MATH 4000/6000 – Homework #6

posted March 23, 2022; due March 30, 2022

The beauty of mathematics only shows itself to more patient followers. – Maryam Mirzakhani

Assignments are expected to be neat and stapled. **Illegible work may not be marked**. Starred problems (*) are required for those in MATH 6000 and extra credit for those in MATH 4000.

1. Exercise 3.2.1.

2. Exercise 3.2.6(a,e).

3. (a) Show that $\mathbb{Q}[\sqrt{2}, i]$ is a splitting field for $x^8 - 1$ over $\mathbb{Q}$.

   Hint. We know from class that $\mathbb{Q}[\cos(2\pi/8) + i\sin(2\pi/8)]$ is a splitting field for $x^8 - 1$ over $\mathbb{Q}$. What are $\cos(2\pi/8)$ and $\sin(2\pi/8)$?

   (b) Let $\zeta = \cos(2\pi/2022) + i\sin(2\pi/2022)$, and let $\sqrt[2022]{3}$ denote the positive real 2022th root of 3. Prove that $\mathbb{Q}[\zeta, \sqrt[2022]{3}]$ is a splitting field for $x^{2022} - 3$ over $\mathbb{Q}$.

4. Exercise 3.3.2(b,c,e,h).

5. Exercise 3.3.4.

6. (*) Exercise 3.3.7.

   Hint. Argue that the Eisenstein criterion can be applied to $f(x + 1)$. Look at Examples 7(c) on p. 110.