1 Question a

1. What are the fundamental modes of the two waveguides? (TM_{mn}, TE_{mn}, TEM)

Solution:

For the parallel plate waveguide (PPW), the fundamental mode is TEM.

For the rectangular waveguide, the fundamental mode is TE_{10}.

2 Question b

1. What are the expressions of the electric- and magnetic field components of the fundamental modes in both waveguides? Draw the electric field distribution in the transverse (x,y)-plane.

Solution:

For Rectangular Waveguide (RWG);

Electric Field Components: For TE10 mode, Electric field has only one component that is transverse to the plane of incidence. Here it is the y direction.

$$E_x^0(x,y) = 0$$

$$E_y^0(x,y) = -j\frac{k\eta}{k_{x1}} H_{10}^z \sin k_{x1}x$$

$$E_z^0(x,y) = 0$$

Magnetic Field Components: For TE10 mode, The Magnetic field has to be orthogonal to the Electric field and therfore, it lies on the plane of incidence and hence it has both x and z components.

$$H_x^0(x,y) = j\frac{k_z}{k_{x1}} H_{10}^z \sin k_{x1}x$$

$$H_y^0(x,y) = 0$$

$$H_z^0(x,y) = H_{10}^z \cos k_{x1}x$$

The direction of electric field is given by the following figure. (For TE10 mode)

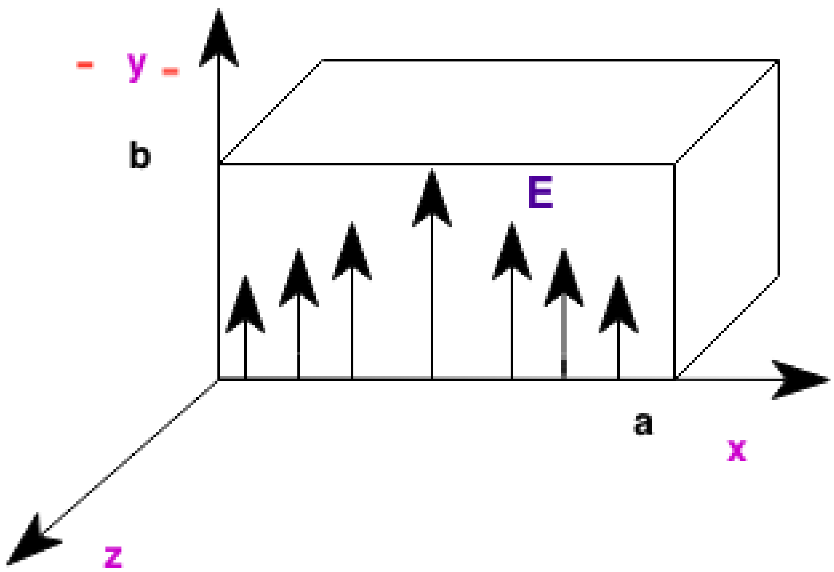


Figure1: Electric field lines for RWG

For Parallel Plate Waveguide (PPW) Waveguide;

TEM is the fundamental mode because in a PPW there are two separate conductors which allow the propagation of TEM polarized wave. Here, both E_z and H_z are 0. This is exactly like an parallel plate capacitor where the electric field is directed from one plate to the other. Therefore, as stated in the diagram in the question, electric field only has the y component (shown in the figure) and magnetic field has only the x component.

As these fields can be treated as static electric and magnetic fields, one can write these in terms of the electric potential and current.

$$E_y(\vec{z}) = -\frac{1}{d}V^+e^{-jkz}\hat{y}$$

$$H_x(\vec{z}) = \frac{1}{w}I^+e^{-jkz}\hat{x}$$

If the characteristic impedance is given by $Z_0 = \frac{V^+}{I^+}$, we know from transmission line problems that, $Z_0 = \frac{1}{\eta d}$ where η is the wave impedance in free space.

Therefore, we can write the magnetic field component as:

$$H_x(\vec{z}) = \frac{1}{\eta d}V^+e^{-jkz}\hat{x}$$

The terms with e^{jkz} are not present because this parallel plate waveguide is assumed to be semi-infinite and there is no regressive field reflected from the other end.

