MATH 3100 – Homework #1

posted January 9, 2020; due at the **start of class** on January 17, 2020

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

1. §1.1: Exercise 4.
2. §1.1: Exercise 5.
3. §1.1: Exercise 8.
4. §1.2: Exercise 5.
5. §1.2: Exercise 10.
6. §1.2: Exercise 14(b).
7. §1.2: Exercise 19.
8. Define real numbers α and β by $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$.
   (a) Check that α and β are roots of the polynomial $x^2 - x - 1$.
   (b) Using (a), deduce that $\alpha^{n+1} = \alpha^n + \alpha^{n-1}$ and $\beta^{n+1} = \beta^n + \beta^{n-1}$, for every integer n. (You don't need induction for this step, just algebra.)
   (c) Recall that the Fibonacci sequence $\{F_n\}$ is defined by $F_1 = 1$, $F_2 = 1$, and the recurrence $F_{n+1} = F_n + F_{n-1}$ for $n \geq 2$.
   Use complete induction to prove that $\frac{\alpha^n - \beta^n}{\sqrt{5}} = F_n$ for all natural numbers n.
   Hint: The result of (b) will be useful.
9. The following argument is an *alleged* proof that any finite group of people all have the same height:

   > Let S be the set of natural numbers n for which the statement "every group of n people share the same height" is true. Obviously the statement is true if there is just one person, so $1 \in S$. Now we suppose that $n \in S$, and we prove that $n+1 \in S$. Take any group of $n+1$ people, say A_1, …, A_{n+1}. Since $n \in S$, it must be that $A_1, \ldots, A_n$ all share the same height, and similarly for $A_2, \ldots, A_{n+1}$. But these two groups overlap; for instance, the second person A_2 is in both. So all of our $n+1$ people have the same height (indeed, everyone is the same height as A_2). Thus, $n+1 \in S$. So by induction, S is all of the natural numbers.

   Clearly explain the mistake in the proof.