MATH 3100 – Homework #8

posted November 12, 2021; due by 5 PM on Friday, November 19

Section and exercise numbers correspond to our course notes.

Required problems

1. Recall our Key Lemma on convergence of power series: *If $\sum_{k=0}^{\infty} a_k x^k$ converges for the real number $x = x_0$, then it converges absolutely for all x with $|x| < |x_0|$.* This lemma was used to characterize all possibilities for the domain of convergence of a power series centered at 0. In class, I left one case of the characterization as an exercise for you. This is that exercise!

   Suppose that $\sum_{k=0}^{\infty} a_k x^k$ is a power series centered at 0 with domain of convergence D. Suppose D is bounded above and let $R = \text{lub } D$. Assume also that $R > 0$.

   (a) Using the key lemma, show that $\sum_{k=0}^{\infty} a_k x^k$ converges for every real number x with $|x| < R$.

   (b) Using the key lemma, show that $\sum_{k=0}^{\infty} a_k x^k$ diverges for every real number x with $|x| > R$.

   Hence, the domain of convergence is one of $[-R, R]$, $(-R, R)$, $(-R, R]$, or $[-R, R)$.

2. §3.1: 6(a–d)

3. §3.1: 7

4. §3.1: 10

5. §3.1: 11

6. Suppose $\sum_{k=0}^{\infty} a_k x^k$ is a power series centered at 0 with radius of convergence 1. Prove that the series $\sum_{k=1}^{\infty} \frac{a_k}{k^2}(-2)^k$ diverges.

   Hint. Proceed by contradiction. Assuming convergence, argue that the original power series must converge when $x = 3/2$, contradicting that its radius of convergence is 1. It might help to look back at the proof of the Key Lemma. Don't assume for this problem that a power series and its derivative always have the same radius of convergence.

Recommended problems

Play with the `geogebra` applets by peressamuel at `https://www.geogebra.org/m/CuPC2QuT` and `https://www.geogebra.org/m/XJ27HcuH`. Make sure you understand how the applets illustrate the claims made there about uniform convergence.