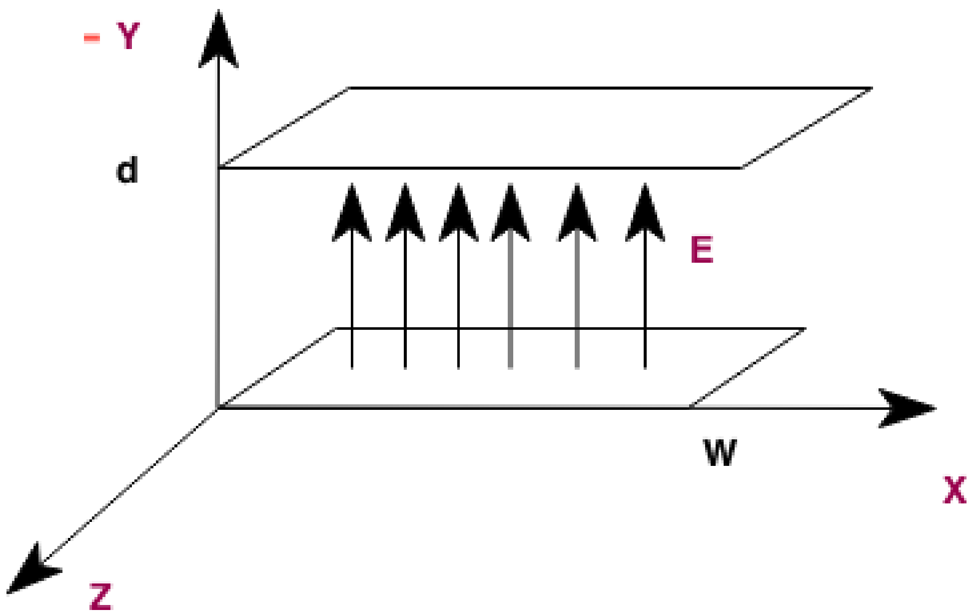


Figure2: Electric field lines for PPW

3 Question c

1. What are the expressions of the propagation constants in the two waveguides?

Solution:

For the RWG;

$$kz = k\sqrt{1 - (\frac{c}{2af})^2}$$

Or can be expressed in terms of k_{x1} where for a TE01 mode, $k_{x1} = \frac{\pi}{a}$

$$kz = \sqrt{k^2 - k_{x1}^2}$$

For PPW;

The propagation constant is simply a real quantity and is given by, $k_z = \frac{\omega}{c} = \frac{2\pi f}{c}$

4 Question d

1. What are the cut-off frequencies of the two fundamental modes? Plot the real and imaginary parts of the propagation constants from $2GHz < f < 22GHz$. Can you explain what happens in the regions $f < f_c$ and $f > f_c$?

Solution:

For the RWG;

The cut-off frequency is the frequency at which k becomes equals to k_{x1} Therefore

$$k_c = k_{x1} = \frac{\pi}{a}$$

$$\implies \frac{2\pi f_c}{c} = \frac{\pi}{a}$$

$$\implies f_c = \frac{c}{2a}$$

This f_c is the minimum frequency required for the wave to propagate inside the waveguide.

From frequencies $0 < f < f_c$, the imaginary part of the propagation constant is finite and real part is 0. Similarly, for $f > f_c$, the real part is finite and the imaginary part is 0.

