NeutronDecay

December 17, 2019

[1]:
```
from sympy import *
```

[2]:
```
import heppackv0 as hep
```

```
Reading heppackv0.py

Done
```

[3]:
```
print('The CM system is used')
E,E0,M,M1,M2,p,Delta,m=symbols('E E0 M M1 M2 p Delta m',positive=True)
theta,alpha,omega,=symbols('theta alpha omega',real=True)
```

```
The CM system is used
```

We consider neutron decay $n \to pe^- \bar{\nu}_e$ or d quark decay $d \to ue^- \bar{\nu}_e$ in the approximation where the recoil of the outgoing neutron (u quark) is neglected.

[4]:
```
ki=[M,M,0,0]
pout=[E,m,theta,0]
kf=[M1,M1,0,0]
pf=[omega,0,0,0]
```

1) Neutrons as elementary fermions

The matrix elements are calculated from

$$T_{fi} = \frac{4G_\mathrm{F}}{\sqrt{2}} \bar{u}(k_f)\gamma^\mu \frac{1-\gamma^5}{2} u(k_i) \bar{u}(p_\mathrm{out})\gamma_\mu \frac{1-\gamma^5}{2} v(p_i) \quad .$$

The weak currents, e.g. $\bar{u}(k_f)\gamma^\mu \frac{1-\gamma^5}{2} u(k_i)$ are contained in the heppackv0 package. In this ansatz the neutrons are treated as elementary fermions.

[5]:
```
dec11=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),hep.ubuw(kf,1,ki,1)));dec11
```

[5]:

$$-2\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(\sqrt{E-m}+\sqrt{E+m}\right)\cos\left(\frac{\theta}{2}\right)$$

[6]:
```
dec12=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),hep.ubuw(kf,1,ki,1)));dec12
```
[6]:

$$2\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(\sqrt{E-m}-\sqrt{E+m}\right)\sin\left(\frac{\theta}{2}\right)$$

[7]:
```
simplify(dec11**2+dec12**2)
```
[7]:

$$8MM_1\omega\left(E-2\sqrt{E^2-m^2}\sin^2\left(\frac{\theta}{2}\right)+\sqrt{E^2-m^2}\right)$$

[8]:
```
F=8*omega*M*M1*(E+p*cos(theta));F
```
[8]:

$$8MM_1\omega\left(E+p\cos\left(\theta\right)\right)$$

[9]:
```
dec21=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),hep.ubuw(kf,-1,ki,1)));dec21
```
[9]:

$$-2\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(\sqrt{E-m}+\sqrt{E+m}\right)\sin\left(\frac{\theta}{2}\right)$$

[10]:
```
dec22=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),hep.ubuw(kf,-1,ki,1)));dec22
```
[10]:

$$2\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(-\sqrt{E-m}+\sqrt{E+m}\right)\cos\left(\frac{\theta}{2}\right)$$

[11]:
```
simplify(dec21**2+dec22**2)
```
[11]:

$$8MM_1\omega\left(E-2\sqrt{E^2-m^2}\cos^2\left(\frac{\theta}{2}\right)+\sqrt{E^2-m^2}\right)$$

[12]:
```
GT=8*omega*M*M1*(E-p*cos(theta));GT
```
[12]:

$$8MM_1\omega\left(E-p\cos\left(\theta\right)\right)$$

[13]:
```
Tsqav=simplify(F+GT)/2;Tsqav
```
[13]:

$$8EMM_1\omega$$

or including the G_F term

$$\overline{\sum_i}|T_i|^2 = 64G_F^2 E\omega MM_1 \ .$$

The differential decay rate for a three body decay is calculated from

$$\mathrm{d}\Gamma = \frac{1}{2M}\int |T_{fi}|^2 \mathrm{d}L$$

with

$$\mathrm{d}L = (2\pi)^4\delta^4(P-p_1-p_2-p_3)\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3p_3}{(2\pi)^3 2E_3} \ .$$

In our case the formula for $\mathrm{d}\Gamma$ reduces to

$$\mathrm{d}\Gamma = \frac{4G_F^2}{(2\pi)^5}\int \delta^4(k_i - p_{\mathrm{out}} - p_f - k_f)\mathrm{d}^3p\mathrm{d}^3p_f\mathrm{d}^3k_f \ .$$

