ggtogg

March 15, 2021

[1]:
```
from sympy import *
import heppackv0 as hep
```

```
Reading heppackv0.py

Done
```

[2]:
```
s,theta=symbols('s theta',real=True)
p=symbols('p',positive=True)
u,t,s=symbols('u t s',real=True)
```

[3]:
```
p1=[p,0,0,0]
p2=[p,0,pi,pi]
p3=[p,0,theta,0]
p4=[p,0,pi-theta,pi]
```

[4]:
```
sCM=4*p*p;sCM
```

[4]:

$$4p^2$$

[5]:
```
tCM=-sCM*sin(theta/2)**2;tCM
```

[5]:

$$-4p^2\sin^2\left(\frac{\theta}{2}\right)$$

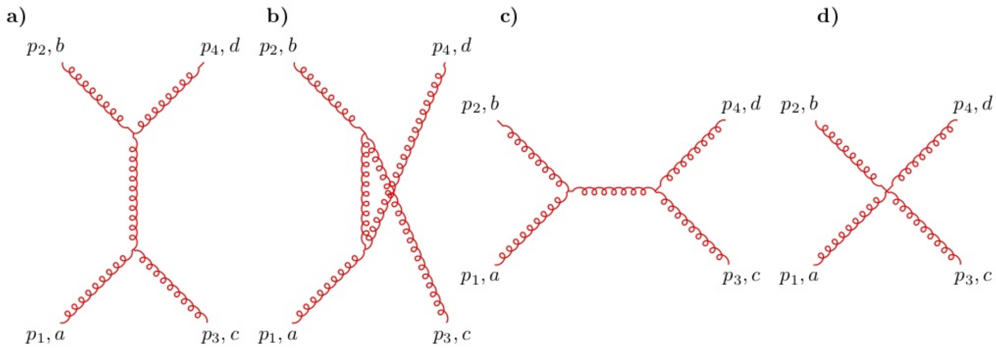
[6]:
```
uCM=-sCM*cos(theta/2)**2;uCM
```

[6]:

$$-4p^2\cos^2\left(\frac{\theta}{2}\right)$$

Basics

There are 4 Feynman-Diagrams: a) t-channel, b) u-channel, c) s-channel, d) Four-Gluon-Vertex with their momenta and color indices:



The color factors of the four-gluon Vertex correspond exactly to the color factors of the separate three-gluon-channels. The package heppackv0.py provides the necessary routines, which (hopefully) need no further comments. V3gtchannel calculates e.g. the amplitude a). There are 16 helicity amplitudes, 8 amplitudes are independent, further 8 amplitudes are related by parity invariance. As usual we omitt the coupling g_S^2

The amplitudes

*

1) $T(g_-g_+ \to g_-g_-)$

[7]:
```
s11=hep.V3gschannel(p1,-1,p2,1,p3,-1,p4,-1)/sCM;s11
```

[7]:

$$0$$

[8]:
```
s12=hep.V4gschannel(p1,-1,p2,1,p3,-1,p4,-1);s12
```

[8]:

$$0$$

[9]:
```
t11=hep.V3gtchannel(p1,-1,p2,1,p3,-1,p4,-1)/tCM;t11
```

[9]:

$$-\frac{(-\cos(\theta)+1)\sin^2(\theta)}{8\sin^2\left(\frac{\theta}{2}\right)}$$

[10]:
```
t12=hep.V4gtchannel(p1,-1,p2,1,p3,-1,p4,-1);t12
```

[10]:

$$\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}-\frac{1}{2}\right)$$

[11]:
```
simplify(t11+t12)
```

[11]:

$$0$$

[12]:
```
u11=hep.V3guchannel(p1,-1,p2,1,p3,-1,p4,-1)/uCM;u11
```

[12]:

$$-\frac{(\cos(\theta)+1)\sin^2(\theta)}{8\cos^2\left(\frac{\theta}{2}\right)}$$

[13]:
```
u12=hep.V4guchannel(p1,-1,p2,1,p3,-1,p4,-1);u12
```

[13]:

$$-\left(-\frac{\cos\left(\theta\right)}{2}-\frac{1}{2}\right)\left(-\frac{\cos\left(\theta\right)}{2}+\frac{1}{2}\right)$$

