Lab8_Yunting

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1 Lab08 - Neural Networks and Deep Learning

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2 Install the required packages

[105]:
```
import matplotlib.pyplot as plt
import math
import numpy as np
import scipy
#from sklearn import mixture
#from sklearn.model_selection import train_test_split
import tensorflow as tf
```

3 a

Generate and plot a data set X_1 (training set) containing 100 points from w_1 (50 points from each associated Guassian) and 150 points from w_2 (again 50 points from each associated Gaussian). In the same way, generate an additional set X_2 (test set). ## Training set X_1

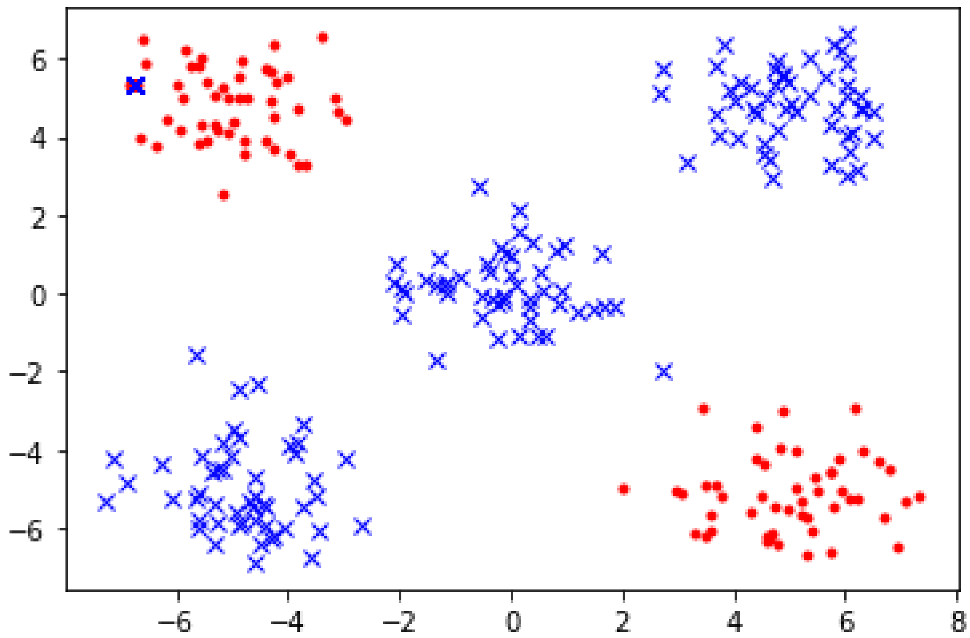
[106]:
```
# cov = var * identity matrix
# var = 1
# w1
w11_train = np.random.multivariate_normal(mean = [-5, 5], cov = [[1, 0], [0,␣
↪1]], size = 50)
w12_train = np.random.multivariate_normal(mean = [5, -5], cov = [[1, 0], [0,␣
↪1]], size = 50)

# w2
w21_train = np.random.multivariate_normal(mean = [-5, -5], cov = [[1, 0], [0,␣
↪1]], size = 50)
w22_train = np.random.multivariate_normal(mean = [0, 0], cov = [[1, 0], [0,␣
↪1]], size = 50)
w23_train = np.random.multivariate_normal(mean = [5, 5], cov = [[1, 0], [0,␣
↪1]], size = 50)
```

[107]:
```
W1 = np.vstack((w11_train, w12_train))
W2 = np.vstack((w21_train, w22_train, w23_train))
X1 = np.vstack((W1, W2))
#len(W2)
```

3.0.1 Label each observation and plot

we assume W1 label is 1, and W2 label is 0

[108]:
```
y1 = np.concatenate((np.ones(shape=(1, len(W1))), np.zeros(shape=(1,␣
↪len(W2)))), axis=-1)

# Training Plot
_ = plt.plot(X1[np.where(y1 == 1), 0], X1[np.where(y1 == 1), 1], 'r.', X1[np.
↪where(y1 == 0), 0], X1[np.where(y1 == 0), 1], 'bx')
```



[109]:
```
"""
# label the data point, w1 = +1, w2 = -1
w1 = np.vstack((w11_train, w12_train))
x_w1, y_w1 = w1, np.ones((len(w1), 1))

w2 = np.vstack((w21_train, w22_train, w23_train))
x_w2, y_w2 = w2, np.zeros((len(w2), 1))
for i in range(len(y_w2)):
  if y_w2[i] == 0:
    y_w2[i] = -1
```

```
W1 = np.concatenate((x_w1, y_w1), axis=1)
W2 = np.concatenate((x_w2, y_w2), axis=1)

#data = np.vstack((W1, W2, W1, W2))
#print(data.shape)
#print(data[:, 2:])
X_1 = np.vstack((W1, W2)) # Training set
#np.random.shuffle(X1) # shuffle
#X2 = np.vstack((W1, W2)) # testing set
#np.random.shuffle(X2) # shuffle

#data = np.vstack((X1, X2))
#print(X1[:, 2:])
"""
```