The integral can be evaluated without discussing the contours of the Dalitzplot. Integrating over d^3k_f uses up 3 dimensions of the δ-function,

$$\mathrm{d}\Gamma = \frac{4G_F^2}{(2\pi)^5}\int \delta(M - M_1 - \omega - E)\mathrm{d}^3p\mathrm{d}^3p_f \ .$$

Using $\mathrm{d}^3p_f = |\vec{p}_f|^2\mathrm{d}|\vec{p}_f|\mathrm{d}\Omega = 4\pi\omega^2\mathrm{d}\omega$ the d^3p_f integral is easily performed

$$\mathrm{d}\Gamma = \frac{8G_F^2}{(2\pi)^4}\int (M-M_1-E)^2\mathrm{d}^3p = \frac{8G_F^2}{(2\pi)^3}\int (M-M_1-E)^2p^2\mathrm{d}p\mathrm{d}\cos\theta \ .$$

Finally using the abbreviation $M - M_1 = \Delta$ the electron energy spectrum is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}p} = \frac{2G_F^2}{\pi^3}p^2(\Delta - E)^2 \ .$$

For massless electrons the total width is easily calculated:

[14]:
```
Gam0=integrate(E**2*(Delta-E)**2,(E,0,Delta));Gam0
```

[14]:

$$\frac{\Delta^5}{30}$$

resulting in

$$\Gamma_0 = \frac{G_F^2\Delta^5}{15\pi^3} \ ,$$

whereas the integral for finite m looks ugly.

[15]:
```
Gam=simplify(integrate(p**2*(Delta-sqrt(p**2+m**2))**2,
(p,0,sqrt(Delta**2-m**2))));Gam
```

[15]:

$$\frac{\Delta^4\sqrt{\Delta^2-m^2}}{30}-\frac{3\Delta^2m^2\sqrt{\Delta^2-m^2}}{20}+\frac{\Delta m^4\operatorname{asinh}\left(\frac{\sqrt{\Delta^2-m^2}}{m}\right)}{4}-\frac{2m^4\sqrt{\Delta^2-m^2}}{15}$$

[16]:
```
print('numerical evaluation for neutron decay')
h1=Gam.subs(m,0.511).subs(Delta,1.3) ;h1
```

```
numerical evaluation for neutron decay
```

[16]:

$$0.0589505418402686$$

[17]:
```
print('numerical evaluation for neutron decay with m=0')
h2=Gam0.subs(Delta,1.3);h2
```

```
numerical evaluation for neutron decay with m=0
```

[17]:

$$0.123764333333333$$

[18]:
```
print('Correction factor for neutron decay')
h1/h2
```

```
Correction factor for neutron decay
```

[18]:

$$0.476312845975566$$

2) Hadronic neutron currents

The resulting formula

$$\Gamma_0 = 0.476\frac{G_F^2\Delta^5}{15\pi^3}$$

is not in agreement with the measured neutron decay width. Leptonic weak currents differ from hadronic weak currents

$$\bar{u}(k_f)\gamma^\mu\frac{1-\alpha\gamma^5}{2}u(k_i)\ ,$$

where α is a number $\mathcal{O}(1)$ to be taken from experiment. Weak currents of the general form

$$\bar{u}(k_f)\gamma^\mu\frac{C_V-C_A\gamma^5}{2}u(k_i)\ ,$$

are also contained in heppackv0.py. C_V and C_A, respectively 1 and C_A/C_V have to be given as 5th and 6th argument

[19]:
```
T11=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),
hep.ubuva(kf,1,ki,1,1,alpha)));T11
```

[19]:

$$-\sqrt{M}\sqrt{M_1}\sqrt{\omega}\,(\alpha+1.0)\left(\sqrt{E-m}+\sqrt{E+m}\right)\cos\left(\frac{\theta}{2}\right)$$

[20]:
```
T12=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),
hep.ubuva(kf,1,ki,1,1,alpha)));T12
```

[20]:

$$\sqrt{M}\sqrt{M_1}\sqrt{\omega}\,(\alpha+1.0)\left(\sqrt{E-m}-\sqrt{E+m}\right)\sin\left(\frac{\theta}{2}\right)$$

