MATH 3100 – Course Outline

The following is a topical outline of what was covered this semester. Items indicated with a (*) are theorems we did not prove in class; while you should know these results and how to apply them, you will not be tested on their proofs.

1. Chapter I: Sequences
   (a) Explicit vs. recursive definitions of sequences
   (b) Review of mathematical induction, with applications to recursively defined sequences
   (c) Fundamental concepts associated to sequences: increasing/decreasing, bounded above, bounded below, upper and lower bounds
   (d) Subsequences and their properties
   (e) Definition of convergence, and proving convergence from this definition
   (f) Geometric sequences and the "master theorem"
   (g) Rules for computing limits: sum rule, product rule, quotient rule, constant multiple rule, L'Hôpital's rule (*), and the use of continuity to move limits inside functions
   (h) Every sequence has a monotone subsequence. Every bounded sequence has a convergent subsequence.
   (i) The Completeness Axiom of the real numbers (*)
   (j) Cauchy sequences, and the equivalence between being convergent and being Cauchy
   (k) the Least Upper Bound Property of the real numbers (every nonempty subset of the real numbers that is bounded above has a least upper bound)
   (l) Intermediate Value Theorem
   (m) Maximum Value Theorem
2. Chapter II: Series
   (a) Definition of convergence of an infinite series, in terms of partial sums
   (b) The kth term test for divergence
   (c) Description of when geometric series converge and what they converge to
   (d) Comparison tests: Basic comparison test, eventual comparison test, limit comparison test
   (e) Integral comparison test
   (f) The concept of absolute convergence. Absolute convergence implies convergence.
   (g) Alternating series test
   (h) Ratio test
   (i) Riemann's rearrangement theorem (*)
   (j) The concept of a power series and its associated domain of convergence (d.o.c.)

(k) Application of the ratio test to determine domains of convergence

(l) Theoretical description of the possible domain of convergence of a power series. Definition of the radius of convergence

(m) Finding closed forms for power series based on geometric series

3. Chapter III: Sequences and series of functions

   (a) The concept of pointwise convergence for a sequence of functions

   (b) Bad behavior ("annoyances") of pointwise convergence, as regards continuity and integration

   (c) Distance between functions and the definition of uniform convergence of a sequence of functions

   (d) Uniform convergence implies pointwise convergence, but not vice versa

   (e) Uniform convergence plays nice with continuity and integration

   (f) Series of functions and uniform convergence of series of functions

   (g) The Weierstrass M-test as a sufficient condition for uniform convergence of a series of functions

   (h) If a power series centered at 0 has radius of convergence R, it converges uniformly on $[-r, r]$ for any positive number $r < R$.

   (i) Manipulating power series via integration and differentiation

   (j) Recognizing numerical series as special values of power series

   (k) Definition of Maclaurin series

   (l) Characterization theorem for Maclaurin polynomials

   (m) Rules for computing Maclaurin polynomials: sums, products, and compositions.

   (n) Taylor's theorem (*)

   (o) $\sin x, \cos x, e^x$ are everywhere represented by their Maclaurin series

   (p) Applications of Taylor's theorem

Not a bad semester's work, eh?