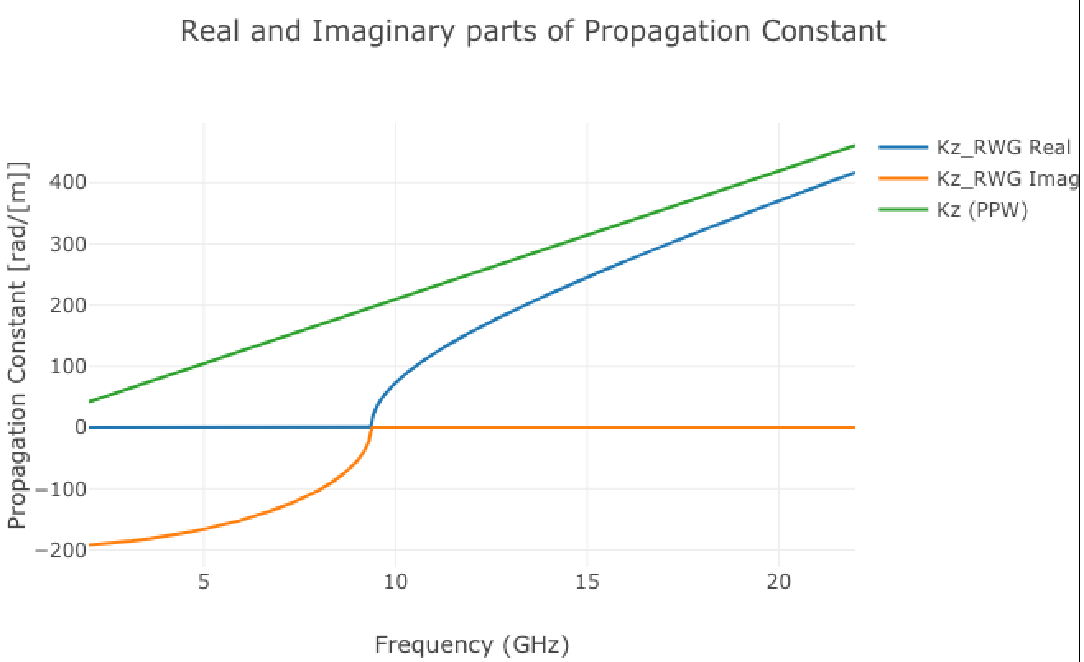
This explains that below the cut off frequency, the propagation constant ($\beta = -j\sqrt{k_{x1}^2 - k^2}$) is purely imaginary. This imaginary β makes the propagation term on the electric field and magnetic field ($e^{-j\beta}$) as a real quantity, indicating no propagation, but only attenuation of the wave.

In short, no wave can be present below the cut-off frequency. Above the cut-off frequency, the propagation constant is real($\beta = \sqrt{k^2 - k_{x1}^2}$) and it allows the wave to propagate inside the waveguide. For this problem the cut-off frequency for RWG was found to be $f_c = 9.3685GHz$.

For the PPW;

For PPW, the fundamental mode is TEM. Therefore, the cut-off frequency is $f_c = 0Hz$.

Plot of the propagation constant real and imaginary parts with respect to frequency are given below for PPW and RWG. (From $2GHz$ to $22GHz$)



/3/3

Figure3: Propagation constant with respect to frequency in RWG and PPW if filled with air.

5 Question e

1. Can the electric- and magnetic fields in these waveguides be related to each- other by an impedance? And if so, give the expressions and plot the impedances from $2GHz < f < 22GHz$.

Solution:

For the RWG:

Yes, the Electric and Magnetic fields in the waveguide are related by an impedance. If we divide the tangential components of E and H from question (b), we find,

$$Z_{TE} = \frac{k\eta}{k_z}$$

An alternate formula is,

$$Z_{TE} = \frac{\omega\mu}{k_z}$$

Where kz is given by the equations in question (c); η is the wave impedance in free space and k is the propagation constant in free space.

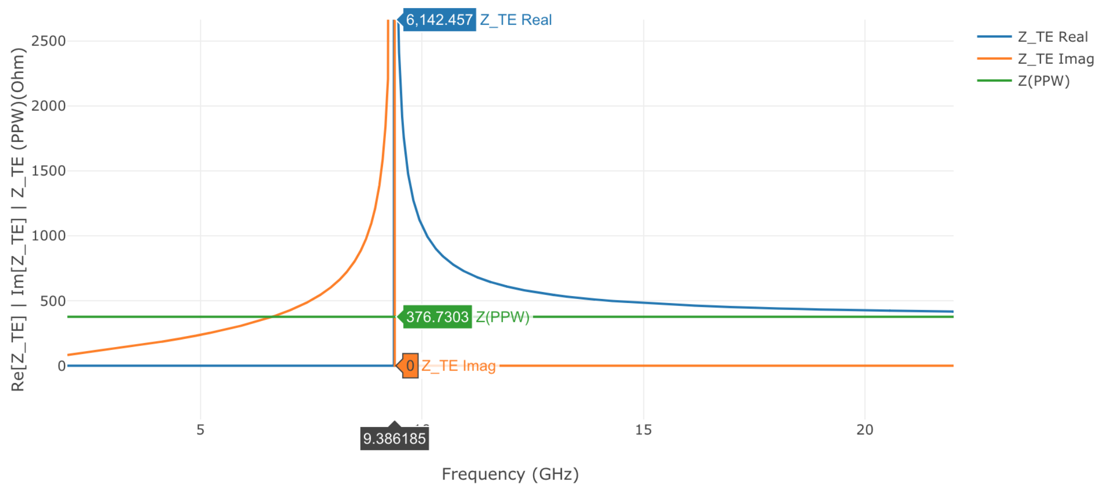
The real, imaginary and the absolute values of Z_{TE} are plotted with respect to frequency. (From $2GHz$ to $22GHz$)

Here also, above the cut-off frequency, Z_{TE} is real and below the cut-off frequency, Z_{TE} is imaginary. The more closer the frequency to the cut-off frequency, the more is the impedance.

