MATH 3100 – Homework #7

posted November 4, 2022; due by 5 PM on Friday, November 11

Section and exercise numbers correspond to our course notes.

Required problems

1. Let $\{f_n\}$ be a sequence of functions on A, and let f be a function on A. Suppose that $f_n \to f$ uniformly on A, with the definition of uniform convergence given in class. Prove that the following holds:

$$(\forall \epsilon > 0)\ (\exists N \in \mathbf{N})\ (\forall x \in A)\ (|f_n(x) - f(x)| < \epsilon \text{ whenever } n \geq N).$$

[In fact, this sentence could have been taken as the *definition* of uniform convergence. That is the approach followed in your course notes!]

2. §3.1: 4(a,c,d,e,f)

3. §3.1: 6(a–d)

4. §3.1: 7

5. §3.1: 10

6. §3.1: 11

7. In this exercise we consider the series of functions $\sum_{k=0}^{\infty} x^k$ on $A = (-1, 1)$. Recall from our discussion of power series that this series converges on A with sum function

$$s(x) = \frac{1}{1-x}.$$

   (a) Let s_n be the nth partial sum function. That is, $s_n(x) = \sum_{k=0}^{n} x^k$. Find a simple formula for $s_n(x)$.

   (b) Show that $d(s_n, s) = \infty$ for every n. Deduce that $\sum_{k=0}^{\infty} x^k$ does **not** converge uniformly on $(-1, 1)$.

Recommended problems

§3.1: 4(b,g)

Play with the `geogebra` applets by peressamuel at `https://www.geogebra.org/m/CuPC2QuT` and `https://www.geogebra.org/m/XJ27HcuH`. Make sure you understand how the applets illustrate the claims made there about uniform convergence.