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Final Exam Part 2

1. Thesis Replication

Replication of *Acemoglu, D., Johnson, S., Robinson, J. A., Yared, P. (2008).* ***Income and democracy.*** *American Economic Review, 98(3), 808-42.*

1.0 Overview

This seminal paper investigates the relationship between income and democracy – a much-abused ideal in some sense. While previous literature do indeed confirm a clear cross-country correlation between income and democracy, they do not control for potential problems of endogeneity. This study shows that when these factors are factored out via IV estimation, the correlation between income and democracy disappear significantly.

My intention is to replicate columns (1) through (5) of **Table 2**, which regress a specific measure of democracy on various (lagged) dependent variables given various model specifications. It should be noted that 3 datasets that differ in frequency of observations are used for construction of the entire table, as can be confirmed by the three distinguished sections. My focus shall be on the first section, which utilizes a dataset with observations taken at 5-year intervals.

Table 2
Fixed Effects Results using Freedom House Measure of Democracy

	Base Sample, 1960-2000							
	5-year data					Annual data	10-year data	
	Pooled OLS	Fixed Effects OLS	Anderson-Hsiao IV	Arellano-Bond GMM	Fixed Effects OLS	Fixed Effects OLS	Fixed Effects OLS	Arellano-Bond GMM
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent Variable is Democracy							
Democracy $_{t-1}$	0.706	0.379	0.469	0.489		[0.00]	-0.025	0.226
	(0.035)	(0.051)	(0.100)	(0.085)			(0.088)	(0.123)
Log GDP per Capita $_{t-1}$	0.072	0.010	-0.104	-0.129	0.054	[0.33]	0.053	-0.318
	(0.010)	(0.035)	(0.107)	(0.076)	(0.046)		(0.066)	(0.180)
Hansen J Test				[0.26]				[0.07]
AR(2) Test				[0.45]				[0.96]
Observations	945	945	838	838	958	2895	457	338
Countries	150	150	127	127	150	148	127	118
R-squared	0.73	0.80			0.76	0.93	0.77	

1.1 Data

The measure of democracy – our main independent variable – is borrowed from the Freedom House Political Rights index, calculated from a set of questions that represent the various values of democracy. This is transformed to a scale of 0 to 1, with 1 being the most democratic. Log GDP per capita (in constant 1990 dollars) is used as the measure of income. The total dataset comprises of 211 nations across 1960-2000, resulting in 2,321 observations. Note that those observation without a designated democracy index or suffice lagged variable are dropped from estimation when necessary.

The final dataset used for estimation in this section is provided in an attached `.csv` file.

1.2 Empirical Model

The main empirical model that the authors construct is:

$$d_{it} = \alpha d_{it-1} + \gamma y_{it-1} + x'_{it-1}\beta + \mu_t + \delta_i + u_{it}$$

where d_{it} is the democracy score of country i in year t. Note that lagged values d_{it-1} on the right hand side are included to capture persistence in inertia. The main variable of interest is y_{it-1}, which denotes the log income of country i at time $t-1$.

In estimation, a variety of strategies are deployed to rigorously obtain the effect of income on democracy. Those include pooled OLS (column (1)), inclusion of fixed effects (column (2)), time differencing IV estimation (column (3)), and GMM estimation (column(4)). A detailed account of model specifications and their underlying assumptions can be found in Acemoglu et al.(2008).

1.3 Replication Results

Results of replication are given as follows. Note that coefficients for year fixed effects and country fixed effects are omitted here; full estimates can be found in the `R-script`.

Column (1):

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-0.3468646	0.0626704	-5.5347	4.049e-08	***
lag1_fhp	0.7063698	0.0282351	25.0175	< 2.2e-16	***
lag1_gdp	0.0723185	0.0093252	7.7552	2.303e-14	***

Column (2):

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-0.0416887	0.1547515	-0.2694	0.7876992	
lag1_fhp	0.3786284	0.0434607	8.7120	< 2.2e-16	***
lag1_gdp	0.0104150	0.0224057	0.4648	0.6421782	

Column (3):

```
                 Estimate Std. Error t value  Pr(>|t|)    
(Intercept)      0.061739   0.050635  1.2193 0.2230870    
dif2_fhp         0.468659   0.134997  3.4716 0.0005441 ***
dif2_gdp        -0.103579   0.278689 -0.3717 0.7102362    
```

