**MATH 4400 – Learning objectives to meet for the Final Exam**

The final exam is cumulative and will cover all of the material in the course up to and including the material on Farey sequences and Diophantine approximation. The application to square-triangular numbers and Pell's equation is not examinable.

This review sheet covers only topics not on previous exams. Please refer to the review sheets for Exams #1 and #2 for earlier material.

## What to be able to state

### Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- multiplicative arithmetic function
- the functions $\phi$, $\sigma$, $\tau$, and $\mu$
- perfect number
- the Farey sequence $\mathfrak{F}_n$ of order $n$

### Big theorems

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class or on homework, describe the components and main ideas of the proof.

- If $f(n)$ is multiplicative, then so is $g(n) = \sum_{d|n} f(d)$.
- If $g(n) = \sum_{d|n} f(d)$ is multiplicative, then $f(n)$ is multiplicative.
- Euclid's perfect number theorem: If $2^n - 1$ is prime, then $2^{n-1}(2^n - 1)$ is perfect.
- Euler's perfect number theorem: If $N$ is even and perfect, then $N = 2^{n-1}(2^n - 1)$ where $2^n - 1$ is prime.
- Möbius inversion formula
- formula for $\mu(n)$ in terms of the prime factorization of $n$
- If $\frac{a}{b} < \frac{c}{d}$ are consecutive elements in a Farey sequence, then $bc - ad = 1$.
- If $\frac{a}{b} < \frac{c}{d}$ are consecutive elements in a Farey sequence, then the first element to come between them is $\frac{a+c}{b+d}$, and this is the unique fraction between them of denominator $b + d$.
- Let $\alpha$ be an irrational number. For every $N$, there is a fraction $\frac{a}{q} \in \mathfrak{F}_N$ with $|\frac{a}{q} - \alpha| < \frac{1}{q(N+1)}$.
- Let $\alpha$ be an irrational number. For infinitely many positive integers $q$, there is an integer $p$ with $|\frac{p}{q} - \alpha| < \frac{1}{q^2}$.

### What to be able to do

You can expect $\approx 8$ problems on the exam. The first problem — worth $\approx 25\%$ of the points — is purely computational. You are expected to be able to do all of the following computations (representative but not comprehensive):

- Find values of multiplicative functions given the prime factorization of $n$
- Apply the Möbius inversion formula: If $g(n) = \sum_{d|n} f(d)$, and $g(n)$ is given, compute values of $f(n)$