[109]:
```
'\n# label the data point, w1 = +1, w2 = -1\nw1 = np.vstack((w11_train,
w12_train))\nx_w1, y_w1 = w1, np.ones((len(w1), 1))\n\nw2 =
np.vstack((w21_train, w22_train, w23_train))\nx_w2, y_w2 = w2,
np.zeros((len(w2), 1))\nfor i in range(len(y_w2)):\n  if y_w2[i] == 0:\n
y_w2[i] = -1\n\n\nW1 = np.concatenate((x_w1, y_w1), axis=1)\nW2 =
np.concatenate((x_w2, y_w2), axis=1)\n\n#data = np.vstack((W1, W2, W1,
W2))\n#print(data.shape)\n#print(data[:, 2:])\nX_1 = np.vstack((W1, W2)) #
Training set\n#np.random.shuffle(X1) # shuffle\n#X2 = np.vstack((W1, W2)) #
testing set \n#np.random.shuffle(X2) # shuffle\n\n#data = np.vstack((X1,
X2))\n#print(X1[:, 2:])\n'
```

3.1 Testing set X_2

[110]:
```
# cov = var * identity matrix
# var = 1
# w1
w11_test = np.random.multivariate_normal(mean = [-5, 5], cov = [[1, 0], [0,␣
 ↪1]], size = 50)
w12_test = np.random.multivariate_normal(mean = [5, -5], cov = [[1, 0], [0,␣
 ↪1]], size = 50)

# w2
w21_test = np.random.multivariate_normal(mean = [-5, -5], cov = [[1, 0], [0,␣
 ↪1]], size = 50)
w22_test = np.random.multivariate_normal(mean = [0, 0], cov = [[1, 0], [0, 1]],␣
 ↪size = 50)
w23_test = np.random.multivariate_normal(mean = [5, 5], cov = [[1, 0], [0, 1]],␣
 ↪size = 50)
```

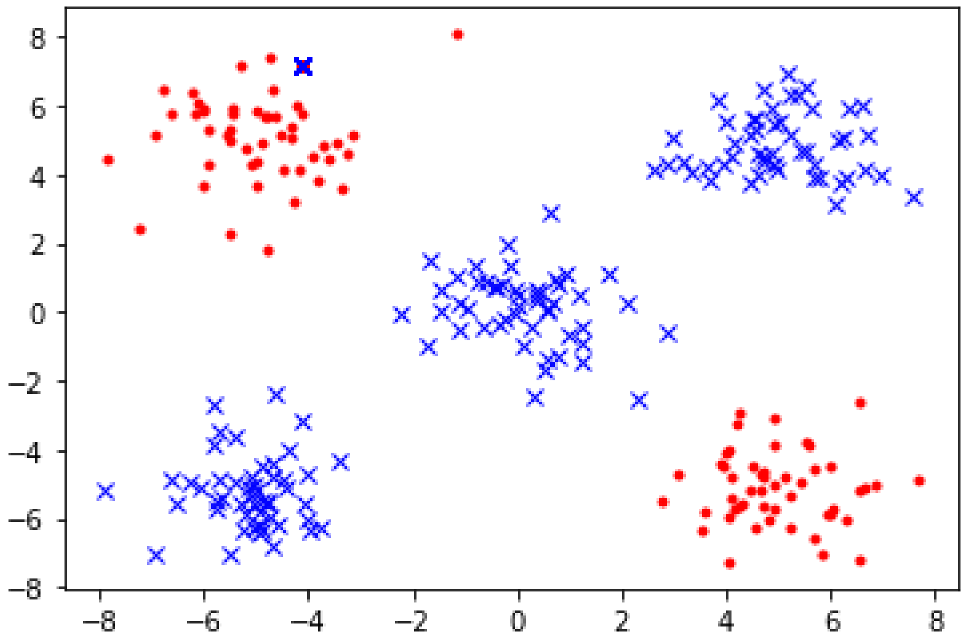
[111]:
```
W1 = np.vstack((w11_test, w12_test))
W2 = np.vstack((w21_test, w22_test, w23_test))
```

```
X2 = np.vstack((W1, W2))
```

3.1.1 Label each observation and plot

we assume `W1` label is 1, and `W2` label is 0

[112]:
```
y2 = np.concatenate((np.ones(shape=(1, len(W1))), np.zeros(shape=(1,␣
↪len(W2)))), axis=-1)

# Training Plot
_ = plt.plot(X2[np.where(y2 == 1), 0], X2[np.where(y2 == 1), 1], 'r.', X2[np.
↪where(y2 == 0), 0], X2[np.where(y2 == 0), 1], 'bx')
```



I didn't set a seed to generate random numbers, so we can see that `X1` and `X2` are distinct.

[113]:
```
print(X1[:5, :]==X2[:5, :])
```

```
[[False False]
 [False False]
 [False False]
 [False False]
 [False False]]
```

