2D Poiseuille Exact Coherent State equations (the other streamfunction)

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1 The full equations

$$T_{ij} = \frac{1}{\mathrm{Wi}}(C_{ij} - \mathbf{I}_{ij}) \tag{1}$$

$$u = \partial_y \psi \tag{2}$$

$$v = -\partial_x \psi \tag{3}$$

1.1 ψ

$$\begin{aligned} 0 = &-\mathrm{Re}\ \nu\partial_x \nabla^2\psi + \mathrm{Re}\ u\partial_x\nabla^2\psi + \mathrm{Re}\ v\partial_y\nabla^2\psi \\ &- \beta\nabla^4\psi + (1-\beta)\left(\partial_{xx}T_{xy} + \partial_{xy}(T_{yy} - T_{xx}) - \partial_{yy}T_{xy}\right) \end{aligned} \tag{4}$$

1.2 C_{xx}

$$\begin{aligned} 0 = &\ \nu\partial_x C_{xx} - (u\partial_x + v\partial_y)C_{xx} \\ &+ 2C_{xx}\partial_x u + 2C_{xy}\partial_x v - T_{xx} \end{aligned} \tag{5}$$

1.3 C_{yy}

$$\begin{aligned} 0 = &\ \nu\partial_x C_{yy} - (u\partial_x + v\partial_y)C_{yy} \\ &+ 2C_{xy}\partial_y u + 2C_{yy}\partial_y v - T_{yy} \end{aligned} \tag{6}$$

1.4 C_{xy}

$$\begin{aligned} 0 = &\ \nu\partial_x C_{xy} - (u\partial_x + v\partial_y)C_{xy} \\ &+ C_{xx}\partial_y u + C_{yy}\partial_x v - T_{xy} \end{aligned} \tag{7}$$

1.5 ψ_0

$$\begin{aligned} 0 = &-\mathrm{Re}\ \overline{v\partial_y v} \\ &+ \beta\overline{\partial_{yyy}\psi} + (1-\beta)\overline{\partial_y T_{xy}} + 2 \end{aligned} \tag{8}$$

2 The 1st order terms in a Taylor expansion

These are the equations as they will need to be entered into the Jacobian matrix.

$$\psi = \Psi + \delta\psi \tag{9}$$
$$C_{ij} = C_{ij} + \delta c_{ij} \tag{10}$$

2.1 ψ

$$\begin{aligned} 0 = &-\nu\mathrm{Re}\ \partial_x\nabla^2\delta\psi + \mathrm{Re}\ (\partial_x\nabla^2\Psi)\partial_y\delta\psi + \mathrm{Re}\ U\partial_x\nabla^2\delta\psi \\ &- \mathrm{Re}\ (\partial_y\nabla^2\Psi)\partial_x\delta\psi + \mathrm{Re}\ V\partial_y\nabla^2\delta\psi - \beta\nabla^4\delta\psi \\ &+ \frac{(1-\beta)}{\mathrm{Wi}}\left(\partial_{xx}\delta c_{xy} + \partial_{xy}(\delta c_{yy} - \delta c_{xx}) - \partial_{yy}\delta c_{xy}\right) \end{aligned} \tag{11}$$

2.2 C_{xx}

$$\begin{aligned} 0 = &\ \nu\partial_x\delta c_{xx} - (\mathbf{V}\cdot\nabla)\delta c_{xx} \\ &- (\partial_x C_{xx})\partial_y\delta\psi + (\partial_y C_{xx})\partial_x\delta\psi \\ &+ 2(\partial_x U)\delta c_{xx} + 2(\partial_x V)\delta c_{xy} \\ &+ 2C_{xx}\partial_{xy}\delta\psi - 2C_{xy}\partial_{xx}\delta\psi - \frac{1}{\mathrm{Wi}}\delta c_{xx} \end{aligned} \tag{12}$$

2.3 C_{yy}

$$\begin{aligned} 0 = &\ \nu\partial_x\delta c_{yy} - (\mathbf{V}\cdot\nabla)\delta c_{yy} \\ &- (\partial_x C_{yy})\partial_y\delta\psi + (\partial_y C_{yy})\partial_x\delta\psi \\ &+ 2(\partial_y U)\delta c_{xy} + 2(\partial_y V)\delta c_{yy} \\ &+ C_{xy}\partial_{yy}\delta\psi - C_{yy}\partial_{xy}\delta\psi - \frac{\delta c_{yy}}{\mathrm{Wi}} \end{aligned} \tag{13}$$

2.4 C_{xy}

$$\begin{aligned} 0 = &\ \nu\partial_x\delta c_{xy} - (\mathbf{V}\cdot\nabla)\delta c_{xy} \\ &- (\partial_x C_{xy})\partial_y\delta\psi + (\partial_y C_{xy})\partial_x\delta\psi \\ &+ (\partial_y U)\delta c_{xx} + (\partial_x V)\delta c_{yy} \\ &+ C_{xx}\partial_{yy}\delta\psi - C_{yy}\partial_{xx}\delta\psi - \frac{\delta c_{xy}}{\mathrm{Wi}} \end{aligned} \tag{14}$$

2.5 ψ_0

$$\begin{aligned} 0 = &+\mathrm{Re}\ \overline{(\partial_x\Psi)\partial_{yy}\delta\psi} + \mathrm{Re}\ \overline{(\partial_{yy}\Psi)\partial_x\delta\psi} \\ &+ \beta\overline{\partial_{yyy}\delta\psi} + \frac{1-\beta}{\mathrm{Wi}}\overline{\partial_y\delta c_{xy}} \end{aligned} \tag{15}$$