Homework #1

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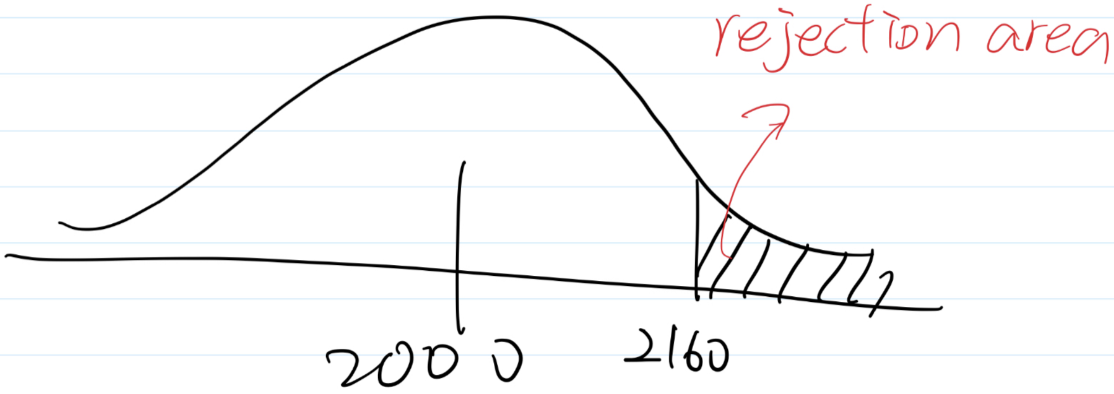
Review of Estimation and Hypothesis Testing (handouts, your old notes, . . .)

When α is not given, use the p-value approach to make your conclusions. When it's difficult to conclude, use $\alpha = 0.05$. For two-sample problems, use the F-test to decide which t-test to use.

1. The manufacturer of a certain brand of household light bulbs claims that the bulbs produced by his factory have an average life of at least 2,000 hours. The mean and standard deviation of 20 light bulbs selected from the manufacturer's production process were calculated to be 2,160 and 142 hours, respectively.

(a) Do the data represent sufficient evidence to support the manufacturer's claim? How can you interpret your answer?

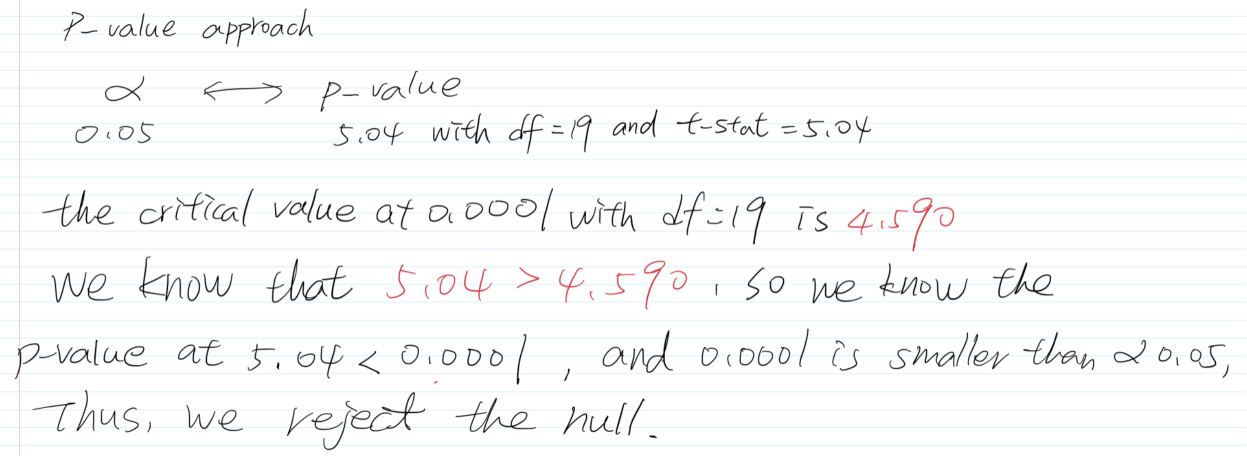


- Let we set H0: mu < 2000, and Ha: mu >= 2000. Note, this is a one-sided t-test with $\alpha = 0.05$, and degree of freedom: n-1 = 20-1 = 19. According to the information given, the sample mean is 2160, and the sample standard deviation is 142. We use t-statistics to conduct the statistical inference.