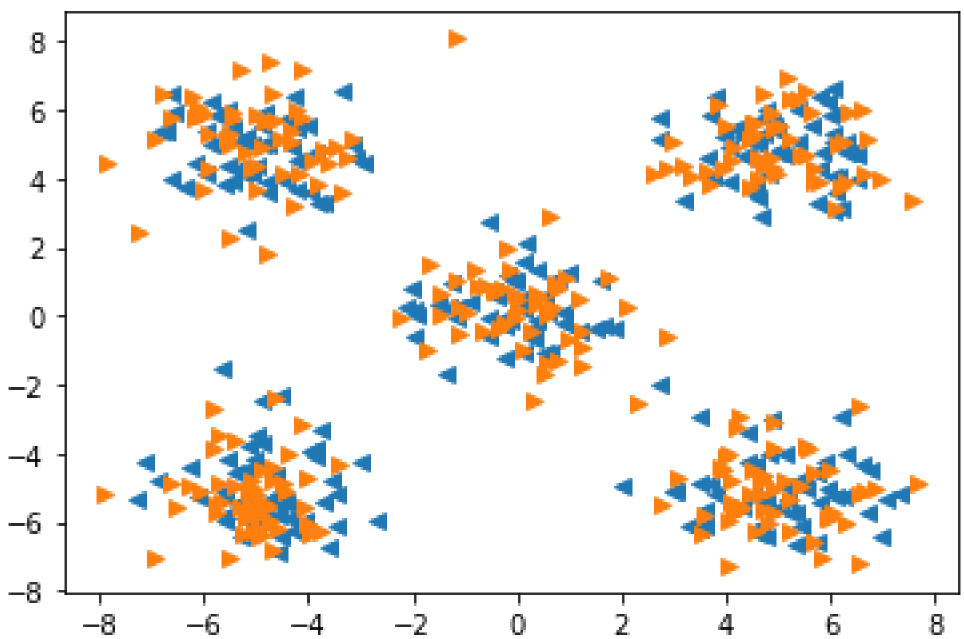
3.2 Data Shape

[114]:
```
#X1, X2, y1, y2 = train_test_split(data[:, 0:2], data[:, 2:], test_size= 0.5,␣
↪random_state=100) # 50% for training, 50 for of testing, ans shuffle
#x_train = X1[:, 0:2]
#y_train = X1[:, 2:]
#x_test = X2[:, 0:2]
#y_test = X2[:, 2:]
print("-----the shape of each dataset--------")
print(X1.shape)
print(X2.shape)
print(y1.shape) # should transpose when fitting model
print(y2.shape) # should transpose when fitting model
```

```
-----the shape of each dataset--------
(250, 2)
(250, 2)
(1, 250)
(1, 250)
```

3.3 Plot X_1 and X_2

[115]:
```
# plt.scatter(x_train[:, 0], x_train[:, 1], c = 'blue')
# plt.scatter(x_test[:, 0], x_test[:, 1], c = 'green')
plt.figure(1)
_ = plt.plot(X1[:, 0], X1[:, 1], '<', X2[:, 0], X2[:, 1], '>')
#_ = plt.plot(X1[np.where(y1 == 1), 0], X1[np.where(y1 == 1), 1], 'r.', X1[np.
↪where(y1 == -1), 0], X1[np.where(y1 == -1), 1], 'bx')
```



4 B

Based on the training set X_1, train a two-layer neural network with two nodes in the hidden layer, each one having the rectified linear activation function or ReLU and a single output node with linear activation function using the standard backpropagation algorithm for 6000 iterations and step size equal to 0.01. Compute the training and test errors, based on X_1 and X_2 respectively. Also, plot the test points as well as the decision lines formed by the network.

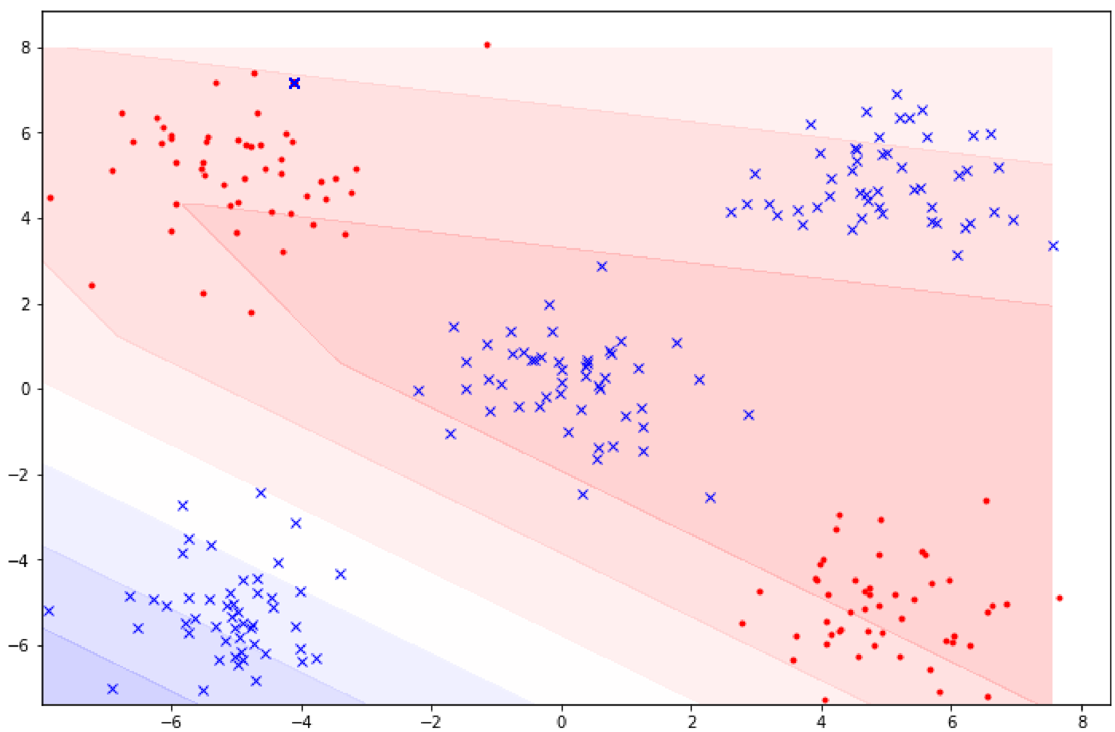
4.1 Definition and training of the network

