MATH 3200 – Learning objectives to meet for Exam #2

The exam will cover Chapters 3 and 4 of our textbook, discussing induction and set theory, respectively. While you *are* responsible for reading the book, the exam will not ask about definitions or concepts not covered in class or on homework. In particular, sections 4.8 and 4.9 are not examinable.

Due to the special circumstances we find ourselves in, the exam will **not be timed or proctored**. The exam will also be **open book/open notes/open HW solutions**. However, you **may not** discuss the exam questions with your others, and you **may not** look up answers in outside sources.

You will have 24 hours to turn in the exam, starting from Friday, 12 PM Eastern Time. The exam will be distributed by email; **you will need to scan your solutions and upload them through Gradescope**. I will send more detailed instructions about exam submissions later this week; note that you will have to format your submissions in a special way (for instance, writing each solution on a separate page, and putting your name and student ID at the top of each page).

What to be able to state

Basic definitions

Be able to give concise, complete, and precise definitions of each of the following terms.

- Axiom of Mathematical Induction
- Axiom of Strong Induction
- basic set definitions: what it means for two sets to be **equal**, what it means for one set to be a **subset** of another, and what it means for one set to be a **proper subset** of another; also, what it means for two sets to be **disjoint**
- set operations: intersection, union, complement, set difference; you are also expected to know the symbols denoting these operations
- Commutative Property (Axiom 4.64), Associative Property (Statement 4.65), Distributive Property (Statement 4.67), and de Morgan's laws (Statement 4.69)

What to be able to do

You may be called on to perform any or all of the following tasks.

- Use the method of induction, or the method of strong induction, to prove mathematical statements. Make sure to follow the instructions in the Induction Pointers document (which you may refer to during the exam).
- Recognize the output of basic set operations (as in HW #4, problems 1, 3)
- Prove set equalities, or set containments, using the method of element chasing (as on HW#4, problems 4, 6, 7, 8, and 11)

- Establish set equalities using the "beyond element chasing" method (as on HW #4, problems 9 and 10).

  In "beyond element chasing" proofs, you must label whenever you use the Commutative Property, Associative Property, Distributive Property, or de Morgan's laws. However, you do not have to label your use of simple set properties. Here "simple set properties" includes the Transitive Property (Statement 4.54), the fact that $A \setminus B = A \cap B^c$, and all of the items appearing under Exercise 4.61.

What to expect on the exam

You can expect 4 or 5 (possibly multi-part) questions on the exam. These will include:

- a proof by induction
- a proof by strong induction
- at least one problem asking you to apply the element chasing method to establish a set equality or containment (possibly in the style of Problem #8 on HW 4)

Extra practice problems

1. Prove that $1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$ for all natural numbers n.
2. Prove that $(1-\frac{1}{2})(1-\frac{1}{3})\cdots(1-\frac{1}{n}) = \frac{1}{n}$ for all integers $n \geq 2$.
3. Let $d_0 = 2, d_1 = 5$, and define
$$d_{n+1} = 5d_n - 6d_{n-1} \quad \text{for each natural number } n.$$
   Prove that $d_n = 2^n + 3^n$ for every integer $n \geq 0$.
4. Recall that the Fibonacci sequence is defined by $F_1 = F_2 = 1$, and
$$F_{n+1} = F_n + F_{n-1} \quad \text{for all integers } n \geq 2.$$
   Prove that for every natural number n,
$$F_{n+3} = 2F_{n+1} + F_n.$$
5. Show that $\{12k + 10 : k \in \mathbb{Z}\} \subseteq \{4k + 2 : k \in \mathbb{Z}\}$. Are these two sets equal? Justify your answer.
6. Let A, B be sets. Prove that $A \cup B = B$ if and only if $A \subseteq B$. Remember that "if and only if" means you have to prove two things: If $A \cup B = B$, then $A \subseteq B$, and if $A \subseteq B$, then $A \cup B = B$.
7. Use the method of element chasing to prove the following: Let A and B be sets satisfying $A \times \mathbb{Z} \subseteq B \times \mathbb{Z}$. Then $A \subseteq B$.

   [Remember that $\mathbb{Z}$ denotes the set of integers.]
8. Let A and B be sets with $A \times \mathbb{Z} \subseteq \mathbb{Z} \times B$. Prove: $A \subseteq B$.

9. Let A, B, C be sets. Prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

10. Use the "beyond element chasing" method to prove the following: If A, B, C are three sets, all subsets of a universal set U, then

$$(A \cup B) \setminus (A \cup C) = B \setminus (A \cup C).$$