Column (4)*:

```
               Estimate     Std. Error    t value       Pr(>|t|)
(Intercept)    -3.4297e-01   6.2823e-02   -5.4593e+00   4.7797e-08
lag1_fhp        6.9151e-01   3.2923e-02    2.1004e+01   6.0750e-98
lag1_gdp        7.3981e-02   9.8151e-03    7.5375e+00   4.7891e-14
```

Column (5):

```
                Estimate  Std. Error t value  Pr(>|t|)  
(Intercept)   -2.9759e-01  1.8053e-01 -1.6485 0.0996525 .
lag1_gdp       5.3588e-02  2.6143e-02  2.0498 0.0407088 *
```

1.4 Caveats

First, the inconsistency of standard errors in replication should be noted. I believe this lies on different algorithms on generating clustered standard errors. To generate standard errors clustered by country, I apply White's estimator by giving the argument `vcovHC(type="HC0")`. While this manipulates the standard errors in the correct direction, the exact values were as shown in the paper were difficult to obtain.

Another critical matter of concern is the inconsistency of the sample size with the results of **Table 2**. I believe that this comes from differences in (1) how *NA* values were handled and (2) the construction of lagged values. In estimation, observations with *NA* values are dropped *en bloc*. However, having problems with the `lag()` function, I choose to construct the lagged variables pre-regression, which may have resulted in said inconsistencies.

(*) For column (4), I was unable to obtain coefficients that resemble those of the authors'. I believe there may have been inaccurate specifications whilst constructing the arguments for the `gmm()` function.

2. $J = n\bar{m}_n(\hat{\theta}_{GMM})'\hat{A}^{-1}\bar{m}_n(\hat{\theta}_{GMM})$

wts $J \xrightarrow{d} \chi^2_{(L-K)}$

recall, $\hat{\beta}_{GMM} = \beta + (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'u$

plug in for $\hat{u} = Y - X\hat{\beta}_{GMM} = Y - X\beta - X(X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'u$

recall, $\bar{m}_n(\beta) = \frac{1}{n}Z'(Y - X\beta)$

therefore $\bar{m}_n(\hat{\beta}_{GMM}) = \frac{1}{n}Z'(Y - X\hat{\beta}_{GMM}) = \frac{1}{n}Z'\hat{u}$

plug in for $\bar{m}_n(\hat{\beta}) = \frac{1}{n}(Z'u - Z'X(X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'u)$

$= \frac{1}{n}[I - P_Z]Z'u$

$$J = n\frac{1}{n}u'Z[I-P_Z]\hat{A}^{-1}[I-P_Z]\frac{1}{n}Z'u$$

$$= \left[\hat{A}^{-\frac{1}{2}}[I-P_Z]\frac{1}{\sqrt{n}}Z'u\right]'\left[\hat{A}^{-\frac{1}{2}}[I-P_Z]\frac{1}{\sqrt{n}}Z'u\right]$$

$$\xrightarrow{d} A^{-\frac{1}{2}}[I-P_Z]N(0,A)\cdot A^{-\frac{1}{2}}[I-P_Z]N(0,A)$$

$$= N(0,I)\cdot[I-P_Z]\cdot N(0,I)$$

$$\sim \chi^2_{tr(I-P_Z)} = \chi^2_{L-K}.$$

QED

3. MLE

3.1 ARMAX(1,0,0)

Results of MLE are given as follows:

```
Maximum likelihood estimation

Call:
mle2(minuslogl = logftn, start = list(b0 = 0, b1 = 0, b2 = 2,
    sigma2 = 1))

Coefficients:
       Estimate Std. Error z value     Pr(z)    
b0     0.134991   0.035671  3.7844 0.0001541 ***
b1     0.508625   0.063638  7.9924 1.323e-15 ***
b2     2.476268   0.368487  6.7201 1.816e-11 ***
sigma2 0.094786   0.014342  6.6089 3.872e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

3.2 ARMAX(1,0,1)

Results of MLE are given as follows:

```
Maximum likelihood estimation

Call:
mle2(minuslogl = logftn, start = list(b0 = 0, b1 = 0, b2 = 2,
    b3 = 1, sigma2 = 1))

Coefficients:
        Estimate Std. Error z value     Pr(z)    
b0      0.081532   0.022943  3.5537 0.0003799 ***
b1      0.710273   0.052134 13.6240 < 2.2e-16 ***
b2      1.634233   0.284569  5.7428 9.310e-09 ***
b3     -0.407268   0.080766 -5.0426 4.593e-07 ***
sigma2  0.056614   0.010980  5.1559 2.525e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```