Name: ____________________

MATH 3100 Exam 2 extra credit

October 18, 2021

Instructions: You may choose up to three questions for me to grade. **Indicate which questions you have chosen by circling their numbers below, in the "Question column".** If you indicate more than 3 problems, I will grade (only) the first three. You will receive the higher of the two scores (original submission vs. replacement submission) on each problem.

You may use your course notes as well as the course textbook. You may also consult old homework solutions. However, you may **not** collaborate with anyone on these problems or look up answers on the internet. **Getting outside help will be considered a violation of the academic honesty policy.**

Turn in the exam by 5 PM on Monday, November 1. **You must include your original exam with your resubmission.**

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	20	
2	12	
3	16	
4	16	
5	16	
Total:	80	

1. Which of the following series converge or diverge? You may assume any results discussed in class. If using a convergence test, explain *briefly* which one and how it applies.

   (a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2021n+1}$.

   (b) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+1}}$.

(c) [5 points] $\displaystyle\sum_{n=1}^{\infty}(-1)^n \arctan(n)/n^2$.

(d) [5 points] $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{e^{n^3}}$.

2. [12 points] Let $\{a_n\}$ be a Cauchy sequence. Define $b_n = 3a_n + 2$, for each $n \in \mathbb{N}$. Prove, from the definition of a Cauchy sequence, that $\{b_n\}$ is Cauchy. You may not assume that Cauchy sequences converge or vice versa.

3. Let $a_1 = 1$, and let $a_{n+1} = 2 + \frac{a_n}{2}$ for all natural numbers n.

   (a) [6 points] Prove that $0 \leq a_n \leq 4$ for all $n \in \mathbb{N}$.

   (b) [6 points] Prove that $\{a_n\}$ is an increasing sequence.

   (c) [4 points] Explain how you know the sequence $\{a_n\}$ converges (2 pts). What is its limit (2 pts)?

4. [16 points] Recall that a sequence $\{x_n\}$ diverges to infinity if, for every real number Z, there is an $N \in \mathbb{N}$ such that $x_n > Z$ whenever $n \geq N$.

   Suppose that $\sum_{k=1}^{\infty} a_k$ is a series of nonnegative terms that diverges. Give a careful proof that the corresponding sequence $\{s_n\}$ of partial sums diverges to infinity.

5. [16 points] Using the reasoning behind the proof of the integral test, prove that
$$\sum_{n=101}^{1000} \frac{1}{n^3} < \frac{1}{20000}.$$
If you use a picture as part of your proof, make sure that you explain what the picture shows.