For the PPW:

For PPW, the impedance is just the wave impedance and it is given by,

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 376.7303[Ohm]$$



/3/3

Figure4: Wave impedance with respect to frequency in RWG and PPW if filled with air.

6 Question f

1. f) What are the attenuation constants in the waveguides due to lossy, but good, conductors? Plot the attenuation in the waveguides, in dB/cm, for a conductivity of $\sigma = 5{*}10^7[S/m]$ (From $2GHz$ to $22GHz$). Explain why the conductor losses in a rectangular WG are higher close to the cut-off frequency.

Solution:

For the RWG:

The attenuation constant is found out by the following equation.

$$\alpha = \frac{R_s}{\eta b\sqrt{1-(f_c/f)^2}}[1+\frac{2b}{a}(f_c/f)^2]$$

Where $R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$

If we express this loss in dB/cm, we have,

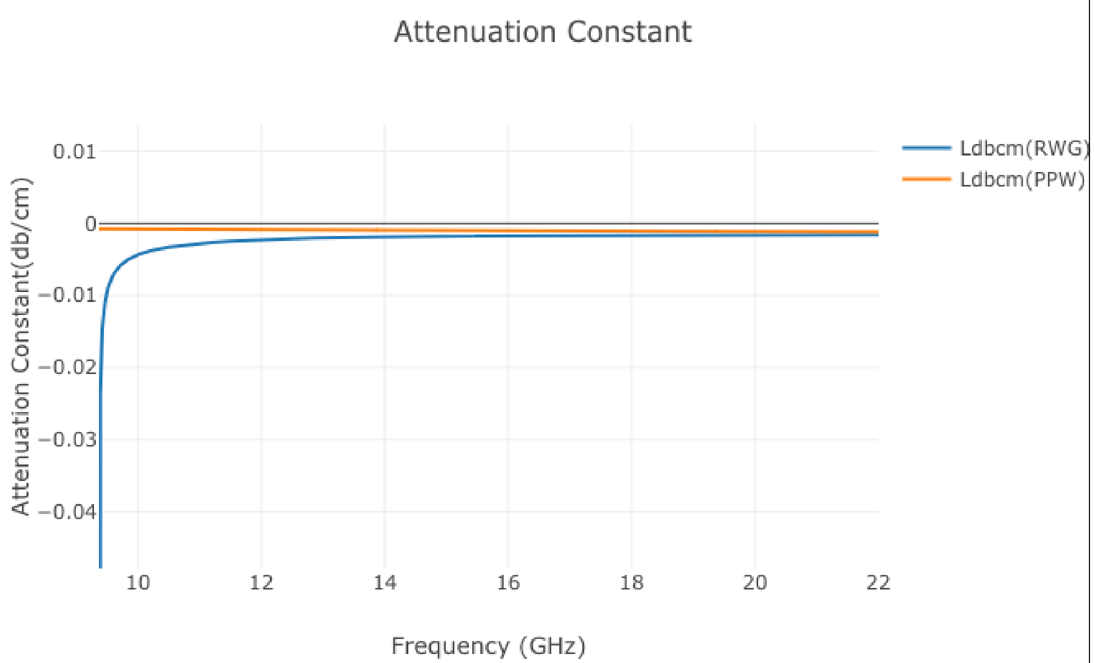
$$L_{dbcm} = -20\log_{10}(e) * \alpha * 10^{-2}$$
$$\implies L_{dbcm} = -8.686 * \alpha * 10^{-2}$$

There is no need to plot the attenuation constant below the cut-off frequency, because below the cut-off frequency the wave doesn't exist and the losses due to the conducting nature of the waveguide is not considered.

For the PPW:

The attenuation constant in air is given by,

$$\alpha = R_s/(b\eta_0)$$



/3/3

Figure5: Attenuation constant with respect to frequency in RWG and PPW if filled with air.

