Linear stability equations (Inverted)

February 28, 2013

$$(v_x, v_y, v_z) = (U, V, W) + (\delta u, \delta v, \delta w)e^{ikx+\lambda t} \tag{1}$$

$$T_{ij} = C_{ij} + \delta c_{ij} e^{ikx+\lambda t} \tag{2}$$

NAVIER STOKES X DIRECTION:

$$\begin{aligned} Re\lambda\delta u = -Re&\left[ikU\delta u + V\frac{\partial\delta u}{\partial y} + W\frac{\partial\delta u}{\partial z} + \delta v\frac{\partial U}{\partial y} + \delta w\frac{\partial U}{\partial z}\right] \\ &- ik\delta p + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta u \\ &+ \frac{(1-\beta)}{W_i}\left[ik\delta c_{xx} + \frac{\partial\delta c_{xy}}{\partial y} + \frac{\partial\delta c_{xz}}{\partial z}\right] \end{aligned} \tag{3}$$

NAVIER STOKES Y DIRECTION:

$$\begin{aligned} Re\lambda\delta v = -Re&\left[ikU\delta v + V\frac{\partial\delta v}{\partial y} + W\frac{\partial\delta v}{\partial z} + \delta v\frac{\partial V}{\partial y} + \delta w\frac{\partial V}{\partial z}\right] \\ &- \frac{\partial\delta p}{\partial y} + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta v \\ &+ \frac{(1-\beta)}{W_i}\left[ik\delta c_{xy} + \frac{\partial\delta c_{yy}}{\partial y} + \frac{\partial\delta c_{yz}}{\partial z}\right] \end{aligned} \tag{4}$$

NAVIER STOKES Z DIRECTION:

$$\begin{aligned} Re\lambda\delta w = -Re&\left[ikU\delta w + V\frac{\partial\delta w}{\partial y} + W\frac{\partial\delta w}{\partial z} + \delta v\frac{\partial W}{\partial y} + \delta w\frac{\partial W}{\partial z}\right] \\ &- \frac{\partial\delta p}{\partial z} + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta w \\ &+ \frac{(1-\beta)}{W_i}\left[ik\delta c_{xz} + \frac{\partial\delta c_{yz}}{\partial y} + \frac{\partial\delta c_{zz}}{\partial z}\right] \end{aligned} \tag{5}$$

INCOMPRESSABILITY:

$$0 = ik\delta u + \frac{\partial\delta v}{\partial y} + \frac{\partial\delta w}{\partial z} \tag{6}$$

XX EQUATION:

$$\begin{aligned} \lambda\delta c_{xx} = -\frac{1}{W_i}\delta c_{xx} &- \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xx} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xx} \\ &+ 2ik\delta u C_{xx} + 2C_{xy}\frac{\partial\delta u}{\partial y} + 2C_{xz}\frac{\partial\delta u}{\partial z} + 2\delta c_{xy}\frac{\partial U}{\partial y} + 2\delta c_{xz}\frac{\partial U}{\partial z} \end{aligned} \tag{7}$$

YY EQUATION:

$$\lambda \delta c_{yy} = -\frac{1}{W_i}\delta c_{yy} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{yy} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{yy}$$
$$+ 2ik\delta v C_{xy} + 2C_{yy}\frac{\partial \delta v}{\partial y} + 2C_{yz}\frac{\partial \delta v}{\partial z} + 2\delta c_{yy}\frac{\partial V}{\partial y} + 2\delta c_{yz}\frac{\partial V}{\partial z} \tag{8}$$

ZZ EQUATION:

$$\lambda \delta c_{zz} = -\frac{1}{W_i}\delta c_{zz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{zz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{zz}$$
$$+ 2ik\delta w C_{xz} + 2C_{yz}\frac{\partial \delta w}{\partial y} + 2C_{zz}\frac{\partial \delta w}{\partial z} - 2\delta c_{yz}\frac{\partial W}{\partial y} + 2\delta c_{zz}\frac{\partial W}{\partial z} \tag{9}$$

XY EQUATION:

$$\lambda \delta c_{xy} = -\frac{1}{W_i}\delta c_{xy} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xy} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xy}$$
$$+ C_{yy}\frac{\partial \delta u}{\partial y} + C_{yz}\frac{\partial \delta u}{\partial z} + \delta c_{yy}\frac{\partial U}{\partial y} + \delta c_{yz}\frac{\partial U}{\partial z}$$
$$+ ik\delta v C_{xx} + C_{xz}\frac{\partial \delta v}{\partial z} + \delta c_{xy}\frac{\partial V}{\partial y} + \delta c_{xz}\frac{\partial V}{\partial z} - C_{xy}\frac{\partial \delta w}{\partial z} \tag{10}$$

XZ EQUATION:

$$\lambda \delta c_{xz} = -\frac{1}{W_i}\delta c_{xz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xz}$$
$$+ C_{yz}\frac{\partial \delta u}{\partial y} + C_{zz}\frac{\partial \delta u}{\partial z} + \delta c_{yz}\frac{\partial U}{\partial y} + \delta c_{zz}\frac{\partial U}{\partial z}$$
$$+ ik\delta w C_{xx} + C_{xy}\frac{\partial \delta w}{\partial y} + \delta c_{xy}\frac{\partial W}{\partial y} + \delta c_{xz}\frac{\partial W}{\partial z} - C_{xz}\frac{\partial \delta v}{\partial y} \tag{11}$$

YZ EQUATION:

$$\lambda \delta c_{yz} = -\frac{1}{W_i}\delta c_{yz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{yz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{yz}$$
$$+ ik\delta v C_{xz} + C_{zz}\frac{\partial \delta v}{\partial z} + \delta c_{zz}\frac{\partial V}{\partial z}$$
$$+ ik\delta w C_{xy} + C_{yy}\frac{\partial \delta w}{\partial y} + \delta c_{yy}\frac{\partial W}{\partial y} + ik\delta u C_{yz} \tag{12}$$