MATH 3100 – Homework #6

posted October 22, 2021; due by 5 PM on November 1, 2021

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

Required problems

1. §2.3: 2
2. §2.3: 3(b,c,f,g)
3. §2.3: 9
4. §2.3: 11
5. Let $\{a_n\}$ be a sequence and let $\{c_n\}$ and $\{d_n\}$ be the subsequences given by $c_n = a_{2n}$ and $d_n = a_{2n-1}$. Assume that $\lim c_n = L$ and that $\lim d_n = L$ for the same real number L. Prove, **using the limit definition**, that $\lim a_n = L$.

   Hint. This could have been a problem in Chapter 1; nothing about series is used.

6. In this problem we give an "alternate" proof of the AST[1].

   Suppose that $\sum_{k=1}^{\infty} a_k$ satisfies the three conditions of the alternating series test. We assume, without loss of generality, that $a_k \geq 0$ for odd k and $a_k \leq 0$ for even k.

   Let $s_n = a_1 + a_2 + \cdots + a_n$ be the nth partial sum of $\sum_{k=1}^{\infty} a_k$.

   Let $c_n = s_{2n}$ and let $d_n = s_{2n-1}$.

   (a) Explain why $\{c_n\}$ and $\{d_n\}$ bounded.

   Hint. You may take as already known that $\{s_n\}$ is bounded (we proved in class that $|s_n| \leq |a_1|$ for all n). Your proof should be very short!

   (b) Show that $\{c_n\}$ is increasing and $\{d_n\}$ is decreasing.

   Hint: Use the pairing idea described in class.

   (c) Explain why $\lim c_n = L$ and $\lim d_n = L'$ for some real numbers L and L'.

   (d) Show that $L = L'$.

   Hint. First show that $c_n - d_n = a_{2n}$. Now take the limit as $n \to \infty$.

   (e) Deduce from problem (6) that $\lim s_n = L$. (Hence, $\sum_{k=1}^{\infty} a_k$ converges to L.)

7. §2.4: 1(b,e,f,h,k,l)

Recommended problems

§2.3: 3(a,d,e), 7, 10
§2.4: 1(a,c,d,g,i,j)

[1] pun intended