[137]:
```
"""
This session is adapted from Dr. Zois Boukouvalas
iter = iteration, lr = learning rate, k = number of hidden layer nodes
"""

def NN(iter, k, lr):
  np.random.seed(100)
  model1 = tf.keras.models.Sequential([
  tf.keras.layers.Dense(2, activation=None, input_dim=2), # model will take as␣
 ↪input arrays 2 and output shape 2
  tf.keras.layers.Dense(k, activation="relu"),
  tf.keras.layers.Dense(1, activation=None) # Output layer
  ])
  opt = tf.keras.optimizers.SGD(learning_rate=lr)
  # set model configurations
  model1.compile(optimizer=opt,
                loss='binary_crossentropy',
                metrics=['binary_accuracy'],)

  # give test set as validation data if you want to see accuracy and loss for␣
 ↪the test set during training
  batch_size = 6
  # Verbosity mode. 0 = silent, 1 = progress bar, 2 = one line per epoch
  history = model1.fit(X1, y1.T, batch_size=batch_size, epochs=int(iter/
 ↪batch_size), verbose=0)
  # Test the trained model on the test set. If you passed the test set as␣
 ↪validation data during training,
  # test loss & accuracy will be the same to those of the last training epoch␣
 ↪for the validation data
  pe_train, accu_train = model1.evaluate(X1, y1.T)
  pe_test, accu_test = model1.evaluate(X2, y2.T)
  print("The accuracy score of training set is {}".format(accu_train))
  print("The accuracy score of testing set is {}".format(accu_test))
```

```
amin, bmin = X2.min(axis=0) - 0.1
amax, bmax = X2.max(axis=0) - 0.1
hticks = np.linspace(amin,amax,101)
vticks = np.linspace(bmin, bmax, 101)
aa, bb = np.meshgrid(hticks, vticks)
ab= np.c_[aa.ravel(), bb.ravel()] #flatten
#make prediction
c=model1.predict(ab)
Z=c.reshape(aa.shape)
plt.figure(figsize=(12,8))
# plot contour
plt.contourf(aa,bb,Z, cmap='bwr', alpha = 0.2)
_=plt.plot(X2[np.where(y2 == 1), 0], X2[np.where(y2 == 1), 1], 'r.', X2[np.
↪where(y2 == 0), 0], X2[np.where(y2 == 0), 1], 'bx')
```

[138]:
```
NN(iter=6000, k=2, lr=0.01)
```

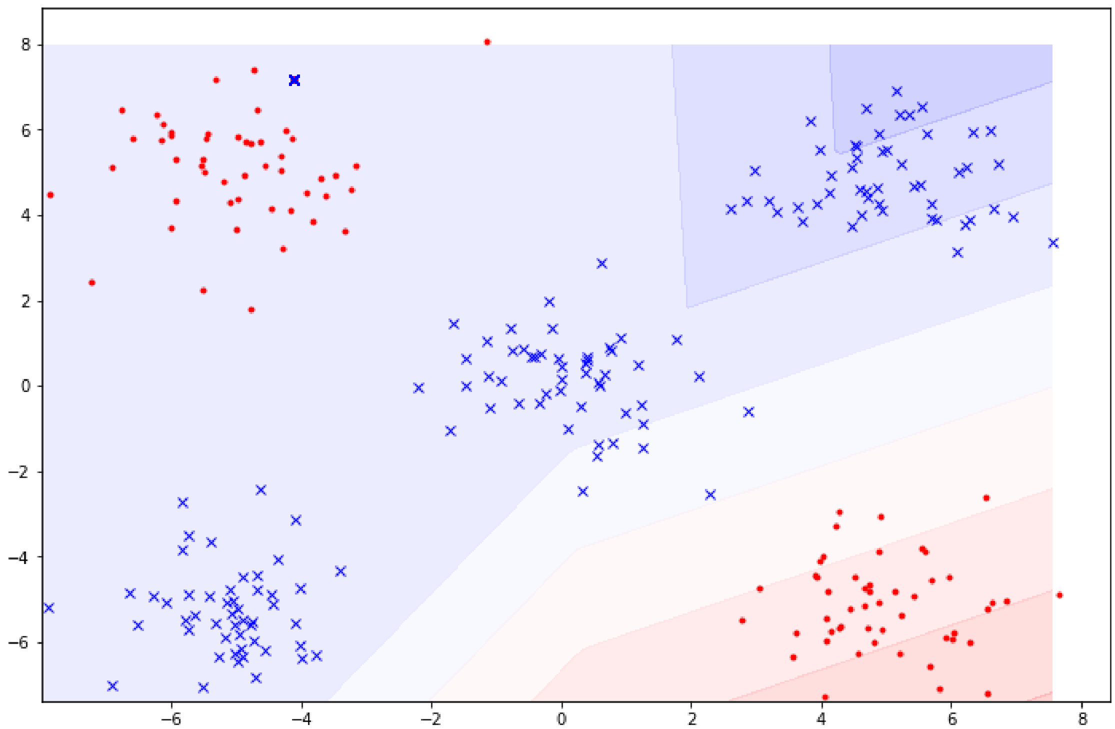
```
8/8 [==============================] - 0s 2ms/step - loss: 6.1700 -
binary_accuracy: 0.6000
8/8 [==============================] - 0s 2ms/step - loss: 6.1700 -
binary_accuracy: 0.6000
The accuracy score of training set is 0.6000000238418579
The accuracy score of testing set is 0.6000000238418579
```



