Dirac-tutorial

December 18, 2019

[1]:
```
from sympy import *
```

[2]:
```
import heppackv0 as hep
```

```
Reading heppackv0.py
```

```
Done
```

[3]:
```
E,M,m,p,theta,phi=symbols('E M m p theta phi',real=True)
```

pe= electron in xz plane, pe4 its fourvector, peneg=-pe, pez= electron along z xis, pexyz = electron unrestricted

[4]:
```
pe=[E,m,theta,0]
```

[5]:
```
peneg=[E,m,pi-theta,pi]
```

[6]:
```
pe0=[m,m,0,0]
```

[7]:
```
pe4=hep.fourvec(pe);pe4
```

[7]:

$$\left[E,\quad 2\sqrt{E^2-m^2}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right),\quad 0,\quad \sqrt{E^2-m^2}\left(-\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)\right)\right]$$

[8]:
```
pez=[E,m,0,0]
```

[9]:
```
pexyz=[E,m,theta,phi]
```

1) helicity spinors

[69]:
```
hep.v(pe,1)
```

[69]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[11]:
```
hep.v(pexyz,1)
```

[11]:

$$\begin{bmatrix} \sqrt{E-m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[12]:
```
hep.vbar(pe,1)
```

[12]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) & -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) & \sqrt{E+m}\sin\left(\frac{\theta}{2}\right) & -\sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

Check the normalization (not Bjorken Drell)

[13]:
```
norm=hep.ubar(pe,1)*hep.u(pe,1);norm
```

[13]:

$$\left[-\left(E-m\right)\sin^{2}\left(\frac{\theta}{2}\right)-\left(E-m\right)\cos^{2}\left(\frac{\theta}{2}\right)+\left(E+m\right)\sin^{2}\left(\frac{\theta}{2}\right)+\left(E+m\right)\cos^{2}\left(\frac{\theta}{2}\right)\right]$$

[14]:
```
simplify(norm)
```

[14]:

$$\left[2m\right]$$

2) Dirac Equation for spinors

[15]:
```
dir=hep.dag(pe4)*hep.u(pe,1);dir
```

[15]:

$$\begin{bmatrix} E\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)-\sqrt{E-m}\sqrt{E^{2}-m^{2}}\left(-\sin^{2}\left(\frac{\theta}{2}\right)+\cos^{2}\left(\frac{\theta}{2}\right)\right)\cos\left(\frac{\theta}{2}\right)-2\sqrt{E-m}\sqrt{E^{2}-m^{2}}\sin^{2}\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \\ E\sqrt{E+m}\sin\left(\frac{\theta}{2}\right)+\sqrt{E-m}\sqrt{E^{2}-m^{2}}\left(-\sin^{2}\left(\frac{\theta}{2}\right)+\cos^{2}\left(\frac{\theta}{2}\right)\right)\sin\left(\frac{\theta}{2}\right)-2\sqrt{E-m}\sqrt{E^{2}-m^{2}}\sin\left(\frac{\theta}{2}\right)\cos^{2}\left(\frac{\theta}{2}\right) \\ -E\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)+\sqrt{E+m}\sqrt{E^{2}-m^{2}}\left(-\sin^{2}\left(\frac{\theta}{2}\right)+\cos^{2}\left(\frac{\theta}{2}\right)\right)\cos\left(\frac{\theta}{2}\right)+2\sqrt{E+m}\sqrt{E^{2}-m^{2}}\sin^{2}\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \\ -E\sqrt{E-m}\sin\left(\frac{\theta}{2}\right)-\sqrt{E+m}\sqrt{E^{2}-m^{2}}\left(-\sin^{2}\left(\frac{\theta}{2}\right)+\cos^{2}\left(\frac{\theta}{2}\right)\right)\sin\left(\frac{\theta}{2}\right)+2\sqrt{E+m}\sqrt{E^{2}-m^{2}}\sin\left(\frac{\theta}{2}\right)\cos^{2}\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[16]:
```
tst1=simplify(dir[0]);tst1
```

[16]:

$$\left(E\sqrt{E+m}-\sqrt{E-m}\sqrt{E^{2}-m^{2}}\right)\cos\left(\frac{\theta}{2}\right)$$

[17]:
```
tst2=E*sqrt(E+m)-sqrt(E-m)*sqrt(E-m)*sqrt(E+m);tst2
```

[17]:

$$E\sqrt{E+m}-(E-m)\sqrt{E+m}$$

All algebraic programmes have difficulties to simplify this into the obvious result $m\sqrt{E+m}\cos\theta/2$ and thus proving $(\not{p}-m)u(p_e,1)=0$

