MATH 3100 – Learning objectives to meet for the final exam

The exam is cumulative and all of the material discussed during the semester is fair game. **The following review sheet covers only material introduced after Midterm #3.**

What to be able to state (since last exam)

Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- Maclaurin series of f
- nth Maclaurin polynomial of f

Big theorems

Be familiar with all of the following theorems, making sure that you can give full statements for those not already spelled out below. For results proved in class, describe the components and main ideas of the proof.

- if f is represented by a power series (centered at 0) near 0, that power series is the Maclaurin series of f
- $P_n^f(x)$ is the "best approximation" to f near 0 of any polynomial of the form $a_0 + a_1 x + \cdots + a_n x^n$
- rules for computing Maclaurin polynomials (as given in Theorem 3.2.6(i)–(iii))
- Taylor's theorem

What to expect on the final exam

The format of the final exam is similar to your three midterms, but the length is approximately double (meaning, 9 or 10 question). **Among other things**, there will be ...

- A problem asking you to establish the value of a limit directly from the definition of a limit.
- A problem asking you to determine convergence/divergence of given series (and possibly absolute.conditional convergence).
- A problem asking you to apply Taylor's theorem.

Practice problems for §3.2

1. Find $P_5^{1/(1-x^3)}(x)$. Then use this to find $P_5^f(x)$ if $f(x) = \sin(x)/(1-x^3)$.
2. (a) Using Taylor's theorem, prove that for all $x \in [0,1]$, and all positive integers n, we have
$$\left| e^x - \sum_{k=0}^{n} \frac{x^k}{k!} \right| \le e \frac{x^{n+1}}{(n+1)!}.$$

(b) Write down the inequality obtained by taking $x = 1$ and $n = 2$ in part (a). Use this to show that $e \leq 3$.

(c) Using the results of (a) and (b), show that e is within 10^{-4} of $\sum_{k=0}^{8} 1/k!$.

3. (a) Let $f(x) = \sqrt{4+x}$. Find $P_2^f(x)$.

   (b) Show that $\sqrt{5}$ is within $\frac{3}{8} \cdot \frac{1}{2^5}$ of $P_2^f(1)$. Is $P_2^f(1)$ larger or smaller than $\sqrt{5}$? Justify your answers **using Taylor's theorem**.

4. Let $f(x) = \sin x$. Prove that if $|x| < 0.4$, then

$$|\sin x - (x - x^3/6)| < 10^{-4}.$$

Hint: $(0.4)^5 < 5!/10^4$.