5 C

[139]:
```
NN(iter=6000, k=2, lr=0.0001)
```

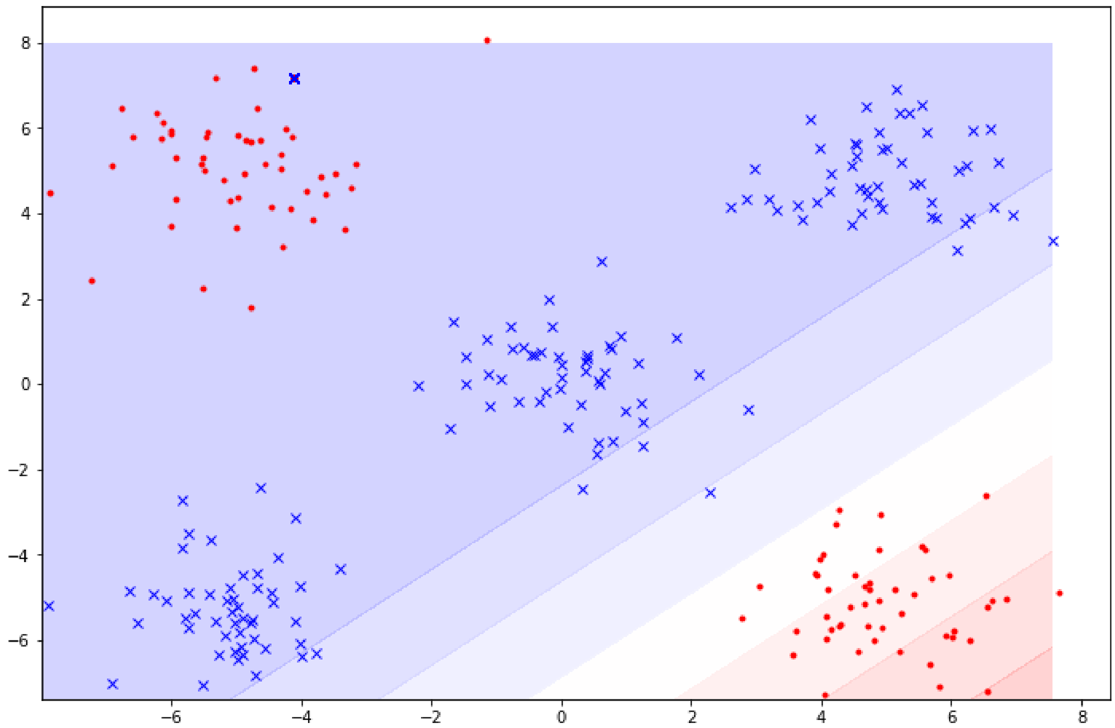
```
8/8 [==============================] - 0s 2ms/step - loss: 3.0883 -
binary_accuracy: 0.8000
8/8 [==============================] - 0s 2ms/step - loss: 3.0916 -
binary_accuracy: 0.7960
The accuracy score of training set is 0.800000011920929
The accuracy score of testing set is 0.7960000038146973
```



6 D

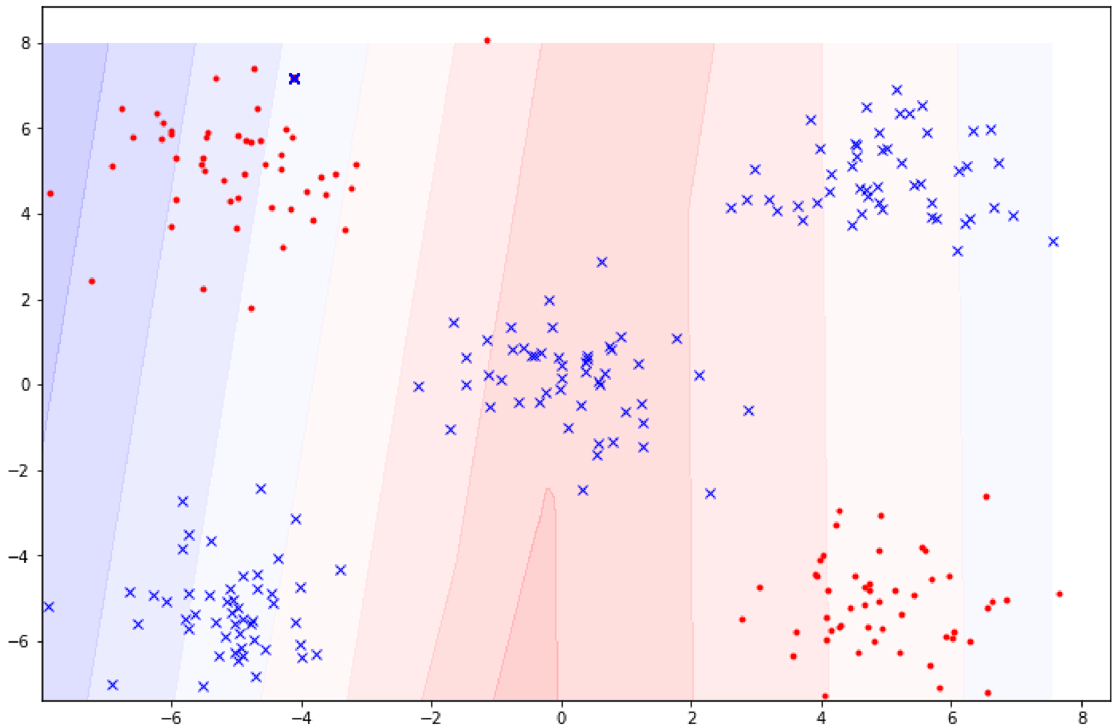
[140]:
```
NN(iter=6000, k=1, lr=0.0001)
```

```
8/8 [==============================] - 0s 2ms/step - loss: 3.0920 -
binary_accuracy: 0.8000
8/8 [==============================] - 0s 2ms/step - loss: 3.0923 -
binary_accuracy: 0.8000
The accuracy score of training set is 0.800000011920929
The accuracy score of testing set is 0.800000011920929
```