[14]:
```
simplify(u11+u12)
```

[14]:

$$0$$

i.e. $T(g_-g_+ \to g_-g_-) = 0$

2) $T(g_-g_+ \to g_-g_+)$

[15]:
```
s21=hep.V3gschannel(p1,-1,p2,1,p3,-1,p4,1)/sCM;s21
```

[15]:

$$0$$

[16]:
```
s22=hep.V4gschannel(p1,-1,p2,1,p3,-1,p4,1);s22
```

[16]:

$$0$$

i.e. $T^s(g_-g_+ \to g_-g_+) = 0$

[17]:
```
t21=hep.V3gtchannel(p1,-1,p2,1,p3,-1,p4,1)/tCM;t21
```

[17]:

$$-\frac{-5\sin^2\left(\theta\right)+\cos^3\left(\theta\right)+7\cos\left(\theta\right)+8}{8\sin^2\left(\frac{\theta}{2}\right)}$$

[18]:
```
t22=hep.V4gtchannel(p1,-1,p2,1,p3,-1,p4,1);t22
```

[18]:

$$\left(-\frac{\cos\left(\theta\right)}{2}-\frac{1}{2}\right)\left(\frac{\cos\left(\theta\right)}{2}+\frac{1}{2}\right)$$

[19]:
```
a1=simplify(t21+t22);a1
```

[19]:

$$\frac{4\cos\left(\theta\right)+\cos\left(2\theta\right)+3}{2\cos\left(\theta\right)-2}$$

a_1 should be expressed by invariants. The proof of the following formula is easiest done by hand.

[20]:
```
a1=2*u*u/t/s;a1
```

[20]:

$$\frac{2u^2}{st}$$

i.e. $T^t(g_-g_+ \to g_-g_+) = \frac{2u^2}{ts}$

[21]:
```
u21=hep.V3guchannel(p1,-1,p2,1,p3,-1,p4,1)/uCM;u21
```

[21]:

$$-\frac{\frac{17\cos(\theta)}{2}+\cos(2\theta)-\frac{\cos(3\theta)}{2}+7}{16\cos^2\left(\frac{\theta}{2}\right)}$$

[22]:
```
u22=hep.V4guchannel(p1,-1,p2,1,p3,-1,p4,1);u22
```

[22]:

$$-\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)^2$$

[23]:
```
simplify(u21+u22)
```

[23]:

$$-\frac{4\cos(\theta)+\cos(2\theta)+3}{2\cos(\theta)+2}$$

[24]:
```
a2=2*u/s;a2
```

[24]:

$$\frac{2u}{s}$$

i.e. $T^u(g_-g_+\to g_-g_+)=\frac{2u}{s}$
The following routine shows the nonvanishing color factors for $a=1$, which in python means $a=0$.

