eepipi-tutorial

December 18, 2019

[1]:
```python
from sympy import *
```

[2]:
```python
import heppackv0 as hep
```

```
Reading heppackv0.py

Done
```

[3]:
```python
M,m,theta,phi, alpha=symbols('M m theta phi alpha',real=True)
t,t0,s,s0,p=symbols('t t0,s,s0,p',real=True)
E,P,beta,xi=symbols('E P beta xi',positive=True)
```

This tutorial will be less verbose than eemmuu.ipynb. Because the amplitudes are very simple we can afford to include a possible ϕ dependence right from the beginning.

[4]:
```python
pin=[E,m,0,0]
ki=[E,m,pi,pi]
pf=[E,M,theta,phi]
kf=[E,M,pi-theta,pi+phi]
```

[5]:
```python
pf4=simplify(hep.fourvec(pf));pf4
```

[5]:

$$\left[E,\quad 2\sqrt{E^2-M^2}\sin\left(\frac{\theta}{2}\right)\cos\left(\phi\right)\cos\left(\frac{\theta}{2}\right),\quad 2\sqrt{E^2-M^2}\sin\left(\phi\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right),\quad \sqrt{E^2-M^2}\left(-\sin^2\left(\frac{\theta}{2}\right)+\right.\right.$$

[6]:
```python
kf4=hep.fourvec(kf);kf4
```

[6]:

$$\left[E,\quad -2\sqrt{E^2-M^2}\sin\left(\frac{\theta}{2}\right)\cos\left(\phi\right)\cos\left(\frac{\theta}{2}\right),\quad -2\sqrt{E^2-M^2}\sin\left(\phi\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right),\quad \sqrt{E^2-M^2}\left(\sin^2\left(\frac{\theta}{2}\right)\right.\right.$$

The Feynman rules require $p_f^\mu - k_f^\mu$ at the outgoing vertex. Here this is done by hand because Jupyter refuses subtraction of lists. One also learns that Jupyter starts the indexing of vectors etc with 0.

[7]:
```
p4=[0,2*simplify(hep.fourvec(pf)[1]),2*simplify(hep.
↪fourvec(pf)[2]),2*simplify(hep.fourvec(pf)[3])];p4
```

[7]:

$$\left[0, \sqrt{E^2 - M^2}\left(-\sin\left(\phi - \theta\right) + \sin\left(\phi + \theta\right)\right), \sqrt{E^2 - M^2}\left(\cos\left(\phi - \theta\right) - \cos\left(\phi + \theta\right)\right), 2\sqrt{E^2 - M^2}\cos\left(\theta\right)\right]$$

[8]:
```
tmp=hep.dotprod4(hep.vbu(ki,1,pin,-1),p4)/4/E**2;tmp
```

[8]:

$$\frac{\sqrt{E^2 - M^2}\left(i\sin\left(\phi\right) - \cos\left(\phi\right)\right)\sin\left(\theta\right)}{E}$$

[9]:
```
t1=tmp.subs(E**2-M**2,P**2);t1
```

[9]:

$$\frac{P\left(i\sin\left(\phi\right) - \cos\left(\phi\right)\right)\sin\left(\theta\right)}{E}$$

[10]:
```
t1sq=simplify((t1*conjugate(t1)).subs(P**2,E**2-M**2));t1sq
```

[10]:

$$\frac{\left(E^2 - M^2\right)\sin^2\left(\theta\right)}{E^2}$$

[11]:
```
t2=hep.dotprod4(hep.vbu(ki,1,pin,1),p4)/4/E**2;t2
```

[11]:

$$-\frac{m\sqrt{E^2 - M^2}\cos\left(\theta\right)}{E^2}$$

The amplitudes may depend on ϕ but the cross section fpr unpolarized beams is independent of ϕ.

[12]:
```
t2sq=t2**2;t2sq
```

[12]:

$$\frac{m^2\left(E^2 - M^2\right)\cos^2\left(\theta\right)}{E^4}$$

[13]:
```
tsqav=simplify((t1sq+t2sq)/2);tsqav
```

[13]:

$$\frac{\left(E^2 - M^2\right)\left(E^2\sin^2\left(\theta\right) + m^2\cos^2\left(\theta\right)\right)}{2E^4}$$

The invariants t, t_0, s expressed in CM variables:

[14]:
```
tCM=m**2+M**2-2*E**2+2*sqrt(E**2-m**2)*sqrt(E**2-M**2)*cos(theta);tCM
```
[14]:

$$-2E^2 + M^2 + m^2 + 2\sqrt{E^2 - M^2}\sqrt{E^2 - m^2}\cos(\theta)$$

[15]:
```
t0CM=tCM-M**2-m**2;t0CM
```
[15]:

$$-2E^2 + 2\sqrt{E^2 - M^2}\sqrt{E^2 - m^2}\cos(\theta)$$

[16]:
```
sCM=4*E**2;sCM
```
[16]:

$$4E^2$$

Guessing from the results of eemumu we try

[17]:
```
tsqav_inv=2*(s*m**2-s*t-t0**2)/s**2;tsqav_inv
```
[17]:

$$\frac{2m^2s - 2st - 2t_0^2}{s^2}$$

[18]:
```
tsqav_invv1=tsqav_inv.subs(s,sCM)
tsqav_invv2=tsqav_invv1.subs(t,tCM)
tsqav_invv3=tsqav_invv2.subs(t0,t0CM)
proof=simplify(tsqav_invv3-tsqav)
proof
```
[18]:

$$0$$

and prove it. Another useful form is

[19]:
```
tsqav_alt=beta**2*sin(theta)**2/2+m**2*beta**2*cos(theta)**2/2/E**2;tsqav_alt
```
[19]:

$$\frac{\beta^2\sin^2(\theta)}{2} + \frac{\beta^2 m^2\cos^2(\theta)}{2E^2}$$

[20]:
```
simplify(tsqav_alt.subs(beta**2,1-M**2/E**2)-tsqav)
```
[20]:

$$0$$

In contrast to the procedure in the eemumu notebook we here have pions travelling in the θ, ϕ direction produced from positrons and electrons with spins up and down along the x axis. The columns of the transformation matrix yield for $\theta = \pi/2, \phi = 0$

