

## MATH 3100 – Homework #5

posted October 4, 2021; due by 5 PM on October 11, 2021

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

### Required problems

1. §1.7: 7

2. Let  $f$  be a continuous function on a closed interval  $[a, b]$ .

(a) Prove the **Minimum Value Theorem**: There is a  $c \in [a, b]$  with the property that  $f(x) \geq f(c)$  for all  $x \in [a, b]$ .

*Hint:* Apply the Maximum Value Theorem to the function  $-f$ .

(b) Prove that there are real numbers  $m, M$  with  $m \leq M$  such that

$$\{f(x) : x \in [a, b]\} = [m, M].$$

Here  $[m, M]$  means (as usual) the closed interval from  $m$  to  $M$ ; when  $m = M$ , we understand  $[m, M]$  as the set with sole element  $m$ .

*Hint:* Combine the Maximum Value, Minimum Value, and Intermediate Value Theorems.

3. §2.1: 4

*Remark:* Our notes call a sequence  $\{a_k\}$  **summable** if  $\sum_{k=1}^{\infty} a_k$  converges.

4. §2.1: 9

5. §2.1: 13

6. §2.1: 14

7. §2.1: 15

8. §2.2: 1(a,c,d,f,h,j,l)

*Hint:* None of these parts require the integral test!

9. §2.2: 2

*Hint:* First prove that  $a_n^2 \leq a_n$  eventually. Then finish the problem using the eventual comparison test.

10. §2.2: 3

### Recommended problems (NOT to turn in)

§2.1: 1, 2, 3, 5, 6, 8, 10

§2.2: 1(b,e,g,i,k)