[18]:
```
factor(tst2)
```

[18]:

$$m\sqrt{E+m}$$

[19]:
```
hep.u(pe,1)[0]
```

[19]:

$$\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)$$

[20]:
```
simplify(dir[0])
```

[20]:

$$\left(E\sqrt{E+m}-\sqrt{E-m}\sqrt{E^2-m^2}\right)\cos\left(\frac{\theta}{2}\right)$$

3) Spin Matrices and more

explicit form of γ^μ and $\gamma_5=\gamma^5$:

[21]:
```
hep.g3
```

[21]:

$$\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\\ -1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

[22]:
```
hep.g5
```

[22]:

$$\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$\sigma^{\mu\nu}=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$ here for σ^{ij}

[23]:
```
S1=I/2*(hep.g2*hep.g3-hep.g3*hep.g2);S1
```

[23]:

$$\begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

[24]:
```
S2=I/2*(hep.g3*hep.g1-hep.g1*hep.g3);S2
```

[24]:

$$\begin{bmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{bmatrix}$$

[25]:
```
S3=I/2*(hep.g1*hep.g2-hep.g2*hep.g1);S3
```

[25]:

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$$

The S_i form a fourdimensional representation of the Pauli matrices. Proof:

[26]:
```
S1*S1+S2*S2+S3*S3
```

[26]:

$$\begin{bmatrix}3 & 0 & 0 & 0\\ 0 & 3 & 0 & 0\\ 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 3\end{bmatrix}$$

[27]:
```
I/2*(S1*S3-S3*S1)
```

[27]:

$$\begin{bmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{bmatrix}$$

$S_3/2 = \frac{\mathrm{i}}{2}(\gamma^1\gamma^2 - \gamma^2\gamma^1)$ may serve as the spin operator for solutions of the Dirac equation in the rest system. There are four solutions, the normalization is already included.

[28]:
```
w1=hep.u(pe0,1);w1
```

[28]:

$$\begin{bmatrix}\sqrt{2}\sqrt{m}\\0\\0\\0\end{bmatrix}$$

[29]:
```
w2=hep.u(pe0,1)
```

[30]:
```
w3=hep.v(pe0,-1);w3
```

[30]:

$$\begin{bmatrix}0\\0\\\sqrt{2}\sqrt{m}\\0\end{bmatrix}$$

[31]:
```
w4=hep.v(pe0,1)
```

w_1, w_2 are solutions of positive energy and spins up and down along the z-axis. w_3 and w_4 are likewise negative energy solutions. The spin assignment can be checked by equations like the following one:

[32]:
```
S3*w3-w3
```

[32]:

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

The correct interpretation of the w_i in terms of particles and antiparticles can be read off the rhs of the definitions for the w_i. Note w_3 is an antiparticle with spin down along the z-axis. The operator $\gamma_5 \not{s}$ (where s is the spin four vector) handles the spin assignments properly. In the rest system we have $s^\mu = (0,0,0,1)$ and $p^\mu = (m,0,0,0)$ with the characteristic invariants $p^2 = m^2, s^2 = -1, p \cdot s = 0$. The spin operator H_3 in the z direction is therefore

[33]:
```
H3=-hep.g5*hep.g3
```

[34]:
```
H3*hep.v(pe0,1)
```

[34]:

$$\begin{bmatrix}0\\0\\0\\\sqrt{2}\sqrt{m}\end{bmatrix}$$

$\gamma_5 \;\; /s$ is written invariantly and consequently is also the proper helicity operator in the Lab. A Lorentz boost transforms the momentum fourvector of the restsystem to $(E, 0, 0, p)$ and the spin vector to $(p/m, 0, 0, E/m)$ with p an abbreviation for $\sqrt{E^2 - m^2}$. Forgetting the denominator m and the factor 1/2 for convenience we get

[35]:
```
H3=hep.g5*hep.g0*sqrt(E**2-m**2)-hep.g5*hep.g3*E;H3
```

[35]:

$$\begin{bmatrix} E & 0 & -\sqrt{E^2-m^2} & 0 \\ 0 & -E & 0 & -\sqrt{E^2-m^2} \\ \sqrt{E^2-m^2} & 0 & -E & 0 \\ 0 & \sqrt{E^2-m^2} & 0 & E \end{bmatrix}$$

