MATH 595
Advanced Topics in Elementary Number Theory

M/W/F 12:00 PM-12:50 PM, Altgeld 345
Instructor: Paul Pollack, Altgeld 301

Prerequisites:
- Elementary real analysis (at the level of 'baby' Rudin)
- Elementary abstract algebra (a first course in groups, rings, and fields should suffice)
- Elementary number theory (at the level of MATH 453)

Course description:

The purpose of this class is to highlight the beauty and power of elementary number theory.

In a first course in number theory, there are many theorems mentioned but not proved; these often, include Gauss's characterization of the integers which are sums of three squares, Dirichlet's theorem on primes in arithmetic progressions, Waring's assertion on sums of kth powers of integers, the prime number theorem, etc. Many of these problems have solutions which, while intricate, do not require advanced ideas for their solution.

Possible topics we will cover include:

- Euler's prime producing polynomial and unique factorization in $\mathbf{Z}[(1+sqrt(-163))/2]$ (following T. Zaupper and E. Gyarmati)
- Gauss, cyclotomy, and quadratic and cubic reciprocity (following Gauss, Jacobi, and the modern-day mathematician Z.-H. Sun)
- Elementary prime number theorem, at the level of Chebyshev and Mertens
- An elementary proof of Dirichlet's theorem (following H. N. Shapiro and A. O. Gelfond)
- Gauss's characterization of the integers which are sums of three squares (following J. Wojcik)
- The Erdos-Selberg proof of the prime number theorem (following V. Nevanlinna)
- Elementary sieve methods and their applications (following Hooley and Ford & Halberstam)
- Waring's problem on sums of powers (following the approach of D. Hilbert, as modified by F. Dress)
- Perfect numbers (following Erdos, Pomerance, Wirsing and others).

Textbook: While each of these topics has received serious study, many of the arguments we will discuss have never appeared in textbook form. As such, there is no suitable textbook. The instructor is in the process of writing such a book (currently under consideration for publication by Springer-Verlag and the AMS). Handouts from this book will be distributed in class.

Final note:

Please come see me if you have any questions about this course. Shy people can e-mail me at pppollac@illinois.edu