MATH 3100 – Homework #7

posted November 3, 2021; due by 5 PM on Wednesday, November 10

Section and exercise numbers correspond to our course notes.

Required problems

1. Prove the following special case of the squeeze theorem: If $a_n \leq b_n \leq c_n$ for all n, and $\lim a_n = \lim c_n = 0$, then $\lim b_n = 0$. (The purpose of this problem is to give you further practice with the limit definition; the problem itself could have appeared in Chapter 1.)

2. §2.4: 2 [“Find a closed form” means “find a simple formula for the sum”.]

3. §2.4: 3

4. §2.4: 5

5. §2.4: 6

6. §3.1: 4(a,c,d,e,f)

7. Let $\{f_n(x)\}$ be a sequence of functions on A, and let $f(x)$ be a function on A. Suppose that $f_n(x) \to f(x)$ uniformly on A, with the definition of uniform convergence given in class. Prove that the following holds:

$$(\forall \epsilon > 0)\ (\exists N \in \mathbf{N})\ (\forall x \in A)\ (|f_n(x) - f(x)| < \epsilon \text{ whenever } n \geq N).$$

(In fact, this sentence can be taken as the *definition* of uniform convergence. That is the approach followed in your course notes!)

Recommended problems

§2.4: 4
§3.1: 4(b,g)