Analytic Number Theory (Fall 2018) – Homework #3
posted October 14, 2018; due October 25, 2018

Problems: References are to *Not always buried deep*; "Exercise A.B" means Exercise B at the end of Chapter A. Point values are listed in brackets. You *may* use outside resources, including published papers, but your write-up should mention which references you consulted.

1. [10] Exercise 4.11
2. [10] Exercise 4.13
3. [15] Exercise 4.14
4. [20] Exercise 4.20
5. [5] Exercise 4.24
6. [5] Exercise 4.26
7. [10] Let $f(x)$ be a continuous, positive-valued function defined and decreasing for $x \geq 1$. (Do **not** assume that $f(x)$ is differentiable.) Show that there is a constant γ_f such that, for all positive integers N,

$$-f(N+1) \leq \sum_{n \leq N} f(n) - \left(\int_1^{N+1} f(t)\, dt + \gamma_f \right) \leq 0.$$

[From this, one can deduce the theorem proved in class proved in class about the partial sums of the harmonic series, as well as the estimate for $\sum_{n \leq x} \frac{1}{\sqrt{n}}$ used in our proof of Dirichlet's theorem.]