Teaching philosophy

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On the face of it, my most influential teacher seems an unlikely role model. Arnold Ross was elderly – already over ninety years old in 1998 when I attended his summer mathematics program for high school students. He had an archaic style of speaking and a penchant for convoluted metaphors. He was hard of hearing; he would ask students to correct his arithmetic mistakes (allegedly made on purpose to help him gauge students' attentiveness), but sometimes not notice when they did. And it seemed like each lecture raised more questions than it answered. Yet when the summer of 1998 came to an end, I realized that over the course of his eight-week program, I had gone from a dabbler impressed by the magic of number theory to a full-fledged apprentice. Much of my teaching philosophy stems from my attempts, over the past dozen years, to put my finger on what made this transformation possible.

J. H. Conway has said that a mathematician is a magician who reveals his secrets. Arnold Ross's secret was, as he put it in his own teaching philosophy statement, "setting the stage for telling flashes of awareness." At Ross's program, this stage-setting was facilitated by an intricate infrastructure. Ross's lectures were supplemented by daily problem sets (Moore-method style), and students at the program worked on these sets back in their Ohio State University dorm rooms, relying on their counselors (normally college students, also living in the dorms) to grade their sets and answer questions. I had my first teaching experience working at the Ross program. Over my four-year tenure there, first as a junior counselor and then as a full counselor, I earned a reputation for certain idiosyncratic responses to students' questions. Whenever I was asked about something for the first time, I would be sure to question the questioner: "Well, what do you think?" Based on the answer, I would try to say *just enough* to put the student back on the the path to discovery. One might expect students to be annoyed by this (and certainly sometimes they proved impatient!), but by the end of the summer, they realized that there was more behind my approach than sadism. I had left enough to do for them to honestly stake a claim on their newfound knowledge.

The situation of the Ross program is far removed from that of the typical classroom, and different methods are required to provoke "telling flashes of awareness". One of my successes in this arena which I am most proud of came when I was teaching Finite Mathematics as a Dartmouth graduate student. To initiate our study of graph theory, I devoted twenty minutes to having my students attempt at the board to find a solution to the (unsolvable!) 'gas-water-electricity' problem. By the end of this, many of them were convinced that $K_{3,3}$ is not planar, without having been told the definition of a "planar graph" or even of "$K_{3,3}$." This made it

easier for them to later understand the definitions in graph theory, and it provided motivation for considering the subject in the first place.

Arnold Ross was keen on emphasizing that mathematics is a language. It is a rare soul who can pick up a language without regularly conversing with others. To ensure that my students have a conversation partner, I make myself accessible outside of the classroom. Whenever I teach, I go out of my way to emphasize to students that the time I schedule for my office hours has been reserved for them, and that they will not be interrupting me if they show up. I have had very positive responses to this invitation; often, enough people show up that we have to move from my office to a place with better seating. When I taught Fundamental Mathematics (an introduction-to-proofs class) at Illinois last Fall, it was necessary to schedule a classroom for the term just for office hours!

I also try to encourage conversation within the classroom. This takes different forms, depending on the flexibility allowed by the syllabus. At the very least, it means that I encourage students to stop me and ask questions about anything I say that doesn't make sense. Often it goes further: for example, when covering enumerative combinatorics in Finite Math, instead of putting the formulas on the board and then presenting a list of amenable problems, I would open the class with problems and as a class we would try to inch our way to a solution. This not only forced the students to put their thoughts into words, but I am convinced that the the progression from concrete to abstract made the material more memorable; rather than memorizing an answer, they could mentally refer back to an entire process.

Not all of my teaching is connected with the classroom. During the summer of 2010, I served as a mentor for two Illinois graduate students as part of the REGS program (Research Experiences for Graduate Students). After a few weeks and just as many false starts, we settled on a project. A short, pretty paper in the *Journal of Number Theory* had caught my eye during my regular journal scanning, and I thought it might be susceptible to generalization. I put on the board two possible theorems that seemed attackable and asked my students to mount an offensive. Advising someone seems to me somewhat akin to describing directions to a place you've never been. Moreover, you may have a vague idea how to get there, but by no means may you confirm your suspicions by going there yourself! To my chagrin, the approach I had in mind for the second theorem was a bust. This didn't stop one of my students from proving it anyway, although he did comment that it seemed remarkably hard! This experience taught me that as an advisor, it's OK not to have all the answers; it's more important to have good questions. Also, one should have faith in one's students. Each week for the rest of the summer, the students would meet with me and explain what they had accomplished. I would offer my thoughts, sometimes suggesting how they might overcome a thorny technical issue, and sometimes proposing that they take the project in a slightly different direction. The summer ended with enough results for a paper, which I helped edit. This paper, which would be the first of their careers, is currently under consideration at a research journal.

As of late last year, my teaching has gone global: The American Mathematical Society published my first book, *Not Always Buried Deep: A Second Course in Elementary Number*

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Theory. This textbook, also suitable for self-study, is something of a letter to my former self; it introduces concepts from my research area (elementary analytic number theory) to undergraduates and others possessing only a modest background. While this sort of number theory is well-represented in research journals, there are few textbook treatments. When writing the book, I made a conscious effort to capture not just the relevant mathematics, but also to get across the way people who work in the area think about these problems and to indicate some of why they find them so fascinating. The feedback from readers has been very positive. I received an e-mail saying the book was being used over Skype for an informal seminar among a few recent Swarthmore graduates: "your text seemed written for us". And one (professional mathematician) reader has labeled me the Lady Gaga of elementary number theory! During Spring of 2010, I was pleased to have the opportunity to teach a graduate course at Illinois based on a draft of the text; almost concurrently, Carl Pomerance used a draft to teach a similar course at Dartmouth College. There is a connection to Ross's program here also; the text began its life (in 2003 or so) as a collection of notes for counselors and junior counselors there.

I have chosen to structure this essay around the influence of attending Arnold Ross's program. Much could also be said about the impact of the many excellent teachers who have crossed my path since. I will close with just one lesson I have learned from them. As mathematicians, there is a great temptation to idealize mathematics as a thing of cold, austere beauty. There is truth to this, but it is just one side of a coin. Much of what makes mathematics enjoyable is intimately tied up with the people who do it; there is immense pleasure both in watching someone discover for herself a well-known theorem, and in listening to an established mathematician lecture on a subject on which she is an expert. This human side of mathematics seems to me to be worth emphasizing while teaching, not only because it is true to how mathematicians experience the world, but because it has an obvious emotional hook with students.

I look forward to a long teaching career and all the lessons I still have to learn from my fellow travelers on the road of mathematics, both students and colleagues.