Mode Matching Technique

With 2 circular cross-section waveguides

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Field distribution of one circular cross-section waveguide

- Dimensions and properties:
  - Radius = 2.03 [cm]
  - Relative Permittivity $\epsilon_r = 1$
  - Relative Permeability $\mu_r = 1$
  - Covered with PEC walls

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Field distribution of one circular cross-section waveguide (2)

$TE^z \quad modes$

$$E_\rho = -\frac{1}{\epsilon\rho}\frac{\partial F_z}{\partial\phi} = -A_{mn}\frac{m}{\epsilon\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi)+D_2\cos(m\phi)]e^{-j\beta_z z}$$

$$E_\phi = \frac{1}{\epsilon}\frac{\partial F_z}{\partial\rho} = A_{mn}\frac{\beta_\rho}{\epsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

$$H_\rho = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 F_z}{\partial\rho\partial z} = -A_{mn}\frac{\beta_\rho\beta_z}{\omega\mu\epsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

$$H_\phi = -j\frac{1}{\omega\mu\epsilon}\frac{1}{\rho}\frac{\partial^2 F_z}{\partial\phi\partial z} = -A_{mn}\frac{m\beta_z}{\omega\mu\epsilon}\frac{1}{\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi)+D_2\cos(m\phi)]e^{-j\beta_z z}$$

$$H_z = -j\frac{1}{\omega\mu\epsilon}\left(\frac{\partial^2}{\partial z^2}+\beta^2\right)F_z = -jA_{mn}\frac{\beta_\rho^2}{\omega\mu\epsilon}J_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

$TM^z \quad modes$

$$H_\rho = \frac{1}{\mu\rho}\frac{\partial A_z}{\partial\phi} = B_{mn}\frac{m}{\mu\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi)+D_2\cos(m\phi)]e^{-j\beta_z z}$$

$$H_\phi = \frac{1}{\mu}\frac{\partial A_z}{\partial\rho} = -B_{mn}\frac{\beta_\rho}{\mu}J'_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

$$E_\rho = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 A_z}{\partial\rho\partial z} = -B_{mn}\frac{\beta_\rho\beta_z}{\omega\mu\epsilon}J'_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

$$E_\phi = -j\frac{1}{\omega\mu\epsilon}\frac{1}{\rho}\frac{\partial^2 A_z}{\partial\phi\partial z} = -B_{mn}\frac{m\beta_z}{\omega\mu\epsilon}\frac{1}{\rho}J_m(\beta_\rho\rho)[-C_2\sin(m\phi)+D_2\cos(m\phi)]e^{-j\beta_z z}$$

$$E_z = -j\frac{1}{\omega\mu\epsilon}\left(\frac{\partial^2}{\partial z^2}+\beta^2\right)A_z = -jB_{mn}\frac{\beta_\rho^2}{\omega\mu\epsilon}J_m(\beta_\rho\rho)[C_2\cos(m\phi)+D_2\sin(m\phi)]e^{-j\beta_z z}$$

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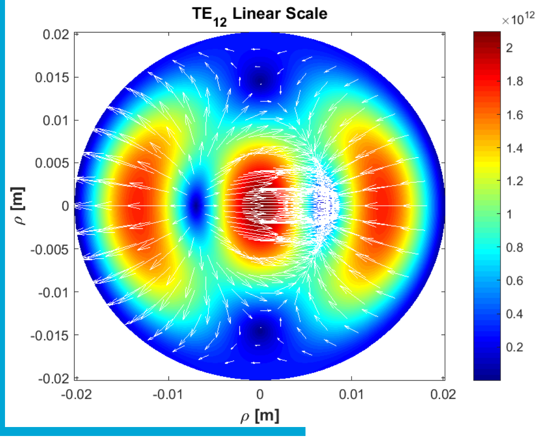
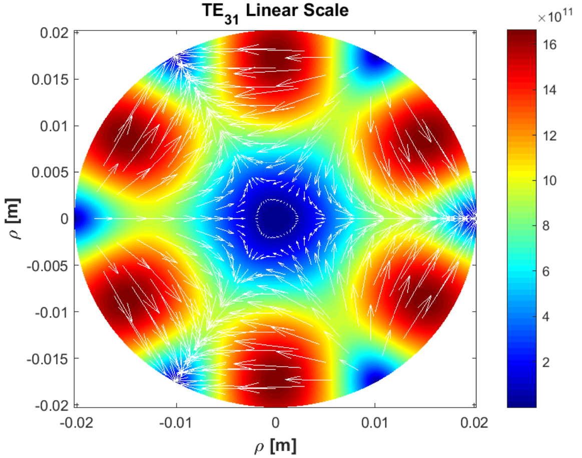
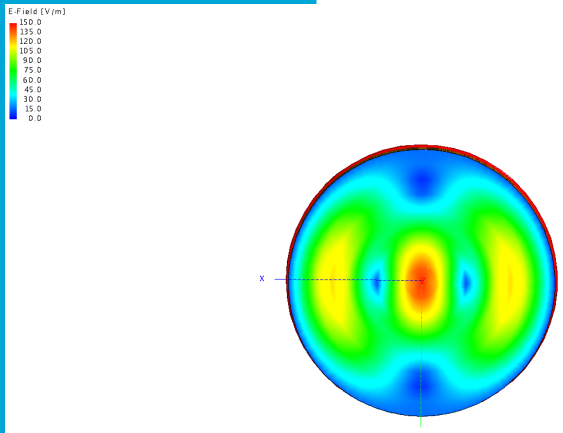
Field distribution of one circular cross-section waveguide (3) Results



