MATH 3100 – Learning objectives to meet for Exam #1

The exam will cover §1.1–§1.5 of the course notes, through the example concluding at the top of p. 60. This should essentially coincide with what is covered by the end of class on Monday, September 13.

What to be able to state

Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- sequence, terms of a sequence, graph of a sequence
- increasing, decreasing, monotone, and the "strictly" and "eventually" variants
- bounded above, bounded below, upper bound, lower bound, bounded
- subsequence
- $\{a_n\}$ converges to L, where L is a real number
- $\{a_n\}$ diverges
- geometric sequence
- continuous function

Big theorems

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class, describe the components and main ideas of the proof.

- Principal of mathematical induction, complete mathematical induction
- A convergent sequence has a **unique** limit
- Convergent sequences are bounded
- Subsequences of convergent sequences converge to the original limit
- Behavior of geometric sequences (Proposition 1.4.15)
- If $\{a_n\}$ is bounded above by U and $a_n \to L$, then $L \leq U$
- (Bounded) $\cdot$ (going to 0) goes to 0
- Sum rule for limits, product rule for limits, quotient rule for limits
- Continuous functions "commute" with limits (Proposition 1.5.10)

What to expect on the exam

You can expect 5 questions on the exam, including

- one problem testing mathematical induction
- one problem testing your ability to compute the limit of a specific sequence directly from the ϵ-N definition

The rest of the exam is designed to test your comfort level working with the basic definitions. I am not interested in having you regurgitate proofs of results from the notes; I want to know if you have internalized the ideas enough to solve similar problems.

Sample problems

1. Prove that $\lim_{n\to\infty} \frac{3n^2}{n^2+n-5} = 3$ directly from the definition of a limit.

2. (a) What does it mean to say a sequence is **bounded above**? **Bounded below**? **Bounded**?

   (b) Suppose that $\{a_n\}$ and $\{b_n\}$ are bounded above. Define a new sequence $\{c_n\}$ by $c_n = a_n$ for $n \leq 100$ and $c_n = b_n$ for $n > 100$. Give a careful proof that $\{c_n\}$ is bounded above.

3. Determine, with brief explanations, each of the following limits. Do **not** use the limit definition; rather, use known limits and limit rules established in class. For each part, explain which rules you use.

   (a) $\lim_{n\to\infty} \frac{n}{n+1}$.

   (b) $\lim_{n\to\infty}(-2/3)^n \cos(n)$.

4. Assuming that $a_n \to 2$, give a direct proof from the limit definition that $\lim_{n\to\infty}(3a_n - 1) = 5$. You may **not** use the product or sum rule.

5. Let $\{a_n\}$ be a sequence.

   (a) What does it mean to say that $\{a_n\}$ is **eventually nonnegative** ?

   (b) Suppose $\{a_n\}$ is eventually nonnegative. Suppose also that $\{a_n\}$ converges to the real number L. Prove that $L \geq 0$.