[36]:
```
H3*hep.v(pez,1)
```

[36]:

$$\begin{bmatrix} 0 \\ E\sqrt{E-m} - \sqrt{E+m}\sqrt{E^2-m^2} \\ 0 \\ E\sqrt{E+m} - \sqrt{E-m}\sqrt{E^2-m^2} \end{bmatrix}$$

which Jupyter refuses to simplify (see line 17) but which is after dividing by m given by

[37]:
```
hep.v(pez,1)
```

[37]:

$$\begin{bmatrix} 0 \\ -\sqrt{E-m} \\ 0 \\ \sqrt{E+m} \end{bmatrix}$$

which it should be. The textbooks usually quote the helicity projection operator $\Sigma = (1 + \gamma_5 \;\; / \; s)/2$. Another projection operator is $(1 + \gamma_5)/2$ which selects righthandel massless particles

[38]:
```
hep.projpl*hep.u(pe,1).subs(m,0)
```

[38]:

$$\begin{bmatrix} \sqrt{E}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[39]:
```
hep.projpl*hep.u(pe,-1).subs(m,0)
```

[39]:

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

γ_5 generically transforms a lefthanded particle into a righthanded antiparticle and vice versa

[40]:
```
hep.g5*hep.u(pexyz,-1)
```

[40]:

$$\begin{bmatrix}\sqrt{E-m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\-\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)\\-\sqrt{E+m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)\end{bmatrix}$$

[41]:
```
hep.v(pexyz,1)
```

[41]:

$$\begin{bmatrix}\sqrt{E-m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\-\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)\\-\sqrt{E+m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)\end{bmatrix}$$

4) C,P,T

Charge conjugation C transforms a particle with given helicity into an antiparticle with the same helicity. $C = i\gamma^2$ with an subsequent complex conjugate transformation does the job:

[42]:
```
I*hep.g2*hep.v(pexyz,1)
```

[42]:

$$\begin{bmatrix}\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)\\\sqrt{E+m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\\\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)\\\sqrt{E-m}e^{-i\phi}\sin\left(\frac{\theta}{2}\right)\end{bmatrix}$$

[43]:
```
hep.u(pexyz,1)
```

[43]:

$$\begin{bmatrix}\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)\\\sqrt{E+m}e^{i\phi}\sin\left(\frac{\theta}{2}\right)\\\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)\\\sqrt{E-m}e^{i\phi}\sin\left(\frac{\theta}{2}\right)\end{bmatrix}$$

In the xz plane $C = i\gamma^2$

[44]:
```
C=I*hep.g2;C
```

[44]:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[45]:
```
C*hep.u(pe,1)
```

[45]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[46]:
```
hep.v(pe,1)
```

[46]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

that is $Cu(p_e, 1) = v(p_e, 1)$. The parity operation P transforms $\vec{p} \to -\vec{p}$ and $\lambda \to -\lambda$. For spinors this is done by $P = \gamma^0$.

[47]:
```
P=hep.g0;P
```

[47]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

[48]:
```
P*hep.v(pe,1)
```

[48]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[49]:
```
hep.v(peneg,-1)
```

[49]:

$$\begin{bmatrix} \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

that is $Pv(E,\vec{p},1) = v(E,-\vec{p},-1)$. Time reversal T transforms $\vec{p} \to -\vec{p}$ leaving λ unchanged. In applying T to spinors one has to consider the antiunitarian character of the time reversal operator. Ths means that $T = -\gamma^1\gamma^3$ has to be applied to the complex conjugate spinors (see Bjorken Drell). In the xz plane complex conjugation is not relevant and thus:

[50]:
```
T=-hep.g1*hep.g3;T
```

[50]:

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

[51]:
```
T*hep.v(peneg,-1)
```

[51]:

$$\begin{bmatrix} \sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[52]:
```
hep.v(pe,-1)
```

[52]:

$$\begin{bmatrix} \sqrt{E-m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E-m}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+m}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

In summary this means $TPC = \gamma_5$

[53]:
```
T*P*C-hep.g5
```

[53]:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The S matrix is CPT invariant (CPT theorem). Therefore scattering amplitudes of fermions are invariant with respect to replacing particles by antiparticles with reversed helicities.

