MATH 3100 – Homework #5

posted October 3, 2022; due by 5 PM on October 10, 2021

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

Required problems

1. §1.7: 7
2. Let f be a continuous function on a closed interval $[a, b]$.
   (a) Prove the Minimum Value Theorem: There is a $c \in [a, b]$ with the property that $f(x) \geq f(c)$ for all $x \in [a, b]$.

   Hint: Apply the Maximum Value Theorem to the function $-f$.

   (b) Prove that there are real numbers m, M with $m \leq M$ such that

   $$\{f(x) : x \in [a, b]\} = [m, M].$$

   Here $[m, M]$ means (as usual) the closed interval from m to M; when $m = M$, we understand $[m, M]$ as the set with sole element m.

   Hint: Combine the Maximum Value, Minimum Value, and Intermediate Value Theorems.
3. §2.1: 4

   Remark: Our notes call a sequence $\{a_k\}$ **summable** if $\sum_{k=1}^{\infty} a_k$ converges.
4. §2.1: 9
5. §2.1: 13
6. §2.1: 14
7. §2.1: 15
8. §2.2: 1(a,c,d,f,h,j,l)

   Hint: None of these parts require the integral test!
9. §2.2: 2

   Hint: First prove that $a_n^2 \leq a_n$ eventually. Then finish the problem using the eventual comparison test.
10. §2.2: 3

Recommended problems (NOT to turn in)

§2.1: 1, 2, 3, 5, 6, 8, 10
§2.2: 1(b,e,g,i,k)