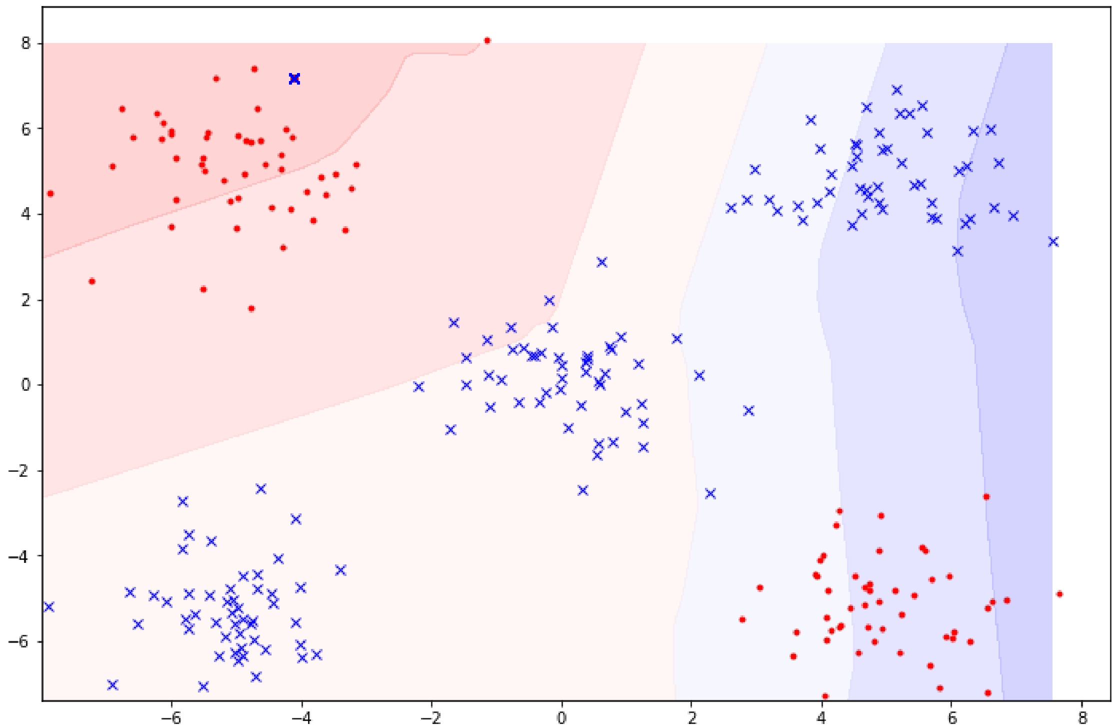
[141]:
```
NN(iter=6000, k=4, lr=0.0001)
```

```
8/8 [==============================] - 0s 2ms/step - loss: 6.1700 -
binary_accuracy: 0.6000
8/8 [==============================] - 0s 2ms/step - loss: 6.1700 -
binary_accuracy: 0.6000
The accuracy score of training set is 0.6000000238418579
The accuracy score of testing set is 0.6000000238418579
```