[21]:
```
simplify(T11**2+T12**2)
```

[21]:

$$2MM_1\omega\,(\alpha+1.0)^2\left(E-2\sqrt{E^2-m^2}\sin^2\left(\frac{\theta}{2}\right)+\sqrt{E^2-m^2}\right)$$

[22]:
```
F1=2*(1+alpha)**2*omega*M*M1*(E+p*cos(theta));F1
```

[22]:

$$2MM_1\omega\,(E+p\cos(\theta))\,(\alpha+1)^2$$

[23]:
```
T21=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),
hep.ubuva(kf,-1,ki,1,1,alpha)));T21
```

[23]:

$$-2\sqrt{M}\sqrt{M_1}\alpha\sqrt{\omega}\left(\sqrt{E-m}+\sqrt{E+m}\right)\sin\left(\frac{\theta}{2}\right)$$

[24]:
```
T22=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),
hep.ubuva(kf,-1,ki,1,1,alpha)));T22
```

[24]:

$$2\sqrt{M}\sqrt{M_1}\alpha\sqrt{\omega}\left(-\sqrt{E-m}+\sqrt{E+m}\right)\cos\left(\frac{\theta}{2}\right)$$

[25]:
```
simplify(T21**2+T22**2)
```

[25]:

$$8MM_1\alpha^2\omega\left(E-2\sqrt{E^2-m^2}\cos^2\left(\frac{\theta}{2}\right)+\sqrt{E^2-m^2}\right)$$

[26]:
```
GT1=8*omega*M*M1*alpha**2*(E-p*cos(theta));GT1
```
[26]:

$$8MM_1\alpha^2\omega\left(E - p\cos\left(\theta\right)\right)$$

[27]:
```
T31=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),
hep.ubuva(kf,1,ki,-1,1,alpha)));T31
```
[27]:

$$0$$

[28]:
```
T32=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),
hep.ubuva(kf,1,ki,-1,1,alpha)));T32
```
[28]:

$$0$$

[29]:
```
T41=simplify(hep.dotprod4(hep.ubvw(pout,-1,pf,1),
hep.ubuva(kf,-1,ki,-1,1,alpha)));T41
```
[29]:

$$\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(\alpha - 1.0\right)\left(\sqrt{E-m} + \sqrt{E+m}\right)\cos\left(\frac{\theta}{2}\right)$$

[30]:
```
T42=simplify(hep.dotprod4(hep.ubvw(pout,1,pf,1),
hep.ubuva(kf,-1,ki,-1,1,alpha)));T42
```
[30]:

$$-\sqrt{M}\sqrt{M_1}\sqrt{\omega}\left(\alpha - 1.0\right)\left(\sqrt{E-m} - \sqrt{E+m}\right)\sin\left(\frac{\theta}{2}\right)$$

[31]:
```
simplify(T41**2+T42**2)
```
[31]:

$$2MM_1\omega\left(\alpha - 1.0\right)^2\left(E - 2\sqrt{E^2-m^2}\sin^2\left(\frac{\theta}{2}\right) + \sqrt{E^2-m^2}\right)$$

[32]:
```
F2=2*(1-alpha)**2*omega*M*M1*(E+p*cos(theta));F2
```
[32]:

$$2MM_1\omega\left(E + p\cos\left(\theta\right)\right)\left(-\alpha + 1\right)^2$$

[33]:
```
Tdecsq=simplify(F1+F2+GT1)/2;Tdecsq
```
[33]:

$$2MM_1\omega\left(3E\alpha^2 + E - \alpha^2 p\cos\left(\theta\right) + p\cos\left(\theta\right)\right)$$

Repeating the discussion above the evaluation of $\mathrm{d}\Gamma/\mathrm{d}p$ needs a $\mathrm{d}\cos\theta$ integration. With $\int_{-1}^{1}\cos\theta \mathrm{d}\cos\theta = 0$ the formula for the momentum spectrum has simply to be multiplied by the ratio

$$\frac{2E\omega MM_1(1+3\alpha^2)}{8E\omega MM_1}$$

of the average matrix elements squared with the result

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}p} = \frac{G_F^2}{2\pi^3}(1+3\alpha^2)p^2(\Delta - E)^2 \ .$$

and

$$\Gamma = 0.476\frac{G_F^2\Delta^5}{60\pi^3}(1+3\alpha^2)$$

Finally taking the flavor structure of the weak currents into account the neutron decay width is given by

$$\Gamma = 0.476\frac{G_F^2\Delta^5}{60\pi^3}(1+3\alpha^2)\cos\theta_C \ ,$$

where θ_C is the Cabbibo angle.