TE_{12}
MATLAB

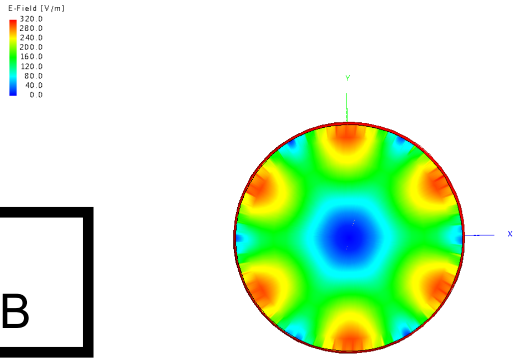
TE_{31}
MATLAB





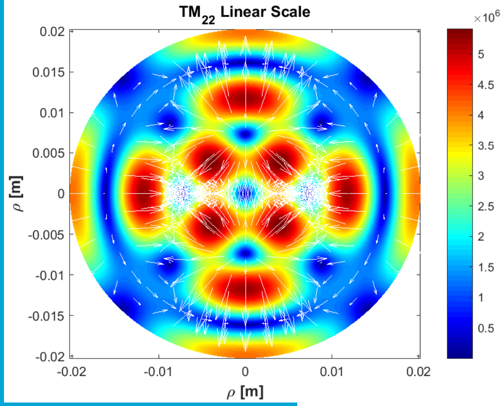
TE_{12}
Feko
2018

TE_{31}
Feko
2018



Surface: E field
Quiver: H field For MATLAB

Field distribution of one circular cross-section waveguide (4) Results

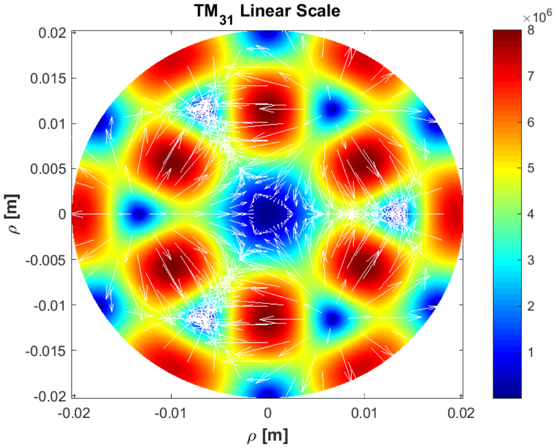


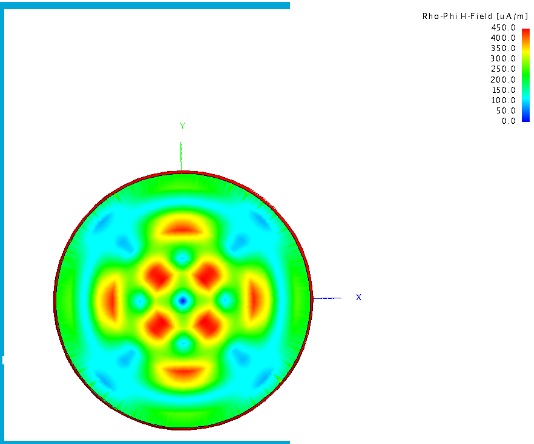
TM_{22}

MATLAB

TM_{22}

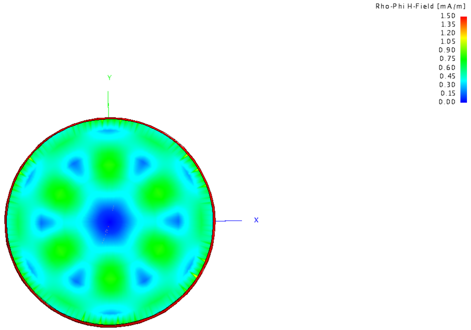
MATLAB





TM_{22}
Feko
2018

TM_{31}
Feko
2018



Surface: H field
Quiver: E field For MATLAB

Normalization Constant - (1)

Normalization constant for a single circular cross section waveguide (Q_n):

$$\iint_{A_{area}} (\overrightarrow{E}_n^{area} \times \overrightarrow{H}_m^{area}) \,.\, \hat{z} dS = Q_n^{area}\delta_{nm}, \quad \delta_{nm}(m=n) = 1, \quad 0 \quad otherwise$$

$$\Longrightarrow \quad Q_n = \iint_{A_{area}} [\overrightarrow{E}_{\rho n}^{area}\overrightarrow{H}_{\phi m}^{area} - \overrightarrow{E}_{\phi n}^{area}\overrightarrow{H}_{\rho m}^{area}]\rho d\rho d\phi$$

