Chapter 1: The Role Of Algorithms In Computing

Exercise 1.1 – 1

Give a real-world example that requires sorting or a real-world example that requires computing a convex hull.

- English dictionary requires all the words to be arranged in their lexicographical order.
- Keeping track of the spatial extent of a disease outbreak could be done using the convex hull.

Exercise 1.1 – 2

Other than speed, what other measures of efficiency might one use in a real-world setting?

- Memory utilization is another measure of efficiency that one might use in a real-world setting.

Exercise 1.1 – 3

Select a data structure that you have seen previously, discuss its strengths and weaknesses.

- Array is a container that can hold a fixed number of elements (variables of same data type) and where each element can be accessed using its index.
- Strengths:
  Efficient data access.
  Can be used to implement other, more complex data structures.
- Weaknesses:
  We must know the size of array in advance.
  Does not support insertion or deletion of elements.

Exercise 1.1 – 4

How are shortest-path and travelling-salesman problems given above similar? How are they different?

- Shortest-path and travelling-salesman problems are similar because both aim to find the shortest path from one point to another.
- Difference is, in shortest-path, source and destination are two different points. We might only need to go through a subset of given points when travelling from source to the destination.
- On the other hand, in travelling-salesman problem, destination is same as the starting point. We are required to cover all the other points before ending up back at the point we began our journey from.

Exercise 1.1 – 5

Come up with a real-world problem in which only the best solution will do. Then come up with one in which a solution that is "approximately" the best is good enough.

- In case of calculating the trajectory needed to enter a geostationary orbit for satellite launch, only the best solution will do. On the other hand, when calculating age of a person, rounding down to the closest integer is a good enough solution.

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Exercise 1.2 – 1

Give an example of an application that requires algorithmic content at the application level, and discuss the function of the algorithms involved.

- Communication through popular social media platforms like Facebook, Instagram, WhatsApp etc. requires algorithmic content at their application level. For user privacy, every message is encrypted before it is sent through the communication channel. On the receiving end, this message is then decrypted back to the original message for the receiver to read.

Exercise 1.2 – 2

Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in 64nlgn steps. For which values of n does insertion sort beat merge sort?

- Since n represents input size, it can only take integer values starting from 0. We need to prove:
  $8n^2 < 64n * lgn$
  Or $n < 8 * lgn$
- This condition is satisfied only if n belongs to the set [2, 43].

Exercise 1.2 – 3

What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

- We need to find the smallest value of n that satisfies the equation:
  $100n^2 < 2^n$
- When n takes the value 0, the above equation is satisfied. But if we are told to find a non-zero value, then 15 will be the minimum value n can take to satisfy the equation.

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Problem 1.1

For each function f(n) and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes f(n) microseconds.

- The values in the below table are approximations since I have assumed there are 30 days in every month of the year.

	1 second	**1 minute**	**1 hour**	**1 day**	**1 month**	**1 year**	**1 century**
lgn	$2^{10\text{^}6}$	$2^{6 * 10\text{^}7}$	$2^{3.6 * 10\text{^}9}$	$2^{8.64 * 10\text{^}10}$	$2^{2.59 * 10\text{^}12}$	$2^{3.15 * 10\text{^}13}$	$2^{3.15 * 10\text{^}15}$
√n	10^{12}	$3.6 * 10^{15}$	$1.29 * 10^{19}$	$7.46 * 10^{21}$	$6.72 * 10^{24}$	$9.95 * 10^{26}$	$9.96 * 10^{30}$
n	10^{6}	$6 * 10^{7}$	$3.6 * 10^{9}$	$8.64 * 10^{10}$	$2.59 * 10^{12}$	$3.15 * 10^{13}$	$3.16 * 10^{15}$
nlgn	62746	2801417	133378058	2755147513	71870856404	797633893349	$6.86 * 10^{13}$
n^2	1000	7745	60000	293938	1609968	5615692	56176151
n^3	100	391	1532	4420	13736	31593	146679
2^n	19	25	31	36	41	44	51
n!	9	11	12	13	15	16	17