**MATH 4000/6000 – Final Exam review sheet**

The final exam is **cumulative**! It is scheduled for December 6, 2018, from 8 AM – 11 AM in our usual classroom (Boyd 222). This review sheet discusses **only** the material of §5.1; previous topics are discussed in your review sheets for Exams 1–3.

## What to be able to state

### Basic definitions

You should be able to give precise descriptions of all of the following. Assume in what follows that $K$ is a field extension of $F$.

- $\alpha_1, \dots, \alpha_n \in K$ span $K$ over $F$
- $\alpha_1, \dots, \alpha_n \in K$ are linearly independent over $F$
- $\alpha_1, \dots, \alpha_n \in K$ are a basis for $K$ over $F$
- $K$ has finite degree over $F$
- the degree of $K$ over $F$, and the notation $[K : F]$

### Big theorems

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class, describe the components and main ideas of the proof.

- If $K/F$ is an extension of finite degree, then $K/F$ possesses a basis with finitely many elements. Any two bases for $K/F$ the same number of elements. (You do not have to know the proofs of these claims, but you need to know and understand the statements.)
- If $L/K$ and $K/F$ are field extensions, then $[L : F]$ is finite $\iff$ $[L : K]$ and $[K : F]$ are both finite. Moreover, if $[L : F]$ is finite, then $[L : K][K : F] = [L : F]$. This last fact is referred to as *multiplicativity of degree in towers*.
- If $K/F$ is a field extension, and $\alpha \in K$ is the root of an irreducible polynomial $p(x) \in F[x]$ with $\deg p(x) = n$ then $[F[\alpha] : F] = n$. In fact, $1, \alpha, \dots, \alpha^{n-1}$ is a basis for $F[\alpha]$ over $F$.
- If $L/F$ is a finite extension, say $[L : F] = n$, then every $\alpha \in L$ is the root of a nonconstant polynomial in $F[x]$. If $p(x)$ is an irreducible polynomial with $p(\alpha) = 0$, then $\deg p(x)$ divides $n$.

## What to be able to do

You are expected to know how to use the methods described in class to solve problems similar to the following.

§5.1: 11(a,c,f,g,i), 13, 15, 16