$$\Longrightarrow \quad Q_n = \iint_{A_{area}} K_{mn}\left[\frac{m^2}{\rho^2}J_m^2(\beta_{\rho(m,n)}\rho)\sin^2(m\phi) + \beta_{\rho(m,n)}^2 J_m'^2(\beta_{\rho(m,n)}\rho)\right]\rho d\rho d\phi$$

Where: $$K_{mnTE} = \frac{A_{mn}^2\beta_{z,mn}C_2^2}{\omega\mu\epsilon^2} \qquad K_{mnTM} = \frac{B_{mn}^2\beta_{z,mn}C_2^2}{\omega\epsilon\mu^2}$$

Normalization Constant - (3)

Normalization constant (Using some Bessel Function properties):

$$Q_n^{area}\delta_{nm} = K_{mn}\frac{\beta^2_{\rho(m,n)}}{4}\left[(I_A + I_C)(I_{sin} + I_{cos}) + 2I_B(I_{sin} - I_{cos})\right]$$

Where:

$$I_A = \int_0^r J^2_{m-1}(\beta_{\rho(m,n)}\rho)\rho d\rho \qquad I_C = \int_0^r J^2_{m+1}(\beta_{\rho(m,n)}\rho)\rho d\rho$$

$$I_B = \int_0^r J_{m-1}(\beta_{\rho(m,n)}\rho)J_{m+1}(\beta_{\rho(m,n)}\rho)\rho d\rho$$

$$I_{sin} = \int_0^{2\pi} sin^2(m\phi)d\phi = \pi$$

$$I_{cos} = \int_0^{2\pi} cos^2(m\phi)d\phi = \pi$$

Normalization constant (simplified):

$$Q_n^{area}\delta_{nm} = K_{mn(TE/TM)}\frac{\beta^2_{\rho(m,n)}}{4}\left[(I_A + I_C)(I_{sin} + I_{cos})\right]$$

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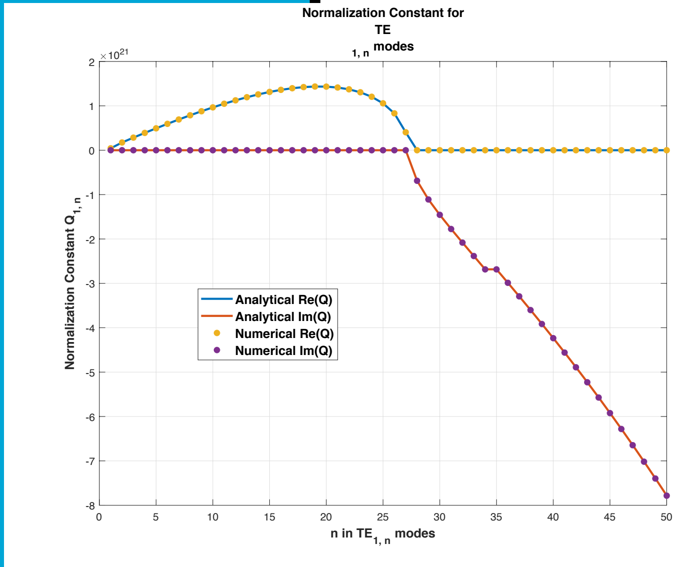
Normalization Constant - (2)