$$t = \frac{\hat{\mu} - \mu}{s/\sqrt{n}} = \frac{2160 - 2000}{142/\sqrt{20}} = 5.04$$

- The critical value for $\alpha = 0.05$ at df = 19 is 1.729 according to the t-distribution table. As 5.04 > 1.729, so we reject the null and claim that we have sufficient evidence to support the manufacturer's claim with a 5 % probability that I am going to reject the null when it is true.

p-value approach



P-value approach

α ⟷ p-value

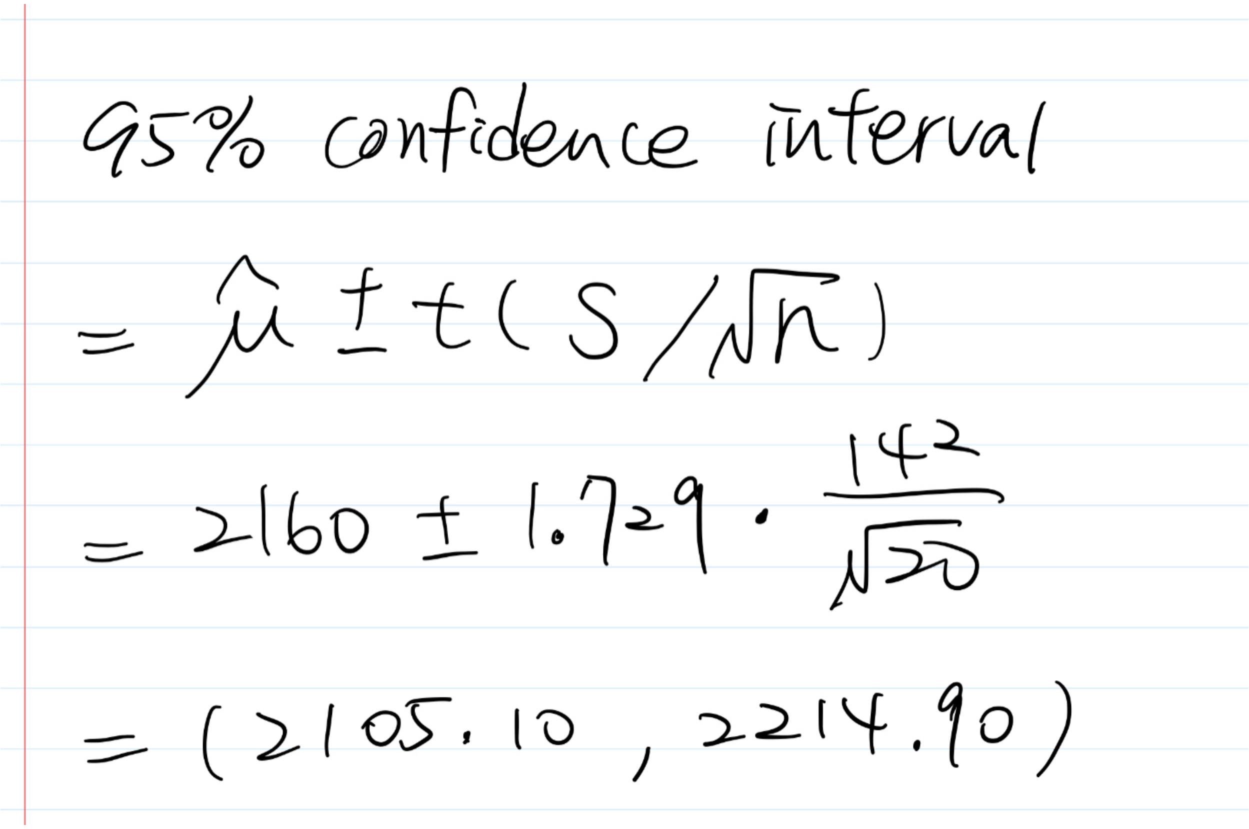
0.05 5.04 with df=19 and t-stat=5.04

the critical value at 0.0001 with df=19 is 4.590

we know that 5.04 > 4.590, so we know the p-value at 5.04 < 0.0001, and 0.0001 is smaller than α 0.05,

Thus, we reject the null.

(b) Construct a 95% confidence interval for the mean lifetime of household light bulbs.



95% confidence interval

$$= \hat{\mu} \pm t(S/\sqrt{n})$$

$$= 2160 \pm 1.729 \cdot \frac{142}{\sqrt{20}}$$

$$= (2105.10, 2214.90)$$

2. There are two manufacturing processes, old and new, that produce the same product. The defect rate has been measured for 20 days for the old process, and for 14 days for the new process, resulting in the following sample summaries.

The firm is interested in switching to the new process only if it can be demonstrated convincingly that the new process reduces the defect rate. Is there significant evidence of that? Use $\alpha = 5\%$; assume that the collected data represent two random samples from Normal distributions. Use the method of testing that is appropriate for this situation.

- This is a one-sided t-test for the difference in mean. As the question mentioned, the new process of manufacturing can reduces the defect rate so Ho: muNEW = muOLD; Ha: muNEW > muOLD.

$$t = \frac{\hat{\mu}_{old} - \hat{\mu}_{new} - 0}{\sqrt{\frac{S^2_{old}}{n_{old}}} + \sqrt{\frac{S^2_{new}}{n_{new}}}} = \frac{4.7-2.3-0}{\sqrt{\frac{6.8^2}{20} + \frac{5.0^2}{14}}} = 1.186$$

3. (Required for Stat-615, optional for Stat-415) An account on server A is more expensive than an account on server B. However, server A is faster. To see if whether it's optimal to go with the faster but more expensive server, a manager needs to know how much faster it is. A certain computer algorithm is executed 30 times on server A and 20 times on server B with the following results,

