MATH 3200 – Homework #4

posted October 9, 2024; due at the **start of class** on October 18, 2024

Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.** Assume — unless explicitly told otherwise — that **you are expected to include clear and concise explanations of your reasoning, expressed in complete sentences.**

1. Let $A = \{3, \{\pi, 3\}, \mathbf{Z}, \pi, \{3\}\}$.

   (a) Is $\mathbf{Z}$ an element or a subset of A?

   (b) Give an example of an element of A.

   (c) Give an example of a (nontrivial) subset of A.

   (d) Is it possible to give the same answer to the preceding two questions?

   (e) Compute $|A|$.

   (f) How many elements do each of the following have: $\{\}$, $\emptyset$, and $\{\emptyset\}$?

   Explain your answer to (d). For the other parts, the answer alone is sufficient.

2. Let $A = \{x \in \mathbf{Z} : 5 \nmid x - 3\}$ and $B = \{x \in \mathbf{Z} : x = 15k + 3 \text{ for some } k \in \mathbf{Z}\}$. Prove that $A \cap B = \emptyset$.

   Recall that to show $A \cap B = \emptyset$, it is enough to assume there is some element in A and B and to derive a contradiction.

3. Let $A = \{2, 3, 4, 5\}$ and $B = \{1, 5\}$ be subsets of the universal set $U = \{1, 2, 3, 4, 5\}$. Write out the following sets.

   (a) A^c

   (b) B^c

   (c) $A \cup B$

   (d) $A \cap B$

   (e) $A \setminus B$

   (f) $B \setminus A$

   [No justification required.]

4. Call an integer n *round* if $3 \mid n$, *strange* if $n = 3k + 1$ for some $k \in \mathbf{Z}$, and *weird* if $n + 1$ is round. Let R, S, and W be the sets of round, strange, and weird integers, respectively.

   Let $T = \{n \in \mathbf{Z} : n - 1 \in S\}$. Prove that $W = T$.

   Suggestion. Use the method of element-chasing.

5. Suppose A, B, and C are subsets of a universal set U. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (This is one of our distributive laws.)

6. Prove or disprove both of the following. Give a counterexample if the statement is false and a proof via element-chasing if the statement is true.

   (a) Let A, B, and C be sets with $A \subseteq B$ and $B \subseteq C$. Then $A \subseteq C$.

   (b) Let A, B, and C be sets where $A \subseteq B$ and $B \in C$. Then $A \in C$.

7. Prove or disprove: Let A, B, and X be sets. If $X \subseteq A \cap B$, then $X \subseteq A$ and $X \subseteq B$. Give a counterexample if the statement is false and a proof via element-chasing if the statement is true.

8. Suppose A, B, C are subsets of a universal set U.

   Use the "beyond element-chasing method" to prove that $A \cap (A \cap B)^c = A \setminus B$. Indicate whenever you use the Commutative Property, the Associative Property, the Distributive Property, and de Morgan's laws.

9. Suppose A, B, C are subsets of a universal set U.

   Use the "beyond element-chasing method" to prove that $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$. Indicate whenever you use the Commutative Property, the Associative Property, the Distributive Property, and de Morgan's laws.

10. Let A, B, C, D be sets. Prove that

$$(A \setminus B) \times (C \setminus D) = (A \times C) \setminus [(A \times D) \cup (B \times C)].$$