[25]:
```
lamvec=[hep.lam1,hep.lam2,hep.lam3,hep.lam4,hep.lam5,hep.lam6,hep.lam7,hep.lam8]
a=0
ct=0
cu=0
cs=0
reslst=[]
#
for b in range(8):
    for c in range(8):
        for d in range(8):
            for i in range(8):
                ctm=hep.fsu3(a,c,i)*hep.fsu3(b,d,i)
                ct=ct+ctm
                cum=hep.fsu3(a,d,i)*hep.fsu3(b,c,i)
                cu=cu+cum
                csm=hep.fsu3(a,b,i)*hep.fsu3(c,d,i)
                cs=cs+csm
            if ct!=0 or cu!=0 or cs!=0:
                print(b+1,c+1,d+1,ct,cu,cs)
            ct=0
            cu=0
            cs=0
```

```
1 2 2 1 1 0
1 3 3 1 1 0
```

```
1 4 4 1/4 1/4 0
1 5 5 1/4 1/4 0
1 6 6 1/4 1/4 0
1 7 7 1/4 1/4 0
2 1 2 0 -1 1
2 2 1 -1 0 -1
2 4 5 1/4 -1/4 1/2
2 5 4 -1/4 1/4 -1/2
2 6 7 -1/4 1/4 -1/2
2 7 6 1/4 -1/4 1/2
3 1 3 0 -1 1
3 3 1 -1 0 -1
3 4 6 -1/4 1/4 -1/2
3 5 7 -1/4 1/4 -1/2
3 6 4 1/4 -1/4 1/2
3 7 5 1/4 -1/4 1/2
4 1 4 0 -1/4 1/4
4 2 5 1/2 1/4 1/4
4 3 6 -1/2 -1/4 -1/4
4 4 1 -1/4 0 -1/4
4 5 2 1/4 1/2 -1/4
4 6 3 -1/4 -1/2 1/4
4 6 8 -sqrt(3)/4 0 -sqrt(3)/4
4 8 6 0 -sqrt(3)/4 sqrt(3)/4
5 1 5 0 -1/4 1/4
5 2 4 -1/2 -1/4 -1/4
5 3 7 -1/2 -1/4 -1/4
5 4 2 -1/4 -1/2 1/4
5 5 1 -1/4 0 -1/4
5 7 3 -1/4 -1/2 1/4
5 7 8 -sqrt(3)/4 0 -sqrt(3)/4
5 8 7 0 -sqrt(3)/4 sqrt(3)/4
6 1 6 0 -1/4 1/4
6 2 7 -1/2 -1/4 -1/4
6 3 4 1/2 1/4 1/4
6 4 3 1/4 1/2 -1/4
6 4 8 -sqrt(3)/4 0 -sqrt(3)/4
6 6 1 -1/4 0 -1/4
6 7 2 -1/4 -1/2 1/4
6 8 4 0 -sqrt(3)/4 sqrt(3)/4
7 1 7 0 -1/4 1/4
7 2 6 1/2 1/4 1/4
7 3 5 1/2 1/4 1/4
7 5 3 1/4 1/2 -1/4
7 5 8 -sqrt(3)/4 0 -sqrt(3)/4
7 6 2 1/4 1/2 -1/4
7 7 1 -1/4 0 -1/4
7 8 5 0 -sqrt(3)/4 sqrt(3)/4
```

```
8 4 6 sqrt(3)/4 sqrt(3)/4 0
8 5 7 sqrt(3)/4 sqrt(3)/4 0
8 6 4 sqrt(3)/4 sqrt(3)/4 0
8 7 5 sqrt(3)/4 sqrt(3)/4 0
```

All non vanishing squared matrix elements $|c_t T^t + c_u T^u|^2$ for $a = 1$ are stored in the array reslst. The final result has to be multiplied by 8.

[26]:
```
lamvec=[hep.lam1,hep.lam2,hep.lam3,hep.lam4,hep.lam5,hep.lam6,hep.lam7,hep.lam8]
a=0
ct=0
cu=0
#cs=0
reslst=[]
#
for b in range(8):
    for c in range(8):
        for d in range(8):
            for i in range(8):
                ctm=hep.fsu3(a,c,i)*hep.fsu3(b,d,i)
                ct=ct+ctm
                cum=hep.fsu3(a,d,i)*hep.fsu3(b,c,i)
                cu=cu+cum
                #csm=hep.fsu3(a,b,i)*hep.fsu3(c,d,i)
                #cs=cs+csm
            if ct!=0 or cu!=0:
                elm=a1*ct+a2*cu
                #a1print(b,c,d,ct,cu)
                reslst.append(elm*elm)
            ct=0
            cu=0
```

[27]:
```
reslst
```

[27]:

$$\left[\left(\frac{2u}{s} + \frac{2u^2}{st}\right)^2, \ \left(\frac{2u}{s} + \frac{2u^2}{st}\right)^2, \ \left(\frac{u}{2s} + \frac{u^2}{2st}\right)^2, \ \left(\frac{u}{2s} + \frac{u^2}{2st}\right)^2, \ \left(\frac{u}{2s} + \frac{u^2}{2st}\right)^2, \ \left(\frac{u}{2s} + \frac{u^2}{2st}\right)^2, \ \frac{4u^2}{s^2}, \ \frac{4}{s^2}\right.$$