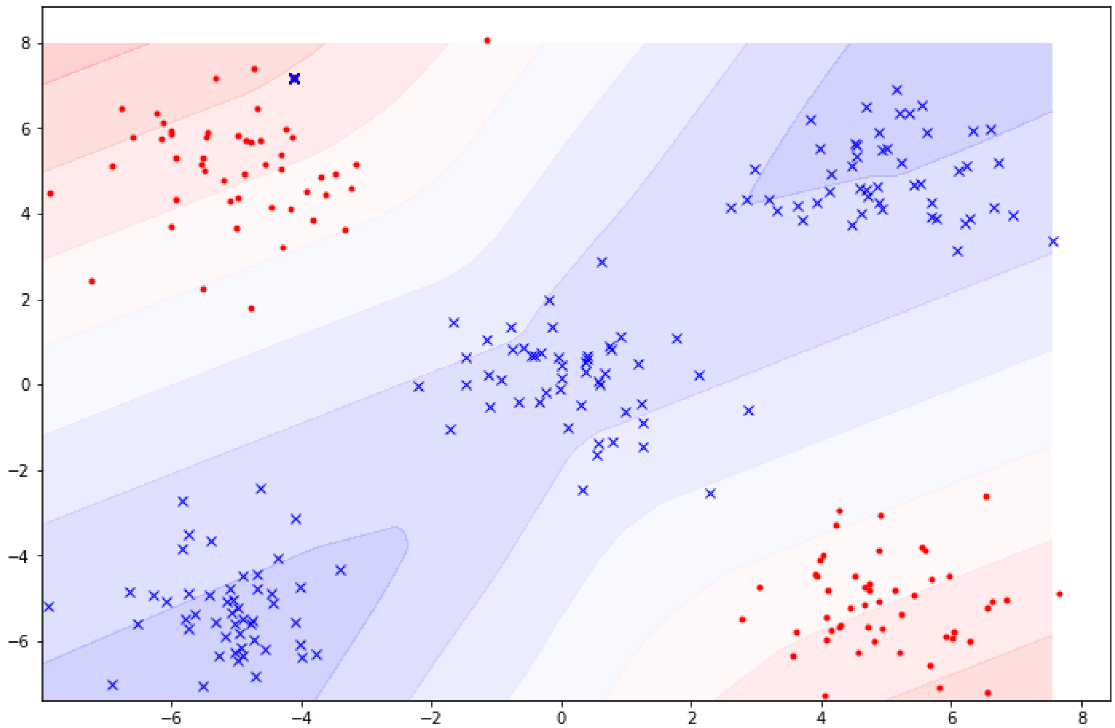
[142]:
```
NN(iter=6000, k=20, lr=0.0001)
```

```
8/8 [==============================] - 0s 2ms/step - loss: 3.0874 -
binary_accuracy: 0.8000
8/8 [==============================] - 0s 2ms/step - loss: 3.0885 -
binary_accuracy: 0.8000
The accuracy score of training set is 0.800000011920929
The accuracy score of testing set is 0.800000011920929
```



[149]:
```
NN(iter=6000, k=50, lr=0.0001)
```

```
8/8 [==============================] - 0s 2ms/step - loss: 0.0059 -
binary_accuracy: 1.0000
8/8 [==============================] - 0s 2ms/step - loss: 0.0101 -
binary_accuracy: 0.9960
The accuracy score of training set is 1.0
The accuracy score of testing set is 0.9959999918937683
```



7 Conclusion

In the model, the learning rate and the number of nodes in the hidden layer (k) are important. When the number of k is small, the accuracy is poor. If k is too large, the model will consume an inordinate amount of computing time. Also, when compared to `learning rate = 0.01` in question B, the smaller learning rate `learning rate = 0.0001` appers to perform better, but it is just a case. It is worth thinking about how to define the balance to set up the number of nodes in the hidden layer and learning rate.

8 References

- https://towardsdatascience.com/gaussian-mixture-models-with-python-36dabed6212a
- https://towardsdatascience.com/lets-code-a-neural-network-in-plain-numpy-ae7e74410795
- https://machinelearninggeek.com/backpropagation-neural-network-using-python/
- https://github.com/SebastianMantey/Deep-Learning-Tutorial/blob/master/notebooks/04%20-%20Backpropagation%20with%20NumPy.ipynb
- https://www.tensorflow.org/guide/keras/sequential_model

9 Testing Zone

[123]:
```
"""
from sklearn.neural_network import MLPClassifier
clf = MLPClassifier(hidden_layer_sizes=2, max_iter=6000, learning_rate_init=0.
↪01, activation='relu').fit(x_train, y_train)
#clf.predict_proba(x_test[:1])
#clf.predict(x_test[:5, :])
clf.score(x_test, y_test)
"""
```

[123]:
```
"\nfrom sklearn.neural_network import MLPClassifier\nclf =
MLPClassifier(hidden_layer_sizes=2, max_iter=6000, learning_rate_init=0.01,
activation='relu').fit(x_train,
y_train)\n#clf.predict_proba(x_test[:1])\n#clf.predict(x_test[:5,
:])\nclf.score(x_test, y_test)\n"
```

[124]:
```
"""
# Initialize variables
np.random.seed(4321)
learning_rate = 0.0001
iterations = 6000
N = y_train.size # 400

# number of input features
input_size = 2

# number of hidden layers neurons
hidden_size = 2

# number of neurons at the output layer
output_size = 2

results = pd.DataFrame(columns=["mse", "accuracy"])

# initializing weight for the hidden layer
W1 = np.random.normal(scale=0.5, size=(input_size, hidden_size))


# initializing weight for the output layer
W2 = np.random.normal(scale=0.5, size=(hidden_size , output_size))
"""
```

[124]:
```
'\n# Initialize variables\nnp.random.seed(4321)\nlearning_rate =
0.0001\niterations = 6000\nN = y_train.size # 400\n\n# number of input
features\ninput_size = 2\n\n# number of hidden layers neurons\nhidden_size =
2\n\n# number of neurons at the output layer\noutput_size = 2\n\nresults =
pd.DataFrame(columns=["mse", "accuracy"])\n\n# initializing weight for the
```

```
hidden layer\nW1 = np.random.normal(scale=0.5, size=(input_size, hidden_size))
\n\n\n# initializing weight for the output layer\nW2 =
np.random.normal(scale=0.5, size=(hidden_size , output_size)) \n'
```