7 Question g

1. Consider now the waveguides to be filled with Teflon, characterized by $\epsilon_r = 2.1$ and $\tau = 0.001$

What happens with the propagation constants of the fundamental mode? Support your answer with a plot with your updated propagation constants. What are the new cut-off frequencies?

Solution:

For the RWG:

If the waveguide is filled with a dielectric, the propagation constant becomes,

$$kz = \sqrt{k^2 - k_{x1}^2}$$

where

$$k = \omega\sqrt{\epsilon_r \epsilon_0 \mu_0}\sqrt{1 - j\tau}$$

where ϵ_r is the relative dielectric constant and $\tau = \frac{\epsilon''}{\epsilon'}$. The ϵ' is the real part of the equivalent dielectric constant and ϵ'' is the imaginary part of the dielectric constant in the medium.

The plot for rectangular waveguide shows that the cut-off frequency is now lesser than the cut-off frequency when it was filled with air. The new value is $f_{teflon} = \frac{c}{2an_{teflon}} = 6.47GHZ$. This happens due to the fact that the velocity of the electromagnetic wave drops with an order of the refractive index of the medium (n_{teflon}) which is equal to the square root of ϵ_r.

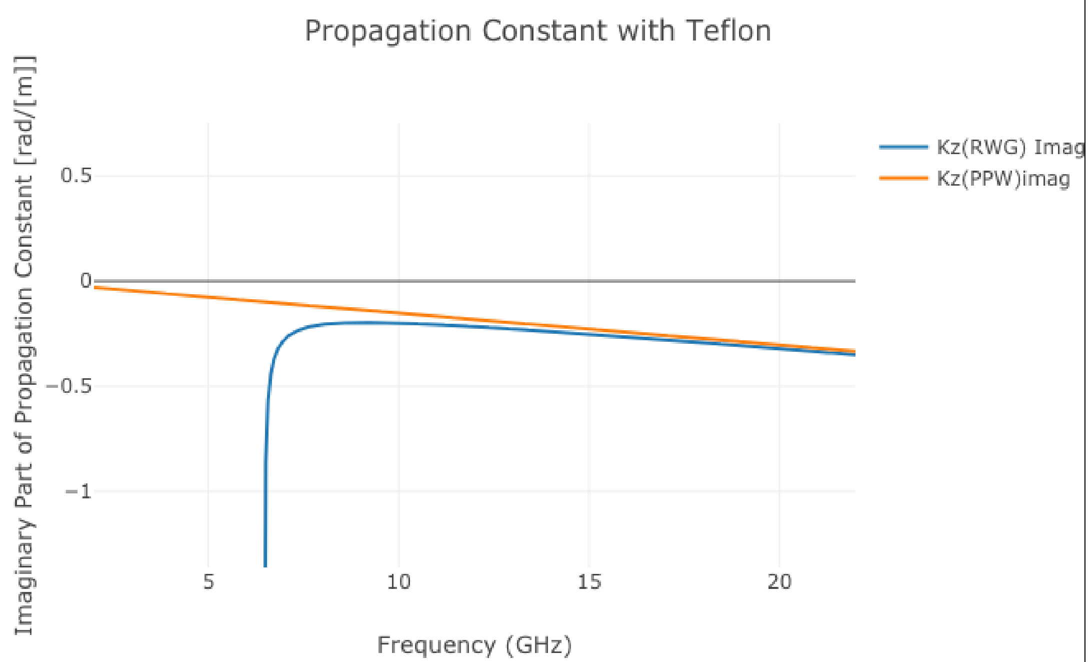
Furthermore, the imaginary part is finite even after the cut-off frequency and doesn't go to zero that fast (asymptotic towards 0). This happens because even after the cut-off frequency, the dielectric losses are present.