3) Momentum spectrum and polarization of the decay electrons

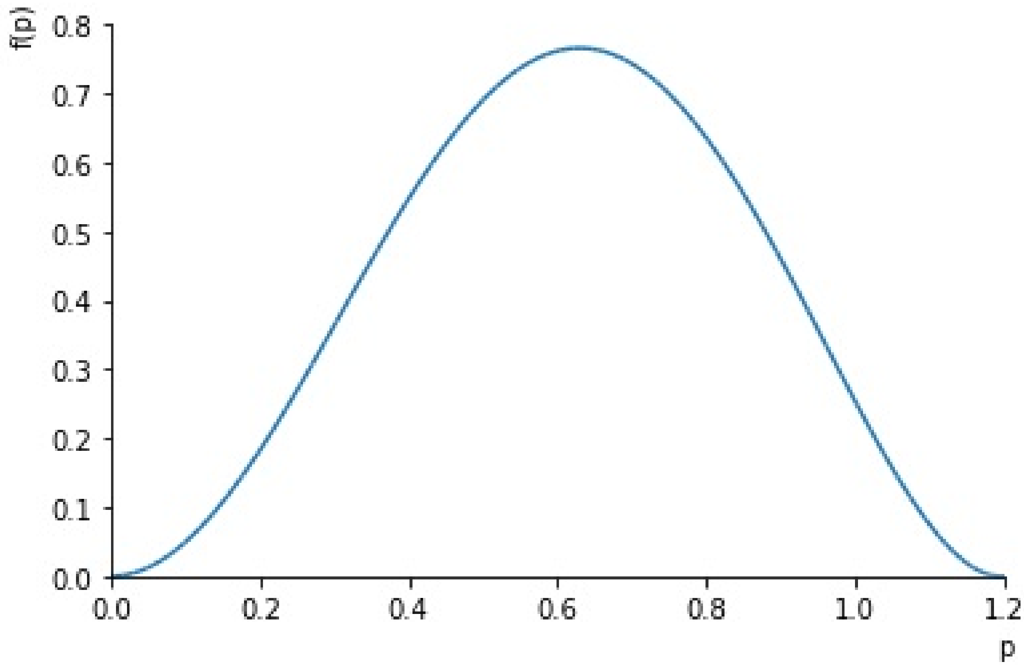
In the next step the momentum spectrum with a normalization close to 1 is plotted:

[34]:
```
gamplot=p**2*(Delta-sqrt(p**2+m**2))**2*30/Delta**5;gamplot
```

[34]:

$$\frac{30p^2\left(\Delta - \sqrt{m^2+p^2}\right)^2}{\Delta^5}$$

[39]:
```
plot(gamplot.subs(m,0.511).subs(Delta,1.3),(p,0,1.2))
```



[39]: <sympy.plotting.plot.Plot at 0x116aafe80>

The average polarization of the decay electrons can simply be calculated from the matrix elements given in chapter 1.

[36]:
```
polF=simplify(dec12**2-dec11**2);polF
```

[36]:

$$8MM_1\omega\left(-2E\cos^2\left(\frac{\theta}{2}\right)+E-\sqrt{E^2-m^2}\right)$$

[37]:
```
polGT=simplify(dec22**2-dec21**2);polGT
```

[37]:

$$8MM_1\omega\left(-2E\sin^2\left(\frac{\theta}{2}\right)+E-\sqrt{E^2-m^2}\right)$$

[38]:
```
pol=simplify((polF+polGT)/2/Tsqav);pol
```

[38]:

$$-\frac{\sqrt{E^2-m^2}}{E}$$

that is $-\beta_e$.

```
[ ]:
```