[28]:
```
simplify(sum(reslst))
```

[28]:

$$\frac{36u^2\left(t^2 + tu + u^2\right)}{s^2t^2}$$

Using $2ut = s^2 - t^2 - u^2$ and multiplying by 8 we get the result for $|T(g_- g_+ \to g_- g_+)|^2$

[29]:
```
144*u*u*(s*s+t*t+u*u)/s/s/t/t
```

[29]:

$$\frac{144u^2\left(s^2 + t^2 + u^2\right)}{s^2t^2}$$

3) $T(g_- g_+ \to g_+ g_-)$

[30]:
```
s31=hep.V3gschannel(p1,-1,p2,1,p3,1,p4,-1)/sCM;s31
```

[30]:

$$0$$

[31]:
```
s32=hep.V4gschannel(p1,-1,p2,1,p3,1,p4,-1);s32
```

[31]:

$$0$$

[32]:
```
t31=hep.V3gtchannel(p1,-1,p2,1,p3,1,p4,-1)/tCM;t31
```

[32]:

$$-\frac{-\frac{17\cos(\theta)}{2}+\cos(2\theta)+\frac{\cos(3\theta)}{2}+7}{16\sin^2\left(\frac{\theta}{2}\right)}$$

[33]:
```
t32=hep.V4gtchannel(p1,-1,p2,1,p3,1,p4,-1);t32
```

[33]:

$$\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}-\frac{1}{2}\right)$$

[34]:
```
simplify(t31+t32)
```

[34]:

$$\frac{-\sin^2(\theta)-2\cos(\theta)+2}{\cos(\theta)-1}$$

[35]:
```
b1=-2*t/s;b1
```

[35]:

$$-\frac{2t}{s}$$

[36]:
```
u31=hep.V3guchannel(p1,-1,p2,1,p3,1,p4,-1)/uCM;u31
```

[36]:

$$-\frac{-5\sin^2(\theta)-\cos^3(\theta)-7\cos(\theta)+8}{8\cos^2\left(\frac{\theta}{2}\right)}$$

[37]:
```
u32=hep.V4guchannel(p1,-1,p2,1,p3,1,p4,-1);u32
```

[37]:

$$-\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)^2$$

[38]:
```
simplify(u31+u32)
```

[38]:

$$\frac{\sin^2(\theta)+2\cos(\theta)-2}{\cos(\theta)+1}$$

[39]:
```
b2=-2*t *t/u/s;b2
```

[39]:

$$-\frac{2t^2}{su}$$

[40]:
```
lamvec=[hep.lam1,hep.lam2,hep.lam3,hep.lam4,hep.lam5,hep.lam6,hep.lam7,hep.lam8]
a=0
ct=0
cu=0
#cs=0
reslst=[]
#
for b in range(8):
    for c in range(8):
        for d in range(8):
            for i in range(8):
                ctm=hep.fsu3(a,c,i)*hep.fsu3(b,d,i)
                ct=ct+ctm
                cum=hep.fsu3(a,d,i)*hep.fsu3(b,c,i)
                cu=cu+cum
                #csm=hep.fsu3(a,b,i)*hep.fsu3(c,d,i)
                #cs=cs+csm
            if ct!=0 or cu!=0:
                elm=b1*ct+b2*cu
                #a1print(b,c,d,ct,cu)
                reslst.append(elm*elm)
            ct=0
            cu=0
```

[41]:
```
simplify(sum(reslst))
```

[41]:

$$\frac{36t^2\left(t^2+tu+u^2\right)}{s^2u^2}$$

i.e. $|T(g_-g_+ \to g_+g_-)|^2$ is given by

[42]:
```
144*t*t*(s*s+t*t+u*u)/s/s/u/u
```

[42]:

$$\frac{144t^2\left(s^2+t^2+u^2\right)}{s^2u^2}$$

4) $T(g_-g_+ \to g_+g_+)$

[43]:
```
s41=hep.V3gschannel(p1,-1,p2,1,p3,1,p4,1)/sCM;s41
```

[43]:

$$0$$

[44]:
```
s42=hep.V4gschannel(p1,-1,p2,1,p3,1,p4,1);s42
```

[44]:

$$0$$

[45]:
```
t41=hep.V3gtchannel(p1,-1,p2,1,p3,1,p4,1)/tCM;t41
```
[45]:

$$-\frac{(-\cos(\theta)+1)\sin^2(\theta)}{8\sin^2\left(\frac{\theta}{2}\right)}$$

