STAT 614 - HW 3

1. Triceps skinfold thickness is an upper arm measurement that has been used as a proxy measure of body fat. The table below gives the mean and standard deviations of tricep skinfold thickness (in cm) for two populations of adult males, those with chronic airflow limitation (such as COPD, a type of obstructive lung disease) and those without any airflow limitation. A study comparing tricep skinfold thickness is being planned in these populations using the respective sample sizes (*n*), also given in the last column of the table.

Population	μ	σ	*n*
Chronic airflow limitation	0.92	0.4	32
No airflow limitation	1.35	0.5	40

   a. Consider a random sample $y_1, y_2, y_3, \ldots, y_n$ from the chronic airflow limitation population with mean μ and standard deviation σ as given in the table. What is the standard deviation of the sample mean, $\bar{y}$? (Note, this is often called the "standard error" of the mean, especially when an estimate for σ is used.)

The standard error for mean with chronic airflow limitation is SE = $\frac{S}{\sqrt{n}} = \frac{0.4}{\sqrt{32}} = 0.07071068$

   b. Assume the Central Limit Theorem is applicable. What does it suggest about potential values of the sample mean, $\bar{y}$, the researchers can expect in their study?

The Central Limit Theorem suggests that for the chronic airflow limitation population, the mean triceps skin-fold thickness in samples of size *n* = 32 follows a normal distribution with a mean, 0.92 cm, and standard deviation estimated to be 0.07071068 cm (or a variance estimated to be $0.07071068^2 = 0.005$ cm^2). Thus, we expect observed sample means to be no more than about 0.071 from the true population mean (on average).

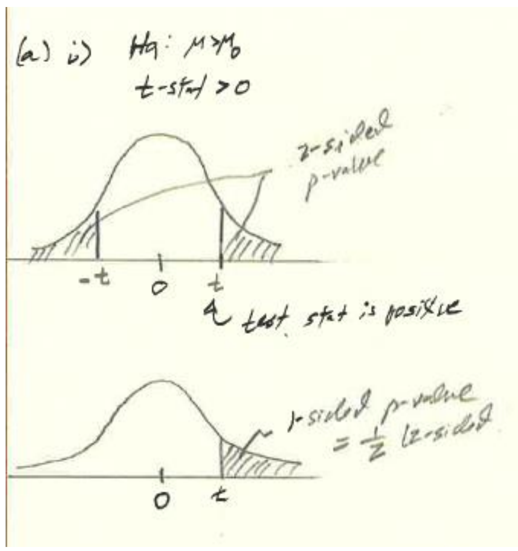
2. A human resources manager for a large company takes a random sample of 50 employees from the company database. She calculates the mean time that they have been employed. She records this value and then repeats the process: She takes another random sample of 50 names and calculates the mean employment time. After she has done this 1000 times, she makes a histogram of the mean employment times. Is this histogram a display of the population distribution, the distribution of a sample, or the sampling distribution of mean?

The human resources manager has created a display of the sampling distribution of the (sample) mean (i.e. or equivalently the distribution of the sample mean).

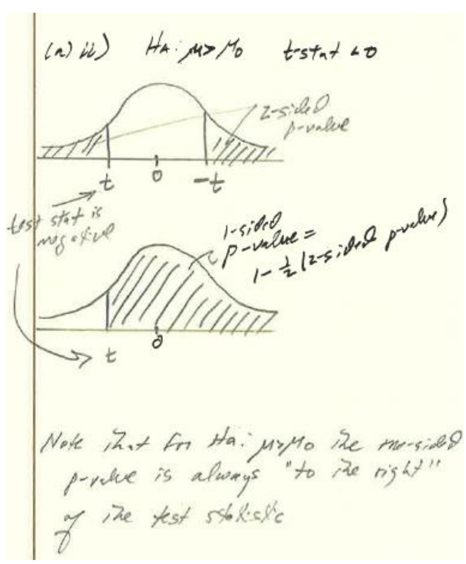
3. In most software, the default p-values are computed based on a 2-sided alternative hypothesis (H_A: $\mu \neq \mu_0$). (Note: R's t.test() functions allow one-sided tests!) However, we may want to use a 1-sided alternative in some problems. Hence, we need to be able to compute the correct 1-sided p-values from the reported 2-sided version. Sketch a graph for each of the following to demonstrate that each is the correct procedure.

