**MATH 3100 – Learning objectives to meet for Exam #3**

The exam will cover §2.3–§3.1 of the course notes, up through the end of lecture on Wednesday, November 17.

## What to be able to state

### Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- Absolute convergence and conditional convergence
- Power series centered at 0, power series centered at $a$
- Domain of convergence, radius of convergence
- Pointwise convergence of a sequence of functions $f_n$ to $f$ on a set $A$
- Distance $d(f, g)$ between two functions $f$ and $g$ on a set $A$
- Uniform convergence of a sequence of functions $f_n$ to $f$ on a set $A$
- Uniform convergence of a series of functions $\sum_{n=1}^{\infty} f_n(x)$ on a set $A$

### Big theorems

Be familiar with all of the following theorems, making sure that you can give full statements for those not already spelled out below. For results proved in class, describe the components and main ideas of the proof.

- Alternating series test
- Ratio test
- Key Lemma: If $\sum_{n=0}^{\infty} a_n x^n$ converges at $x = x_0$, it converges absolutely for all $x$ with $|x| < |x_0|$
- Characterization of the domain of convergence of a power series centered at 0 (Proposition 2.4.4)
- Uniform convergence preserves continuity (Theorem 3.1.11; in fact, we proved something a bit more general than the statement in the notes, since we didn't require that $A$ be a closed interval $[a, b]$)
- Uniform convergence plays nice with integration on closed intervals (Proposition 3.1.12)
- Applications of uniform convergence to power series (Theorems 3.1.13 and 3.1.14)
- Weierstrass $M$-test

## What to expect on the exam

There will be five questions on the exam, possibly having multiple parts. These will include...

- A problem testing you on a basic definition and requiring you to give a proof using this definition
- A problem or problem part asking you to determine the domain and/or radius of convergence of (one or more) power series, possibly centered at a nonzero point
- At least one problem asking you to determine a pointwise limit and to further determine whether or not the convergence to the limit is uniform

## Practice problems

1. Find the domain of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n(n+1)}(x-1)^n$. Do the same for $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}x^n$.

2. Consider the functions $f_n$ defined on $\mathbf{R}$ by $f_n(x) = \frac{x^2+nx}{n}$. Determine the pointwise limit $f$ of the $f_n$. Does $f_n$ converge uniformly to $f$ on $A = \mathbf{R}$? Does the answer change if instead $A = [0, 1]$?

3. Consider the functions $f_n$ defined on $A = (-1, 1)$ by $f_n(x) = \frac{1-x^{n+1}}{1-x}$. Determine the pointwise limit $f$ of the $f_n$. Does $f_n$ converge uniformly to $f$ on $A$?

   *Remark.* This problem is asking you (in disguise) to determine whether the power series $\sum_{k=0}^{\infty} x^k$ converges uniformly on its entire domain of convergence. Do you see how?

4. State the Weierstrass $M$-test. Use that result to show that the series $\sum_{k=1}^{\infty} e^{-kx}/k^2$ converges uniformly on $A = [0, \infty)$. If we set $s(x) = \sum_{k=1}^{\infty} e^{-kx}/k^2$, explain why $s(x)$ is continuous on $[0, \infty)$.

5. We will show on Friday that $\arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for $|x| < 1$. Assuming this, find the exact value of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}.$$

   Simplify your answer as much as possible.

6. Find a power series representing $\displaystyle\int_0^x \frac{dt}{1-t^3}$ for $|x| < 1$. You **do not** have to evaluate $\displaystyle\int_0^x \frac{dt}{1-t^3}$ in closed form (though it is possible to do so!).