Stat 615/415 (Regression)

1. **(5.2)** For the matrices below, obtain (1) $A+C$, (2) $A-C$, (3) $B'A$, (4) AC', (5) $C'A$.

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 6 \\ 9 \\ 3 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 8 \\ 8 & 6 \\ 5 & 1 \\ 2 & 4 \end{pmatrix}$$

Solution.

(1) $A+C = \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 3 & 8 \\ 8 & 6 \\ 5 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ 11 & 11 \\ 10 & 8 \\ 6 & 12 \end{pmatrix}$

(2) $A-C = \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 8 \\ 8 & 6 \\ 5 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & -7 \\ -5 & -1 \\ 0 & 6 \\ 2 & 4 \end{pmatrix}$

(3) $B'A = \begin{pmatrix} 6 & 9 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 12+27+15+4, & 6+45+21+8 \end{pmatrix} = \begin{pmatrix} 58 & 80 \end{pmatrix}$

(4) $AC' = \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix} \begin{pmatrix} 3 & 8 & 5 & 2 \\ 8 & 6 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 14 & 22 & 11 & 8 \\ 49 & 54 & 20 & 26 \\ 71 & 82 & 32 & 38 \\ 76 & 80 & 28 & 40 \end{pmatrix}$

(5) $C'A = \begin{pmatrix} 3 & 8 & 5 & 2 \\ 8 & 6 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 5 & 7 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 6+24+25+8 & 3+40+35+16 \\ 16+18+5+16 & 8+30+7+32 \end{pmatrix} = \begin{pmatrix} 63 & 94 \\ 55 & 77 \end{pmatrix}$

2. **(5.10).** Find the inverse of each of the following matrices:

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$$

Check that these are correct inverse matrices by calculating AA^{-1} and $B^{-1}B$.

Solution.

For $A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$, $\det(A) = 2-12 = -10$, *and* $A^{-1} = \frac{1}{-10}\begin{pmatrix} 1 & -4 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -0.1 & 0.4 \\ 0.3 & -0.2 \end{pmatrix}$.

Check: $AA^{-1} = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -0.1 & 0.4 \\ 0.3 & -0.2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$

For $B = \begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$, $\det(B) = 20-18 = 2$, *and* $B^{-1} = \frac{1}{2}\begin{pmatrix} 5 & -3 \\ -6 & 4 \end{pmatrix} = \begin{pmatrix} 2.5 & -1.5 \\ -3 & 2 \end{pmatrix}$.

Check: $B^{-1}B = \begin{pmatrix} 2.5 & -1.5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$

3. **(6.2)** Set up the X matrix and $\boldsymbol{\beta}$ vector for each of the following regression models (that is, write the model as $Y = X\boldsymbol{\beta} + \varepsilon$ and write the vector Y and matrix X explicitly):

   (a) $Y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$

   (b) $\sqrt{Y_i} = \beta_0 + \beta_1 X_{i1} + \beta_2 \log_{10} X_{i2} + \varepsilon_i$

   Solution.

$$(a)\quad \begin{pmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_n \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} & X_{11}^2 \\ X_{21} & X_{22} & X_{21}^2 \\ \cdots & \cdots & \cdots \\ X_{n1} & X_{n2} & X_{n1}^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{pmatrix}$$

$$(b)\quad \begin{pmatrix} \sqrt{Y}_1 \\ \sqrt{Y}_2 \\ \cdots \\ \sqrt{Y}_n \end{pmatrix} = \begin{pmatrix} 1 & X_{11} & \log_{10} X_{12} \\ 1 & X_{21} & \log_{10} X_{22} \\ \cdots & \cdots & \cdots \\ 1 & X_{n1} & \log_{10} X_{n2} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{pmatrix}$$