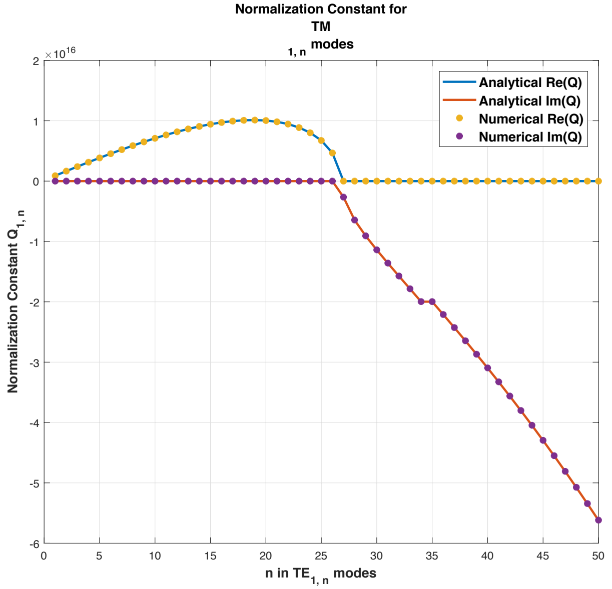
Solution to the integrals (Lommel's Integrals) $I_A \quad and \quad I_C$

$$\int_0^a J_\nu^2(\beta_{\rho(m,n)}\rho)\rho d\rho = \frac{1}{2}a^2(J_\nu(\beta_{\rho(m,n)}a)^2 - J_{\nu-1}(\beta_{\rho(m,n)}a)J_{\nu+1}(\beta_{\rho(m,n)}a));$$



TE (1, n) modes at 200 GHz MATLAB

TM(1, n) modes at 200 GHz MATLAB



Inner Cross Product (2 Waveguides) - (1)

Inner cross product (X) : (Between waveguide P and waveguide R)

$$X = \iint_{A_R} (\vec{E}^R_{pm,pn} \times \vec{H}^P_{rm,rn}) \,.\, \hat{z} dS = (Q^R_{rm})^{\frac{1}{2}} (Z^R_{rm})^{\frac{1}{2}} \chi_{pr} (Y^P_{pm})^{\frac{1}{2}} (Q^P_{pm})^{\frac{1}{2}}$$

Where χ_{pr} is a frequency dependent term.

$$\chi_{pr} = \iint_{A_R} (\vec{\Phi}^R_{E_{pm,pn}} \times \vec{\Phi}^P_{H_{rm,rn}}) \,.\, \hat{z} dS = \iint_{A_R} (\vec{\Phi}^R_{E_{pm,pn}} \,.\, \vec{\Phi}^P_{E_{rm,rn}}) dS = \iint_{A_R} (\vec{\Phi}^R_{H_{pm,pn}} \,.\, \vec{\Phi}^P_{H_{rm,rn}}) dS$$

Where:

$$\vec{\Phi}^{TE}_{E_{m,n}} = \nabla_t \Phi^{TE}_{m,n} \times \hat{z}$$

$$\vec{\Phi}^{TE}_{H_{m,n}} = \nabla_t \Phi^{TE}_{m,n}$$

$$\vec{\Phi}^{TM}_{H_{m,n}} = \hat{z} \times \nabla_t \Phi^{TE}_{m,n}$$

$$\vec{\Phi}^{TM}_{E_{m,n}} = \nabla_t \Phi^{TM}_{m,n}$$

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Inner Cross Product (2 Waveguides) - (2)

Scalar potentials:

$$\Phi_{m,n}^{TE} = (N_{mn}^{TE})^{\frac{1}{2}} J_m(\beta_{\rho,mn}\rho)\cos(m\phi) \quad \Phi_{m,n}^{TM} = (N_{mn}^{TM})^{\frac{1}{2}} J_m(\beta_{\rho,mn}\rho)\cos(m\phi)$$

