MATH 3200 – Homework #2

posted January 22, 2020; due at the **start of class** on January 29, 2020

All numbering corresponds to the course textbook, A TeXas-Style Introduction to Proof. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.** Assume — unless explicitly told otherwise — that **you are expected to include clear and concise explanations of your reasoning, expressed in complete sentences.**

The following exercises assume you have read Definition 2.27 of the textbook, concerning what it means to say that "m divides n", where m and n are integers. An exercise of the form "Statement X.YY" means you are to prove Statement X.YY, *or* to decide that Statement X.YY is false and disprove it.

1. Statement 2.32.
2. Statement 2.33.
3. Statement 2.37.
4. Exercise 2.47.
5. Prove or disprove: Let a, b, c be integers. If $a \nmid bc$, then $a \nmid b$.
6. Prove or disprove: Let a, b be integers. If ab and $a + b$ are even, then both a and b are even.
7. Prove or disprove: An integer is even if and only if it can be expressed as a sum of two odd integers.

   [Recall from p. 27 that "P if and only if Q" means "P implies Q **and** Q implies P". So to prove the above statement, you would need to show that if x is an even integer, then x is a sum of two odd integers, **and** that if x is a sum of two odd integers, then x is even.]
8. Which integers divide 0? Which integers does 0 divide? Prove your answers are correct.

For the next two exercises, we recall our ASSUMPTIONS/AXIOMS concerning inequalities. For all real numbers a, b, c, we assume:

R0. $1 \neq 0$
R1. If $a > b$ and $b > c$, then $a > c$.
R2. If $a > b$ then $a + c > b + c$.
R3. If $a > b$ and $c > 0$, then $ac > bc$.
R4. If $a > b$ and $c < 0$, then $ac < bc$.
R5. For every real number a, **exactly one** of the following is true: $a > 0$, $a = 0$, or $a < 0$.

9. Let x be a real number. Show that if $x > 0$, then $\frac{1}{x} > 0$. Explicitly say where and how you use each of the rules above.
10. Let x be a real number. Show that if $x \neq 0$, then $x^2 > 0$. Explicitly say where and how you use each of the rules above.