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\rangle + |\tfrac{-1}{2}\rangle\right)$$

$$|{\downarrow}\rangle = \frac{1}{\sqrt{2}}\left(-|\tfrac{1}{2}\rangle + |\tfrac{-1}{2}\rangle\right)$$

leading to the helicity states

$$\begin{aligned} |e^+\uparrow\rangle &= \frac{1}{\sqrt{2}}\left(|e^+,R\rangle + |e^+,L\rangle\right) \\ |e^-\downarrow\rangle &= \frac{1}{\sqrt{2}}\left(-|e^-,L\rangle + |e^-,R\rangle\right)\ . \end{aligned}$$

The incoming state is calculated from the product of the helicity staes. For $m = 0$ only

$$|e^+\uparrow\rangle\,|e^-\downarrow\rangle = \frac{1}{2}\left(|e^+,L\rangle|e^-,R\rangle - |e^+,R\rangle|e^-,L\rangle\right)$$

survives. Thus the incoming current has to be calculatet from $\bar{v}_L(k_i)\gamma^\mu u_R(p_i) - \bar{v}_R(k_i)\gamma^\mu u_L(p_i)$.

[21]:
```
hep.vbu(ki,-1,pin,1)
```

[21]:

$$[0,\quad -2E,\quad -2iE,\quad 0]$$

[22]:
```
hep.vbu(ki,1,pin,-1)
```

[22]:

$$[0,\quad 2E,\quad -2iE,\quad 0]$$

[23]:
```
vbudif=[0,-2*E,0,0];vbudif
```

[23]:

$$[0,\quad -2E,\quad 0,\quad 0]$$

The calculation of the amplitude is very simple via

[24]:
```
tmp=simplify(vbudif[1]*p4[1])/4/E**2;tmp
```

[24]:

$$-\frac{\sqrt{E^2-M^2}\sin(\theta)\cos(\phi)}{E}$$

which gives (up to kinematical factors) the polarized cross section after squaring and polishing

[25]:
```
polsqP=tmp**2;polsqP
```

[25]:

$$\frac{(E^2-M^2)\sin^2(\theta)\cos^2(\phi)}{E^2}$$

[26]:
```
sigP=polsqP.subs(E**2-M**2,E**2*beta**2);sigP
```

[26]:

$$\beta^2 \sin^2(\theta) \cos^2(\phi)$$

which is the result of the standard reference Y.S.Tsai, PRD12, 3533 (1975). Combining with the unpolarized cross section (next line) we obtain the result for beams with a partial polarization ξ^2

[27]:
```
sigU=beta**2*sin(theta)**2/2;sigU
```

[27]:

$$\frac{\beta^2 \sin^2(\theta)}{2}$$

[28]:
```
(1-xi**2)*sigU+xi**2*sigP
```

[28]:

$$\beta^2\xi^2 \sin^2(\theta) \cos^2(\phi) + \frac{\beta^2\left(-\xi^2+1\right)\sin^2(\theta)}{2}$$

[29]:
```
sig=sigU-(xi**2*beta**2*sin(theta)**2*cos(2*phi))/2;sig
```

[29]:

$$-\frac{\beta^2\xi^2 \sin^2(\theta)\cos(2\phi)}{2} + \frac{\beta^2 \sin^2(\theta)}{2}$$

and finally after conventionally switching to a magnetic field along the y axis the cross section $d\sigma/d\Omega$

[30]:
```
dsigdOm=simplify(alpha**2/16/E**2*sig);print('dsigma/dOmega='); dsigdOm
```

dsigma/dOmega=

[30]:

$$\frac{\alpha^2\beta^2\left(-\xi^2\cos(2\phi)+1\right)\sin^2(\theta)}{32E^2}$$

The amplitudes squared for $e\pi$ scattering is obtained via crossing $s \leftrightarrow t$ with a minus sign because only one fermion line is crossed

[31]:
```
tsqav_sc=-2*(t*m**2-s*t-s0**2)/t**2;tsqav_sc
```

[31]:

$$\frac{-2m^2t+2st+2s_0^2}{t^2}$$

[32]:

```
dsigdt=16*pi**2*alpha**2/(16*pi*s0**2*(1-4*m**2*M**2/s0**2))*tsqav_sc;
↪print('dsigma/dt='); dsigdt
```

```
dsigma/dt=
```

[32]:

$$\frac{\pi\alpha^2\left(-2m^2t+2st+2s_0^2\right)}{s_0^2t^2\left(-\frac{4M^2m^2}{s_0^2}+1\right)}$$

The Mott cross section is obtained in the limit $M \to \infty$. It is identical to the result of eemumu notebook. The basic difference between pions and protons is the magnetic moment of the proton which is proportional to $1/M$.

[ ]:

```

```