Where:

$$N_{mn}^{TE} = \frac{1}{|\epsilon\frac{\pi}{2}((\beta_{\rho,mn}r)^2 - m^2)J_m^2(\beta_{\rho,mn}r)|}$$

$$N_{mn}^{TM} = \frac{1}{|\epsilon\frac{\pi}{2}(\beta_{\rho,mn}r)^2 J_m'^2(\beta_{\rho,mn}r)|}$$

Inner Cross Product (2 Waveguides) - (3)

$\chi_{m,n}$ For different configurations:

$\chi_{m,n}$	TE_P	TM_P
TE_R	$\iint_{A_R} \nabla_t \Phi_R \cdot \nabla_t \Phi_P dS$	$\iint_{A_R} (\nabla_t \Phi_R \times \nabla_t \Phi_P) \cdot \hat{z} dS$
TM_R	0	$\iint_{A_R} \nabla_t \Phi_R \cdot \nabla_t \Phi_P dS$

Inner Cross Product (2 Waveguides) - (4)

Analytical expressions for $\chi_{m,n}$ for the combination of TE/TE or TM/TM:

$$\chi_{pr} = \iint_{A_R} \nabla_t \Phi_R \,.\, \nabla_t \Phi_P dS = (N^{TE/TM}_{(pm,pn)} N^{TE/TM}_{(rm,rn)})^{\frac{1}{2}} \frac{\beta_{\rho(pm,pn)}\beta_{\rho(rm,rn)}}{4} \left[(I_A + I_D)(I_{cos} + I_{sin}) \right]$$

Where:

$$I_A = \int_0^r J_{pm-1}(\beta_{\rho(pm,pn)}\rho) J_{rm-1}(\beta_{\rho(rm,rn)}\rho)\rho d\rho$$

$$I_D = \int_0^r J_{pm+1}(\beta_{\rho(pm,pn)}\rho) J_{rm+1}(\beta_{\rho(rm,rn)}\rho)\rho d\rho$$

$$I_{sin} = \int_0^{2\pi} sin(pm\phi)sin(rm\phi)d\phi = \pi, \quad pm = rm$$

$$I_{cos} = \int_0^{2\pi} cos(pm\phi)cos(rm\phi)d\phi = \pi, \quad pm = rm$$

I_A And. I_D Can be solved by the following Lommel's integral:

$$\int_0^r J_\nu(\beta_\nu \rho) J_\nu(\beta_\mu \rho)\rho d\rho = \frac{r}{(\beta_\nu^2 - \beta_\mu^2)} \Big(-\beta_\nu J_{\nu-1}(\beta_\nu r) J_\nu(\beta_\mu r) + \beta_\mu J_\nu(\beta_\nu r) J_{\nu-1}(\beta_\nu r) \Big)$$

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Mode Matching (2 Waveguides)

Boundary conditions at the junction (z = 0):

Electric field boundary condition:

$$Q_P(a_P + b_P) = X^t(a_R + b_R)$$

Magnetic field boundary condition:

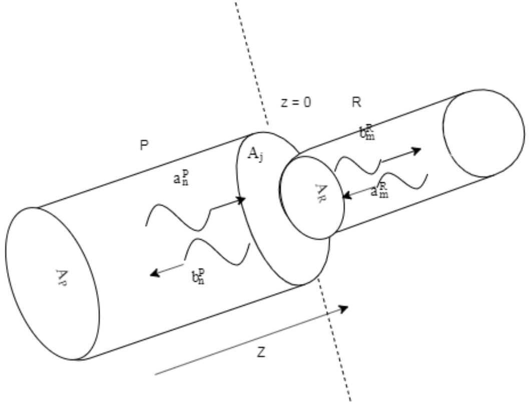
$$Q_R(b_R - a_R) = X(a_P - b_P)$$

General Scattering matrix:

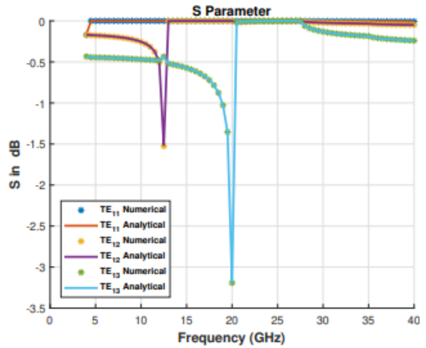
$$GSM = \begin{bmatrix} Q_P^{-1}X^tFX - I_P & Q_P^{-1}X^tFQ_R \\ FX & FQ_R - I_R \end{bmatrix}$$

