

Compvirt2020

January 6, 2021

```
[1]: from sympy import *  
import heppackv0 as hep
```

Reading heppackv0.py

Done

```
[2]: theta,u,t,t0=symbols('theta u t t0',real=True)  
E1,E2,p,M,s,s0=symbols('E1 E2 p M s s0',positive=True)
```

1 Kinematics

Amplitudes and cross section for Compton scattering $\gamma_V + e \rightarrow \gamma + e$ with an incoming virtual photon of mass M . Energy momentum conservation reads $k_i + p_i = k_f + p_f$. The CM system is used where E_2, θ denote the outgoing photon energy and scattering angle and E_1 the energy of the incoming electron. Obviously E_2 is also the energy of the outgoing electron and therefore $E_\gamma = 2E_2 - E_1$ is taken from energy conservation. With $t = -4E_1E_2 \sin^2(\theta/2)$, $u = -4E_1E_2 \cos^2(\theta/2)$ and $s = 4E_2^2$ we get

$$M^2 = s + t + u = 4E_2^2 - 4E_1E_2 ,$$

which in calling the Python routines is named M_0 . We also use the abbreviation $s_0 = s - M^2 = 4E_1E_2$.

```
[3]: M0=2*sqrt(E2**2-E1*E2);M0
```

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[3]:
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$$2\sqrt{-E_1E_2 + E_2^2}$$

