MATH 3100 – Homework #3
posted February 12, 2020; due at the **start of class** on February 19, 2020

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

Required problems

1. §1.4: 23
2. §1.5: 1(a,c,e,g,i,k,m,o)
3. §1.5: 3
4. §1.5: 5; your answer for (a) should use the limit laws, with each step carefully explained as in Example 1.5.6
5. §1.5: 6
6. Let $\{a_n\}$ be the sequence defined recursively by $a_1 = 2$ and
$$a_{n+1} = \frac{1}{2}(a_n + 2/a_n)$$
for each natural number n. (Recall that this is the sequence we introduced in class in order to prove that $\sqrt{2}$ exists as a real number.)
   (a) Show that $1 \leq a_n \leq 2$ for each natural number n. *Hint:* Induction.
   (b) Show that for each natural number n,
$$a_{n+1}^2 - 2 = \frac{1}{4}(a_n - 2/a_n)^2.$$
   (c) Deduce from (b) that $a_n^2 \geq 2$ for every natural number n. *Note:* Make sure your argument covers the case $n = 1$.
   (d) Show that the sequence $\{a_n\}$ is decreasing. The result of part (c) will be useful.
7. §1.6: 2
[We will do some of this in class. Write up and turn in the complete argument, not just the part left to you!]
8. §1.6: 5
9. §1.6: 8
Hint: You may need to use Propositions 1.4.16 and 1.4.17.

Recommended problems (NOT to turn in)

§1.4: 17
§1.5: 7(a), 9, 10, 12
§1.6: 9, 10, 12