

# 2D Exact Coherent State equations

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## 1 The full equations

$$T_{ij} = \frac{1}{\text{Wi}} (C_{ij} - \mathbf{I}_{ij}) \quad (1)$$

$$u = -\partial_y \psi \quad (2)$$

$$v = \partial_x \psi \quad (3)$$

### 1.1 $\psi$

$$\begin{aligned} 0 = & \text{Re } \nu \partial_x \nabla^2 \psi - \text{Re } u \partial_x \nabla^2 \psi - \text{Re } v \partial_y \nabla^2 \psi \\ & + \beta \nabla^4 \psi - \frac{(1-\beta)}{\text{Wi}} (\partial_{xx} C_{xy} + \partial_{xy} (C_{yy} - C_{xx}) - \partial_{yy} C_{xy}) \end{aligned} \quad (4)$$

### 1.2 $C_{xx}$

$$\begin{aligned} 0 = & \nu \partial_x C_{xx} - (u \partial_x + v \partial_y) C_{xx} \\ & + 2C_{xx} \partial_x u + 2C_{xy} \partial_x v - T_{xx} \end{aligned} \quad (5)$$

### 1.3 $C_{yy}$

$$\begin{aligned} 0 = & \nu \partial_x C_{yy} - (u \partial_x + v \partial_y) C_{yy} \\ & + 2C_{xy} \partial_y u + 2C_{yy} \partial_y v - T_{yy} \end{aligned} \quad (6)$$

### 1.4 $C_{xy}$

$$\begin{aligned} 0 = & \nu \partial_x C_{xy} - (u \partial_x + v \partial_y) C_{xy} \\ & + C_{xx} \partial_y u + C_{yy} \partial_x v - T_{xy} \end{aligned} \quad (7)$$

### 1.5 $\psi_0$

$$\begin{aligned} 0 = & -\text{Re } \overline{(\partial_y \psi)} \partial_{xy} \psi + \text{Re } \overline{(\partial_x \psi)} \partial_{yy} \psi \\ & + \beta \overline{\partial_{yyy} \psi} + \frac{(1-\beta)}{\text{Wi}} \overline{\partial_y C_{xy}} + 2 \end{aligned} \quad (8)$$

## 2 The 1st order terms in a Taylor expansion

These are the equations as they will need to be entered into the Jacobian matrix.

$$\psi = \Psi + \delta\psi \quad (9)$$

$$C_{ij} = C_{ij} + \delta c_{ij} \quad (10)$$

### 2.1 $\psi$

$$\begin{aligned} 0 = & \nu \text{Re } \partial_x \nabla^2 \delta\psi + \text{Re } (\nabla^2 \Psi) \partial_y \delta\psi - \text{Re } U \partial_x \nabla^2 \delta\psi \\ & - \text{Re } (\partial_y \nabla^2 \Psi) \partial_x \delta\psi - \text{Re } V \partial_y \nabla^2 \delta\psi + \beta \nabla^4 \delta\psi \\ & - \frac{(1-\beta)}{\text{Wi}} (\partial_{xx} \delta c_{xy} + \partial_{yy} (\delta c_{yy} - \delta c_{xx}) - \partial_{yy} \delta c_{xy}) \end{aligned} \quad (11)$$

### 2.2 $C_{xx}$

$$\begin{aligned} 0 = & \nu \partial_x \delta c_{xx} - (\mathbf{V} \cdot \nabla) \delta c_{xx} \\ & + (\partial_x C_{xx}) \partial_y \delta\psi - (\partial_y C_{xx}) \partial_x \delta\psi \\ & + 2(\partial_x U) \delta c_{xx} + 2(\partial_x V) \delta c_{xy} \\ & - 2C_{xx} \partial_{xy} \delta\psi + 2C_{xy} \partial_{xx} \delta\psi - \frac{1}{\text{Wi}} \delta c_{xx} \end{aligned} \quad (12)$$

### 2.3 $C_{yy}$

$$\begin{aligned} 0 = & \nu \partial_x \delta c_{yy} - (\mathbf{V} \cdot \nabla) \delta c_{yy} \\ & + (\partial_x C_{yy}) \partial_y \delta\psi - (\partial_y C_{yy}) \partial_x \delta\psi \\ & + 2(\partial_y U) \delta c_{xy} + 2(\partial_y V) \delta c_{yy} \\ & - C_{xy} \partial_{yy} \delta\psi + C_{yy} \partial_{xy} \delta\psi - \frac{\delta c_{yy}}{\text{Wi}} \end{aligned} \quad (13)$$

### 2.4 $C_{xy}$

$$\begin{aligned} 0 = & \nu \partial_x \delta c_{xy} - (\mathbf{V} \cdot \nabla) \delta c_{xy} \\ & + (\partial_x C_{xy}) \partial_y \delta\psi - (\partial_y C_{xy}) \partial_x \delta\psi \\ & + (\partial_y U) \delta c_{xx} + (\partial_x V) \delta c_{yy} \\ & - C_{xx} \partial_{yy} \delta\psi + C_{yy} \partial_{xx} \delta\psi - \frac{\delta c_{xy}}{\text{Wi}} \end{aligned} \quad (14)$$

### 2.5 $\psi_0$

$$\begin{aligned} 0 = & -\text{Re } (\overline{\partial_{xy} \Psi}) \partial_y \delta\psi - \text{Re } (\overline{\partial_y \Psi}) \partial_{xy} \delta\psi \\ & + \text{Re } (\overline{\partial_x \Psi}) \partial_{yy} \delta\psi + \text{Re } (\overline{\partial_{yy} \Psi}) \partial_x \delta\psi \\ & + \beta \overline{\partial_{yyy} \delta\psi} + \frac{1-\beta}{\text{Wi}} \overline{\partial_y \delta c_{xy}} \end{aligned} \quad (15)$$