5) Weyl representation

The popular Weyl representation is reached by the unitary transformation

[54]:
```
U=(hep.one+hep.g5*hep.g0)/sqrt(2);U
```

[54]:

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

[55]:
```
Uinv=U.inv();Uinv
```

[55]:

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

Basically γ_5 and γ^0 are interchanged

[56]:
```
g5W=U*hep.g5*Uinv;g5W
```

[56]:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[57]:
```
g0W=U*hep.g0*Uinv;g0W
```

[57]:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

[58]:
```
g1W=U*hep.g1*Uinv
```

[59]:
```
g2W=U*hep.g2*Uinv
```

[60]:
```
g3W=U*hep.g3*Uinv
```

A typical Weyl spinor looks complicated at first sight

[61]:
```
uW=U*hep.u(pe,1);uW
```
[61]:

$$\begin{bmatrix} -\frac{\sqrt{2}\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)}{2} + \frac{\sqrt{2}\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)}{2} \\ -\frac{\sqrt{2}\sqrt{E-m}\sin\left(\frac{\theta}{2}\right)}{2} + \frac{\sqrt{2}\sqrt{E+m}\sin\left(\frac{\theta}{2}\right)}{2} \\ \frac{\sqrt{2}\sqrt{E-m}\cos\left(\frac{\theta}{2}\right)}{2} + \frac{\sqrt{2}\sqrt{E+m}\cos\left(\frac{\theta}{2}\right)}{2} \\ \frac{\sqrt{2}\sqrt{E-m}\sin\left(\frac{\theta}{2}\right)}{2} + \frac{\sqrt{2}\sqrt{E+m}\sin\left(\frac{\theta}{2}\right)}{2} \end{bmatrix}$$

but with the help of e.g.

[62]:
```
simplify(uW[0]**2)
```
[62]:

$$\left(E - \sqrt{E-m}\sqrt{E+m}\right)\cos^2\left(\frac{\theta}{2}\right)$$

we obtain simple expressions for the 4 Weyl helicity spinors

[63]:
```
uWR=Matrix([[sqrt(E-p)*cos(theta/2)],[sqrt(E-p)*sin(theta/2)],
            [sqrt(E+p)*cos(theta/2)],[sqrt(E+p)*sin(theta/2)]]);uWR
```
[63]:

$$\begin{bmatrix} \sqrt{E-p}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E-p}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+p}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+p}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[64]:
```
uWL=Matrix([[-sqrt(E+p)*sin(theta/2)],[sqrt(E+p)*cos(theta/2)],
            [-sqrt(E-p)*sin(theta/2)],[sqrt(E-p)*cos(theta/2)]]);uWL
```
[64]:

$$\begin{bmatrix} -\sqrt{E+p}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+p}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E-p}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E-p}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[65]:
```
vWL=Matrix([[-sqrt(E-p)*cos(theta/2)],[-sqrt(E-p)*sin(theta/2)],
            [sqrt(E+p)*cos(theta/2)],[sqrt(E+p)*sin(theta/2)]]);vWL
```
[65]:

$$\begin{bmatrix} -\sqrt{E-p}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E-p}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E+p}\cos\left(\frac{\theta}{2}\right) \\ \sqrt{E+p}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}$$

[66]:

```
vWR=Matrix([[sqrt(E+p)*sin(theta/2)],[-sqrt(E+p)*cos(theta/2)],
                [-sqrt(E-p)*sin(theta/2)],[sqrt(E-p)*cos(theta/2)]]);vWR
```

[66]:

$$\begin{bmatrix} \sqrt{E+p}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{E+p}\cos\left(\frac{\theta}{2}\right) \\ -\sqrt{E-p}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{E-p}\cos\left(\frac{\theta}{2}\right) \end{bmatrix}$$

The Weyl representation is particularly attractive for massless particles, because the spinors are reduced to 2 components only, the Dirac equation is replaced by 2 Weyl equations

[67]:

```
uWL.subs(p,E)
```

[67]:

$$\begin{bmatrix} -\sqrt{2}\sqrt{E}\sin\left(\frac{\theta}{2}\right) \\ \sqrt{2}\sqrt{E}\cos\left(\frac{\theta}{2}\right) \\ 0 \\ 0 \end{bmatrix}$$

[68]:

```
vWR.subs(p,E)
```

[68]:

$$\begin{bmatrix} \sqrt{2}\sqrt{E}\sin\left(\frac{\theta}{2}\right) \\ -\sqrt{2}\sqrt{E}\cos\left(\frac{\theta}{2}\right) \\ 0 \\ 0 \end{bmatrix}$$

[ ]: