Analytic Number Theory (Fall 2018) – Homework #4
posted November 18, 2018; due December 12, 2018

Problems: References are to *Not always buried deep*; "Exercise A.B" means Exercise B at the end of Chapter A. Point values are listed in brackets. You *may* use outside resources, including published papers, but your write-up should mention which references you consulted.

For most of the problems on this homework set, Brun's sieve or Brun's pure sieve will suffice. But for Problem 6.21, you will want to keep in mind the result we referred to as **Brun's upper bound sieve**:

Theorem. *Fix a positive integer k. Let $x \geq 1$, and suppose that for all primes $p \leq x$, we are given a set $\mathcal{E}_p$ that is a union of $\nu(p)$ residue classes modulo p, where $\nu(p) \leq k$. Then*

$$\# \bigcap_{p \leq x} ([1, x] \setminus \mathcal{E}_p) \ll_k x \prod_{p \leq x} \left(1 - \frac{\nu(p)}{p}\right).$$

1. [10] 6.1

2. [15] 6.5

3. [10] 6.6

4. [10] 6.8

5. [10] 6.14

6. [10] 6.21

7. [10] 6.30

8. [10] 6.31 (look at 6.30 first)