(a) Is there a significant difference between the two servers?
(b) Is server A significantly faster?

4. Micro-project. Data on 522 recent home sales are available on our Blackboard web site The following variables are included.
Use software to find out if there is significant evidence that:

```r
# read the dataset
library(tidyverse)
```

```
## -- Attaching packages ---- tidyverse 1.3.0 --

## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.2
## v tidyr   1.1.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0

## -- Conflicts ------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```r
homeSales <- read_csv("./data/HOME_SALES(1).csv")
```

```
## Parsed with column specification:
## cols(
##   ID = col_double(),
##   SALES_PRICE = col_double(),
##   FINISHED_AREA = col_double(),
##   BEDROOMS = col_double(),
##   BATHROOMS = col_double(),
##   GARAGE_SIZE = col_double(),
##   YEAR_BUILT = col_double(),
##   STYLE = col_double(),
##   LOT_SIZE = col_double(),
##   AIR_CONDITIONER = col_character(),
##   POOL = col_character(),
##   QUALITY = col_character(),
##   HIGHWAY = col_character()
## )
```

```
homeSales
```

```
## # A tibble: 522 x 13
##       ID SALES_PRICE FINISHED_AREA BEDROOMS BATHROOMS GARAGE_SIZE YEAR_BUILT
##    <dbl>       <dbl>         <dbl>    <dbl>     <dbl>       <dbl>      <dbl>
##  1     1         360          3032        4         4           2       1972
##  2     2         340          2058        4         2           2       1976
##  3     3         250          1780        4         3           2       1980
##  4     4         206.         1638        4         2           2       1963
##  5     5         276.         2196        4         3           2       1968
##  6     6         248          1966        4         3           5       1972
##  7     7         230.         2216        3         2           2       1972
##  8     8         150          1597        2         1           1       1955
##  9     9         195          1622        3         2           2       1975
## 10    10         160          1976        3         3           1       1918
## # ... with 512 more rows, and 6 more variables: STYLE <dbl>, LOT_SIZE <dbl>,
## #   AIR_CONDITIONER <chr>, POOL <chr>, QUALITY <chr>, HIGHWAY <chr>
```