[46]:
```
t42=hep.V4gtchannel(p1,-1,p2,1,p3,1,p4,1);t42
```
[46]:

$$\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}+\frac{1}{2}\right)$$

[47]:
```
simplify(t41+t42)
```
[47]:

$$0$$

[48]:
```
u41=hep.V3guchannel(p1,-1,p2,1,p3,1,p4,1)/uCM;u41
```
[48]:

$$-\frac{(\cos(\theta)+1)\sin^2(\theta)}{8\cos^2\left(\frac{\theta}{2}\right)}$$

[49]:
```
u42=hep.V4guchannel(p1,-1,p2,1,p3,1,p4,1);u42
```
[49]:

$$-\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)$$

[50]:
```
simplify(u41+u42)
```
[50]:

$$0$$

i.e. $T(g_-g_+ \to g_+g_+) = 0$.
Now follows the most complicated 3rd non vanishing amplitude

5) $T(g_-g_- \to g_-g_-)$

[51]:
```
s51=hep.V3gschannel(p1,-1,p2,-1,p3,-1,p4,-1)/sCM;s51
```
[51]:

$$\cos(\theta)$$

[52]:
```
s52=hep.V4gschannel(p1,-1,p2,-1,p3,-1,p4,-1);s52
```
[52]:

$$\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)^2-\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)^2$$

[53]:
```
simplify(s51+s52)
```
[53]:

$$2\cos(\theta)$$

[54]:
```
t51=hep.V3gtchannel(p1,-1,p2,-1,p3,-1,p4,-1)/tCM;t51
```

[54]:

$$-\frac{31\cos(\theta)-22\cos(2\theta)+\cos(3\theta)+54}{32\sin^2\left(\frac{\theta}{2}\right)}$$

[55]:
```
t52=hep.V4gtchannel(p1,-1,p2,-1,p3,-1,p4,-1);t52
```

[55]:

$$\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}-\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}+1$$

[56]:
```
simplify(t51+t52)
```

[56]:

$$\frac{-2\cos(\theta)+\cos(2\theta)-3}{-\cos(\theta)+1}$$

[57]:
```
u51=hep.V3guchannel(p1,-1,p2,-1,p3,-1,p4,-1)/uCM;u51
```

[57]:

$$-\frac{-31\cos(\theta)-22\cos(2\theta)-\cos(3\theta)+54}{32\cos^2\left(\frac{\theta}{2}\right)}$$

[58]:
```
u52=hep.V4guchannel(p1,-1,p2,-1,p3,-1,p4,-1);u52
```

[58]:

$$-\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)^2+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}+1$$

[59]:
```
simplify(u51+u52)
```

[59]:

$$\frac{2\cos(\theta)+\cos(2\theta)-3}{\cos(\theta)+1}$$

[60]:
```
c1=2*u*u/t/s+6*u/s;c1
```

[60]:

$$\frac{6u}{s}+\frac{2u^2}{st}$$

[61]:
```
c2=2*t*t/u/s+6*t/s;c2
```

[61]:

$$\frac{2t^2}{su}+\frac{6t}{s}$$

[62]:
```
c3=-2*u/s+2*t/s;c3
```

[62]:

