

Pointers for writing or rewriting induction proofs

- **Remember that $P(n)$ is a statement, not a number or a function.**

When describing statements, do not use an “=” sign.

$P(n)$ is a statement about the particular number n , not about all natural numbers at once.

Good:

For each natural number n , let $P(n)$ be the statement “ $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.”

For every integer $n \geq 7$, let $P(n)$ be the statement “ $2^n < n!$ ”.

Bad:

Let $P(n) = \frac{n(n+1)}{2}$ for all natural numbers n .

Let $P(n) = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$.

Let $P(n)$ be the statement “ $2 + 4 + 6 + \dots + 2n = n(n + 1)$ for every $n \geq 1$ ”.

- **Be precise about your induction hypothesis!** This includes being careful to specify the nature of the n being considered.

Good:

Assume $P(n)$, where n is a natural number.

Assume $P(n)$, where n is some integer with $n \geq 7$.

Assume $P(1), P(2), \dots, P(n)$ all hold, where n is a natural number.

Assume $P(3), P(4), \dots, P(n)$ all hold, where n is some integer with $n \geq 3$.

Bad:

Assume $P(n)$.

Assuming $P(n)$ says $n > 3$.

Assume $P(1), \dots, P(n)$.

- **The conclusion of the induction step is $P(n+1)$, not $P(n)$.**
- **After completing the induction step, end your proof with the correct conclusion.**

Good:

By the Axiom of Mathematical Induction, $P(n)$ holds for all integers $n \geq 13$.

By the Axiom of Complete Mathematical Induction, $P(n)$ holds for all natural numbers n .

Bad:

Thus, $P(n)$ is true.

- **Write neatly! Many assignments were bordering on illegible this time around.**