(a) For H_A: $\mu > \mu_0$: The alternative hypothesis tells which direction we use to find a p-value. Recall that the p-value is a summary of the evidence *against* Ho *in favor* of Ha so "in favor of Ha" in this case would be "to the right" on the distribution because Ha considers the population mean to be *above* some given value μ_0. So, no matter what value the observed test statistic is, we accumulate probability by taking the area under the distribution curve *to the right of the test statistic!*

i. if test statistic $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$ is positive, the (1-sided p-value) = 0.5*(2-sided p-value).

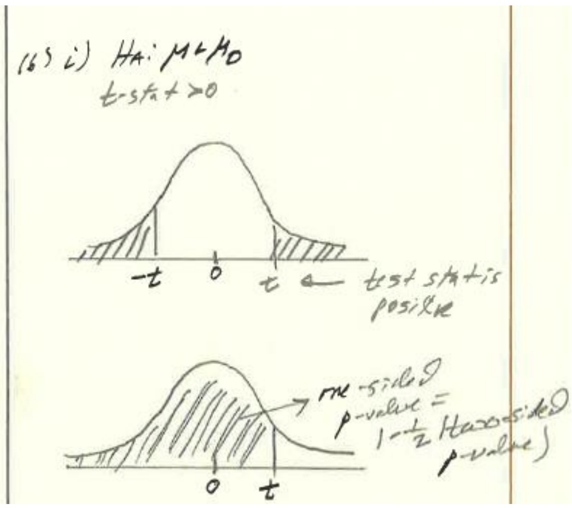


ii. if test statistic $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$ is negative, (1-sided p-value) = 1 – 0.5*(2-sided p-value).

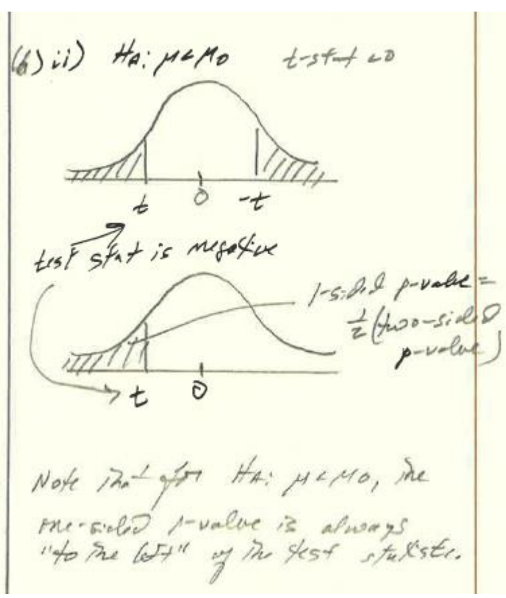


(b) For H_A: $\mu < \mu_0$: "in favor of Ha" in this case would be "to the left" on the distribution because Ha considers the population mean to be *below* some given value μ_0. Thus, no matter what the observed test statistic values is, we compute the p-value by taking the area *to the left* of it.

i. if test statistic $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ is positive, (1-sided p-value) = 1 – 0.5*(2-sided p-value).



ii. if test statistic $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ is negative, (1-sided p-value) = 0.5*(2-sided p-value).



4. Suppose the following statement is made in the conclusions section of a paper: "A comparison of breathing capacities of individuals in households with low nitrogen dioxide levels and individuals in households with high nitrogen dioxide levels indicated that there is **no difference** in means (two-sided p-value = 0.24)."

   a. Give a reasonable null and alternative hypothesis that was being tested in this scenario. Carefully define the population parameters of interest being tested.

μ_1 = population mean breathing capacity of the population of individuals in households with low nitrogen dioxide

μ_2 = population mean breathing capacity of the population of individuals in households with high nitrogen dioxide

H_0: $\mu_1 = \mu_2$ vs. H_A: $\mu_1 > \mu_2$

   b. Why is this statement an inaccurate summary of the hypothesis test?

There is a subtle distinction between what is written and what is appropriate to conclude from this (or any) analysis. Here they conclude that the population means are not different when failing to reject the null hypothesis. But, failing to reject a null hypothesis just tells us the data are *consistent* with the null hypothesis, suggesting that a value of 0 (no difference in population means) is one of several values the data are consistent with. That isn't actually saying much because the data are consistent with *many* different values. And the confidence interval gives *all* values that the data are consistent with! (This goes to the point that it is always a stronger statement to reject the null hypothesis than to fail to reject it.)

   c. Re-write the statement so that it is properly summarizing the results of the hypothesis test.