$$\frac{2t}{s}-\frac{2u}{s}$$

[63]:
```
lamvec=[hep.lam1,hep.lam2,hep.lam3,hep.lam4,hep.lam5,hep.lam6,hep.lam7,hep.lam8]
a=0
ct=0
```

```python
cu=0
cs=0
reslst=[]
#
for b in range(8):
    for c in range(8):
        for d in range(8):
            for i in range(8):
                ctm=hep.fsu3(a,c,i)*hep.fsu3(b,d,i)
                ct=ct+ctm
                cum=hep.fsu3(a,d,i)*hep.fsu3(b,c,i)
                cu=cu+cum
                csm=hep.fsu3(a,b,i)*hep.fsu3(c,d,i)
                cs=cs+csm
            if ct!=0 or cu!=0 or cs!=0:
                elm=c1*ct+c2*cu+c3*cs
                #a1print(b,c,d,ct,cu)
                reslst.append(elm*elm)
            ct=0
            cu=0
            cs=0
```

[64]:
```python
h3=simplify(sum(reslst));h3
```

[64]:

$$\frac{36\left(t^6+5t^5u+11t^4u^2+14t^3u^3+11t^2u^4+5tu^5+u^6\right)}{s^2t^2u^2}$$

[65]:
```python
h4=h3*s**2*t**2*u**2/36;h4
```

[65]:

$$t^6+5t^5u+11t^4u^2+14t^3u^3+11t^2u^4+5tu^5+u^6$$

[66]:
```python
h5=simplify(h4.subs(t,-s-u));h5
```

[66]:

$$s^4\left(s^2+su+u^2\right)$$

[67]:
```python
final=36*h5/s**2/t**2/u**2;final
```

[67]:

$$\frac{36s^2\left(s^2+su+u^2\right)}{t^2u^2}$$

i.e. $|T(g_-g_- \to g_-g_-)|^2$ is given by

[68]:
```python
144*s*s*(s*s+t*t+u*u)/t**2/u**2
```

[68]:

$$\frac{144s^2\left(s^2+t^2+u^2\right)}{t^2u^2}$$

6) $T(g_-g_- \to g_-g_+)$

[69]:
```
s61=hep.V3gschannel(p1,-1,p2,-1,p3,-1,p4,1)/sCM;s61
```

[69]:

$$0$$

[70]:
```
s62=hep.V4gschannel(p1,-1,p2,-1,p3,-1,p4,1);s62
```

[70]:

$$0$$

[71]:
```
t61=hep.V3gtchannel(p1,-1,p2,-1,p3,-1,p4,1)/tCM;t61
```

[71]:

$$-\frac{(-\cos(\theta)+1)\sin^2(\theta)}{8\sin^2\left(\frac{\theta}{2}\right)}$$

[72]:
```
t62=hep.V4gtchannel(p1,-1,p2,-1,p3,-1,p4,1);t62
```

[72]:

$$\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}+\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}$$

[73]:
```
simplify(t61+t62)
```

[73]:

$$0$$

[74]:
```
u61=hep.V3guchannel(p1,-1,p2,-1,p3,-1,p4,1)/uCM;u61
```

[74]:

$$-\frac{(\cos(\theta)+1)\sin^2(\theta)}{8\cos^2\left(\frac{\theta}{2}\right)}$$

[75]:
```
u62=hep.V4guchannel(p1,-1,p2,-1,p3,-1,p4,1);u62
```

[75]:

$$-\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}$$

[76]:
```
simplify(u61+u62)
```

[76]:

$$0$$

i.e. $T(g_-g_- \to g_-g_+) = 0$

7) $T(g_-g_- \to g_+g_-)$

[77]:
```
s71=hep.V3gschannel(p1,-1,p2,-1,p3,1,p4,-1)/sCM;s71
```

[77]:

$$0$$

[78]:
```
h2=hep.V3gOutOut(p3,1,p4,-1);h2
```

[78]:

$$\left[0,\quad -4p\left(-\sin^4\left(\frac{\theta}{2}\right)+\sin^2\left(\frac{\theta}{2}\right)-\frac{\sin^2(\theta)}{4}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right),\quad 0,\quad \left(-p\left(-\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)\right)+p\left(\right.\right.$$

