eeWWHE

December 17, 2021

[1]:
```
from sympy import *
import heppackv0 as hep
```

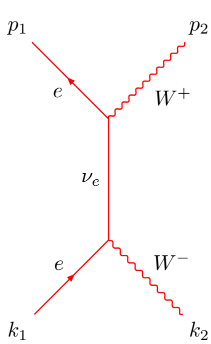
```
Reading heppackv0.py

Done
```

[2]:
```
E1,M1,m,g=symbols('E1 M1 m g',positive=True)
theta,c1,c2=symbols('theta c1 c2',real=True)
```

Production of longitudinal W-Bosons, investigation of amplitudes violating unitarity.

1) Standard weak interaction



$$e^-_+(k_1) + e^+_+(p_1) \to W^-_0(k_2) + W^+_0(p_2)$$

[3]:
```
k1=[E1,m,0,0]
p1=[E1,m,pi,pi]
k2=[E1,M1,theta,0]
p2=[E1,M1,pi-theta,pi]
```

The amplitude is calculated from
$-\imath T_{fi} = \bar{v}(p_1)\frac{-\imath g}{\sqrt{2}}\gamma^\mu\frac{1-\gamma_5}{2}\epsilon^*_\mu(p_2)\frac{\imath \not{q}}{q^2}\epsilon^*_\nu(k_2)\frac{-\imath g}{\sqrt{2}}\gamma^\nu\frac{1-\gamma_5}{2}u(k_1)$
$T_{fi} = \frac{g^2}{2}\bar{v}(p_1)\frac{1+\gamma_5}{2}\ \not{\epsilon}^*(p_2)\frac{\not{q}}{q^2}\ \not{\epsilon}^*(k_2)\frac{1-\gamma_5}{2}u(k_1)$
valid for all helicities of the fermions and bosons.

[4]:
```
kin=hep.fourvec(k1)
kout=hep.fourvec(k2)
```

[5]:
```
qdag=hep.dag(kout)-hep.dag(kin)
```

[6]:
```
epsk2dag=hep.dag(hep.polbar(k2,0))
epsp2dag=hep.dag(hep.polbar(p2,0))
```

[7]:
```
core=epsp2dag*qdag*epsk2dag
```

Evaluate $(q^2 2/g^2)T_{fi}$ for the helicities quoted above after line [2]

[8]:
```
T1v1=simplify(hep.vbar(p1,1)*hep.projpl*core*hep.projm*hep.u(k1,1));T1v1
```

[8]:

$$\Big[-\frac{m\Big(4E_1^2\sqrt{E_1^2-M_1^2}\sin^2\left(\frac{\theta}{2}\right)\sin^2(\theta)+2E_1^2\sqrt{E_1^2-M_1^2}\sin^2(\theta)\cos(\theta)-2E_1^2\sqrt{E_1^2-M_1^2}\sin^2(\theta)+2E_1^2\sqrt{E_1^2-m^2}\sin^2(\theta)-2M_1^2\sqrt{E_1^2-M_1^2}\sin^2\left(\frac{\theta}{2}\right)+M_1^2\sqrt{E_1^2-}}{M_1^2}$$

multiply by $M_1^2 = M_W^2$ and divide by m in order to get the high energy limit of the long expression in brackets.

[9]:
```
T1v2=simplify((T1v1*M1**2/m).subs(m,0).subs(M1,0));T1v2
```

[9]:

$$\left[-2E_1^3\sin^2(\theta)\right]$$

Divide by $q^2 = -2E_1^2(1-\cos\theta)$ and restore the factors $m, M_1^2, g^2/2$. Sympy cannot simplify $\sin^2\theta/(1-\cos\theta)$ to $(1+\cos\theta)$, therefore next line inserted by hand.

[10]:
```
T1=g**2/2*m/M1**2*E1*(1+cos(theta));T1
```

[10]:

$$\frac{E_1 g^2 m(\cos(\theta)+1)}{2M_1^2}$$

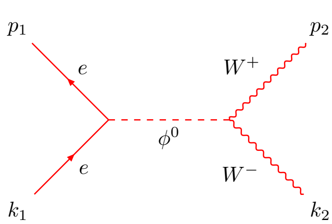
[11]:
```
T10=g**2*m*E1/2/M1**2;T10
```

[11]:

$$\frac{E_1 g^2 m}{2M_1^2}$$

Amplitude grows $\sim E_1$. The isotropic part $T_{1,0} = g^2 m E_1/2M_1^2$ may be cancelled by a scalar particle s-channel amplitude.