The following would be better: "A comparison of breathing capacities of individuals in households with low nitrogen dioxide levels and individuals in house-holds with high nitrogen dioxide levels indicated that there is **no statistical evidence of** a difference in the means (two-sided p-value = 0.24)" Or, "The data are consistent with the hypothesis of no difference in population means (two-sided p-value = 0.24)."

5. **Use the lead.sav data set from HW 1.** Revisit HW 1 for a description of this data set and study.For this problem you will compare the Wechsler full-scale IQ scores (the variable IQF) between the different lead exposure groups, denoted by the GROUP variable.

   a. Compute the mean, standard deviation, standard error, and 95% confidence interval for the population mean IQ score for **each** lead exposure group, separately. Summarize each confidence interval.

The following table gives the requested summary statistics. Each confidence interval gives reasonable estimates for the unknown respective population mean. For example, for the exposure group, the 95% confidence interval for the population mean IQF score is 84.40 to 91.65. For the control group, reasonable values for the population mean IQF score are 89.42 to 96.34, at a 95% confidence level.

Population	Sample Mean	Standard Dev.	Standard Error	95% CI for μ or $\mu_2 - \mu_1$
1	88.02	12.21	1.80	84.40 to 91.65

2	92.88	15.34	1.74	89.42 to 96.34
Diff = 1 - 2	-4.86	N/A	2.65 2.50	-10.11 to 0.39 (equal vars) -9.32 to 0.09 (unequal vars)

b. Researchers were interested in assessing the difference in the mean IQ score between the two exposure group populations. Give the estimate mean difference, the standard error, and the 95% confidence interval for the **difference** in population mean IQ scores. Summarize the confidence interval.

I have included the summaries for the differences in the table above taking the exposure group mean minus the control group mean as that is what the software did by default. I have included both the equal variances t-procedure results along with the adjustment if you do not assume equal variances.

(Note: I did not expect you to have checked the assumptions for this problem because we had not covered Chapter 3 – now that we know about them, it is worth assessing. Either is fine for this problem – the sample standard deviations are 12.21 and 15.34 respectively. These are different – they always will be – but not so different that I would be concerned about this assumption. That said, it isn't necessarily wrong to use the unequal variances – there just tends to be advantages to use the former.)

The confidence interval suggests that with 95% confidence, the population mean IQ score for the control population is estimated to be 0.39 lower to 10.11 points higher than the population mean IQ score for the exposed population (assuming equal variances and results are similar for unequal variances with a slightly narrower CI).

c. Researchers hypothesized that an exposed population (GROUP = 1) would have a lower population mean IQ score than a control population (GROUP = 2). Set up and conduct a statistical hypothesis test to address the research hypothesis. Carefully state the null and alternative hypotheses to be tested. Give the parameter of interest, the estimate of this parameter, the standard error of the estimate, the test statistic, and the p-value. Summarize the results of the test.

μ_1 = population mean IQF score of the exposed population

μ_2 = population mean IQF score of the control population

H_0: $\mu_1 = \mu_2$ vs. H_A: $\mu_1 < \mu_2$

This is equivalent to H_0: $\mu_1 - \mu_2 = 0$ vs. H_A: $\mu_1 - \mu_2 < 0$

(Note: I am using group 1 – group 2 solely because the software defaults to that!)

For the equal variances assumed t-procedure:
Parameter of Interest = Par = $\mu_1 - \mu_2$
Estimate of the Parameter = Est = difference in sample means = -4.86
Standard Error of the Estimate = SE(Est) = 2.65
Test statistics = (Est – 0) / SE = -1.8334
P-value = 0.069/2 = 0.035 (Note the 1-sided hypothesis!)

With this small p-value we conclude there is compelling evidence against the null hypothesis in favor of the alternative hypothesis. That is, there is evidence that the population mean IQ score for the control population exceeds the population mean IQ score of the exposed population. Our best estimate is that the mean IQ score in the control population is 4.86 units above the mean score in the exposed population. This *may* seem contradictory to the confidence interval because the confidence interval includes zero, but this is the one-sided hypothesis test and the confidence interval is two-sided (there are one-sided confidence intervals!! We just won't cover them in this class.)

Results are similar for the unequal variances t-procedure:
Parameter of Interest = Par = μ_1 - μ_2
Estimate of the Parameter = Est = difference in sample means = -4.86
Standard Error of the Estimate = SE(Est) = 2.50
Test statistics = (Est – 0) / SE = -1.9439
P-value = 0.054/2 = 0.027 (Note the 1-sided hypothesis!)