For the PPW:

For the PPW, the propagation constant when Teflon is used becomes,

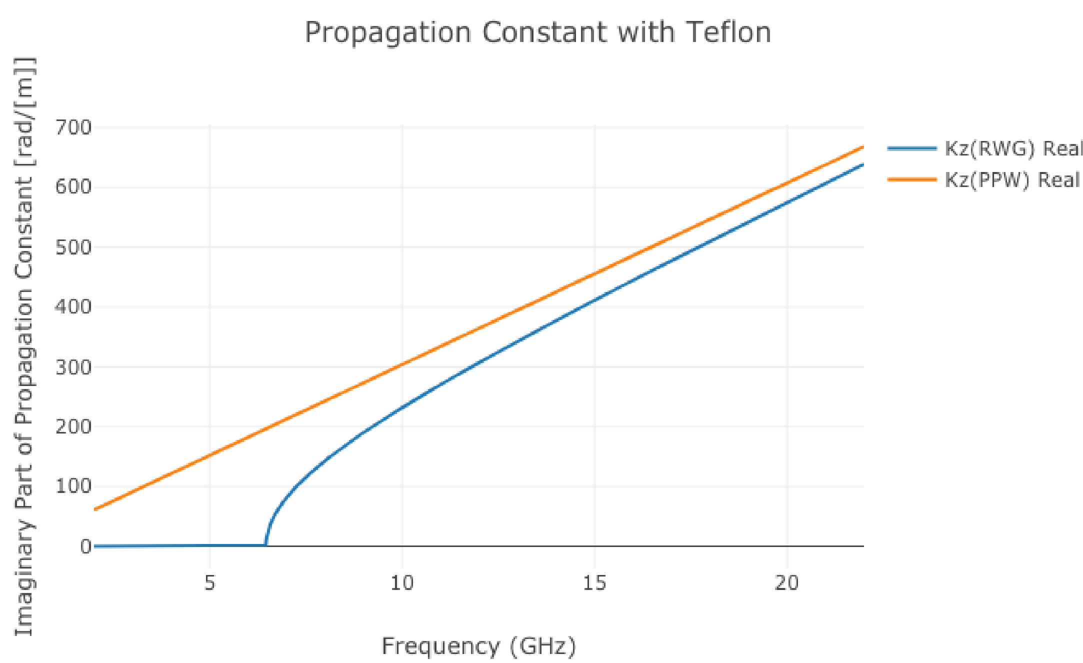
$$kz = \omega\sqrt{\epsilon_r \epsilon_0 \mu_0}\sqrt{1 - j\tau}$$

The Real and Imaginary parts of propagation constant for PPW and RWG were plotted separately. Now, the imaginary part here as well is not suddenly zero after the cut-off frequency, but it has some finite value due to dielectric losses.



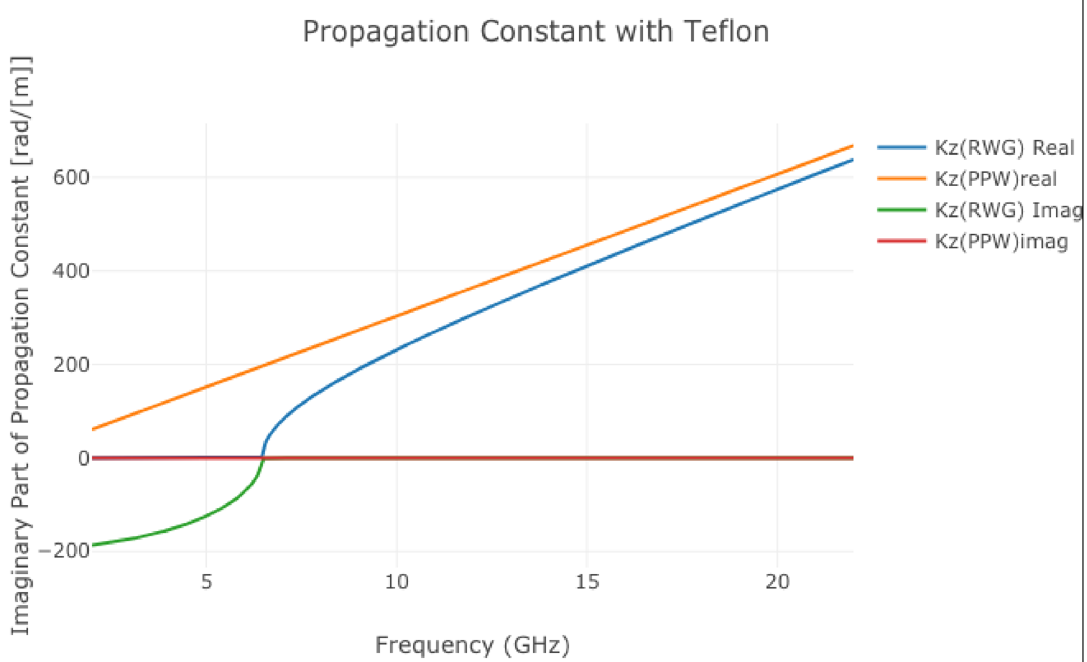
/3/3

Figure6: Imaginary part of Propagation constant with respect to frequency in RWG and PPW if filled with Teflon.