[79]:
```
s72=simplify(hep.V4gschannel(p1,-1,p2,-1,p3,1,p4,-1));s72
```

[79]:

$$0$$

[80]:
```
t71=hep.V3gtchannel(p1,-1,p2,-1,p3,1,p4,-1)/tCM;t71
```

[80]:

$$-\frac{(-\cos(\theta)+1)\sin^2(\theta)}{8\sin^2\left(\frac{\theta}{2}\right)}$$

[81]:
```
t72=hep.V4gtchannel(p1,-1,p2,-1,p3,1,p4,-1);t72
```

[81]:

$$\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}-\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}$$

[82]:
```
simplify(t71+t72)
```

[82]:

$$0$$

[83]:
```
u71=hep.V3guchannel(p1,-1,p2,-1,p3,1,p4,-1)/uCM;u71
```

[83]:

$$-\frac{(\cos(\theta)+1)\sin^2(\theta)}{8\cos^2\left(\frac{\theta}{2}\right)}$$

[84]:
```
u72=hep.V4guchannel(p1,-1,p2,-1,p3,1,p4,-1);u72
```

[84]:

$$-\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}$$

[85]:
```
simplify(u71+u72)
```

[85]:

$$0$$

i.e. $T(g_-g_-\to g_+g_-)=0$

8) $T(g_-g_-\to g_+g_+)$

[86]:
```
s81=hep.V3gschannel(p1,-1,p2,-1,p3,1,p4,1)/sCM;s81
```

[86]:

$$\cos(\theta)$$

[87]:
```
s82=simplify(hep.V4gschannel(p1,-1,p2,-1,p3,1,p4,1));s82
```

[87]:

$$-\cos(\theta)$$

[88]:
```
t81=hep.V3gtchannel(p1,-1,p2,-1,p3,1,p4,1)/tCM;t81
```

[88]:

$$-\frac{-\frac{17\cos(\theta)}{2}+\cos(2\theta)+\frac{\cos(3\theta)}{2}+7}{16\sin^2\left(\frac{\theta}{2}\right)}$$

[89]:
```
t82=hep.V4gtchannel(p1,-1,p2,-1,p3,1,p4,1);t82
```

[89]:

$$\left(-\frac{\cos(\theta)}{2}-\frac{1}{2}\right)\left(\frac{\cos(\theta)}{2}+\frac{1}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}+1$$

[90]:
```
simplify(t81+t82)
```

[90]:

$$0$$

[91]:
```
u81=hep.V3guchannel(p1,-1,p2,-1,p3,1,p4,1)/uCM;u81
```

[91]:

$$-\frac{\frac{17\cos(\theta)}{2}+\cos(2\theta)-\frac{\cos(3\theta)}{2}+7}{16\cos^2\left(\frac{\theta}{2}\right)}$$

[92]:
```
u82=hep.V4guchannel(p1,-1,p2,-1,p3,1,p4,1);u82
```

[92]:

$$-\left(-\frac{\cos(\theta)}{2}+\frac{1}{2}\right)^2+\sin^4\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)+\frac{\sin^2(\theta)}{4}+1$$

[93]:
```
simplify(u81+u82)
```

[93]:

$$0$$

i.e. $T(g_-g_-\to g_+g_+)=0$

The cross section

Summing up the squared amplitudes, multiplying by 2 for the remaining 8 amplitudes, dividing by 4 for averaging over polarizations and dividing by 64 for averaging of colors we obtain the averaged sum of squared matrix elements

[94]:
```
sig=(s**2/t**2/u**2+t**2/s**2/u**2+u**2/t**2/s**2)*(s**2+t**2+u**2)*9/8;sig
```

[94]:

$$\frac{9\left(s^2+t^2+u^2\right)\left(\frac{s^2}{t^2u^2}+\frac{t^2}{s^2u^2}+\frac{u^2}{s^2t^2}\right)}{8}$$

[95]:
```
sigalt=(3-t*u/s**2-s*u/t**2-s*t/u**2);sigalt
```

[95]:

$$-\frac{st}{u^2}-\frac{su}{t^2}+3-\frac{tu}{s^2}$$

[96]:
```
h7=sig-sigalt*9/2
simplify(h7.subs(s,-t-u))
```

[96]:

$$0$$

[ ]:
```

```