MATH 4000/6000 – Final Exam review sheet

The final exam is **cumulative**! It is scheduled for May 8, from 8 AM – 11 AM in our usual classroom. This review sheet discusses **only** the material of §5.1; previous topics are discussed in your review sheets for Exams 1–3.

What to be able to state

Basic definitions

You should be able to give precise descriptions of all of the following. Assume in what follows that K is a field extension of F.

- $\alpha_1, \ldots, \alpha_n \in K$ span K over F
- $\alpha_1, \ldots, \alpha_n \in K$ are linearly independent over F
- $\alpha_1, \ldots, \alpha_n \in K$ are a basis for K over F
- K has finite degree over F
- the degree of K over F, and the notation $[K : F]$

Big theorems

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class, describe the components and main ideas of the proof.

- If K is an extension of F of finite degree, then K has a basis over F. Any two bases for K over F have the same number of elements. (You do not have to know the proofs of these claims, but you need to know and understand the statements.)
- If L is a field extension of K, and K is a field extension of F, then $[L : F]$ is finite $\Longleftrightarrow$ $[L : K]$ and $[K : F]$ are both finite. Moreover, if $[L : F]$ is finite, then $[L : K][K : F] = [L : F]$. This last fact is referred to as *multiplicativity of degree in towers.*
- If K is a field extension of F, and $\alpha \in K$ is a root of an irreducible polynomial $p(x) \in F[x]$ with $\deg p(x) = n$ then $[F[\alpha] : F] = n$. In fact, $1, \alpha, \ldots, \alpha^{n-1}$ is a basis for $F[\alpha]$ over F.
- If L is a finite extension of F, say $[L : F] = n$, then every $\alpha \in L$ is the root of a nonconstant polynomial in $F[x]$. If $p(x)$ is an irreducible polynomial with $p(\alpha) = 0$, then $\deg p(x)$ divides n.

What to be able to do

You are expected to know how to solve problems resembling the examples worked in class, as well as ones similar to the following.

§5.1: 11(a,c,f,g,i), 13, 15, 16