/3/3

Figure7: Real part of Propagation constant with respect to frequency in RWG and PPW if filled with Teflon.



/3/3

Figure8: Real and Imaginary parts of Propagation constant with respect to frequency in RWG and PPW if filled with Teflon. [For a better picture]

8 Question h

1. Consider now the waveguides to be filled with Teflon, characterized by $\epsilon_r = 2.1$ and $\tau = 0.001$

   What are the expressions of the attenuation constant in the waveguides due to lossy dielectrics? Plot the attenuation constant in dB/cm (From $2GHz$ to $22GHz$)

Solution:

For the RWG:

The expression for propagation constant is derived below if the waveguide is filled with some dielectric. From the power equation, we have the general expression for attenuation constant as:

$$\alpha = \frac{P_{loss}}{2P_t}$$

Where,

$$P_{loss} = \int\int \epsilon'' \left|\vec{Ey}\right|^2 dS$$

and,

$$P_t = \frac{|E_y|^2 ab}{4Z_{TE}}$$

using the above equations, if we solve for the attenuation constant α, we can get the following equation,

$$\alpha = \frac{\tau \epsilon_r k_0^2}{2\beta_z}$$

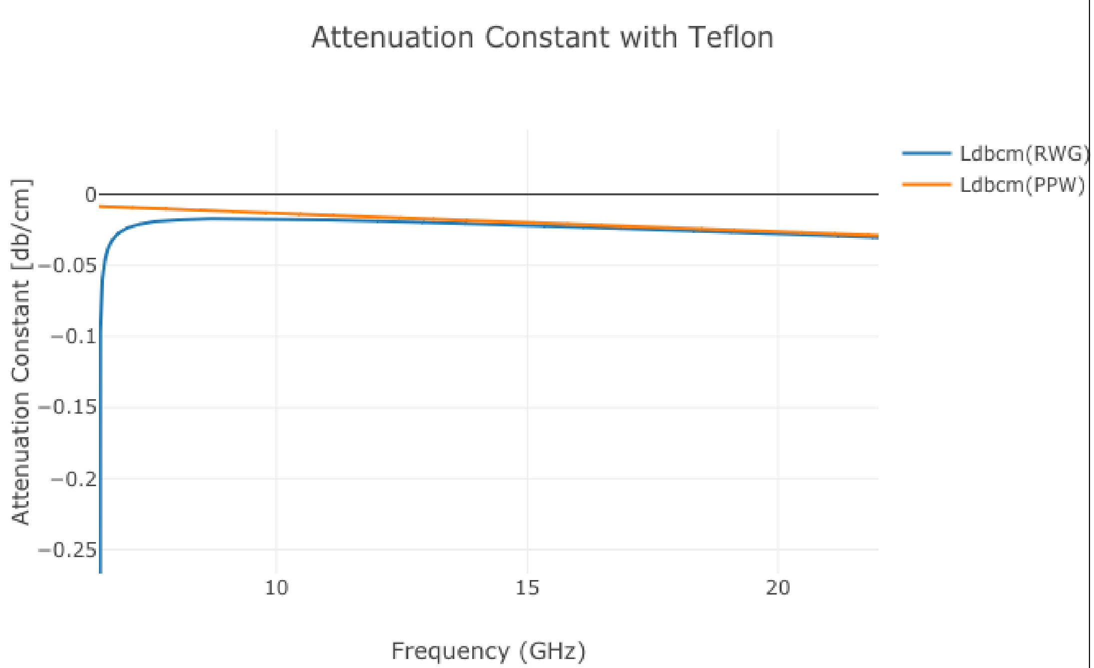
Where

$$\beta_z = \sqrt{k_0^2\epsilon_r - k_{10}^2} = \sqrt{k_0^2\epsilon_r - (\frac{\pi}{a})^2}$$

For the PPW:

For parallel plate waveguide, the attenuation constant is given by,

$$\alpha = \frac{\tau k_0 \sqrt{\epsilon_r}}{2}$$



/3/3

Figure8: Attenuation Constant in dbcm with respect to frequency in RWG and PPW if filled with Teflon.