2) New particle in s-channel



$$e^-_+(k_1) + e^+_+(p_1) \to W^-_0(k_2) + W^+_0(p_2)$$

$-\imath T_{fi} = \bar{v}(k_1)u(p_1)(-\imath g_{\phi ee})\frac{-\imath}{s-M_\phi^2}(-\imath g_{\phi WW})g^{\mu\nu}\epsilon_\mu^*(k_2)\epsilon_\nu^*(p_2)$
$T_{fi} = -g_{\phi ee}g_{\phi WW}\bar{v}(k_1)u(p_1)\epsilon^*(k_2)\cdot\epsilon^*(p_2)/(s-M_\phi^2)$

[12]:
```
T2v1=hep.vbar(k1,1)*hep.u(p1,1)*hep.dotprod4(hep.polbar(k2,0),hep.polbar(p2,0));
↪T2v1
```

[12]:

$$\left[2\sqrt{E_1-m}\sqrt{E_1+m}\left(\frac{2E_1^2}{M_1^2}-1\right)\right]$$

Divide by propagator $4E_1^2 - M_\phi^2$ and get the high energy limit of the amplitude Amplitude without sign an couplings

[13]:
```
T2v2=E1/M1**2;T2v2
```

[13]:

$$\frac{E_1}{M_1^2}$$

$T_2 = -g_{\phi ee}g_{\phi WW}E_1/M_1^2$.
$T_{1,0} + T_2 = 0$ for $g_{\phi ee}g_{\phi WW} = T_{1,0}/T_{2,v2}$
$c_{12} = g_{\phi ee}g_{\phi WW}$

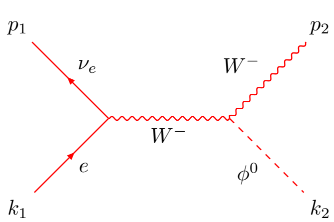
[14]:
```
c12=simplify(T10/T2v2);c12
```

[14]:

$$\frac{g^2 m}{2}$$

T_2 determines only the product of the scalar couplings. The study of $W^-\phi^0$ production helps to calculate $g_{\phi ee}$ and $g_{\phi WW}$ separately.

3) $\bar{v}e \to W^-\phi^0$ via s-channel



$$e_+^-(k_1) + \bar{v}(p_\nu) \to W_0^-(p_2) + \phi^0(k_2)$$

$-\imath T_{fi} = \bar{v}(p_\nu)\frac{-\imath g}{\sqrt{2}}\gamma^\mu\frac{1-\gamma_5}{2}u(k_1)(-\imath)\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{s-M_W^2}(-\imath g_{\phi WW})g^{\nu\sigma}\epsilon_\sigma^*(p_2)$
$T_{fi} = -\frac{g}{\sqrt{2}}j_w^\mu\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{s-M_W^2}g_{\phi WW}\epsilon^{*\nu}(p_2)$
with the weak current j_w given by
$j_w^\mu = \bar{v}(p_\nu)\gamma^\mu\frac{1-\gamma_5}{2}u(k_1)$

[15]:
```
pnu=[E1,0,pi,pi]
```

[16]:
```
jweak=hep.vbuw(pnu,1,k1,1);jweak
```

[16]:

$$\left[\sqrt{E_1}\left(\sqrt{E_1-m}-\sqrt{E_1+m}\right),\quad 0,\quad 0,\quad \sqrt{E_1}\left(-\sqrt{E_1-m}+\sqrt{E_1+m}\right)\right]$$

With $\sqrt{E_1+m} \to \sqrt{E_1}\left(1+\frac{m}{2E_1}\right)$ this reduces at high energies to

[17]:
```
jweak=[-m,0,0,m];jweak
```

[17]:

$$[-m, \quad 0, \quad 0, \quad m]$$

which does not depend on E_1. Therefore $j^{\mu}_{w} g_{\mu\nu} \epsilon^{*\nu}/s$ is $\sim 1/E_1$ whereas the second term in the propagator makes the amplitude grow $\sim E_1$.

