Linear Stability Equations for 2D flow between two plates

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$$(v_x, v_y) = (U, V) + (\delta u, \delta v)e^{ikx+\lambda t} \tag{1}$$

$$\tau_{ij} = T_{ij} + \delta\tau_{ij}e^{ikx+\lambda t} \tag{2}$$

NAVIER STOKES X DIRECTION:

$$- Re\left[ikU\delta u + \delta v\frac{\partial U}{\partial y}\right] - ik\delta p + \beta\left(-k^2 + \frac{\partial^2}{\partial y^2}\right)\delta u + (1-\beta)\left(ik\delta\tau_{xx} + \frac{\partial \delta\tau_{xy}}{\partial y}\right) = Re\lambda\delta u \tag{3}$$

NAVIER STOKES Y DIRECTION:

$$- ReikU\delta v - \frac{\partial \delta p}{\partial y} + \beta\left(-k^2 + \frac{\partial^2}{\partial y^2}\right)\delta v + (1-\beta)\left(ik\delta\tau_{xy} + \frac{\partial \delta\tau_{yy}}{\partial y}\right) = Re\lambda\delta v \tag{4}$$

INCOMPRESSIBILITY:

$$ik\delta u + \frac{\partial \delta v}{\partial y} = 0 \tag{5}$$

XX EQUATION:

$$W_i\left[Uik\delta\tau_{xx} + \delta v\frac{\partial T_{xx}}{\partial y} - 2ik\delta u T_{xx} - 2T_{xy}\frac{\partial \delta u}{\partial y} - 2\delta\tau_{xy}\frac{\partial U}{\partial y}\right] - 2ik\delta u + \delta\tau_{xx} = -W_i\lambda\delta\tau_{xx} \tag{6}$$

YY EQUATION:

$$W_i\left[Uik\delta\tau_{yy} + \delta v\frac{\partial T_{yy}}{\partial y} - 2ik\delta v T_{xy} - 2\frac{\partial \delta v}{\partial y}T_{yy}\right] - 2\frac{\partial \delta v}{\partial y} + \delta\tau_{yy} = -W_i\lambda\delta\tau_{yy} \tag{7}$$

XY EQUATION:

$$W_i\left[ikU\delta\tau_{xy} + \delta v\frac{\partial T_{xy}}{\partial y} - ikT_{xx}\delta v - T_{yy}\frac{\partial \delta u}{\partial y} - \delta\tau_{yy}\frac{\partial U}{\partial y}\right] - ik\delta v - \frac{\partial \delta u}{\partial y} + \delta\tau_{xy} = -W_i\lambda\delta\tau_{xy} \tag{8}$$