Where:

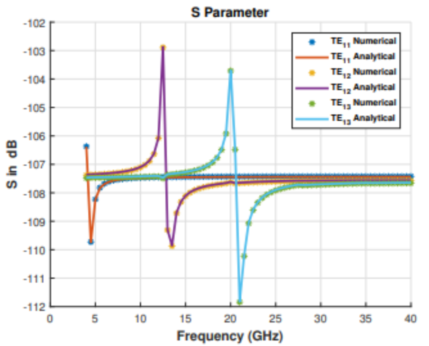
$$F = 2(Q_R + XQ_P^{-1}X^t)^{-1}$$



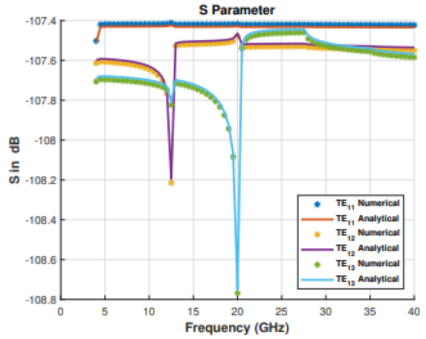
Results: MATLAB (TE) - (1)



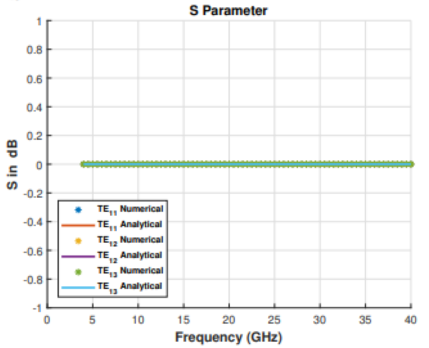
(a) S_{pp} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}



(b) S_{pr} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}

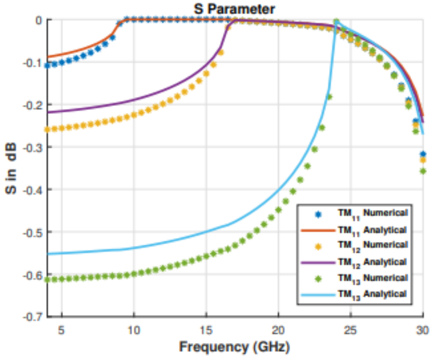


(c) S_{rp} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}

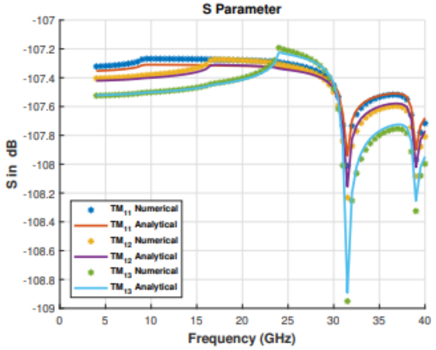


(d) S_{rr} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}

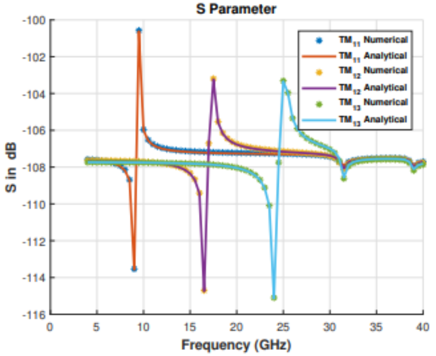
Results: MATLAB (TM) - (2)



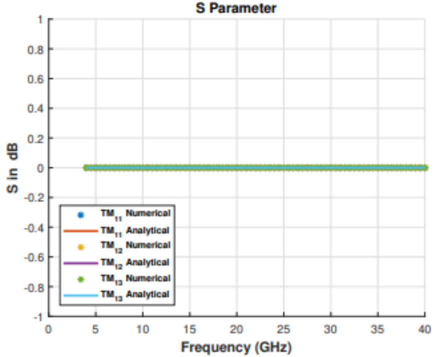
(a) S_{pp} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}