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[4]: kin=[2*E2-E1,M0,0,0]  
kout=[E2,0,theta,0]  
pin=[E1,0,pi,pi]  
pout=[E2,0,pi-theta,pi]
```

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[5]: E1divE2=s0/s;E1divE2
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[5]:
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$$\frac{s_0}{s}$$

[6]: `costhetahalf=sqrt(-u/s0);costhetahalf`

[6]:
$$\frac{\sqrt{-u}}{\sqrt{s_0}}$$

[7]: `sinthetahalf=sqrt(-t/s0);sinthetahalf`

[7]:
$$\frac{\sqrt{-t}}{\sqrt{s_0}}$$

[8]: `Etwo=sqrt(s)/2;Etwo`

[8]:
$$\frac{\sqrt{s}}{2}$$

2 The Amplitudes

Due to the additional helicity $\lambda_1 = 0$ for the incoming photons we have now 12 independent amplitudes $T_{fi} = T_{\lambda_3 \lambda_4; \lambda_1 \lambda_2}$. The outgoing photon is real ($\lambda_3 = \pm 1$). Neglecting the mass of the electrons the calculation is easy.

$$T1 = T_{--;--} :$$

[9]: `h1=simplify(hep.compt(kin,-1,pin,-1,kout,-1,pout,-1));h1`

[9]:
$$\frac{4\sqrt{E_1} \cos\left(\frac{\theta}{2}\right)}{\sqrt{E_2} (\cos(\theta) + 1)}$$

[10]: `h2=2*sqrt(E1)/sqrt(E2)/cos(theta/2);h2`

[10]:
$$\frac{2\sqrt{E_1}}{\sqrt{E_2} \cos\left(\frac{\theta}{2}\right)}$$

[11]: `T1sq=4*E1divE2/costhetahalf**2;T1sq`

[11]:
$$-\frac{4s_0^2}{su}$$

$$T2 = T_{-+;--} :$$

[12]: `T2=simplify(hep.compt(kin,-1,pin,-1,kout,-1,pout,1));T2`

[12]:
$$0$$

$$T3 = T_{++;--} :$$

[13]: `T3=simplify(hep.compt(kin,-1,pin,-1,kout,1,pout,1));T3`

[13]:
$$0$$

$$T4 = T_{+-;--} :$$

[14]: `T4=simplify(hep.compt(kin,-1,pin,-1,kout,1,pout,-1));T4`

[14]:

$$\frac{4(E_1 - E_2) \sin^2\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)}{\sqrt{E_1} \sqrt{E_2} (\cos(\theta) + 1)}$$

[15]: `T4=simplify(2*sinthetahalf**2/costhetahalf*(E1divE2-1)/sqrt(E1divE2));T4`

[15]:

$$\frac{2t(s-s_0)}{\sqrt{ss_0}\sqrt{-u}}$$

[16]: `T4sq=simplify(T4**2);T4sq`

[16]:

$$-\frac{4t^2(s-s_0)^2}{ss_0^2u}$$

[17]: `T4sq=-4*t**2*M**4/s/s0**2/u;T4sq`

[17]:

$$-\frac{4M^4t^2}{ss_0^2u}$$

$$T5 = T_{--,+-} :$$

[18]: `T5=simplify(hep.compt(kin,1,pin,-1,kout,-1,pout,-1));T5`

[18]:

$$0$$

$$T6 = T_{-+;+-} :$$

[19]: `T6=simplify(hep.compt(kin,1,pin,-1,kout,-1,pout,1));T6`

[19]:

$$0$$

$$T7 = T_{++;+-} :$$

[20]: `T7=simplify(hep.compt(kin,1,pin,-1,kout,1,pout,1));T7`

[20]:

$$0$$

$$T8 = T_{+-;+-} :$$

[21]: `T8=simplify(hep.compt(kin,1,pin,-1,kout,1,pout,-1));T8`

[21]:

$$\frac{2\sqrt{E_2} \cos\left(\frac{\theta}{2}\right)}{\sqrt{E_1}}$$

[22]: `T8sq=4*E2*cos(theta/2)**2/E1;T8sq`

[22]:

$$\frac{4E_2 \cos^2\left(\frac{\theta}{2}\right)}{E_1}$$

[23]: `T8sq=simplify(4*cos(0.5*theta)**2/E1/E2);T8sq`

[23]:

$$-\frac{4su}{s_0^2}$$

$T_9 = T_{--,0-}$:

[24]: `T9=simplify(hep.compt(kin,0,pin,-1,kout,-1,pout,-1));T9`

[24]:

$$\frac{\sqrt{2} \left(4E_1 \sin^4\left(\frac{\theta}{2}\right) - 12E_1 \sin^2\left(\frac{\theta}{2}\right) + E_1 \sin^2(\theta) - 4E_1 \cos(\theta) + 4E_1 - 4E_2 \sin^4\left(\frac{\theta}{2}\right) + 4E_2 \sin^2\left(\frac{\theta}{2}\right) - E_2 \sin^2(\theta) \right) \sin(\theta)}{4\sqrt{E_1}\sqrt{-E_1+E_2}(\cos(\theta)+1)}$$

It is hard to convince Sympy that the terms in the brackets of the numerator add up to 0. One does it by hand by collecting all trigonometric function multiplying E_1 and E_2 separately and proving that both are identical 0.

[25]: `T9=0;T9`

[25]:

$$0$$

$T_{10} = T_{-+,0-}$:

[26]: `T10=simplify(hep.compt(kin,0,pin,-1,kout,-1,pout,1));T10`

[26]:

$$0$$

$T_{11} = T_{++,0-}$:

[27]: `T11=simplify(hep.compt(kin,0,pin,-1,kout,1,pout,1));T11`

[27]:

$$0$$

$T_{12} = T_{+-,0-}$:

[28]: `h3=simplify(hep.compt(kin,0,pin,-1,kout,1,pout,-1));h3`

[28]:

$$-\frac{2\sqrt{-2E_1+2E_2}\sin\left(\frac{\theta}{2}\right)}{\sqrt{E_1}}$$

[29]: `h4=-2*sqrt(2)*simplify(sqrt(1-E1/E2)*sin(0.5*theta)/sqrt(E1/E2));h4`

[29]:

$$-\frac{2\sqrt{2}\sqrt{-t}\sqrt{s-s_0}}{s_0}$$

[30]: `T12sq=h4**2;T12sq`

[30]:

$$-\frac{8t(s-s_0)}{s_0^2}$$

[31]: `T12sq=-8*t*M**2/s0**2;T12sq`

[31]:

$$-\frac{8M^2t}{s_0^2}$$

3 Cross section

[32]: `sig1=T1sq+T4sq+T8sq+T12sq;sig1`

[32]:

$$-\frac{4M^4t^2}{ss_0^2u} - \frac{8M^2t}{s_0^2} - \frac{4su}{s_0^2} - \frac{4s_0^2}{su}$$

[33]: `simplify(sig1.subs(t,-s0-u).subs(s0,s-M**2))`

[33]:

$$\frac{-8M^4 + 8M^2s + 8M^2u - 4s^2 - 4u^2}{su}$$

[34]: `sig=-8*M**2*t/s/u-4*s/u-4*u/s;sig`

[34]:

$$-\frac{8M^2t}{su} - \frac{4s}{u} - \frac{4u}{s}$$

[]: