :-  (-104.349839)ar + (333.333333)aQ + (134.164079)aq
```

4.

```
 "1 cartesian to cylindrical"

 "2 cylindrical to cartesian"

 "3 cartesian to spherical"

 "4 spherical to cartesian"
Enter your choice (1,2,3 or 4) : 4


 "Enter a point P(r,Q,q) in spherical coordinates"
r = 1

Q = -4

q = 6

point P(x,y,z) in cartesian coordinates :  x=-0.069374  y=-0.007292  z=0.997564

 "Enter the vector A(Ar,AQ,Aq) in spherical coordinates"
Ar = 7

AQ = 0

Aq = -4

vector A(Ax,Ay,Az) in cartesian coordinates :-  (-0.067507)ax + (-4.029128)ay + (6.982948)az
```

Result : a point P and a vector A both are transformed into various coordinate systems from one to another. i.e from cartesian to cylindrical, cylindrical to cartesian, cartesian to spherical, cylindrical to spherical.

Discussion : A point or vector can be represented in any curvilinear coordinate system, which may be orthogonal or non orthogonal. An orthogonal system is one in which the coordinates arc mutually perpendicular. Non Orthogonal systems are hard to work with and they are of little or no practical use. Examples of orthogonal coordinate systems include the Cartesian (or rectangular), the circular cylindrical, the spherical, the elliptic cylindrical, the parabolic cylindrical, the conical, the prolate spheroidal, the oblate spheroidal, and the ellipsoidal.