2D Poiseuille Exact Coherent State equations (Cambridge)

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1 The full equations

$$
\begin{aligned}
T_{ij} &= \frac{1}{\mathrm{Wi}}(C_{ij} - \mathbf{I}_{ij}) && (1)\\
u &= -\partial_y \psi && (2)\\
v &= \partial_x \psi && (3)
\end{aligned}
$$

1.1 ψ

$$
\begin{aligned}
0 = &\mathrm{Re}\ \nu \partial_x \nabla^2 \psi - \mathrm{Re}\ u \partial_x \nabla^2 \psi - \mathrm{Re}\ v \partial_y \nabla^2 \psi \\
&+ \beta \nabla^4 \psi - \frac{(1-\beta)}{\mathrm{Wi}} \left(\partial_{xx} C_{xy} + \partial_{xy}(C_{yy} - C_{xx}) - \partial_{yy} C_{xy} \right) && (4)
\end{aligned}
$$

1.2 C_{xx}

$$
\begin{aligned}
0 = &\nu \partial_x C_{xx} - (u\partial_x + v\partial_y) C_{xx} \\
&+ 2C_{xx}\partial_x u + 2C_{xy}\partial_x v - T_{xx} && (5)
\end{aligned}
$$

1.3 C_{yy}

$$
\begin{aligned}
0 = &\nu \partial_x C_{yy} - (u\partial_x + v\partial_y) C_{yy} \\
&+ 2C_{xy}\partial_y u + 2C_{yy}\partial_y v - T_{yy} && (6)
\end{aligned}
$$

1.4 C_{xy}

$$
\begin{aligned}
0 = &\nu \partial_x C_{xy} - (u\partial_x + v\partial_y) C_{xy} \\
&+ C_{xx}\partial_y u + C_{yy}\partial_x v - T_{xy} && (7)
\end{aligned}
$$

1.5 ψ_0

$$
\begin{aligned}
0 = &-\mathrm{Re}\ \overline{u\partial_x u} - \mathrm{Re}\ \overline{v \partial_y v} \\
&- \beta \overline{\partial_{yyy}\psi} + \frac{(1-\beta)}{\mathrm{Wi}} \overline{\partial_y C_{xy}} + 2 && (8)
\end{aligned}
$$

2 The 1st order terms in a Taylor expansion

These are the equations as they will need to be entered into the Jacobian matrix.

$$\psi = \Psi + \delta\psi \tag{9}$$

$$C_{ij} = C_{ij} + \delta c_{ij} \tag{10}$$

2.1 ψ

$$\begin{aligned} 0 = &\nu \mathrm{Re}\ \partial_x \nabla^2 \delta\psi + \mathrm{Re}\ (\partial_x \nabla^2 \Psi)\partial_y \delta\psi - \mathrm{Re}\ U \partial_x \nabla^2 \delta\psi \\ &- \mathrm{Re}\ (\partial_y \nabla^2 \Psi)\partial_x \delta\psi - \mathrm{Re}\ V \partial_y \nabla^2 \delta\psi + \beta \nabla^4 \delta\psi \\ &- \frac{(1-\beta)}{\mathrm{Wi}} \left(\partial_{xx}\delta c_{xy} + \partial_{yy}(\delta c_{yy} - \delta c_{xx}) - \partial_{yy}\delta c_{xy}\right) \end{aligned} \tag{11}$$

2.2 C_{xx}

$$\begin{aligned} 0 = &\nu \partial_x \delta c_{xx} - (\mathbf{V}\cdot\nabla)\delta c_{xx} \\ &+ (\partial_x C_{xx})\partial_y \delta\psi - (\partial_y C_{xx})\partial_x \delta\psi \\ &+ 2(\partial_x U)\delta c_{xx} + 2(\partial_x V)\delta c_{xy} \\ &- 2C_{xx}\partial_{xy}\delta\psi + 2C_{xy}\partial_{xx}\delta\psi - \frac{1}{\mathrm{Wi}}\delta c_{xx} \end{aligned} \tag{12}$$

2.3 C_{yy}

$$\begin{aligned} 0 = &\nu \partial_x \delta c_{yy} - (\mathbf{V}\cdot\nabla)\delta c_{yy} \\ &+ (\partial_x C_{yy})\partial_y \delta\psi - (\partial_y C_{yy})\partial_x \delta\psi \\ &+ 2(\partial_y U)\delta c_{xy} + 2(\partial_y V)\delta c_{yy} \\ &- C_{xy}\partial_{yy}\delta\psi + C_{yy}\partial_{xy}\delta\psi - \frac{\delta c_{yy}}{\mathrm{Wi}} \end{aligned} \tag{13}$$

2.4 C_{xy}

$$\begin{aligned} 0 = &\nu \partial_x \delta c_{xy} - (\mathbf{V}\cdot\nabla)\delta c_{xy} \\ &+ (\partial_x C_{xy})\partial_y \delta\psi - (\partial_y C_{xy})\partial_x \delta\psi \\ &+ (\partial_y U)\delta c_{xx} + (\partial_x V)\delta c_{yy} \\ &- C_{xx}\partial_{yy}\delta\psi + C_{yy}\partial_{xx}\delta\psi - \frac{\delta c_{xy}}{\mathrm{Wi}} \end{aligned} \tag{14}$$

2.5 ψ_0

$$\begin{aligned} 0 = &-\mathrm{Re}\ \overline{(\partial_{xy}\Psi)\partial_y\delta\psi} - \mathrm{Re}\ \overline{(\partial_y\Psi)\partial_{xy}\delta\psi} \\ &+ \mathrm{Re}\ \overline{(\partial_x\Psi)\partial_{yy}\delta\psi} + \mathrm{Re}\ \overline{(\partial_{yy}\Psi)\partial_x\delta\psi} \\ &- \beta\overline{\partial_{yyy}\delta\psi} + \frac{1-\beta}{\mathrm{Wi}}\overline{\partial_y\delta c_{xy}} \end{aligned} \tag{15}$$