(a) The sales price depends on the air conditioner in the house.

- We set the houses with air conditioner as 1, the houses without air conditioner as 0. The small p-value indicates the sales price is significantly associated with air conditioner in the house.

```
# mutate AIR_CONDITIONER as dummy variable
homeSales %>%
mutate(AIR_CONDITIONER = recode(AIR_CONDITIONER, "YES" = "1", "NO" = "0"),
       AIR_CONDITIONER = as.double(AIR_CONDITIONER)) -> homeSales01

acPrice <- lm(SALES_PRICE ~ AIR_CONDITIONER, data = homeSales01)
summary(acPrice)
```

```
##
## Call:
## lm(formula = SALES_PRICE ~ AIR_CONDITIONER, data = homeSales01)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -171.80  -90.88  -36.19   52.57  624.20
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)       189.58      14.09  13.455  < 2e-16 ***
## AIR_CONDITIONER   106.22      15.45   6.873  1.8e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 132.2 on 520 degrees of freedom
## Multiple R-squared:  0.08329,    Adjusted R-squared:  0.08152
## F-statistic: 47.24 on 1 and 520 DF,  p-value: 1.8e-11
```

(b) On the average, homes with an air conditioner are more expensive.

- The model explains sales price of air conditioner homes is 106.22 thousand dollars more expensive than non-air conditioner homes, on average.

(c) On the average, homes with an air conditioner are larger.

- According to linear model below, the homes is 3283 thousand dollars less than non-air conditioner homes, on average. And the small p-value 0.0161 points out we have sufficient evidence to reject the null.

```
acSize <- lm(LOT_SIZE ~ AIR_CONDITIONER, data = homeSales01)
summary(acSize)
```

```
##
## Call:
## lm(formula = LOT_SIZE ~ AIR_CONDITIONER, data = homeSales01)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -21434  -6931  -1705   2754  63014
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)        27100       1240  21.858   <2e-16 ***
## AIR_CONDITIONER    -3283       1360  -2.415   0.0161 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11630 on 520 degrees of freedom
## Multiple R-squared:  0.01109,    Adjusted R-squared:  0.009187
## F-statistic: 5.831 on 1 and 520 DF,  p-value: 0.01609
```

(d) The sales price depends on the proximity to a highway.

- With a large p-value of 0.245, we do not have sufficient evidence to conclude that the sales price and its proximity to highway have an association.

```
# mutate HIGHWAY as dummy variable
homeSales01 %>%
mutate(HIGHWAY = recode(HIGHWAY, "YES" = "1", "NO" = "0"),
       HIGHWAY = as.double(HIGHWAY)) -> homeSales02

hwPrice <- lm(SALES_PRICE ~ HIGHWAY, data = homeSales02)
summary(hwPrice)
```

```
##
## Call:
## lm(formula = SALES_PRICE ~ HIGHWAY, data = homeSales02)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -194.92  -96.92  -48.92   56.08  641.08
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  278.925      6.099  45.731   <2e-16 ***
## HIGHWAY      -48.897     42.016  -1.164    0.245
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 137.9 on 520 degrees of freedom
## Multiple R-squared:  0.002598,   Adjusted R-squared:  0.0006797
## F-statistic: 1.354 on 1 and 520 DF,  p-value: 0.2451
```

(e) On the average, homes are cheaper when they are close to a highway.

(f) On the average, homes are cheaper when they are far from a highway.