[18]:
```
qvec=[2*E1,0,0,0];qvec
```

[18]:

$$[2E_1, \quad 0, \quad 0, \quad 0]$$

[19]:
```
-hep.dotprod4(jweak,qvec)*hep.dotprod4(qvec,hep.polbar(p2,0))/M1**2
```

[19]:

$$\frac{4E_1^2 m\sqrt{E_1^2-M_1^2}}{M_1^3}$$

[20]:
```
f3=g/sqrt(2)*m*E1/M1**3;f3
```

[20]:

$$\frac{\sqrt{2}E_1 g m}{2M_1^3}$$

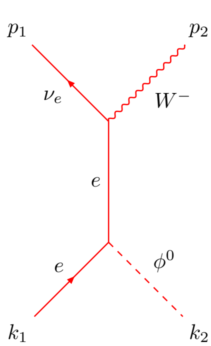
$c_2 = g_{\phi WW}$

[21]:
```
T3=-f3*c2;T3
```

[21]:

$$-\frac{\sqrt{2}E_1 c_2 g m}{2M_1^3}$$

4) $\bar{\nu}e \to W^-\phi^0$ via t-channel



$$e^-_+(k_1) + \bar{\nu}(p_\nu) \to \phi^0(k_2) + W^-_0(p_2)$$

$-\imath T_{fi} = \bar{v}(p_\nu)\frac{-\imath g}{\sqrt{2}}\gamma^\nu\frac{1-\gamma_5}{2}\epsilon^*_\nu(p_2)\frac{\imath \not{q}}{q^2}(-\imath g_{\phi ee})u(k_1)$

$T_{fi} = \frac{g}{\sqrt{2}}g_{\phi ee}\bar{v}(p_\nu)\frac{1+\gamma_5}{2}\ \not{\epsilon}^*(p_2)\frac{\not{q}}{q^2}u(k_1)$

[22]:
```
core1=epsp2dag*qdag
```

[23]:
```
T4v1=simplify(hep.vbar(pnu,1)*hep.projpl*core1*hep.u(k1,1));T4v1
```
[23]:

$$\left[\frac{\sqrt{E_1}\left(E_1\left(-\sqrt{E_1^2-M_1^2}+\sqrt{E_1^2-m^2}\cos\left(\theta\right)\right)+\sqrt{E_1^2-M_1^2}\left(\sqrt{E_1^2-M_1^2}\cos\left(\theta\right)-\sqrt{E_1^2-m^2}\right)\right)\left(\sqrt{E_1-m}+\sqrt{E_1+m}\right)}{M_1}\right]$$

[24]:
```
T4v2=simplify((M1*T4v1[0]).subs(m,0).subs(M1,0));T4v2
```
[24]:

$$4E_1^3\left(\cos\left(\theta\right)-1\right)$$

[25]:
```
f4=simplify(T4v2*g/sqrt(2)/M1/(-2*E1**2*(1-cos(theta))));f4
```
[25]:

$$\frac{\sqrt{2}E_1g}{M_1}$$

$c1 = g_{\phi ee}$

[26]:
```
T4=f4*c1;T4
```
[26]:

$$\frac{\sqrt{2}E_1c_1g}{M_1}$$

Unitarity condition reads now $T_3 + T_4 \to 0$ for $E_1 \to \infty$ resulting in $f_4 g_{\phi ee} - f_3 g_{\phi WW}$=0.
Therefore $g_{\phi WW}^2 = c_{12} f_3 / f_4$

[27]:
```
c1=simplify(sqrt(f3/f4*c12));c1
```
[27]:

$$\frac{gm}{2M_1}$$

[28]:
```
c2=simplify(f4/f3*c1);c2
```
[28]:

$$M_1g$$

The final result
$g_{\phi ee} = \frac{gm}{2M_W}$
$g_{\phi WW} = gM_W$
agrees with the couplings determined from spontaneous symmetry breaking. This is certainly not an accident.

[ ]: