1a) $w_1 = 1$

$w_2 = 1$

$w_3 = -1$

$w_4 = 0.5$

$w_5 = 1$

$w_6 = 2$

$z_1 = x w_1 = 4 \cdot 1 = 4$

$z_2 = x w_2 = 4 \cdot 1 = 4$

$z_3 = x w_3 = 4 \cdot -1 = -4$

→ hidden layers = (z_1, z_2, z_3)

↳ use activation ftn to get output for hidden layers

$z_1 = 4$

$z_1 \cdots$

$z_2 = 4$

$z_3 = 0$

→ find z

$$z = (z_1 w_4) + (z_2 w_5) + (z_3 w_6)$$

$$= (4 \cdot 0.5) + (4 \cdot 1) + (0 \cdot 2)$$

$$= 2 + 4$$

$$= 6$$

→ use activation function for output node

$$a = \sigma(x) = \frac{1}{1 - e^{-x}}$$

$$\sigma(z = 6) = \frac{1}{1 + e^{-6}}$$

$= \boxed{0.998}$

1b) calculate the loss

$$\text{loss} = (y - \hat{y})^2$$

$$= (y - a)^2 \qquad \hat{y} = a = 0.99752$$

$$= (y - 0.99752)^2$$

$$= (0 - 0.99752)^2$$

$$= 0.995046 = \boxed{0.996}$$

1c) $$\frac{\partial L}{\partial a} = -2(y - a)$$

$$= -2(0 - 0.9975)$$

$$= -2(-0.9975)$$

$$= 1.9950$$

$$\frac{\partial a}{\partial z} = \frac{\partial}{\partial z}(\sigma(z))$$

$$= a(1-a)$$

$$= (0.9975)(1-0.9975)$$

$$= 0.0024$$

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z}$$

$$= 1.995 \cdot 0.0024$$

$$= 0.0049$$

$$\frac{\partial L}{\partial w_4} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w_4}$$

$$= 0.0099$$

$$\frac{\partial L}{\partial w_5} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w_5}$$

$$= 0.0049 \cdot z_2'$$

$$= 0.0199$$

$$\frac{\partial L}{\partial w_6} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w_6}$$

$$= 0.0049 \cdot z_3'$$

$$= 0$$

$$\frac{dL}{dw_5} = \frac{dL}{dz} \cdot \frac{dz}{dw_5} = 0.019$$

$$\frac{dL}{dw_5} = \frac{dL}{dz} \cdot \frac{dz}{dw_6} = 0$$

$$z = z_1' w_4 + w_2' w_5 + z_1' w_6$$

$$\frac{\partial L}{\partial z_1} = w_4$$

$$\frac{\partial z}{\partial z_2'} = w_5$$

$$\frac{\partial z}{\partial z_3'} = w_6$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial z_1} = 0.0049 \cdot 0.5 = 0.0024$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial z_2} = 0.0049 \cdot 1 = 0.0049$$

$$\frac{\partial L}{\partial z_3} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial z_3} = 0.0049 \cdot 2 = 0.0098$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial x_1'} \cdot \frac{\partial z_1'}{\partial z_1}$$

$$= 0.0024 \cdot 1$$

$$= 0.0024$$

$$\frac{\partial L}{\partial z_2} = 0.0049$$

$$\frac{\partial L}{\partial z_3} = 0.0098$$

$$z_1 = w_1 x \rightarrow \frac{\partial z_1}{\partial w_1} = x$$

$$z_2 = w_2 x \rightarrow \frac{\partial z_2}{\partial w_2} = x$$

$$z_3 = w_3 x \rightarrow \frac{\partial z_3}{\partial w_3} = x$$

$$\frac{\partial L}{\partial w_1} \times \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} = 0.0024 \cdot 4 = 0.0096$$

$$\frac{\partial L}{\partial w_2} \cdot \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_2} = 0.0049 \cdot 4 = 0.0196$$

$$\frac{\partial L}{\partial w_3} \cdot \frac{\partial L}{\partial z_3} \cdot \frac{\partial z_3}{\partial w_3} = 0.0098 \cdot 4 = 0.392$$

Updation:

$$w_1 = w_1 - \alpha \frac{\partial L}{\partial w_1}$$, with α = learning rate

$\rightarrow \alpha = 0.1$

$$w_1 = 1 - (0.1)(0.0096)$$

$$= 1 - 0.00096$$

$$= 0.99904$$

$$w_2 = 1 - (0.1 \cdot 0.0196)$$

$$= 0.99804$$

$$w_3 = -1 - (0.1 \cdot 0.0392)$$

$$= -1.00392$$

$$w_4 = 0.5 - (0.1 \cdot 0.0199)$$

$$= 0.49801$$

$$w_5 = 1 - (0.1 \cdot 0.0199)$$

$$= 0.99801$$

$$w_6 = 2 - (0.1 \cdot 0)$$

$$= 2$$

The nearly calculated weights are:

$\rightarrow w_1 = 0.991$

$\rightarrow w_2 = 0.998$

$\rightarrow w_3 = -1.004$

$\rightarrow w_4 = 0.498$

$\rightarrow w_5 = 0.998$

$\rightarrow w_6 = 2.000$

Now using forward propagation:

$$z_1 = x w_1 = 4 \cdot 0.9904$$
$$= 3.99616$$

$$z_2 = x w_2 = 3.99216$$

$$z_3 = x w_3 = -4.01568$$

$\rightarrow z_1' = 3.996$

$\rightarrow z_2' = 3.992$

$\rightarrow z_3' = 0$

$$Z = z_1' \cdot w_4 + z_2' \cdot w_5 + z_3' \cdot w_6$$

$$= (3.996 \cdot 0.5) + (3.99216 \cdot 1) + (0 \cdot 2)$$

$$= 1.99808 + 3.99216 + 0$$

$$= 5.99024$$

$$Z = 5.990$$

1d) $\boxed{\begin{aligned} a &= \sigma(2) \\ &= \frac{1}{1+e^{-2}} \\ &= 0.998 \end{aligned}}$

$$L(y,a) = (y-a)^2$$

$$= (0 - 0.9975031)^2$$

$$= 0.99501$$

1e)

→ first output $= 0.99752$

→ output after update $= 0.9975031$

$Loss_1 = 0.99504$

$Loss_2 = 0.99504$

$Loss_2 < Loss_1$ and output after update is closer to target (0).