[125]:
```
"""
# ref: https://machinelearninggeek.com/
 ↪backpropagation-neural-network-using-python/
for itr in range(iterations):

  # feedforward propagation
  # on hidden layer
  Z1 = np.dot(x_train, W1)
  A1 = ReLU(Z1)

  # on output layer
  Z2 = np.dot(A1, W2)
  A2 = ReLU(Z2)

  # Calculating error
  mse = mean_squared_error(A2, y_train)
  acc = accuracy(A2, y_train)
  results=results.append({"mse":mse, "accuracy":acc},ignore_index= True)

  # backpropagation
  E1 = A2 - y_train
  dW1 = E1 * A2 * (1 - A2)

  E2 = np.dot(dW1, W2.T)
  dW2 = E2 * A1 * (1 - A1)

  # weight updates
  W2_update = np.dot(A1.T, dW1) / N
  W1_update = np.dot(x_train.T, dW2) / N

  W2 = W2 - learning_rate * W2_update
  W1 = W1 - learning_rate * W1_update

results.mse.plot(title="Mean Squared Error")
results.accuracy.plot(title="Accuracy")
"""
```

[125]:
```
'\n# ref: https://machinelearninggeek.com/backpropagation-neural-network-using-
python/\nfor itr in range(iterations):    \n\n  # feedforward propagation\n  #
on hidden layer\n  Z1 = np.dot(x_train, W1)\n  A1 = ReLU(Z1)\n\n  # on output
layer\n  Z2 = np.dot(A1, W2)\n  A2 = ReLU(Z2)\n    \n  # Calculating error\n
mse = mean_squared_error(A2, y_train)\n  acc = accuracy(A2, y_train)\n
results=results.append({"mse":mse, "accuracy":acc},ignore_index= True)\n\n  #
backpropagation\n  E1 = A2 - y_train\n  dW1 = E1 * A2 * (1 - A2)\n\n  E2 =
```

```
np.dot(dW1, W2.T)\n  dW2 = E2 * A1 * (1 - A1)\n\n  # weight updates\n  W2_update
= np.dot(A1.T, dW1) / N\n  W1_update = np.dot(x_train.T, dW2) / N\n\n  W2 = W2 -
learning_rate * W2_update\n  W1 = W1 - learning_rate *
W1_update\n\nresults.mse.plot(title="Mean Squared
Error")\nresults.accuracy.plot(title="Accuracy")\n'
```

[126]:
```
"""
# rectified linear function (ReLU)
def ReLU(x):
  return np.maximum(0, x)

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def mean_squared_error(y_pred, y_true):
    return ((y_pred - y_true)**2).sum() / (2*y_pred.size)

def accuracy(y_pred, y_true):
    acc = y_pred.argmax(axis=1) == y_true.argmax(axis=1)
    return acc.mean()
"""
```

[126]:
```
'\n# rectified linear function (ReLU)\ndef ReLU(x):\n  return np.maximum(0,
x)\n\ndef sigmoid(x):\n    return 1 / (1 + np.exp(-x))\n\ndef
mean_squared_error(y_pred, y_true):\n    return ((y_pred - y_true)**2).sum() /
(2*y_pred.size)\n    \ndef accuracy(y_pred, y_true):\n    acc =
y_pred.argmax(axis=1) == y_true.argmax(axis=1)\n    return acc.mean()\n'
```

[127]:
```
"""
#Create testing data
w11_test = np.random.multivariate_normal(mean = [-5, 5], cov = [[1, 0],␣
↪[0,1]],size = 50)
w12_test = np.random.multivariate_normal(mean = [5, -5], cov = [[1, 0],␣
↪[0,1]],size = 50)
w21_test = np.random.multivariate_normal(mean = [-5, -5], cov = [[1, 0],␣
↪[0,1]],size = 50)
w22_test = np.random.multivariate_normal(mean = [0, 0], cov = [[1, 0], [0,␣
↪1]],size = 50)
w23_test = np.random.multivariate_normal(mean = [5, 5], cov = [[1, 0], [0,␣
↪1]],size= 50)
W1_test = np.vstack((w11_test, w12_test))
W2_test = np.vstack((w21_test, w22_test, w23_test))
X2 = np.vstack((W1_test, W2_test))
y2 = np.concatenate((np.ones(shape=(1, 100)), np.zeros(shape=(1,␣
↪150))),axis=-1) # Class labels
print(X2.shape)
"""
```

[127]:
```
'\n#Create testing data\nw11_test = np.random.multivariate_normal(mean = [-5,
5], cov = [[1, 0], [0,1]],size = 50)\nw12_test =
np.random.multivariate_normal(mean = [5, -5], cov = [[1, 0], [0,1]],size =
50)\nw21_test = np.random.multivariate_normal(mean = [-5, -5], cov = [[1, 0],
[0,1]],size = 50)\nw22_test = np.random.multivariate_normal(mean = [0, 0], cov =
[[1, 0], [0, 1]],size = 50)\nw23_test = np.random.multivariate_normal(mean = [5,
5], cov = [[1, 0], [0, 1]],size= 50)\nW1_test = np.vstack((w11_test,
w12_test))\nW2_test = np.vstack((w21_test, w22_test, w23_test))\nX2 =
np.vstack((W1_test, W2_test))\ny2 = np.concatenate((np.ones(shape=(1, 100)),
np.zeros(shape=(1, 150))),axis=-1) # Class labels\nprint(X2.shape)\n'
```

10 Output

[151]:
```
# should access the Google Drive files before running the chunk
%%capture
!sudo apt-get install texlive-xetex texlive-fonts-recommended␣
↪texlive-plain-generic
!jupyter nbconvert --to pdf "/content/drive/MyDrive/American_University/
↪2021_Fall/DATA-642-001_Advanced Machine Learning/GitHub/Labs/08/submit/
↪Lab8_Yunting.ipynb"
```