Linear stability equations for Viscoelastic SSP code

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$$(v_x, v_y, v_z) = (U, V, W) + (\delta u, \delta v, \delta w)e^{ikx+\lambda t} \tag{1}$$

$$T_{ij} = C_{ij} + \delta c_{ij} e^{ikx+\lambda t} \tag{2}$$

NAVIER STOKES X DIRECTION:

$$\begin{aligned} Re\lambda\delta u = -Re\left[ikU\delta u + V\frac{\partial\delta u}{\partial y} + W\frac{\partial\delta u}{\partial z} + \delta v\frac{\partial U}{\partial y} + \delta w\frac{\partial U}{\partial z}\right] \\ - ik\delta p + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta u \\ + \frac{(1-\beta)}{W_i}\left[ik\delta c_{xx} + \frac{\partial\delta c_{xy}}{\partial y} + \frac{\partial\delta c_{xz}}{\partial z}\right] \end{aligned} \tag{3}$$

NAVIER STOKES Y DIRECTION:

$$\begin{aligned} Re\lambda\delta v = -Re\left[ikU\delta v + V\frac{\partial\delta v}{\partial y} + W\frac{\partial\delta v}{\partial z} + \delta v\frac{\partial V}{\partial y} + \delta w\frac{\partial V}{\partial z}\right] \\ - \frac{\partial\delta p}{\partial y} + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta v \\ + \frac{(1-\beta)}{W_i}\left[ik\delta c_{xy} + \frac{\partial\delta c_{yy}}{\partial y} + \frac{\partial\delta c_{yz}}{\partial z}\right] \end{aligned} \tag{4}$$

NAVIER STOKES Z DIRECTION:

$$\begin{aligned} Re\lambda\delta w = -Re\left[ikU\delta w + V\frac{\partial\delta w}{\partial y} + W\frac{\partial\delta w}{\partial z} + \delta v\frac{\partial W}{\partial y} + \delta w\frac{\partial W}{\partial z}\right] \\ - \frac{\partial\delta p}{\partial z} + \beta\left[-k^2 + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\delta w \\ + \frac{(1-\beta)}{W_i}\left[ik\delta c_{xz} + \frac{\partial\delta c_{yz}}{\partial y} + \frac{\partial\delta c_{zz}}{\partial z}\right] \end{aligned} \tag{5}$$

INCOMPRESSABILITY:

$$0 = ik\delta u + \frac{\partial\delta v}{\partial y} + \frac{\partial\delta w}{\partial z} \tag{6}$$

XX EQUATION:

$$\lambda\delta c_{xx} = -\frac{1}{W_i}\delta c_{xx} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xx} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xx} + 2ik\delta u C_{xx} + 2C_{xy}\frac{\partial \delta u}{\partial y} + 2C_{xz}\frac{\partial \delta u}{\partial z} + 2\delta c_{xy}\frac{\partial U}{\partial y} + 2\delta c_{xz}\frac{\partial U}{\partial z} \quad (7)$$

YY EQUATION:

$$\lambda\delta c_{yy} = -\frac{1}{W_i}\delta c_{yy} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{yy} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{yy} + 2ik\delta v C_{xy} + 2C_{yy}\frac{\partial \delta v}{\partial y} + 2C_{yz}\frac{\partial \delta v}{\partial z} + 2\delta c_{yy}\frac{\partial V}{\partial y} + 2\delta c_{yz}\frac{\partial V}{\partial z} \quad (8)$$

ZZ EQUATION:

$$\lambda\delta c_{zz} = -\frac{1}{W_i}\delta c_{zz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{zz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{zz} + 2ik\delta w C_{xz} + 2C_{yz}\frac{\partial \delta w}{\partial y} + 2C_{zz}\frac{\partial \delta w}{\partial z} + 2\delta c_{yz}\frac{\partial W}{\partial y} + 2\delta c_{zz}\frac{\partial W}{\partial z} \quad (9)$$

XY EQUATION:

$$\lambda\delta c_{xy} = -\frac{1}{W_i}\delta c_{xy} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xy} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xy} + C_{yy}\frac{\partial \delta u}{\partial y} + C_{yz}\frac{\partial \delta u}{\partial z} + \delta c_{yy}\frac{\partial U}{\partial y} + \delta c_{yz}\frac{\partial U}{\partial z} + ik\delta v C_{xx} + C_{xz}\frac{\partial \delta v}{\partial z} + \delta c_{xy}\frac{\partial V}{\partial y} + \delta c_{xz}\frac{\partial V}{\partial z} - C_{xy}\frac{\partial \delta w}{\partial z} \quad (10)$$

XZ EQUATION:

$$\lambda\delta c_{xz} = -\frac{1}{W_i}\delta c_{xz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{xz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{xz} + C_{yz}\frac{\partial \delta u}{\partial y} + C_{zz}\frac{\partial \delta u}{\partial z} + \delta c_{yz}\frac{\partial U}{\partial y} + \delta c_{zz}\frac{\partial U}{\partial z} + ik\delta w C_{xx} + C_{xy}\frac{\partial \delta w}{\partial y} + \delta c_{xy}\frac{\partial W}{\partial y} + \delta c_{xz}\frac{\partial W}{\partial z} - C_{xz}\frac{\partial \delta v}{\partial y} \quad (11)$$

YZ EQUATION:

$$\lambda\delta c_{yz} = -\frac{1}{W_i}\delta c_{yz} - \left[ikU + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z}\right]\delta c_{yz} - \left[\delta v\frac{\partial}{\partial y} + \delta w\frac{\partial}{\partial z}\right]C_{yz} + ik\delta v C_{xz} + C_{zz}\frac{\partial \delta v}{\partial z} + \delta c_{zz}\frac{\partial V}{\partial z} + ik\delta w C_{xy} + C_{yy}\frac{\partial \delta w}{\partial y} + \delta c_{yy}\frac{\partial W}{\partial y} - ik\delta u C_{yz} \quad (12)$$