Carmichael numbers of various shapes

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Let $k \geq 1$ be an odd number. If $N = 2^n k + 1$ is a Carmichael number, then

$$n < 2^{2\times 10^6 \tau(k)^2 (\log k)^2 \omega(k)}.$$

The proof of this result uses the Subspace Theorem. Further, the smallest odd k such that $2^n k + 1$ is Carmichael for some n is $k = 27$ ($1729 = 2^6 \times 27 + 1$ is a Carmichael number). These results have obtained in joint work with J. Cilleruelo (Madrid) and A. Pizarro (Valparaiso). In the same spirit, in work in progress, we prove jointly with Banks, Finch, Pomerance and Stănică that the set

$$\{k \text{ odd} : k = (N-1)/2^n \text{ for some Carmichael number } N \text{ and some positive integer } n\}$$

is of asymptotic density zero. These results together with some of the main steps of their proofs will be presented during the talk.