Analytic Number Theory (Fall 2018) – Homework #1
posted August 21, 2018; due August 30, 2018

Problems: References are to *Not always buried deep*; "Exercise A.B" means Exercise B at the end of Chapter A. Point values are listed in brackets. You *may* use outside resources, including published papers, but your write-up should mention which references you consulted.

1. [10] Exercise 1.4.

2. [10] Exercise 1.8.

3. [10] Exercise 1.10.

4. [10] Exercise 1.11.

5. [10] Exercise 1.13.

6. [15] Exercises 1.18 and 1.19.

7. [10] Exercise 1.33.

8. [15] We showed in class that for all $x \geq 2$ and all positive integers k,

$$\sum_{p \leq x} \frac{1}{p} \leq k!^{1/k} \cdot \left(1 + \log(x^k)\right)^{1/k}.$$

How sharp an upper bound for $\sum_{p \leq x} \frac{1}{p}$ can you derive from this?