MATH 3100 – Learning objectives to meet for Exam #2

The exam will cover §1.6–§2.2 of the course notes, through the end of the lecture on Monday, March 2. The material from the end of §1.5 (on L'Hôpital's rule, etc.) is not examinable.

What to be able to state

Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- Completeness property of the real numbers
- Cauchy sequence
- Upper bound, least upper bound of a set of real numbers
- Continuous function
- Infinite series, the corresponding sequence of terms
- Sequence of partial sums of an infinite series
- Convergence of an infinite series
- Geometric series

Big theorems

Be familiar with all of the following theorems, making sure that you can give full statements for those not already spelled out below. For results proved in class, describe the components and main ideas of the proof.

- Every sequence has a monotone subsequence.
- Every bounded sequence has a convergent subsequence.
- Cauchy sequences are bounded.
- A sequence converges if and only if it is a Cauchy sequence.
- Every nonempty subset of the real numbers that is bounded above has a least upper bound.
- Intermediate value theorem (both the simpler version appearing as Theorem 1.7.3 and the extension proved as Exercise 1.7.1)
- Maximum value theorem
- kth term test (i.e., Proposition 2.1.10)
- Sum rule and constant multiple rule for series (Proposition 2.1.12)
- Convergence properties of geometric series (Proposition 2.1.14)

- A series with nonnegative terms converges if and only if its sequence of partial sums is bounded above.
- Comparison test
- Eventual comparision test
- Limit comparison test
- Integral comparison test

What to expect on the exam

- At least one problem testing you on a basic definition and requiring you to give a proof using this definition
- At least one multipart problem asking you to test concrete examples of series for convergence/divergence
- At least one problem requiring you to apply the Intermediate Value Theorem

A few extra problems

Look at the extra practice problems from the last two assignments. Here are a few others to consider.

1. Give an example of a sequence $\{a_n\}$ where $a_{n+1} - a_n \to 0$ but $\{a_n\}$ is **not** a Cauchy sequence.

   Hint: A closely-related problem is to find an example of a series where the terms go to zero but the series does not converge.

2. Show that the series
$$\sum_{n=100}^{\infty} \frac{1}{n \cdot \ln n \cdot (\ln \ln n)}$$
   diverges but that
$$\sum_{n=100}^{\infty} \frac{1}{n \cdot \ln n \cdot (\ln \ln n)^2}$$
   converges.

3. Use the idea behind the proof of the integral test to show that
$$\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n} \quad \text{for all } n \in \mathbf{N}.$$
   (You did this before by induction.)

4. Exercise 2.2.1.

5. Exercise 2.2.2.

6. Exercise 2.2.3.