(b) S_{pr} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}

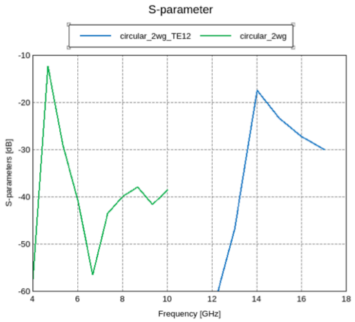


(c) S_{rp} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}

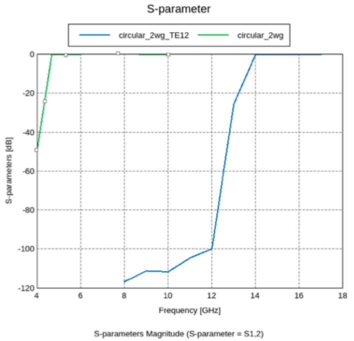


(d) S_{rr} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}

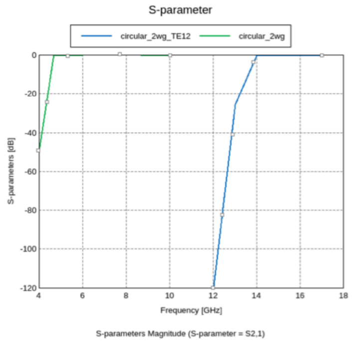
Results: FEKO(TE) - (1)



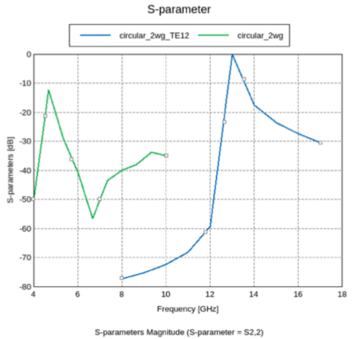
(a) S_{pp} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}



(b) S_{pr} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}



(c) S_{rp} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}



(d) S_{rr} for coupling of modes TE_{11} with TE_{11}, TE_{12} with TE_{12} and TE_{13} with TE_{13}

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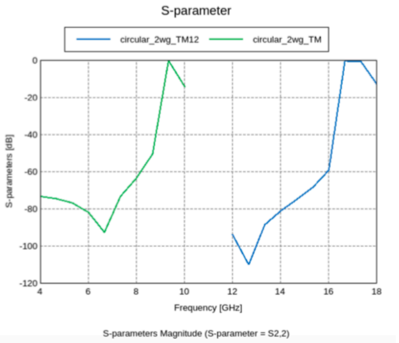
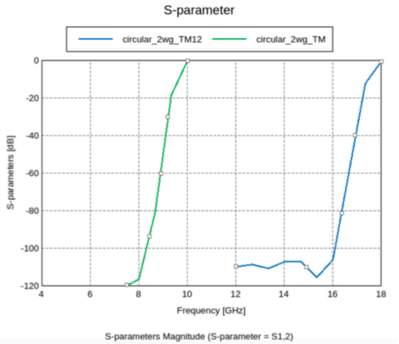
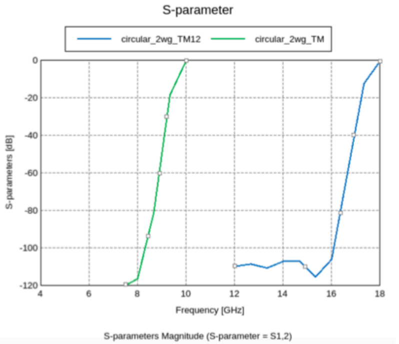
Results: FEKO(TM) - (2)



(a) S_{pp} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}



(b) S_{pr} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}



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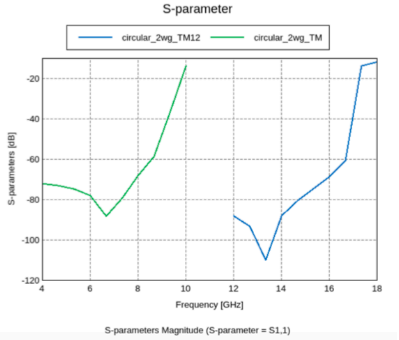

(d) S_{rr} for coupling of modes TM_{11} with TM_{11}, TM_{12} with TM_{12} and TM_{13} with TM_{13}

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Conclusion

- Field pattern for one waveguide matches Feko results
- S parameter with Mode matching technique doesn’t match Feko results.
    - Somehow S11 results from Mode Matching are similar to the S12 results from the Feko simulation. Which is not desirable.
    - Should evanescent modes be included in the S11, S12 matrices from Spp and Spr matrices of General Scattering matrix?
    - More investigation is needed.

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